

# Double-charm spectroscopy

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Based on recent work with J. Vijande, A. Valcarce,  
E. Hiyama, M. Oka & A. Hosaka,  
older work with J.P. Ader, P. Taxil, S. Zouzou, J.L. Ballot, S. Fleck,  
C. Gignoux, B. Silvestre-Brac, Fl. Stancu, M. Genovese,  
Cafer Ay, & Hyam Rubinstein,



# Introduction

- After years of (interesting)  $Q\bar{Q}$  physics
- More attention now on the  $QQ\dots$  sector
- Some milestones
  - 1970 GIM
  - 1974 October revolution
  - 1974-75 Gaillard, Lee and Rosner
  - 1977 Upsilon discovery and French's baryonium
  - **1981**  $QQ\bar{q}\bar{q}$  becomes stable in the large  $M/m$  limit (ART)
  - **1988** First serious quark model of  $QQq$
  - 2002 Double-charmonium production in  $e^+e^-$
  - 2002-2005  $\Xi_{cc}$  seen by SELEX in two decay modes
  - 2002 COMPASS Workshop at CERN, Cooper, Moinester, R. insist on double charm
  - 2003  $X(3872)$
  - 2017  $\Xi_{cc}^{++}$  at LHCb
  - 2017  $QQ\bar{q}\bar{q}$  "reinvented" at Chicago and Tel-Aviv



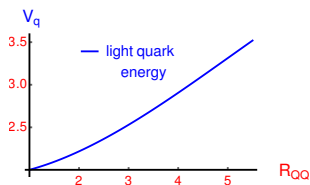
# Doubly-heavy baryons

- Obviously  $r(QQ) \ll r(Qq)$  in  $(QQq)$  for large  $M/m$
- The two heavy quarks are clustered in the ground state
- But the **naive diquark** model is misleading
- The diquark internal energy is **modified** by the third quark,
- The first excitations are within  $QQ$
- For instance, with a **linear potential**, masses  $M/m = 5$ ,
- $E_{\text{var}} = 4.940$     $E_{\text{BO}} = 4.938$     $E_{Dq} = 4.749$  (arbitrary units)



# Doubly-heavy baryons

- **Born-Oppenheimer** is more appropriate
- As for  $H_2^+$  in atomic physics
- Solve for  $q$  at fixed  $cc$
- Effective  $cc$  potential, the  $QQ$  analogue of  $Q\bar{Q}$



Born-Oppenheimer potential for  $(QQq)$ ,  $M/m = 5$ ,  $V \propto \sum_{i<j} r_{ij}$   
 Fleck, R., PTP 82 (1989) 760



# Summary for QQq

- LHCb mass favored as compared to SELEX
- Hyperfine splitting about 120 MeV or more for  $ccq$
- $bcq$  has more states
- An effective QQ system
- First excitations within  $cc$
- $cc$  effective potential = analog of the  $c\bar{c}$  effective potential of charmonium (gluon and light-quark degrees of freedom integrated out)



# History

- Many candidates
- Many predictions
- Baryonium
- Z baryon (light pentaquark)
- Dibaryon  $H$ , ...
- Heavy pentaquark  $\bar{c}uuds$   $\bar{c}ddus$ ,  $\bar{c}ssud$  (Gignoux et al., Lipkin)

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## EVIDENCE FOR A NARROW WIDTH BOSON OF MASS 2.95 GeV

Bari–Bonn–CERN–Daresbury–Glasgow–Liverpool–Milano–Purdue–Vienna Collaboration

# Early speculations

- Based on duality (Rosner)
- Based on chromomagnetism
  - $\sum_{i < j} A(r_{ij}) \tilde{\lambda}_i \cdot \tilde{\lambda}_j \sigma_i \sigma_j / m_i m_j$
  - Scalar mesons,  $H$  dibaryon (Jaffe),  $\bar{Q}qqqq$  (Gignoux et al., Lipkin)
  - Also present in  $QQ\bar{q}\bar{q}$  vs.  $Q\bar{q} + Q\bar{q}$
- Yukawa interaction
  - Deuteron
  - $D\bar{D}^*$ ,  $B\bar{B}^*$  (Törnqvist, ...)
  - $DD^*$ ,  $BB^*$  (Manohar, ...)





# Doubly heavy tetraquarks $QQ\bar{q}\bar{q}$

- First proposed by Ader et al. in 1981

## Do narrow heavy multiquark states exist?

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(Received 11 August 1981)

- And many others: Heller et al., Rosina et al., Brink et al., Lipkin, Barnea et al., Vijande et al., Oka et al., Bicudo et al., etc.
- Early papers somewhat forgotten in the recent literature!



# (QQ $\bar{q}\bar{q}$ )

- (QQ $\bar{q}\bar{q}$ ) becomes stable if  $M/m$  large
- As shown **37 years ago** by Ader et al.
- And many others: Heller et al., Rosina et al., Brink et al., Lipkin, Barnea et al., Vijande et al., Oka et al., Bicudo et al., etc.
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~~Chicago 2017~~



Geneva 1981



# Why ( $QQq̄q̄$ ) becomes stable?

- Even in the pure **chromoelectric limit**, stable for large  $M/m$ ,
- Very close analogy with atomic physics

## Stable multiquarks: Lessons from atomic physics

J.M. Richard (LPSC, Grenoble), 1992, 8 pp.

Published in *In "Bad Honnef 1992, Quark cluster dynamics" 84-91*

Prepared for Conference: [C92-06-29 3](#), p.84-91 [Proceedings](#)

71, NUMBER 9

PHYSICAL REVIEW LETTERS

30 AUGUST 1993

### Proof of Stability of the Hydrogen Molecule

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(Received 24 May 1993)*

We sketch two rigorous proofs of the stability of the hydrogen molecule in quantum mechanics. The first one is based on an extrapolation of variational estimates of the ground state energy of a positronium molecule to arbitrary mass ratios. The second one is an extension of Heitler-London theory to nuclei of finite mass.

$$H = \left( \frac{1}{4M} + \frac{1}{4m} \right) \sum p_i^2 + V + \left( \frac{1}{4M} - \frac{1}{4m} \right) [p_1^2 + p_2^2 - p_3^2 - p_4^2]$$

$$= H_{\text{even}} + H_{\text{odd}}$$

- With **the same threshold** for  $H$  and  $H_{\text{even}}$ .
- $C$ -symmetry breaking:  $E(H) \leq E(H_{\text{even}})$ .
- In atomic physics  $H_2$  more stable than  $Ps_2$
- Quark models with flavor indep.:  $QQq̄q̄$  becomes stable.



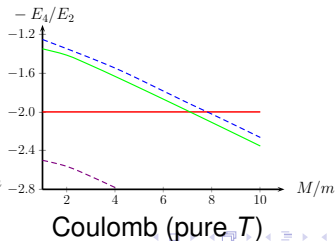
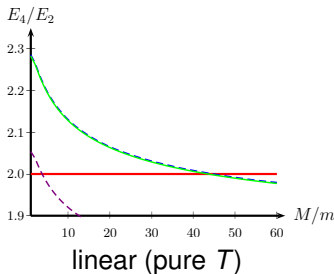
# Illustration with a toy model

- Pure chromo-electric

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m_i} - \text{c.o.m.} - \frac{3}{16} \sum_{i < j} \tilde{\lambda}_i \cdot \tilde{\lambda}_j v(r_{ij}),$$

with masses  $\{m_i\} = \{M, M, m, m\}$ .

- Two color wave functions (notation by Chan H-M et al. in the 70s)  
 $T = \bar{33}$  and  $M = 6\bar{6}$
- Assume either pure  $T$ , or pure  $M$  or include color-mixing
- Stability reached and improved as  $M/m \nearrow$

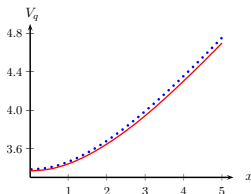


# Lessons from the toy model

- The critical  $M/m$  depends on the shape of the potential
- Perfect control of the 4-body dynamics  $E_{\text{low}} < E < E_{\text{Variational}}$
- The diquark *approximation* fails
- Born-Oppenheimer very good, again
- **Questions**: spin-corrections? color mixing? 3- and 4-body forces?

# Lessons from the toy model: Born-Oppenheimer approximation

- Works very well
- $V_{\text{eff}}(QQ\bar{q}\bar{q}) \simeq V_{\text{eff}}(QQq) + C^t$
- with  $C^t = Qqq - Q\bar{q}$
- i.e., Eichten and Quigg's identity when one solves for QQ
- $(QQ\bar{q}\bar{q})_{\bar{3}3} \simeq QQq + Qqq - Q\bar{q}$



B.O. potential for  $QQ\bar{q}\bar{q}$  (solid red line) and shifted  $QQq$  (dotted blue line).



# Lessons from the toy model: Hall-Post lower bound

- Invented in the 50's for few-nucleon systems
- Discovered independently in studies of boson systems (Fisher-Ruelle, Dyson-Lenard, Lévy-Leblond, ...)
- And for comparing mesons and baryons (Ader et al., Nussinov, ...)
- Simple form

$$H_3(m) = \sum \left[ \frac{\mathbf{p}_1^2}{4m} + \frac{\mathbf{p}_2^2}{4m} + V_{12} \right] = \sum_{i < j} H_2^{(i,j)}(2m)$$

- Implies (g.s.)

$$E_3(m) \geq 3 H_2(2m)$$

- Many refinements to remove c.m. motion and optimize the decomposition to improve the lower bound (Basdevant, Martin, R., Wu, Zouzou, Krikeb, ...)





## Application to all-heavy $QQQ̄Q̄$

- Hall-Post method shows rigorously that with the  $T$  color wave function,  $QQQ̄Q̄$  is unbound
- Equal masses  $m$ ,  $T$  color wavefunction

$$H_4(m) = \frac{1}{2} h_{12}(m) + \frac{1}{2} h_{34}(m) + \frac{1}{4} \sum_{\substack{i=1,2 \\ j=3,4}} h_{ij}(m)$$

where  $h$  is the 2-body Hamiltonian, thus

$$E_4(m) \geq 2E_2(m)$$

- Removing the center-of-mass properly leads to the better

$$E_4(m) \geq E_2(m) + E_2(m/2)$$

e.g.,  $E_4 \geq 2.26 E_2$  for a linear potential

- Numerical calculations show that  $M$  state is also unbound, and also the ground-state with  $T - M$  mixing



# (QQ $\bar{q}\bar{q}$ ) spin effects

## The $T_{cc} = DD^*$ Molecular State

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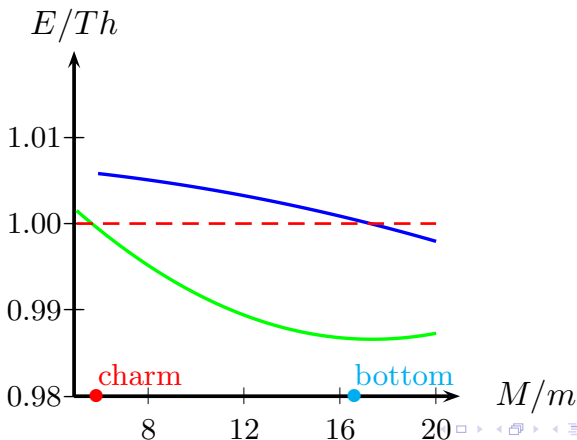
Published online December 9, 2004; © Springer-Verlag 2004

**Abstract.** We show that the molecule-like configuration of  $DD^*$  enables weak binding with two realistic potential models (Bhaduri and Grenoble AL1). Three-body forces may increase the binding and strengthen the  $cc$  diquark configuration. As a signature we propose the branching ratio between radiative and pionic decay.



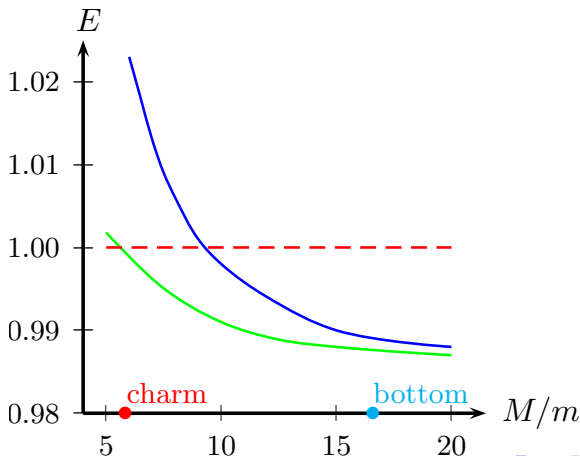
## (QQ $\bar{q}\bar{q}$ ) spin effects

- Use an explicit model tuned to ordinary hadrons, and including an explicit short-range spin-spin term
- **Chromoelectric** interaction favors (QQ $\bar{q}\bar{q}$ ) vs. (Q $\bar{q}$ ) + (Q $\bar{q}$ )
- **Chromomagnetic** interactions also helps in some cases, e.g.,  $1^+$



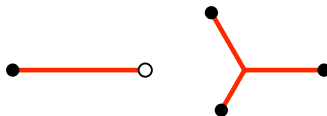
# Toy model of (QQ $\bar{q}\bar{q}$ ) color mixing

- Chromoelectric and chromomagnetic transitions from  $T$  to  $M$  type of states
- Crucial in particular near the critical  $M/m$  ratio



# Improved chromoelectric model

- Based on the string model
- Linear confinement interpreted as



- Not very visible in baryon spectroscopy as compared to

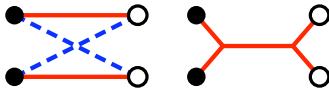
$$V_{\text{conf}} = \frac{1}{2}(r_{12} + r_{23} + r_{31})$$

of the naive additive model.

# String potential for $QQQ̄Q̄$

- Instead of  $\propto \sum \tilde{\lambda}_i \cdot \tilde{\lambda}_j r_{ij}$ , use

$$V = \min \left\{ r_{13} + r_{24}, r_{14} + r_{23}, \min_{J,K} (r_{1J} + r_{2J} + r_{JK} + r_{K3} + r_{K4}) \right\},$$



- Not so difficult (one does not need to compute the location of the junctions (Ay, R., Rubinstein (2009), Bicudo et al.))
- gives **more attraction** (R., Vijande and Valcarce, 2007), and even binding for equal masses **not submitted to the Pauli principle**, say  $(QQ'Q̄Q̄')$  with  $M(Q) = M(Q')$  but  $Q \neq Q'$ .
- This restriction was forgotten in some recent papers



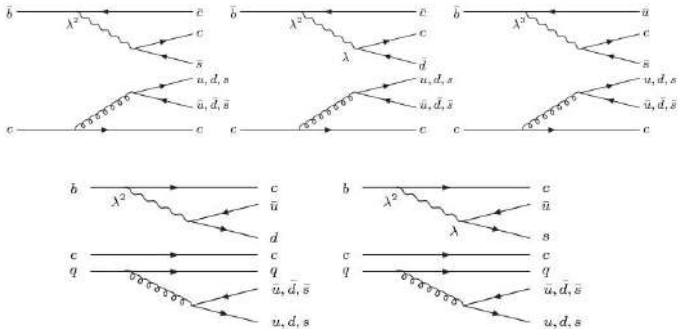
# Summary for QQ $\bar{q}\bar{q}$

- $bb\bar{q}\bar{q}$  stable. Variety of states with various spin and isospin.  
Weak decay
- $cc\bar{u}\bar{d}$  with  $I = 0$  and  $J^{PC} = 1^{++}$  at the edge
- Thus several scenarios
  - $DD\pi$  resonance
  - Sharp  $DD\gamma$
  - Stable vs. strong and EM if one can combine SR quark physics and LR Yukawa physics coherently (to be studied)
  - $bc\bar{u}\bar{d}$  and perhaps  $bc\bar{q}\bar{q}$  good candidates
- Of course, SF scenario welcome: cascade  $bb\bar{u}\bar{d} \rightarrow bc\bar{u}\bar{d} + x \rightarrow cc\bar{u}\bar{d} + \dots$  welcome



# Production of double charm tetraquarks

- $e^+e^-$  at 10 GeV (Oka et al.) around .3 fb
- In hadron colliders, two  $c\bar{c}$  pairs plus rearrangement into  $DD^*$
- $B$  decay (e.g., Roma group)





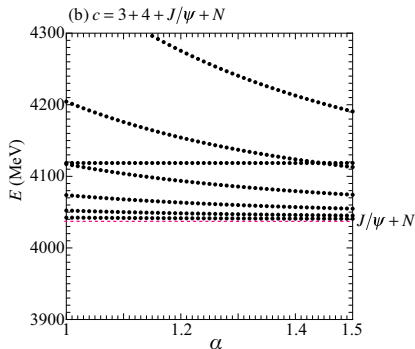
# Hidden-charm pentaquark bound states

- Valcarce, Vijande, R., Phys. Lett. B774 (2017) 710-714 [arXiv:1710.08239]
- ( $\bar{c}cqqq$ ) with  $I = 1/2$  and  $J = 5/2$  below the lowest S-wave threshold  $\bar{D}^*\Sigma_c^*$  (but above  $N\eta_c$  in D-wave)
- For  $I = 3/2$  and  $J = 1/2, 3/2$  binding below S- and D-wave thresholds
- Both chromo-electric and -magnetic parts necessary for binding
- More complicated final states, perhaps, than the LHBb  $P_c$



# Hidden-charm pentaquark resonances

- Hiyama et al. (recent paper): real scaling, borrowed from electron-atom and electron-molecule scattering to separate, among the energies above the threshold, actual resonances from fictitious states produced by the variational method. Looks promising.
- Similar to Lüscher criteria for lattice, stability plateau in QCDSR



# Outlook

- The three- and four-body problems are delicate, even for simple models
- Naive clustering assumptions usually do not work
- Already 37 years of study of  $(QQ\bar{q}\bar{q})$
- Stable if  $M/m$  large enough
- Even for a pure chromo-electric interaction, but chromomagnetism helps for  $1^{++}$
- $(cc\bar{u}\bar{d})$  with  $1^+$  at the edge in some specific models
- Main uncertainty: extrapolation from  $QQ\bar{Q}'$  and  $QQ'Q''$  to multiquarks.
- Multibody forces suggested in the string model give interesting features
- Weak decay of bottom hadrons looks promising, again

