Unified description of heavy baryons in the covariant confined quark model

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Plan of the Talk

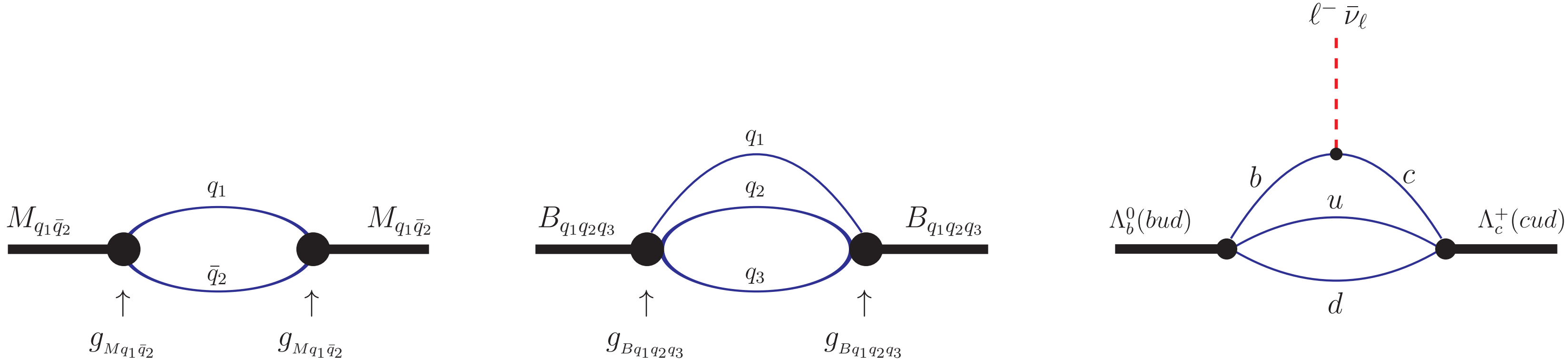
- Introduction
- Baryon Structure in the Covariant Confined Quark Model
  – Formalism
  – Application to electromagnetic, strong and weak decays of heavy baryons
- Conclusions

Introduction

- Bound States in Quantum Field Theory
- Bethe, Salpeter (1951) Relativistic equation for bound state problem using S-matrix formalism $\Longrightarrow$ Integral equation for WF of two interacting fermions
- Jouvet (1956) QFT approach for Fermi coupling bounding the hadrons $\Longrightarrow$ compositeness condition $Z = |\langle\psi_{\rm bare}|\psi_{\rm phys}\rangle|^2 = 0$
  Probability to find "bare" state in "physical (dressed)" state is zero
- Both methods are equivalent
- Salam (1962), Weinberg (1963)
  Master equation relating mass spectrum and coupling constant

$$Z = 1 - g^2_{Hq\bar{q}}\Pi'(m_H) = 0$$

Here Π' is the derivative of mass operator of hadron on mass shell cd



Baryon Structure in CCQM: formalism

- Unified Description of Light and Heavy Baryons
- Quantum Numbers of Heavy Baryons:
  J^P spin-parity, I isospin, quark content
- Interpolating currents (QCD sum rules)
  $J = \varepsilon^{abc}\Gamma_1 q_1^a\,(q_2^b\, C\,\Gamma_2 q_3^c)$
  Here q_i^a is quark field, $C = \gamma^0\gamma^2$ is charge conjugation matrix, Γ_i is gamma matrix
- Proton = $d \oplus \{uu\}_{1^+} = u \oplus [ud]_{0^+}$

$$
\begin{aligned}
J^V &= \varepsilon^{abc}\gamma^\mu\gamma^5\, d^a\,(u^b\, C\gamma_\mu\, u^c) \\
&= J^P - J^S - \frac{J^A}{2} \\
&= \varepsilon^{abc}\, u^a\,(u^b\, C\gamma_5\;\; d^c) - \varepsilon^{abc}\,\gamma^5 d^a\,(u^b\, C\, u^c) + \frac{1}{2}\varepsilon^{abc}\,\gamma^\mu d^a\,(u^b\, C\,\gamma_5\gamma_\mu u^c)
\end{aligned}
$$

- $\Delta^{++}(1232)$ = $u \oplus \{uu\}_{1^+}$

$$
J^{V,\mu} = \varepsilon^{abc}\, u^a\,(u^b\, C\gamma_\mu\, u^c)
$$

Baryon Structure in CCQM: formalism

- $\Lambda_Q = Q \oplus [ud]_{0+}$

$$
\begin{aligned}
J^P &= \varepsilon^{abc} Q^a \, (u^b \, C\gamma_5 \, d^c) \\
J^S &= \varepsilon^{abc} \gamma^5 \, Q^a \, (u^b \, C \, d^c) \\
J^A &= \varepsilon^{abc} \, \gamma^\mu \, Q^a \, (u^b \, C\gamma_5\gamma_\mu \, d^c)
\end{aligned}
$$

J^P and J^A degenerate in NRL and HQL $m_Q \to \infty$
J^S vanishes

- Σ_Q, Σ_Q^* = $Q \oplus [uu]_{1+}$

$$
\begin{aligned}
J^V &= \varepsilon^{abc} \gamma^\mu \gamma^5 \, Q^a \, (u^b \, C\gamma_\mu \, u^c) \\
J^T &= \frac{1}{2} \varepsilon^{abc} \sigma^{\mu\nu} \gamma^5 \, Q^a \, (u^b \, C\sigma_{\mu\nu} \, u^c)
\end{aligned}
$$

Baryon Structure in CCQM: formalism

- $\Xi_{QQ}, \Xi^*_{QQ} = q \oplus [QQ]_{1^+}$

$$
\begin{aligned}
J^V_{1/2^+} &= \varepsilon^{abc} \gamma^\mu \gamma^5 \, q^a \, (Q^b \, C\gamma_\mu \, Q^c) \\
J^T_{1/2^+} &= \frac{1}{2} \, \varepsilon^{abc} \sigma^{\mu\nu} \gamma^5 \, q^a \, (Q^b \, C\sigma_{\mu\nu} \, Q^c) \\
J^{V,\mu}_{3/2^+} &= \varepsilon^{abc} \, q^a \, (Q^b \, C\gamma_\mu \, Q^c)
\end{aligned}
$$

- $\Xi_{bcc}, \Xi^*_{bcc} = b \oplus \{cc\}_{1^+}$

$$
\begin{aligned}
J^V_{1/2^+} &= \varepsilon^{abc} \gamma^\mu \gamma^5 \, b^a \, (c^b \, C\gamma_\mu \, c^c) \\
J^T_{1/2^+} &= \frac{1}{2} \, \varepsilon^{abc} \sigma^{\mu\nu} \gamma^5 \, b^a \, (c^b \, C\sigma_{\mu\nu} \, c^c) \\
J^{V,\mu}_{3/2^+} &= \varepsilon^{abc} \, b^a \, (c^b \, C\gamma_\mu \, c^c)
\end{aligned}
$$

Baryon Structure in CCQM: formalism

- Interpolating currents and quantum numbers of light baryons

Baryon	J^P	J^{abc}	Mass (MeV)
p	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 d^a u^b C\gamma_\mu u^c$	938.27
n	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 u^a d^b C\gamma_\mu d^c$	939.57
Λ	$\frac{1}{2}^+$	$s^a u^b C\gamma_5 d^c$	1115.68
Σ^+	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 s^a u^b C\gamma_\mu u^c$	1189.37
Σ^0	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 s^a u^b C\gamma_\mu d^c$	1192.64
Σ^-	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 s^a d^b C\gamma_\mu d^c$	1197.45
Ξ^0	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 u^a s^b C\gamma_\mu s^c$	1314.86
Ξ^-	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 d^a s^b C\gamma_\mu s^c$	1314.86
Δ^{++}	$\frac{3}{2}^+$	$u^a u^b C\gamma_\mu u^c$	1230.55
Δ^+	$\frac{3}{2}^+$	$d^a u^b C\gamma_\mu u^c$	1234.90
Δ^0	$\frac{3}{2}^+$	$u^a d^b C\gamma_\mu d^c$	1231.30
Δ^-	$\frac{3}{2}^+$	$d^a d^b C\gamma_\mu d^c$	1230.55
$\Sigma^{+\star}$	$\frac{3}{2}^+$	$s^a u^b C\gamma_\mu u^c$	1382.80
$\Sigma^{0\star}$	$\frac{3}{2}^+$	$s^a u^b C\gamma_\mu d^c$	1383.70
$\Sigma^{-\star}$	$\frac{3}{2}^+$	$s^a d^b C\gamma_\mu d^c$	1387.20
$\Xi^{0\star}$	$\frac{3}{2}^+$	$u^a s^b C\gamma_\mu s^c$	1531.80
$\Xi^{-\star}$	$\frac{3}{2}^+$	$d^a s^b C\gamma_\mu s^c$	1535.00
Ω^-	$\frac{3}{2}^+$	$s^a s^b C\gamma_\mu s^c$	1672.45

Baryon Structure in CCQM: formalism

- Interpolating currents and quantum numbers of heavy baryons

Baryon	J^P	J^{abc}	Mass (MeV)
Λ_c	$\frac{1}{2}^+$	$c^a u^b C\gamma_5 d^c$	2286.46
Σ_c^{++}	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 c^a u^b C\gamma_\mu u^c$	2453.97
Σ_c^{+}	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 c^a u^b C\gamma_\mu d^c$	2452.90
Σ_c^{0}	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 c^a d^b C\gamma_\mu d^c$	2453.75
Ξ_c^{+}	$\frac{1}{2}^+$	$c^a u^b C\gamma_5 s^c$	2467.87
Ξ_c^{0}	$\frac{1}{2}^+$	$c^a u^b C\gamma_5 s^c$	2470.87
$\Xi_c^{\prime +}$	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 c^a u^b C\gamma_\mu s^c$	2577.40
$\Xi_c^{\prime 0}$	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 c^a d^b C\gamma_\mu s^c$	2578.80
Ω_c	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 c^a s^b C\gamma_\mu s^c$	2695.20
$\Omega_c^{\star}$	$\frac{3}{2}^+$	$c^a s^b C\gamma_\mu s^c$	2765.90

Baryon Structure in CCQM: formalism

- Interpolating currents and quantum numbers of heavy baryons

Baryon	J^P	J^{abc}	Mass (MeV)
Λ_b	$\frac{1}{2}^+$	$b^a u^b C\gamma_5 d^c$	5619.40
Σ_b^+	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 b^a u^b C\gamma_\mu u^c$	5811.30
Σ_b^0	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 b^a u^b C\gamma_\mu d^c$	5813.40
Σ_b^-	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 b^a d^b C\gamma_\mu d^c$	5815.50
Ξ_b^0	$\frac{1}{2}^+$	$b^a u^b C\gamma_5 s^c$	5791.90
Ξ_b^-	$\frac{1}{2}^+$	$b^a d^b C\gamma_5 s^c$	5794.50
$\Xi_b^{'0}$	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 b^a u^b C\gamma_\mu s^c$	5935.02
$\Xi_b^{'-}$	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 b^a d^b C\gamma_\mu s^c$	5936.00
Ω_b	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 b^a s^b C\gamma_\mu s^c$	6046.40
Ξ_b^{*-}	$\frac{3}{2}^+$	$b^a u^b C\gamma_\mu s^c$	5955.33
Ω_b^*	$\frac{3}{2}^+$	$b^a s^b C\gamma_\mu s^c$	6088.00

Baryon Structure in CCQM: formalism

- Interpolating currents and quantum numbers of double heavy baryons

Baryon	J^P	J^{abc}	Mass (MeV)
Ξ_{cc}^{++}	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 u^a c^b C\gamma_\mu c^c$	3621.40 (LHCb)
Ω_{cc}^{+}	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 s^a c^b C\gamma_\mu c^c$	3778
Ξ_{cc}^{*++}	$\frac{3}{2}^+$	$u^a c^b C\gamma_\mu c^c$	3727
Ω_{cc}^{*+}	$\frac{3}{2}^+$	$s^a c^b C\gamma_\mu c^c$	3872
Ξ_{bb}^{0}	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 u^a b^b C\gamma_\mu b^c$	10202
Ω_{bb}^{-}	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 s^a b^b C\gamma_\mu b^c$	10359
Ξ_{bb}^{*0}	$\frac{3}{2}^+$	$u^a b^b C\gamma_\mu b^c$	10237
Ω_{bb}^{*-}	$\frac{3}{2}^+$	$s^a b^b C\gamma_\mu b^c$	10389
Ξ_{cb}^{+}	$\frac{1}{2}^+$	$u^a c^b C\gamma_5 b^c$	6933
$\Xi_{cb}^{'+}$	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 u^a c^b C\gamma_5 b^c$	6963
Ω_{cb}^{0}	$\frac{1}{2}^+$	$d^a c^b C\gamma_5 b^c$	7088
$\Omega_{cb}^{'0}$	$\frac{1}{2}^+$	$\gamma^\mu\gamma^5 s^a c^b C\gamma_5 b^c$	7116
Ξ_{cb}^{*+}	$\frac{3}{2}^+$	$u^a c^b C\gamma_\mu b^c$	6980
Ω_{cb}^{*0}	$\frac{3}{2}^+$	$s^a c^b C\gamma_\mu b^c$	7130

EFG (Ebert, Faustov, Galkin) PRD 86, 014008 (2002)

Baryon Structure in CCQM: formalism

- Interaction Lagrangins of Baryons with Quarks

Σ_c^{++} :

$$\mathcal{L}_{\rm int}^{\Sigma_c^{++}}(x) = g_{\Sigma_c^{++}}\, \bar\Sigma_c^{++}(x)\cdot J_{\Sigma_c^{++}}(x) + {\rm H.c.}\,,$$

$$J_{\Sigma_c^{++}}(x) = \int dx_1 \ldots \int dx_3 F_{\Sigma_c^{++}}(x;x_1x_2x_3)\epsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 c^{a_1}(x_1)\, u^{a_2}(x_2) C\gamma_\mu u^{a_3}(x_3)$$

Ξ_{cc}^{++} :

$$\mathcal{L}_{\rm int}^{\Xi_{cc}^{++}}(x) = g_{\Xi_{cc}^{++}}\, \bar\Xi_{cc}^{++}(x)\cdot J_{\Xi_{cc}^{++}}(x) + {\rm H.c.}\,,$$

$$J_{\Xi_{cc}^{++}}(x) = \int dx_1 \ldots \int dx_3 F_{\Xi_{cc}^{++}}(x;x_1x_2x_3)\epsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 u^{a_1}(x_1)\, c^{a_2}(x_2) C\gamma_\mu c^{a_3}(x_3)$$

The vertex function F is chosen to be of the form

$$F(x;x_1,x_2,x_3) \;=\; \delta^{(4)}(x - \sum_{i=1}^{3} w_i x_i)\; \Phi\Big(\sum_{i<j}(x_i - x_j)^2\Big)$$

Baryon Structure in CCQM: formalism

- Correlation Function Φ - correlation function involving the three constituent quarks with coordinates x_1, x_2, x_3 and with masses m_1, m_2, m_3. The variable w_i is defined by $w_i = m_i/(m_1 + m_2 + m_3)$.
  In momentum space
  $$\Phi(-k_1^2 - k_2^2) = \exp\left[\frac{k_1^2 + k_2^2}{\Lambda^2}\right]$$
  size parameter Λ is fixed from data.
- Free fermion propagator for quarks
  $$S_q(p) = \frac{1}{m_q - \not{k}}\,, \quad q = u, d, s, c, b$$
- Heavy Quark Limit
  $$S_Q(k+p) = \frac{1}{m_Q - \not{k} - \not{p}} = -\frac{1 + \not{v}}{2(kv + \bar{\Lambda}_{q_1 q_2})} + \mathcal{O}(1/m_Q)$$
  where $v = p/m_B$ is the heavy baryon 4-velocity,
  $m_{B_{Q q_1 q_2}} = m_Q + \bar{\Lambda}_{q_1 q_2} + \mathcal{O}(1/m_Q)$.

Baryon Structure in CCQM: formalism

- Infrared Confinement
  α-representation for each local quark propagator and integrating out the loop momenta, one can write the resulting matrix element expression as an integral which includes integrations over a simplex of the α-parameters and an integration over a scale variable extending from zero to infinity.
  After doing the loop integrations one obtains

$$\Pi = \int_0^\infty d^n\alpha \, F(\alpha_1, \ldots, \alpha_n) \,,$$

  where F stands for the whole structure of a given diagram.
  The set of Schwinger parameters α_i can be turned into a simplex by introducing an additional t–integration via the identity

$$1 = \int_0^\infty dt \, \delta(t - \sum_{i=1}^n \alpha_i)$$

Baryon Structure in CCQM: formalism

- Additional scale

$$\Pi = \int\limits_0^\infty dt t^{n-1} \int\limits_0^1 d^n\alpha \, \delta\Big(1 - \sum_{i=1}^n \alpha_i\Big) \, F(t\alpha_1, \ldots, t\alpha_n) \, .$$

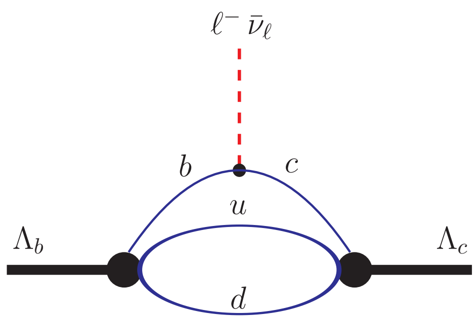
We cut off the upper integration at $1/\lambda^2$ and obtain

$$\Pi^{\rm conf} = \int\limits_0^{1/\lambda^2} dt t^{n-1} \int\limits_0^1 d^n\alpha \, \delta\Big(1 - \sum_{i=1}^n \alpha_i\Big) \, F(t\alpha_1, \ldots, t\alpha_n)$$

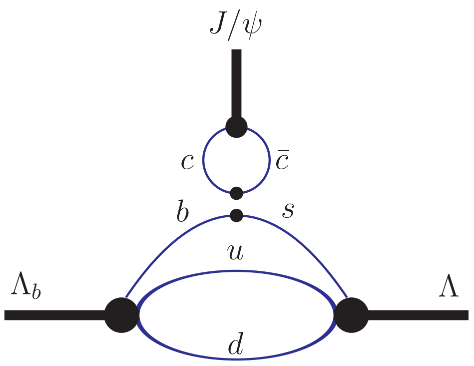
IR cutoff removes all possible thresholds in the quark loop diagram. We take the cutoff parameter $\lambda = 181$ MeV (corresponding to $1/\lambda \sim 1$ fm), which is the same in all physical processes. It has analogy with holographic description of particle interactions in AdS/QCD. One can then interpret z as an extra space coordinate and the upper integration limit $z_{\rm IR} = 1/\lambda$ as the infrared scale where quarks are confined and hadronized.
Truncation over the holographic coordinate z is necessary in order to break conformal invariance and to incorporate confinement in the infrared region.

Baryon Structure in CCQM: Weak Processes

- Typical Feynman Diagram
- Semileptonic Processes

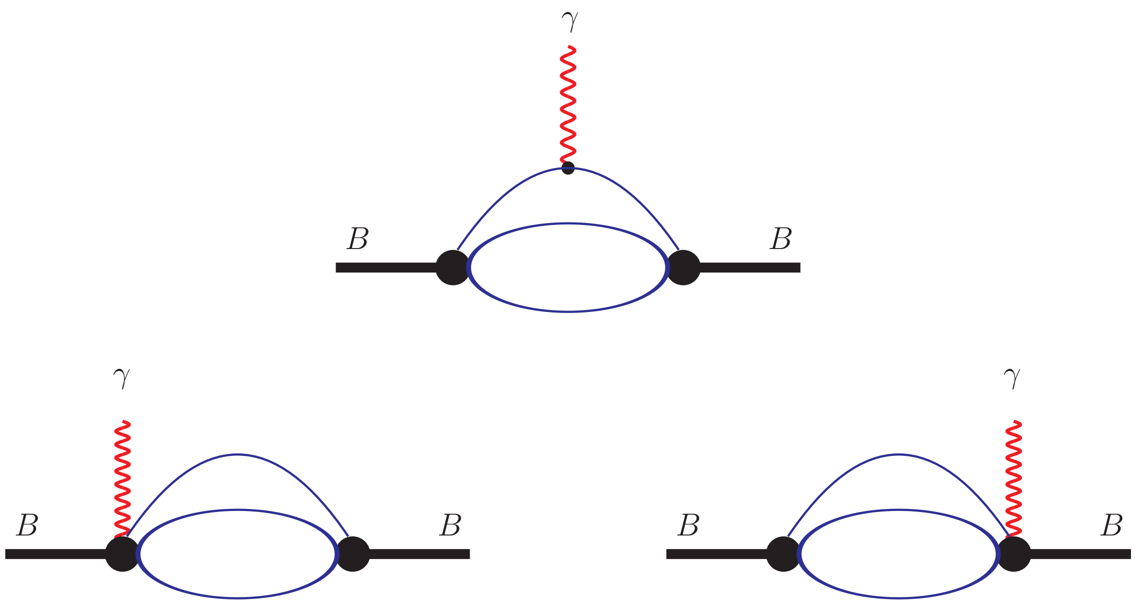


- Nonleptonicleptonic Processes



Baryon Structure in CCQM: EM Processes

- ELectromagnetic Process
- Gauge invariance restoration: $q(x_i) \to \exp\left[ie_q \int\limits_{x_i}^{x} dz_\mu A^\mu(z)\right] q(x_i)$



- Ward-Takahashi indentity

$$q_\mu \, \Lambda^\mu_B(p,p') = e_B \left[\Sigma_B(p) - \Sigma_B(p')\right]$$

Baryon Structure in CCQM: Parameters

- Quark masses and IR cutoff

m_u	m_s	m_c	m_b	λ	
0.242	0.428	1.672	5.046	0.181	GeV

- Hadronic scale parameters Λ_H

N, Δ	Hyperons	B_c	B_b	B_{QQ}	B_{QQQ}	
0.360	0.493	0.868	0.571	0.750	1.179	GeV

- HQL parameters

$\bar{\Lambda}$	$\bar{\Lambda}_s$	$\bar{\Lambda}_{ss}$	
0.600	0.750	0.900	GeV

Magnetic Moments

- Heavy baryon wave functions and magnetic moments in NRL QM

Baryon	Wave function	Magnetic moment
$\Lambda_{Q[ud]}$	$\frac{1}{\sqrt{2}}\,Q(ud-du)\;\;\chi_A$	$\frac{e_Q}{2m_Q}$
$\Xi_{Q[qs]}$	$\frac{1}{\sqrt{2}}\,Q(qs-sq)\;\;\chi_A$	$\frac{e_Q}{2m_Q}$
$\Sigma_{Q\{qq'\}}$	$\frac{1}{\sqrt{2}}\,Q(qq'+q'q)\;\;\chi_S$	$-\frac{e_Q}{6m_Q}+\frac{e_q}{3m_q}+\frac{e_{q'}}{3m_{q'}}$
$\Omega_{Q\{ss\}}$	$Qss\;\;\chi_S$	$-\frac{e_Q}{6m_Q}+\frac{2e_s}{3m_s}$
$\Xi_{q\{QQ'\}}$	$\frac{1}{\sqrt{2}}q(QQ'+Q'Q)\;\;\chi_S$	$-\frac{e_q}{6m_q}+\frac{e_Q}{3m_Q}+\frac{e'_Q}{3m'_Q}$
$\Omega_{s\{QQ\}}$	$sQQ\;\;\chi_S$	$-\frac{e_s}{6m_s}+\frac{2e_Q}{3m_Q}$
$\Xi_{q[cb]}$	$\frac{1}{\sqrt{2}}\,q(cb-bc)\;\;\chi_A$	$\frac{e_q}{2m_q}$
$\Xi_{q\{cb\}}$	$\frac{1}{\sqrt{2}}\,q(cb+bc)\;\;\chi_S$	$-\frac{e_q}{6m_q}+\frac{e_c}{3m_c}+\frac{e_b}{3m_b}$
$\Omega_{s[cb]}$	$\frac{1}{\sqrt{2}}\,s(cb-bc)\;\;\chi_A$	$\frac{e_s}{2m_s}$
$\Omega_{s\{cb\}}$	$\frac{1}{\sqrt{2}}\,s(cb+bc)\;\;\chi_S$	$-\frac{e_s}{6m_s}+\frac{e_c}{3m_c}+\frac{e_b}{3m_b}$
$\Omega_{b\{cc\}}$	$bcc\;\;\chi_S$	$-\frac{e_b}{6m_b}+\frac{2e_c}{3m_c}$
$\Omega_{c\{bb\}}$	$cbb\;\;\chi_S$	$-\frac{e_c}{6m_c}+\frac{2e_b}{3m_b}$

$$\chi_A = \sqrt{\frac{1}{2}}\left\{\uparrow(\uparrow\downarrow-\downarrow\uparrow)\right\}, \qquad \chi_S = \sqrt{\frac{1}{6}}\left\{\uparrow(\uparrow\downarrow+\downarrow\uparrow)-2\downarrow\uparrow\uparrow\right\}$$

Magnetic Moments

- Magnetic moments of single heavy baryons (in units of μ_N)

B	Our (Full)	Our (HQL)	NRQM	Others
Λ_c^+	0.42 (0.41; 0.01)	0.37 (0.37; 0)	0.37 (0.37; 0)	0.35; 0.38; 0.40
Λ_b^0	-0.06 (-0.06; 0.002)	-0.06 (-0.06; 0)	-0.06 (-0.06; 0)	- 0.18
Ξ_c^+	0.41 (0.40; 0.01)	0.37 (0.37, 0)	0.37 (0.37; 0)	0.38
Ξ_c^0	0.39 (0.40; -0.01)	0.37 (0.37; 0)	0.37 (0.37; 0)	0.35; 0.38
Ξ_b^0	-0.06 (-0.06; 0.002)	-0.06 (-0.06; 0)	-0.06 (-0.06; 0)	
Ξ_b^-	-0.06 (-0.06; -0.003)	-0.06 (-0.06; 0)	-0.06 (-0.06; 0)	0.35; 0.38

- Comparison with quark models:
  Jena, Rath, PRD 34 (1986) 196; Glozman, Riska, NPA 603 (1996) 326
  QCD Sum Rules: Zhu et al., PRD 56 (1997) 7273

Magnetic Moments

- Magnetic moments of single heavy baryons (in units of μ_N)

B	Our (Full)	Our (HQL)	NRQM	Others
$\Xi_c^{\prime +}$	0.47 (-0.11; 0.58)	0.08 (-0.12; 0.20)	0.51 (-0.12; 0.63)	0.76; 0.65
$\Xi_c^{\prime 0}$	-0.95 (-0.11; -0.84)	-0.37 (-0.12; -0.25)	-0.98 (-0.12; -0.86)	-1.13; -1.18
$\Xi_b^{\prime 0}$	0.66 (0.02; 0.64)	0.22 (0.02; 0.20)	0.65 (0.02; 0.63)	0.90
$\Xi_b^{\prime -}$	-0.91 (0.02; -0.93)	-0.23 (0.02; -0.25)	-0.84 (0.02; -0.86)	-1
Σ_c^{++}	1.76 (-0.11; 1.87)	0.53 (-0.12; 0.65)	1.86 (-0.12; 1.98)	2.41; 2.33; 2.1
Σ_c^{+}	0.36 (-0.11; 0.47)	0.04 (-0.12; 0.16)	0.37 (-0.12; 0.49)	0.50; 0.49; 0.6
Σ_c^{0}	-1.04 (-0.11; -0.93)	-0.44 (-0.12; -0.32)	-1.11 (-0.12; -0.99)	- 1.38; - 1.35; - 1.6
Σ_b^{+}	2.07 (0.02; 2.05)	0.67 (0.02; 0.65)	2.01 (0.02; 1.99)	2.55
Σ_b^{0}	0.53 (0.02; 0.51)	0.18 (0.02; 0.16)	0.52 (0.02; 0.50)	0.65
Σ_b^{-}	-1.01 (0.02; -1.03)	-0.30 (0.02; -0.32)	-0.97 (0.02; -0.99)	- 1.24
Ω_c^{0}	-0.85 (-0.11; -0.74)	-0.31 (-0.12; -0.19)	-0.85 (-0.12; -0.73)	- 0.89; -1.02
Ω_b^{-}	-0.82 (0.02; -0.84)	-0.17 (0.02; -0.19)	-0.71 (0.02; -0.73)	- 0.75

- Comparison with quark models:
  Jena, Rath, PRD 34 (1986) 196; Glozman, Riska, NPA 603 (1996) 326
  QCD Sum Rules: Zhu et al., PRD 56 (1997) 7273

Magnetic Moments

- Magnetic moments of double and triple heavy baryons (in units of μ_N)

B	Our (Full)	Our (HQL)	NRQM	Others
Ξ_{cc}^{++}	0.13 (0.52; -0.38)	0.25 (0.51; -0.26)	-0.01 (0.49; -0.50)	-0.25
Ξ_{cc}^{+}	0.71 (0.52; 0.19)	0.64 (0.51; 0.13)	0.74 (0.49; 0.25)	0.85
Ξ_{bb}^{0}	-0.54 (-0.06; -0.48)	-0.42 (-0.08; -0.34)	-0.58 (-0.08; -0.50)	- 0.84
Ξ_{bb}^{-}	0.18 (-0.06; 0.24)	0.09 (-0.08; 0.17)	0.17 (-0.08; 0.25)	0.26
Ω_{cc}^{+}	0.67 (0.53; 0.14)	0.60 (0.50; 0.10)	0.67 (0.49; 0.18)	0.78
Ω_{bb}^{-}	0.04 (-0.08; 0.12)	0.14 (-0.06; 0.20)	0.10 (-0.08; 0.18)	0.19
Ξ_{cb}^{+}	1.52 (0.002; 1.52)	0.75 (0.001; 0.75)	1.49 (0; 1.49)	0.69
Ξ_{cb}^{0}	-0.76 (0.002, -0.76)	-0.38 (0.001; -0.38)	-0.74 (0; -0.74)	- 0.59
$\Xi_{cb}^{\prime +}$	-0.12 (0.24; -0.36)	0.18 (0.42; -0.24)	-0.29 (0.21; -0.50)	-0.54
$\Xi_{cb}^{\prime 0}$	0.42 (0.24; 0.18)	0.54 (0.42; 0.12)	0.46 (0.21; 0.25)	0.56
Ω_{cb}^{0}	-0.61 (0.002; -0.61)	-0.26 (0.001; -0.26)	-0.55 (0; -0.55)	0.24
$\Omega_{cb}^{\prime 0}$	0.45 (0.25; 0.20)	0.50 (0.42; 0.08)	0.39 (0.21; 0.18)	0.49
Ω_{ccb}^{+}	0.53 (0.02; 0.51)	0.14 (0.02; 0.12)	0.51 (0.02; 0.49)	
Ω_{cbb}^{0}	-0.20 (-0.08; -0.12)	-0.13 (-0.05; -0.08)	-0.20 (-0.08; -0.12)	

- Comparison with HBChPT: Phys.Rev. D96 (2017) 076011

Radiative Decays

- Radiative Decay Widths (in keV)

Mode	Our	Others
$\Sigma_c^+ \to \Lambda_c + \gamma$	60.7	93 [1]
$\Sigma_c^{*+} \to \Lambda_c^+ + \gamma$	151	
$\Sigma_c^{*+} \to \Sigma_c^+ + \gamma$	0.14	
$\Xi_c^{\prime +} \to \Xi_c^+ + \gamma$	12.7	16 [1]
$\Xi_c^{\prime 0} \to \Xi_c^0 + \gamma$	0.17	0.30 [1]
$\Xi_c^{*+} \to \Xi_c^+ + \gamma$	54	
$\Xi_c^{\prime 0} \to \Xi_c^0 + \gamma$	0.68	
$\Lambda_c(2593)^+ \to \Lambda_c^+ + \gamma$	115	191 c_{RT}^2 [2]; 16 [3]
$\Lambda_c(2593)^+ \to \Sigma_c^+ + \gamma$	77	127 c_{RS}^2 [2]
$\Lambda_c(2593)^+ \to \Sigma_c^{*+} + \gamma$	6	6 c_{RS}^2 [2]
$\Lambda_c(2625)^{*+} \to \Lambda_c^+ + \gamma$	151	253 c_{RT}^2 [2]; 21 [3]
$\Lambda_c(2625)^{*+} \to \Sigma_c^+ + \gamma$	35	58 c_{RS}^2 [2]
$\Lambda_c(2625)^{*+} \to \Sigma_c^{*+} + \gamma$	46	54 c_{RS}^2 [2]
$\Xi_c(2815)^{*+} \to \Xi_c^+ + \gamma$	190	
$\Xi_c(2815)^{*0} \to \Xi_c^0 + \gamma$	497	
$\Lambda_b(5933)^0 \to \Lambda_b^0 + \gamma$	128	90 [3]
$\Lambda_b(5966)^{*0} \to \Lambda_b^0 + \gamma$	172	119 [3]

- [1] Cheng et al, PRD 47 (1993) 1030
  [2] Cho, PRD 50 (1994) 3295
  [3] Chow, PRD 54 (1996) 3374

Strong Decays

- Strong Decay Widths (in MeV)

Mode	Our	Data
P-wave		
$\Sigma_c^{++} \to \Lambda_c + \pi^+$	2.85	$2.34 \pm 0.13 \pm 0.45$ (CDF); $1.84 \pm 0.04^{+0.07}_{-0.20}$ (Belle)
$\Sigma_c^0 \to \Lambda_c + \pi^+$	2.65	$1.65 \pm 0.11 \pm 0.49$ (CDF); $1.76 \pm 0.04^{+0.09}_{-0.21}$ (Belle)
$\Sigma_c^{*++} \to \Lambda_c + \pi^+$	21.21	$15.03 \pm 2.12 \pm 1.36$ (CDF); $14.77 \pm 0.25^{+0.18}_{-0.30}$ (Belle)
$\Sigma_c^{*0} \to \Lambda_c + \pi^-$	22.00	$12.51 \pm 1.82 \pm 1.37$ (CDF); $15.41 \pm 0.41^{+0.20}_{-0.32}$ (Belle)
$\Xi_c^{*+} \to \Xi_c^0 + \pi^+$	1.78	2.14 ± 0.19
$\Xi_c^{*0} \to \Xi_c^+ + \pi^-$	2.11	$2.35 \pm 0.18 \pm 0.13$
S-wave		
$\Lambda_c(2593) \to \Sigma_c^{++} + \pi^-$	0.79	0.62 ± 0.20
$\Lambda_c(2815) \to \Sigma_c^{++} + \pi^-$	0.08	< 0.1
D-wave		
$\Xi_c^*(2625) \to \Xi_c^0 + \pi^+$	0.08	

Radiative Decays of DHBs

- Radiative Decay Widths of DHBs (in keV)

Decay mode	Exact results	HQL	NQM
$\Xi'_{bc} \to \Xi_{bc}$ $\Omega'_{bc} \to \Omega_{bc}$ $\Xi^*_{bc} \to \Xi'_{bc}$	1.56×10^{-2} 1.26×10^{-2} 0.28×10^{-2}	0 0 0	1.35×10^{-2} 1.10×10^{-2} 0.25×10^{-2}
$\Omega^*_{bc} \to \Omega'_{bc}$	0.16×10^{-2}	0	0.14×10^{-2}
$\Xi^{*++}_{cc} \to \Xi^{++}_{cc}$	23.46 ± 3.33	20.53 ± 0.79	36.22
$\Xi^{*+}_{cc} \to \Xi^{+}_{cc}$	28.79 ± 2.51	5.13 ± 0.20	35.65
$\Omega^*_{cc} \to \Omega_{cc}$	2.11 ± 0.11	$\simeq 0.29$	2.42
$\Xi^{*+}_{bc} \to \Xi^{+}_{bc}$	0.49 ± 0.09	$\simeq 0.27$	0.67
$\Xi^{*0}_{bc} \to \Xi^{0}_{bc}$	0.24 ± 0.04	$\simeq 0.07$	0.30
$\Omega^*_{bc} \to \Omega_{bc}$	0.12 ± 0.02	$\simeq 0.03$	0.13
$\Xi^{*+}_{bc} \to \Xi^{l+}_{bc}$	0.46 ± 0.10	$\simeq 0.37$	0.69
$\Xi^{*0}_{bb} \to \Xi^{0}_{bb}$	0.31 ± 0.06	$\simeq 0.11$	0.38
$\Xi^{*-}_{bb} \to \Xi^{-}_{bb}$	5.87×10^{-2}	$\simeq 2.8 \times 10^{-2}$	7.34×10^{-2}
$\Omega^*_{bb} \to \Omega_{bb}$	2.26×10^{-2}	$\simeq 1.0 \times 10^{-2}$	2.36×10^{-2}

Nonleptonic Decays

- Nonleptonic Decay Branchings of Charm Baryons (in units of 10^{-3})
  $c \to s u \bar{d}$ transition

Mode	Our	Data	Theory
$\Lambda_c \to \Lambda\pi^+$	25.5 (7.9)	13 ± 7	7.6 (Körner); 16.7 (Xu); 9.1 (Cheng); 11.2 (Sharma)
$\Lambda_c \to \Lambda\rho^+$	99.2 (44)	< 60	
$\Xi_c \to \Xi\pi^+$	33.4	5.5 ± 1.6	28 (Körner); 49 (Xu); 25 (Cheng); 10.8 (Sharma)
$\Xi_c \to \Xi\rho^+$	208.7		
$\Xi_c' \to \Xi\pi^+$	1.0		
$\Xi_c' \to \Xi\rho^+$	16.5		
$\Omega_c \to \Omega\pi^+$	25.0		10 (Cheng)
$\Omega_c \to \Omega\rho^+$	196.1		36 (Cheng)

Nonleptonic Decays

- Nonleptonic Decay $\Lambda_c \to p\phi$

Branching ratio $B(\Lambda_c \to p\phi)$ in units 10^{-4}

Our result	Theoretical predictions	Data
14.0	19.5 (Cheng); 21.5 (Körner); 9.89 (Zenczykowski); 4.0 (Datta)	10.8 ± 1.4

Branching ratios $B(\Lambda_c \to p\phi\,(\ell^+\ell^-))$ in units of 10^{-7}

Mode	Our results	Data
$\Lambda_c \to p + e^+e^-$	4.11	< 55
$\Lambda_c \to p + \mu^+\mu^-$	4.11	< 440

Nonleptonic Decays

- Nonleptonic Decay Branchings of Λ_b (in units of 10^{-4})
  $b \to cs(d)\bar{c}$ transition

Mode	Our	Data	Theory
$\Lambda_b \to \Lambda_c\, D_s^-$	147.7	110 ± 10	230^{+30}_{-40} (Mannel); 110 (Cheng); 223 (Fayyazuddin); 77 (Mohanta); 129.1 (Giri)
$\Lambda_b \to \Lambda_c\, D_s^{*\,-}$	251.6		173^{+20}_{-30} (Mannel); 91 (Cheng); 326 (Fayyazuddin); 141.4 (Mohanta); 198.3 (Giri)
$\Lambda_b \to \Lambda_c\, D^-$	5.4	4.6 ± 0.6	3 (Mohanta)
$\Lambda_b \to \Lambda_c\, D^{*\,-}$	11.0		4.9 (Mohanta)
$\Lambda_b \to \Lambda J/\psi$	8.3	8.3 ± 1.1	2.1 (Cheng); 1.6 (Cheng); 6.0 (Fayyazuddin); 2.5 (Mohanta); 3.5 ± 1.8 (Chou); 8.4 (Wei); 8.2 (Mott); 7.8 (Fayyazuddin); 3.3 ± 2.0 (Hsiao)
$\Lambda_b \to \Lambda\eta_c$	4.2		1.5 ± 0.9 (Hsiao)

Nonleptonic Decays

- Nonleptonic Decay Rates of Ξ_b (in units of 10^{-4})
  $b \to cs(d)\bar{c}$ transition

Mode	Our	Data	Theory
$\Xi_b \to \Xi_c \, D_s^-$	184.9		
$\Xi_b \to \Xi_c \, D_s^{*\,-}$	320.4		
$\Xi_b \to \Xi_c \, D^-$	4.7		
$\Xi_b \to \Xi_c \, D^{*\,-}$	10.0		
$\Xi_b' \to \Xi_c' \, D_s^-$	74.9		
$\Xi_b' \to \Xi_c' \, D_s^{*\,-}$	69.3		
$\Xi_b' \to \Xi_c' \, D^-$	1.8		
$\Xi_b' \to \Xi_c' \, D^{*\,-}$	2.2		
$\Xi_b \to \Xi J/\psi$	4.6		4.9 ± 3.0 (Hsiao)
$\Xi_b \to \Xi^0 \eta_c$	1.7		2.3 ± 1.4 (Hsiao)
$\Xi_b' \to \Xi J/\psi$	0.7		
$\Xi_b \to \Xi^0 \eta_c$	0.1		

Nonleptonic Decays

- Nonleptonic Decay Rates of Ω_b (in units of 10^{-4})
  $b \to cs(d)\bar{c}$ transition

Mode	Our	Data	Theory
$\Omega_b \to \Omega_c\, D_s^-$	65.1		
$\Omega_b \to \Omega_c\, D_s^{*\,-}$	63.1		
$\Omega_b \to \Omega_c\, D^-$	2.3		
$\Omega_b \to \Omega_c\, D^{*\,-}$	2.8		
$\Omega_b \to \Omega_c^*\, D_s^-$	18.6		
$\Omega_b \to \Omega_c^*\, D_s^{*\,-}$	132.0		
$\Omega_b \to \Omega_c^*\, D^-$	0.7		
$\Omega_b \to \Omega_c^*\, D^{*\,-}$	5.5		
$\Omega_b \to \Omega J/\psi$	18.9		
$\Omega_b \to \Omega \eta_c$	4.7		

Nonleptonic Decays

- Nonleptonic Decay Rates of Ω_b (in units of 10^{-3})
  $b \to cd(s)\bar{u}$ transition

Mode	Our	Data	Theory
$\Lambda_b \to \Lambda_c \pi^-$	5.9	4.9 ± 0.4	$4.6^{+0.2}_{-0.3}$ (Mannel); 3.8 (Cheng); 1.8 (Mohanta); 3.9 (Giri); 5.0 (Lee); 2.85 ± 0.54 (Huber)
$\Lambda_b \to \Lambda_c \rho^-$	17.3		$6.6^{+2.4}_{-4.0}$ (Mannel); 5.4 (Cheng); 4.9 (Mohanta); 10.8 (Giri); 7.2 (Lee); 8.17 ± 1.47 (Huber)
$\Lambda_b \to \Lambda_c K-$	0.4	0.359 ± 0.30	1.3 (Mohanta); 0.37 (Lee); 2.21 ± 0.40 (Huber)
$\Lambda_b \to \Lambda_c K^{*-}$	0.9		0.37 (Lee); 4.22 ± 0.75 (Huber)
$\Xi_b \to \Xi_c \pi^-$	4.7		
$\Xi_b \to \Xi_c \rho^-$	14.1		
$\Xi_b \to \Xi_c K-$	0.4		
$\Xi_b \to \Xi_c K^{*-}$	0.8		
$\Xi'_b \to \Xi'_c \pi^-$	1.3		
$\Xi'_b \to \Xi'_c \rho^-$	3.9		
$\Xi'_b \to \Xi'_c K-$	0.1		
$\Xi'_b \to \Xi'_c K^{*-}$	0.2		
$\Omega_b \to \Omega_c \pi^-$	1.8		
$\Omega_b \to \Omega_c \rho^-$	5.2		
$\Omega_b \to \Omega_c^* \pi^-$	1.6		
$\Omega_b \to \Omega_c^* \rho^-$	5.3		

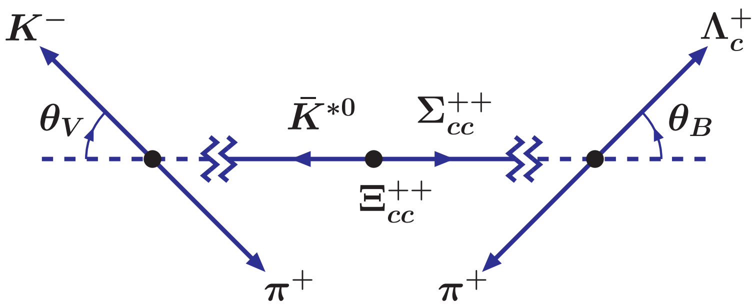
Semileptonic Decays

- Semileptonic Decay Branchings Rates of Single Heavy Baryons (in %)

Mode	Our	Data	Theory
$\Lambda_c^+ \to \Lambda^0 e^+ \nu_e$	2.78	2.9 ± 0.5 (Belle) 3.63 ± 0.58 (BESIII)	3 (Gavela); 3.4 (Perez); 4.4 (Hussain); 2 (Singleton); 1.42 (Cheng); 1.07 (Datta); 1.44 (Luo); 2.64 (Marques); 3.05 (Liu)
$\Lambda_c^+ \to \Lambda^0 \mu^+ \nu_\mu$	2.69	2.7 ± 0.6 (Belle)	2.6 (Perez)
$\Lambda_b^0 \to \Lambda_c^+ e^- \bar{\nu}_e$	6.9	$6.5^{+3.2}_{-2.5}$	6.04 (Azizi); 6.48 (Faustov); 5.81 (Detmold)
$\Lambda_b^0 \to \Lambda_c^+ \tau^- \bar{\nu}_\tau$	2.0	$6.5^{+3.2}_{-2.5}$	1.87 (Azizi); 2.03 Faustov; 1.87 (Detmold)

Cascade decays

- Cascade decay $\Xi_{cc}^{++} \to \Sigma_c^{++}(\to \Lambda_c^+\pi^+) + \bar{K}^{*0}(\to K^-\pi^+)$



Rates

$$\Gamma(\Xi_{cc}^{++} \to \Sigma_c^{++} + \bar{K}^{*0}) = 0.21 \times 10^{12}\,\mathrm{s}^{-1}$$

$$\Gamma(\Xi_{cc}^{++} \to \Sigma_c^{++} + \bar{K}^{0}) = 0.05 \times 10^{12}\,\mathrm{s}^{-1}$$

Branchings (using LHCb prediction $\tau_{\Xi_{cc}^{++}} = 256^{+22}_{-20} \pm 14$ fs, arXiv:1806.02744)

$$B(\Xi_{cc}^{++} \to \Sigma_c^{++} + \bar{K}^{*0}) = \left(\frac{\tau_{\Xi_{cc}^{++}}}{500\,\mathrm{fs}}\right) \cdot 10.5\,\% = 5.4\,\%$$

$$B(\Xi_{cc}^{++} \to \Sigma_c^{++} + \bar{K}^{0}) = \left(\frac{\tau_{\Xi_{cc}^{++}}}{500\,\mathrm{fs}}\right) \cdot 2.5\,\% = 1.3\,\%$$

Cascade decays

- Polarization, longitudinal/transverse helicity fraction and angular decay distributions

$$\frac{d\,\Gamma(\Xi_{cc}^{++} \to \Sigma_c^{++} + \bar{K}^{*0}(\to K^- \pi^+))}{d\cos\theta_V} \quad \sim \quad \frac{3}{2}\cos^2\theta_V\,\mathcal{F}_L + \frac{3}{4}\sin^2\theta_V\,\mathcal{F}_T$$

The angular decay distribution involves the helicity fractions of the $\bar{K}^{*0}$ defined by

$$\mathcal{F}_L = \frac{\mathcal{H}_L}{\mathcal{H}_T + \mathcal{H}_L} = 0.48\,, \qquad \mathcal{F}_T = \frac{\mathcal{H}_T}{\mathcal{H}_T + \mathcal{H}_L} = 0.52 \pm 0.01\,.$$

$$\begin{aligned} \mathcal{H}_T &= |H_{\frac{1}{2}1}|^2 + |H_{-\frac{1}{2}-1}|^2 \\ \mathcal{H}_L &= |H_{\frac{1}{2}0}|^2 + |H_{-\frac{1}{2}0}|^2 \end{aligned}$$

where $H_{\lambda_2\lambda_V}$ is helicity amplitude, λ_2 and $\lambda_1 = \lambda_2 - \lambda_V$ are baryon helicities

Cascade decays

- Longitudinal polarization of Σ_c^{++} depends on polar angle θ_V

$$P_{\Sigma_c^{++}}(\cos\theta_V) = \frac{\frac{3}{4}\sin^2\theta_V\left(|H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2\right) + \frac{3}{2}\cos^2\theta_V\left(|H_{\frac{1}{2}0}|^2 - |H_{-\frac{1}{2}0}|^2\right)}{\frac{3}{4}\sin^2\theta_V\left(|H_{\frac{1}{2}1}|^2 + |H_{-\frac{1}{2}-1}|^2\right) + \frac{3}{2}\cos^2\theta_V\left(|H_{\frac{1}{2}0}|^2 + |H_{-\frac{1}{2}0}|^2\right)}\,.$$

When averaged over $\cos\theta_V$ one has

$$P_{\Sigma_c^{++}} = \frac{\left(|H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2\right) + \left(|H_{\frac{1}{2}0}|^2 - |H_{-\frac{1}{2}0}|^2\right)}{\mathcal{H}_N} = -(0.83 \pm 0.01)\,.$$

Average longitudinal polarization of the Λ_c^+ (we average over $\cos\theta_V$):

$$P_{\Lambda_c^+}(\theta_B) = \frac{|H_{\frac{1}{2}0}|^2 - |H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2}{\mathcal{H}_N}\;\cos\theta_B \;=\; -(0.83 \pm 0.01)\;\cos\theta_B$$

- Longitudinal polarization of the Σ_c^{++}

$$P_{\Sigma_c^{++}}(\Xi_{cc}^{++} \to \Sigma_c^{++} + \bar{K}^0) = \frac{|H_{\frac{1}{2}t}|^2 - |H_{-\frac{1}{2}t}|^2}{\mathcal{H}_S} = -(0.95 \pm 0.02)\,.$$

Summary

- Covariant Confined Quark Model
- Unified description of heavy baryons
- Useful and accurate tool for analysis of data and predictions for planned experiments
- Successful application to mesons, tetraquark, etc.