KAVLI IPMU MATHEMATICS OF THE UNIVERSE

Overview of Lorentz and CPT violation search in the neutrino sector

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I. Introduction to Lorentz Invariance Violation (LV)



Why people like Lorentz invariance

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- Lorentz invariance is one of our most fundamental symmetries
 → underlying all our physics laws!
- What are the consequences of breaking the Lorentz invariance ?
 - 1. Physics depends on the observer referential
 - \rightarrow Measure different mixing angle θ at different time of the year...
 - 2. c may depend on observer referential \rightarrow Tachyons (v > c) are possible
 - 3. Causality can be broken (consequence of #2) :



- <u>Since v > c :</u> see Albert missing before he shot.
- <u>If I send information @v>c :</u> tell Albert he will miss before he shots

You'll miss the hole!

So, why bothering you with Lorentz violation ?

- <u>Pragmatic viewpoint :</u> All fundamental symmetries should be tested. Physics/Lagrangian is Euclidean invariant ?

 → c invariant under boost / rotation (Michelson and Morley)
 → Physics is Minkowski invariant.
- <u>Dreamer/Theorist/My viewpoint:</u> **Predicted in some theories beyond the Standard Model** (string, quantum loop or non-commutative geometry).
- Arises as a consequence of merging SM w/ gravity \rightarrow occurs at the Planck Mass Scale $M_{\rm p}$ = 10^{19} GeV.
- Highly supressed @GeV scale \rightarrow Never observed so far.
- So, how to test its effects ? \rightarrow @low energy (E \ll M_P) \rightarrow Construct an effective theory.

How to build a LV theory at low energy (E < M_P)?

- <u>Naively</u> : Add LV corrections to the SM Lagrangian scaling with ~ E / M_p .
- But, this explicit symmetry breaking violates causality and vacuum stability at high energy !

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 <u>Effective theory</u>: Standard Model Extension = SM Lagrangian + all terms allowing a LV symmetry breaking <u>spontaneous</u> symmetry breaking.
 Example of a <u>LV vector field a^µ =></u> Preferential direction



Neutrino oscillations with Lorentz invariance

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• <u>Arises due to energy difference between mass states / energy eigenstates :</u>



- Phase difference due to energy (so mass) difference between v_1 and v_2 i.e. the Hamiltonian eigenstates : $i \frac{d}{dt} |v_i(t)\rangle = H |v_i(t)\rangle \stackrel{invacuum}{=} E_i |v_i(t)\rangle$
- Phase difference therefore depends on E, but also on L naturally.
- How the Hamiltonian looks like when LV is allowed ?

Hamiltonian for neutrino in SME

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Hamiltonian eigenstates are modified → Neutrino oscillations modified :

Neutrino-Antineutrino basis



- 3-flavour PMNS model modified → Increase/decrease oscillation.
 + Oscillation can happen even for massless neutrinos
- <u>2 types of coefficients</u> : a_L are CPT-odd and c_L are CPT-even coefficient $\rightarrow a_L$ has dimension $\alpha \to Oscillation \ge D$ ifferent than sterile ν .
- Oscillation depends on particle direction p^{μ} <=> Oscillation depends on sidereal time (Earth rotation)
- $v \leftrightarrow \overline{v}$ oscillations possible, v can go faster than light...



Neutrino oscillations with Lorentz invariance

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• <u>Arises due to energy difference between mass states / energy eigenstates :</u>



- LV field couples differently to $\nu_{_1}$ and $\nu_{_2}$
 - \rightarrow New phase difference \rightarrow modify oscillation pattern compared to PMNS
- LV coupling of v_1 and v_2 varies with E \rightarrow additional E dependency of oscillation pattern.
- LV couples differently wrt ν direction \rightarrow Osc. depends on sidereal time

<u>II. LV search 1st method :</u> Sidereal modulation search using T2K



The T2K experiment

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• v_{e} appearance in a v_{μ} beam and v_{μ} disappearance \rightarrow See A. Cervera's talk



- But this time, we focus on the INGRID near detector \rightarrow @280m.
- @280m : No standard PMNS oscillation can happen at this distance.
- <u>If any oscillation @INGRID</u> : Sterile neutrino, Lorentz violation effect....

Sidereal time dependent oscillations @280m

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Oscillation probability at near detectors :

 $P_{\nu_{\mu}\to\nu_{x}} = \left(\frac{L}{hc}\right)^{2} |(C_{\mu x}) + (A_{s})_{\mu x} \sin(\omega_{\oplus}T_{\oplus}) + (A_{c})_{\mu x} \cos(\omega_{\oplus}T_{\oplus}) + (B_{s})_{\mu x} \sin(2\omega_{\oplus}T_{\oplus}) + (B_{c})_{\mu x} \cos(2\omega_{\oplus}T_{\oplus})|^{2}$

 T_{\oplus} the sidereal time, and $\omega_{\oplus} = \frac{2\pi}{23^{h}56^{m}04.0982^{s}}$ the sidereal angular phase

- L² dependency (neutrino baseline).
- 5 effective parameters C, Ac, As, Bc, Bs $\rightarrow (C)_{ab} = (C)^{(0)}_{ab} + E(C)^{(1)}_{ab}$ With :
- 28 SME parameters $a_{L}^{(GeV)}$, $c_{L}^{(GeV)}$

- E (e.g. C^0) and E^2 (e.g. C^1) dependency
- All parameters are direction depdendent except a^{T} and c^{TT} .

• Focus on $v_{\underline{\mu}}$ disappearance (higher statistics) \rightarrow Constraint $\mu \rightarrow \tau$ and $\mu \rightarrow e$.

Ingredient #1 : identify your signal

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Oscillation probability at near detectors :

 $P_{\nu_{\mu}\to\nu_{x}} = \left(\frac{L}{hc}\right)^{2} |(C_{\mu x}) + (A_{s})_{\mu x} \sin(\omega_{\oplus}T_{\oplus}) + (A_{c})_{\mu x} \cos(\omega_{\oplus}T_{\oplus}) + (B_{s})_{\mu x} \sin(2\omega_{\oplus}T_{\oplus}) + (B_{c})_{\mu x} \cos(2\omega_{\oplus}T_{\oplus})|^{2}$

 T_{\oplus} the sidereal time, and $\omega_{\oplus} = \frac{2\pi}{23^{h}56^{m}04.0982^{s}}$ the sidereal angular phase



• <u>Time-independent oscillation :</u> alternative hypothesis to sterile to explain short baseline disappearance (LSND, MiniBooNE...)

This work

• <u>Sidereal time oscillation : higher sensitivity</u>

Ingredient #2 : a detector \rightarrow INGRID

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• 14 modules in a cross shape structure + 2 shoulder modules





- **<u>1 Module =</u>** 1.4 m³ sandwich of :
 - 9 iron planes (interaction)
 - 11 X/Y scintillator planes (detection)
- <u>Data sample</u>: (almost) full T2K POT from Run 1 (2010) to Run 4 (2013) : 6.6 x 10^{20} POT \rightarrow Correspond to 6.8 x 10^{6} events

Ingredient #3 : build pure v_{μ} sample



- <u>Ingredient #4 : Correct time-dependent sources (+evaluate systematics) :</u>
 - From v_u beam (tidal effect, re-alignement of beam...)
 - From INGRID : gain, dark noise variation with time etc.



Data distribution with sidereal time

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	Source	Systematic uncertainty (%)			
	Pile-up	0.01			
	MPPC dark noise	0.01			
	MPPC gain variation	0.06			
	Beam position	0.03			
	Rate correction	0.05			
	Total systematic	0.08			
<u>Statistical uncertainty :</u> 0.2 %					

→ Systematic error is negligible
→ Advantage of sidereal dependency

 $P_{\nu_{\mu} \to \nu_{x}} = (\frac{L}{hc})^{2} |(C)_{\mu x} + (A_{s})_{\mu x} \sin(\omega_{\oplus} T_{\oplus}) + (A_{c})_{\mu x} \cos(\omega_{\oplus} T_{\oplus}) + (B_{s})_{\mu x} \sin(2\omega_{\oplus} T_{\oplus}) + (B_{c})_{\mu x} \cos(2\omega_{\oplus} T_{\oplus})|^{2}$

→ Can be developed in 5 harmonics : constant, ω_{\oplus} , $2\omega_{\oplus}$, $3\omega_{\oplus}$ and $4\omega_{\oplus}$

- <u>1st Step</u>: Search for deviation to 3-flavour
 SM = « no oscillation »
 - \rightarrow Fast Fourier transform
 - \rightarrow Compatible with a flat signal within $3\sigma.$



Likelihood fit result

- <u>2nd Step : extract limit on the SME coefficients :</u>
 → Full binned likelihood method to preserve correlations (crucial)
- Too many correlations to perform a fit of the 28 SME parameters : a, c
- Fit all 10 effective parameters C, A_c , A_s , B_c , B_s for $\mu \rightarrow \tau$ and $\mu \rightarrow e$.



- Constraints ~ $10^{20}~GeV \rightarrow$ oscillations able to probe for LV at E > $M_{\rm p}$ = $10^{19}~GeV$!
- World-leading correlated constraints on almost all parameters



II. LV search 2nd method : Time-indepedendent modifications of the 3-flavour standard oscillation with Super-Kamiokande



- Super-Kamiokande is a 50kT water Cherenkov detector \rightarrow G. Pronost talk.
- Large Physics Portfolio \rightarrow Focus here on atmospheric neutrinos.



- <u>Oscillation search</u> : Measure number of $v_u \& v_e$ as a function of L and E.
- L is determined by measuring the zenith angle θ_{Zenith} \rightarrow L varies from 10 km to 13000km.

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• <u>E is determined using sample separation and energy deposited in SK :</u>



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- Evaluate sidereal-time independent effect of Lorentz Violation \rightarrow Deviation to 3v model \rightarrow Complementary to T2K result.
- Long-baseline oscillation hamiltonian can be re-written :



• Measure c^{TT} (effect αLE) and $a^{T} (\alpha L)$ only \rightarrow Not measured by T2K.



• Discrimination using > 5 GeV sample \rightarrow Partially contained and upward



IV. Potential of future experiments to detect Lorentz Violation



Future Long-Baseline oscillation potential

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- Scales with baseline L and energy $E \rightarrow LBL$ experiments, at high energy.
- <u>Near future</u> : T2K-II LBL or NovA.
- Sensitivity to $a_L \sim 10^{-24} \text{ GeV}$ $\rightarrow 4 \text{ orders of magnitudes better}$ than current limits.



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Future generation → Especially DUNE and atmospheric HK





L = 1300 km + higher energy broad-band beam

LV using speed-of-light measurements

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- Time-of-flight using accelerators (MINOS, T2K and OPERA) : neutrino time-of-flight compatible with the speed of light \rightarrow Bound up to $\sim 10^{-4}$ GeV
- Most stringent direct constraints comes from 1987a.



• <u>Ultra-high energy neutrino (astrophysical)</u>



If v > c \rightarrow Vacuum Cherenkov radiation (Couples to Z boson) $\rightarrow \rightarrow$ limits physics up to 10⁻²⁰ GeV

Ultra-High Energy neutrinos @IceCube

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- But can use neutrino oscillation of UHE v (E > 0.5 TeV) \rightarrow Use IceCube
- Absolute flux has large uncertainties $\rightarrow \underline{\text{Use flavour ratio}}: \alpha_{\beta}^{\oplus} = \bar{\phi}_{\beta}^{\oplus} / \sum_{\gamma} \bar{\phi}_{\gamma}^{\oplus}$



Ultra-High Energy neutrinos @IceCube

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• Effect of LV coefficients ~ 10^{-27} ~ SK atmospheric sensitivity.



• <u>Pros</u>: Potentially the world-leading sensitivity $\rightarrow > 10^{-30}$ GeV <u>Cons</u>: Seems model independent ... if believe UHE flavour model ratio !

Conclusions

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- Lorentz violation is predicted by some theories beyond the SM.
- Highly supressed @GeV scale ... But interferences experiment, as neutrino oscillation, can probe for it !
- <u>Can be searched through different signatures :</u>
 - Modifications of PMNS mixing.
 - <u>NB</u> : @short baseline, can mimic sterile v (w/ different E-dependency)
 - Sidereal-time-dependent oscillations.
 - Speed-of-light measurements (lower sensitivity).
 -
- Has been searched extensively in this generation of experiments : \rightarrow No LV up to $E{\sim}10^{20}-10^{27}~GeV$
- There is no sign of new physics @TeV for now → Let's search it at the Planck scale in the next years !

Additonal slides

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Effective parameters expression in terms of SME parameters

$$(\mathcal{C}^{(1)})_{ab} = (\tilde{a}_{L})_{ab}^{T} - \hat{N}^{Z} (\tilde{a}_{L})_{ab}^{Z} - \frac{1}{2} (3 - \hat{N}^{Z} \hat{N}^{Z}) E(\tilde{c}_{L})_{ab}^{TT} + 2\hat{N}^{Z} E(\tilde{c}_{L})_{ab}^{TZ} + \frac{1}{2} (1 - 3\hat{N}^{Z} \hat{N}^{Z}) E(\tilde{c}_{L})_{ab}^{ZZ} , (\mathcal{A}_{s}^{(1)})_{ab} = \hat{N}^{Y} (\tilde{a}_{L})_{ab}^{X} - \hat{N}^{X} (\tilde{a}_{L})_{ab}^{Y} - 2\hat{N}^{Y} E(\tilde{c}_{L})_{ab}^{X} + 2\hat{N}^{X} E(\tilde{c}_{L})_{ab}^{TY} + 2\hat{N}^{Y} \hat{N}^{Z} E(\tilde{c}_{L})_{ab}^{XZ} - 2\hat{N}^{X} \hat{N}^{Z} E(\tilde{c}_{L})_{ab}^{YZ} , (\mathcal{A}_{c}^{(1)})_{ab} = -\hat{N}^{X} (\tilde{a}_{L})_{ab}^{X} - \hat{N}^{Y} (\tilde{a}_{L})_{ab}^{Y} + 2\hat{N}^{X} E(\tilde{c}_{L})_{ab}^{XZ} - 2\hat{N}^{Y} \hat{N}^{Z} E(\tilde{c}_{L})_{ab}^{YZ} , (\mathcal{A}_{c}^{(1)})_{ab} = -\hat{N}^{X} (\tilde{a}_{L})_{ab}^{X} - \hat{N}^{Y} (\tilde{a}_{L})_{ab}^{X} - 2\hat{N}^{X} \hat{N}^{Z} E(\tilde{c}_{L})_{ab}^{XZ} - 2\hat{N}^{Y} \hat{N}^{Z} E(\tilde{c}_{L})_{ab}^{YZ} , (\mathcal{B}_{s}^{(1)})_{ab} = \hat{N}^{X} \hat{N}^{Y} E((\tilde{c}_{L})_{ab}^{XX} - (\tilde{c}_{L})_{ab}^{YY}) - (\hat{N}^{X} \hat{N}^{X} - \hat{N}^{Y} \hat{N}^{Y}) E(\tilde{c}_{L})_{ab}^{XY} , (\mathcal{B}_{c}^{(1)})_{ab} = -2\hat{N}^{X} \hat{N}^{Y} E(\tilde{c}_{L})_{ab}^{XY} - \frac{1}{2} (\hat{N}^{X} \hat{N}^{X} - \hat{N}^{Y} \hat{N}^{Y}) E((\tilde{c}_{L})_{ab}^{XX} - (\tilde{c}_{L})_{ab}^{YY}) .$$
 (47)

The T2K Experiment

- Observation of $\nu_{_{e}}$ appearance in a $\nu_{_{\mu}}$ beam and $\nu_{_{\mu}}$ disappearance & their antineutrino equivalents



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- <u>Null hypothesis = no sidereal modulation of neutrino event with LSP</u>
- Detection threshold $\leftrightarrow 3\sigma$ deviation from null hypothesis
 - \rightarrow 1 detection threshold for each of the 5 magnitude associated to the 5 harmonics (constant,

 $\omega_{\oplus}, 2\omega_{\oplus}, 3\omega_{\oplus} \text{ and } 4\omega_{\oplus}$

- Method :
- 1. Generate 10,000 toy experiment without LV signal
- 2. For each toy \rightarrow FFT \rightarrow Magnitude for each 5 harmonics
- 3. After 10,000 toys \rightarrow For each 5 magnitudes



• Open the data :



Magnitude of the Fourier mode amplitude in data



Conclusions :

- Compatible with a flat signal within 30
- No evidence for Lorentz violation •

Benjamin Quilain Parameter extraction using Fourier transform²⁰

$$P_{\nu_{\mu} \to \nu_{x}} = (\frac{L}{hc})^{2} |(C)_{\mu x} + (A_{s})_{\mu x} \sin(\omega_{\oplus} T_{\oplus}) + (A_{c})_{\mu x} \cos(\omega_{\oplus} T_{\oplus}) + (B_{s})_{\mu x} \sin(2\omega_{\oplus} T_{\oplus}) + (B_{c})_{\mu x} \cos(2\omega_{\oplus} T_{\oplus})|^{2}$$

- Associated constraints are evaluated for each of the 14 SME parameters : a, c
- \rightarrow Generate signal toys \rightarrow FFT \rightarrow test when crossing the 3 σ threshold

 \rightarrow « A la MINOS » \rightarrow all parameters = 0 except one \rightarrow assume that the full 30 effect comes from 1 parameter

World leading existing limits : <u>Deduce T2K 3 σ upper limits</u> a^{T} : a^{X} : a^{Y} : a^{Z} : c^{TT} : c^{TX} : c^{TY} : c^{TZ} : c^{XX} : c^{XY} : c^{XZ} : c^{YY} : c^{YZ} : c^{ZZ} $\times 10^{-20}$ $\times 10^{-20}$ a_L^X 4.8 GeV a_L^Y 4.8 GeV 10 $\frac{c_L^{TX}}{c_L^{TX}}$ $\frac{c_L^{XX}}{c_L^{XZ}}$ $\frac{c_L^{YZ}}{c_L^{YZ}}$ $\frac{c_L^{TY}}{c_L^{XY}}$ $\frac{c_L^{XY}}{c_L^{YY}}$ 0.9 0.9 3.8 1.6 3.1 3.8 3.1 T2K MINOS MB DC

« Raw » sensitivity results

10⁻³

- T2K is more sensitive than MiniBooNE but less than MINOS : MINOS baseline $\sim 1 \rm km$
 - MINOS higher flux energy (~3 GeV) \rightarrow higher sensitivity to c^{ij} coefficients

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• **World leading existing limits : MiniBooNE** and **MINOS** experiments :

	x 10 ⁻²⁰	x 10 ⁻²⁰
	$a_{L}^{Y}=4.0 \text{ GeV} (5.9 \text{ GeV}) (2.2 \text{ GeV})$	a_{L}^{X} =4.0 GeV (5.6 GeV) (2.2 GeV)
than MiniBooNE but	$c_{L}^{TY}=0.8$ (4.9) (0.009)	$c_{L}^{TX} = 0.8 (4.6) (0.009)$
less than MINOS	$c_{L}^{XY} = 1.6 (/) (0.45)$	c ^{XX} _L =3.1 (/) (0.46)
	c ^{YY} _L =3.1 (/) (0.11)	$c_{L}^{XZ} = 2.3$ (6.2) (0.11)
MINOS baseline : 1km		c_{L}^{YZ} =2.6 (6.5) (0.22)

- MINOS / T2K (INGRID) : FFT = most sensitive technique to sidereal variations...
- ... It does not allow to extract correct limits on coefficients
- **Assume no correlation :** but 14 parameters for 4 observables ...

=> There are (large) correlations between parameters

Correlations between the SME parameters

- \rightarrow But... 14 parameters for 4 observables... \rightarrow Expect very important correlations !
- <u>Probability to detect a LV effect (a deviation > 3</u> σ from null hypothesis)



- If c^{TX}>0 → T2K has less detection ability than if c^{TX}=0
 → Here : neglect correlations → over-estimate the sentivity !!
- <u>Here :</u> two parameters, and only 2-points correlations... → reality much more complex !!
- Correlations depends on position on Earth & beamline direction → change w/ experiments !

Conclusions :

<u>1. Large correlations between parameters</u>

<u>2. Wrong to consider « uncorrelated sensitivity »</u>: \rightarrow if larger correlation at MINOS (different direction) \rightarrow remove a signal that might be seen @T2K !

Results of Lorentz violation search

- Extraction of the 3σ constraints on parameters : « à la MINOS » \rightarrow assumes no correlations



- <u>T2K sensitivity within best in the world</u> (more sensitive than MiniBooNE but less than MINOS)...
- ... assuming no correlations ! but 14 parameters for 4 observables ...
- \rightarrow Large correlations between parameters
- 2. Likelihood method of 5 effective parameters C, A_c , A_s , B_c , B_s & use correlations



Conclusions :

- No evidence for Lorentz violation
- Higher sensitivity than MiniBooNE (slightly higher error but 5 param. fit)
- High correlations between parameters is confirmed

Future sensitivity of near detectors

<u>1. Near detectors :</u> INGRID \rightarrow used the FFT & MINOS method for simplicity

	x 10 ⁻²⁰		x 10 ⁻²⁰
a ^x	$4.8 \rightarrow 2.1 \text{ GeV}$	a ^Y	4.8 → 2.1 GeV
\mathbf{c}^{TX}	$0.9 \rightarrow 0.05$	cTY	$0.9 \rightarrow 0.05$
$\mathbf{c}^{\mathbf{X}\mathbf{X}}$	$3.8 \rightarrow 2.0$	c ^{YY}	$3.8 \rightarrow 2.0$
$\mathbf{c}^{\mathbf{X}\mathbf{Y}}$	1.6 → 1.0	c ^{YZ}	3.1 → 1.6
\mathbf{c}^{XZ}	3.1 → 1.6		

→ Small improvement for (a_L) coefficients, one order of magnitude for (c_L) (energy dependent)

 \rightarrow Improve this analysis using different energy samples :



 \rightarrow Disentangle the effect of the different parameters \rightarrow reduce correlations

→ But Gain ~ order of magnitude on top of the <u>INGRID $2x10^{22}$ POT table...</u>

Future sensitivity of far detector

2. At far detector : Lorentz violation in Super-K

 $\nu_{_{\!\!\!\!\!\!\!\!\!}}$ appearance probability @SK~Disappearance probability @SK :



• L dependency (L² dependency for ND) & linear in C, Ac, As, Bc, Bs \rightarrow Large effect @SK





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Benjamin Quilain Plans for the future : far detector

Event distribution @SK for T2K-II statistics



 \rightarrow Clear LV effect \rightarrow sensitivity ~ 10^{-24}~GeV

Experimental contraints on parameters : « no correlation sensitivity »



 \rightarrow Will be the world leading constraints !!!

<u>And this is very conservative :</u> $v \& \overline{v} \rightarrow$ separation will increase with SK-Gadolinium

 $\rightarrow v$ contamination in \overline{v} -mode might be reduced \rightarrow increase focusing horn current: 250kA \rightarrow 320kA

SK atmospheric neutrinos

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Six fits are performed for the real and imaginary parts of a^T and c^{TT} in the three sectors, $e\mu$, $e\tau$, and $\mu\tau$. The real and imaginary parts of each coefficient are fit simultaneously, but otherwise the coefficients are fit independently following the procedure typical for SME analyses 12.

LV	Parameter	Limit at 95% C.L	. Best Fit	No LV $\Delta \chi^2$	Previous Limi	it
$e\mu$	$\operatorname{Re}\left(a^{T}\right)$	$1.8\times 10^{-23}~{\rm GeV}$	$1.0\times 10^{-23}~{\rm GeV}$	1.4	$4.2\times 10^{-20}~{\rm GeV}$	58
	$\operatorname{Im}\left(a^{T}\right)$	$1.8\times 10^{-23}~{\rm GeV}$	$4.6\times 10^{-24}~{\rm GeV}$			00
	$\operatorname{Re}\left(c^{TT}\right)$	8.0×10^{-27}	1.0×10^{-28}	0.0	9.6×10^{-20}	58
	$\operatorname{Im}\left(c^{TT}\right)$	8.0×10^{-27}	1.0×10^{-28}			
$e\tau$	$\operatorname{Re}\left(a^{T}\right)$	$4.1 \times 10^{-23} \text{ GeV}$	$2.2 \times 10^{-24} \text{ GeV}$	0.0	7.8×10^{-20} GeV	50
	$\operatorname{Im}\left(a^{T}\right)$	$2.8\times 10^{-23}~{\rm GeV}$	$1.0\times 10^{-28}~{\rm GeV}$	0.0	7.8 × 10 Gev	09
	$\operatorname{Re}\left(c^{TT}\right)$	9.3×10^{-25}	1.0×10^{-28}	0.3	1.3×10^{-17}	59
	$\operatorname{Im}\left(c^{TT}\right)$	1.0×10^{-24}	3.5×10^{-25}			
μτ	$\operatorname{Re}\left(a^{T}\right)$	$6.5 \times 10^{-24} \text{ GeV}$	$3.2 \times 10^{-24} \text{ GeV}$	0.0		
	$\operatorname{Im}\left(a^{T}\right)$	$5.1 \times 10^{-24} \text{ GeV}$	$1.0\times 10^{-28}~{\rm GeV}$	0.9	—	
	$\operatorname{Re}\left(c^{TT}\right)$	4.4×10^{-27}	1.0×10^{-28}	0.1		
	$\operatorname{Im}\left(c^{TT}\right)$	4.2×10^{-27}	7.5×10^{-28}	0.1	—	

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• But can use neutrino oscillation of UHE v (E > 0.5 TeV) \rightarrow Use IceCube

8. Standard flavour triangle diagram

