KU LEUVEN

10 ps TOF-PET

- TOF sinograms
- no more (iterative) reconstruction?
- attenuation, scatter, randoms
- resolution:
  - image resolution and angular sampling
- SNR
- estimate attenuation from emission with MLAA
  - relax or eliminate need for attenuation measurement
  - illustrates need for accurate system calibration

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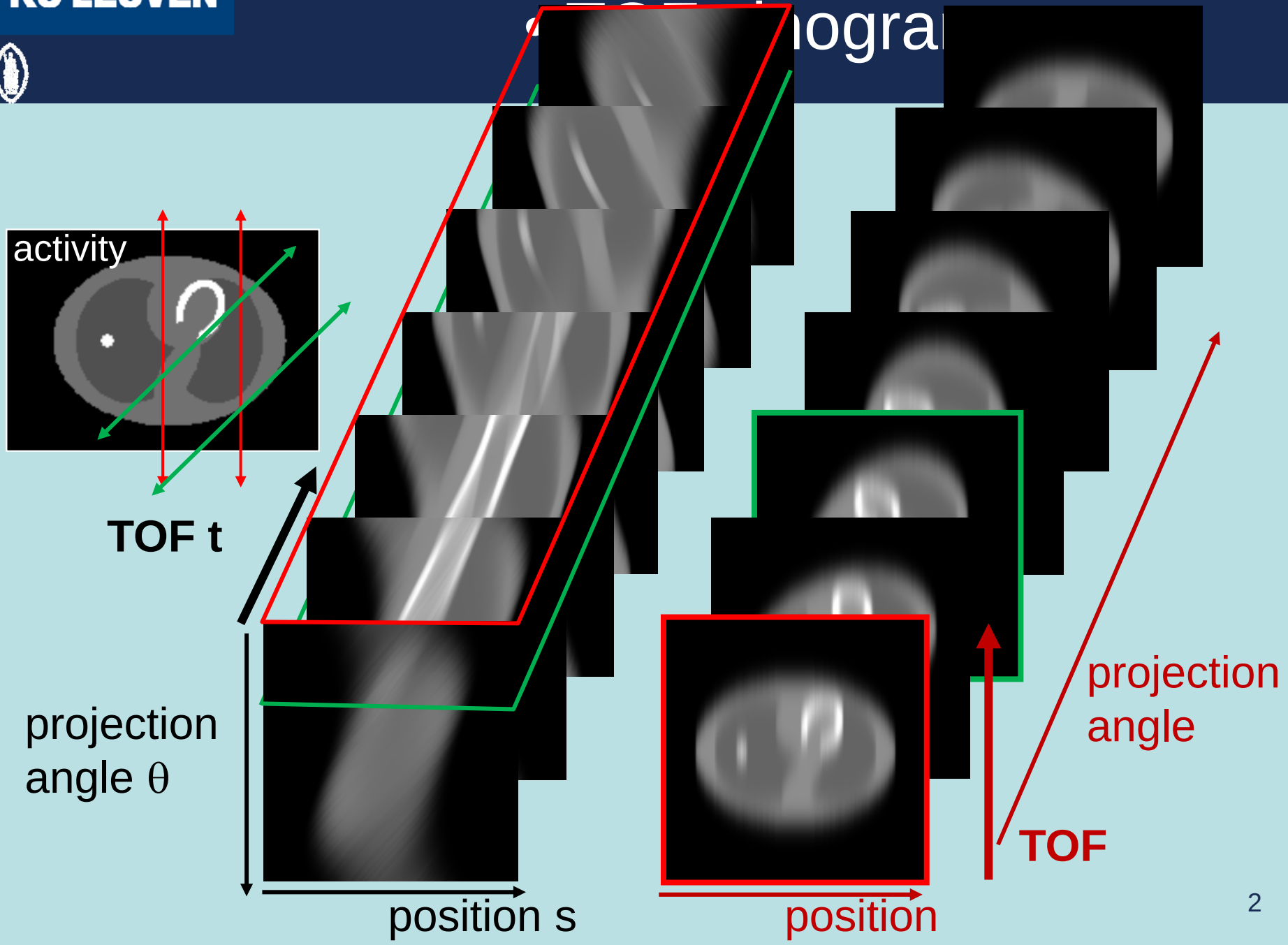
TOF sinogra



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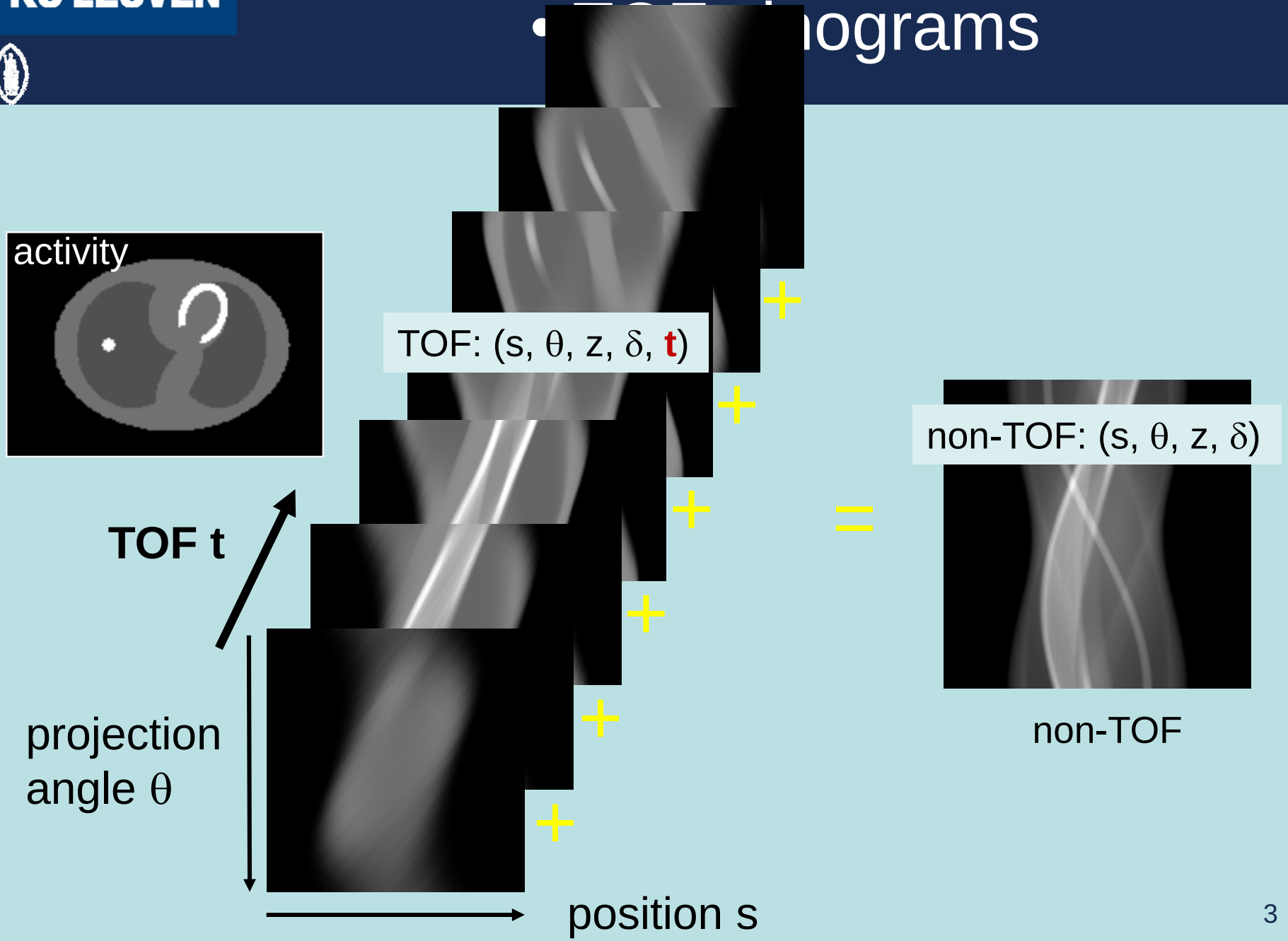
• TOF sinograms



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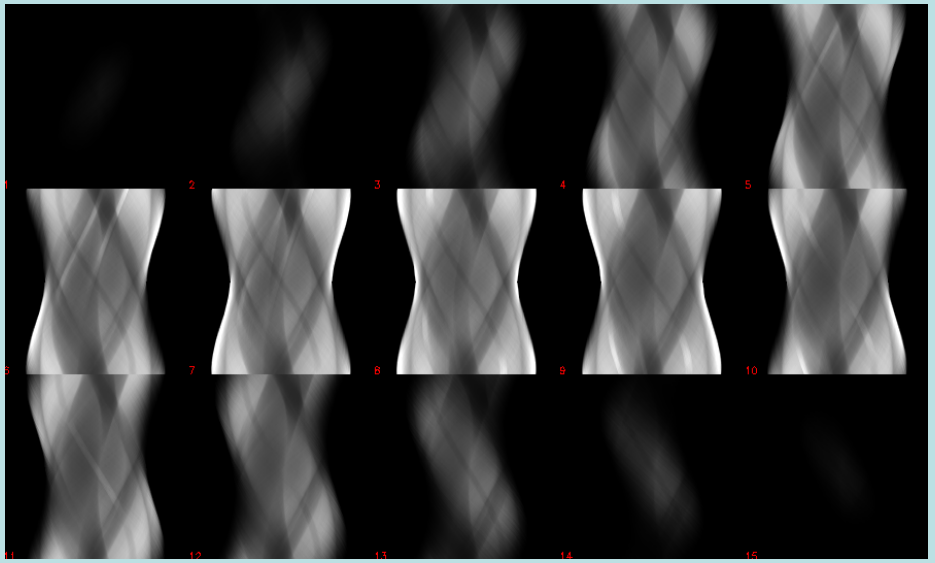
TOF sinograms

non-TOF

TOF



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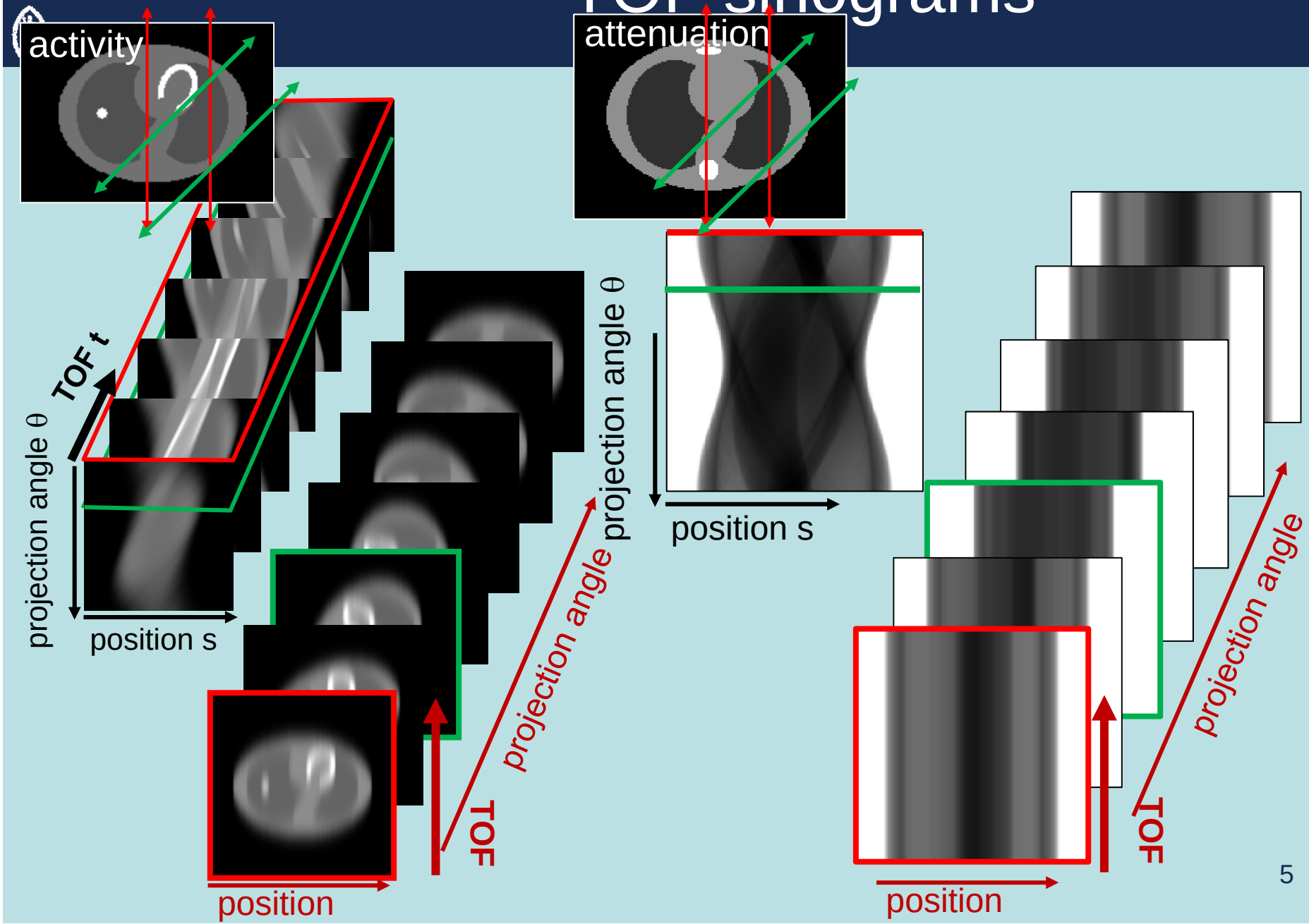
TOF sinograms



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10 ps TOF-PET

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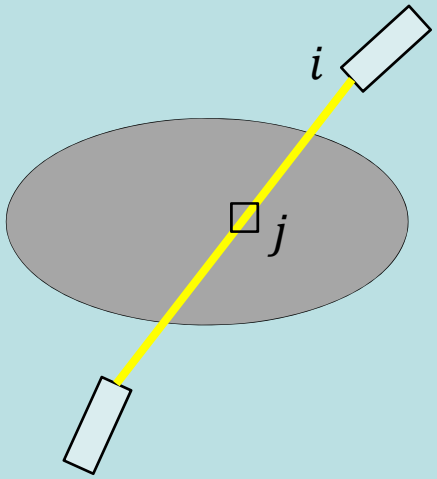
no more (iterative) reconstruction?



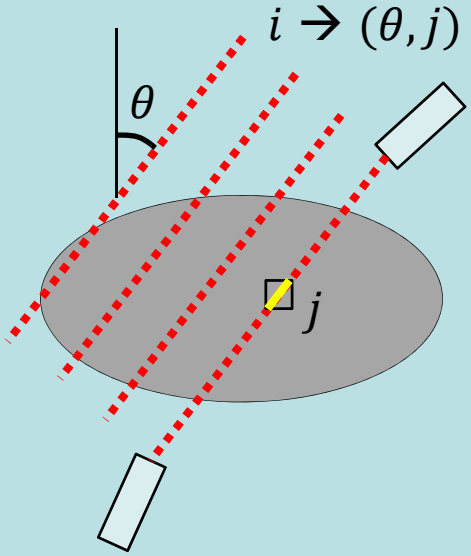
no TOF
or
10^2 ps TOF

projection: $y_i = \sum_j a_{ij}\lambda_j$

backprojection: $b_j = \sum_i a_{ij} y_i$

MLEM: $\lambda_j^{(n+1)} = \frac{\lambda_j^{(n)}}{\sum_i a_{ij}} \sum_i a_{ij} \frac{y_i}{\sum_\xi a_{i\xi}\lambda_\xi^{(n)}}$

10 ps TOF



projection: $y_{\theta j} = a_{\theta j}\,\lambda_j$

backprojection: $b_j = \sum_\theta a_{\theta j} y_{\theta j}$

MLEM: $\lambda_j^{(n+1)} = \frac{\lambda_j^{(n)}}{\sum_\theta a_{\theta j}} \sum_\theta a_{\theta j} \frac{y_{\theta j}}{a_{\theta j}\lambda_j^{(n)}}$

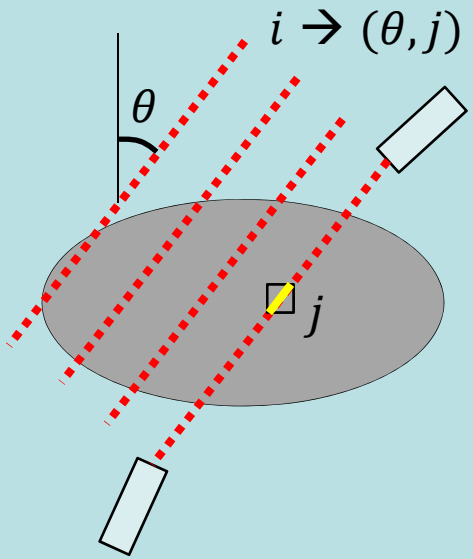
non-iterative!

$= \frac{\sum_\theta y_{\theta j}}{\sum_\theta a_{\theta j}}$

KU LEUVEN

no more (iterative) reconstruction?

10 ps TOF



scatter

projection: $y_{\theta j} = a_{\theta j}\,\lambda_j + s_{\theta j}$

backprojection: $b_j = \sum_{\theta} a_{\theta j} y_{\theta j}$

MLEM: $\lambda_j^{(n+1)} = \frac{\lambda_j^{(n)}}{\sum_{\theta} a_{\theta j}} \sum_{\theta} a_{\theta j} \frac{y_{\theta j}}{a_{\theta j}\lambda_j^{(n)} + s_{\theta j}}$

iterative!

detector resolution

projection: $y_{\theta k} = \sum_{j} d_{\theta k j}\, a_{\theta j}\, \lambda_j$

backprojection: $b_j = \sum_{\theta k} d_{\theta k j}\, a_{\theta j}\, y_{\theta k}$

iterative!

MLEM: $\lambda_j^{(n+1)} = \frac{\lambda_j^{(n)}}{\sum_{\theta k} d_{\theta k j} a_{\theta j}} \sum_{\theta k} d_{\theta k j} a_{\theta j} \frac{y_{\theta k}}{\sum_{\xi} d_{\theta k \xi} a_{\theta \xi} \lambda_{\xi}^{(n)}}$

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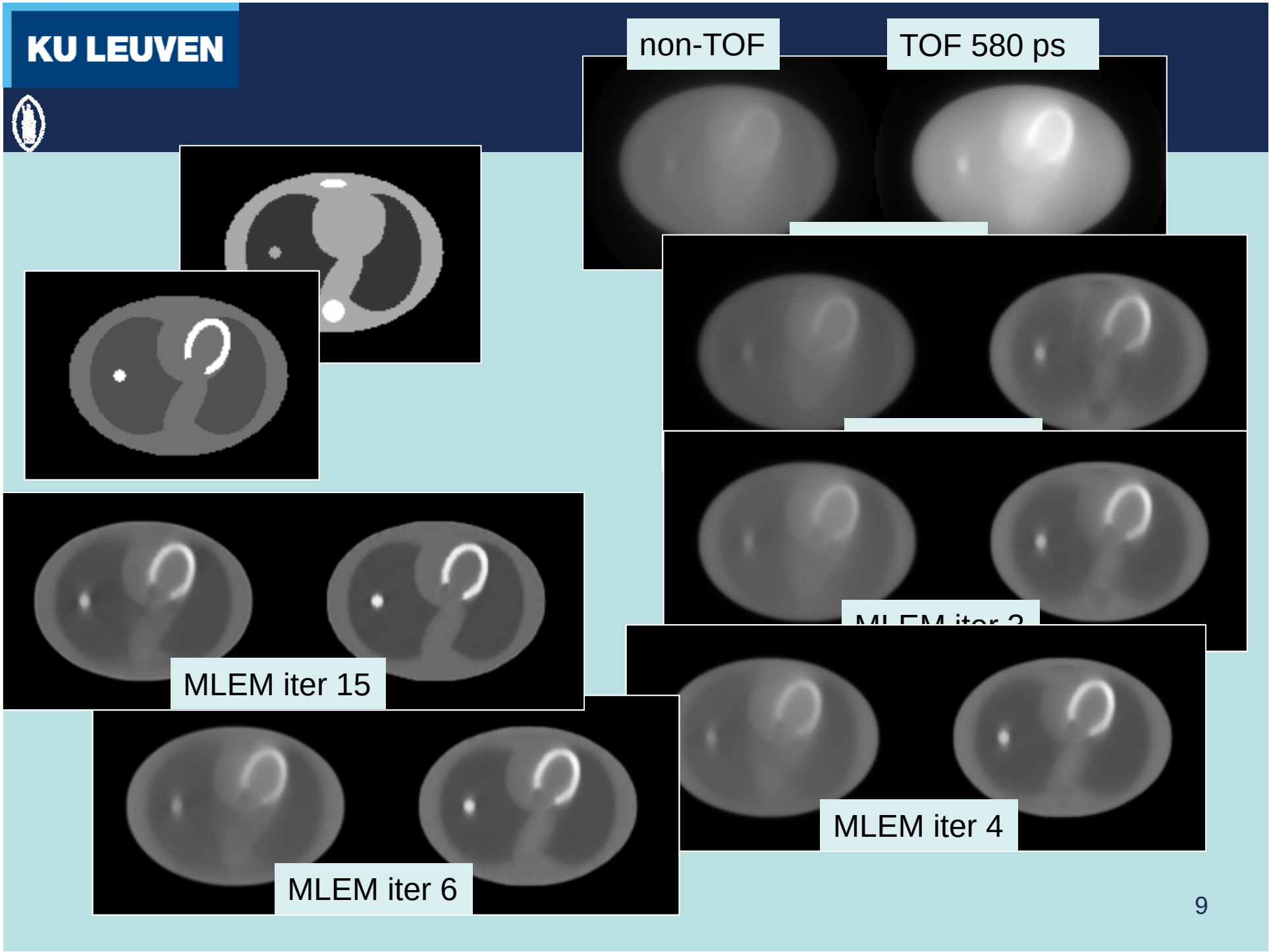

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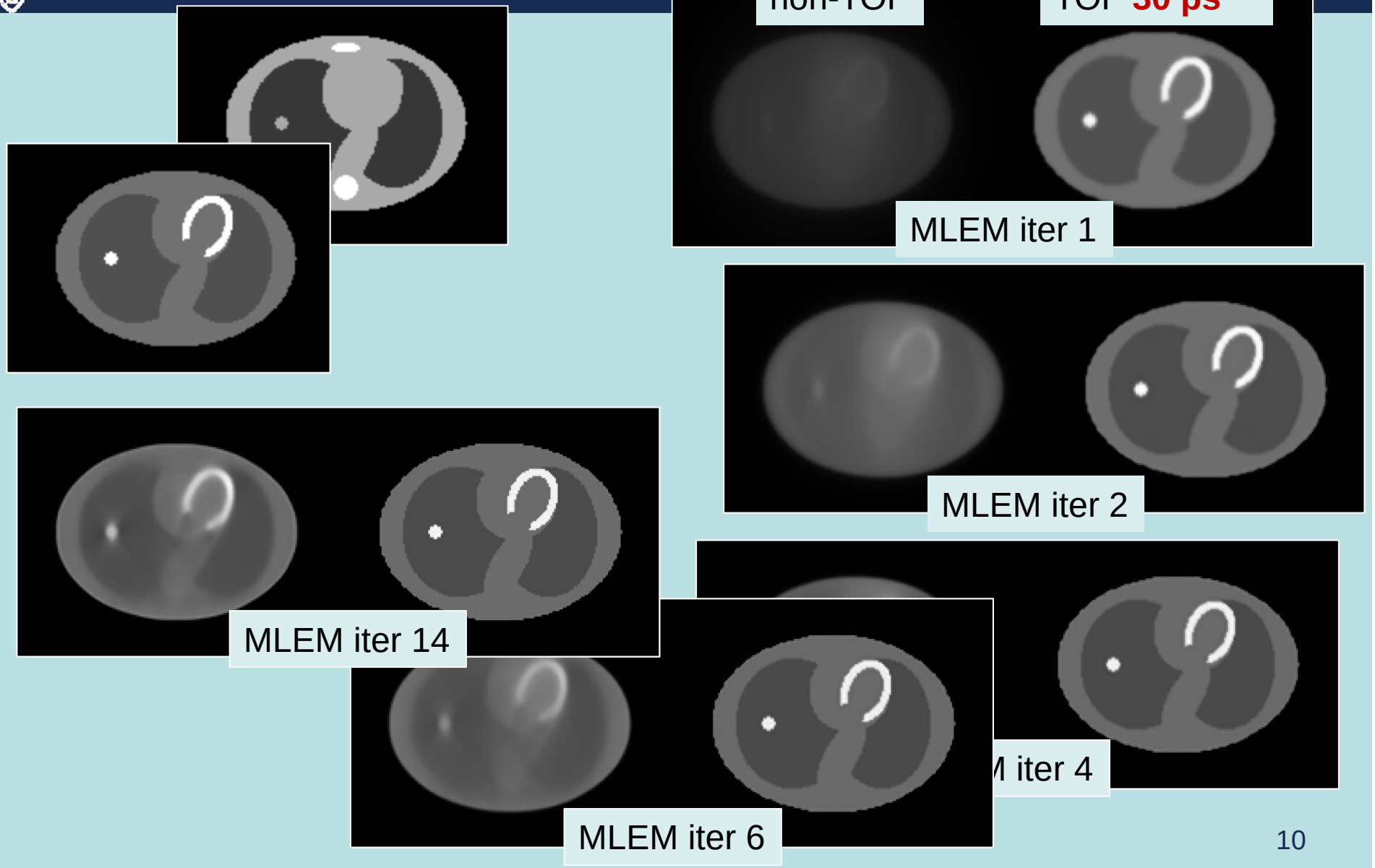
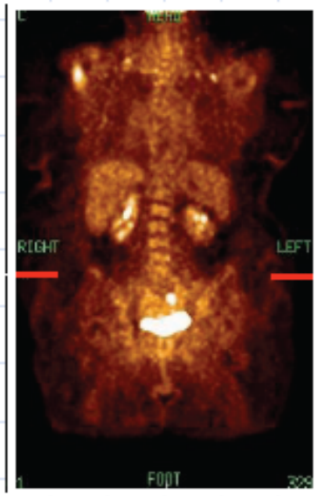
TOF-PET



Improvement in lesion detectability with TOF

Heavy-weight patient study

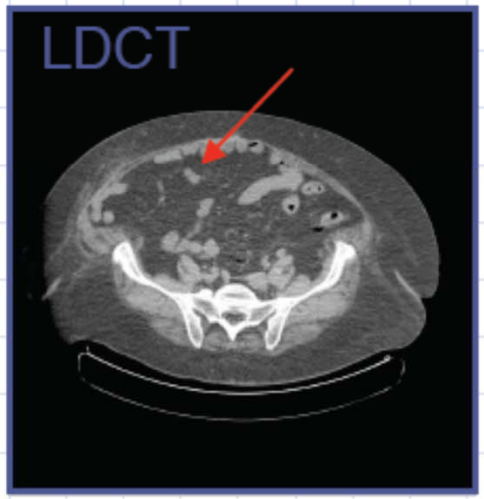


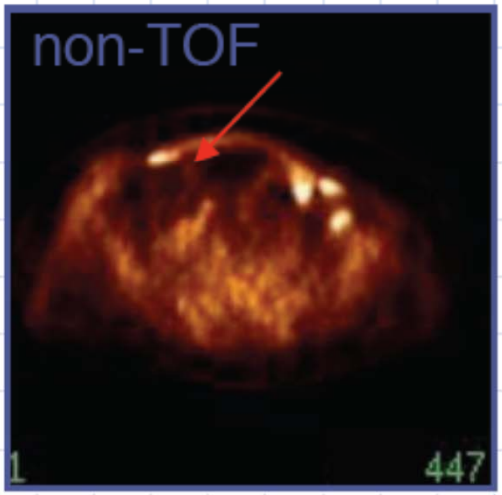
Colon cancer

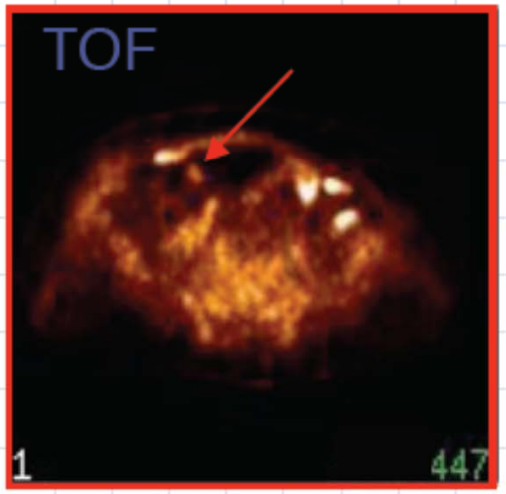
119 kg, BMI = 46.5

13 mCi, 2 hr post-inj
3 min/bed

*Gemini TF







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Gemini TF, time resol 585 ps .. 700 ps

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  - image resolution and angular sampling
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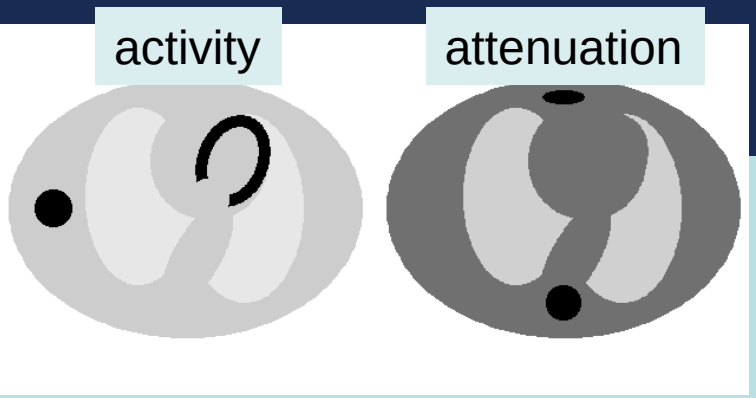
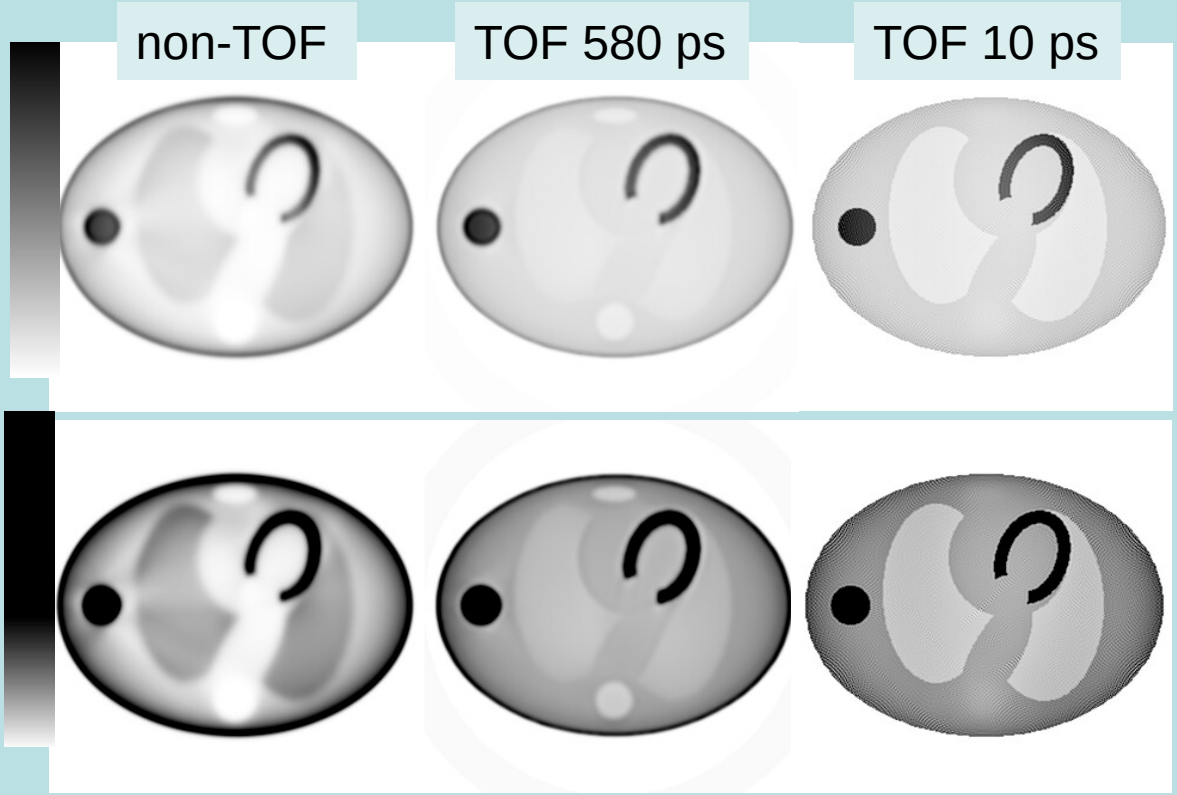
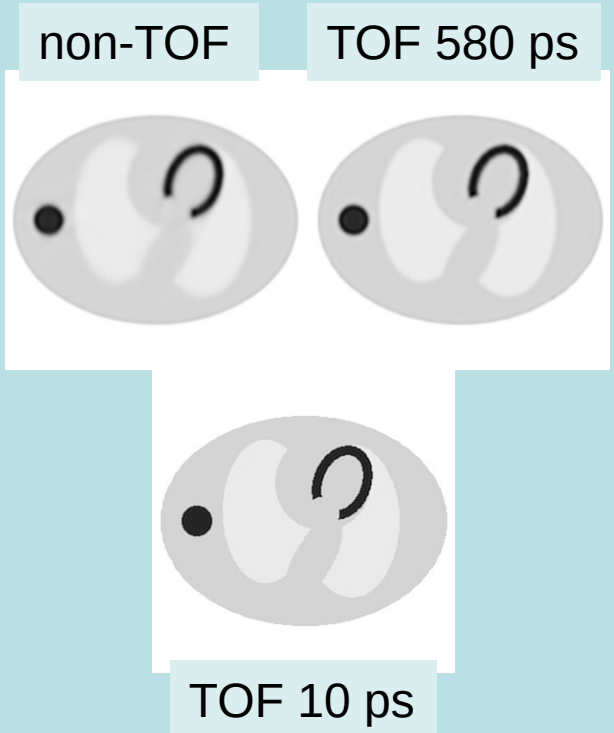
attenuation



no attenuation corr.



with attenuation corr.



Conti M 2011 Why is TOF PET reconstruction a more robust method in the presence of inconsistent data? *Phys Med Biol* 2011, 56: 155-168

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scatter and randoms

randoms fraction ~ total coincidence window ~ diameter of the FOV

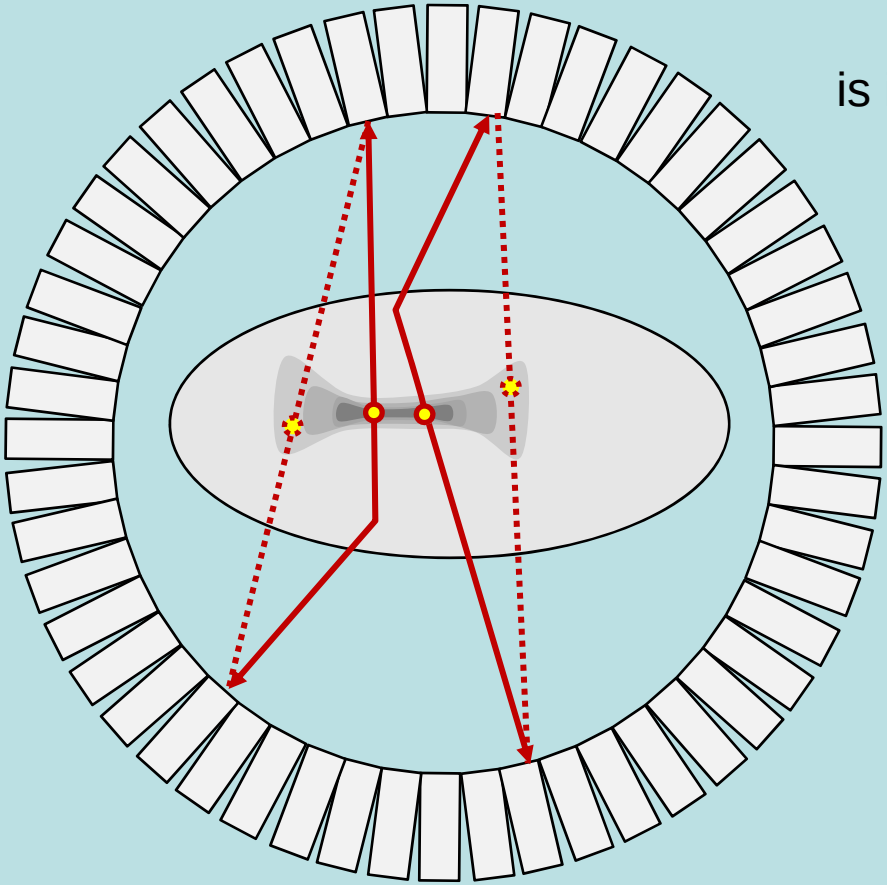
- independent of TOF
- “easily” and accurately estimated
  - from delayed coincidences
  - or from singles
- randoms fraction up to 50% in clinical scans

scatter fraction ~ energy resolution

- dependent on TOF
- very difficult to estimate
  - accelerated Monte Carlo
  - energy measurement?
- scatter fraction 40 .. 50% in clinical scans

scatter and randoms

scatter estimation software usually
assumes that scatter is smooth



is it smooth in TOF direction?

Maybe not

- harder to estimate
- may contain more information

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10 ps TOF-PET

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- **resolution:**
  - **image resolution and angular sampling**
- SNR
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  - relax or eliminate need for attenuation measurement
  - illustrates need for accurate system calibration

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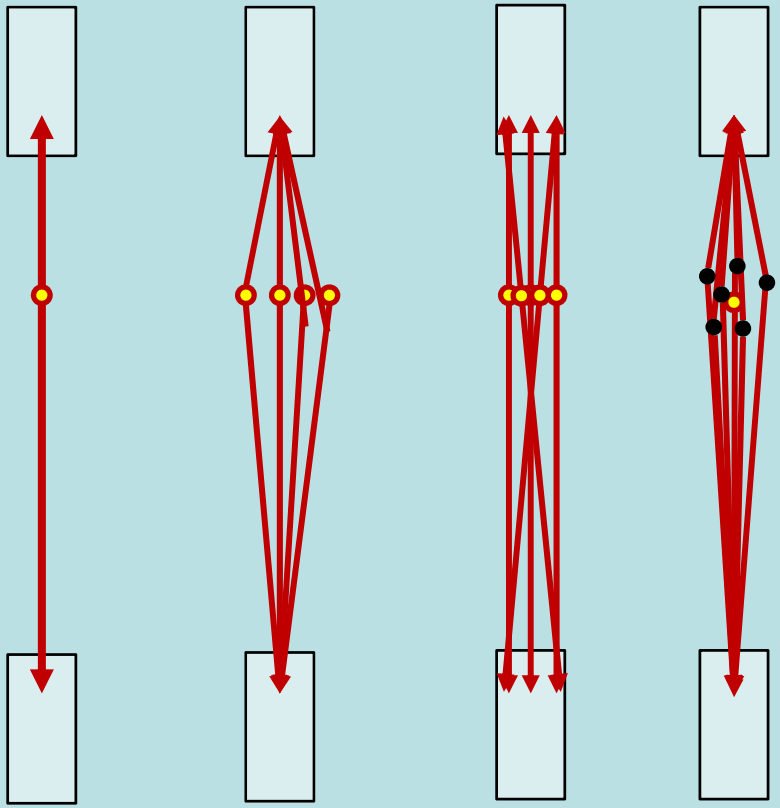
resolution



Depth of interaction:
measured to achieve 10 ps TOF

acollinearity:
2D blurring perpendicular to TOF-axis
~ 0.0022 mm/mm * diameter

detector resolution:
2D blurring perpendicular to TOF-axis
~ crystal_width / 2

positron range:
3D blurring!

isotope	*mean [mm]*	*max [mm]*
^{11}C:	1.1	4.1
^{18}F:	0.6	2.4
^{68}Ga:	2.9	8.2
^{82}Rb:	5.9	14.1

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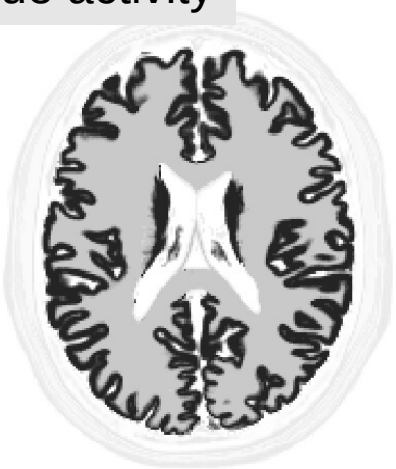
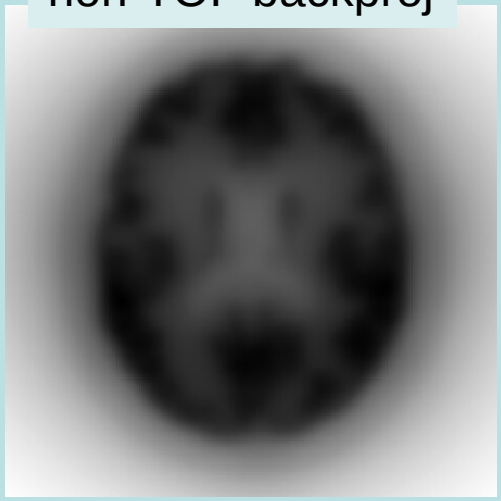
resolution

resolution in TOF-direction ~1.5 mm, in detector direction 5 mm

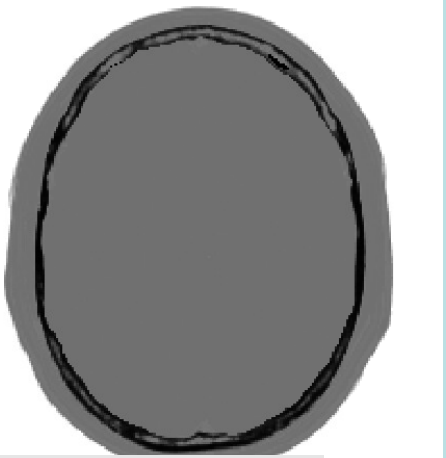
true activity



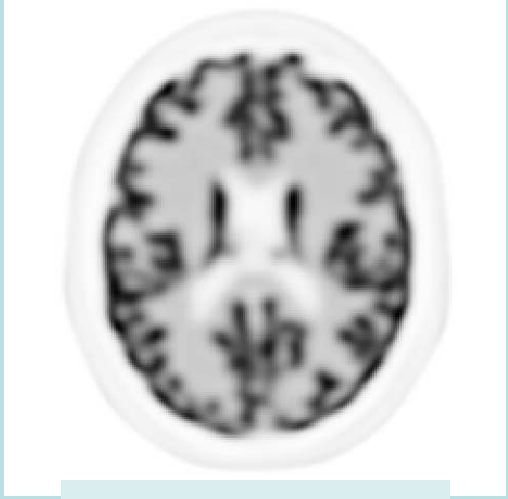
non-TOF backproj



TOF backproj



true attenuation



non-TOF OSEM

TOF OSEM

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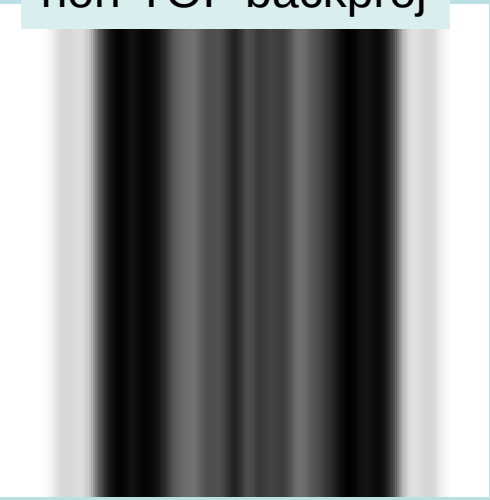
resolution

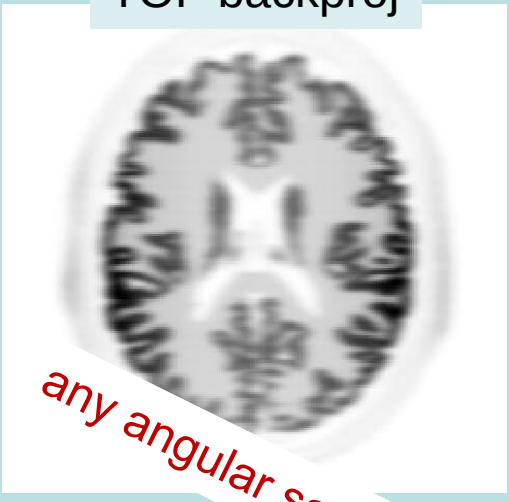
same when only vertical parallel projection is used

true activity

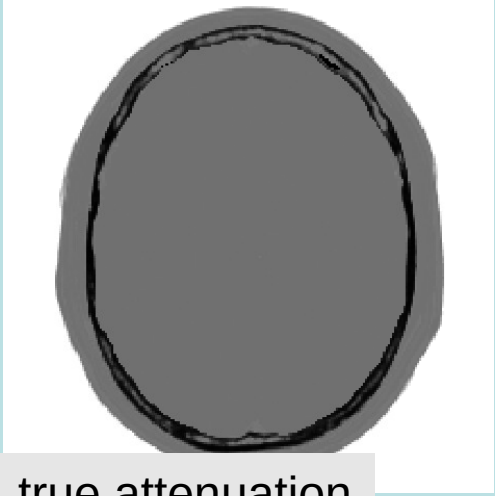
non-TOF backproj



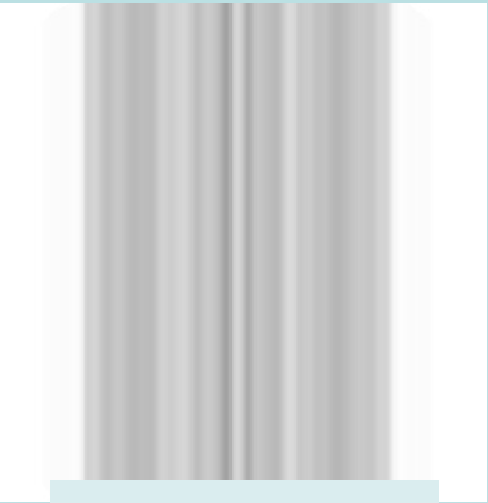
TOF backproj



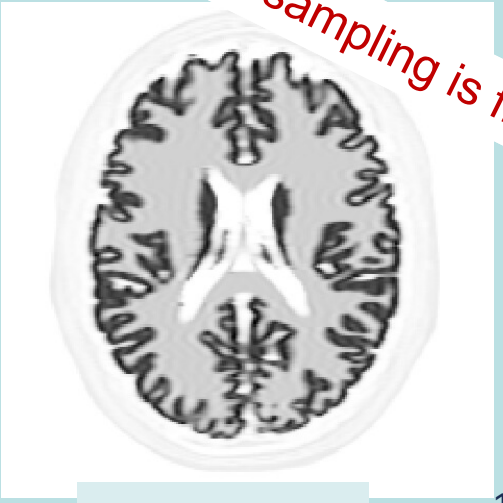
any angular sampling is fine



true attenuation



non-TOF OSEM



TOF OSEM

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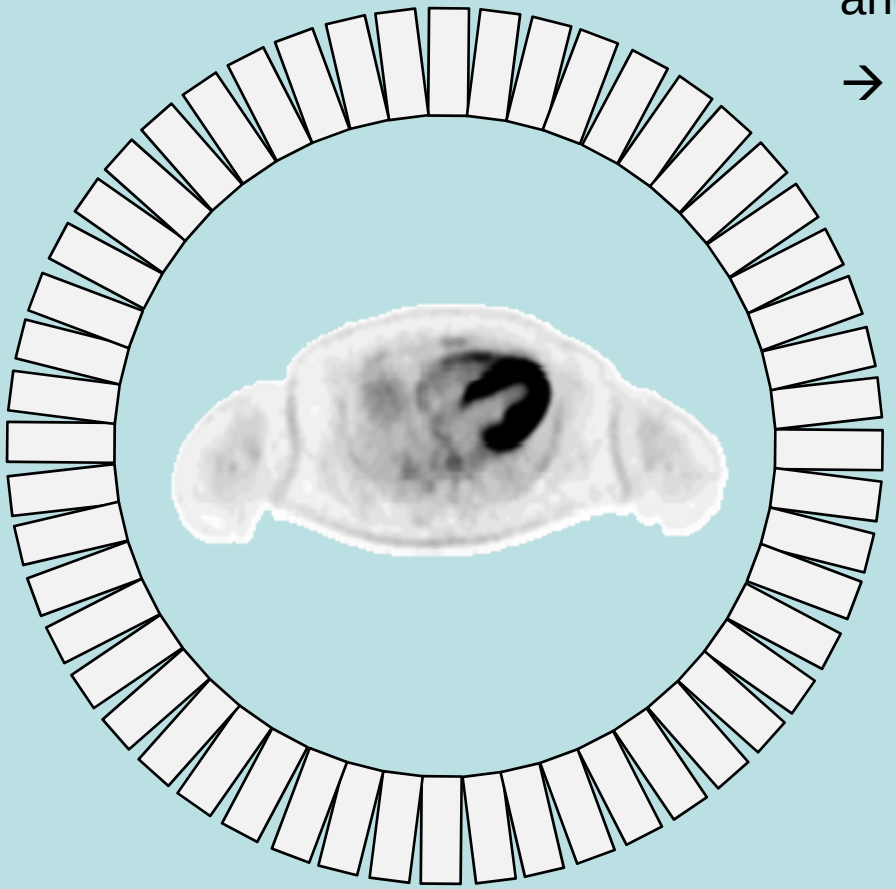
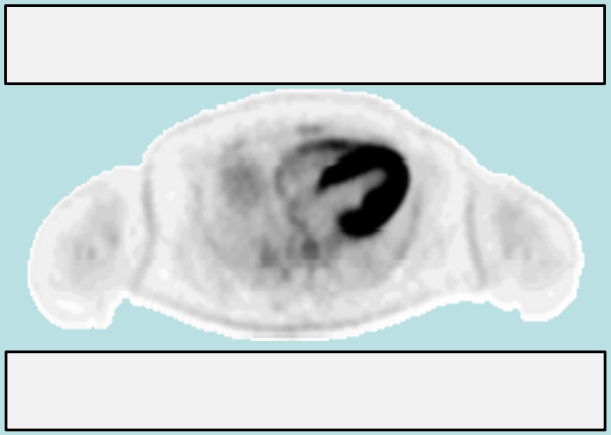
resolution



angular sampling ± irrelevant

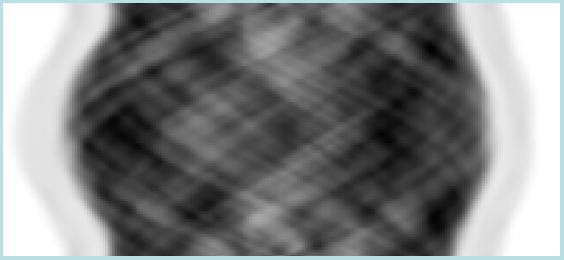
→ optimize image SNR (or NEC)



resolution

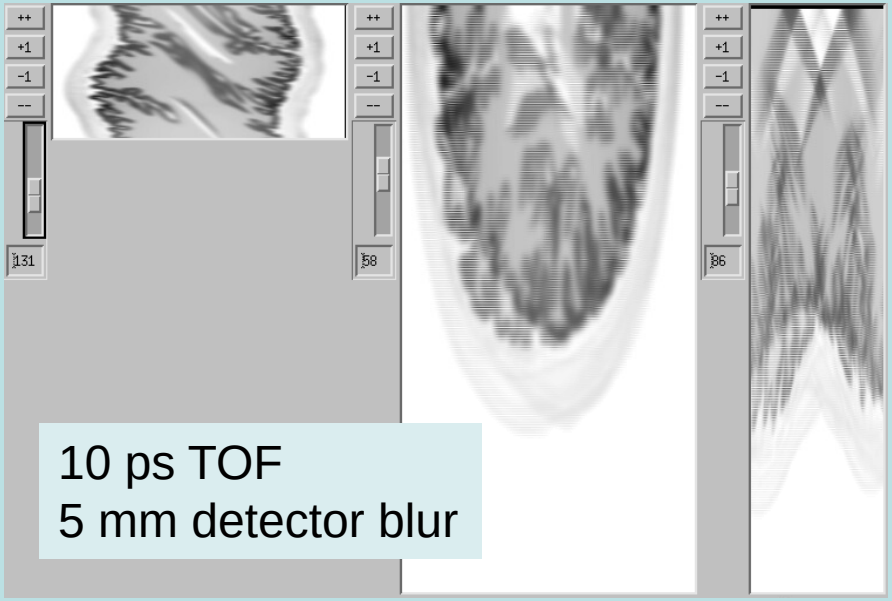
resolution dominated by TOF → use poor detector resolution

5 mm detector resol

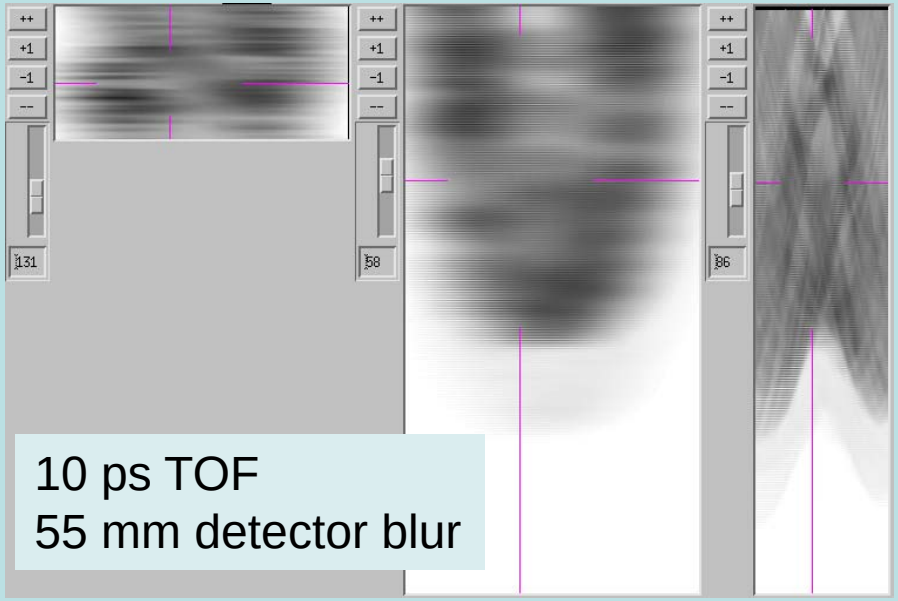


non-TOF sinogram

55 mm detector resol



10 ps TOF
5 mm detector blur



10 ps TOF
55 mm detector blur

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resolution

55 mm detector resol

true activity

non-TOF OSEM

TOF OSEM

M-20-6 – Can Time-of-Flight Information be Used to Mitigate Detector Induced Blur in PET Reconstruction?

M. Toussaint[1,2], J. - P. Dussault[1], R. Lecomte[2]

Session: M-20 - Image Reconstruction II

Date: Saturday, October 28, 2017, 10:20 am

Room: Centennial III ()

IEEE NSS MIC 2017

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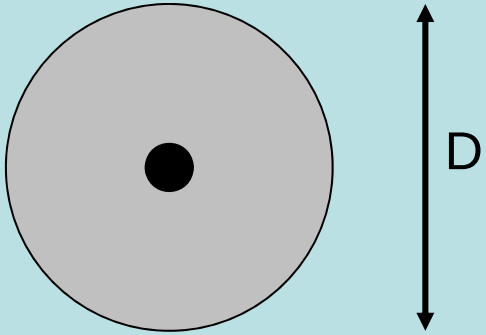
SNR

Improvement in variance with FBP [1]:

for center of uniform disk: $\frac{\mathrm{var}_{\mathrm{nonTOF}}}{\mathrm{var}_{\mathrm{TOF}}} \simeq \sqrt{\frac{2\ln 2}{\pi}}\frac{D}{\Delta x} = 0.66\frac{D}{\Delta x}$ (if $\Delta x << D$)



$$\frac{\mathrm{var}_{\mathrm{TOF}_1}}{\mathrm{var}_{\mathrm{TOF}_2}} \simeq \frac{\Delta x_2}{\Delta x_1}$$

Similar findings for MLEM

[1] Tomitani, *IEEE TNS* 1981, NS-28; 4582 ff

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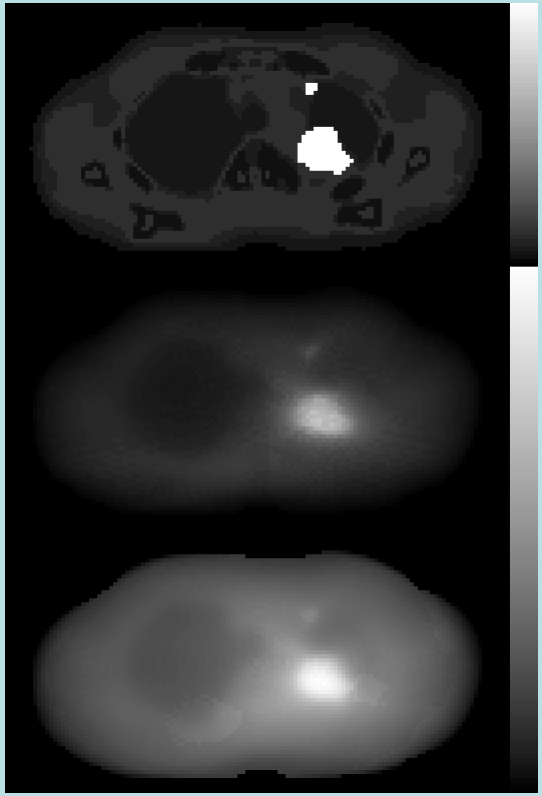
SNR

- 2D realistic thorax phantom
- No scatter, no randoms
- Attenuation modeled
- Intrinsic resolution 0.5 cm
- Pixel size 0.3375 cm
- FOV of 64.8 cm
- Target resolution 1.2 cm FWHM

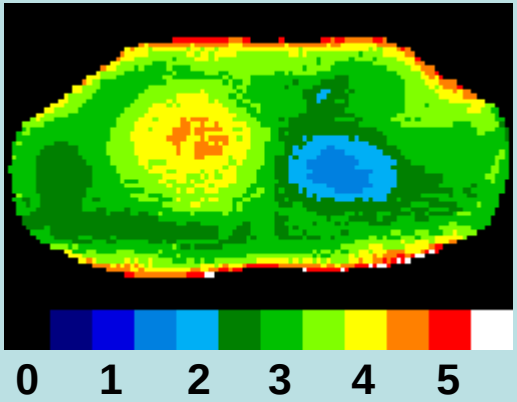
True activity

TOF PET variance
Δt = 500 ps

Non-TOF PET variance



Variance improvement due to TOF information



Kathleen Vunckx et al

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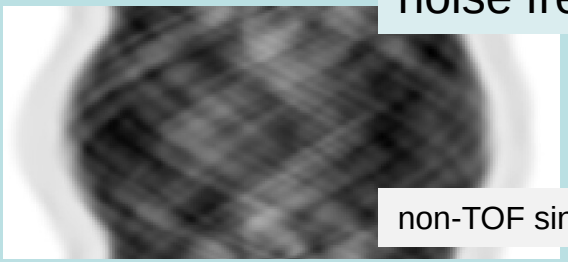
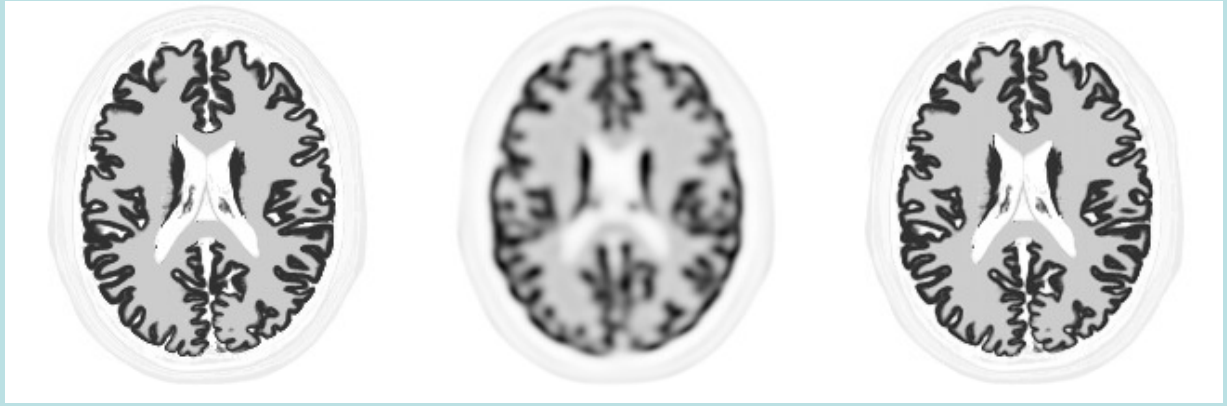
SNR

noise free



non-TOF sino

TOF sino (1 of 440)

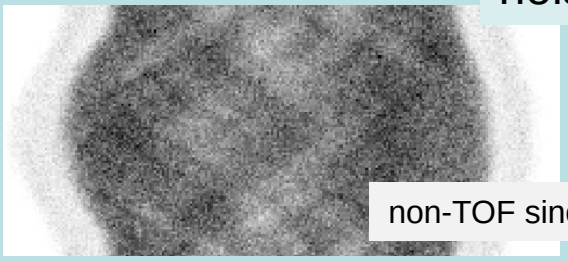


true

non-TOF

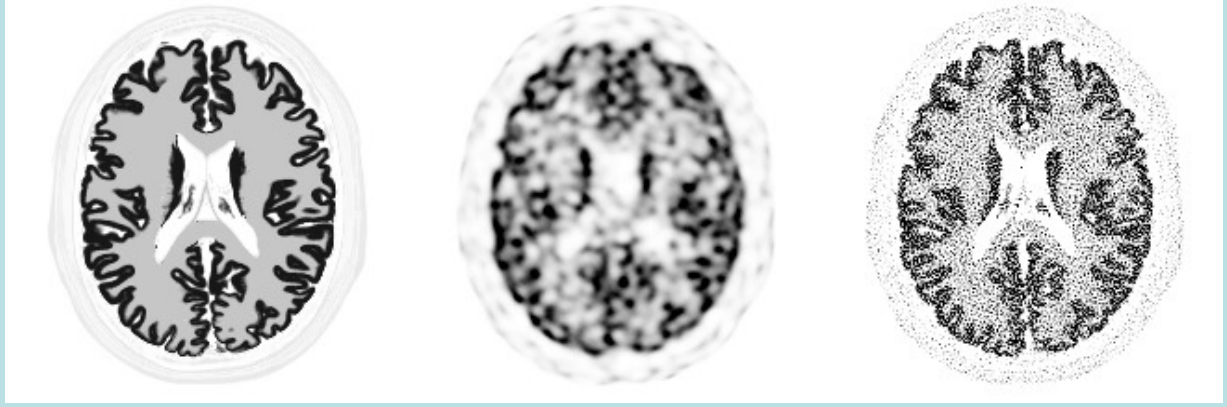
TOF

noisy



non-TOF sino

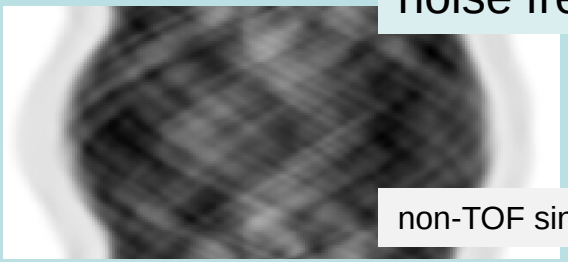
TOF sino (1 of 440)



KU LEUVEN

SNR

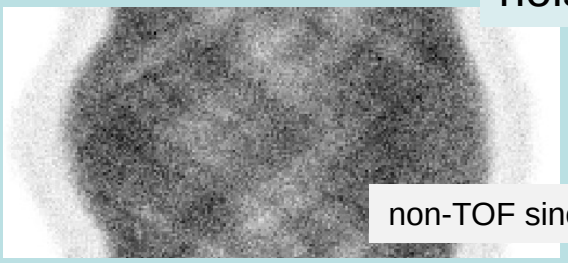
noise free



non-TOF sino

TOF sino (1 of 440)

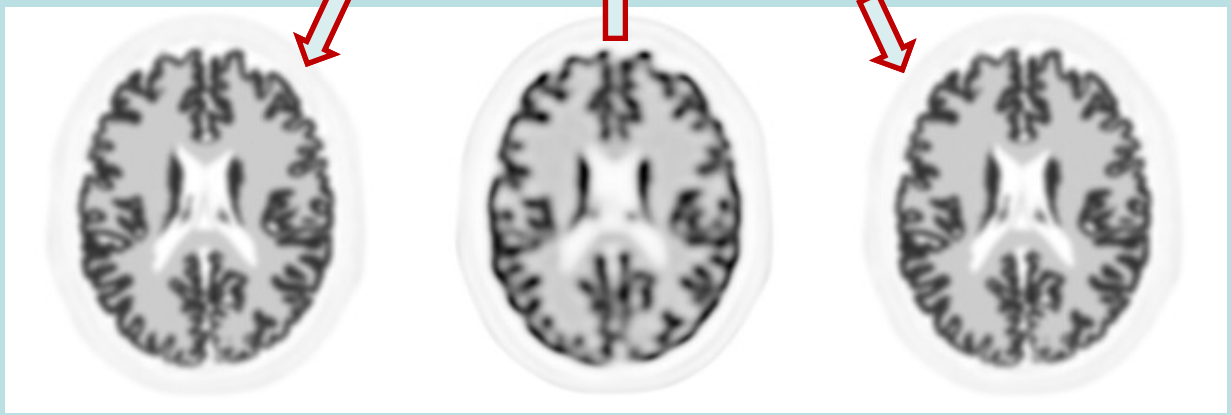
noisy



non-TOF sino

TOF sino (1 of 440)

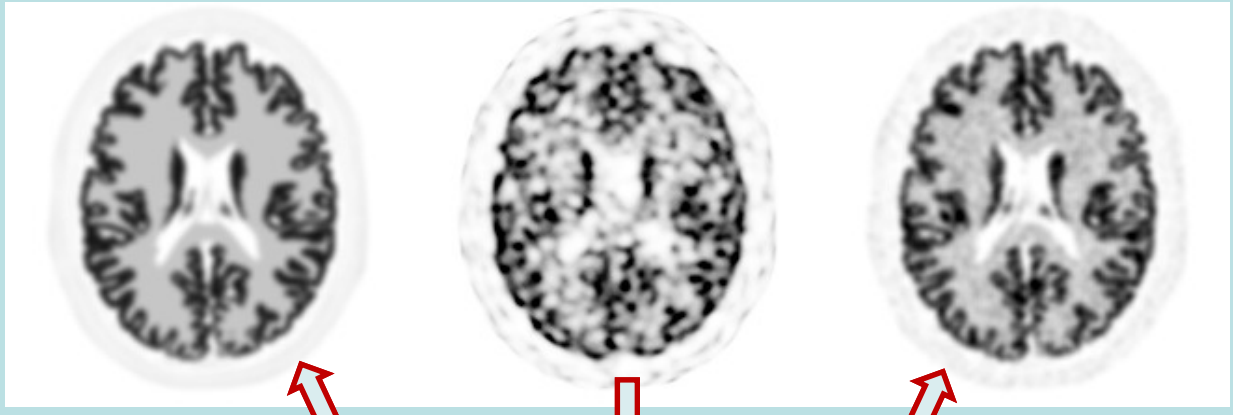
smoothed to ~same resolution



true

non-TOF

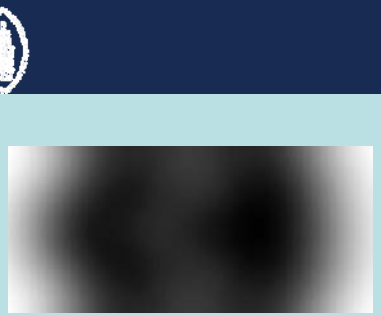
TOF



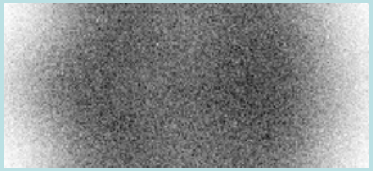
smoothed to ~same resolution

KU LEUVEN

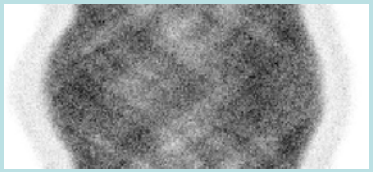
SNR



10 ps TOF and **55 mm** detector blur



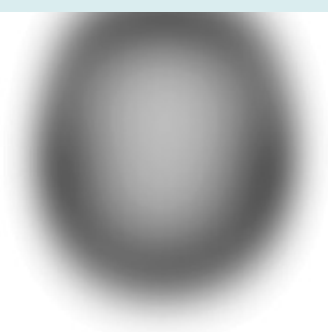
10 ps TOF and **5 mm** detector blur

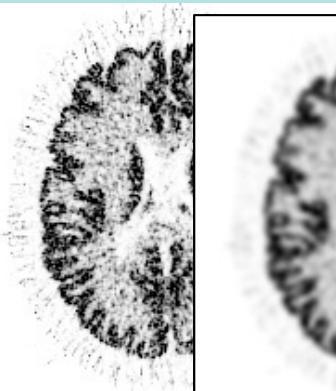


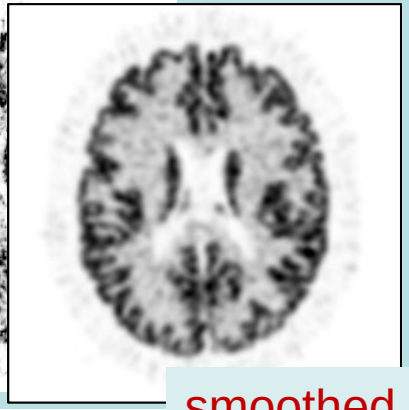
true activity

non-TOF OSEM

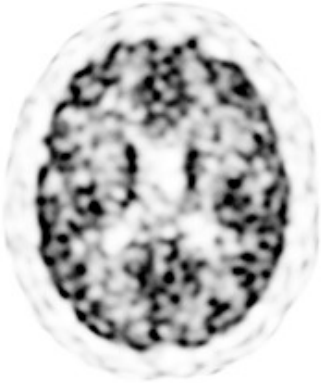
TOF OSEM

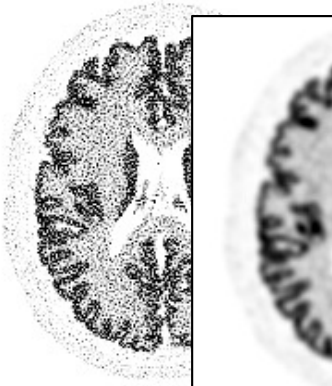






smoothed





KU LEUVEN

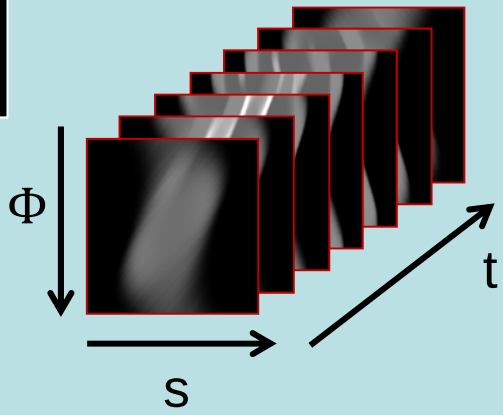
10 ps TOF-PET

- TOF sinograms
- no more (iterative) reconstruction?
- attenuation, scatter, randoms
- resolution:
  - image resolution and angular sampling
- SNR
- **estimate attenuation from emission with MLAA**
  - **relax or eliminate need for attenuation measurement**
  - **illustrates need for accurate system calibration**

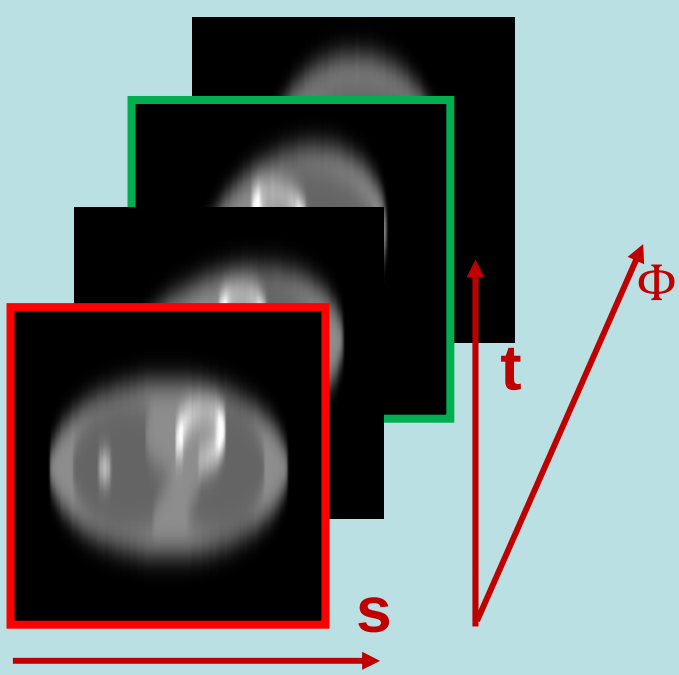
KU LEUVEN

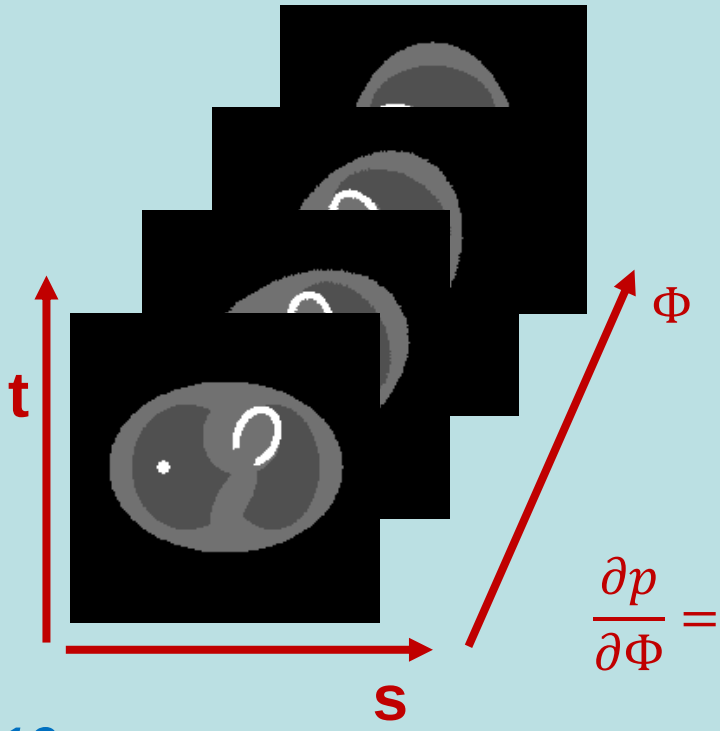
TOF consistency

p is a consistent "2D"-TOF-PET sinogram



$$\Longrightarrow \quad Dp = t\,\frac{\partial p}{\partial s} + \frac{\partial p}{\partial \Phi} - s\,\frac{\partial p}{\partial t} + \sigma^2 \frac{\partial^2 p}{\partial s\,\partial t} = 0$$





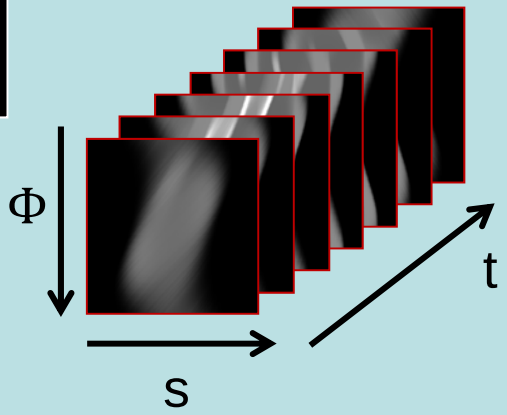
$$\frac{\partial p}{\partial \Phi} = s\,\frac{\partial p}{\partial t} - t\,\frac{\partial p}{\partial s}$$

Defrise et al, Phys Med Biol 2012

KU LEUVEN

TOF consistency

p is a consistent "2D"-TOF-PET sinogram



$$\Longrightarrow \quad Dp = t\,\frac{\partial p}{\partial s} + \frac{\partial p}{\partial \Phi} - s\,\frac{\partial p}{\partial t} + \sigma^2\,\frac{\partial^2 p}{\partial s\,\partial t} = 0$$

set $p_{it} = \dfrac{y_{it}}{a_i}$ $\qquad a_i = e^{-\sum_j l_{ij}\mu_j}$

$$D\frac{y_{it}}{a_i} = t\,\frac{\partial (y_{it}/a_i)}{\partial s} + \frac{\partial (y_{it}/a_i)}{\partial \Phi} - s\,\frac{\partial (y_{it}/a_i)}{\partial t} + \sigma^2\,\frac{\partial^2 (y_{it}/a_i)}{\partial s\,\partial t} = 0$$

$$t\frac{\partial y_{it}}{\partial s}\frac{1}{a_i} - t\frac{y_{it}}{a_i^2}\frac{\partial a_i}{\partial s} + \ \ldots\ = 0$$

$\times\, a_i$ $\qquad$ $t\dfrac{\partial y_{it}}{\partial s} - t\dfrac{y_{it}}{a_i}\dfrac{\partial a_i}{\partial s} + \ \ldots\ = 0$

$$Dy_{it} = y_{it}\left(t\frac{\partial \ln(a_i)}{\partial s} + \frac{\partial \ln(a_i)}{\partial \Phi}\right) + \sigma^2\,\frac{\partial y_{it}}{\partial t}\frac{\partial \ln(a_i)}{\partial s} \qquad t = 1..T$$

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TOF consistency

$$Dy_{it} = \left(t\, y_{it} + \sigma^2 \frac{\partial y_{it}}{\partial t} \right) \frac{\partial \ln(a_i)}{\partial s} + y_{it} \frac{\partial \ln(a_i)}{\partial s} \qquad t = 1..T$$

if $y_{it} > 0$: least squares fit, T equations

2 unknowns $\frac{\partial \ln(a_i)}{\partial s}$ and $\frac{\partial \ln(a_i)}{\partial \Phi}$

$a_i = e^{-\sum_j l_{ij}\mu_j}$ → $\ln(a_i) = -\text{proj}(\mu)$

attenuation sinogram proj(μ) determined up to a constant

=

attenuation factor a determined up to a factor

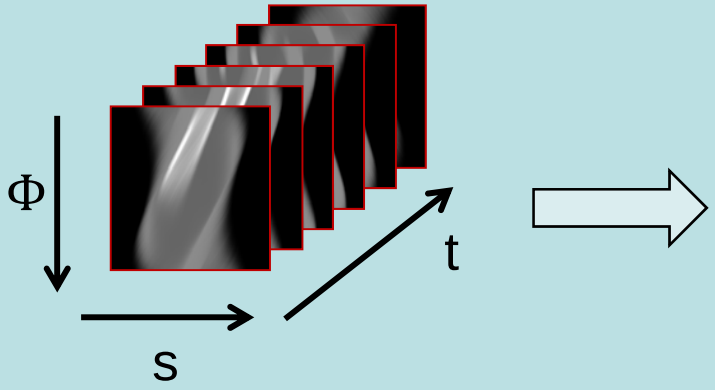
KU LEUVEN

TOF consistency

$$Dy_{it} = \left(t\, y_{it} + \sigma^2 \frac{\partial y_{it}}{\partial t} \right) \frac{\partial \ln(a_i)}{\partial s} + t\, y_{it} \frac{\partial \ln(a_i)}{\partial s} \qquad t = 1..T$$

if $y_{it} > 0$: least squares fit, T equations

2 unknowns $\frac{\partial \ln(a_i)}{\partial s}$ and $\frac{\partial \ln(a_i)}{\partial \Phi}$



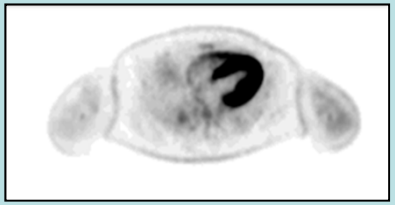
$C \cdot$ 

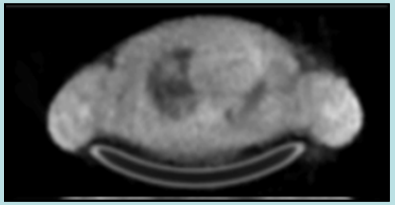
$\frac{1}{C} \cdot$ 

KU LEUVEN

attenuation in TOF-PET

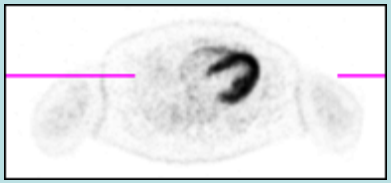
- TOF-PET data
- MLAA

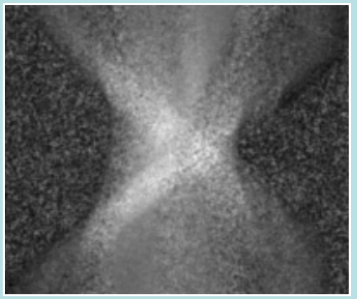




Maximum Likelihood reconstruction of Activity and Attenuation

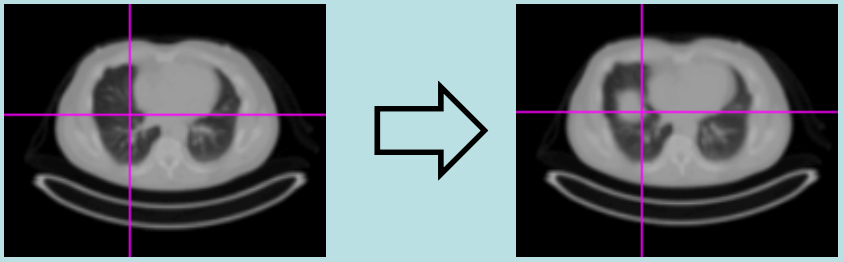
- MLACF





Maximum Likelihood reconstruction of Activity and attenuation Correction Factors

- MLRR



Maximum Likelihood Reconstruction of activity and Registration of attenuation image

KU LEUVEN

TOF-PET MLAA

maximize likelihood $L(y, \boldsymbol{\lambda}, \boldsymbol{\mu})$

$$L = \sum_i \sum_t y_{it} \ln \widehat{y_{it}} - \widehat{y_{it}} \qquad \widehat{y_{it}} = e^{-\sum_k l_{ik} \boldsymbol{\mu_k}} \sum_j c_{itj} \boldsymbol{\lambda_j} + s_{it}$$

Alternated parameter update:

constant $\boldsymbol{\mu}$ $\quad \widehat{y_{it}} = a_i \sum_j c_{itj} \boldsymbol{\lambda_j} + s_{it}$ (TOF-) MLEM

constant $\boldsymbol{\lambda}$ $\quad \widehat{y_{it}} = b_{it}\, e^{-\sum_k l_{ik} \boldsymbol{\mu_k}} + s_{it}$ (TOF-) MLTR

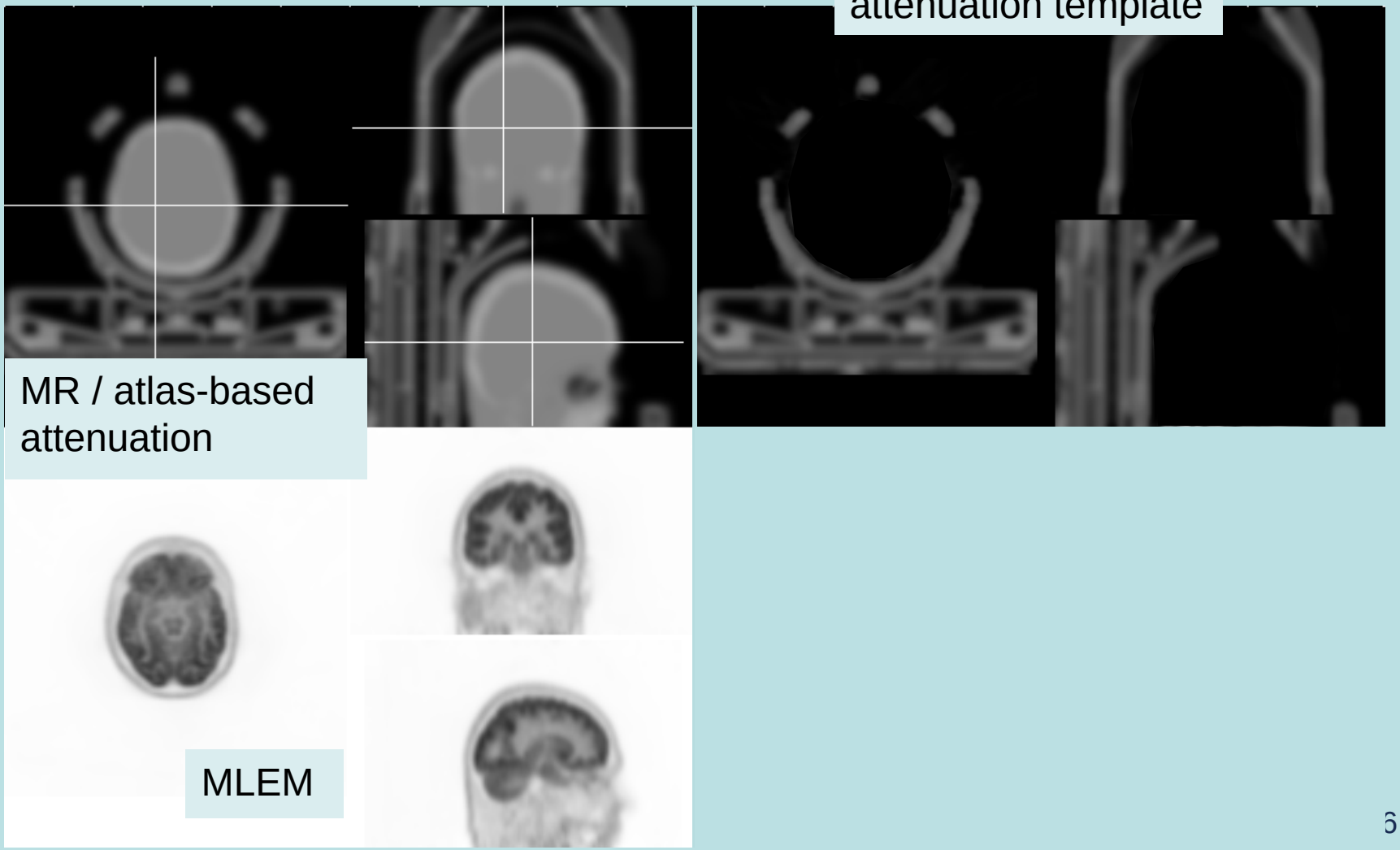
if $s_{it} \sim \widehat{y_{it}}$
or $s_{it} \simeq 0$ $\quad \Rightarrow \quad \widehat{y_i} = b_i\, e^{-\sum_k l_{ik} \boldsymbol{\mu_k}} + s_i$ MLTR

a few MLTR sub-iterations per MLEM (OSEM) sub-iteration

KU LEUVEN

TOF-PET MLAA

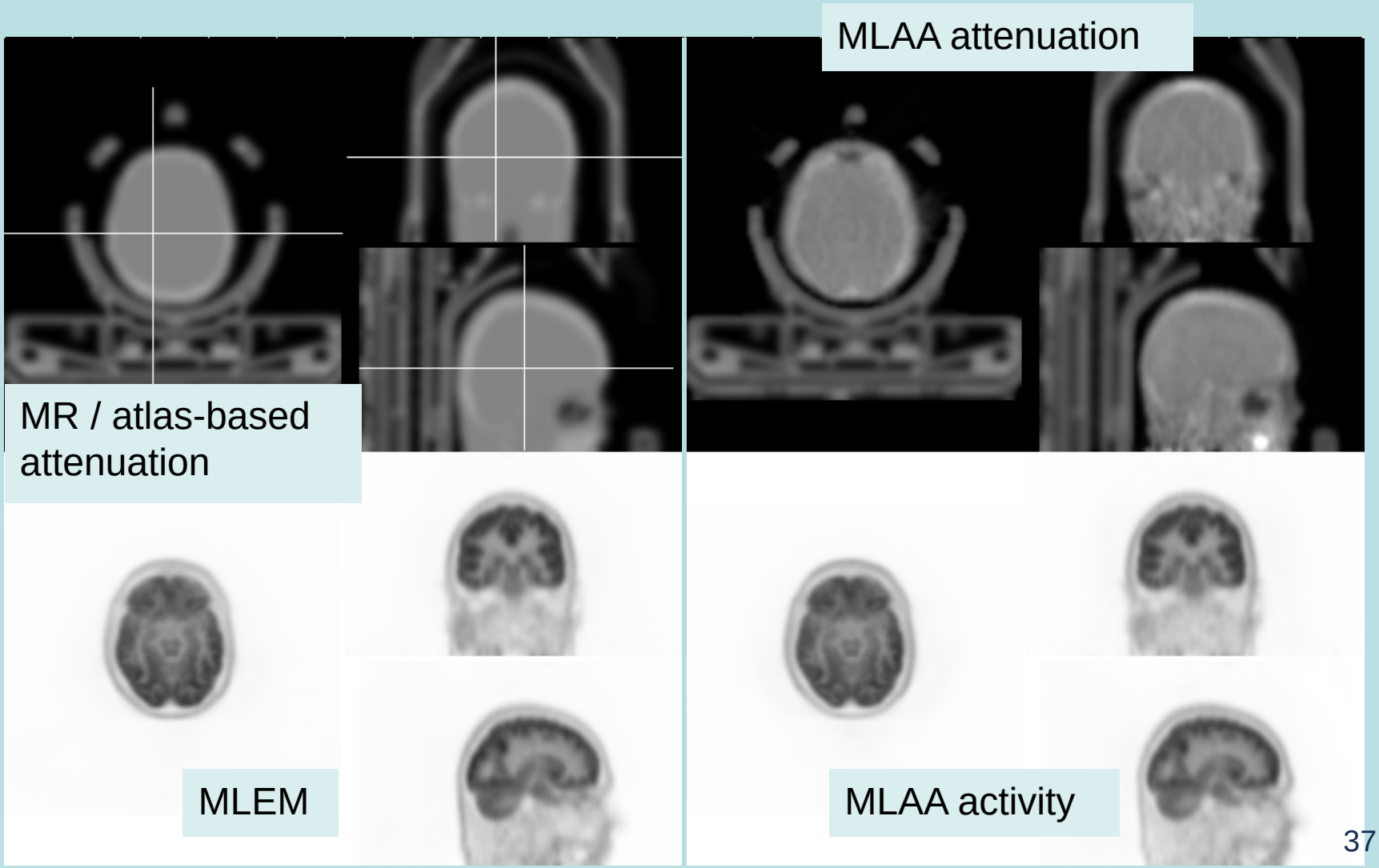
TOF: 380 ps



KU LEUVEN

TOF-PET MLAA

TOF: 380 ps



KU LEUVEN

TOF-PET MLAA

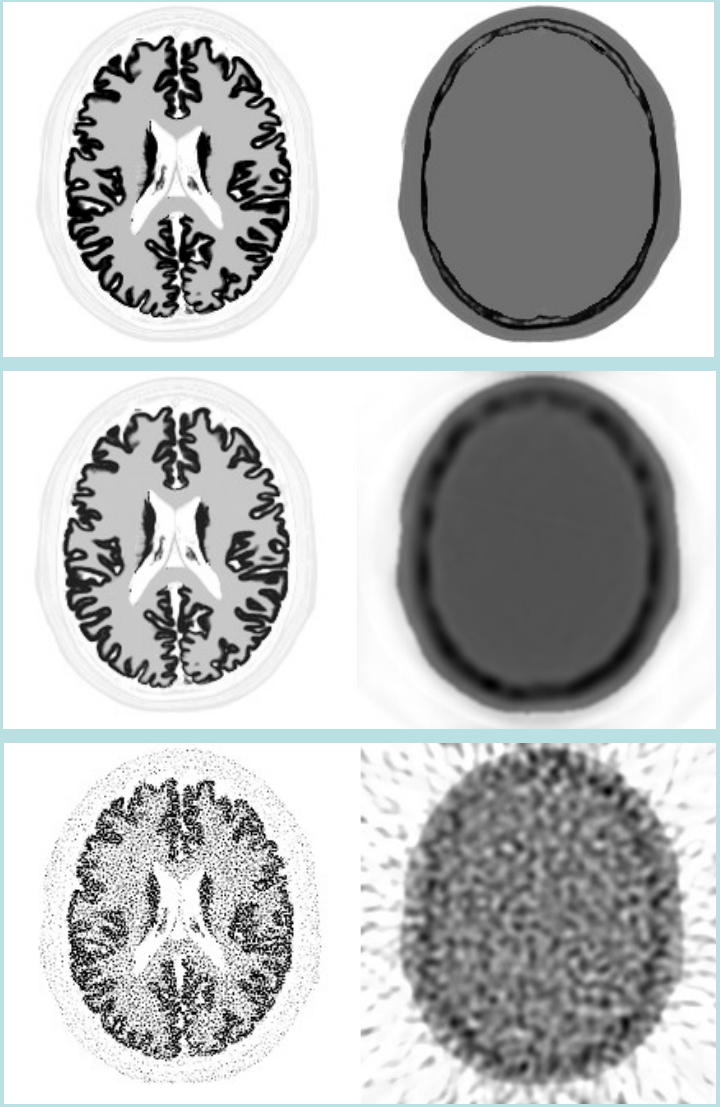
true

MLAA, using known body contour

noise free

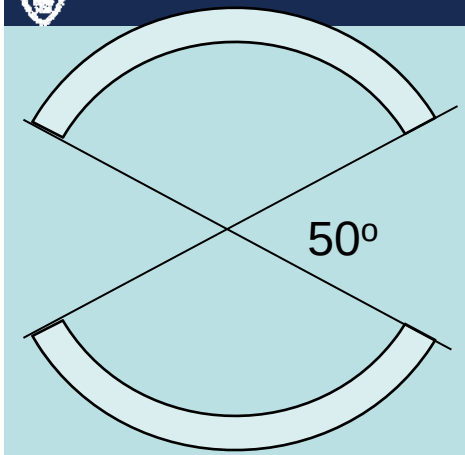
MLAA, using known body contour

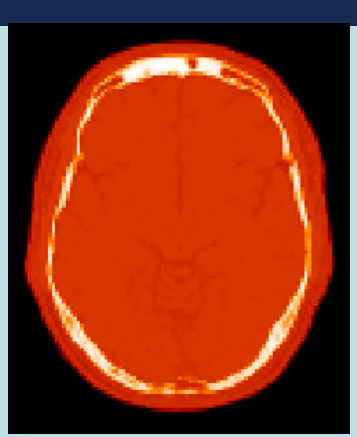
noisy

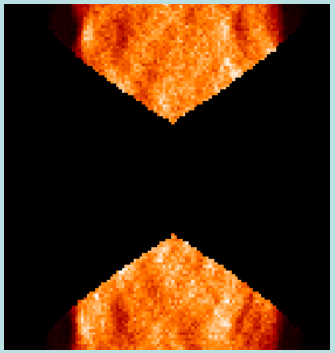


KU LEUVEN

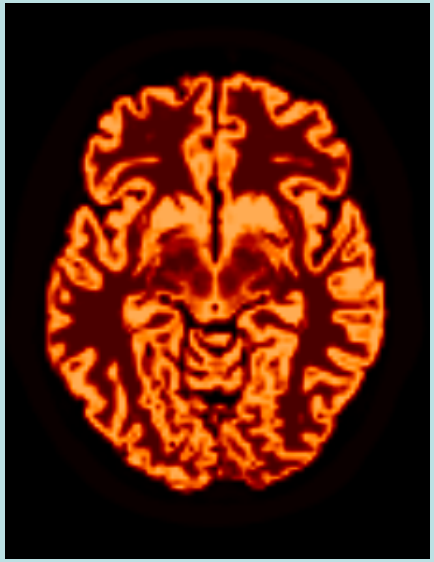
TOF-PET



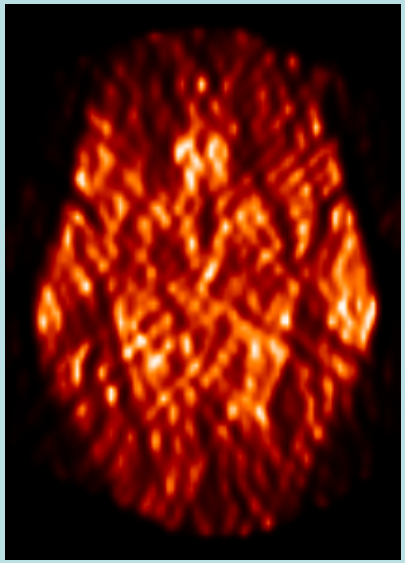




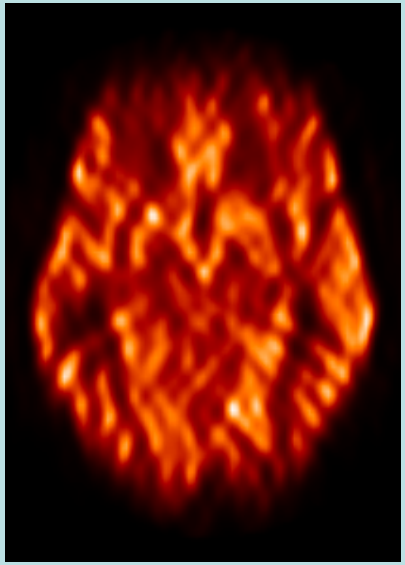
sinogram



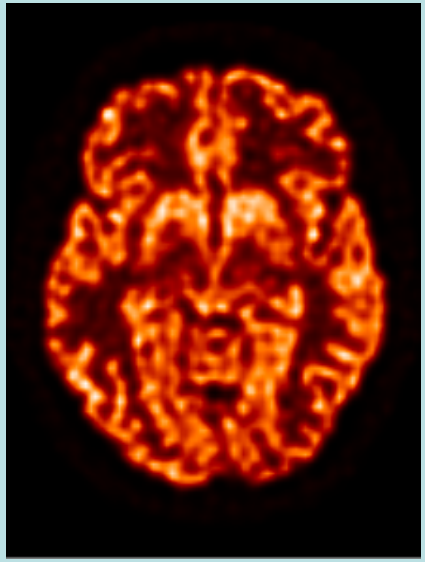
TRUE



non-TOF



TOF 400 ps



TOF 30 ps

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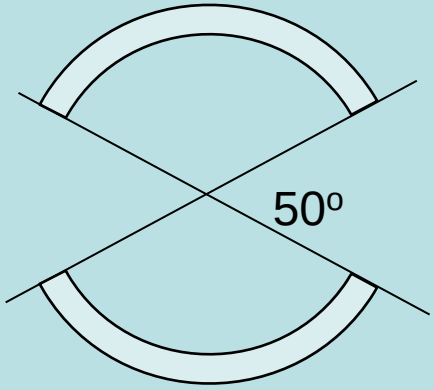
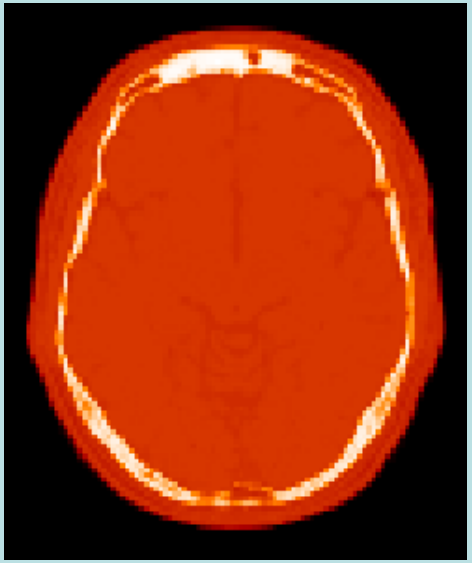
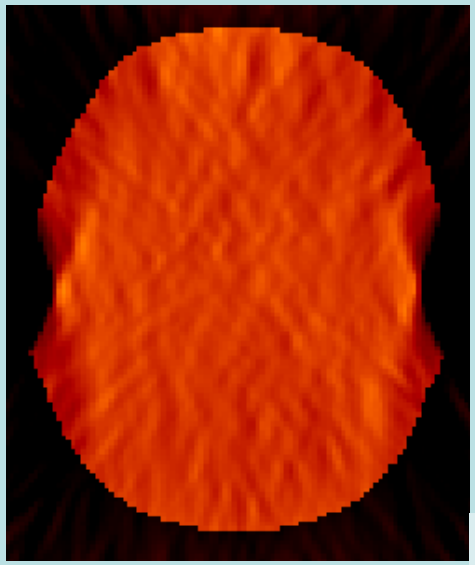
TOF-PET: MLAA

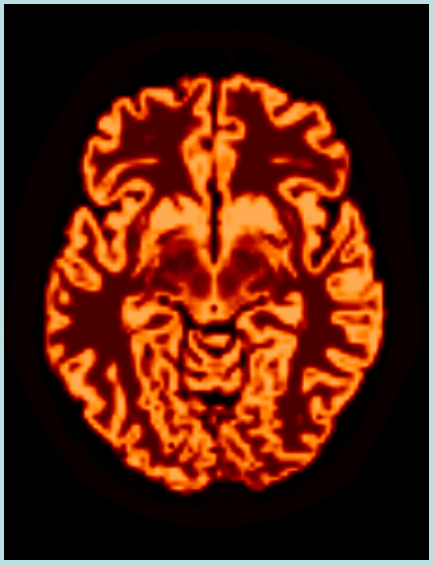


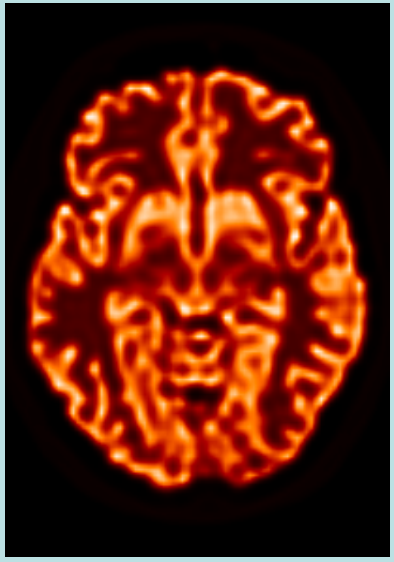
TOF 30 ps





TRUE





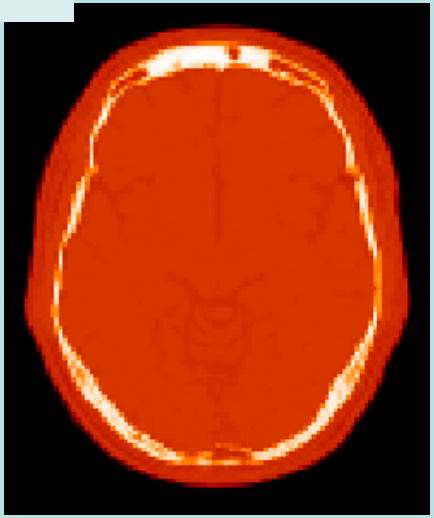
joint estimation

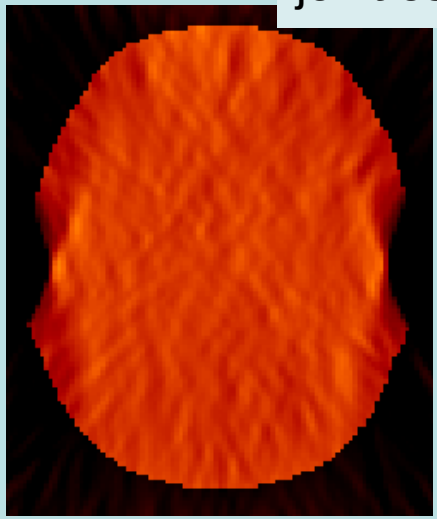
KU LEUVEN

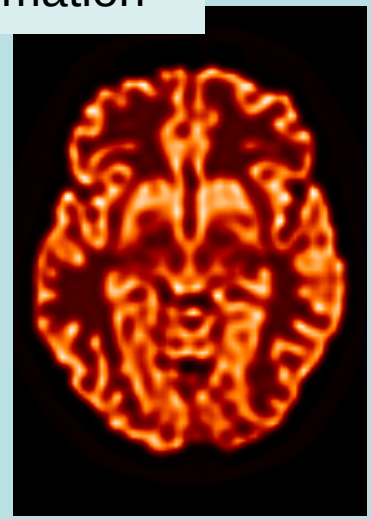
TOF-PET: MLAA

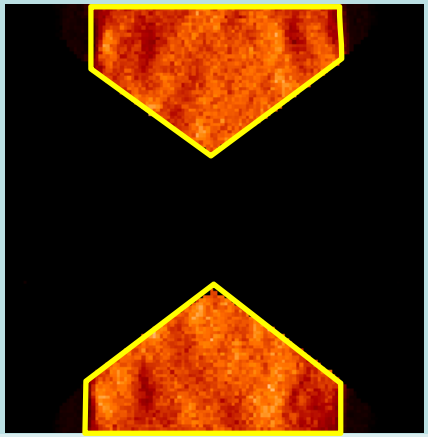
TRUE

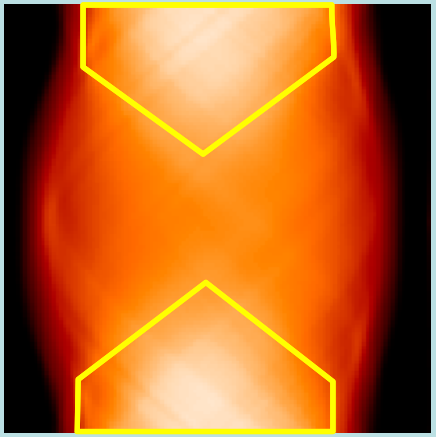
joint estimation

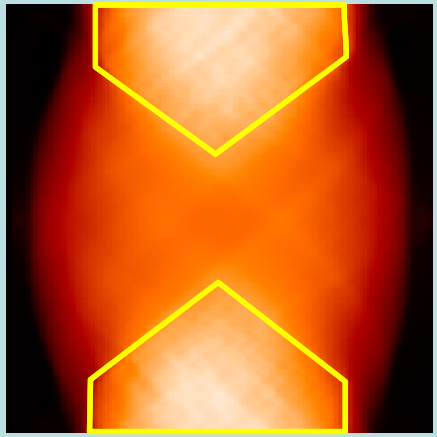


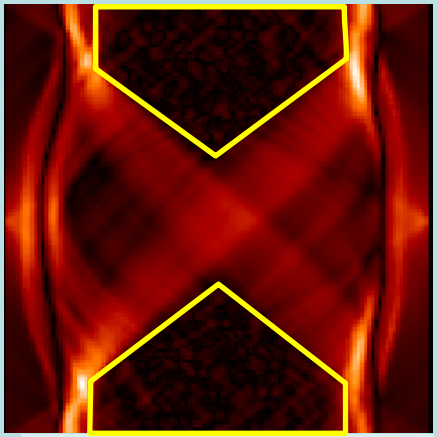












measured activity sinogram

true attenuation

estimated attenuation

difference true and estimated attenuation

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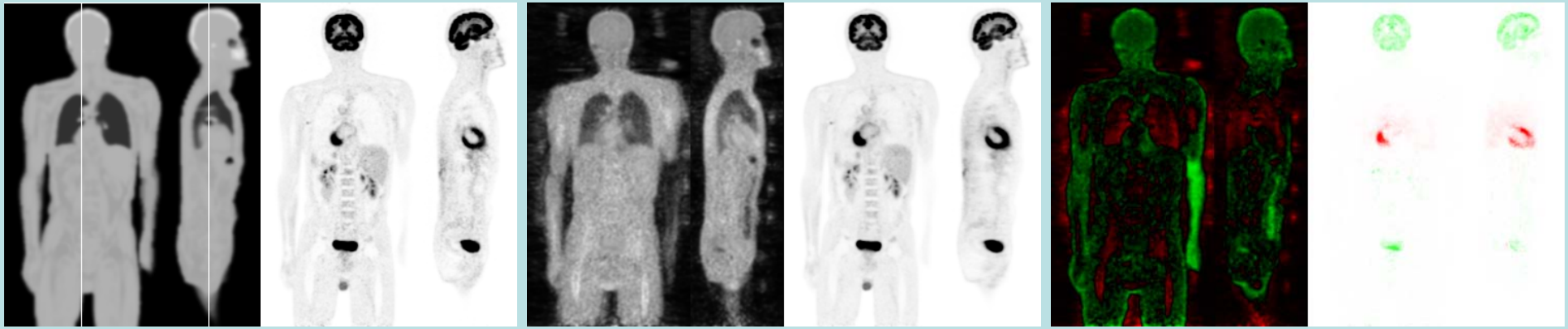
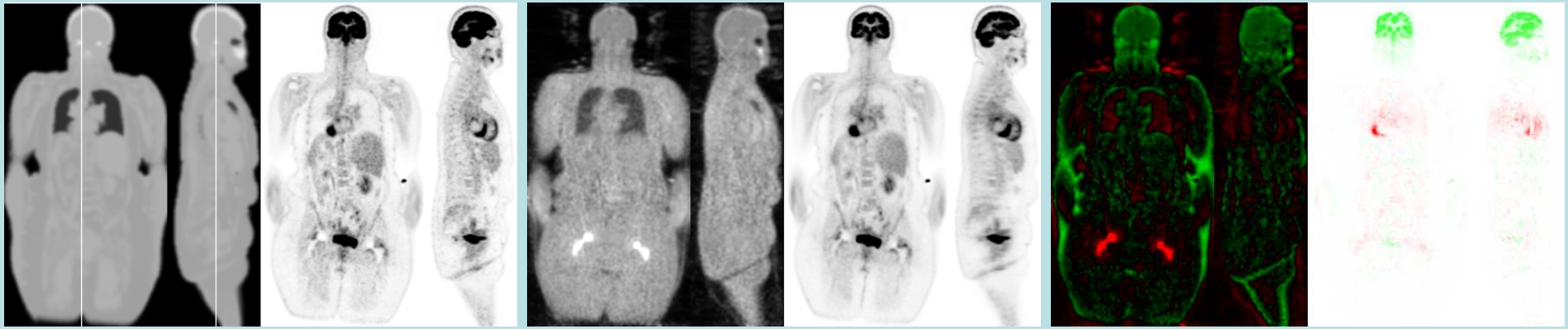
TOF-PET: MLAA

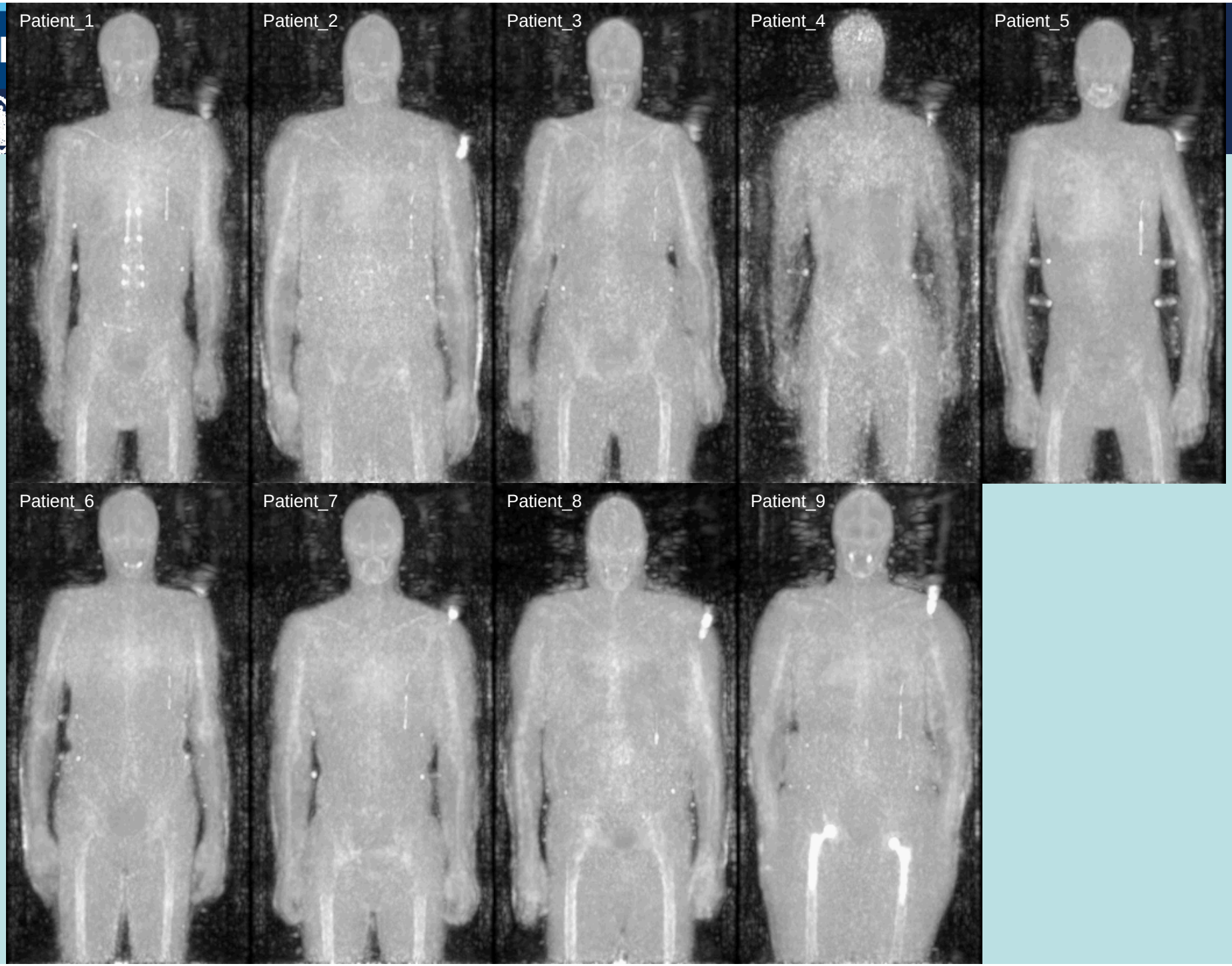
MLEM (CT-based atten) | MLAA | MLAA - MLEM

patient 1



patient 9




Patient_1
Patient_2
Patient_3
Patient_4
Patient_5
Patient_6
Patient_7
Patient_8
Patient_9

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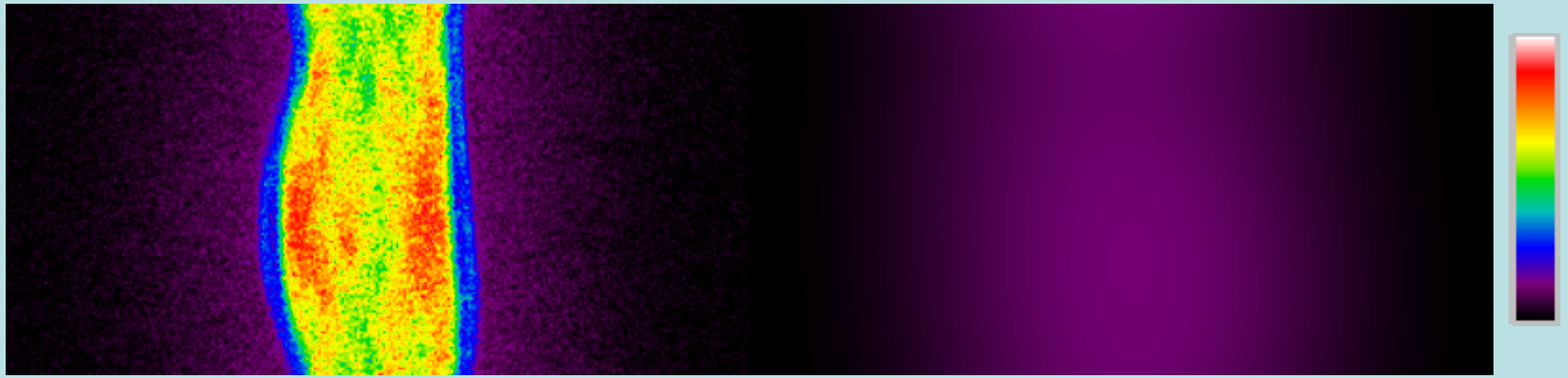
scatter estimation

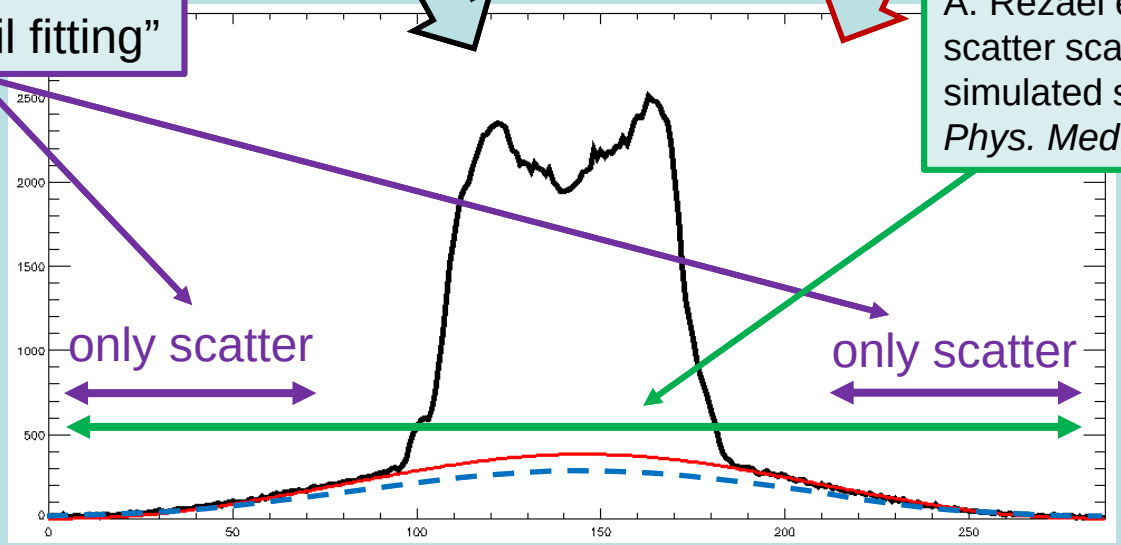


sinogram

estimated scatter

“scatter tail fitting”

A. Rezaei et al. Plane-dependent ML scatter scaling: 3D extension of the 2D simulated single scatter (SSS) estimate. *Phys. Med. Biol*, *62*, pp.6515-6531.



Simulated Single Scatter

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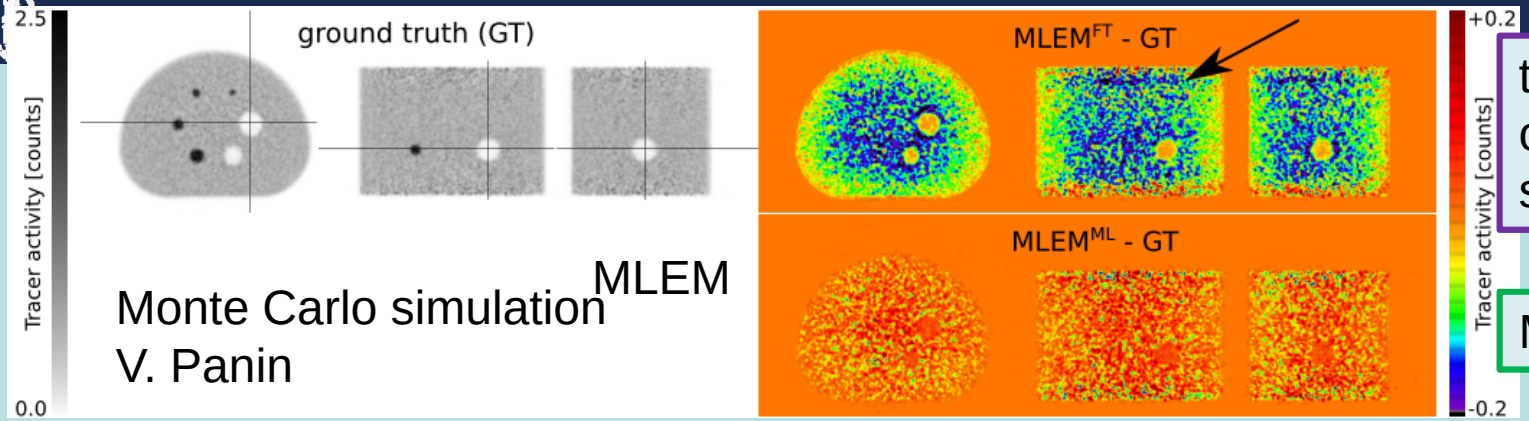
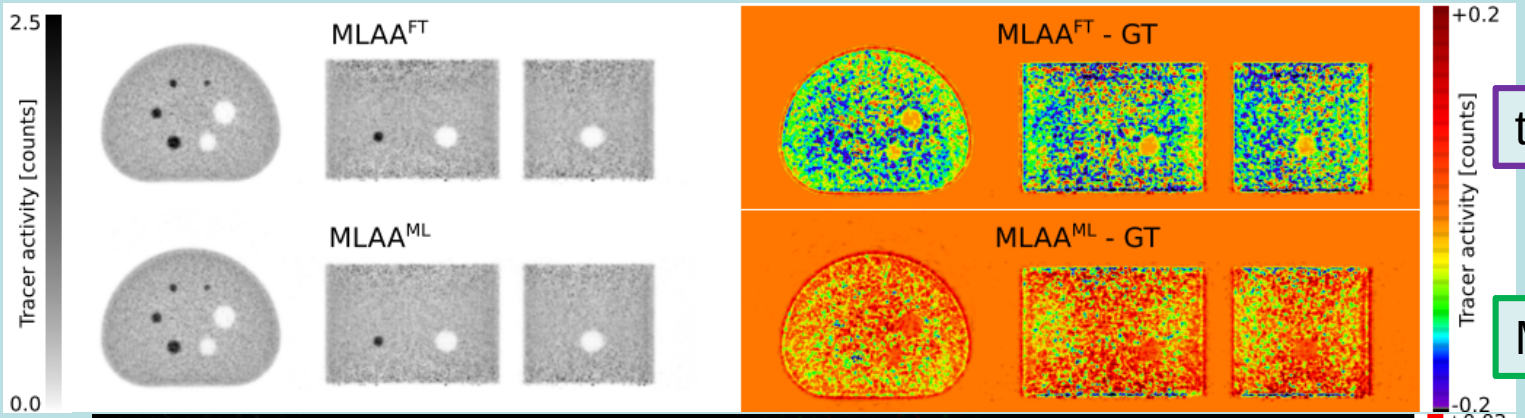
TOF-PET MLAA: scatter



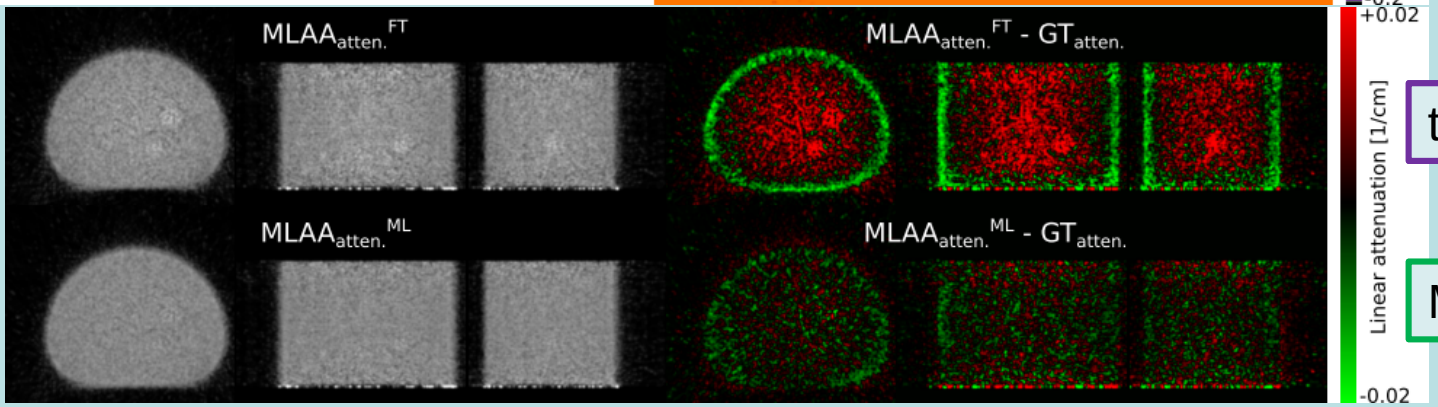
tail fitting overestimated scatter

ML fitting



tail fitting

ML fitting



tail fitting

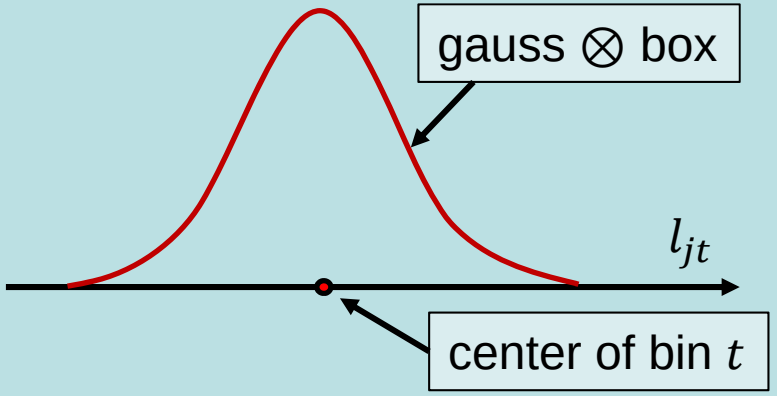
ML fitting

timing and efficiency calibration

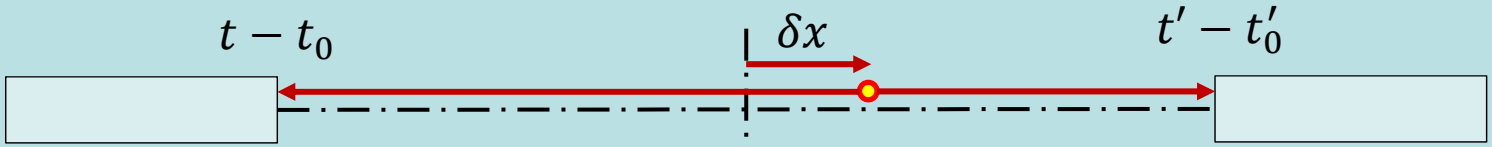
expected data given activity λ:

$$\bar{y}_{it} = a_i \, n_i \sum_j c_{ij} g_{ijt} \lambda_j + n_i s_{it}$$

$$g_{ijt} = \frac{1}{2}\left(\mathrm{erf}\left(\frac{l_{jt}+b}{\sigma\sqrt{2}}\right) - \mathrm{erf}\left(\frac{l_{jt}-b}{\sigma\sqrt{2}}\right)\right)$$



σ: TOF-resolution
b: bin width
l_jt: distance voxel j to center of bin t



$$2/c \cdot \delta x = t - t' = (t - t_0) - (t' - t_0'), \text{ if } t_0 = t_0'$$

g_{ijt} depends on **timing offsets** (l_{jt}) and on **TOF resolution** σ

n_i represents LOR sensitivity

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timing and efficiency calibration

we estimate

- LOR timing offsets
- LOR TOF resolution values
- LOR efficiency corrections to vendor supplied normalization

assuming LOR parameters are determined by its two crystals:

- LOR timing offset: $\delta t_{LOR} = \delta t_{crystal1} - \delta t_{crystal2}$
- LOR TOF resolution: $\sigma_{LOR} = \sqrt{\sigma_{crystal1}^2 + \sigma_{crystal2}^2}$
- LOR efficiency: $\epsilon_{LOR} = \epsilon_{crystal1}\,\epsilon_{crystal2}$

crystal parameters determined during ML-reconstruction of activity using CT-based attenuation correction

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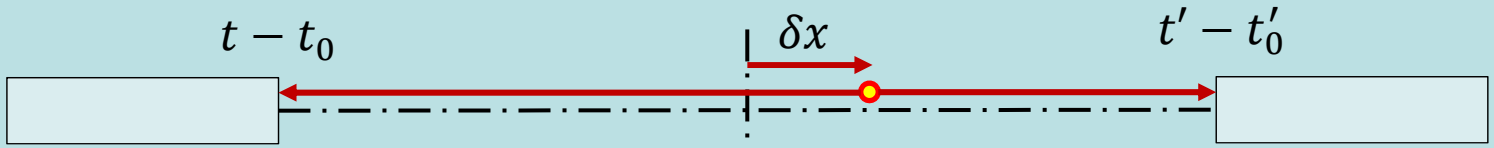
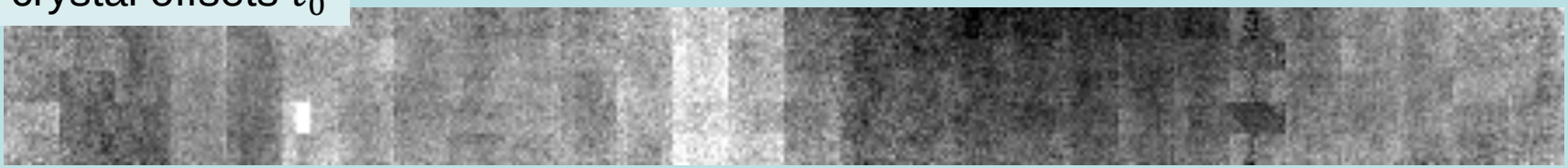
TOF-PET MLAA: TOF offsets



$$2/c \cdot \delta x = t - t' = (t - t_0) - (t' - t_0'), \text{ if } t_0 = t_0'$$

crystal offsets t_0



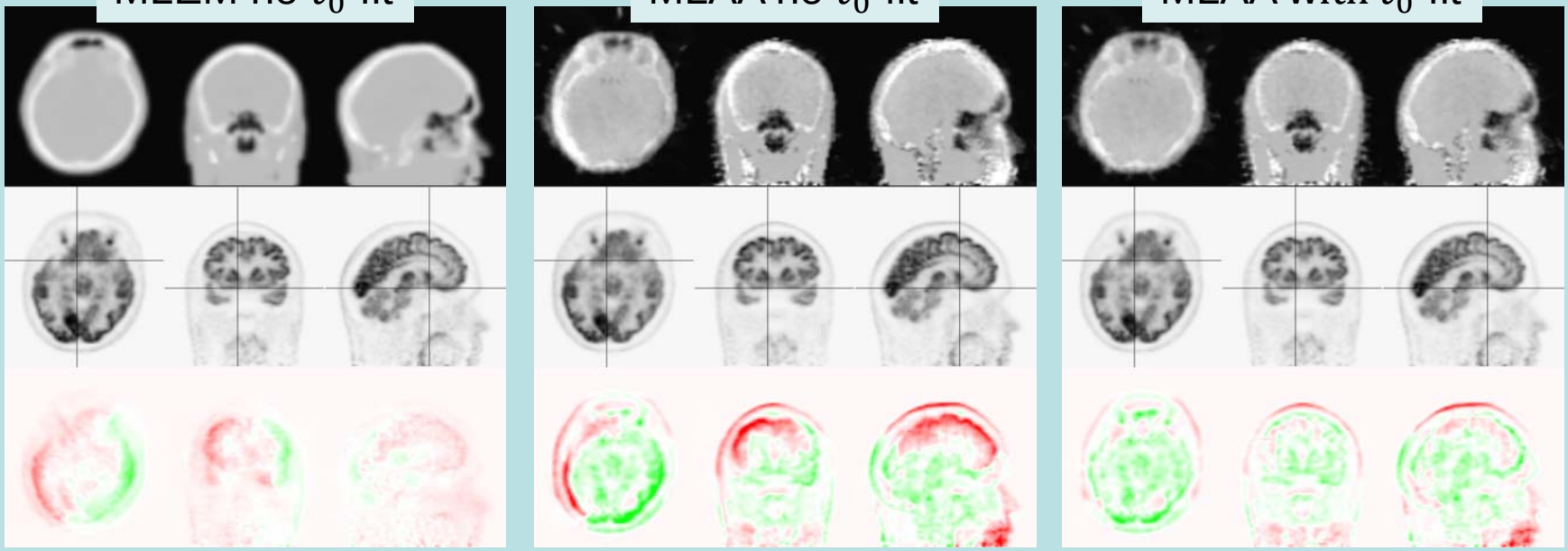
50 ps

-50 ps

MLEM no t_0 fit

MLAA no t_0 fit

MLAA with t_0 fit



+10%

-10%

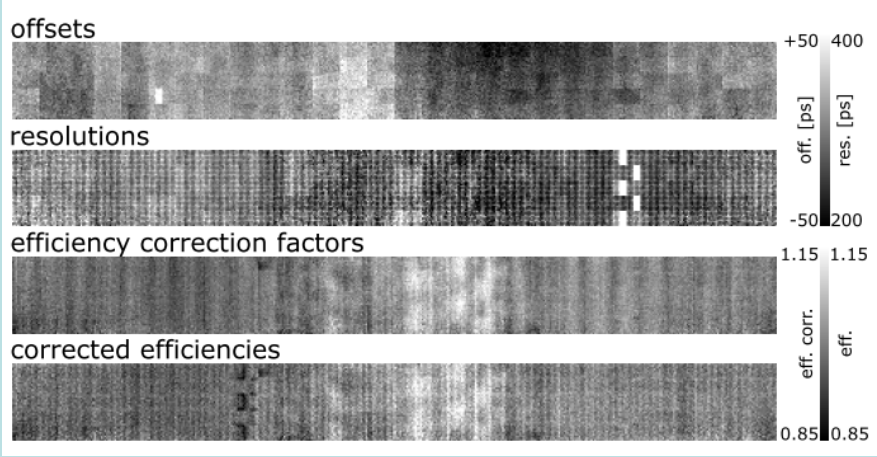
MLEM_no_t_0 – MLEM_t_0

MLAA_no_t_0 – MLEM_no_t_0

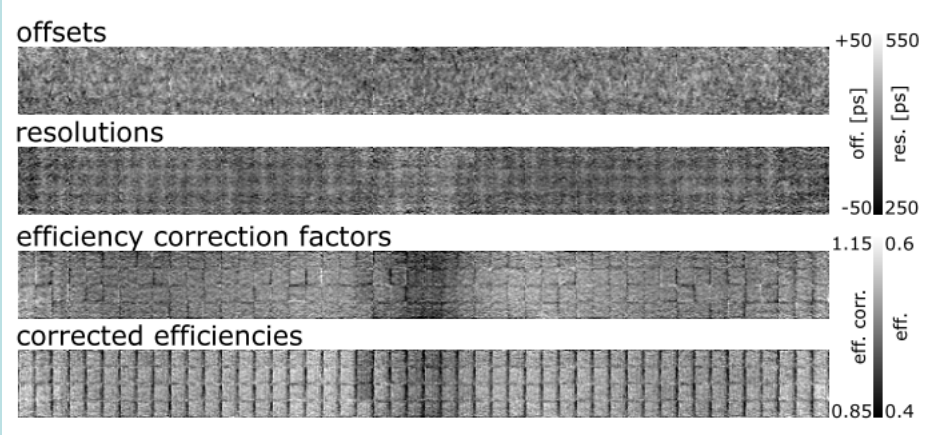
MLAA_t_0 – MLEM_t_0

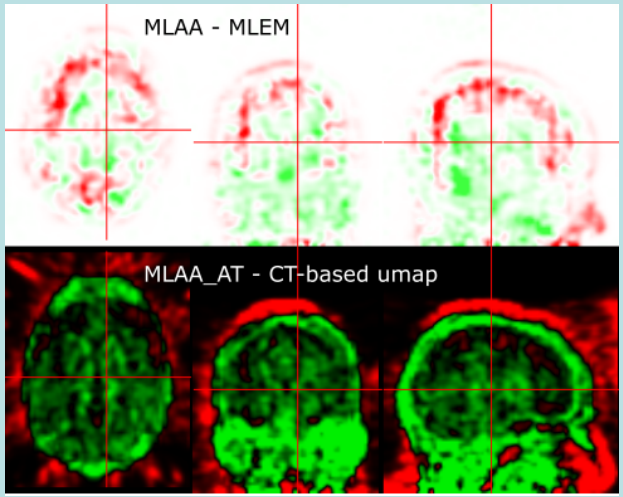
timing and efficiency calibration

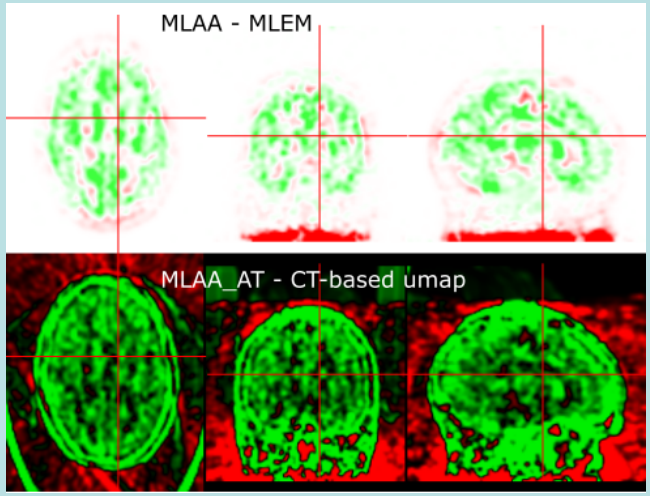
GE Signa (±385 ps)



Siemens mCT (±535 ps)







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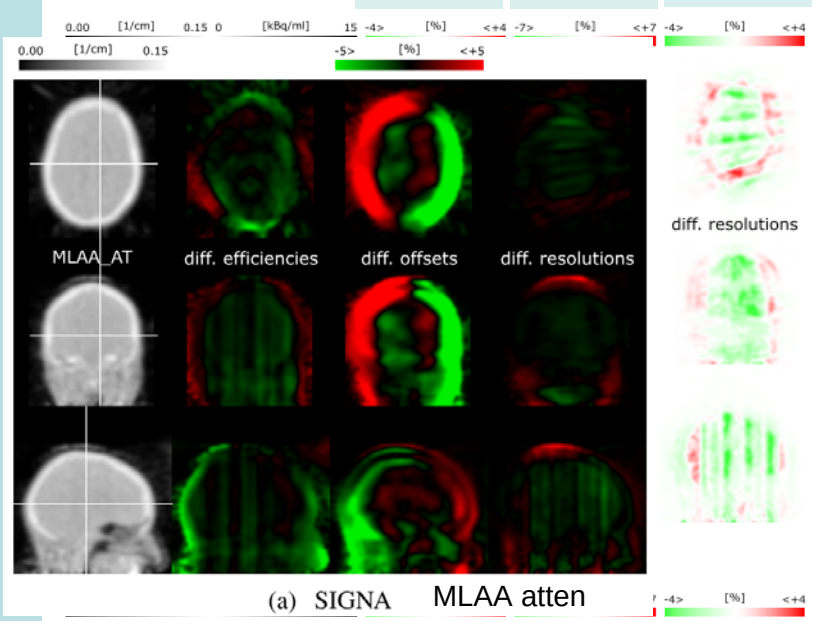
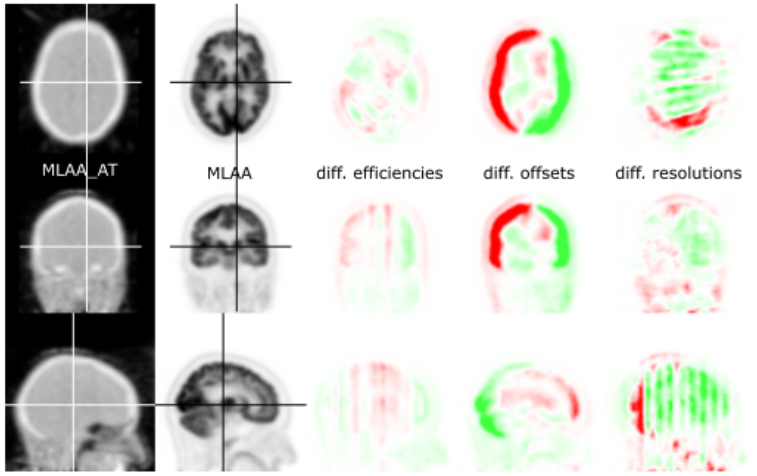
timing and efficiency calibration

GE Signa

ϵ	δt	σ
-4..4%	-7..7%	-4..4%



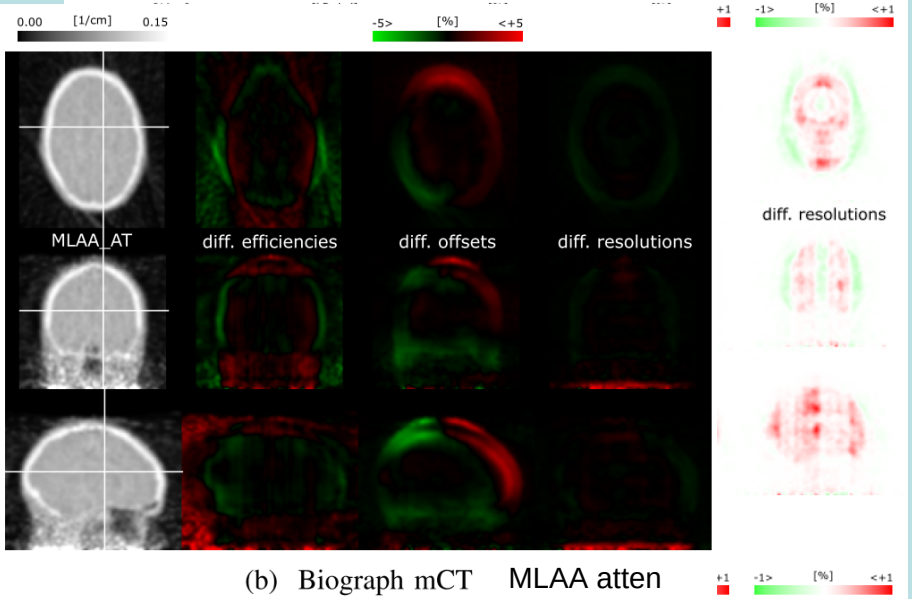
(a) SIGNA MLAA atten



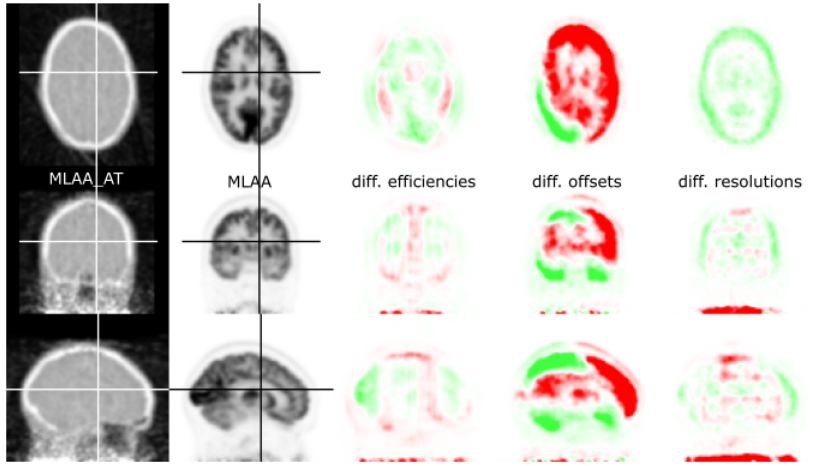
(b) MLAA

Siemens mCT

ϵ	δt	σ
-2..2%	-1..1%	-1..1%



(b) Biograph mCT MLAA atten



(b) MLAA

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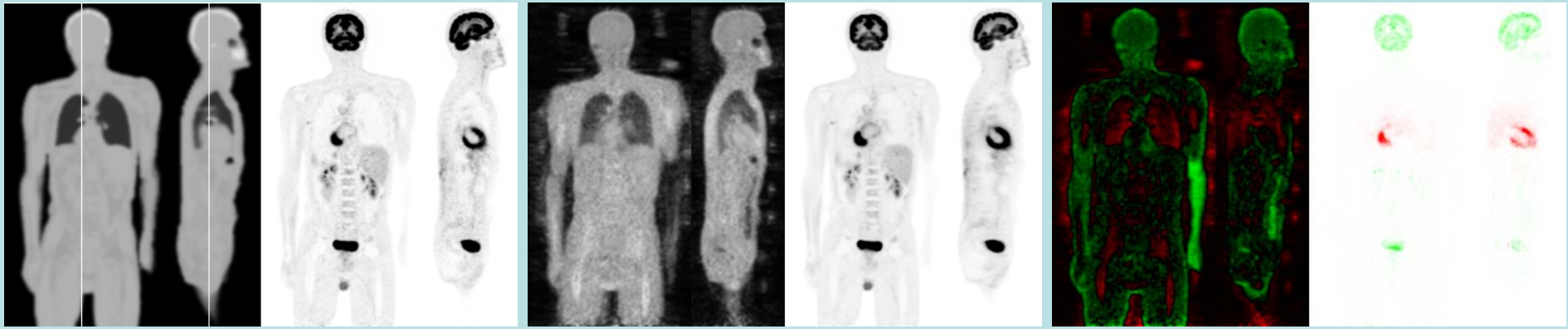
TOF-PET: MLAA

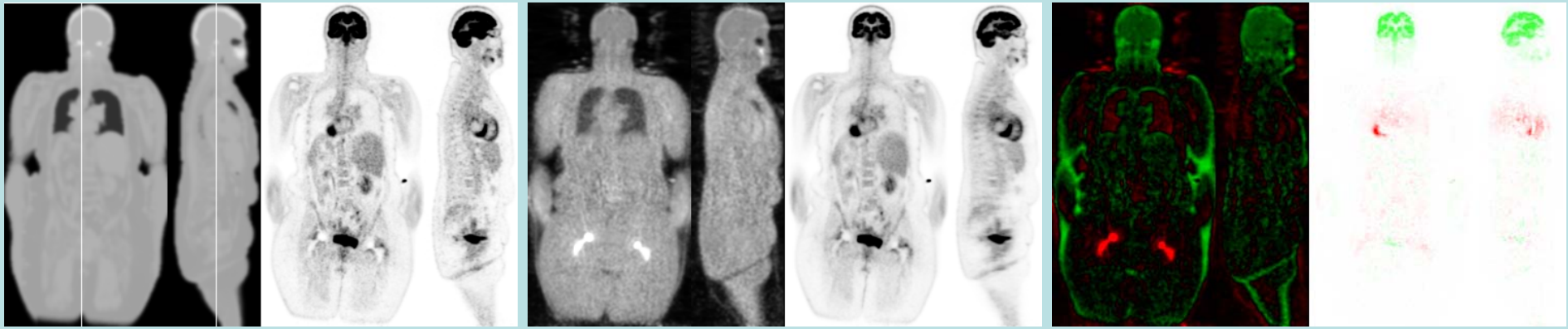
MLEM (CT-based atten) | MLAA | MLAA - MLEM

patient 1



before …

patient 9



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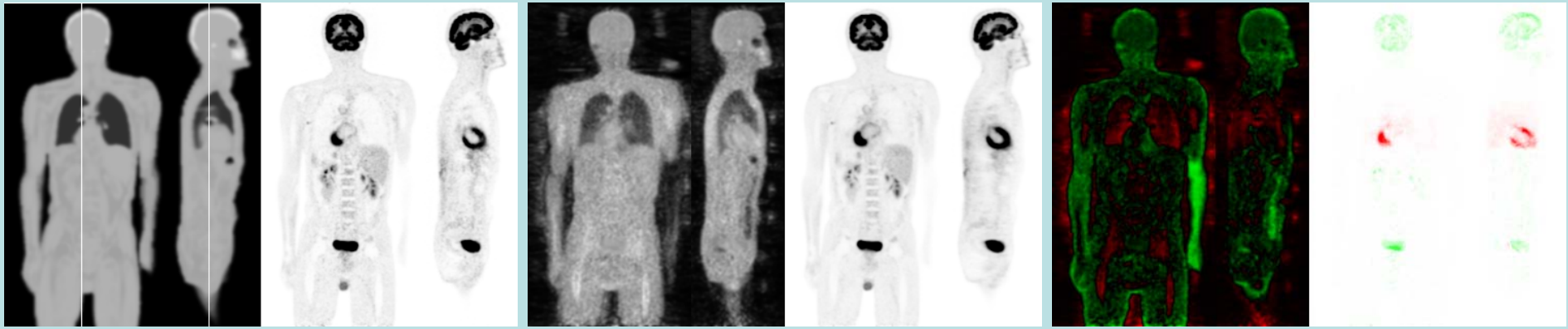
TOF-PET: MLAA

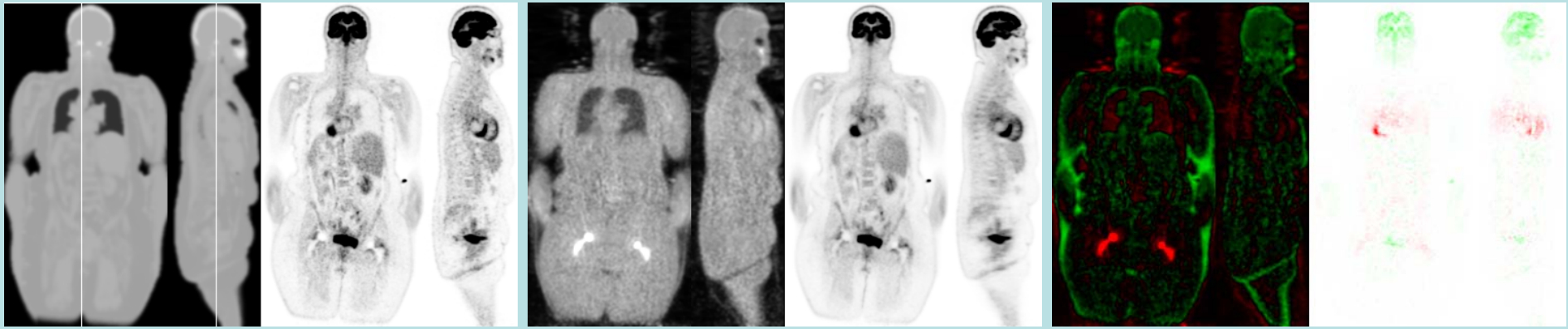
MLEM (CT-based atten) | MLAA | MLAA - MLEM

patient 1



… after

patient 9



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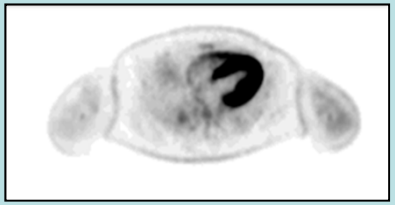
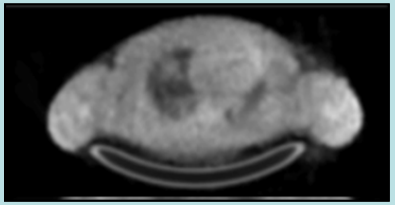
attenuation in TOF-PET

- TOF-PET data

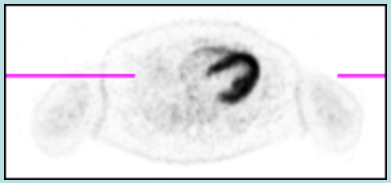
- MLAA

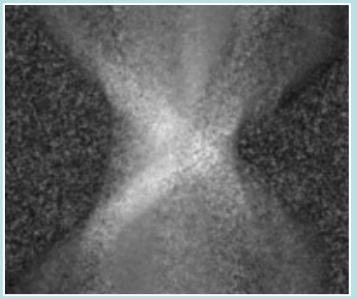




Maximum Likelihood reconstruction of Activity and Attenuation

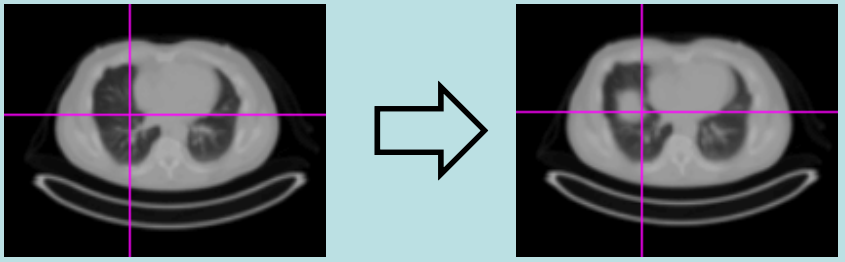
- MLACF





Maximum Likelihood reconstruction of Activity and attenuation Correction Factors

- MLRR

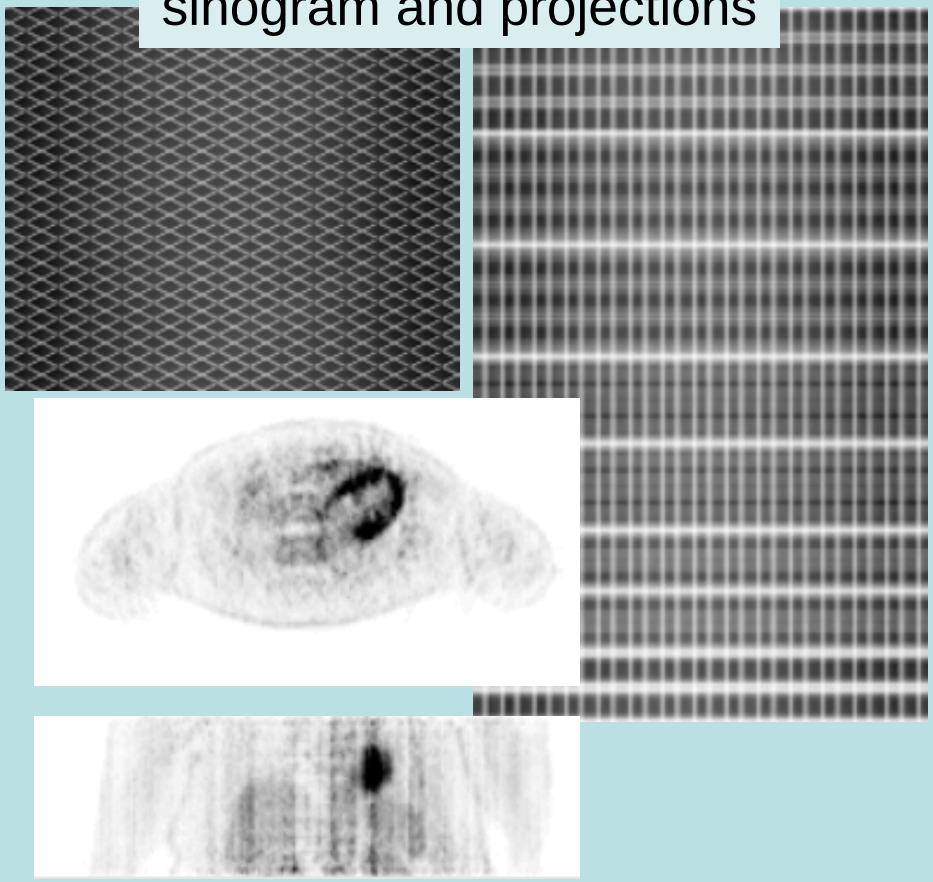


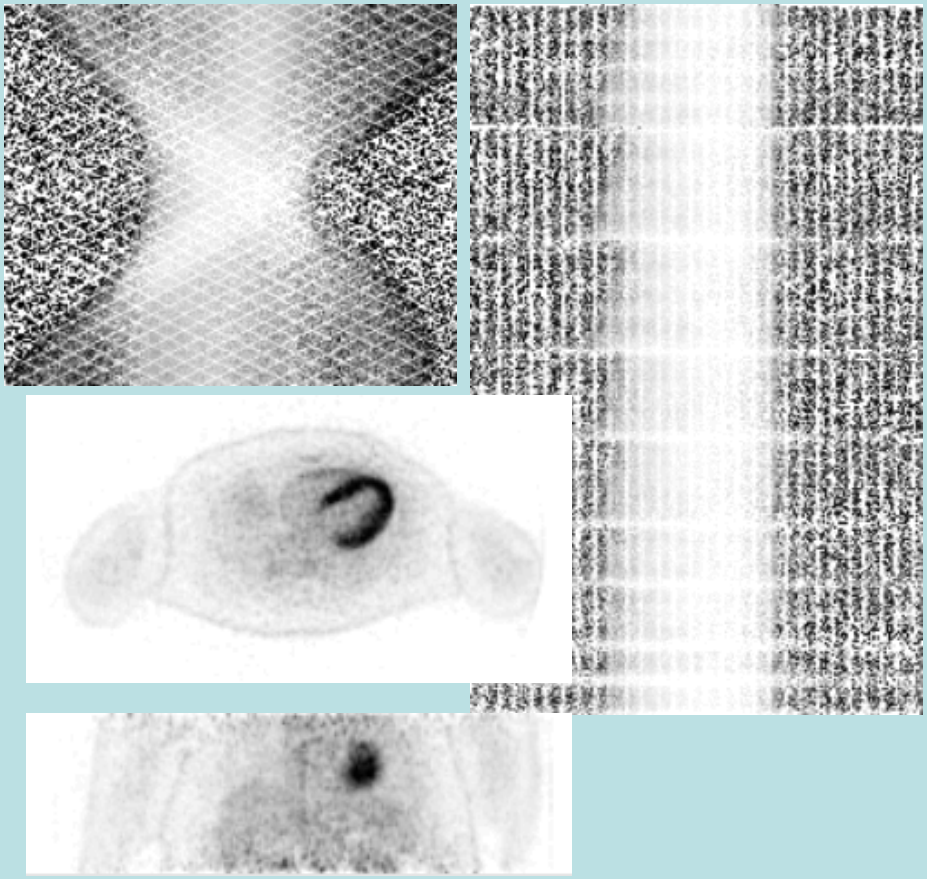
Maximum Likelihood Reconstruction of activity and Registration of attenuation image

MLACF: estimation of λ and a

sensitivity
sinogram and projections

MLACF





MLEM with attenuation corr
without sensitivity corr

MLACF estimates product of both

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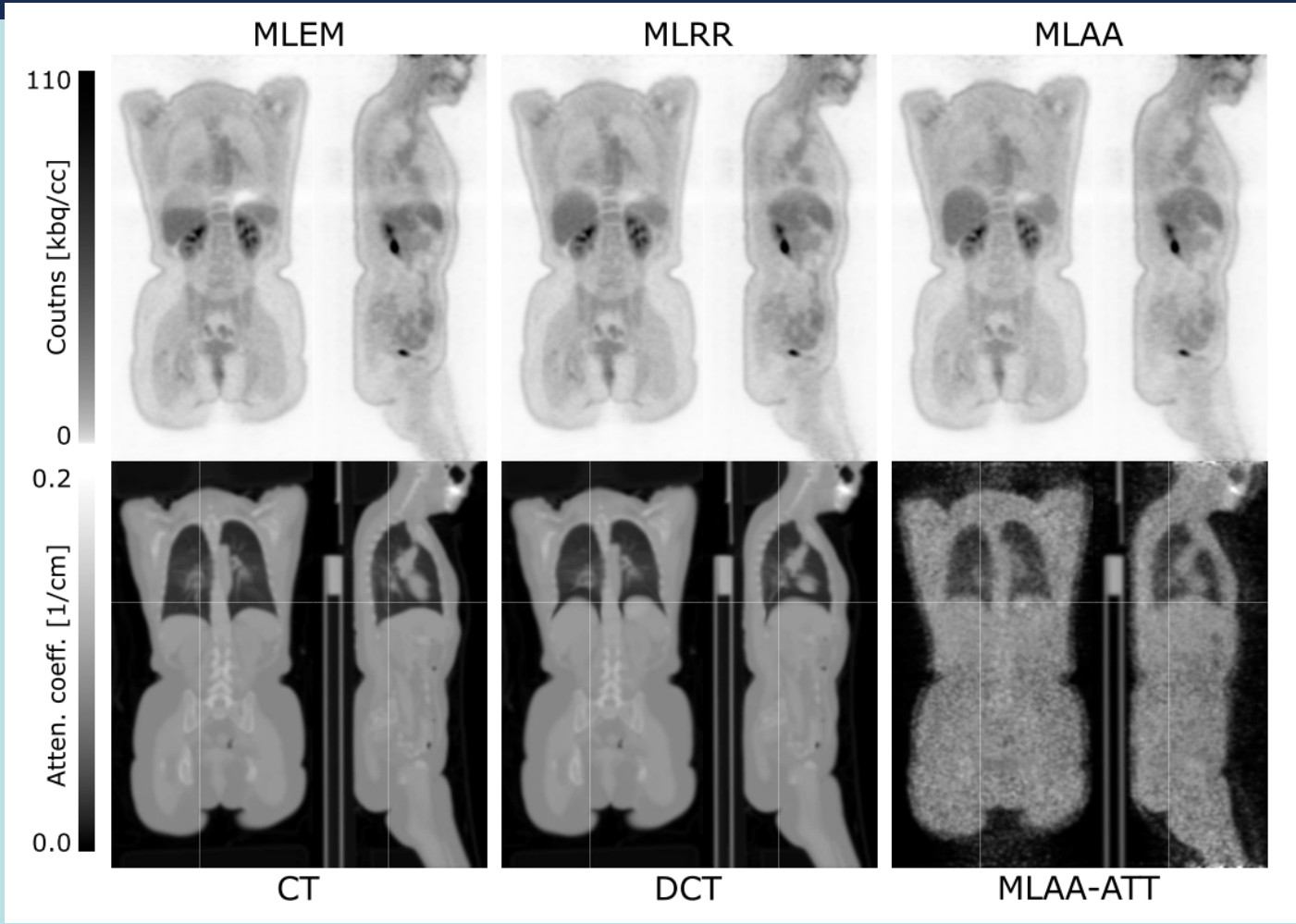
MLRR: estimate λ and CT deformation



from Rezaei et al., to be submitted to *J Nucl Med*

see also: Rezaei et al., *Phys Med Biol* 2016; 61 (4): 1852 – 1874.
Bousse et al., *Phys Med Biol* 2016 (3) L11

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10 ps TOF-PET

10 ps TOF

- reconstruction
  - faster
  - more robust
- (much) better resolution
- limited angle problem eliminated
- (much) better SNR
- better joint estimation of attenuation & activity

- remaining problems
  - scatter
  - positron range
- system calibration essential

thanks!