Parametrizations of three-body hadronic *B*- and *D*-decay amplitudes in terms of analytic and unitary meson-meson form factors

D. Boito[1], J.-P. Dedonder[2], B. El-Bennich[3], R. Escribano[4], R. Kamiński[5], L. Leśniak[5] and B. Loiseau[2], Phys. Rev. D **96**, 113003 (2017)

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Outline

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1. INTRODUCTION
   - Motivations: why study three-body hadronic *B* and *D* decays?
   - QCD Factorization
   - Quasi-two-body factorization
   - *B* and *D* decays for which explicit parametrizations are provided
2. PARAMETRIZED AMPLITUDES $B \to K\pi^+\pi^-$
   - Parametrization of the $B \to K[\pi^\pm\pi^\mp]_S$ amplitudes
   - Parametrization of the $B \to [K\pi^\pm]_S\pi^\mp$ amplitudes
3. PARAMETRIZED AMPLITUDES $D^0 \to K^0_S K^+ K^-$
4. CONCLUDING REMARKS
   - Backup material

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Three-body hadronic B and D decays rich field

- Standard Model, QCD, CP violation, hadron physics.
- Hadron physics: 2-body resonances + interferences $\Leftrightarrow$ weak observables.

$\Rightarrow$ Final state meson-meson interaction $\to$ theoretical constraints: unitarity, analyticity, chiral symmetry + data other than B and D decays.

- Basic Dalitz-plot analyzes $\to$ isobar model or sum of relativistic Breit-Wigner terms representing the different possible implied resonances + non resonant background - S-wave resonance contributions difficult to fit - beyond isobar?

$\Rightarrow$ Replace by parametrizations in terms of unitary two-meson form factors keeping the weak-interaction dynamics governing the flavor-changing process via W-meson exchange.

- Parametrizations based on published results and motivated by analyzes of high-statistics present and forthcoming data: BES III, LHC*b*, Belle II, Super c-tau factory ...
- No three-body decays factorization theorem but major contributions from intermediate resonances $\rho(770)$, $K^*(892)$, $\phi(1020) \Rightarrow$ quasi-two-body decays.
- For instance, $D^0 \to K_S^0\pi^-\pi^+ \to$ quasi-two-body pairs, $[K_S^0\pi^+]_L\ \pi^-$, $[K_S^0\pi^-]_L\ \pi^+$, $K_S^0\ [\pi^+\pi^-]_L$, 2 of 3 mesons: state in $L = S$ or P wave.

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Parametrization based on models of quasi-two-body QCD factorization

- Decays mediated by local four-quark operators $O_i(\mu)$ forming the weak effective nonrenormalizable Hamiltonian $\mathcal{H}_{\rm eff}$. Schematically for $B \to M_1 M_2^* (\to M_3 M_4)$

$$\langle M_1(p_1) M_2^*(p_2)|\mathcal{H}_{\rm eff}|B(p_B)\rangle = \frac{G_F}{\sqrt{2}}\, V_{\rm CKM} \sum_i C_i(\mu) \langle M_1(p_1) M_2^*(p_2)|O_i(\mu)|B(p_B)\rangle$$

G_F Fermi constant, $V_{\rm CKM}$ product Cabibbo-Kobayashi-Maskawa matrix elements $C_i(\mu)$ Wilson coefficients renormalized at scale $\mu \sim m_b$ (or m_c in D decays)

$\Rightarrow$ In the factorization approach with the strong coupling α_s^n at scale μ,

$$\langle M_1 M_2^*|O_i(\mu)|B\rangle = \Big(\langle M_1|J_1^\nu|B\rangle\langle M_2^*|J_{2\nu}|0\rangle + \langle M_1|J_3^\nu|0\rangle\langle M_2^*|J_{4\nu}|B\rangle\Big)\left[1 + \sum_n r_n \alpha_s^n(\mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)\right],$$

r_n strong interaction constant factors, $|0\rangle$ vacuum state.
Leading order: factorization with either weak quark currents J_1, J_2 or J_3, J_4.

$\Rightarrow$ Radiative corrections to a given order $\alpha_s^n(\mu)$.
Nonperturbative corrections to heavy-quark limit $\mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)$ less reliable for D decays, $m_c \sim m_b/3$ so more phenomenological but good starting point.

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Quasi-two-body amplitudes in terms of meson-meson form factors and decay constants

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- $\langle M_1(p_1) | J_1^\nu | B \rangle$ ($= \langle M_1(p_1) \overline{B} | J_1^\nu | 0 \rangle$) transition form factor: Light-front & relativistic constituent quark models - light-cone sum rules - continuum functional QCD - lattice-QCD. Semi-leptonic decays, e.g. $D^0 \to \pi^- e^+ \nu_e$
- $\langle M_2^* | J_{2\nu} | 0 \rangle \propto \langle M_3 M_4 | J_{2\nu} | 0 \rangle$: form factor, creation from a $\bar{q}q$ pair. Dispersion relations + field theory $\to$ form factor known if $M_3 M_4$ strong interaction known at all energies [G. Barton , Introduction to dispersion techniques in field theory, W. A. Benjamin, INC., New York (1965)]. Two-body data + unitarity + asymptotic QCD + chiral symmetry at low energies.
- $\langle M_1 | J_3^\nu | 0 \rangle$: weak decay constant, known from experiment, e.g. f_π, f_K. Evaluated with lattice-regularized QCD and other nonperturbative approaches.
- $\langle M_2^* | J_{4\nu} | B \rangle \propto \langle M_3 M_4 | J_{4\nu} | B \rangle$: complicated - biggest uncertainty. Semi-leptonic processes: $B^0 \to K^+ \pi^- \mu^+ \mu^-$ or $D^0 \to K^- \pi^+ \mu^+ \mu^-$. Soft-collinear effective theory: amplitude can be factorized in terms of generalized B-to-two-body form factor and two-hadron light-cone distribution amplitude. Our model: $\langle M_2^*(p_2) | J_{4\nu} | B \rangle$ related to $\langle M_2^*(p_2) [\to M_3 M_4] | J_{2\nu} | 0 \rangle$ form factor.

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Example: $D^+ \to [K^-\pi^+]_{S,P}\,\pi^+$ [D. R. Boito, R. Escribano, Phys. Rev. D **80**, 054007 (2009)]

- No penguin (loop with W meson), so only effective Wilson-coefficient $a_{1(2)}$ contributions, θ_C being Cabbibo angle,

$$\langle [K^-\pi^+]_{S,P}\,\pi^+ | \mathcal{H}_{\text{eff}} | D^+ \rangle = \frac{G_F}{\sqrt{2}} \cos^2\theta_C$$
$$\times \Big[a_1 \langle [K^-\pi_1^+]_{S,P} | \bar{s}\gamma^\nu (1-\gamma_5) c | D^+ \rangle \langle \pi_2^+ | \bar{u}\gamma_\nu (1-\gamma_5) d | 0 \rangle$$
$$+ a_2 \langle [K^-\pi_1^+]_{S,P} | \bar{s}\gamma^\nu (1-\gamma_5) d | 0 \rangle \langle \pi_2^+ | \bar{u}\gamma_\nu (1-\gamma_5) c | D^+ \rangle \Big] + (\pi_1^+ \leftrightarrow \pi_2^+)$$

$\Rightarrow$ $\langle [K^-\pi_1^+]_{S,P} | \bar{s}\gamma^\nu (1-\gamma_5) d | 0 \rangle$: $K\pi$ form factors,.

$\Rightarrow$ $\langle [K^-\pi_1^+]_{S,P} | \bar{s}\gamma^\nu (1-\gamma_5) c | D^+ \rangle$: less straightforward, but, assuming dominant intermediate resonance R, it can be written in terms of $K\pi$ form factors.
Requires D to R [$R \to K\pi$] transition form factor.
Feature of crucial importance to our proposed parametrizations.

$\Rightarrow$ $\langle \pi_2^+(p) | \bar{u}\gamma_\nu (1-\gamma_5) d | 0 \rangle = -i f_\pi p_\nu$, f_π pion decay constant,
$\langle \pi_2^+ | \bar{u}\gamma_\nu (1-\gamma_5) c | D^+ \rangle$: $D\pi$ transition form factor.

Parametrized amplitudes in terms of analytic and unitary meson-meson form factors

D. Boito, J.-P. Dedonder, B. El-Bennich, R. Escribano, R. Kamiński, L. Leśniak, B. Loiseau, Phys. Rev. D **96**, 113003 (2017), gives parametrizations, based on quasi-two-body factorization, for the following three-body hadronic amplitudes.

- $B^\pm \to \pi^+\pi^-\pi^\pm$ J.-P. Dedonder *et al.*, Acta Phys. Pol. B **42**, 2013 (2011).
- $B \to K\pi^+\pi^-$: A. Furman *et al.*, Phys. Lett. B **622**, 207 (2005); B. El-Bennich *et al.*, Phys. Rev. D **74**, 114009 (2006); B. El-Bennich *et al.*, Phys. Rev. D **79**, 094005 (2009); Erratum-ibid, Phys. Rev. D **83**, 039903 (2011).
- $B^\pm \to K^+K^-K^\pm$: A. Furman *et al.*, Phys. Lett. B **699**, 102 (2011); L. Leśniak and P. Żenczykowski, Phys. Lett. B **737**, 201 (2014).
- $D^+ \to \pi^+\pi^-\pi^+$: D. Boito *et al.*, Phys. Rev. D **79**, 034020 (2009).
- $D^+ \to K^-\pi^+\pi^+$: D. R. Boito and R. Escribano, Phys. Rev. D **80**, 054007 (2009); D. R. Boito, R. Escribano and M. Jamin, Eur. Phys. J. C **59**, 821 (2009).
- $D^0 \to K^0_S\pi^+\pi^-$: J.-P. Dedonder *et al.*, Phys. Rev. D **89**, 094018 (2014).
- $D^0 \to K^0_S K^+ K^-$: J.-P. Dedonder *et al.*, work in progress .

$\Rightarrow$ Here we illustrate parametrizations: $B \to K\pi^+\pi^-$, $D^0 \to K^0_S K^+ K^-$ for meson-meson final states in S wave.

$B \to K[\pi^+\pi^-]_S$ amplitude

$B(p_B) \to K(p_1)\pi^+(p_2)\pi^-(p_3)$, $s_{12} = (p_1+p_2)^2$, $s_{13} = (p_1+p_3)^2$, $s_{23} = (p_2+p_3)^2$
$s_{12}+s_{13}+s_{23} = m_B^2 + m_K^2 + 2m_\pi^2$.

- Parametrized in terms of three complex parameters, b_i^S, $i = 1, 2, 3$, for the different charges $B = B^\pm$, $K = K^\pm$ and $B = B^0(\bar{B}^0)$, $K = K^0(\bar{K}^0)$ or K_S^0,

$$\mathcal{A}_S(s_{23}) \equiv \langle K\,[\pi^+\pi^-]_S|\mathcal{H}_{\text{eff}}|B\rangle$$
$$= b_1^S\left(M_B^2 - s_{23}\right) F_{0n}^{\pi\pi}(s_{23}) + \left(b_2^S F_0^{BK}(s_{23}) + b_3^S\right) F_{0s}^{\pi\pi}(s_{23}).$$

- Non-strange scalar form factor $F_{0n}^{\pi\pi}(s)$: $f_0(500)$, $f_0(980)$, $f_0(1400)$.
  Strange scalar form factor $F_{0s}^{\pi\pi}(s)$: $f_0(980)$, $f_0(1400)$.
- From $B^- \to K^-[\pi^+\pi^-]_S$ [A. Furman *et al.* Phys. Lett. B **622**, 207 (2005)]

$$b_1^{-S} = \frac{G_F}{\sqrt{2}}\left[\chi f_K F_0^{B\to(\pi\pi)_S}(m_K^2)\, U - \tilde{C}\right]$$

$\tilde{C} = f_\pi F_\pi \left(\lambda_u P_1^{GIM} + \lambda_t P_1\right)$, $\lambda_u = V_{ub}V_{us}^*$, $\lambda_t = V_{tb}V_{ts}^*$, F_π $B\pi$ form factor at $m_\pi^2 = 0$, P_1^{GIM}, P_1 complex charming penguin parameters, U short-distance contribution : CKM $\times$ effective Wilson coefficients. χ fitted free parameter.

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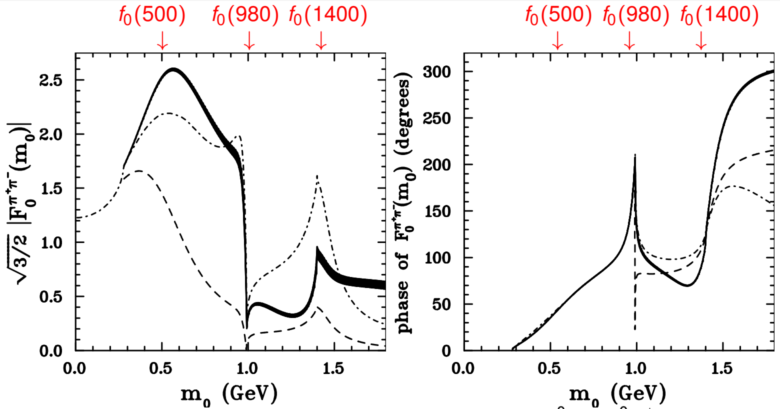
Comparison of unitary non-strange scalar form factors $F_{0n}^{\pi\pi}$



$\Rightarrow$ $F_{0n}^{\pi\pi}(m)$: unitarity + analyticity + $\pi\pi$ data. Dark band: $D^0 \to K_S^0\pi^+\pi^-$ variation with error parameters [J.-P. Dedonder *et al.*, Phys. Rev. D **89**, 094018 (2014)]. Dashed line: $B \to 3\pi$ [J.-P. Dedonder *et al.* Acta Phys. Pol. B **42**, 2013 (2011)]. Dotted-dashed line: B. Moussallam [Eur. Phys. J. C. **14**, 111 (2000)] using Muskhelishvili-Omnès equations.

$B \to [K\pi^\pm]_S\pi^\mp$ amplitude

- In terms of the two complex parameters c_1^S, c_2^S

$$\mathcal{A}_S(s_{12}) \equiv \langle \pi^- \ [K^-\pi^+]_S|\mathcal{H}_{\text{eff}}|B^-\rangle = (c_1^S + c_2^S s_{12}) \frac{F_0^{B\pi}(s_{12}) F_0^{K\pi}(s_{12})}{s_{12}},$$

$F_0^{K\pi}(s)$ [contains $K_0^*(800)$ or κ, $K_0^*(1430)$], $F_0^{B\pi}(s)$, $K\pi$, $B\pi$ scalar form factors.

$\Rightarrow$ Parametrization used with success by R. Aaij *et al.* [LHCb Collaboration], Amplitude analysis of the decay $\bar{B} \to K_S^0\pi^+\pi^-$ and first observation of the *CP* asymmetry in $\bar{B} \to K^*(892)^-\pi^+$, arXiv: 1712.09320 [hep-ex].

- From $B^- \to [K^-\pi^+]_S\pi^-$ [B. El-Bennich *et al.* Phys. Rev. D **79**, 094005 (2009)]

$$c_1^{-S} = \frac{G_F}{\sqrt{2}}(M_B^2 - m_\pi^2)(m_K^2 - m_\pi^2) \times \left[\lambda_u \left(a_4^u(S) - \frac{a_{10}^u(S)}{2} + c_4^u\right) + \lambda_c \left(a_4^c(S) - \frac{a_{10}^c(S)}{2} + c_4^c\right)\right],$$

$\Rightarrow$ $\lambda_c = V_{cb}V_{cs}^*$; $a_i^{u(c)}(S)$, $i = 4, 10$: leading order effective Wilson coefficients + vertex + penguin corrections; $c_4^{u(c)}$ free fitted parameters: non-perturbative + higher order contributions to the penguin diagrams. $\Rightarrow$ Next : $F_0^{K\pi}(s)$ model

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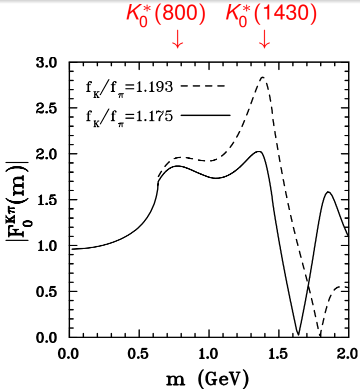
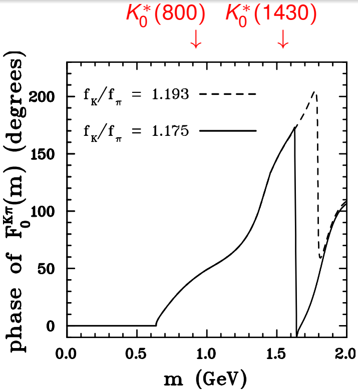
$$\begin{aligned} c_1^{-S} &= \frac{G_F}{\sqrt{2}}(M_B^2 - m_\pi^2)(m_K^2 - m_\pi^2) \\ &\times \left[\lambda_u\left(a_4^u(S) - \frac{a_{10}^u(S)}{2} + c_4^u\right) + \lambda_c\left(a_4^c(S) - \frac{a_{10}^c(S)}{2} + c_4^c\right)\right], \end{aligned}$$

$\Rightarrow$ $\lambda_c = V_{cb}V_{cs}^*$; $a_i^{u(c)}(S)$, $i = 4, 10$: leading order effective Wilson coefficients + vertex + penguin corrections; $c_4^{u(c)}$ free fitted parameters: non-perturbative + higher order contributions to the penguin diagrams. $\Rightarrow$ Next : $F_0^{K\pi}(s)$ model

Scalar $K\pi$ form factor $F_0^{K\pi}(\sqrt{s})$: f_K/f_π 1.193 in fit $B \to [K\pi]\pi$, f_K/f_π=1.175 in fit $D^0 \to K^0_S\pi^+\pi^-$



$\left|F_0^{K^0\pi^-}(m)\right|$ scalar $K\pi$ form factor



Phase of $F_0^{K^0\pi^-}(m)$

$\Rightarrow$ Unitary scalar $K\pi$ form factor: Muskhelishvili-Omnès's 2 coupled channel ($K\pi$, $K\eta'$) equations with experimental $K\pi$ T matrix + chiral symmetry + asymptotic QCD constraints, variation with f_K/f_π [B. Moussallam private communication, see also B. El-Bennich *et al.* Phys. Rev. D **79**, 094005 (2009)]

Parametrization for $D^0 \to K^0_S[K^+K^-]_S$, $D^0 \to [K^0_S K^\pm]_S K^\mp$

$D^0(p_D) \to K^0_S(p_1)K^-(p_2)K^+(p_3)$, $s_{12} = (p_1 + p_2)^2$, $s_{13} = (p_1 + p_3)^2$,
$s_{23} = (p_2 + p_3)^2$, $s_{12} + s_{13} + s_{23} = m^2_{D^0} + m^2_{K^0} + 2m^2_K$.
$[K^+K^-]$: isospin 0 or 1; $[K^0_S K^\pm]$: isospin 1

- With scalar-isocalar $F^{K\bar{K}}_{0n(s)}(s)$ [contains $f_0(980)$, $f_0(1370)$], scalar-isovector $G^{K\bar{K}}_0(s)$ [contains $a_0(980)^0$, $a_0(1450)^0$] form factors,

$$\mathcal{A}^0_{S,0}(s_{23}) = \left(h^S_1 + h^S_2 s_{23}\right) F^{K\bar{K}}_{0n}(s_{23}) + h^S_3 \left(m^2_{K^0} - s_{23}\right) F^{K\bar{K}}_{0s}(s_{23}) + \left(h^S_4 + h^S_5 s_{23}\right) G^{K\bar{K}}_0(s_{23})$$

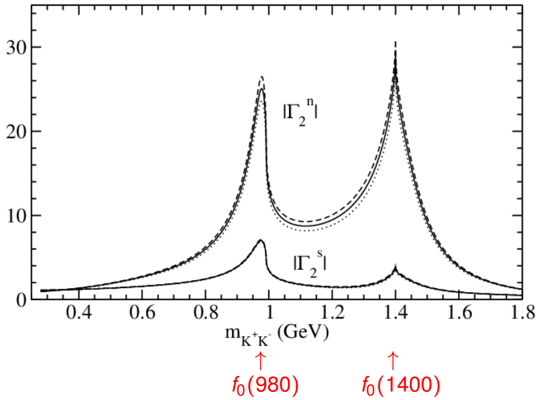
- $G^{K\bar{K}}_0(s)$ [$a_0(980)^-$, $a_0(1450)^-$] $\to \mathcal{A}^0_{S,-}(s_{12}) = (h^S_6 + h^S_7 s_{12})\, G^{K\bar{K}}_0(s_{12})$,
- With $G^{K\bar{K}}_0(s)$ [$a_0(980)^+$, $a_0(1450)^+$] form factor

$$\mathcal{A}^0_{S,+}(s_{13}) = \left[h^S_8 \frac{F^{DK}_0(s_{13})}{s_{13}} + h^S_9(m^2_K - s_{13})\right] G^{K\bar{K}}_0(s_{13})$$

$\Rightarrow$ Next $F^{K\bar{K}}_{0n(s)}(s)$, $G^{K\bar{K}}_0(s)$ models

Unitary scalar-isoscalar $K\bar{K}$ form factors

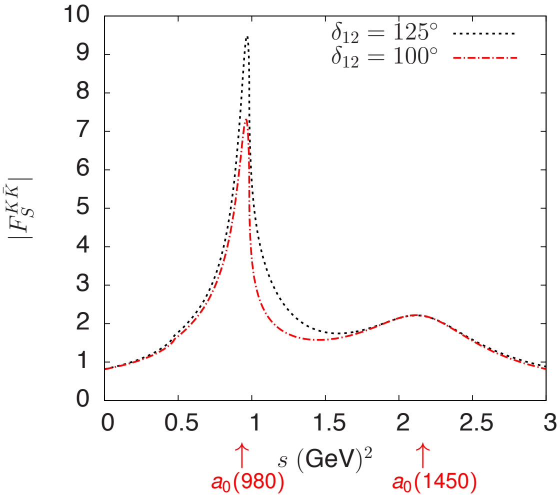
$\to$ Used in preliminary fit $D^0 \to K^0_S K^+ K^-$ [J.-P. Dedonder *et al.]*



Solid lines:
$|\Gamma_2^n(m_{K^+K^-})| = |F_{0n}^{K\bar{K}}(\sqrt{s_{23}})/\sqrt{2}|$,
$|\Gamma_2^s(m_{K^+K^-})| = |F_{0s}^{K\bar{K}}(\sqrt{s_{23}})|$.
Derived by A. Furman, R. Kamiński, L. Leśniak, P. Żenczykowski, [Final state interaction in $B^\pm \to K^+K^-K^\pm$, Phys. Lett. B **699**, 102 (2011)] in
$\to$ solving three coupled channels $\pi\pi$, $K\bar{K}$, 4π (effective 2π-2π or $\sigma\sigma$ or $\eta\eta$ ...),
$\to$ imposing chiral symmetry constraints.

- Dashed and dotted lines: variariations with parameter errors.

Unitary scalar-isovector $G_0^{K\bar{K}}(s)$ [$\equiv F_S^{K\bar{K}}(s)$] form factor



- Unitary model. S-wave coupled channels $\eta\pi$, $K\bar{K}$, asymptotic QCD + chiral symmetry constraints + $a_0(980)$, $a_0(1450)$ $\to$ form factor: Muskhelishvili-Omnès equation (dispersion relation).
  $\delta_{12} \equiv \delta_{11}(\sqrt{s}) + \delta_{22}(\sqrt{s})\big|_{\sqrt{s}=m_{a_0(1450)}}$.
  Channels $\to$ 1 $\equiv \eta\pi$, 2 $\equiv K\bar{K}$.

$\to$ M. Albaladejo, B. Moussallam, Form factors of the isovector scalar current and the $\eta\pi$ scattering phase shifts, Eur. Phys. J. **C75** (2015), arXiv:1507.04526.

Alternatives to isobar-model Dalitz-plot model for weak D, B decays into $\pi\pi\pi$, $K\pi\pi$ and $KK\bar{K}$

- Isobar parametrizations do not respect unitarity and extraction of strong CP phases should be taken with caution. S-wave resonance contribution hard to fit.
- Our parametrizations, not fully three-body unitary, are based on a sound theoretical application of QCD factorization to a hadronic quasi-two-body decay.
- Assume final three-meson state preceded by intermediate resonant states, justified by phenomenological and experimental evidence.

⇒ Analyticity, unitarity, chiral symmetry + correct asymptotic behavior of the two-meson scattering amplitude in S and P waves implemented via analytical and unitary S- and P-wave $\pi\pi$, πK and $K\bar{K}$ form factors entering in hadronic final states of our amplitude parametrizations.

- Parametrized amplitudes can be readily used adjusting parameters in a least-square fit to the Dalitz plot — for a given decay channel — and employing tabulated form factors as functions of momentum squared or energy.

⇒ Explicit amplitude expressions for: $B^\pm \to \pi^+\pi^-\pi^\pm$, $B \to K\,\pi^+\pi^-$, $B^\pm \to K^+K^-K^\pm$, $D^+ \to \pi^-\pi^+\pi^+$, $D^+ \to K^-\pi^+\,\pi^+$, $D^0 \to K^0_S\,\pi^+\,\pi^-$ [previous studies: approach successful] and for $D^0 \to K^0_S\,K^+K^-$ [study in progress].

- In progress: $B^\pm \to K^+K^-\pi^\pm$ [Belle, LHCb] and $B^0 \to K^0_S\,K^+K^-$ [LHCb].

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The weak effective Hamiltonian $\mathcal{H}_{\rm eff}$: sum of local operators $O_i(\mu)$ $\times$ Wilson coefficients $C_i(\mu)$

- Ali *et al.*, Phys. Rev. D **58**, 094009 (1998); M. Beneke *et al.*, Nucl. Phys. **B606**, 245 (2001)

$$\mathcal{H}_{\rm eff}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V^*_{pq} V_{pb} \Big[C_1(\mu) O_1^p(\mu) + C_2(\mu) O_2^p(\mu) + \sum_{i=3}^{10} C_i(\mu) O_i(\mu) + C_{7\gamma}(\mu) O_{7\gamma}(\mu) + C_{8g}(\mu) O_{8g}(\mu) \Big] + \text{h.c.} ,$$

$q = d, s$, V_{ij} CKM matrix elements, Wilson coefficients $C_i(\mu)$: short-distance effects above the renormalization scale μ,
$C_1(\mu) \simeq 1 + \mathcal{O}(\alpha_s(\mu))$, $C_2(\mu) \simeq \mathcal{O}(\alpha_s(\mu))$.

- W exchange $\to O^p_{1(2)}$ two current-current operators with i, j color structure:

$$O_1^p(\mu) = \bar{q}_i \gamma^\mu (1-\gamma_5) p_i \, \bar{p}_j \gamma_\mu (1-\gamma_5) b_j, \quad O_2^p(\mu) = \bar{q}_i \gamma^\mu (1-\gamma_5) p_j \, \bar{p}_j \gamma_\mu (1-\gamma_5) b_i$$

Operators O_i $i = 3$ to 10 from QCD and electroweak penguin diagrams, $O_{7\gamma}$ and O_{8g} electromagnetic and chromomagnetic dipole operators.

$\Rightarrow$ $a_1(\mu) = C_1(\mu) + \frac{1}{N_c} C_2(\mu)$, $a_2(\mu) = C_2(\mu) + \frac{1}{N_c} C_1(\mu)$,
$\uparrow$: color allowed $\uparrow$: color suppressed
Wilson coefficients $C_i(\mu)$ evaluated at renormalization scale $\mu \simeq m_c, m_b$.

$B \to K[\pi^+\pi^-]_P$ amplitude

- Parametrized in terms of complex parameter b_1^P for different charges $B = B^\pm, K = K^\pm$ and $B = B^0(\bar{B}^0), K = K^0(\bar{K}^0)$ or K_S^0,

$$\mathcal{A}_P(s_{12}, s_{13}, s_{23}) \equiv \langle K\ [\pi^+\pi^-]_P | \mathcal{H}_{\text{eff}} | B \rangle = b_1^P\ (s_{13} - s_{12})\ F_1^{\pi\pi}(s_{23}).$$

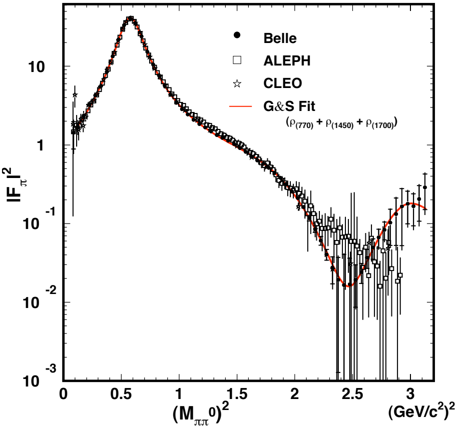
- The pion vector form factor $F_1^{\pi\pi}(s)$: $\rho(770)^0$, $\rho(1450)$ and $\rho(1700)$ contributions.
- From $B^- \to K^-[\pi^+\pi^-]_P$ [B. El-Bennich *et al.* Phys. Rev. D **74**, 114009 (2006)]

$$b_1^{-P} = \frac{G_F}{\sqrt{2} f_\rho} [\, f_K\, A_0^{B\to\rho}(M_K^2)\, (U^- - C^P) + f_\rho\, F_1^{B\to K}(m_\rho^2)\, W^-]$$

$\Rightarrow$ C^p complex charming penguin parameters, U^-, W^- short-distance contributions: CKM $\times$ effective Wilson coefficients.
Decay constants charged ρ, K: mesons f_ρ, f_K.
$A_0^{B\to\rho}(M_K^2)$, $F_1^{B\to K}(m_\rho^2)$, $B\rho$, BK vector form factors.

$\to$ Pion vector form factor $F_1^{\pi\pi}(s)$: extracted from data $\tau^- \to \pi^-\pi^0\nu_\tau$

$\langle[\pi^+(p_2)\pi^-(p_3)]_P|\bar{u}\gamma_\mu(1-\gamma^5)u|0\rangle = -(p_2-p_3)_\mu F_1^{\pi\pi}(q^2)$ - Phenomenological determination



Fit of $\tau^- \to \pi^-\pi^0\nu_\tau$ by Belle Collaboration [Phys. Rev. D **78** 072006 (2008)] with a Gounaris-Sakurai model including $\rho(770)+\rho(1450)+\rho(1700)$

$B \to [K\pi^\pm]_P \pi^\mp$ amplitude

- In terms of one complex parameters c_1^P

$$\mathcal{A}_P(s_{12}, s_{23}) \equiv \langle \pi^- \, [K^-\pi^+]_P | \mathcal{H}_{\text{eff}} | B^- \rangle$$
$$= c_1^P \left(s_{13} - s_{23} - (M_B^2 - m_\pi^2) \, \frac{m_K^2 - m_\pi^2}{s_{12}} \right) F_1^{B\pi}(s_{12}) F_1^{K\pi}(s_{12}).$$

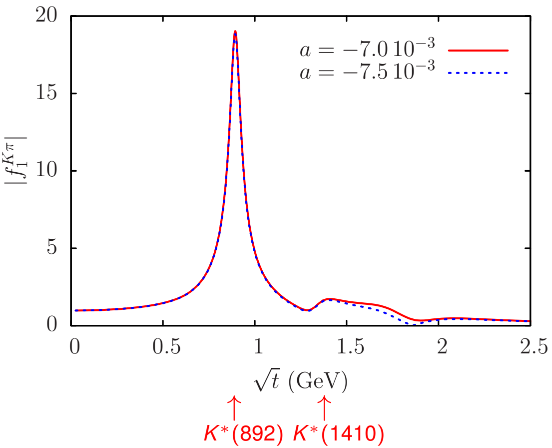
$\Rightarrow$ $F_1^{K\pi}(s)$ [contains $K^*(892), K^*(1410)$], $F_1^{B\pi}(s)$, $K\pi$, $B\pi$ vector form factors.

- From $B^- \to [K^-\pi^+]_S \pi^-$ [B. El-Bennich *et al.* Phys. Rev. D **79**, 094005 (2009)]

$$c_1^{-P} = \frac{G_F}{\sqrt{2}} \left\{ \lambda_u \left(a_4^u(P) - \frac{a_{10}^u(P)}{2} + c_4^u \right) + \lambda_c \left(a_4^c(P) - \frac{a_{10}^c(P)}{2} + c_4^c \right) \right.$$
$$+ \left. 2 \frac{m_{K^*}}{m_b} \frac{f_V^\perp(\mu)}{f_V} \left[\lambda_u \left(a_6^u(P) - \frac{a_8^u(P)}{2} + c_6^u \right) + \lambda_c \left(a_6^c(P) - \frac{a_8^c(P)}{2} + c_6^c \right) \right] \right.$$

$\Rightarrow$ $a_i^{u(c)}(S)$, $i = 4, 6, 10$: leading order effective Wilson coefficients + vertex + penguin corrections; $c_{4,6}^{u(c)}$ free fitted parameters: non-perturbative + higher order contributions to the penguin diagrams; $f_V^\perp(\mu)/f_V$ related to $K^*(892)$ decay constant.

Unitary vector $F_1^{K\pi}(s)$ form factor



- Unitary model. *P*-wave coupled channels $K\pi, K^*\pi, K\rho$ + asymptotic QCD + chiral symmetry constraints + $K\pi$ elastic data+ $K^*(1410)$+ $K^*(1680) \to$ form factor: Muskhelishvili-Omnès equation (dispersion relation).

→ B. Moussallam, Analyticity constraints on the strangeness changing vector current and applications to $\tau \to K\pi\nu_\tau$ and $\tau \to K\pi\pi\nu_\tau$, Eur. Phys. J. C **53**, 401 (2008).

→ Variation with the flavor symmetry breaking parameter *a*.

Parametrization for $D^0 \to K^0_S[K^+K^-]_P$, $D^0 \to [K^0_S K^\pm]_P K^\mp$

- With the vector-isocalar-isovector $F^{K\bar{K}}_{1u}(s)$ [$\omega(782)$, $\omega(1420)$, $\rho(770)^0$, $\rho(1450)^0$, $\rho(1700)^0$], vector-isocalar $F^{K\bar{K}}_{1s}(s)$ [$\phi(1020)$] form factors

$$\mathcal{A}^0_{P,0}(s_{12}, s_{13}, s_{23}) = (s_{12} - s_{13}) \left(h^P_1 F^{K^+K^-}_{1u}(s_{23}) + h^P_2 F^{K^+K^-}_{1s}(s_{23}) \right).$$

- With the vector-isovector $F^{K^-K^0}_1(s)$ [$\rho(770)^-$, $\rho(1450)^-$, $\rho(1700)^-$]

$$\mathcal{A}^0_{P,-}(s_{12}, s_{13}, s_{23}) = h^P_3 \left[s_{23} - s_{13} + \left(m^2_{D^0} - m^2_K\right) \frac{m^2_{K^0} - m^2_K}{s_{12}} \right] F^{K^-K^0}_1(s_{12}).$$

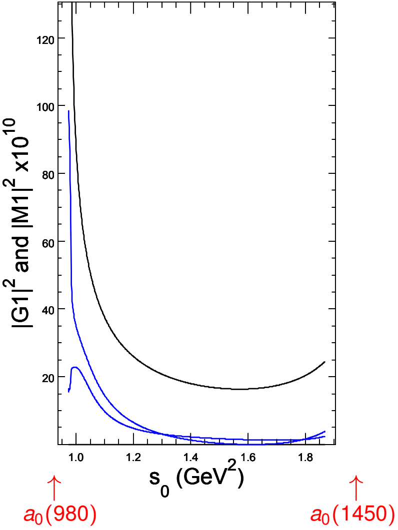
- With the vector-isovector $F^{K^+K^0}_1(s)$ [$\rho(770)^+$, $\rho(1450)^+$, $\rho(1700)^+$]

$$\mathcal{A}^0_{P,+}(s_{12}, s_{13}, s_{23}) = \left[h^P_4 + h^P_5 F^{DK}_1(s_{13}) \right] \times \left[s_{23} - s_{12} + \left(m^2_{D^0} - m^2_K\right) \frac{m^2_{K^0} - m^2_K}{s_{13}} \right] F^{K^+\bar{K}^0}_1(s_{13}).$$

- Full decay amplitude $\mathcal{A}^0 = \mathcal{A}^0_{S,-} + \mathcal{A}^0_{P,-} + \mathcal{A}^0_{S,+} + \mathcal{A}^0_{P,+} + \mathcal{A}^0_{S,0} + \mathcal{A}^0_{P,0} + \cdots$

$\Rightarrow$ Above form factors: vector dominance, quark model assumptions + isospin symmetry [C. Bruch, A. Khodjamirian, J. H. Kühn, Eur. Phys. J. **C39**, 41 (2005)].

Form factor $G_1(s) = G_1(0)F_S^{K\bar{K}}(s)$ in preliminary fit $D^0 \to K^0_S K^+ K^-$ [J.-P. Dedonder *et al.*]



- Unitary model S-wave coupled channels $\eta\pi$, $K\bar{K}$, imposing $a_0(980)$, $a_0(1450)$ poles based on:

→ A. Furman, L. Leśniak, Phys. Lett. B **538**, 266 (2002), arXiv:hep-ph/0203255, *Coupled channel study of a_0 resonances*.

black line: $|S$-wave amplitude $|^2$.

blue Lines: 2 models $|G_1(s_0)|^2$.

Further references on meson-meson form factors

- Scalar $\pi\pi$ form factor: J. T. Daub, C. Hanhart, B. Kubis, A model-independent analysis of final-state interactions in $\bar{B}^0_{d/s} \to J/\psi\pi\pi$, JHEP **1602**, 009 (2016).
- Vector $\pi\pi$ form factor: C. Hanhart, A new parametrization for the vector pion form factor, Phys. Lett. **B** 715, 170 (2012).
  D. Gómez Dumm and P. Roig, Dispersive representation of the pion vector form factor in $\tau \to \pi\pi\nu_\tau$ decays, Eur. Phys. J. C **73**, 2528 (2013).
- Scalar $K\pi$ form factors: M. Jamin, J. A. Oller and A. Pich, Scalar $K\pi$ form factor and light quark masses, Phys. Rev. D **74**, 074009 (2006).
- Vector $K\pi$ form factor: D. R. Boito, R. Escribano and M. Jamin, $K\pi$ vector form factor constrained by $\tau \to K\pi\nu_\tau$ and K_{l3} decays, JHEP **1009**, 031 (2010).
- Scalar KK form factors: B. Moussallam, N_f dependence of the quark condensate from a chiral sum rule, Eur. Phys. J. C **14**, 111 (2000).
- Heavy-to-light transition form factors: M. A. Paracha, B. El-Bennich, M. J. Aslam and I. Ahmed, Ward identities, $B \to V$ transition form factors and applications, J. Phys. Conf. Ser. **630**, 012050 (2015).
  B. El-Bennich, M. A. Ivanov and C. D. Roberts, Flavourful hadronic physics, Nucl. Phys. Proc. Suppl. **199**, 184 (2010).