Testing simple DM phase-space prediction methods on zoom-in simulations I: Formalism

Thomas Lacroix
Collaborators: J. Lavalle, E. Nezri, A. Nuñez, M. Stref

News from the dark 2018

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Phase-space distribution of dark matter: source of theoretical uncertainties

Direct searches

$$\frac{\mathrm{d}R}{\mathrm{d}E} \propto \rho_\odot \int_{v_{\min} \leqslant |\vec{v}| \leqslant v_{\rm esc}} \frac{f_\odot(\vec{v})}{|\vec{v}|} \, \mathrm{d}^3 v$$

Impact at low masses

$v_{\min} \sim v_{\rm esc}$

Speed-dependent annihilation

- $\langle \sigma v \rangle (r) \propto \langle v_{\rm r}^2 \rangle$ **p-wave**
  $\sigma v = s_1 v_{\rm r}^2 \Rightarrow \langle \sigma v \rangle \equiv \langle \sigma v \rangle (r)$
  $$\langle \sigma v \rangle (r) = s_1 \int \mathrm{d}^3 v_1 \mathrm{d}^3 v_2 \, f_r(\vec{v}_1) \, f_r(\vec{v}_2) \, v_{\rm r}^2$$
- $\langle \sigma v \rangle (r) \propto \langle 1/v_{\rm r} \rangle$ or $\langle 1/v_{\rm r}^2 \rangle$
  Sommerfeld

Primordial black holes

- Gravitational microlensing event rates (EROS, MACHO)
  $$\frac{\mathrm{d}\Gamma}{\mathrm{d}t} \propto \rho(r) \int v \, f(\vec{v}, \vec{r}) \, \mathrm{d}^3 v$$
- Merger rates (gravitational waves)

+ DM substructures (test masses), disruption of stellar binaries

Standard approaches 1: "Standard halo model"

Standard halo model (SHM)

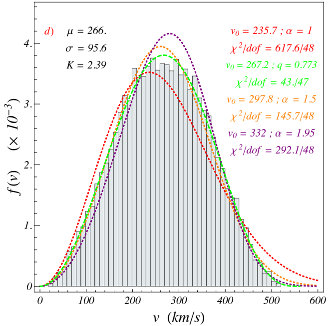
Maxwell-Boltzmann distribution

$$f(\vec{v}) = \frac{1}{v_c^3 \pi^{3/2}} \, e^{-\left(\frac{\vec{v}}{v_c}\right)^2}$$

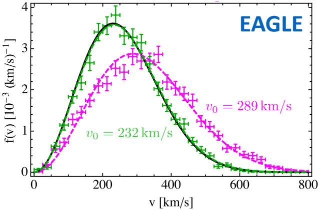
Oversimplification

- Isothermal sphere
- Infinite system
- Ad hoc truncation at v_{esc}

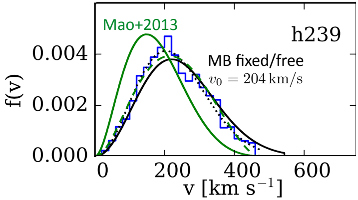
Standard approaches 2: direct fits to simulations



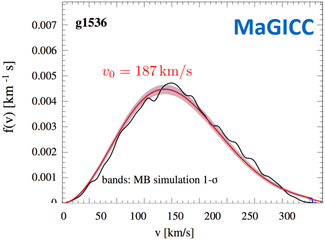
Ling+ 2010, Mollitor+ 2014



Bozorgnia+ 2016



Kelso+ 2016



Sloane+ 2016

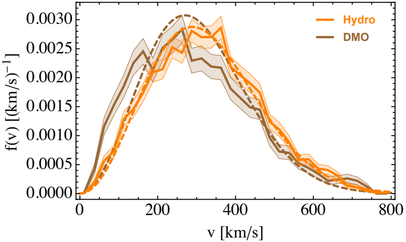
Standard approaches 2: direct fits to simulations

General insight

Generic features found in simulations (e.g., cusp/cores)

But insufficient approach

- Extrapolations based on fits at 8 kpc
- MW constrained systems (e.g., Gaia)
- Subgrid physics (recipes for star formation, ...)



Bozorgnia+ 2017

Self-consistent approach required

Eddington-like methods: next-to-minimal approach

Phase space of dark matter from first principles

Phase-space distribution $f(\vec{v},\vec{r})$: closed system

- Collisionless Boltzmann equation, steady state

$$\{f,H\} = \vec{v}\cdot\frac{\partial f}{\partial\vec{r}} - \frac{\partial\Phi}{\partial\vec{r}}\cdot\frac{\partial f}{\partial\vec{v}} = 0$$

$\longrightarrow$ Jeans' theorem: $f \equiv f(I_1,\ldots,I_N)$ where $\{I_i,H\} = 0$

- Poisson equation

$$\Delta\Phi = 4\pi G\rho \quad \text{with} \quad \rho = \int f(\vec{v},\vec{r})\mathrm{d}^3v$$

Eddington's inversion formula (Eddington 1916)

Isotropic system + spherical symmetry: $f(\vec{v},\vec{r}) \equiv f(\mathcal{E})$

with $\mathcal{E} = \Psi(r) - \frac{v^2}{2}$ and $\Psi(r) = \Phi(R_{\max}) - \Phi(r)$

$$f(\mathcal{E}) = \frac{1}{\sqrt{8}\pi^2}\left[\frac{1}{\sqrt{\mathcal{E}}}\left(\frac{\mathrm{d}\rho}{\mathrm{d}\Psi}\right)_{\Psi=0} + \int_0^{\mathcal{E}}\frac{\mathrm{d}^2\rho}{\mathrm{d}\Psi^2}\frac{\mathrm{d}\Psi}{\sqrt{\mathcal{E}-\Psi}}\right]$$

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Anisotropic extensions

1 free parameter

- Constant anisotropy
  $f_{\beta_0}(\mathcal{E}, L) = G(\mathcal{E}) L^{-2\beta_0}$ Cuddeford 1991
- Osipkov-Merritt: $f(\mathcal{E}, L) = f_{\rm OM}(Q)$ with $Q = \mathcal{E} - \frac{L^2}{2r_{\rm a}^2}$
  Osipkov 1979, Merritt 1985
  $\beta(r) = \frac{r^2}{r^2 + r_{\rm a}^2}$

2 free parameters

$f(\mathcal{E}, L) = G(Q) L^{-2\beta_0}$ Cuddeford 1991

3 free parameters

- $f(\mathcal{E}, L) = w f_{\rm OM}(Q) + (1-w) G(\mathcal{E}) L^{-2\beta_0}$ Bozorgnia+ 2013
- $f(\mathcal{E}, L) = F(\mathcal{E}) \left(1 + \frac{L^2}{2L_0^2}\right)^{-\beta_\infty + \beta_0} L^{-2\beta_0}$ Wojtak+ 2008

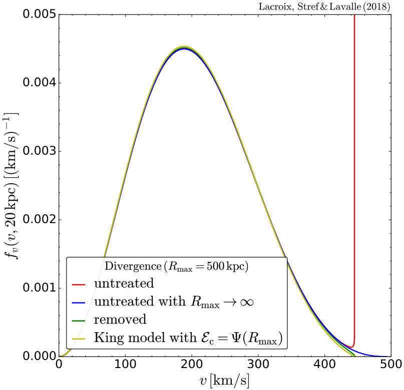
Going beyond spherical spherical symmetry

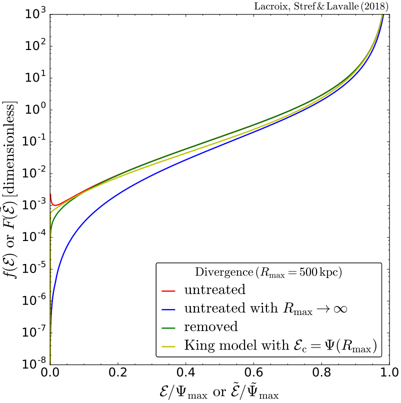
- Angle-action coordinates more suitable coordinate system if no spherical symmetry
  Binney & Tremaine 1987
- Best way to account for complexity revealed by Gaia (see talks by J. Binney and G. Monari)
- Level of refinement not necessarily required for DM searches
  $\rightarrow$ Evaluate astrophysical uncertainties
- Eddington: lower level of technicalities to account for dynamical constraints
- Method applied blindly to direct searches so far
  $\rightarrow$ Study validity range in detail

Theoretical consistency and radial boundary

Imposing a radial boundary

- Finite system ($R_{\rm max}$) $\Rightarrow$ divergence of $f(\vec{r},\vec{v})$ at $v_{\rm esc}$ (from $1/\sqrt{\mathcal{E}}$)
- Phase-space compression
- $v_{\rm esc}$ crucial (direct DM searches at low masses, stellar surveys)





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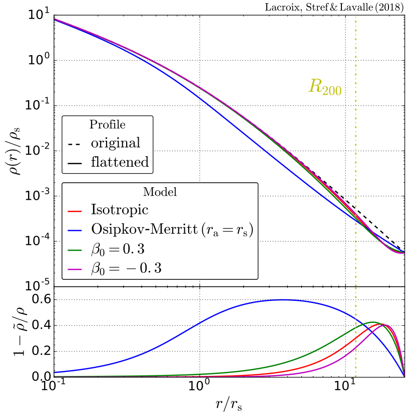
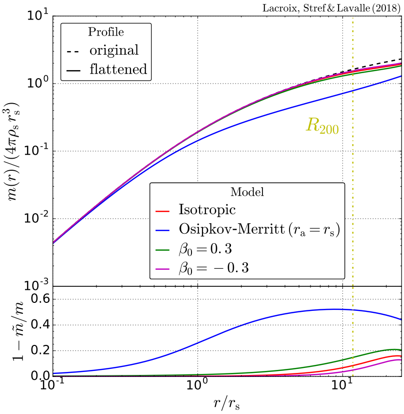
Proper treatment critical for self-consistency

Removing divergence by hand

$\Rightarrow$ approach no longer self-consistent (Poisson)





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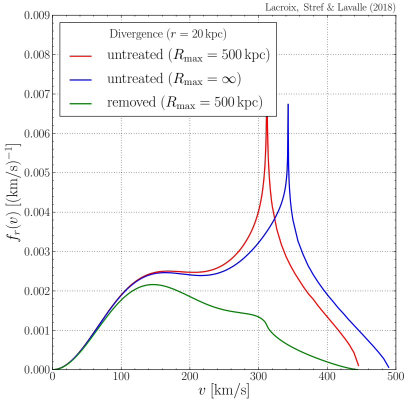
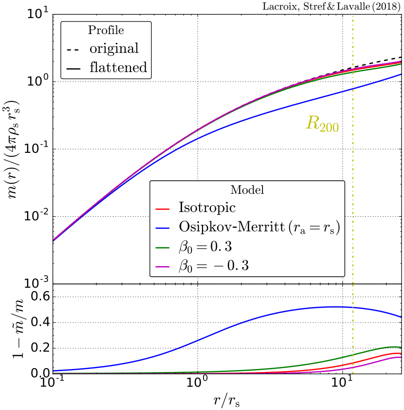
Theoretical consistency and radial boundary – prototypical anisotropic case

Even more critical for the Osipkov-Merritt model





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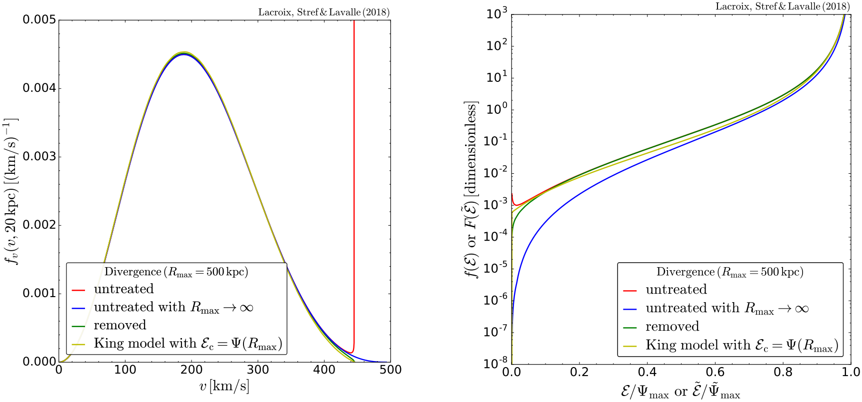
Theoretical consistency and radial boundary

Regularization

- Modified profile, flat at $R_{\max}$
- Energy cutoff (King)



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Not possible for radial anisotropy (e.g., Osipkov-Merritt)

Theoretical consistency: instabilities

Validity range of the method

- Standard criterion: $f \geqslant 0$
- Antonov instabilities for some DM-baryon configurations
- Stable solution if $\dfrac{\mathrm{d}f}{\mathrm{d}\mathcal{E}} > 0 \Leftrightarrow \dfrac{\mathrm{d}^2\rho}{\mathrm{d}\Psi^2} > 0$
  Doremus+ 1971, Kandrup & Sygnet 1985
- Select mass models

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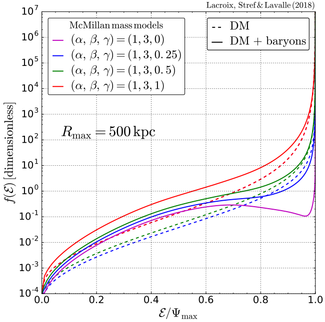
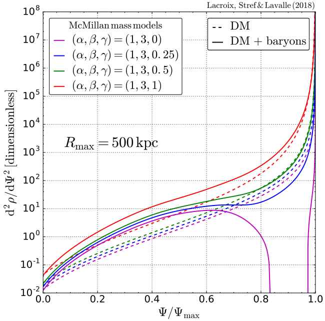
For anisotropic systems criteria against radial perturbations only
Doremus+ 1973



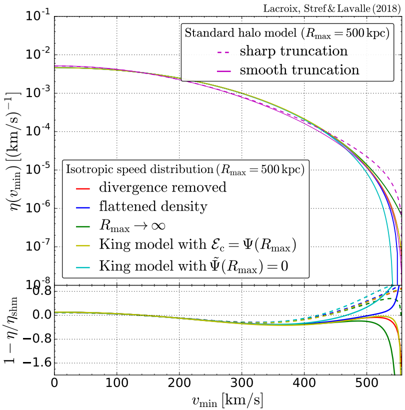


Impact on predictions for DM searches

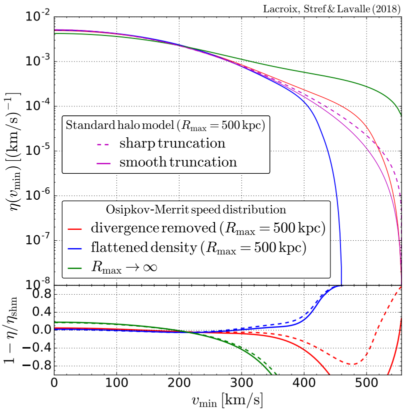
Event rate proportional to

$$\eta(v_{\rm min}) = \int_{v_{\rm min} \leqslant v \leqslant v_\oplus + v_{\rm esc}} \frac{f_{\vec{v},\oplus}(\vec{v})}{v}\, d^3v$$

Isotropic



Osipkov-Merritt



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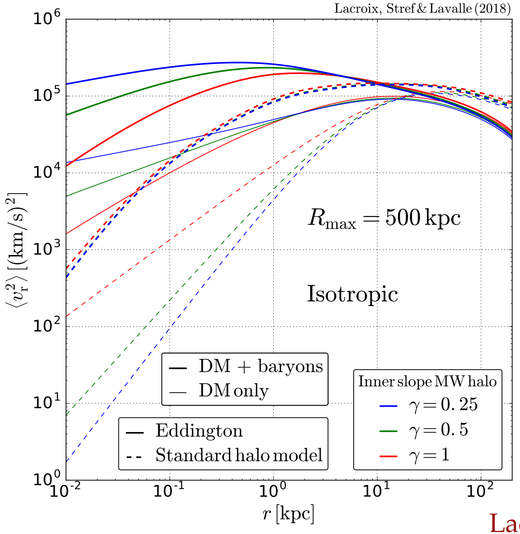
Impact on predictions for DM searches

Prototypical case: p-wave annihilation

$\langle \sigma v \rangle (r) \propto \langle v_r^2 \rangle$



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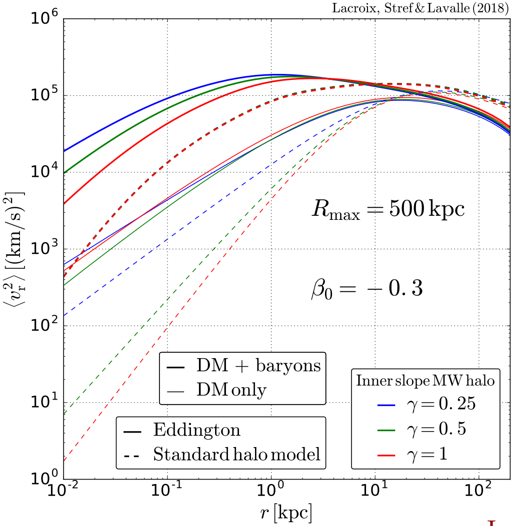
Impact on predictions for DM searches

Extension to anisotropic systems



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Impact on predictions for DM searches

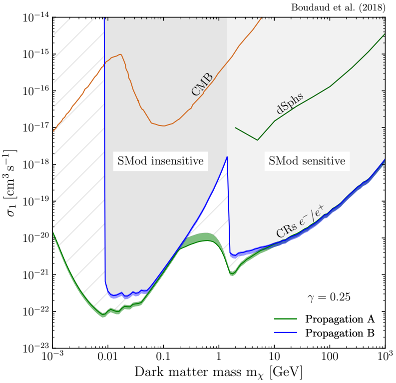
Application: p-wave annihilation

$$\sigma v = \sigma_1 v_{\rm r}^2 \Rightarrow \langle \sigma v \rangle \equiv \langle \sigma v \rangle(r)$$

$$\langle \sigma v \rangle(r) = \sigma_1 \int \mathrm{d}^3 v_1 \mathrm{d}^3 v_2 f_r(\vec{v}_1) f_r(\vec{v}_2)\, v_{\rm r}^2$$

$$\Rightarrow \psi_{\rm e} \neq \langle \sigma v \rangle \int \rho^2(r)\, \mathrm{d}^3 r$$

- Very strong e^+ constraints (Voyager, AMS-02)
- Justifies focusing on Eddington's methods
- Treatment of theoretical uncertainties



Boudaud+ 2018, in prep.

Summary and outlook

Eddington's inversion method

- A few physical assumptions
- Moderate level of technicalities
- Mass model direct input
- Better control astrophysical uncertainties for DM searches
- Dramatic changes wrt Maxwell-Boltzmann

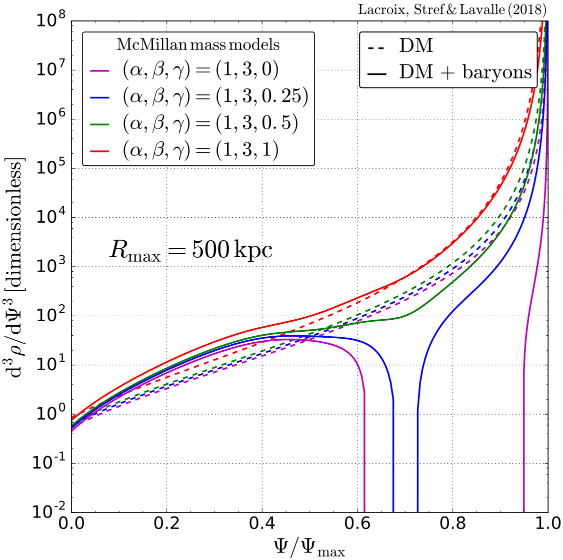
Self-consistency: theoretical validity range

- Radial boundary (direct searches)
- Positive DF + stability

Actual predictivity?

- Testing the method against cosmological simulations
- Not direct fits!!!
- See next talk by A. Nuñez + Lacroix+ 2018b, in prep.

Thank you for your attention!



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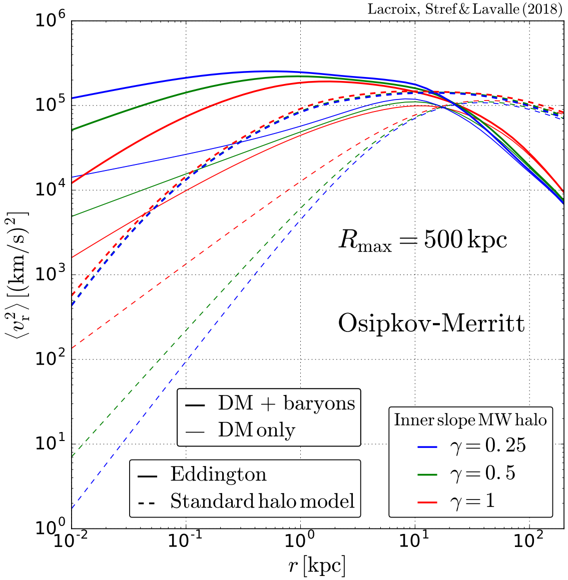

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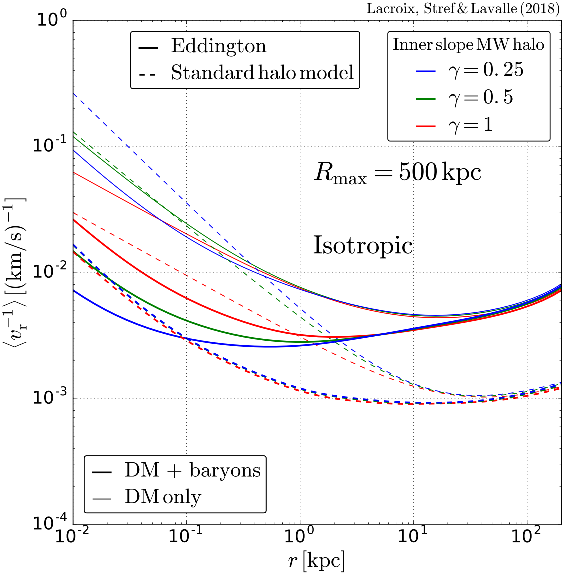

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