Coherent scattering and macroscopic coherence:

Implications for neutrino, DM and axion detection

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Coherent elastic neutrino-nucleus scattering

- Neutrino detection through coherent scattering
  - First observation – a hand-held neutrino detector!
  - Can we do even better (macroscopic coherence)?

(Hint: the answer is mostly negative, though one interesting possibility still exists)

Based on arXiv:1806:10962, in collaboration with Giorgio Arcadi, Manfred Lindner and Stefan Vogl

Coherent elastic neutrino-nucleus scattering

NC – mediated neutrino-nucleus scattering:

$$\nu + A \to \nu + A$$

Incoherent scattering – Probabilities of scattering on individual nucleons add:

$\diamondsuit$ $\sigma \propto (\#$ of scatterers$)$

Coherent scattering on nucleus as a whole – Amplitudes of scattering on individual nucleons add

$\diamondsuit$ $\sigma \propto (\#$ of scatterers$)^2$

Significant increase of the cross sections (but requires small momentum transfer, $q \lesssim R^{-1}$)

(D.Z. Freedman, 1974)

COHERENT experiment

Neutrino energies: $E_\nu \sim 16 - 53\ \mathrm{MeV}$. Nuclear recoil energy: keV - scale.

# of events expected (SM): 173 $\pm$ 48

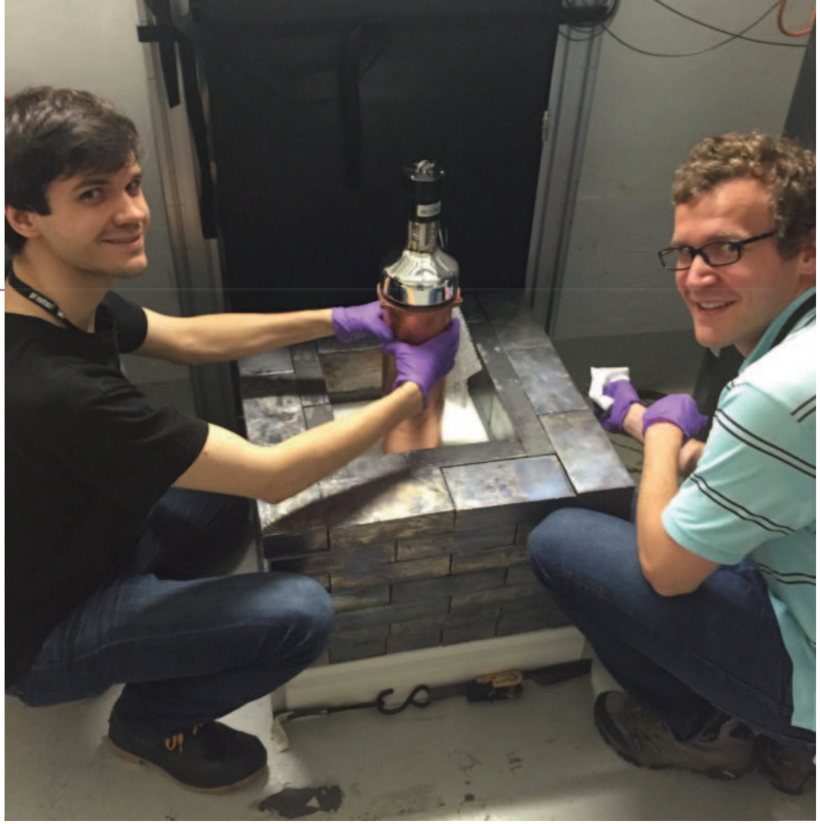
# of events detected: 134 $\pm$ 22

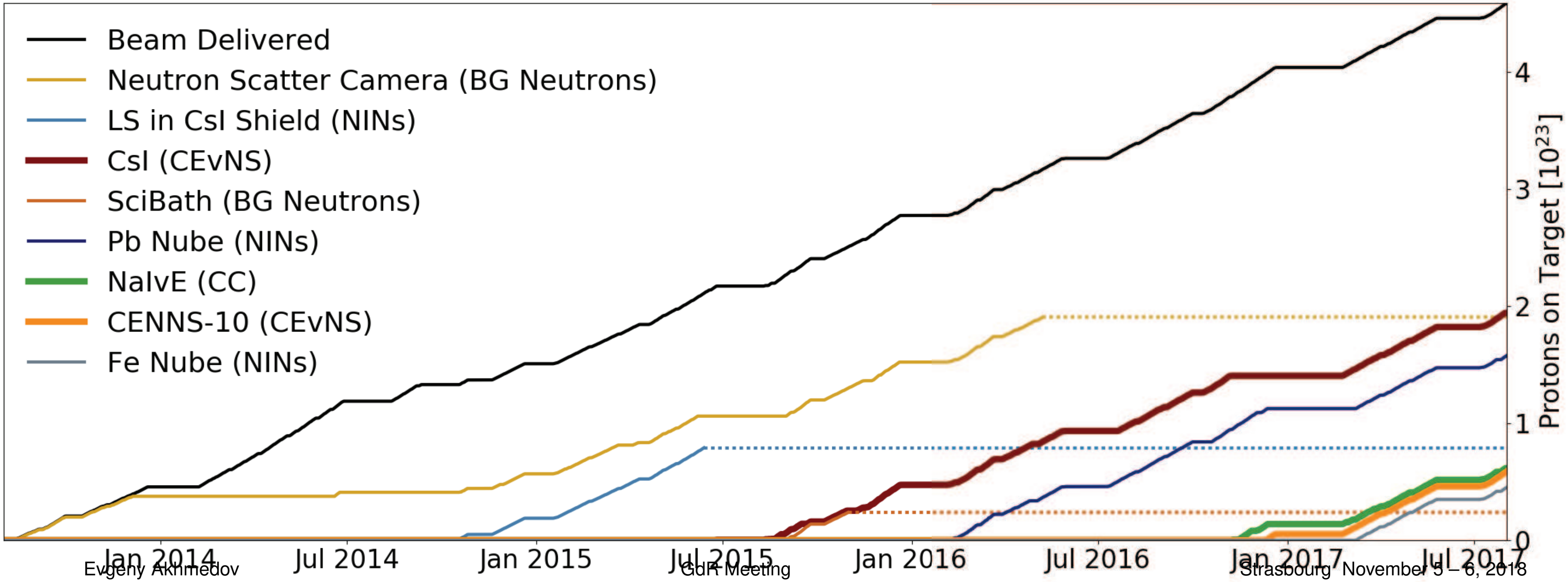
“We report a 6.7 sigma significance for an excess of events, that agrees with the standard model prediction to within 1 sigma”
$\sim 2 \times 10^{23}$ POT; $\sigma \sim 10^{-38}$ cm^2.

D. Akimov et al., Science 10.1126/science.aao0990 (2017).

A hand-held neutrino detector

- 14.6 kg low-background CsI[Na] detector deployed to a basement location of the SNS in the summer of 2015
- ~ 2×10^{23} POT delivered and recorded since CsI began taking data





NC-induced neutrino-nucleus scattering: flavour blind.

$$\diamondsuit \quad \left[\frac{d\sigma_{\nu A}}{d\Omega}\right]_{\rm coh} \simeq \frac{G_F^2}{16\pi^2}[Z(4\sin^2\theta_W - 1) + N]^2 E_\nu^2\,(1+\cos\theta)|F(\vec{q}^{\,2})|^2$$

$F(\vec{q}^{\,2})$ is nuclear formfactor:

$$F_{N(Z)}(\vec{q}^{\,2}) = \frac{1}{N(Z)}\int d^3x \rho_{N(Z)}(\vec{x}) e^{i\vec{q}\vec{x}}, \qquad \vec{q} = \vec{k} - \vec{k}'.$$

For $q \ll R^{-1} \quad \Rightarrow \quad F(\vec{q}^{\,2}) = 1, \qquad \left[d\sigma_{\nu A}/d\Omega\right]_{\rm coh} \propto N^2$.

$$R \simeq 1.2\ {\rm fm}\, A^{1/3}; \quad A \sim 130 \quad \Rightarrow \quad R^{-1} \sim 30\ {\rm MeV}.$$

Recoil energy of the nucleus:

$$E_{rec} \simeq \frac{\vec{q}^{\,2}}{2M_A}, \qquad E_{rec}^{max} = \frac{2E_\nu^2}{M_A + 2E_\nu} \simeq \frac{2E_\nu^2}{M_A}.$$

Can one have coherence on larger scales?

$\diamondsuit$ Coherent neutrino scattering on atoms:

- Advantages – larger number of particles (larger σ)
- CC scattering on electrons contributes – sensitivity to neutrino oscillations!
- Disadvantage: smaller q required $\Rightarrow$ much smaller recoil energies.

For $A \sim 100$:

$$|\vec{q}| \lesssim (\text{a few } a_B)^{-1} \sim 1\,\text{keV} \quad \Rightarrow$$

$$E_{rec} \simeq \frac{\vec{q}^{\,2}}{2m_A} \sim 10^{-5}\,\text{eV}$$

$\sim$ 8 orders of magnitude below currently achieved semsitivity.

$\diamondsuit$ Can one have (at least in principle) macroscopic coherence?

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Simple estimate: consider a target of linear size $\sim$ 1 cm and mass ~ 1 g. For coherent scattering one needs $|\vec{q}| \lesssim q_0 \sim (1\ \mathrm{cm})^{-1} \sim 10^{-5}\ \mathrm{eV}$. Gain: large number of particles in the coherent volume $N \propto 1/q_0^3$.

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For $E_\nu \gg q_0 \sim 10^{-5}\ \mathrm{eV}$ small q $\Rightarrow$ nearly forward ν scattering:

$$\vec{q}^{\,2} = 2E_\nu^2(1 - \cos\theta)$$

$\Rightarrow$ by limiting $\vec{q}^{\,2} < q_0^2$ we constrain the solid angle;

$$\sigma_0 \simeq \frac{G_F^2}{\pi} E_\nu^2 \quad \longrightarrow \quad \frac{G_F^2}{2\pi^2} q_0^2 .$$

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Net enhancement factor $\propto 1/q_0 \propto N^{1/3} \quad \Rightarrow$

$$\sigma_{tot} \propto N^{4/3}\,, \quad \mathrm{not} \quad N^2.$$

Still for $N \sim N_A \simeq 6\times 10^{23}$ a significant enhancement!

Elastic ν scattering on macroscopic bodies

The problem: detection.

Momentum transfers $|\vec{q}| \lesssim q_0 \sim 10^{-5}$ eV to achieve a (1 cm)^3 - scale coherence would mean, for a 1 g target,

$$E_{rec} \simeq \frac{q_0^2}{2M_{tot}} \sim 10^{-43}\,\mathrm{eV}\,!$$

Leaving aside other problems, measuring such small E_{rec} would require energy resolution δE at least of the same order.
But: By time-energy uncertainty relation this would require the measurement time

$$\delta t \sim (\delta E)^{-1} \sim 10^{27}\,\mathrm{sec}$$

– 10 orders of maginutude larger than t_U !

$\Rightarrow$ New ideas are necessary.

Ways around?

One problem: what is detected are typically scintillations and ionization caused by the recoiling target particles that are $\propto E_{rec}$.

$$E_{rec} \simeq \frac{\vec{q}^{\,2}}{2M_{tot}} \ll |\vec{q}|.$$

Can one make use of the recoil momentum $|\vec{q}|$ rather than E_{rec}?

An attempt – Experiments of J. Weber in the 1980s: torsion balance expts.; sapfire crystal. Sources: solar neutrinos; reactor neutrinos; radioactive source.

Combined 2 interesting ideas:

- Force = momentum transfer per unit time $\Rightarrow$ force impinged by neutrinos on the crystal is directly related to $\vec{q}$ rather than to E_{rec}.
- For small enough E_{rec} Mössbauer-type scattering is possible.

Elastic neutrino scattering on crystals

The idea: if the expected recoil energy of individual target atoms $E_R \simeq \frac{\vec{q}^{\,2}}{2m_A}$ is small compared to $T_{\rm Debye} \sim 10~{\rm keV}$, the recoil is given to the crystal as a whole (like in Mössbauer experiments).

Recoil-free fraction

$$f \simeq \exp\left\{-\frac{E_R}{T_D}\left(\frac{3}{2}+\frac{\pi^2 T^2}{T_D^2}\right)\right\}$$

is close to 1 for “would-be” recoil energies $E_R \ll T_D$ – easily satisfied even for $q \sim E_\nu$ as large as a few$\times$(10 MeV).

Individual atoms (or nuclei) do not experience any recoil and so are not tagged. Coherence may occur at macroscopic level!

Positive results claimed, in agreement with the proposed theoretical model. Force exerted on the crystal: $\sim 10^{-5}~{\rm dyn}$.

Weber's approach – criticism

Criticised from several viewpoints

- Ho, 1986: Approach excluded by expts. on neutron scattering on crystals
- Bertsch & Austin, 1986: Excluded by expts. on γ-ray scattering on crystals
- Franson & Jacobs, 1992; McHugh & Keyser, 1993: more sensitive torsion balance experiments with neutrinos – no signal observed
- Criticisms of Weber's theoretical model:
  - Casella, 1986
  - Butler, 1987
  - Smith, 1987
  - Lipkin, 1987 r
  - Trammell & Hannon, 1987
  - Aharonov, Avignone, Casher & Nussinov, 1987

$\Rightarrow$ Cross section oversestimated by $\sim$ 24 orders of magnitude

What was wrong?

Absence of recoil of the individual nuclei is necessary for macroscopic coherence, but not sufficient: It is also necessary that the neutrino waves scattered from different nuclei be in phase with each other.

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[N.B.: If one writes the density of scatterers as $\rho(\vec{x}) = \sum_i \delta^3(\vec{x} - \vec{x}_i)$, factor F takes the familiar form $F(\vec{q}) = \int d^3x \rho(\vec{x}) e^{i\vec{q}\vec{x}}$].

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Now,

$$|F(\vec{q})|^2 \,=\, \sum_{i,j} e^{i\vec{q}(\vec{r}_i-\vec{r}_j)}.$$

In general, for $q\max\{|\vec{r}_i-\vec{r}_j|\} \simeq qL \ll 1$ one has $|F(\vec{q})|^2 \,\simeq\, \sum_{i,j} 1 = N^2$; in the opposite case $qL \gg 1$ only diagonal terms in the sum contribute, $|F(\vec{q})|^2 \,=\, N$.

What was wrong – contd.

For Weber's expts. the condition $|\vec{q}| < L^{-1} \sim 10^{-5}$ eV was violated (only much weaker cond. $|\vec{q}| < (2m_A T_D)^{1/2} \sim 50\ \mathrm{MeV}$ was met).

Crystals are a special case. $|\vec{q}|$ need not be very small! For

$$\vec{q}(\vec{r}_i - \vec{r}_j) = 2\pi n$$

– constructive interference, $d\sigma \propto N^2$. $\Leftrightarrow$ Bragg condition:

$$2d \sin\theta = n\lambda$$

(d is interplanar distance, $\lambda = 2\pi/k$).

But: Bragg maxima lead to $d\sigma \propto N^2$ only in very narrow cones with $\Delta\Omega \propto N^{-2/3}$ and for energy intervals $\Delta E \propto N^{-1/3}$. When integrated over Ω and E_ν lead to the usual $\sigma \propto N$ dependence.

Need a different idea.

A possibility:

Radiative neutrino scattering

$$\nu + A \to \nu + A + \gamma$$

Photon energy ω_γ can be as large as the neutrino momentum transfer (not E_{rec} of the target particle, which can even be zero)! No need to detect tiny recoils.

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An example: radiative νN scattering ($\nu + N \to \nu + N + \gamma$). Discussed in particular in connection with low-energy MiniBooNE events (and much earlier also in connection with some unexplained events in Gargamelle data) – but not as macroscopically coherent process.

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Another possibility – bremsstrahlung on free electrons, $\nu + e \to \nu + e + \gamma$. First considered by Lee and Sirlin (1964) and then by many other people. In all but two papers – also not in connection with macroscopic coherence.

An exception: implications for detection of relic neutrinos.

Astrophysics Preprint Series

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A Detector for the Cosmic Neutrino Background

Abraham Loeb and Glenn D. Starkman
Institute for Advanced Study, Princeton, NJ 08540

Submitted to *Physical Review Letters*

PHYSICAL REVIEW D VOLUME 43, NUMBER 3 1 FEBRUARY 1991

Can cosmic neutrinos be detected by bremsstrahlung from a metal?

Safi Bahcall
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(Received 23 July 1990)

We examine the proposal of Loeb and Starkman to detect cosmic-background neutrinos by coherent bremsstrahlung from electrons in a neutral metal. We show that the positive ions in the metal exert a restoring force which suppresses the radiation by $\sim 10^{-20}$.

I. INTRODUCTION

Loeb and Starkman have recently proposed a method for detecting cosmic-background neutrinos.[1] The method relies on the coherent interaction of neutrinos with bulk matter to get an enhanced cross section for neutrinos bouncing off a slab of metal. The neutrinos accelerate conduction electrons which then radiate low-

many particles acting as a lump. In the soft-photon limit, the probability distribution for radiation is obtained by multiplying the classical radiation distribution (2.1) by the probability that the neutrino will scatter.

The probability for a neutrino to scatter off an extended object, a slab in this case, is found by summing the scattering amplitudes off individual charges times the appropriate phase shift for each scatterer:

Neutrino scattering pushes free electrons inside the metal target. For electron displacement Δz: surface charge density of positive ions $\sigma_c \simeq \Delta z n_e e \quad \Rightarrow$ restoring force per unit mass $\omega_p^2 \Delta z$ ($\omega_p = [4\pi n_e e^2/m_e]^{1/2} \sim 10\ \mathrm{eV}$ is plasma frequency). Coherent scattering gets strongly suppressed:

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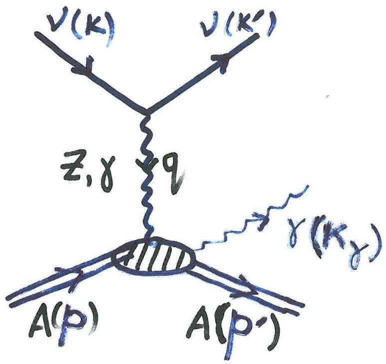
A well-known example: Rayleigh scattering (photon scatt. on bound atomic electrons) *vs* Thomson scattering (scatt. on free electrons).

$$\sigma_R(\omega) \sim \left[\frac{\omega^2}{(\omega^2 - \omega_0^2)}\right]^2 \sigma_T$$

For $\omega \gg \omega_0 \quad \Rightarrow \quad \sigma_R = \sigma_T$; for $\omega \ll \omega_0 \quad \Rightarrow \quad \sigma_R$ suppressed as $(\omega/\omega_0)^4$.

Radiative ν-atom scatt. with $\omega \gtrsim \omega_{\rm char}$



$\nu + A \to \nu + A + \gamma$

Energy-momentum conservation:

$$p + k = p' + k' + k_\gamma$$

The structure factor:

$$F(\vec{k} - \vec{k}') = \sum_i e^{i(\vec{k} - \vec{k}')\vec{r}_i} \longrightarrow \sum_i e^{i(\vec{k} - \vec{k}' - \vec{k}_\gamma)\vec{r}_i}$$

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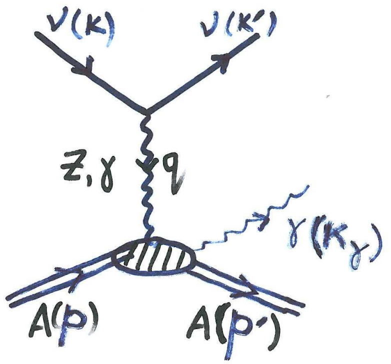

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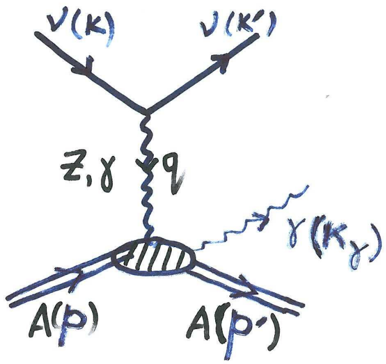

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From momentum conservation: recoil momentum

$$\vec{p}\,' = (\vec{k} - \vec{k}\,') - \vec{k}_\gamma$$

$\Rightarrow$ very small $\vec{k} - \vec{k}\,' - \vec{k}_\gamma$ also means very small $|\vec{p}\,'|$ – exactly what is needed for the process to be coherent!

Advantages:

- The energy of detected photons ω_γ can in principle be as large as momentum transfer to electrons from neutrinos $|\vec{k} - \vec{k}\,'|$.
- Neither $|\vec{k} - \vec{k}\,'|$ nor ω_γ need be small to ensure macroscopic coherence – only their difference needs. For $\omega_\gamma \sim \omega \gg \omega_{at}$ no ω^4 suppression.

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The price to pay:

- Phase-space volume gets severely constrained: $\vec{k}_\gamma$ nearly equals $\vec{k} - \vec{k}\,'$,

$$|\vec{p}\,'| = |\vec{k} - \vec{k}' - \vec{k}_\gamma| < p_0 \lesssim L^{-1}.$$

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Can the increase due to macroscopic coherence compensate for the suppression of the elementary cross section σ_0?

Radiative ν scattering on free electrons

The process:

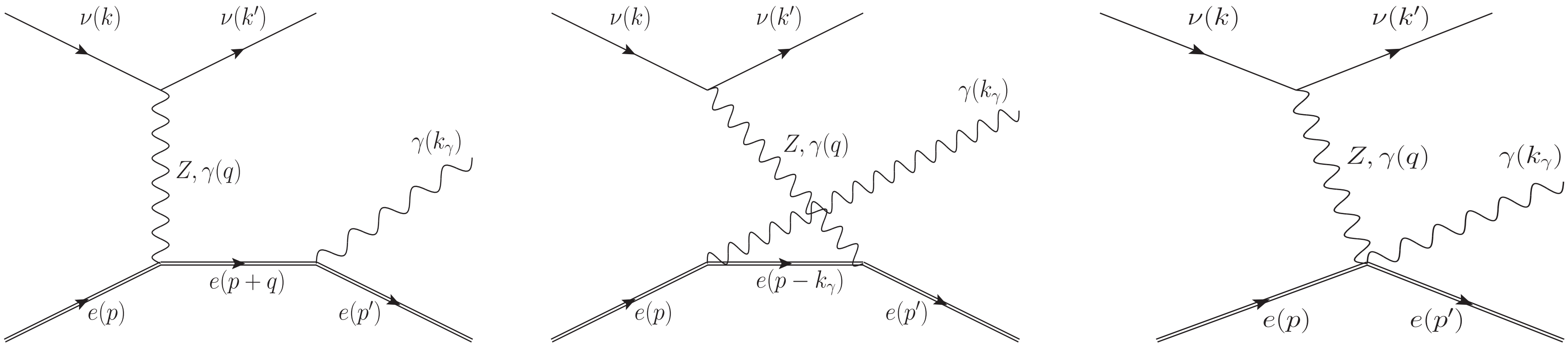
$$\nu(k) + e(p) \to \nu(k') + e(p') + \gamma(k_\gamma)$$

$$k = (\omega, \vec{k})\,, \quad p = (m, \vec{0})\,, \quad k' = (\omega', \vec{k}')\,, \quad p' = (E_{p'}, \vec{p}')\,, \quad k_\gamma = (\omega_\gamma, \vec{k}_\gamma)$$

The amplitude for radiative scattering on a “spinless electron”:

$$\mathcal{M}_w = -i\frac{G_F}{\sqrt{2}} g_V e \epsilon^*_\mu(k_\gamma) Q^{\mu\alpha} j_\alpha \,.$$

$$j_\alpha = \bar{u}(k')\gamma_\alpha(1-\gamma_5)u(k)$$



$$Q^{\mu\alpha} = \left\{ \frac{(2p' + k_\gamma)^\mu (2p + k - k')^\alpha}{2p' \cdot k_\gamma} - \frac{(2p - k_\gamma)^\mu [2p' - (k - k')]^\alpha}{2p \cdot k_\gamma} - 2g^{\mu\alpha} \right\}$$

$Q^{\mu\alpha}$ satisfies the gauge invariance conditions

$$k_{\gamma\mu} Q^{\mu\alpha} = Q^{\mu\alpha}(k - k')_\alpha = 0$$

$$g_V = \begin{cases} 2\sin^2\theta_W + \frac{1}{2}\,, & \nu = \nu_e \qquad (\mathrm{NC}+\mathrm{CC}) \\ 2\sin^2\theta_W - \frac{1}{2}\,, & \nu = \nu_\mu,\, \nu_\tau \quad (\mathrm{NC}) \end{cases}$$

Rad. ν scatt. mediated by weak CC and NC

I. Without constraining $|\vec{p}\,'|$:

$$\frac{d\sigma_w}{d\omega_\gamma} = \frac{G_F^2 g_V^2 e^2}{(2\pi)^3}\frac{1}{m_e^2}\cdot\frac{8}{9}\frac{(\omega-\omega_\gamma)^2}{\omega_\gamma}\{(\omega-\omega_\gamma)^2+\omega^2+2\omega_\gamma^2\}\,.$$

Integrating over ω_γ:

$$\sigma_w(\omega_\gamma>\omega_0) \simeq \frac{G_F^2 g_V^2 e^2}{(2\pi)^3}\frac{1}{m_e^2}\cdot\frac{16}{9}\omega^4\Big[\ln(\omega/\omega_0)-\frac{41}{24}\Big]\,.$$

$\omega_0 \equiv \omega_{\gamma\mathrm{min}}$ is an IR cutoff. Natural choice – from conditions $\omega_\gamma > \omega_{\mathrm{at}}$ (for scattering on electrons in atoms) or $> \omega_p$ (for scattering on free electrons in conductors) to avoid $\sim \omega_\gamma^4$ suppression.

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II. Imposing $|\vec{p}\,'|\le p_0$. Modified kinematics. In particular, for a given ω_γ

$$0\le 1-\cos\theta_\gamma\le\frac{p_0^2+2p_0(\omega-\omega_\gamma)}{2\omega\omega_\gamma}\,.$$

Constraining $|\vec{p}\,'|$ from above by a small p_0

To leading order in p_0:

$$\frac{d\sigma_w}{d\omega_\gamma} = \frac{G_F^2 g_V^2 e^2}{(2\pi)^3} \frac{p_0^4}{4m_e^2} \frac{\omega^2 + (\omega - \omega_\gamma)^2}{\omega^2 \omega_\gamma}\,.$$

Integrated cross section:

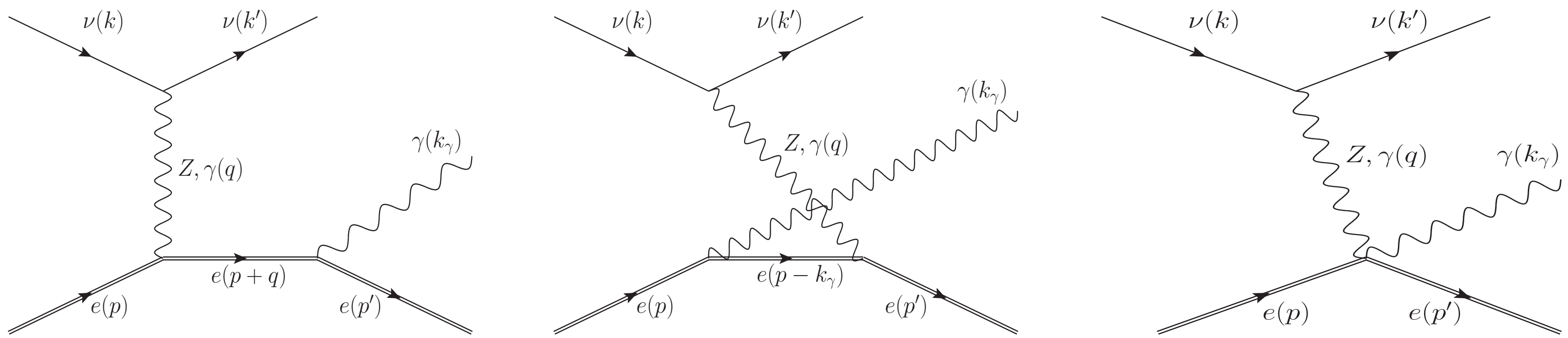
$$\sigma_w(\omega_\gamma > \omega_0) \simeq \frac{G_F^2 g_V^2 e^2}{(2\pi)^3} \frac{p_0^4}{2m_e^2} \left\{ \ln(\omega/\omega_0) - \frac{3}{4} \right\}.$$

Cross sections scale as p_0^4; a factor p_0^3 from the phase space with the electron recoil momentum constrained by $|\vec{p}\,'| \le p_0$, another p_0 from the squared modulus of the transition amplitude.

Problem: Coherent volume scales as $1/p_0^3$!

Radiative ν-e scattering and μ_ν

Process mediated by photon exchange between neutrino and electron:



Energy-momentum conservation gives

$$\vec{k}_\gamma = \vec{k} - \vec{k}\,' - \vec{p}\,' , \qquad \omega_\gamma \simeq \omega - \omega' - \frac{\vec{p}\,'^2}{2m_e}.$$

In the kinematic regime $\vec{p}\,' \approx 0$: the 4-momenta satisfy

$$k_\gamma^\mu \simeq (k - k')^\mu \equiv q^\mu .$$

Final-state photon is on mass shell, $k_\gamma^2 = 0 \quad \Rightarrow \quad q^2 \approx 0$.

Kinematic enhancement due to propagator of intermediate photon.

Radiative ν-e scattering and μ_ν

The amplitude of the process:

$$\mathcal{M}_m = -ie^2 \frac{\mu_\nu}{q^2} \epsilon^*_\mu(k_\gamma) Q^{\mu\alpha} \tilde{j}_\alpha \,, \qquad\qquad q \equiv k - k' \,.$$

$$\tilde{j}_\alpha = \bar{u}(k') \sigma_{\alpha\beta} q^\beta u(k)$$

Without constraining $|\vec{p}\,'|$:

$$\frac{d^2\sigma_m}{d\omega_\gamma d\cos\theta_\gamma} = \frac{\mu_\nu^2 e^4}{(2\pi)^3} \frac{1}{4m_e^2} \cdot \frac{(\omega-\omega_\gamma)^2}{\omega_\gamma} \left(3 - \cos^2\theta_\gamma\right) ,$$

$$\frac{d\sigma_m}{d\omega_\gamma} = \frac{\mu_\nu^2 e^4}{(2\pi)^3} \frac{1}{m_e^2} \cdot \frac{4}{3} \frac{(\omega-\omega_\gamma)^2}{\omega_\gamma} \,.$$

$$\sigma_m(\omega_\gamma > \omega_0) \simeq \frac{\mu_\nu^2 e^4}{(2\pi)^3} \frac{1}{m_e^2} \cdot \frac{4}{3} \omega^2 \left\{ \ln(\omega/\omega_0) - \frac{3}{2} \right\} .$$

For $|\vec{p}\,'| \le p_0$, to leading order in p_0

$$\frac{d\sigma_m}{d\omega_\gamma} = \frac{\mu_\nu^2 e^4}{(2\pi)^3}\frac{1}{m_e^2}\cdot\frac{1}{6}\frac{(\omega-\omega_\gamma)p_0^3}{\omega\omega_\gamma^2}\,,$$

$$\sigma_m(\omega_\gamma>\omega_0) \simeq \frac{\mu_\nu^2 e^4}{(2\pi)^3}\frac{1}{m_e^2}\cdot\frac{1}{6}\frac{p_0^3}{\omega_0} = \frac{\mu_\nu^2\alpha^2}{\pi}\frac{1}{m_e^2}\cdot\frac{1}{3}\frac{p_0^3}{\omega_0}\,.$$

Kinematic enhancement is relatively mild: $d\sigma_m/d\omega_\gamma$ and σ_m scale as p_0^3 rather than p_0^4.

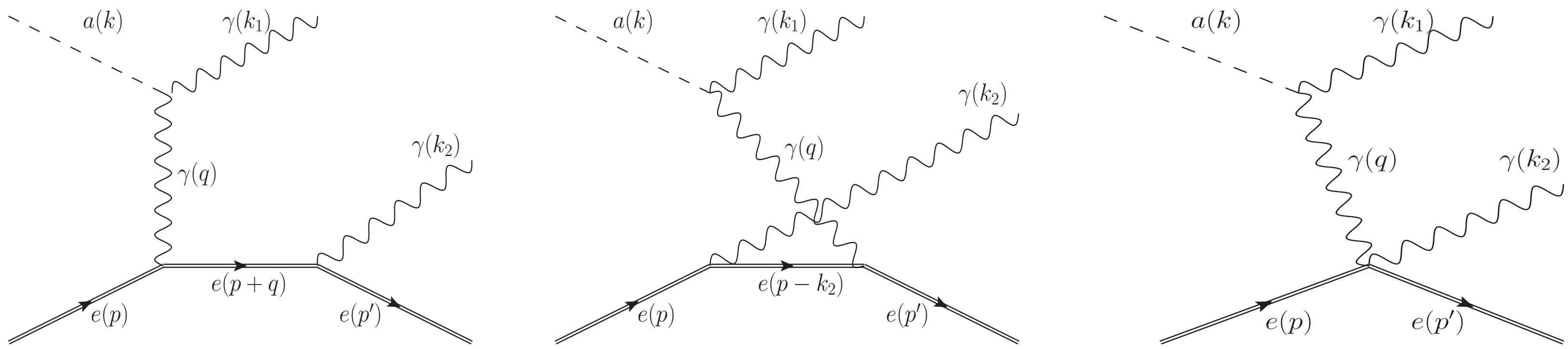
In general, neutrinos may have both the magnetic and electric dipole moments (matrices in flavor space): replace in the amplitude $\mu_\nu \to \tilde{\mu}_{\alpha\beta} \equiv (\mu_\nu + i\epsilon_\nu)_{\alpha\beta}$
In the expressions for the cross sections – replace

$$\mu_\nu^2 \to \sum_\beta |\tilde{\mu}_{\alpha\beta}|^2.$$

Coherent detection of relativistic axions

Radiative inverse Primakoff effect:

$$a(k) + e(p) \to e(p') + \gamma(k_1) + \gamma(k_2)\,.$$



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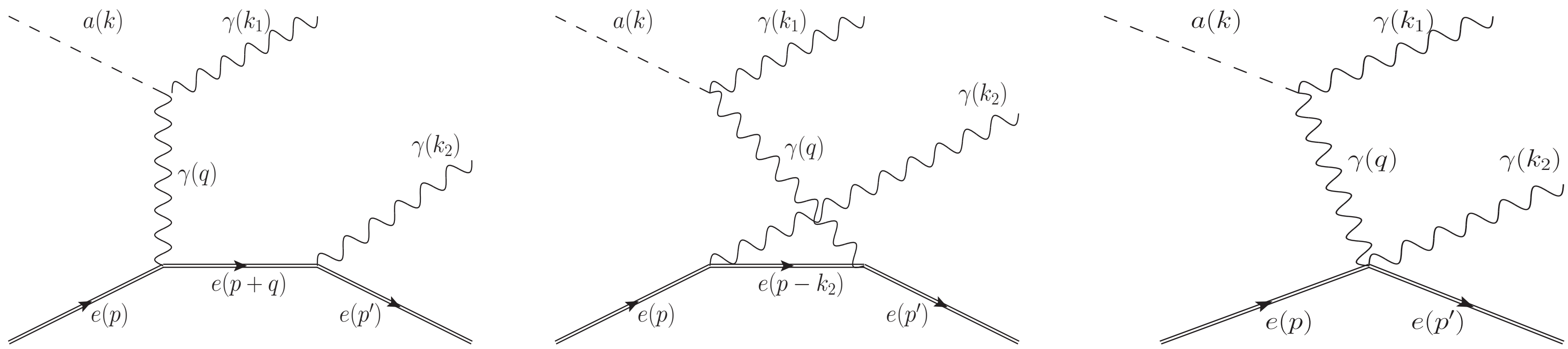

The $a\gamma\gamma$ interaction Lagrangian:

$$\mathcal{L} = \frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$\mathcal{M}_a = -ie^2 g_{a\gamma\gamma} \left\{ \frac{1}{q_1^2} \epsilon_\mu^*(k_1) Q^{\mu\alpha}(p,k;\,p',k_1,k_2)\hat{j}_{2,\alpha} + \frac{1}{q_2^2} \epsilon_\mu^*(k_2) Q^{\mu\alpha}(p,k;\,p',k_2,k_1)\hat{j}_{1,\alpha} \right\}$$

$$\hat{j}_{i,\alpha} = \varepsilon_{\rho\beta\sigma\alpha} k_i^\rho \epsilon^{*\beta}(k_i) q^\sigma\,, \qquad q_i \equiv k - k_i,$$

Coherent axion detection – contd.

I. Without constraining $|\vec{p}\,'|$:

$$\sigma_a \simeq \frac{g_{a\gamma\gamma}^2 e^4}{(2\pi)^3}\frac{\omega^2}{6m_e^2}\left\{\ln(\omega/\omega_0)-\frac{41}{24}\right\}.$$

II. With $|\vec{p}\,'|$ constrained from above by small p_0:

$$\frac{d\sigma_a}{d\omega_1}=\frac{g_{a\gamma\gamma}^2 e^4}{(2\pi)^3}\frac{p_0^3}{48m_e^2\omega^2}\left(\frac{\omega_1}{\omega-\omega_1}+\frac{\omega-\omega_1}{\omega_1}\right)$$

and

$$\sigma_a \simeq \frac{g_{a\gamma\gamma}^2 e^4}{(2\pi)^3}\frac{p_0^3}{24m_e^2\omega}\left[\ln(\omega/\omega)-1\right].$$

Cross sections $\propto p_0^3$ – the sitation similar to radiative neutrino scattering caused by μ_ν.

Effects of atomic binding?

For scattering on atomic electrons in dielectrics the effects of atomic binding should in general be taken into account.

But: in the kinematic regime of interest $(\omega \gg \omega_{\rm at})$ the atomic effects can be neglected. Full analogy with elastic scattering of photons on atoms: when $\omega \gg \omega_{\rm at} \quad \Rightarrow \quad \sigma$ essentially coincides with that for scattering on free electrons.

If in addition $|\vec{p}\,'| \ll L^{-1}$, radiative neutrino scattering on all the electrons of the target is coherent.

Macroscopically coherent DM detection?

The mechanism would not work for non-relativistic projectiles:

Macroscopic coherence requires tiny net recoil momenta $\vec{p}\,'$ of target electrons.

For non-relativistic projectiles $\vec{p}\,' = 0$ is excluded by energy-momentum conservation (would lead to unphysical $\cos\theta_\gamma > 1$).

Small non-zero $\vec{p}\,'$ allowed, but only for $\omega_\gamma \ll |\vec{p}\,'|$ $\Rightarrow$ a strong $\sim \omega_\gamma^4$ suppression.

$\Rightarrow$ The mechanism cannot work for the conventionally discussed DM particles.

But: It may work for relativistic particles that may exist in the dark sector.

Coherent effects and the cross sections

To take possible macroscopic coherence effects into account: multiply the elementary amplitude by the relevant structure factor $F(\vec{p}\,')$ (target dependent). To get simple estimates of of macroscopic coherence effects:

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- Assume all the scatterers in a volume of a linear size L_0 within the target contribute coherently; this requires the net recoil momentum of the scatterer $|\vec{p}\,'| \lesssim p_0 \sim 2\pi L_0^{-1}$.

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- Optimize the choice of p_0 by maximizing the resulting cross section.

Coherent effects and the cross sections

N.B.: One should not forget the issue of observability. A process may be fully coherent but completely unobservable. Example: for elastic ν scattering the optimization requires to choose for the maximum recoil momentum q_0 there the smallest possible value $q_0 \sim L^{-1}$, but the scattering will then be unobservable due to the vanishingly small recoil energy of the target particles. No such problems arise for radiative processes.

I. Radiative νe scattering mediated by weak CC and NC interactions.

$$\bar{\sigma}_w \simeq \sigma_{0w} \times N_{e0}\,, \qquad\qquad N_{0e} \simeq n_e L_0^3 \simeq n_e \left(\frac{2\pi}{p_0}\right)^3,$$

$$\sigma_{0w} \propto p_0^4$$

$\Rightarrow$ $\bar{\sigma}_w \propto p_0$ – maximized for maximal possible value of p_0 which corresponds to the absence of macrosopic coherence.

Macroscopic coherence can be achieved, but the resulting cross sections are much smaller than those in the incoherent case.

μ_ν – mediated process

II. μ_ν (or ϵ_ν) – mediated radiative neutrino scattering.

For small $\vec{p}\,'$ – an enhancement due to the virtual photon propagator being close to its mass-shell pole: $(d\sigma_m/d\omega_\gamma,\, \sigma_m) \propto p_0^3$ rather than $\propto p_0^4$.

Upon multiplication by $N_{e0} \simeq n_e(2\pi/p_0)^3$ the factors p_0^3 cancel out:

$$\frac{d\bar{\sigma}_m}{d\omega_\gamma} \simeq \frac{\mu_\nu^2 e^4}{6}\,\frac{(\omega-\omega_\gamma)}{\omega\omega_\gamma^2}\,\frac{n_e}{m_e^2}\,,$$

$$\bar{\sigma}_m(\omega_\gamma > \omega_0) \simeq \frac{1}{6}\,\mu_\nu^2 e^4\,\frac{n_e}{m_e^2\omega_0} = \frac{8}{3}\pi^2\,\frac{\mu_\nu^2\alpha^2}{m_e^2\omega_0}\,n_e\,.$$

$\Rightarrow$ To leading order in small p_0 the cross sections per target electron are p_0 – independent.

(This is correct when p_0 satisfies $L^{-1} \lesssim p_0 \ll \{\omega,\, \omega_\gamma,\, \omega-\omega_\gamma\}$, which we always assume).

Coherence effects – alternative approach

The simplified approach for evaluating effects of macrosc. coherence (introduce cutoff $p_0 \sim 2\pi/L_0$ on $|\vec{p}\,'|$, replace the structure factors $F(\vec{k}-\vec{k}'-\vec{k}_\gamma) = F(\vec{p}\,')$ within the coherence volume L_0^3 by unity and multiply the obtained σ_0 by the number electrons in the coherent volume), gives a fairly good accuracy.

An alternative (more accurate) approach: replace the summation in the expression for the structure factor F by integration. $\Rightarrow$

$$F(\vec{k}-\vec{k}'-\vec{k}_\gamma) \simeq \frac{N_e}{V}(2\pi)^3\delta^3(\vec{k}-\vec{k}'-\vec{k}_\gamma)$$

($N_e = n_e V$ is the total $\#$ of electrons in the target).

$$\Rightarrow \quad |F(\vec{k}-\vec{k}'-\vec{k}_\gamma)|^2 \simeq N_e n_e (2\pi)^3\delta^3(\vec{k}-\vec{k}'-\vec{k}_\gamma)$$

For the μ_ν – induced process this gives

$$\frac{d\overline{\sigma}_m}{d\omega_\gamma} = \frac{\mu_\nu^2 e^4}{4\pi}\,\frac{(\omega-\omega_\gamma)}{\omega\omega_\gamma^2}\,\frac{n_e}{m_e^2}$$

just smaller than the simple estimate by a factor of $3/2\pi \simeq 0.5$.

Radiative *vs.* elastic μ_ν-induced scattering

The best lab limits on neutrino magnetic moments – from reactor experiments on elastic νe scattering:

$$\frac{d\sigma_m^{el}}{dT} = \frac{\mu_\nu^2 e^2}{4\pi}\Big(\frac{1}{T} - \frac{1}{\omega}\Big) \simeq \frac{\mu_\nu^2 \alpha}{T}$$

(T is the kinetic energy of the recoil electron). To be compared with $d\bar{\sigma}_m/d\omega_\gamma$. In convenient units:

$$n_e = N_A \rho(\mathrm{g/cm^3}) \mathrm{Y_e\ cm^{-3}} \simeq (1.33\ \mathrm{keV})^3\, \rho(\mathrm{g/cm^3}) \qquad \Rightarrow$$

$$\frac{d\overline{\sigma}_m}{d\omega_\gamma} \simeq 4\pi\alpha^2\mu_\nu^2\, \frac{(1.33\ \mathrm{keV})^3}{m_e^2\omega_\gamma^2}\, \rho(\mathrm{g/cm^3}).$$

Need to compare

$$4\pi\alpha^2\mu_\nu^2\,\frac{(1.33\ \mathrm{keV})^3}{m_e^2\omega_\gamma^2}\,\rho(\mathrm{g/cm^3}) \qquad \longleftrightarrow \qquad \frac{\mu_\nu^2\alpha}{T}$$

For $\omega_\gamma \sim 10\ \mathrm{eV}$ and $\rho \sim 1\ \mathrm{g/cm}^3$ – $d\bar{\sigma}_m/d\omega_\gamma$ exceeds $d\sigma_m^{el}/dT$ only for $T \gtrsim 100\ \mathrm{keV}$.

An advantage: the rad. process could in principle allow detection of very low energy neutrinos.

An example: for $\omega \sim 100\ \mathrm{eV}$ in elastic νe scattering $T \leq 2\omega^2/m_e \simeq 0.04\ \mathrm{eV}$ – too small to be measured. But: photons of energy $\sim 100\ \mathrm{eV}$ produced in coherent radiative νe scattering can be easlily detected.

Radiative coherent axion-photon conversion

In many respects similar to the neutrino magnetic moment induced radiative νe scattering.

$$\bar{\sigma}_a \simeq \frac{2}{3}\pi^2\,\frac{g_{a\gamma\gamma}^2\alpha^2}{m_e^2\,\omega}n_e \simeq \frac{2}{3}\,g_{a\gamma\gamma}^2\alpha^2\pi^2\,\frac{(1.33\ \mathrm{keV})^3}{m_e^2\,\omega}\,\rho(\mathrm{g/cm^3})\,.$$

To be compared with axion-photon conversion in an external magnetic field (used in searches for axions from the sun with helioscopes): For axion traveling through a transverse magnetic field B over a length L the conversion probability

$$P \simeq 2.4\times 10^{-21}\left(g_{a\gamma\gamma}\times 10^{10}\,\mathrm{GeV}\right)^2\left(\frac{B}{\mathrm{T}}\right)^2\left(\frac{L}{\mathrm{m}}\right)^2 \qquad (1)$$

$\Rightarrow$ Macroscopically coherent radiative axion conversion on electrons is not competitive with the coherent conversion in a magnetic field.

Summary

An attempt at extending coherent neutrino detection to macroscopic scales. Radiative neutrino scattering on electrons – appears to be an attractive possibility. Advantages:

- Produced photon is easier to detect than (tiny) recoil of target particles
- The photon energy need not be small to ensure macroscopic coherence

The problem: Requires severe restriction of the phase space.

- For weak CC and NC induced processes – enhancement due to macroscopic coherence cannot overcome suppression due to the reduced phase space. Net effect is decrease rather than increase of $\bar{\sigma}_w$.
- For μ_ν-induced radiative νe scattering: Macroscopic coherence gives advantage over the elastic scattering only for $T \gtrsim 100\ \mathrm{keV}$.

  But: it may allow detection of very low-E neutrinos ($\sim 10\ \mathrm{eV} - 10\ \mathrm{keV}$).

Summary – contd.

- Similar mechanism for detection of relativistic axions (coherent radiative axion-photon conversion) is not competitive with axion-photon conversion in external magnetic fields
- The mechanism cannot work for non-relativistic projectiles (e.g. for conventional DM candidates). It still may work for relativistic particles in the dark sector ("boosted" DM, etc.).

No enhancement of neutrino detection by huge factors.

We need a different idea!

Backup slides

Xsection per target particle and coherence

Let the $\#$ of scatterers within one coherent volume L_0^3 be N_0 and the $\#$ of coherent volumes in the target be k. The total number of scatterers in the target is $N = kN_0$.

If σ_0 is the elementary cross section of the process, the cross section corresponding to scattering on all the target particles contained within one coherent volume is $\sigma_0 N_0^2$. The total cross section is $\sigma_0 N_0^2 \times k = \sigma_0 N_0 N$.

The cross section per one target particle is then $\sigma_0 N_0$. In the fully coherent case $(N_0 = N)$ and completely incoherent case $(N_0 = 1)$ the total cross sections are $\sigma_0 N^2$ and $\sigma_0 N$, respectively, and the corresponding cross sections per target particle are $\sigma_0 N$ and σ_0.

Effects of atomic binding

For scattering on atomic electrons in dielectrics the effects of atomic binding should in general be taken into account. But: in the kinematic regime of interest to us ($\omega \gg \omega_{\rm at},\ |\vec{p}\,'| \ll L^{-1}$) the atomic effects can be neglected.

1. For radiative scattering on free electrons the contribution of the first two terms in $Q^{\mu\alpha}$ is small – main contribution comes from the third term.

For small $\vec{p}\,'$ the terms in $Q^{\mu\alpha}$ proportional to $\propto p'^{\mu}p^{\alpha}$ and $p^{\mu}p'^{\alpha}$ nearly cancel each other, and the terms $\propto (k-k')^{\alpha}$ are subleading for non-relativistic electrons and weak NC and CC mediated νe scattering (they vanish exactly for neutrino magnetic moment mediated νe scattering and for axion-photon conversion). The terms $\propto k_{\gamma}^{\mu}$ do not contribute by gauge invariance.

Also true when atomic effects are taken into account: the analogues of the first two terms in $Q^{\mu\alpha}$ are small, and the main contribution comes form the analogue of the third term, which turns out to be largely insensitive to the effects of atomic structure.

Effects of atomic binding – contd.

Full analogy with elastic scattering of photons on atoms: when $\omega_i \gg \omega_{\rm at}$ $\Rightarrow$ σ essentially coincides with that for scattering on free electrons. To leading order, in the Coulomb gauge

$$\mathcal{A} \propto -\frac{1}{m}\sum_n \left\{ \frac{\langle i|e^{-i\vec{k}_f\vec{r}}\vec{\mathrm{p}}\vec{\epsilon}_f^{\,*}|n\rangle\langle n|e^{i\vec{k}_i\vec{r}}\vec{\mathrm{p}}\vec{\epsilon}_i|i\rangle}{E_n - E_i - \omega_i - i\varepsilon} + \frac{\langle i|e^{i\vec{k}_i\vec{r}}\vec{\mathrm{p}}\vec{\epsilon}_i|n\rangle\langle n|e^{-i\vec{k}_f\vec{r}}\vec{\mathrm{p}}\vec{\epsilon}_f^{\,*}|i\rangle}{E_n - E_i + \omega_i - i\varepsilon} \right\}$$

$$+(\vec{\epsilon}_f^{\,*}\cdot\vec{\epsilon}_i)\langle i|e^{i(\vec{k}_i-\vec{k}_f)\vec{r}}|i\rangle .$$

($\vec{\mathrm{p}} = -i\vec{\nabla}$ and we have taken into account that for elastic scattering on a heavy system $\omega_f = |\vec{k}_f|$ coincides with $\omega_i = |\vec{k}_i|$.)

For $\omega_i \ll \omega_{at}$ all three terms are of the same order and nearly cancel each other, leading to the $\sim \omega_i^4$ suppression of σ. <u>But:</u> for $\omega_i \gg \omega_{at}$ the first two terms are small compared to the third term and can be neglected. Also, for spherically symmetric atomic states $|i\rangle$ they tend to cancel each other.

Effects of atomic binding – contd.

Using the closure property of the atomic states and commuting the factors $e^{i\vec{k}_i\vec{r}}$, $e^{-i\vec{k}_f\vec{r}}$ with the momentum operator: for the sum of the first two terms we get in this regime $-(1/m\omega_i)\epsilon_f^{*l}\epsilon_i^s e^{i(\vec{k}_i-\vec{k}_f)\vec{r}}\langle i|k_i^l \mathrm{p}^s + k_f^s \mathrm{p}^l|i\rangle$, which vanishes for spherically symmetric states $|i\rangle$. For coherent scattering on a group of atoms $|i\rangle$ is the ground state of the system $\Rightarrow$ The cancellation happens also when $|i\rangle$ is spherically symmetric (has zero total angular momentum) even if the ground states of the individual atoms are not.

The remaining term $(\vec{\epsilon}_f^{\,*}\cdot\vec{\epsilon}_i)\langle i|e^{i(\vec{k}_i-\vec{k}_f)\vec{r}}|i\rangle$ in general depends on the electron charge distribution in the state $|i\rangle$. For $|\vec{k}_i-\vec{k}_f| \ll R_{at}^{-1}$ it is actually independent of the atomic structure $\Rightarrow$ for photon scattering on a single atom reduces to $Z(\vec{\epsilon}_f^{\,*}\cdot\vec{\epsilon}_i)$ This corresponds to coherent elastic photon-atom scattering.

If the $|\vec{k}_i-\vec{k}_f| \ll L^{-1}$ the scattering on all electrons in the target of linear size L is coherent. Otherwise, one would need to take into account structure factors which depend on the electron distribution in the target.

Effects of atomic binding – contd.

Similar arguments apply to radiative neutrino scattering on atoms. Need to replace

$$\vec{k}_i \to \vec{k} - \vec{k}'\,, \qquad \omega_i \to \omega - \omega'\,, \qquad \vec{k}_f \to \vec{k}_\gamma\,, \qquad \omega_f \to \omega_\gamma\,,$$

$$\vec{\epsilon}_i \to \vec{j} \quad \text{and} \quad \vec{\mathrm{p}}\vec{\epsilon}_i \to \mathrm{p}\cdot\mathrm{j}$$

with $\mathrm{p}^\mu = i\partial^\mu$ the 4-momentum operator and $j^\mu = (j^0, \vec{j}\,)$ the relevant matrix element of the neutrino current. The condition $|\vec{k}_i - \vec{k}_f| \ll R_{at}^{-1}$ is replaced by $|\vec{k} - \vec{k}' - \vec{k}_\gamma| = |\vec{p}\,'| \ll R_{at}^{-1}$, – satisfied with a large margin when macroscopically coherent effects are considered.

With minor modifications (due to the presence of two photons in the final state) the same argument applies also to radiative axion-photon conversion on atoms.

Radiative ν-e scattering and μ_ν

In convenient units:

$$\frac{d\overline{\sigma}_m}{d\omega_\gamma} \simeq 2.06 \times 10^{-56} \left(\frac{\mu_\nu}{10^{-12}\mu_B}\right)^2 \rho(\mathrm{g/cm^3}) \left(\frac{100\ \mathrm{eV}}{\omega_\gamma}\right)^2 \ \mathrm{cm^2/eV}\,.$$

Here $\mu_B = e/2m_e$ is the electron Bohr magneton.

Radiative coherent axion-photon conversion

In many respects similar to the neutrino magnetic moment induced radiative νe scattering.

$$\bar{\sigma}_a \simeq \frac{2}{3}\pi^2 \frac{g_{a\gamma\gamma}^2 \alpha^2}{m_e^2\,\omega} n_e \simeq \frac{2}{3}\, g_{a\gamma\gamma}^2 \alpha^2 \pi^2 \frac{(1.33\ \mathrm{keV})^3}{m_e^2\,\omega}\, \rho(\mathrm{g/cm^3})\,.$$

Compare with axion-photon conversion in an external magnetic field (used in searches for axions from the sun with helioscopes). For axion traveling through a transverse magnetic field B over a length L the conversion probability

$$P = 2.4\times 10^{-21}\left(g_{a\gamma\gamma}\times 10^{10}\,\mathrm{GeV}\right)^2 \left(\frac{B}{\mathrm{T}}\right)^2 \left(\frac{L}{\mathrm{m}}\right)^2 F\,, \tag{2}$$

Form factor

$$F = \left(\frac{2\sin(\frac{qL}{2})}{qL}\right)^2 \sim 1 \tag{3}$$

accounts for the loss of coherence as a function of the momentum transfer q.

The photon production rate: $\Gamma_a = j_a \cdot P \cdot A_{\rm eff}$ (j_a is the axion flux and $A_{\rm eff}$ is the effective area of the detector).
For axion helioscopes (such as CAST): $B \approx 10$ T, $L \approx 10$ m, $A_{\rm eff} \approx 1\,{\rm cm}^2 \quad \Rightarrow$

$$\Gamma_a \approx 2.4 \times 10^{-17}\,{\rm cm}^2\,(g_{a\gamma\gamma} \times 10^{10}\,{\rm GeV})^2 j_a\,.$$

The photon production rate due to the coherent radiative axion-photon conversion mechanism:

$$\Gamma_a \;=\; j_a \bar{\sigma}_a n_e V \;\simeq\; \bar{\sigma}_a (1.33\,{\rm keV})^3 \rho({\rm g/cm}^3) V j_a\,,$$

Take $\omega \simeq 3\,{\rm keV}$ (characteristic energy of axions produced in the sun) and $\rho \simeq 1\,{\rm g/cm}^3$, $\;\Rightarrow\;$ the photon production rate due to coherent radiative inverse Primakoff effect is by far small compared to that due to the axion-photon conversion.

Macroscopically coherent radiative axion conversion on electrons is not competitive with the coherent conversion in a magnetic field.

Kinematics

Consider

$$X(k)+e(p)\to X(k')+e(p')+\gamma(k_\gamma)\,,$$

(X is a projectile particle of mass M).

$$k=\big(\omega,\vec{k}\big)\,,\quad p=(m_e,\vec{0})\,,\quad k'=\big(\omega',\vec{k}'\big)\,,\quad p'=\big(E_{p'},\vec{p}\,'\big)\,,\quad k_\gamma=\big(\omega_\gamma,\vec{k}_\gamma\big)\,,$$

$$\omega=\sqrt{\vec{k}^{\,2}+M^2},\quad \omega'=\sqrt{\vec{k}'^2+M^2}\,,\quad E_{p'}=\sqrt{\vec{p}\,'^2+m_e^2}\,,\quad \omega_\gamma=|\vec{k}_\gamma|\,.$$

Energy and momentum conservation:

$$\omega=\omega'+(E_{p'}-m_e)+\omega_\gamma\,,\quad \vec{k}=\vec{k}'+\vec{p}\,'+\vec{k}_\gamma\,.$$

Assume recoil electron to be non-relativistic (neglect its kinetic energy) $\Rightarrow$ $\omega=\omega'+\omega_\gamma$. Substituting here $\vec{k}\,'$ from mom. conservation:

$$(\vec{k}-\vec{k}_\gamma-\vec{p}\,')^2=\vec{k}^2-2\omega_\gamma\sqrt{\vec{k}^{\,2}+M^2}+\omega_\gamma^2 \qquad \Rightarrow$$

Kinematics – contd.

$$\vec{p}^{\,\prime 2} - 2|\vec{p}^{\,\prime}|R\cos\theta_{\vec{p}^{\,\prime}(\vec{k}-\vec{k}^{\prime})} - 2|\vec{k}|\omega_\gamma\cos\theta_\gamma = -2\omega_\gamma\sqrt{\vec{k}^2+M^2}\,.$$

$$R \equiv |\vec{k}-\vec{k}_\gamma| = \sqrt{\vec{k}^2+\omega_\gamma^2-2\omega_\gamma|\vec{k}|\cos\theta_\gamma}\,.$$

For $M\ne 0$ the quantity $|\vec{p}^{\,\prime}|$ cannot be arbitrarily small: for $\vec{p}^{\,\prime}\to 0$ we get $\cos\theta_\gamma = \sqrt{\vec{k}^{\,2}+M^2}/|\vec{k}| > 1$ – unphysical.

Next, let $|\vec{p}^{\,\prime}|$ be non-zero but small (so that $\vec{p}^{\,\prime 2}$ term can be neglected). Requiring $\cos\theta_\gamma \le 1$:

$$\sqrt{\vec{k}^2+M^2}-|\vec{k}| \;\le\; \frac{||\vec{k}|-\omega_\gamma|}{\omega_\gamma}\cdot|\vec{p}^{\,\prime}|\cos\theta_{\vec{p}^{\,\prime}(\vec{k}-\vec{k}^{\prime})} \qquad\qquad (*)$$

For non-relativistic projectiles

$$M-\frac{|\vec{k}|}{\omega_\gamma}\cdot|\vec{p}^{\,\prime}|\cos\theta_{\vec{p}^{\,\prime}(\vec{k}-\vec{k}^{\prime})} \;\le\; |\vec{k}|\,,$$

$\Rightarrow$ the two terms on the l.h.s. side should nearly cancel.

Kinematics – contd.

This requires

$$\omega_\gamma \simeq \frac{|\vec{k}|}{M} |\vec{p}\,'| \cos\theta_{\vec{p}'(\vec{k}-\vec{k}')} \ll |\vec{p}\,'| \,.$$

To achieve macroscopic coherence we need $|\vec{p}\,'| \lesssim 10^{-5}\ \mathrm{eV} \quad \Rightarrow$ the requirement $\omega_\gamma \gtrsim \omega_{at}$ (or $\omega_\gamma \gtrsim \omega_p$ for scattering on free electrons in a conductor) is badly violated for non-relativistic projectiles. Similarly for moderately relativistic projectiles.

In the ultra-relativistic regime $|\vec{k}| \gg M$ – an upper bound on M from eq. (*):

$$M^2 \,\le\, 2\omega \frac{\omega - \omega_\gamma}{\omega_\gamma} |\vec{p}\,'| \cos\theta_{\vec{p}'(\vec{k}-\vec{k}')} \,.$$

For ω_γ not too close to ω and $\cos\theta_{\vec{p}\,'(\vec{k}-\vec{k}')} \sim 1$: $M^2 \lesssim 2\omega|\vec{p}\,'|$.

For $|\vec{p}\,'| \sim 10^{-5}\ \mathrm{eV}$ and $\omega \sim 1\ \mathrm{keV}$ this gives $M \lesssim 0.14\ \mathrm{eV}$ – readily satisfied when the projectiles are neutrinos or axions.