Lifetimes of charmed hadrons within the heavy-quark expansion

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IPPP Durham

Workshop on singly and doubly charmed baryons
LPNHE Paris
26.06.18

Heavy quark expansion in charm?

B-physics: HQE is well established approach, $\Lambda/m_b \sim 0.2 \ll 1$

$$\Delta\Gamma_s^{\mathrm{exp}} = (0.086 \pm 0.006)\,\mathrm{ps}^{-1}, \qquad \Delta\Gamma_s^{\mathrm{SM}} = (0.088 \pm 0.020)\,\mathrm{ps}^{-1}.$$

[HFLAV '18] [Artuso, Borissov, Lenz '16]

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- $\Delta\Gamma_s$ dominated by $\mathrm{D}_s^{(*)+} D_s^{(*)-}$ final state, momentum release $\sim 3.5\ \mathrm{GeV}$
- D decays dominated by $\mathrm{K}\pi^{(1-3)}$ final state, momentum release $\sim 1.7\ \mathrm{GeV}$
- expected expansion parameter is of the order 0.4

Small enough for convergence?

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Small enough for convergence?

Shut up and calculate!

Outline

- Introduction to HQE
- D-meson lifetimes as testing ground
- Hadronic matrix elements from sum rules
- Singly charmed baryons
- Doubly charmed baryons
- Outlook

The Heavy Quark Expansion (HQE)

Use optical theorem:

$$\Gamma(H_c) = \frac{1}{2M_{H_c}} \left\langle H_c \right| \mathrm{Im} \left(i \int d^4x T \left[\mathcal{H}_{\mathrm{eff}}(x) \mathcal{H}_{\mathrm{eff}}(0) \right] \right) \left| H_c \right\rangle = \frac{1}{2M_{H_c}} \langle H_c | \mathcal{T} | H_c \rangle$$

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OPE for small x, i.e. large momentum release

$$\begin{aligned}\Gamma(H_c \to f) = & \frac{G_F^2 m_c^5}{192\pi^3} |V_{CKM}|^2 \frac{1}{2M_{H_c}} \Bigg[c_3^f \left\langle H_c \right| \bar{c}c \left| H_c \right\rangle \\ & + c_5^f \frac{\left\langle H_c \right| \bar{c} g_s \sigma_{\mu\nu} G^{\mu\nu} c \left| H_c \right\rangle}{m_c^2} \\ & + \sum_i c_{6,i}^f \frac{\left\langle H_c \right| (\bar{c}\Gamma_i q)(\bar{q}\Gamma_i' c) \left| H_c \right\rangle}{m_c^3} \\ & + \mathcal{O}\left(\frac{1}{m_c^4}\right) \Bigg].\end{aligned}$$

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$$+ c_5^f \frac{\langle H_c | \bar{c} g_s \sigma_{\mu\nu} G^{\mu\nu} c | H_c \rangle}{m_c^2}$$
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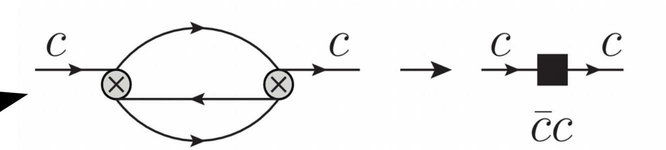

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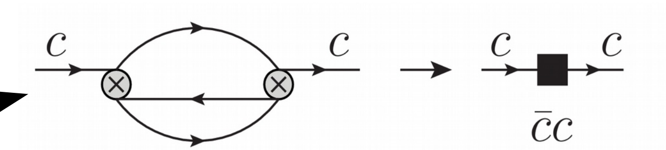

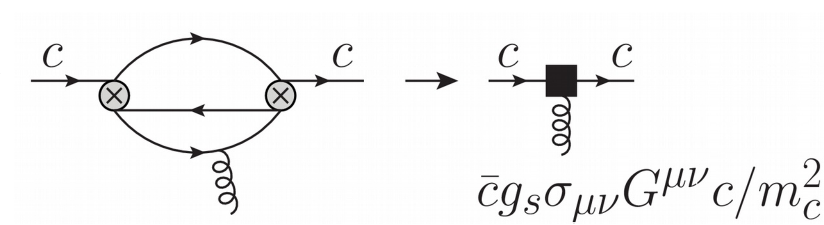

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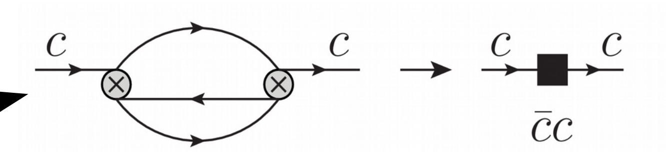
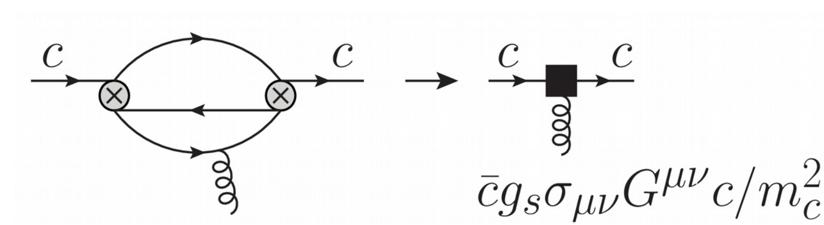
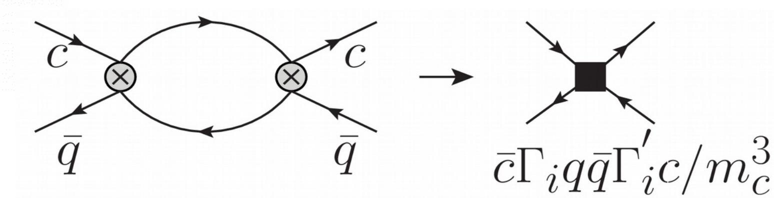


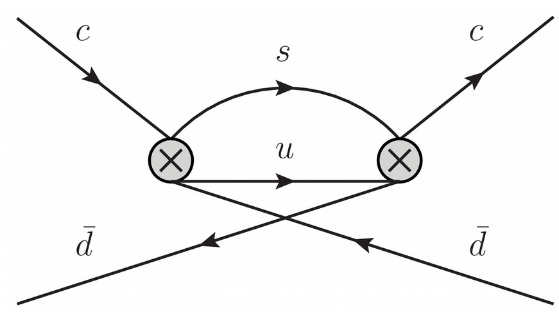




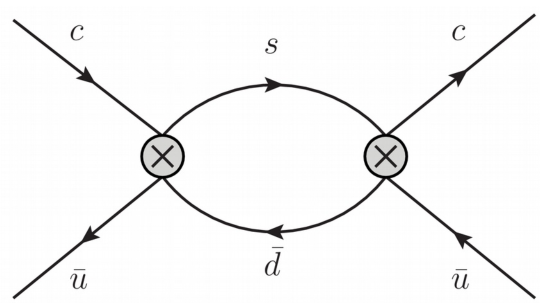
D-meson lifetimes

Large lifetime ratio: $\left(\frac{\tau(D^+)}{\tau(D^0)}\right)_{\text{exp}} = 2.536 \pm 0.019$

Dominant contribution from spectator effects:



Pauli interference



Weak annihilation

Known at NLO in QCD:

[Beneke, Buchalla, Greub, Lenz, Nierste '02]

[Ciuchini, Franco, Lubicz, Mescia '01]

[Franco, Lubicz, Mescia, Tarantino '02]

- Phase-space enhancement of $16\pi^2$, 2 → 2 process instead of 1 → 3
- Large ratio does not contradict convergence: $2.5 \approx 1 + 0.21^3 \times 16\pi^2$

Studied in [Lenz, TR '13] including NLO QCD and $1/m_c$ corrections.

Large hadronic uncertainties from missing lattice input!

Non-perturbative input

Need hadronic matrix elements of the dimension-six operators:

$$
\begin{aligned}
Q^q &= \bar{c}\gamma_\mu(1-\gamma_5)q\ \bar{q}\gamma^\mu(1-\gamma_5)c, & Q_S^q &= \bar{c}(1-\gamma_5)q\ \bar{q}(1+\gamma_5)c,\\
T^q &= \bar{c}\gamma_\mu(1-\gamma_5)T^a q\ \bar{q}\gamma^\mu(1-\gamma_5)T^a c, & T_S^q &= \bar{c}(1-\gamma_5)T^a q\ \bar{q}(1+\gamma_5)T^a c.
\end{aligned}
$$

Commonly parametrized through Bag parameters:

$$
\begin{aligned}
\left\langle D^+|Q^d - Q^u|D^+\right\rangle &= f_D^2 M_D^2\, B_1, & \left\langle D^+|Q_S^d - Q_S^u|D^+\right\rangle &= f_D^2 M_D^2\, B_2,\\
\left\langle D^+|T^d - T^u|D^+\right\rangle &= f_D^2 M_D^2\, \epsilon_1, & \left\langle D^+|T_S^d - T_S^u|D^+\right\rangle &= f_D^2 M_D^2\, \epsilon_2.
\end{aligned}
$$

Inspired by vacuum saturation approximation (VSA):

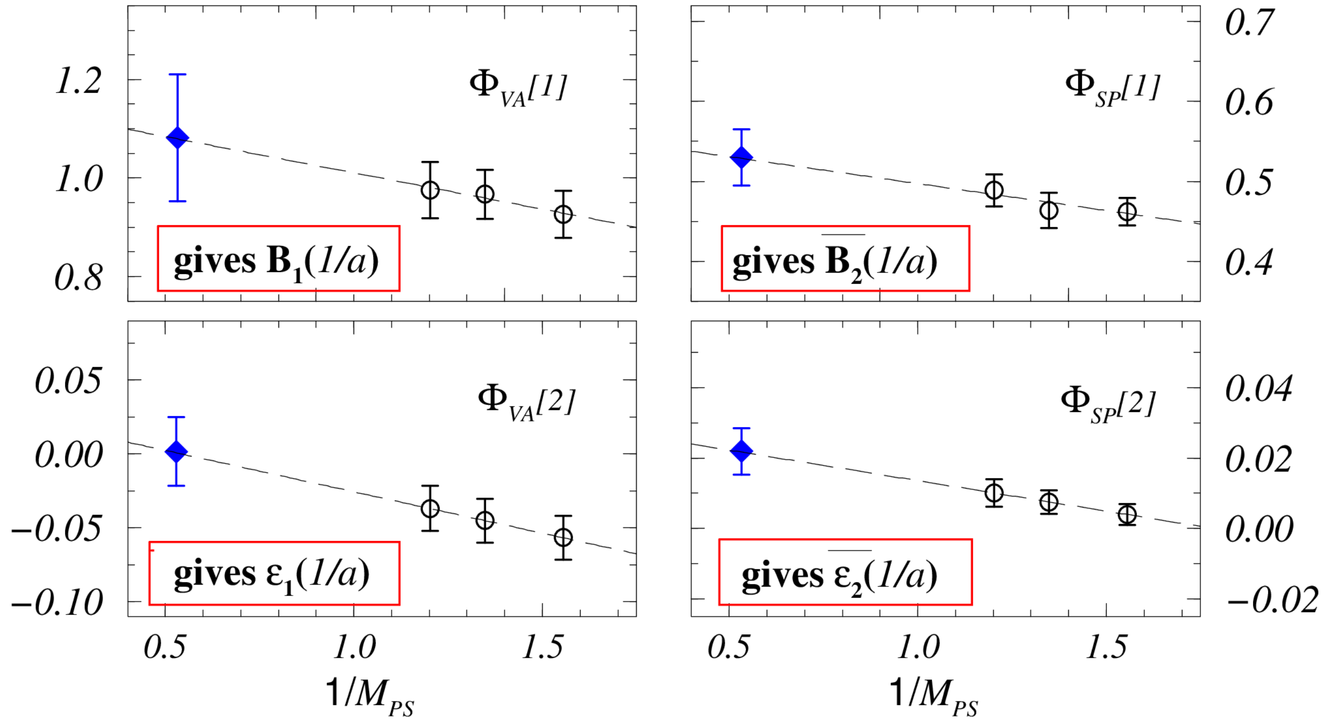
$$
\langle D|\bar{c}\Gamma q\bar{q}\Gamma' c|D\rangle = \sum_X \langle D|\bar{c}\Gamma q|X\rangle\, \langle X|\bar{q}\Gamma' c|D\rangle \approx \langle D|\bar{c}\Gamma q|0\rangle\, \langle 0|\bar{q}\Gamma' c|D\rangle
$$

This yields:

$$
\mathrm{B}_i^{\mathrm{VSA}} = 1 \pm \tfrac{1}{N_c}, \quad \epsilon_i^{\mathrm{VSA}} = 0 \pm \tfrac{1}{N_c}.
$$

Status of lattice for lifetimes

Latest result is from quenched computation for B mesons in 2001 [Becirevic '01]

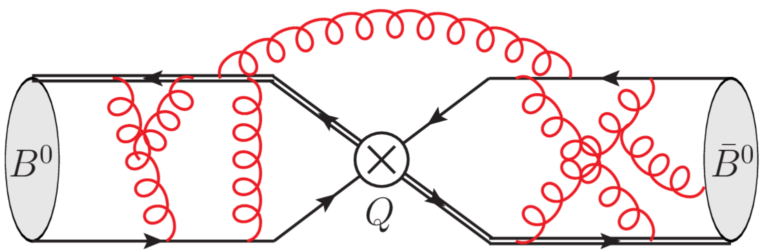


$$\left(\frac{\tau(D^+)}{\tau(D^0)}\right)_{\overline{\mathrm{MS}},\mathrm{VSA}} = 2.2 \pm 1.6^{(\mathrm{hadronic})}\,{}^{+0.3}_{-0.7}{}^{(\mathrm{scale})} \pm 0.1^{(\mathrm{exp})},$$

$$\left(\frac{\tau(D^+)}{\tau(D^0)}\right)_{\overline{\mathrm{MS}},\mathrm{Bec01}} = 2.2 \pm 0.4^{(\mathrm{hadronic})}\,{}^{+0.3}_{-0.7}{}^{(\mathrm{scale})} \pm 0.1^{(\mathrm{exp})}.$$

[Lenz, TR '13]

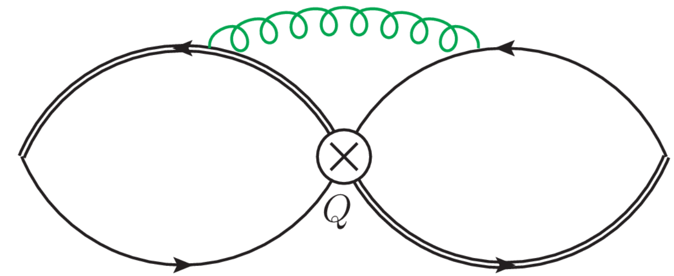
Sum rule determination



Sum rule $\longleftrightarrow$

Quark-hadron duality

Analyticity



Hadronic matrix element

Characteristic scale: Λ_{QCD}

$\alpha_s\left(\Lambda_{\mathrm{QCD}}\right) \sim \mathcal{O}(1)$

$\Rightarrow$ non-perturbative

Correlation function

Characteristic scale: 'virtuality' ω

Choose ω s.t. $\alpha_s\left(\omega\right) \ll 1$

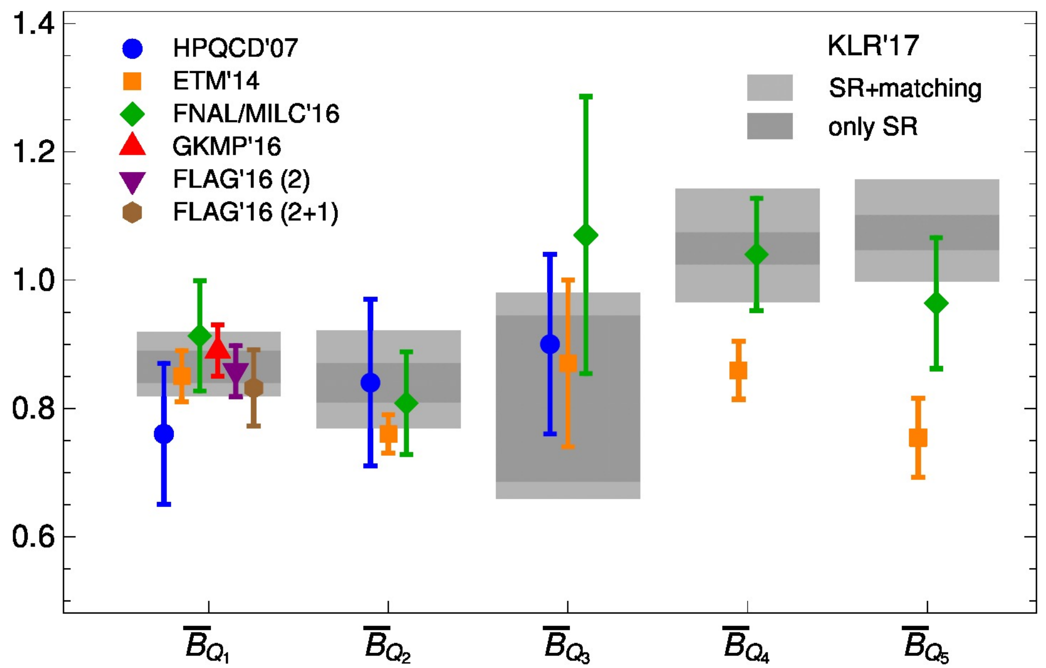
$\Rightarrow$ perturbatively calculable

$$\mathrm{F}^2(\mu)\langle \tilde{\mathcal{O}}(\mu)\rangle\, e^{-\frac{\overline{\Lambda}}{t_1}-\frac{\overline{\Lambda}}{t_2}} = \int\limits_0^{\omega_c} d\omega_1 d\omega_2\, e^{-\frac{\omega_1}{t_1}-\frac{\omega_2}{t_2}}\, \rho^{\mathrm{OPE}}_{\tilde{\mathcal{O}}}(\omega_1,\omega_2).$$

$$\begin{aligned}\rho^{\mathrm{OPE}}_{\tilde{\mathcal{O}}}(\omega_1,\omega_2) = {} & \rho^{\mathrm{pert}}_{\tilde{\mathcal{O}}}\left(\frac{\omega_1}{\omega_2}\right)\omega_1^2\omega_2^2 + \rho^{\langle\bar{q}q\rangle}_{\tilde{\mathcal{O}}}\left(\frac{\omega_1}{\omega_2}\right)\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\left[\omega_2^2\delta(\omega_1)+\omega_1^2\delta(\omega_2)\right] + \\ & \rho^{\langle G^2\rangle}_{\tilde{\mathcal{O}}}\left(\frac{\omega_1}{\omega_2}\right)\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + \rho^{\langle\bar{q}G^2q\rangle}_{\tilde{\mathcal{O}}}\left(\frac{\omega_1}{\omega_2}\right)\langle g_s\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle\left[\delta(\omega_1)+\delta(\omega_2)\right] \\ & + \dots\end{aligned}$$

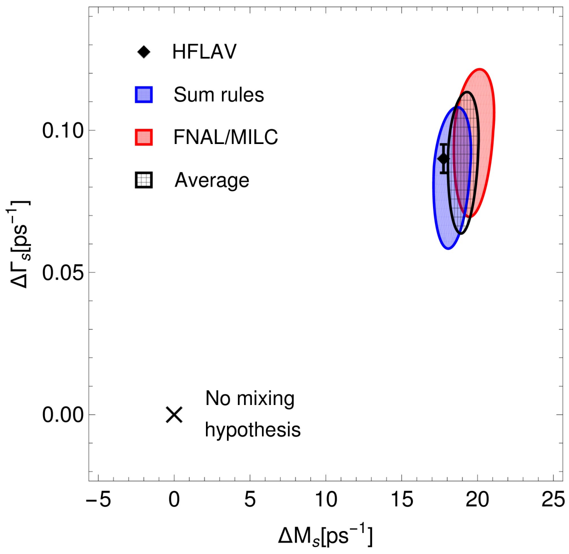
Three-loop HQET master integrals from [Grozin, Lee '08]

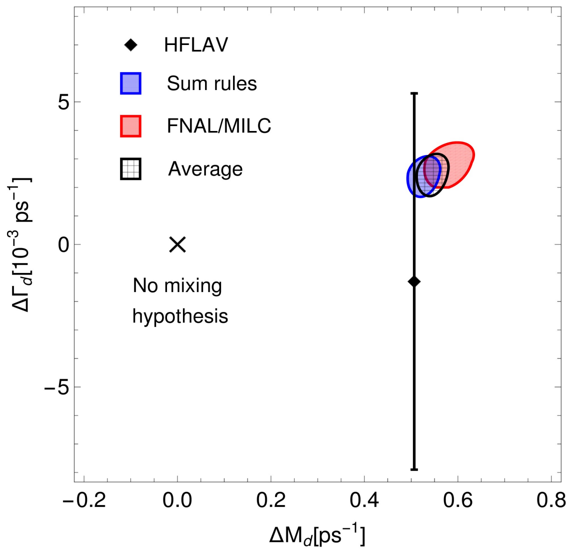
Comparison with lattice & data (B mixing)



- Good agreement with lattice with competitive uncertainties

- Good agreement with experimental data on B mixing



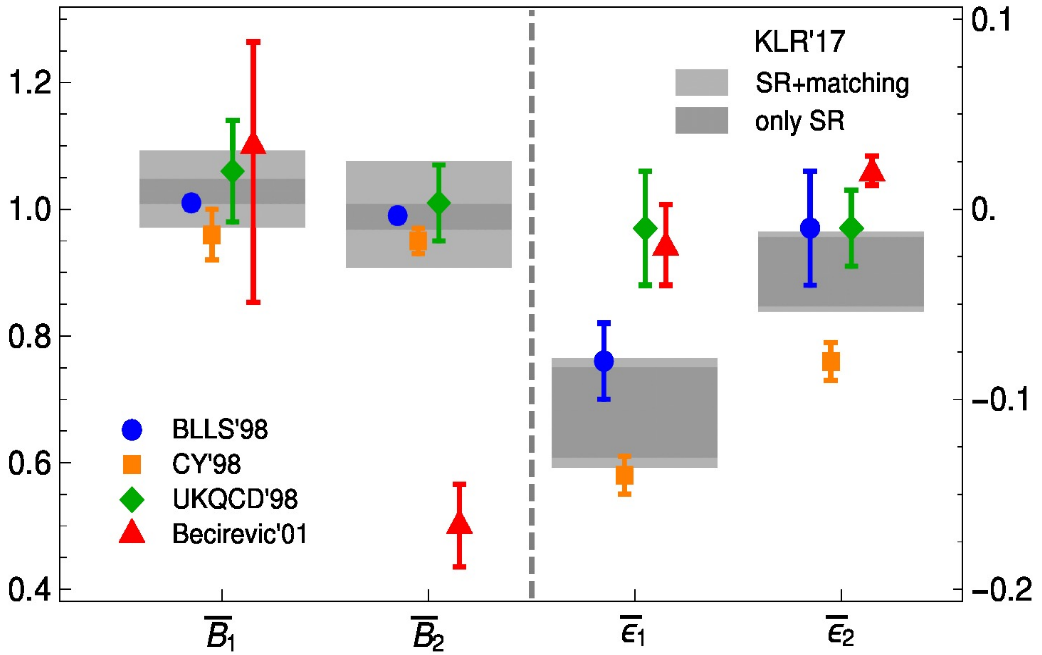


[Kirk, Lenz, TR, '17]

Earlier sum rule study for Q1: [Grozin, Klein, Mannel, Pivovarov, '16]

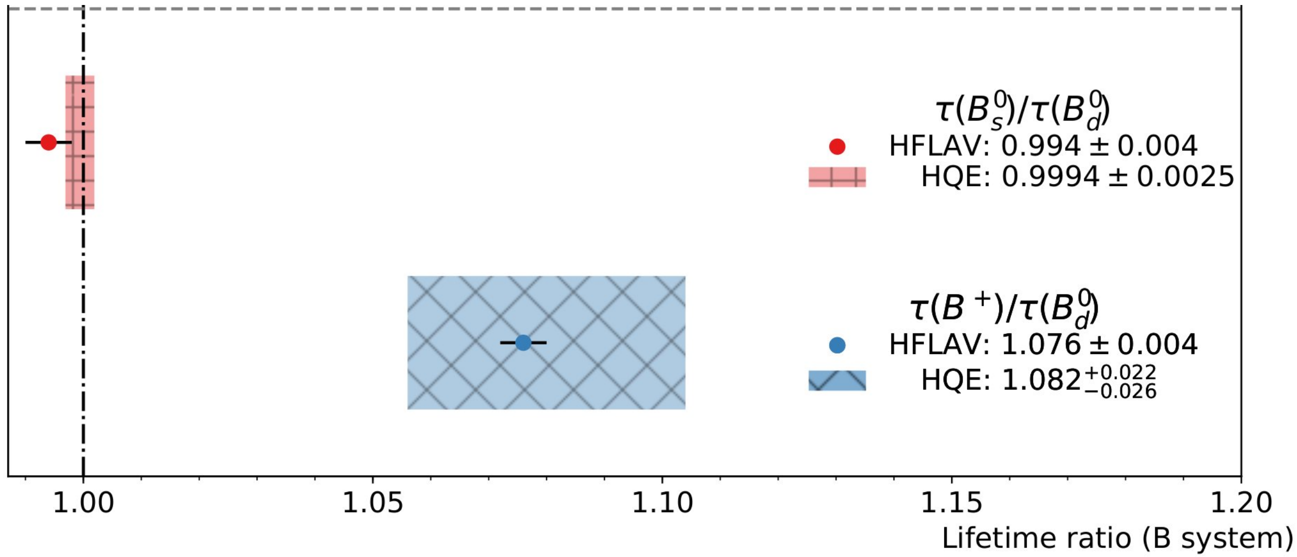
QCD-HQET matching for Q1 at NNLO: [Grozin, Mannel, Pivovarov, '17 - '18]

Comparison with lattice & data (B lifetimes)

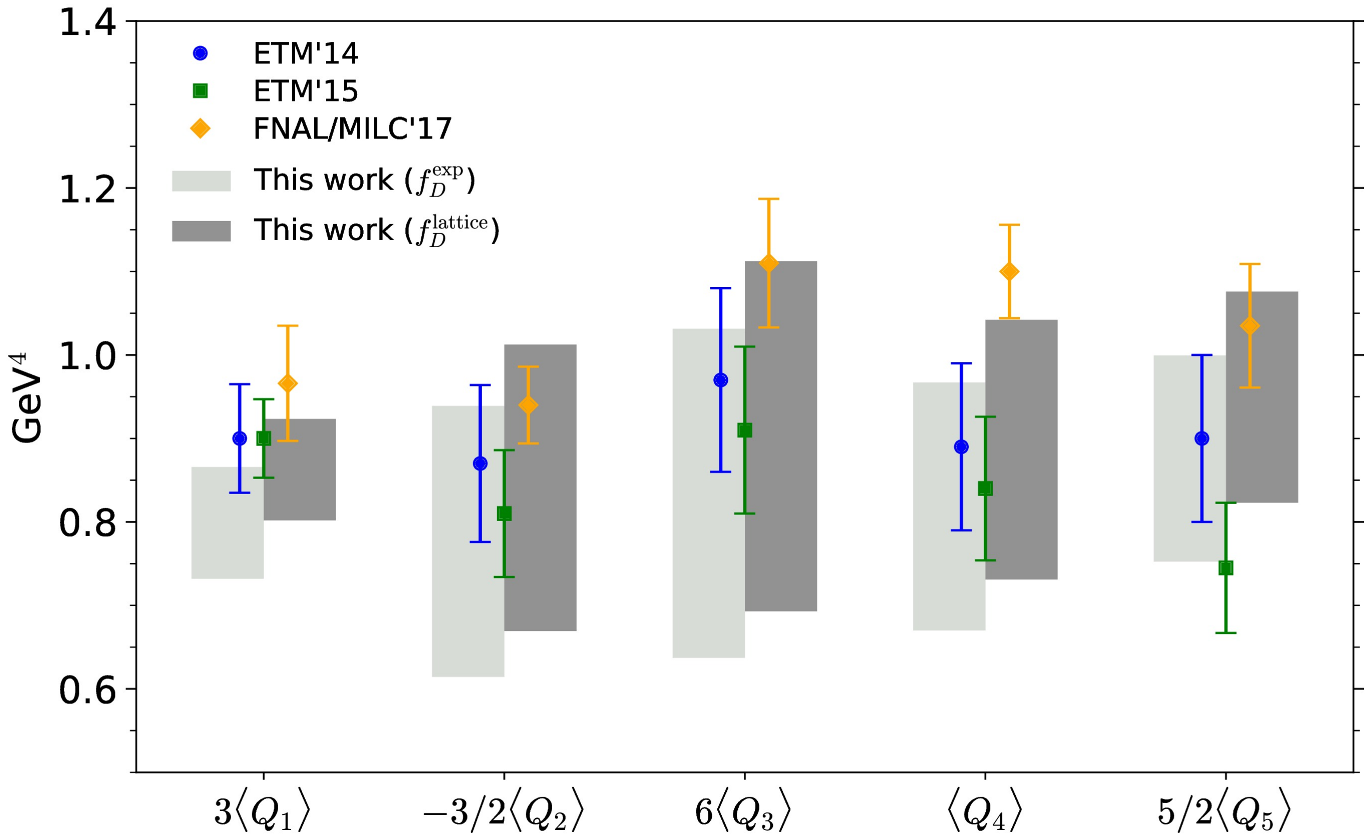


- First state-of the art result for lifetimes
- Good agreement with experimental data on B lifetimes

[Kirk, Lenz, TR, '17]



Comparison with lattice (D mixing)



[Kirk, Lenz, TR, '17]

- Good agreement with lattice when lattice decay constants are used
- Uncertainties larger than in lattice simulations

Results for D lifetime matrix elements

HQET Bag parameters determined at low scale:

- vacuum saturation approximation (VSA) works very well
- small uncertainties from the sum rule

$$\begin{aligned}
\tilde{B}_1(1.5\ \mathrm{GeV}) &= 1.000\,^{+0.020}_{-0.020} = 1.000\,^{+0.000}_{-0.000}(\overline{\Lambda})\,^{+0.020}_{-0.020}(\mathrm{intr.})\,^{+0.002}_{-0.002}(\mathrm{cond.})\,^{+0.000}_{-0.001}(\mu_\rho),\\
\tilde{B}_2(1.5\ \mathrm{GeV}) &= 1.000\,^{+0.020}_{-0.020} = 1.000\,^{+0.000}_{-0.000}(\overline{\Lambda})\,^{+0.020}_{-0.020}(\mathrm{intr.})\,^{+0.002}_{-0.002}(\mathrm{cond.})\,^{+0.000}_{-0.001}(\mu_\rho),\\
\tilde{\epsilon}_1(1.5\ \mathrm{GeV}) &= -0.016\,^{+0.021}_{-0.022} = -0.016\,^{+0.007}_{-0.008}(\overline{\Lambda})\,^{+0.020}_{-0.020}(\mathrm{intr.})\,^{+0.003}_{-0.003}(\mathrm{cond.})\,^{+0.003}_{-0.003}(\mu_\rho),\\
\tilde{\epsilon}_2(1.5\ \mathrm{GeV}) &= 0.004\,^{+0.022}_{-0.022} = 0.004\,^{+0.007}_{-0.008}(\overline{\Lambda})\,^{+0.020}_{-0.020}(\mathrm{intr.})\,^{+0.004}_{-0.004}(\mathrm{cond.})\,^{+0.002}_{-0.002}(\mu_\rho).
\end{aligned}$$

RG evolution and matching to QCD yields:

$$\begin{aligned}
\overline{B}_1(3\,\mathrm{GeV}) &= 0.902\,^{+0.077}_{-0.051} = 0.902\,^{+0.018}_{-0.018}\,(\mathrm{sum\ rule})\,^{+0.075}_{-0.048}\,(\mathrm{matching}),\\
\overline{B}_2(3\,\mathrm{GeV}) &= 0.739\,^{+0.124}_{-0.073} = 0.739\,^{+0.015}_{-0.015}\,(\mathrm{sum\ rule})\,^{+0.123}_{-0.072}\,(\mathrm{matching}),\\
\overline{\epsilon}_1(3\,\mathrm{GeV}) &= -0.132\,^{+0.041}_{-0.046} = -0.132\,^{+0.025}_{-0.026}\,(\mathrm{sum\ rule})\,^{+0.033}_{-0.038}\,(\mathrm{matching}),\\
\overline{\epsilon}_2(3\,\mathrm{GeV}) &= -0.005\,^{+0.032}_{-0.032} = -0.005\,^{+0.011}_{-0.012}\,(\mathrm{sum\ rule})\,^{+0.030}_{-0.030}\,(\mathrm{matching}).
\end{aligned}$$

[Kirk, Lenz, TR, ‘17]

Lifetime ratio

$$\left.\frac{\tau(D^+)}{\tau(D^0)}\right|_{\rm exp} = 2.536 \pm 0.019,$$

$$\left.\frac{\tau(D^+)}{\tau(D^0)}\right|_{\overline{\rm MS}} = 2.61^{+0.72}_{-0.77} = 2.61\,^{+0.70}_{-0.66}\,({\rm had.})\,^{+0.12}_{-0.38}\,({\rm scale}) \pm 0.09\,({\rm param.}),$$

$$\left.\frac{\tau(D^+)}{\tau(D^0)}\right|_{\rm PS} = 2.70^{+0.74}_{-0.82} = 2.70\,^{+0.72}_{-0.68}\,({\rm had.})\,^{+0.11}_{-0.45}\,({\rm scale}) \pm 0.10\,({\rm param.}),$$

$$\left.\frac{\tau(D^+)}{\tau(D^0)}\right|_{\rm 1S} = 2.56^{+0.81}_{-0.99} = 2.56\,^{+0.78}_{-0.74}\,({\rm had.})\,^{+0.22}_{-0.65}\,({\rm scale}) \pm 0.10\,({\rm param.}),$$

$$\left.\frac{\tau(D^+)}{\tau(D^0)}\right|_{\rm kin} = 2.53^{+0.72}_{-0.76} = 2.53\,^{+0.70}_{-0.66}\,({\rm had.})\,^{+0.13}_{-0.37}\,({\rm scale}) \pm 0.10\,({\rm param.}),$$

[Kirk, Lenz, TR, ‘17]

- Good agreement between various mass schemes, leading free charm decay cancels in ratio

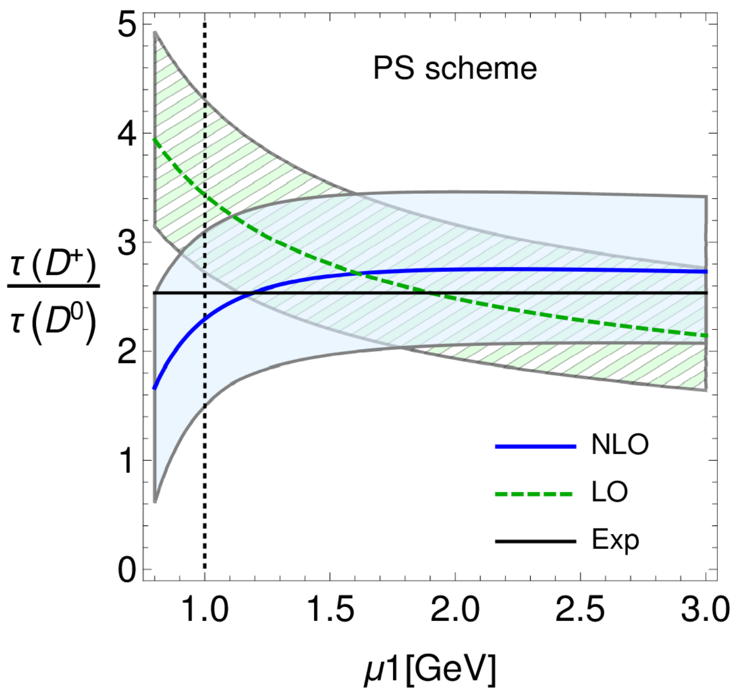
- Good agreement with experimental value

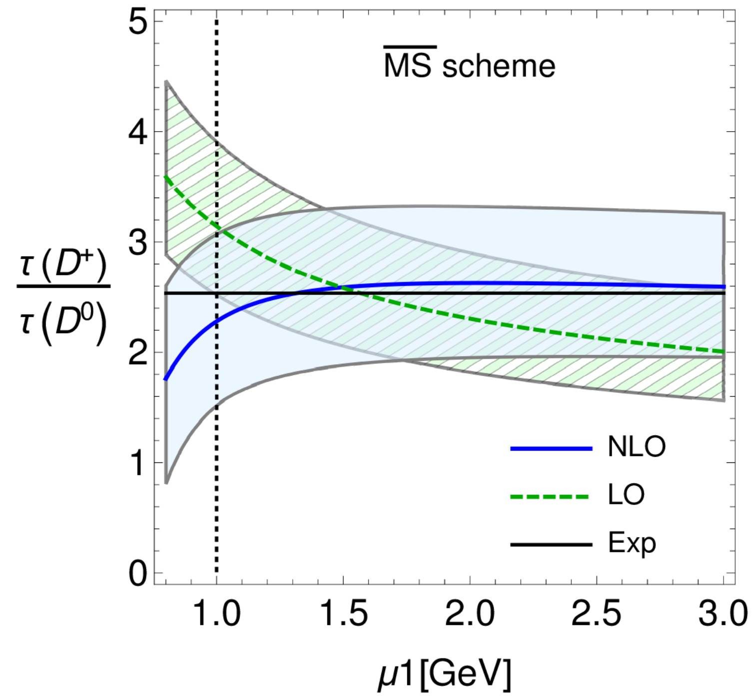
- Good convergence: $1 + 16\pi^2 \times 0.23^3 \times (1 + 0.27 - 0.34)$

NLO QCD

Dimension seven

Lifetime ratio





PS mass scheme

$\overline{\mathrm{MS}}$ mass scheme

Higher precision needs matrix elements from lattice!!!

Further possible improvements: Dimension seven matrix elements and NLO matching coefficients, NNLO QCD-HQET matching.

SINCE YEARS OF BEGGING DID NOT HELP – IT'S TIME TO PROVOKE

Lifetimes are too heavy for lattice physicists!

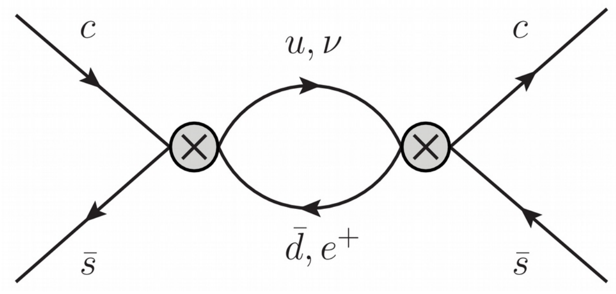
The strongest lattice researcher alive

Arbitrary sum rule researcher

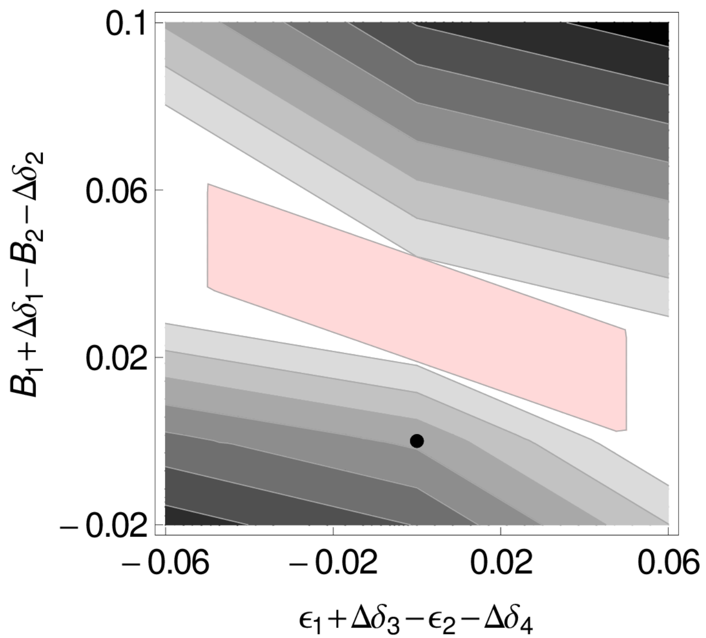
Matrix elements for lifetimes of HEAVY mesons

Ds lifetime and semileptonic rates

D_s^+: SU(3) breaking effects in the matrix elements currently not known from sum rules (w.i.p. [King, Lenz, TR]).



Cabibbo allowed spectator effects in semileptonic rates, used in [Lenz, TR, '13] to constrain combinations of matrix elements



$$\left[\frac{\Gamma(D_s^+\to Xe^+\nu)}{\Gamma(D^0\to Xe^+\nu)}\right]_{\mathrm{exp}} = 0.821 \pm 0.054$$

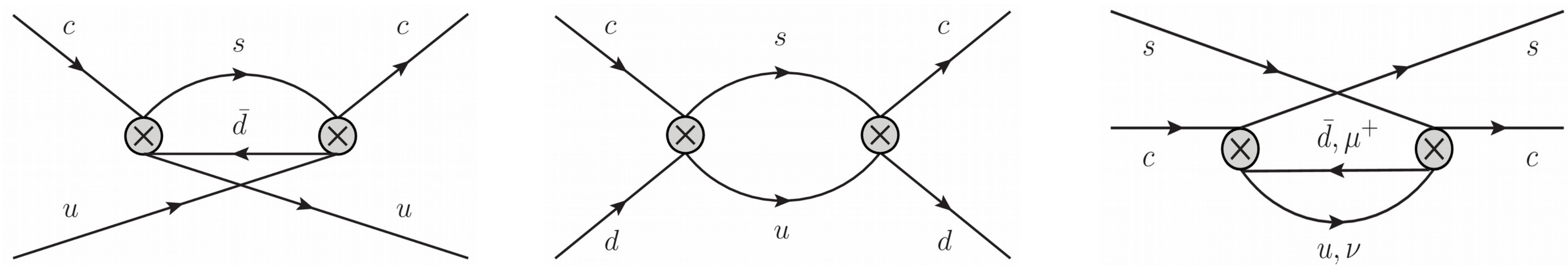
$$\left[\frac{\Gamma(D_s^+\to Xe^+\nu)}{\Gamma(D^0\to Xe^+\nu)}\right]_{\mathrm{th}} = 1 + A(B_1^s - B_2^s) + B(\epsilon_1^s - \epsilon_2^s) + \dots$$

$$\Rightarrow \quad \left(\frac{\overline{\tau}(D_s^+)}{\tau(D^0)}\right)_{\mathrm{exp}} = 1.292 \pm 0.019,$$

$$\left(\frac{\overline{\tau}(D_s^+)}{\tau(D^0)}\right)_{\overline{\mathrm{MS}}} = 1.19 \pm 0.12^{(\mathrm{hadronic})} \pm 0.04^{(\mathrm{scale})} \pm 0.01^{(\mathrm{exp})}.$$

Single charm baryons

We focus on $\Lambda_c^+, \Xi_c^+, \Xi_c^0$ where the two light quarks are in a spin-0 state and the matrix elements $\langle H_c|\bar{c} g_s \sigma_{\mu\nu} G^{\mu\nu} c|H_c\rangle$ vanish. The spectator effects are



Only two independent dimension-six matrix elements due to heavy-quark spin symmetry (holds up to 1/mc corrections). Using results from an "exploratory study" on the lattice [Di Pierro, Sachrajda, Michael, '97]

Observable	HQE estimation	Experiment
$\tau(\Xi_c^+)/\tau(\Lambda_c^+)$	~ 2.1	2.21 ± 0.15
$\tau(\Xi_c^+)/\tau(\Xi_c^0)$	~ 3.2	3.95 ± 0.48
$\Gamma(\Xi_c^+ \to e^+ \text{ anything})/\Gamma(\Lambda_c^+ \to e^+ \text{ anything})$	~ 1.8	$---$
$\Gamma(\Xi_c^0 \to e^+ \text{ anything})/\Gamma(\Lambda_c^+ \to e^+ \text{ anything})$	~ 1.8	$---$

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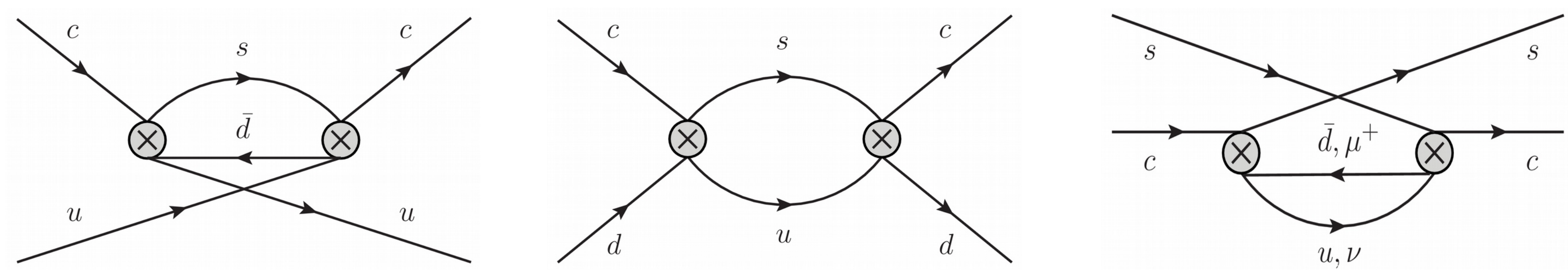

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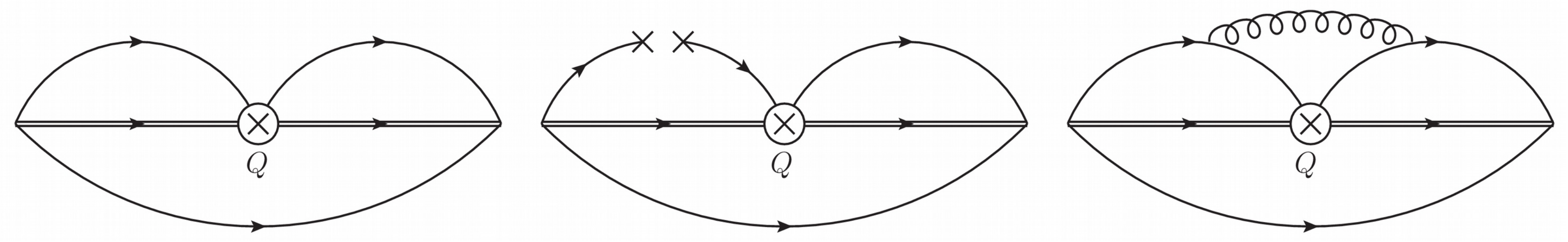
Experimental values would provide a crucial check!!

Matrix elements from sum rules?



Possible, but:

- One extra loop
- No dominant factorizable contribution
- Some arbitrariness in choice of interpolating currents

Cannot expect more than 30-40 % precision!!

Condensates: [Colangelo, de Fazio '96]

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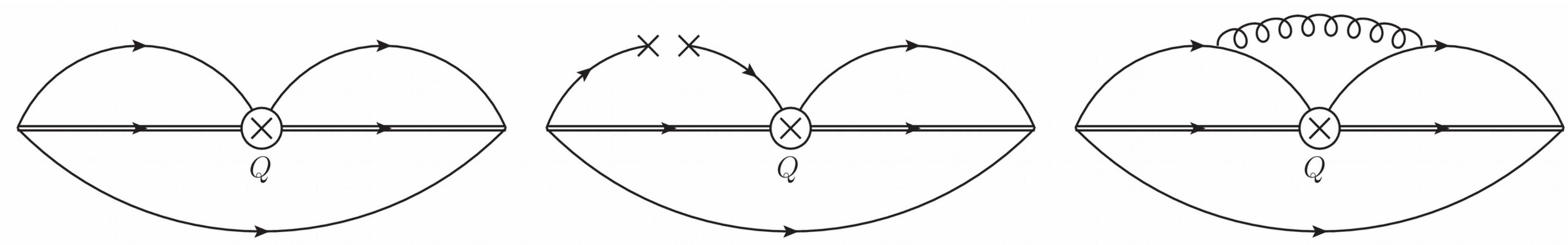

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Consistency check for double-charm baryons

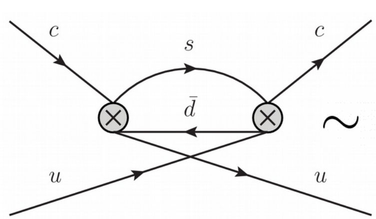
Very recent measurement of $\tau(\Xi_{cc}^{++})$ [LHCb, '18]:

$$\tau(\Xi_{cc}^{++}) = 0.256^{+0.024}_{-0.022}\,(\mathrm{stat}) \pm 0.014\,(\mathrm{syst})\,\mathrm{ps}$$

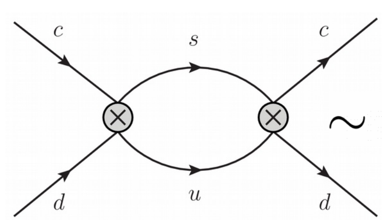
Estimate of free charm quark decay from experimental lifetimes:

$$\Gamma_0(c) \sim 2.4\,\mathrm{ps}^{-1} \quad \Rightarrow \quad \tau_0(\Xi_{cc}^{++}) \sim 0.21\,\mathrm{ps}$$

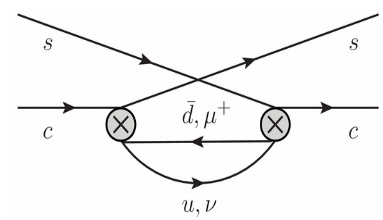
Taking the values of the single-charm baryon sector as naive estimates for the size of spectator effects we find



$\sim -1\,\mathrm{ps}^{-1}$



$\sim 3.6\,\mathrm{ps}^{-1}$



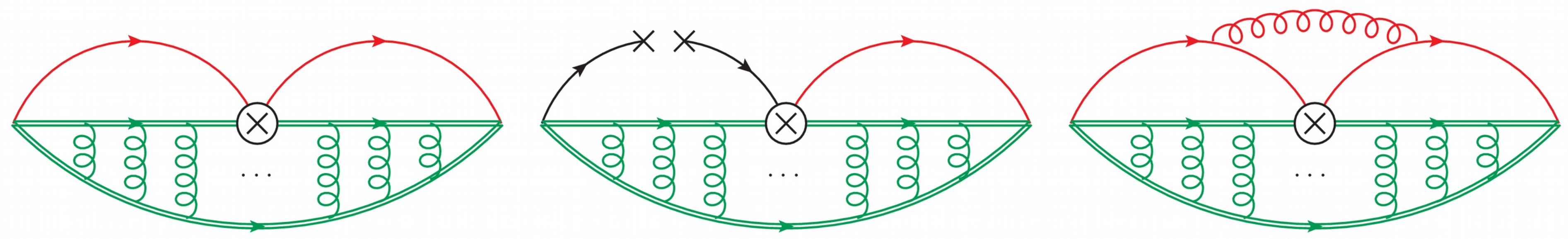
$\sim 1.3\,\mathrm{ps}^{-1}$

	$\tau(\Xi_{cc}^{++})$ [ps]	$\frac{\tau(\Xi_{cc}^{++})}{\tau(\Xi_{cc}^{+})}$	$\frac{\tau(\Xi_{cc}^{++})}{\tau(\Omega_{cc}^{+})}$	$\frac{\Gamma_{\mathrm{sl}}(\Omega_{cc}^{+})}{\Gamma_{\mathrm{sl}}(\Xi_{cc}^{++})}$
Naive	0.36	4	2.5	2.5
Guberina, Melic, Stefancic '99	1.05	5.3	3.5	2.7
Chang, Li, Li, Wang '07	0.67	2.7	3.2	-
Karliner, Rosner '14	0.185	3.5	-	-
Berezhnoy, Likhoded '16	0.46 ± 0.05	2.9 ± 0.8	1.7 ± 0.4	-

Matrix elements from sum rules?

Heavy-heavy system is more complicated than heavy light, multiple scales!

If we assume $\mathrm{m}_c \gg m_c v \gg m_c v^2 \sim \Lambda_{\mathrm{QCD}}$ holds for the ground state, i.e. we have a non-relativistic heavy di-quark of size $1/(\mathrm{m}_c v)$ inside the baryon of size $1/\Lambda_{\mathrm{QCD}}$. Coulomb gluons between charm quarks are perturbative, but must be resummed: $\alpha_s/v \sim 1$



Hierarchy must be questioned, although the $\Upsilon(1S)$ state seems to satisfy [TR, '18]
$\mathrm{m}_b \gg m_b v \gg m_b v^2 \gg \Lambda_{\mathrm{QCD}}$
which suggests that $\mathrm{m}_c v \sim m_b v^2 \gg \Lambda_{\mathrm{QCD}}$
is not impossible for the ground state double charmed mesons.

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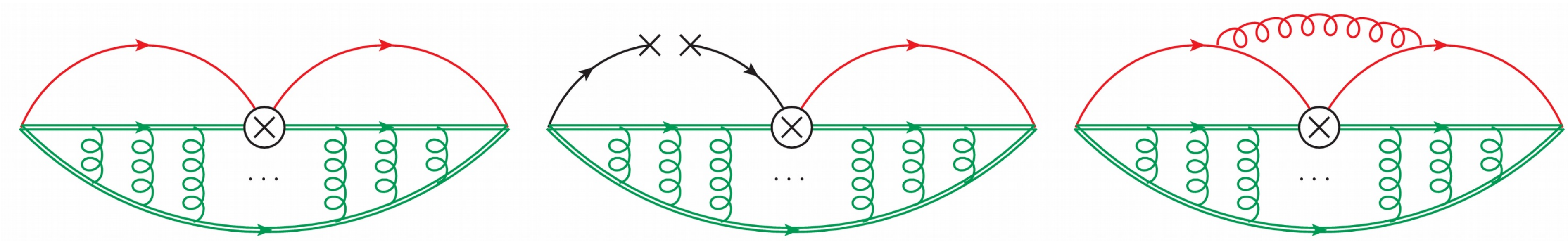

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Conclusions & outlook

- No indication that the HQE for inclusive decays fails anywhere in the charm sector (D mixing requires more work)
- Uncertainties in lifetime ratios dominated by hadronic matrix elements
- First matrix elements for D mesons provided by sum rules, lifetimes are in good agreement with experiment and show good convergence
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- Pattern of experimental single-charm baryon lifetimes is reproduced, hierarchy in semileptonic rates predicted. Getting hadronic matrix elements is challenging
- New LHCb measurement shows that Ξ_{cc}^{++} is fairly short-lived, but not in contradiction with HQE expectation. HQE predicts pattern of lifetimes and semileptonic rates. Getting hadronic matrix elements is extremely challenging

Thank you!!

Backup

$\overline{B}_1$	$\overline{B}_2$	$\bar{\epsilon}_1$	$\bar{\epsilon}_2$	ρ_3	ρ_4	σ_3	σ_4
${}^{+0.07}_{-0.05}$	± 0.00	${}^{+0.52}_{-0.47}$	± 0.017	± 0.05	± 0.00	± 0.46	± 0.00

f_B	μ_1	μ_0	m_c	m_s	α_s	CKM
± 0.08	${}^{+0.07}_{-0.40}$	${}^{+0.08}_{-0.21}$	± 0.08	± 0.00	${}^{+0.07}_{0.06}$	± 0.00

Table 9: Individual errors for the ratio $\tau(D^+)/\tau(D^0)$ in the PS mass scheme.

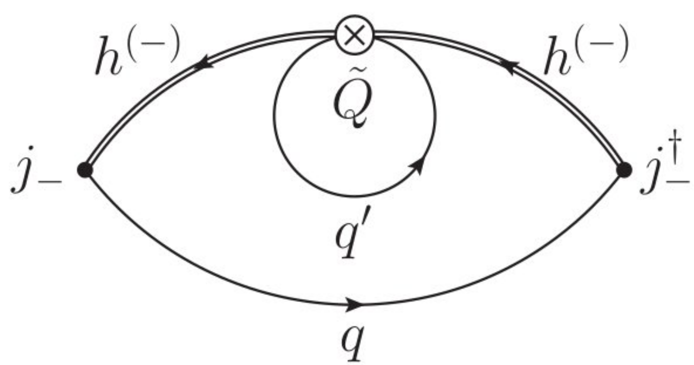


Figure 5: Leading order eye contraction.

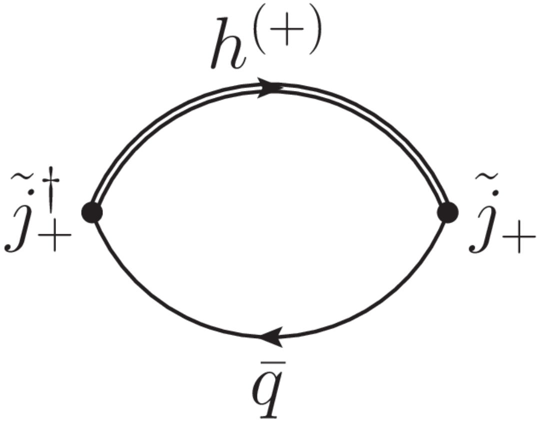
HQET sum rules: decay constant

Sum rules give results which are truly independent from the lattice. Based on:

- Analyticity of correlation functions
- Quark-hadron duality

[Shifman, Vainshtein, Zakharov '79]

First consider the sum rule for the decay constant.
Based on the two-point correlator:



$$\Pi(\omega) = i \int d^d x e^{ipx} \left\langle 0 \middle| \mathrm{T} \left[\tilde{j}_+^\dagger(0) \tilde{j}_+(x) \right] \middle| 0 \right\rangle$$

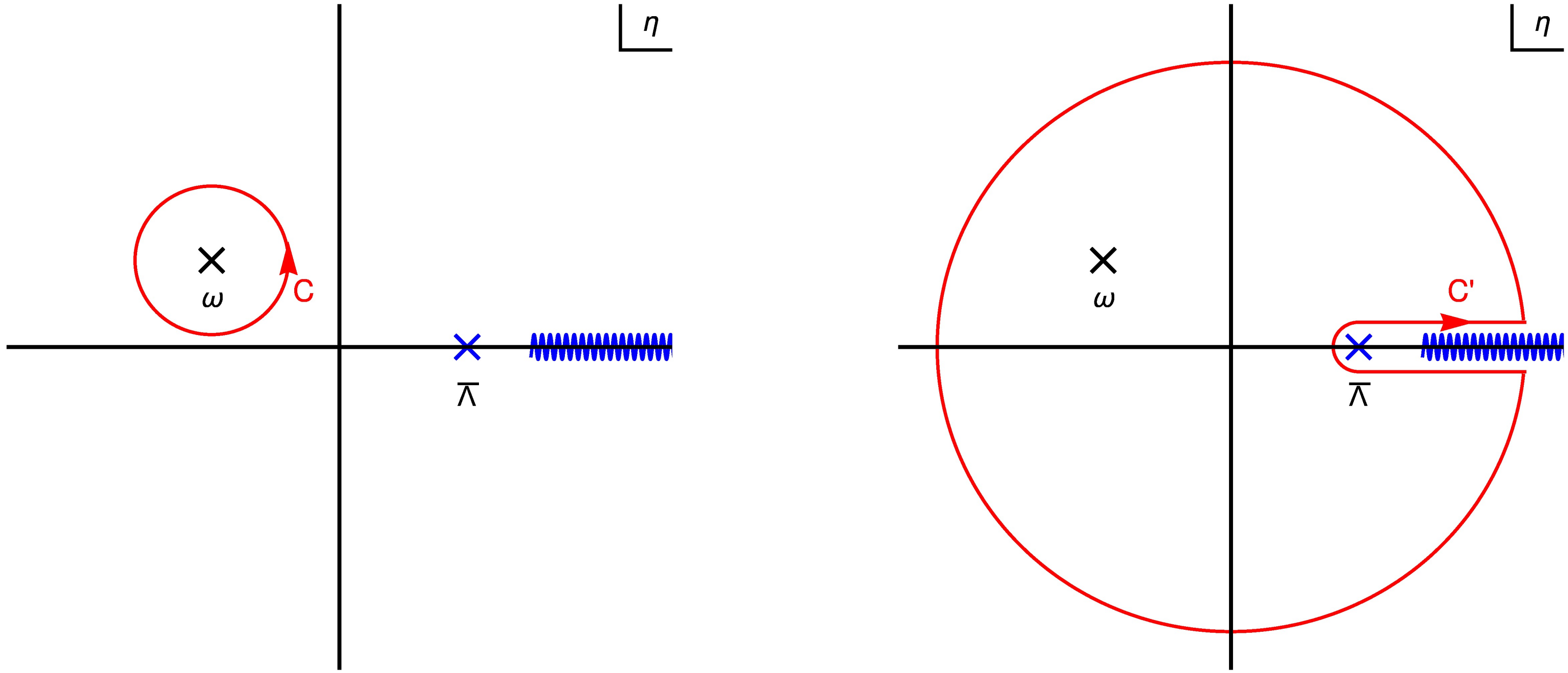
$$\tilde{j}_+ = \bar{q}\gamma^5 h^{(+)} \qquad \omega = p \cdot v$$

Use Cauchy to derive a dispersion relation:

$$\Pi(\omega) = \frac{1}{2\pi i} \oint_C d\eta \, \frac{\Pi(\eta)}{\eta - \omega}$$

HQET sum rules: decay constant

Deform the contour:



$$\Pi(\omega) = \int_0^\infty d\eta \, \frac{\rho_\Pi(\eta)}{\eta - \omega} + \oint d\eta \, \frac{\Pi(\eta)}{\eta - \omega}$$

HQET sum rules: decay constant

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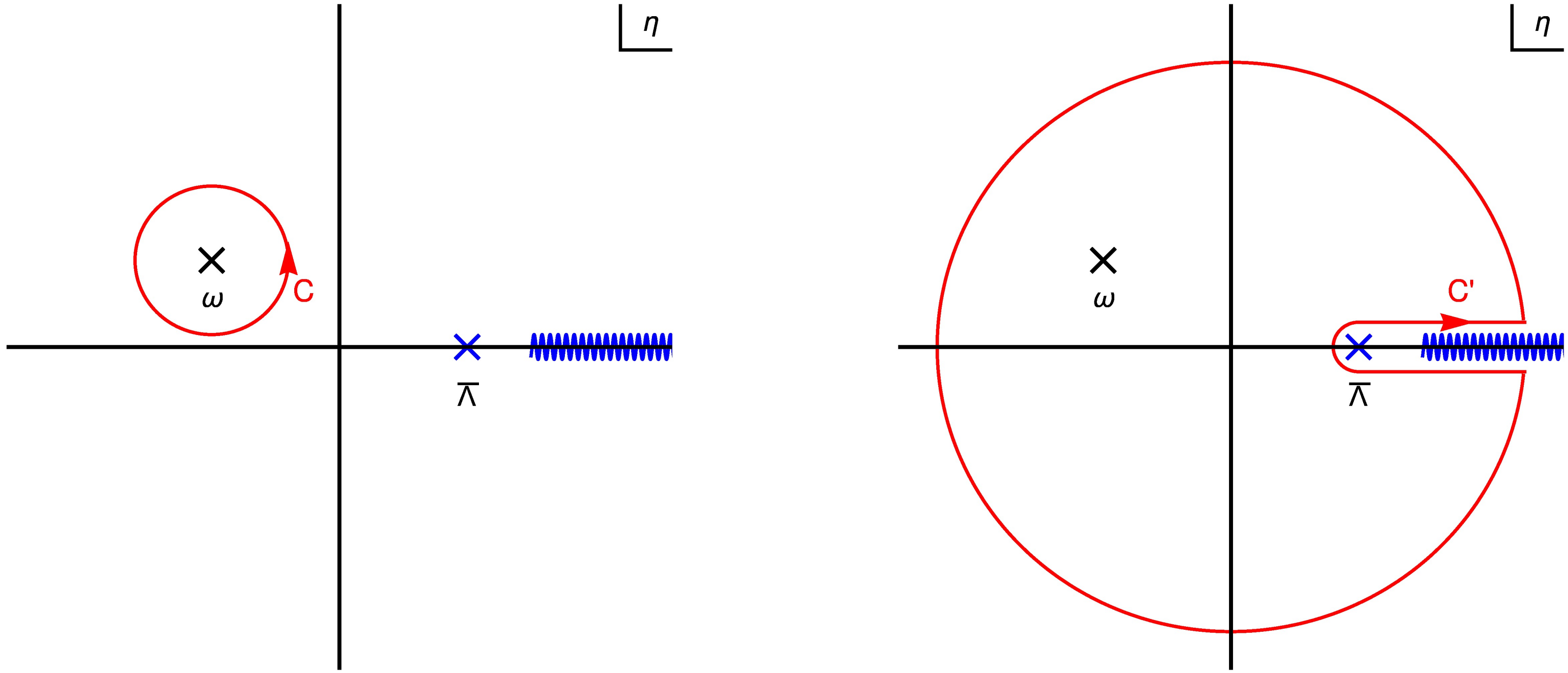

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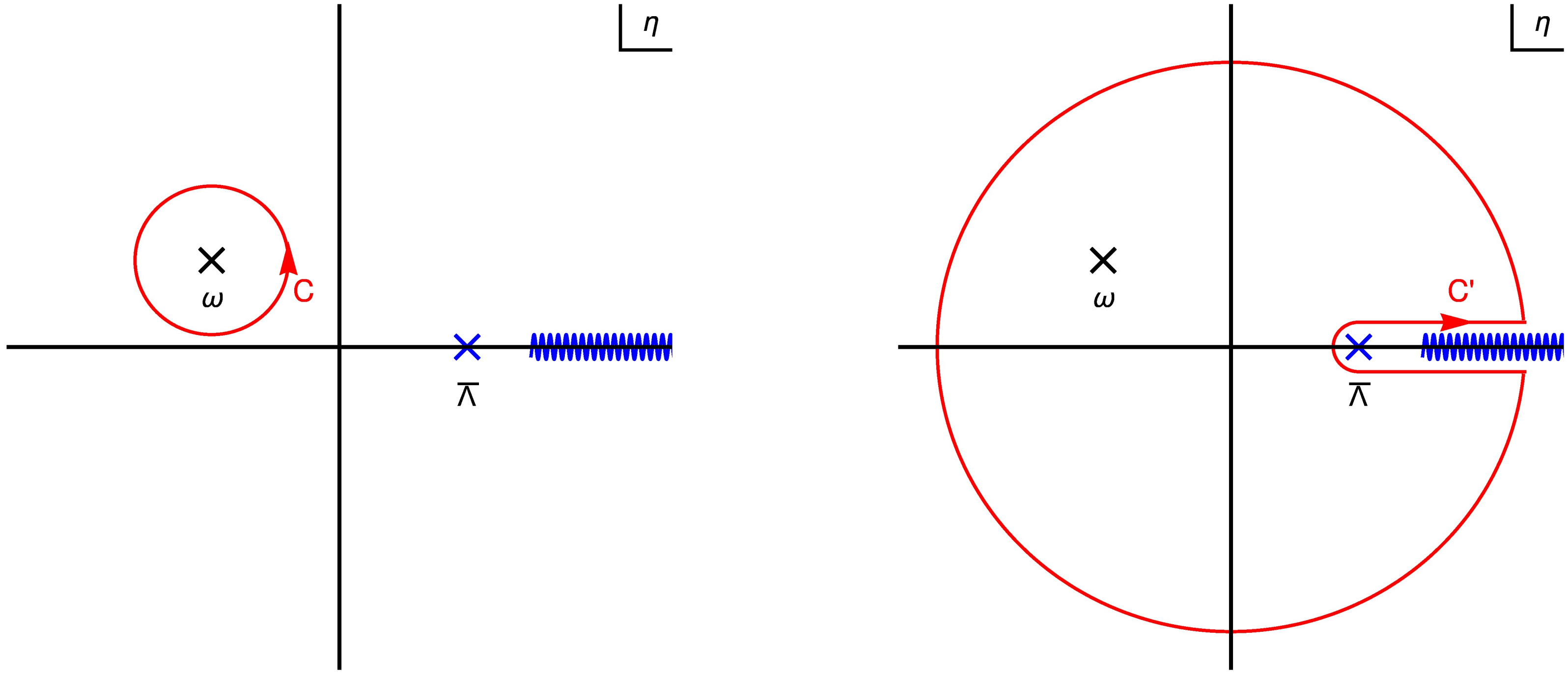

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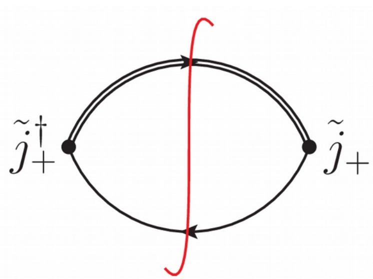
Discontinuity



$$\rho_\Pi^{\text{had}}(\omega) = F^2(\mu)\delta(\omega - \overline{\Lambda}) + \rho_\Pi^{\text{cont}}(\omega)$$

HQET decay constant

HQET sum rules: decay constant

Applying a Borel transform and a cutoff on the continuum part we obtain:

$$F^2(\mu)e^{-\frac{\overline{\Lambda}}{t}} = \int_0^{\omega_c} d\omega e^{-\frac{\omega}{t}} \rho_{\Pi}^{\mathrm{OPE}}(\omega)$$

[Broadhurst,Grozin '92; Bagan, Ball, Braun,Dosch '92; Neubert '92]

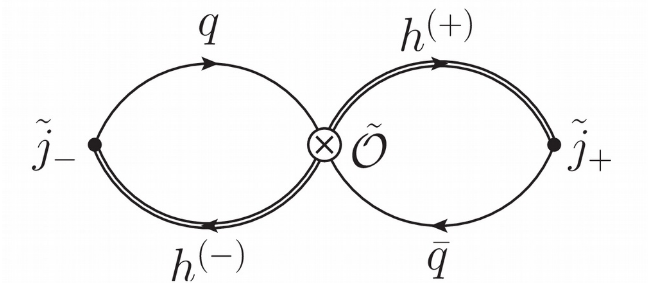
[PDG '16]

Reference	Method	N_f	f_{B^+}(MeV)	f_{B_s}(MeV)	f_{B_s}/f_{B^+}
ETM 13 [85] *,†	LQCD	2+1+1	196(9)	235(9)	1.201(25)
HPQCD 13 [86]	LQCD	2+1+1	184(4)	224(5)	1.217(8)
Average	LQCD	2+1+1	184(4)	224(5)	1.217(8)
Aoki 14 [87] *,‡	LQCD	2+1	218.8(6.5)(30.8)	263.5(4.8)(36.7)	1.193(20)(44)
RBC/UKQCD 14 [88]	LQCD	2+1	195.6(6.4)(13.3)	235.4(5.2)(11.1)	1.223(14)(70)
HPQCD 12 [89] *	LQCD	2+1	191(1)(8)	228(3)(10)	1.188(12)(13)
HPQCD 12 [89] *	LQCD	2+1	189(3)(3)*	–	–
HPQCD 11 [90]	LQCD	2+1	–	225(3)(3)	–
Fermilab/MILC 11 [69]	LQCD	2+1	196.9(5.5)(7.0)	242.0(5.1)(8.0)	1.229(13)(23)
Average	LQCD	2+1	189.9(4.2)	228.6(3.8)	1.210(15)
Our average	LQCD	Both	187.1(4.2)	227.2(3.4)	1.215(7)
Wang 15 [71] §	QCD SR		194(15)	231(16)	1.19(10)
Baker 13 [91]	QCD SR		186(14)	222 (12)	1.19(4)
Lucha 13 [92]	QCD SR		192.0(14.6)	228.0(19.8)	1.184(24)
Gelhausen 13 [72]	QCD SR		$207(^{+17}_{-9})$	$242(^{+17}_{-12})$	$1.17(^{+3}_{-4})$
Narison 12 [73]	QCD SR		206(7)	234(5)	1.14(3)
Hwang 09 [75]	LFQM		–	270.0(42.8)¶	1.32(8)

Sum rules are in good agreement with lattice, but have larger uncertainties

HQET sum rules: Bag parameters

Consider the three-point correlator:



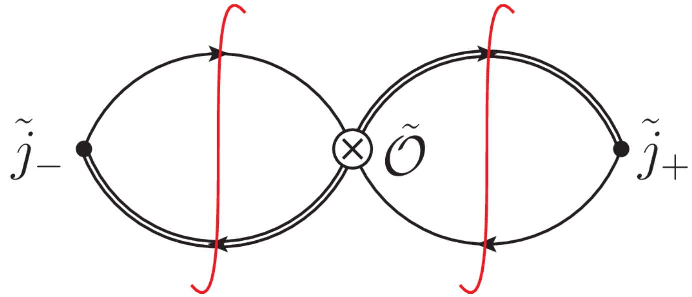
$$K_{\tilde{Q}}(\omega_1,\omega_2) = \int d^d x_1 d^d x_2 e^{ip_1\cdot x_1 - ip_2\cdot x_2} \left\langle 0 \left| \mathrm{T}\left[\tilde{j}_+(x_2)\tilde{Q}(0)\tilde{j}_-(x_1)\right]\right| 0\right\rangle$$

Going through the same steps one obtains the sum rule: [Chetyrkin, Kataev, Krasulin, Pivovarov '86]

$$F^2(\mu)\langle\tilde{Q}(\mu)\rangle e^{-\frac{\overline{\Lambda}}{t_1}-\frac{\overline{\Lambda}}{t_2}} = \int_0^{\omega_c} d\omega_1 d\omega_2 e^{-\frac{\omega_1}{t_1}-\frac{\omega_2}{t_2}} \rho_{\tilde{Q}}^{\mathrm{OPE}}(\omega_1,\omega_2)$$

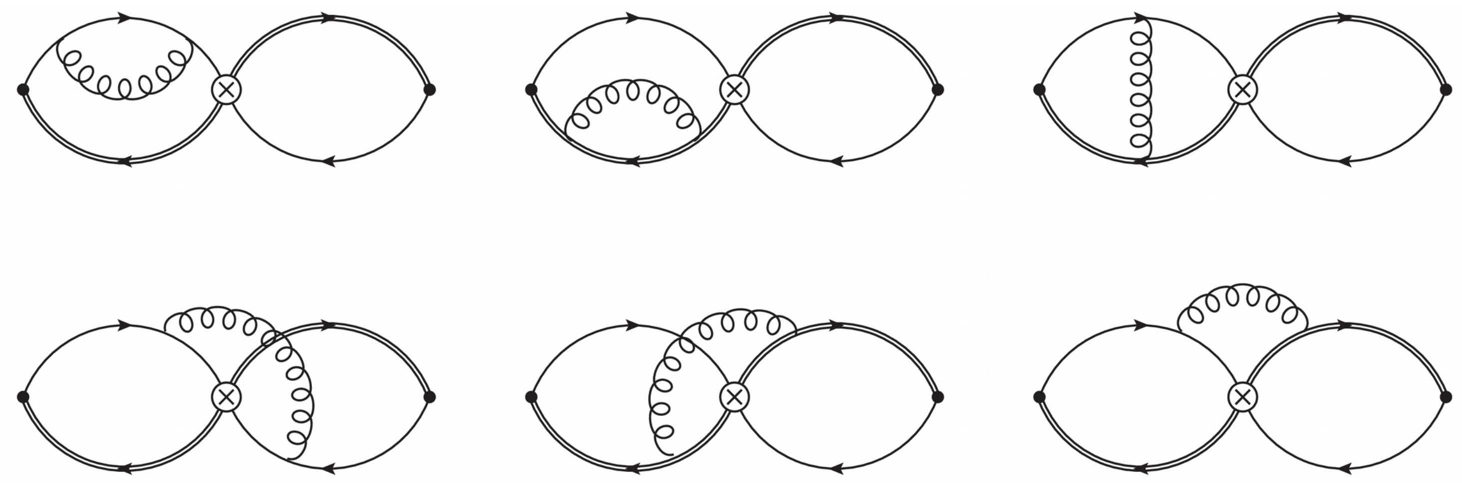
$$\rho_{\tilde{Q}}^{\mathrm{OPE}}(\omega_1,\omega_2) = \rho_{\tilde{Q}}^{\mathrm{pert}}(\omega_1,\omega_2) + \rho_{\tilde{Q}}^{\langle\bar{q}q\rangle}(\omega_1,\omega_2)\langle\bar{q}q\rangle + \rho_{\tilde{Q}}^{\langle\alpha_s G^2\rangle}(\omega_1,\omega_2)\langle\alpha_s G^2\rangle + \dots$$

In practise we compute the correlator and then take its double discontinuity



Three-point correlator

NLO accuracy in the perturbative part requires a three-loop calculation:



Master integrals:
[Grozin, Lee '08]

Operator Q1:
[Grozin, Mannel, Klein, Pivovarov '16]

All dimension six operators:
[Kirk, Lenz, TR '17]

$$\rho^{\mathrm{pert}}_{\tilde{Q}_i}(\omega_1,\omega_2) = A_{\tilde{Q}_i}\rho_\Pi(\omega_1)\rho_\Pi(\omega_2) + \frac{\omega_1^2\omega_2^2}{\pi^4}\frac{\alpha_s}{4\pi}\, r_{\tilde{Q}_i}\left(\frac{\omega_2}{\omega_1}, L_\omega\right)$$

Non-factorizable contribution

Factorizable contribution, reproduces the vacuum saturation approximation B=1 (VSA)

$$\begin{aligned}
r_{\tilde{Q}_1}(x,L_\omega) &= 8 - \frac{a_2}{2} - \frac{8\pi^2}{3},\\
r_{\tilde{Q}_2}(x,L_\omega) &= 25 + \frac{a_1}{2} - \frac{4\pi^2}{3} + 6L_\omega + \phi(x),\\
r_{\tilde{Q}_4}(x,L_\omega) &= 16 - \frac{a_3}{4} - \frac{4\pi^2}{3} + 3L_\omega + \frac{\phi(x)}{2},\\
r_{\tilde{Q}_5}(x,L_\omega) &= 29 - \frac{a_3}{2} - \frac{8\pi^2}{3} + 6L_\omega + \phi(x).
\end{aligned}$$

Sum rule for Bag parameters

Formulate sum rule for deviation $\Delta B_{\tilde{Q}}(\mu) = B_{\tilde{Q}}(\mu) - 1$ from the HQET Bag parameters $\langle \tilde{Q}(\mu) \rangle = A_{\tilde{Q}}\, F^2(\mu)\, B_{\tilde{Q}}(\mu)$.

$$\begin{aligned}
\Delta B_{\tilde{Q}_i} &= \frac{1}{A_{\tilde{Q}_i} F(\mu)^4} \int\limits_0^{\omega_c} d\omega_1 d\omega_2 e^{\frac{\overline{\Lambda}-\omega_1}{t_1}+\frac{\overline{\Lambda}-\omega_2}{t_2}} \Delta\rho_{\tilde{Q}_i}(\omega_1,\omega_2) \\
&= \frac{1}{A_{\tilde{Q}_i}} \frac{\int\limits_0^{\omega_c} d\omega_1 d\omega_2 e^{-\frac{\omega_1}{t_1}-\frac{\omega_2}{t_2}} \Delta\rho_{\tilde{Q}_i}(\omega_1,\omega_2)}{\left(\int\limits_0^{\omega_c} d\omega_1 e^{-\frac{\omega_1}{t_1}} \rho_\Pi(\omega_1)\right)\left(\int\limits_0^{\omega_c} d\omega_2 e^{-\frac{\omega_2}{t_2}} \rho_\Pi(\omega_2)\right)}.
\end{aligned}$$

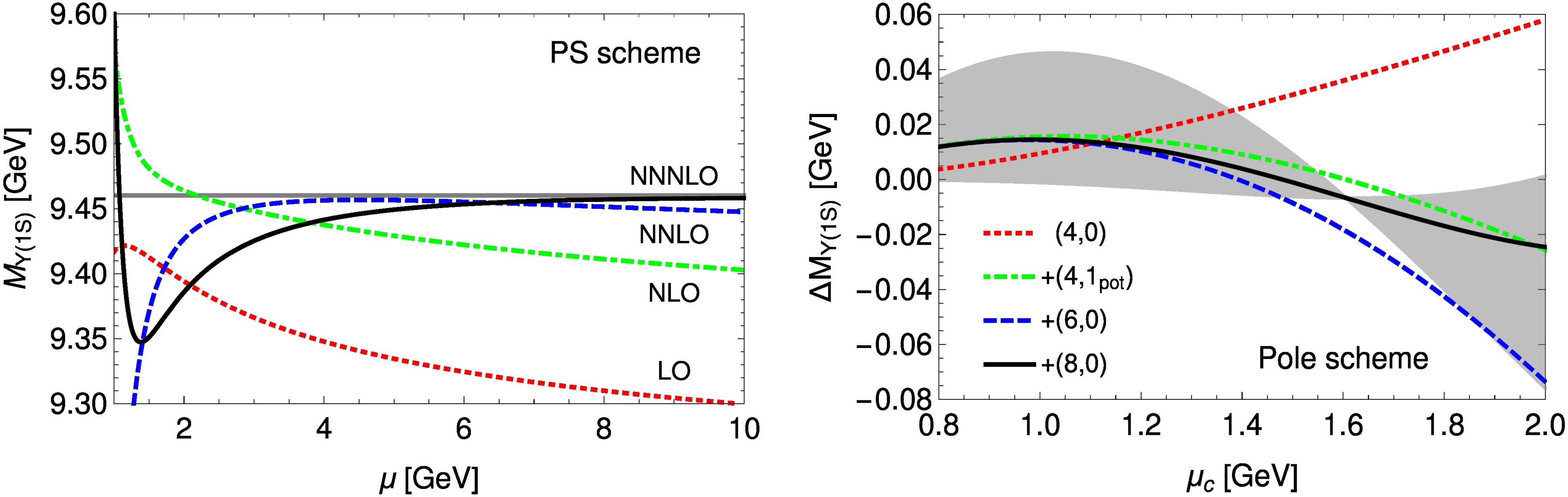
Dispersion relation is not violated by arbitrary analytical weight function (Note of caution: Duality breaks down for pathological choices)

$$F^4(\mu) e^{-\frac{\overline{\Lambda}}{t_1}-\frac{\overline{\Lambda}}{t_2}} w(\overline{\Lambda},\overline{\Lambda}) = \int\limits_0^{\omega_c} d\omega_1 d\omega_2 e^{-\frac{\omega_1}{t_1}-\frac{\omega_2}{t_2}} w(\omega_1,\omega_2)\rho_\Pi(\omega_1)\rho_\Pi(\omega_2) + \dots .$$

With an appropriate choice we obtain an analytic result for the pert contribution:

$$\Delta B^{\mathrm{pert}}_{\tilde{Q}_i}(\mu_\rho) = \frac{4}{N_c^2 A_{\tilde{Q}_i}} \frac{\alpha_s(\mu_\rho)}{4\pi}\, r_{\tilde{Q}_i}\left(1, \log\frac{\mu_\rho^2}{4\overline{\Lambda}^2}\right).$$

Upsilon(1S) mass



[TR '18]

$$
\begin{aligned}
M_{\Upsilon(1S)}^{\exp} &= 9\,460.30 \pm 0.26\,\mathrm{MeV}, \\
M_{\Upsilon(1S)} &= 9\,437\,^{+61}_{-114}\ \mathrm{MeV} \\
&= 9\,437\,^{+28}_{-74}\,(\mu)\,^{+25}_{-75}\,(m_b)\,^{+0}_{-1}\,(\alpha_s)\ \pm 9\,(m_c) \\
&\quad \pm 36\,(\mu_c)\,^{+29}_{-14}\,(O_0)\,^{+4}_{-18}\,(O_1)\,^{+10}_{-1}\,(O_2)\ \mathrm{MeV},
\end{aligned}
$$

$$1.5\,\mathrm{GeV} \leq \mu \leq 6\,\mathrm{GeV},$$

$$0.8\,\mathrm{GeV} \leq \mu_c \leq 2\,\mathrm{GeV}.$$