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K - π scattering : the dispersive and analytic approach

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In collaboration with
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Phys.Rev. D93 (2016) no.7, 074025

Eur.Phys.J. C77 (2017) no.2, 91

Workshop on multibody charmless B-decays. LPNHE, Paris
June 6-7th, 2018

Motivation to study πK scattering

- π, K appear as final products of almost all hadronic strange processes:
Examples: B, D, decays, CP violation studies, etc...
- π, K are Goldstone Bosons of QCD \rightarrow Test Chiral Symmetry Breaking
- Many light resonances appear \rightarrow Strange SPECTROSCOPY

Particularly interesting for this workshop on Bound States:

- $\kappa/K_0^*(800)$ light scalar meson. “needs confirmation”@PDG.
Light scalar mesons longstanding candidates for non-ordinary mesons.
But still some controversy

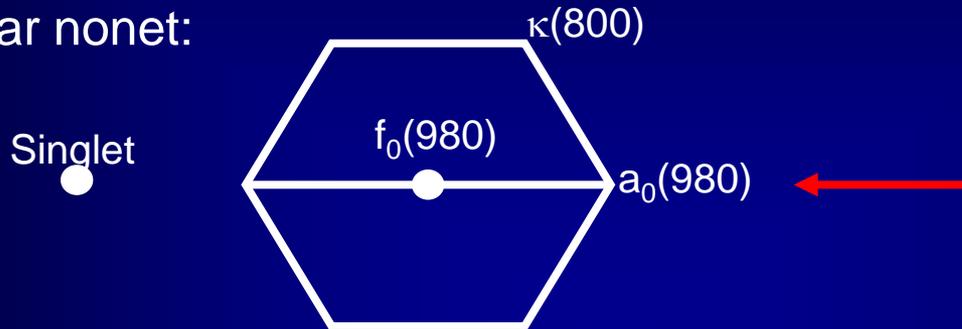
The light scalar controversy. The theory side... classification

Scalar SU(3) multiplets identification controversial

- Too many resonances for many years....
But there is an emerging picture



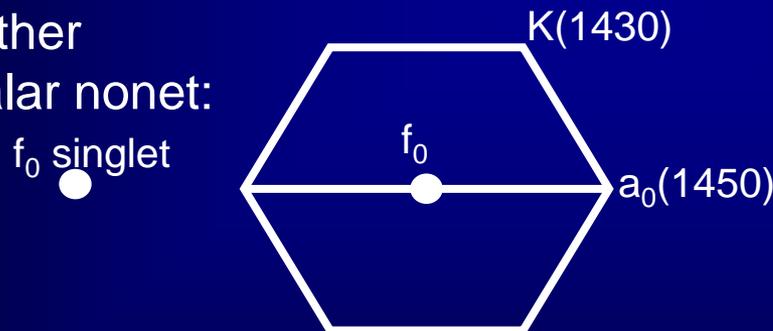
A Light scalar nonet:



Non-strange heavier!!
Inverted hierarchy problem
For quark-antiquark

$f_0(500)$ and $f_0(980)$ are
really octet/singlet mixtures

+ Another
heavier scalar nonet:



+ glueball



Enough f_0 states have been observed: $f_0(1370)$, $f_0(1500)$, $f_0(1700)$.
The whole picture is complicated by mixture between them (lots of works here)

Only the $\kappa(800)$ or $K0^*(800)$ "Needs Confirmation" @ PDG

Data on πK scattering:

Most reliable sets:

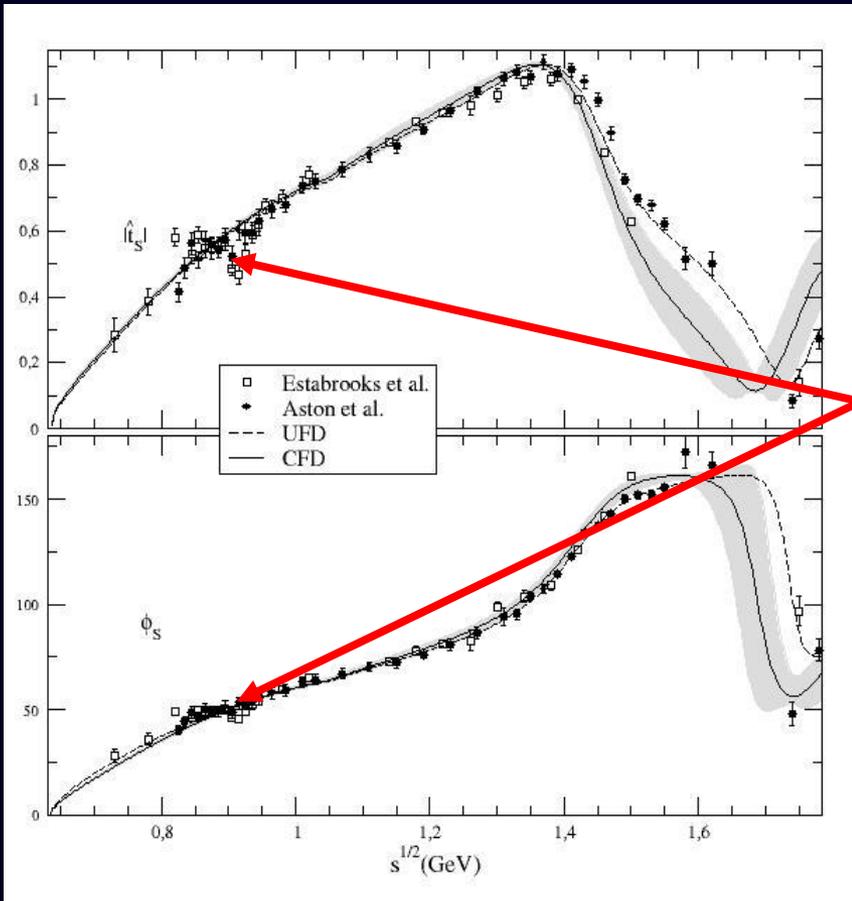
Estabrooks et al. 78 (SLAC)

Aston et al. 88 (SLAC-LASS)

$l=1/2$ and $3/2$ combination

No clear “peak” or phase movement
of $\kappa/K_0^*(800)$ resonance

Definitely NO BREIT-WIGNER shape



Mathematically correct to use POLES

Strong support for $K_0^*(800)$ from decays of heavier mesons, but rigorous model-independent extractions absent. Often inadequate Breit-Wigner formalism

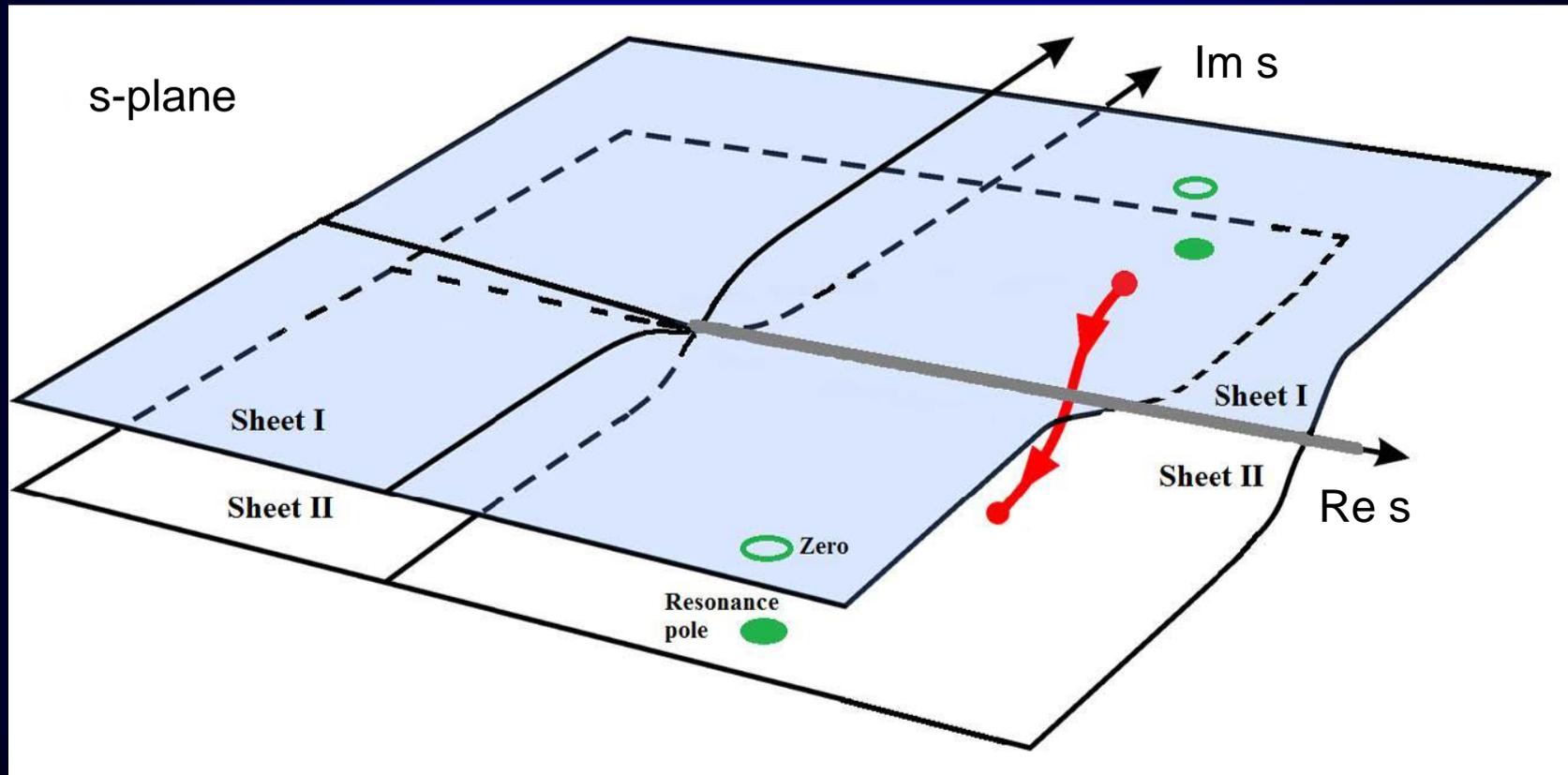
Resonances as poles

The Breit-Wigner shape is just an approximation for narrow and isolated resonances

The universal features of resonances are their pole positions and residues *

$$\sqrt{s_{pole}} \approx M - i \Gamma/2$$

*in the Riemann sheet obtained from an analytic continuation through the physical cut



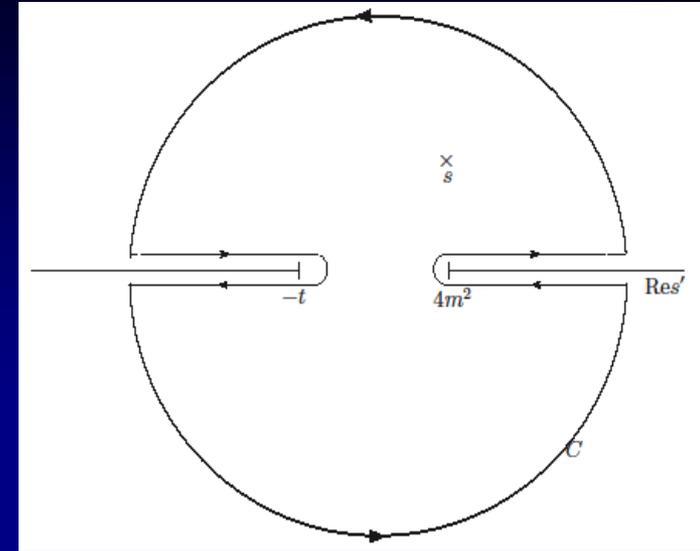
Why use dispersion relations?

CAUSALITY:

Amplitudes $T(s,t)$ are ANALYTIC in complex s plane but for cuts for thresholds.

Crossing implies **left cut** from u-channel threshold

Cauchy Theorem determines $T(s,t)$ at ANY s , from an INTEGRAL on the contour



If $T \rightarrow 0$ fast enough at high s , curved part vanishes

$$T(s, t) = \int_{th}^{\infty} \frac{Im T(s', t)}{s - s'} ds' + LC$$

Otherwise, determined up to polynomial (subtractions)

Left cut usually a problem

Good for:

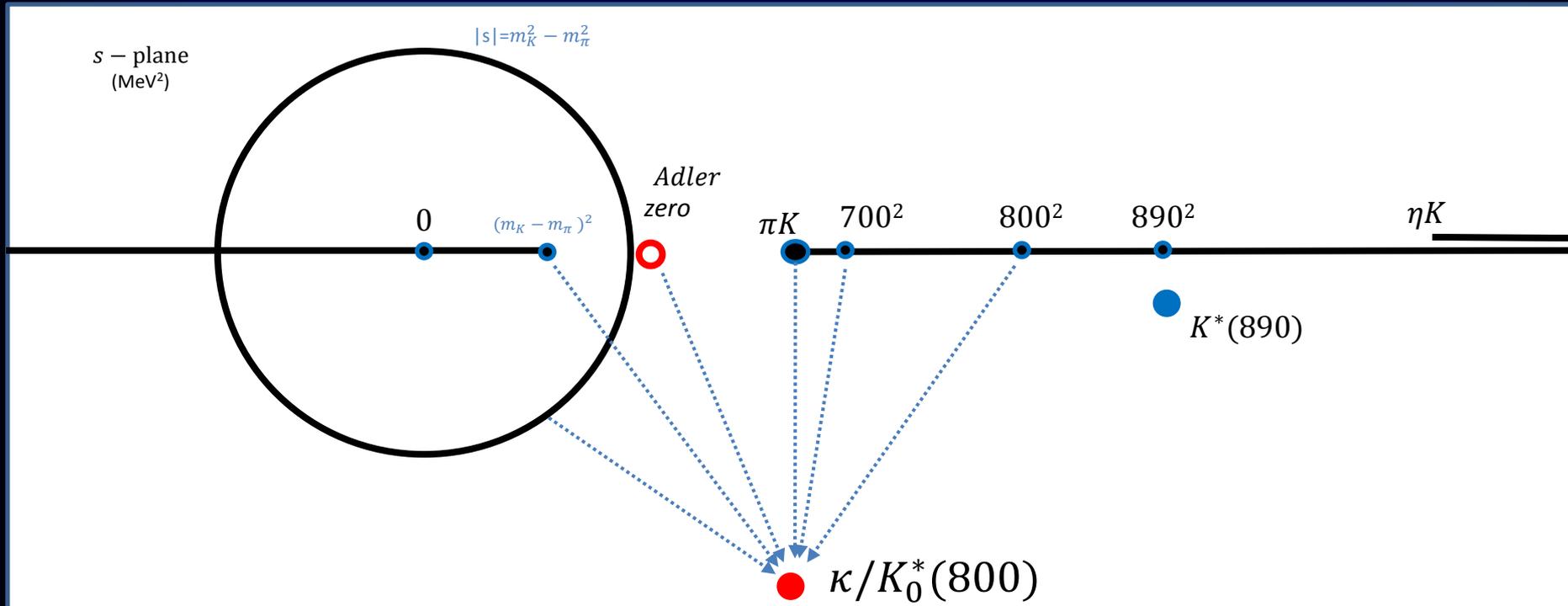
- 1) Calculating $T(s,t)$ where there is not data
- 2) Constraining data analysis
- 3) ONLY MODEL INDEPENDENT extrapolation to complex s -plane without extra assumptions

Why so much worries about low energy and CORRECT ANALYTIC STRUCTURE?

Analyticity is expressed in the s -variable, not in \sqrt{s}

Why so much worries about low energy and CORRECT ANALYTIC STRUCTURE?

Analyticity is expressed in the s -variable, not in \sqrt{s}



Important for
the $\kappa/K_0^*(800)$

- Threshold behavior (chiral symmetry)
- Subthreshold behavior (chiral symmetry \rightarrow Adler zeros)
- Other cuts (Left & circular)
- Avoid spurious singularities

Problem shared
by lattice!

Less important for other resonances...

So, we need to get rid of ONE VARIABLE to write CAUCHY THEOREM in terms of the other one

TWO MAIN APPROACHES

- 1) Integrate one variable and keep the other
(partial wave dispersion relations)

- Analytic structure complicated if unequal masses (Circular cuts)
- For **elastic** region second Riemann sheet is easy to obtain.

Due to elastic unitarity:

$$S^{II}(s) = \frac{1}{S^I(s)}$$

Recalling $s(s) = 1 + 2i\sigma t(s), \quad \sigma(s) = \frac{k}{2\sqrt{s}}$

The second sheet is then:

$$t^{II}(s) = \frac{t^I(s)}{1 + 2i\sigma t^I(s)}$$

Looking for resonance poles
is nothing but looking for a zero in that denominator
on the first Riemann sheet accessible with the pw DR

The problem is the left (and circular) cut

Partial Wave Dispersion Relations: Unitarized ChPT

90's Truong, Dobado, Herrero, JRP, Oset, Oller, Ruiz Arriola, Nieves, Meissner,...

Unitarized ChPT

Uses Chiral Perturbation Theory amplitudes inside dispersion relation.

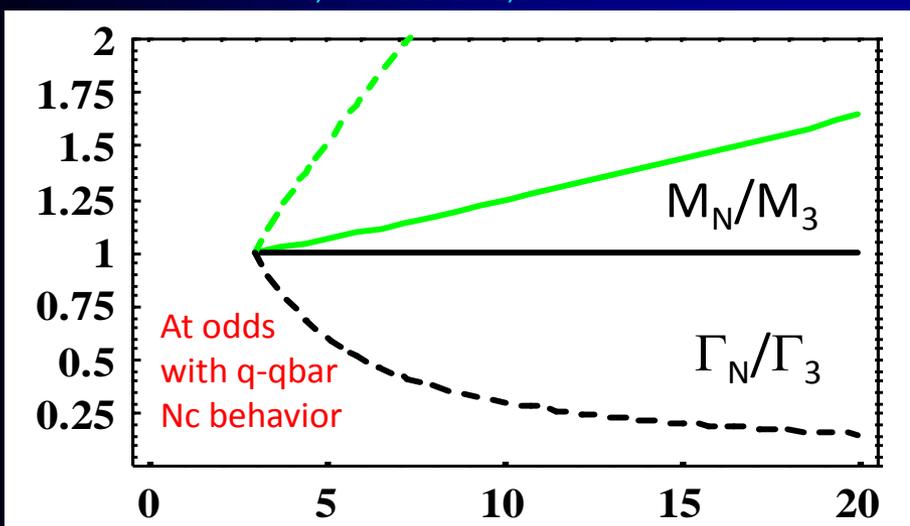
Relatively simple, although different levels of rigour. Generates all scalars

LEFT CUT APPROXIMATED, not so good for precision: $(753 \pm 52) - i(235 \pm 33) \text{ MeV}$

But good for connecting with QCD. Strong hints of non-ordinary nature:

N_c behavior

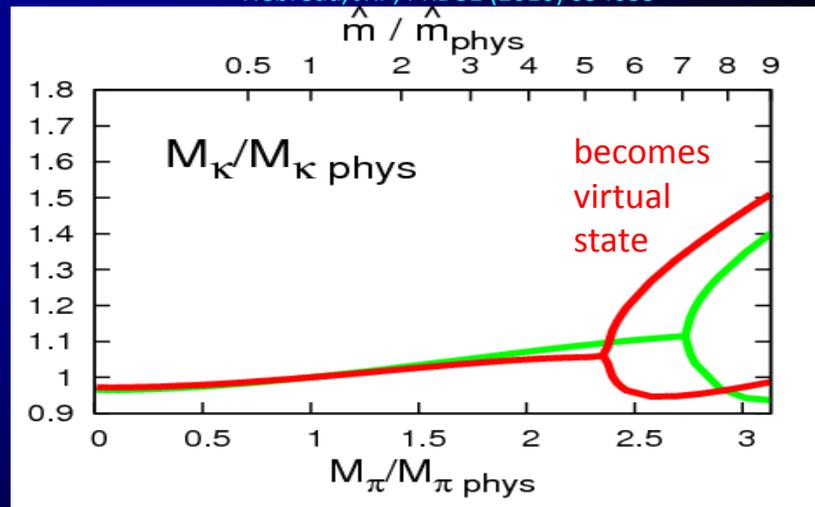
JRP, PRL. 92:102001,2004



Correct behavior obtained for vectors

m_q dependence

Nebreda, JRP, PRD81 (2010) 054035



Virtual state recently found on lattice

Dudek, Edwards, Thomas, Wilson, PRL. 113 (2014) 18, 182001

Both suggest important "molecular" component

● Roy-like equations. 70's Roy, Basdevant, Pennington, Petersen...

00's Ananthanarayan, Caprini, Colangelo, Gasser, Leutwyler, Moussallam, Descotes Genon, Lesniak, Kaminski, JRP, Ruiz de Elvira, Yndurain...

LEFT CUT WITH PRECISION.

PRICE: Infinite set of coupled integral equations. VALIDITY LIMITED at ~1.1 GeV

Use data on all waves + high energy . Optional: ChPT predictions for subtraction constants

The most precise and model independent pole determinations

$f_0(500)$ and $K_0^*(800)$ existence, mass and width
firmly established with precision

$(658 \pm 13) - i(278.5 \pm 12)$ MeV

Descotes-Genon, B. Moussallam

Listed @PDG, but not enough for PDG

We have been asked for an independent
dispersive analysis to trigger the PDG revision

Two strategies

- SOLVE equations: (Ananthanarayan, Colangelo, Gasser, Leutwyler, Caprini, Moussallam, Stern...)

S and P wave solution for Roy-like equations unique at low energy if high-energy, higher waves and scattering lengths known. (in isospin limit)

NO scattering DATA used at low energies ($\sqrt{s} \leq 1 \text{ GeV}$)

Good if interested in low energy scattering and do not trust data.

Uses ChPT/other input for threshold parameter

Already followed by Paris group (B. Moussallam et al.) . Most reliable determination so far.

- Impose Dispersion Relations on fits to data. (García-Martín, Kaminski,JRP, Ruiz de Elvira, Ynduráin)

Use any functional form and fit to DATA imposing DR within uncertainties.

Also needs input on other waves and high energy.

(But you can use physical inspiration for clever choices of parameterizations)

THIS IS OUR APPROACH

So, we need to get rid of ONE VARIABLE to write CAUCHY THEOREM in terms of the other one

TWO MAIN APPROACHES

- 1) Integrate one variable and keep the other
(partial wave dispersion relations)
- 2) Fix one variable in terms of the other
(fixed-t, hyperbolic relations...)

Fixed-t Dispersion Relations (DR)

Simple analytic structure in s-plane, simple derivation and use

Left cut: With crossing can be rewritten in terms of physical region

Most popular: $t_0=0$, **FORWARD DISPERSION RELATIONS (FDRs)**.

(Kaminski, Pelaez , Yndurain, Garcia Martin, Ruiz de Elvira, Rodas)

One equation per amplitude.

High Energy part known since Forward Amplitude~ Total cross section

Calculated up **1.7 GeV for πK** (and 1400 MeV for $\pi\pi$)

JRP, A .Rodas, Phys.Rev. D93 (2016) no.7, 074025

Not directly usable for unphysical sheets but very useful to constraint physical amplitudes up to relatively high energies

Forward dispersion relations for $K\pi$.

Since interested in the resonance region, we use minimal number of subtractions

Defining the $s \leftrightarrow u$ symmetric and anti-symmetric amplitudes at $t=0$

$$T^+(s) = \frac{T^{1/2}(s) + 2T^{3/2}(s)}{3} = \frac{T^{I_t=0}(s)}{\sqrt{6}},$$
$$T^-(s) = \frac{T^{1/2}(s) - T^{3/2}(s)}{3} = \frac{T^{I_t=1}(s)}{2}.$$

We need one subtraction for the symmetric amplitude

$$\text{Re}T^+(s) = T^+(s_{\text{th}}) + \frac{(s - s_{\text{th}})}{\pi} P \int_{s_{\text{th}}}^{\infty} ds' \left[\frac{\text{Im}T^+(s')}{(s' - s)(s' - s_{\text{th}})} - \frac{\text{Im}T^+(s')}{(s' + s - 2\Sigma_{\pi K})(s' + s_{\text{th}} - 2\Sigma_{\pi K})} \right],$$

And none for the antisymmetric

$$\text{Re}T^-(s) = \frac{(2s - 2\Sigma_{\pi K})}{\pi} P \int_{s_{\text{th}}}^{\infty} ds' \frac{\text{Im}T^-(s')}{(s' - s)(s' + s - 2\Sigma_{\pi K})}.$$

where $\Sigma_{\pi K} = m_{\pi}^2 + m_K^2$

Dispersive analysis of πK scattering DATA up to 1.6 GeV

(not a solution of dispersion relations, but a constrained fit)

A.Rodas & JRP, PRD93,074025 (2016)

First observation:

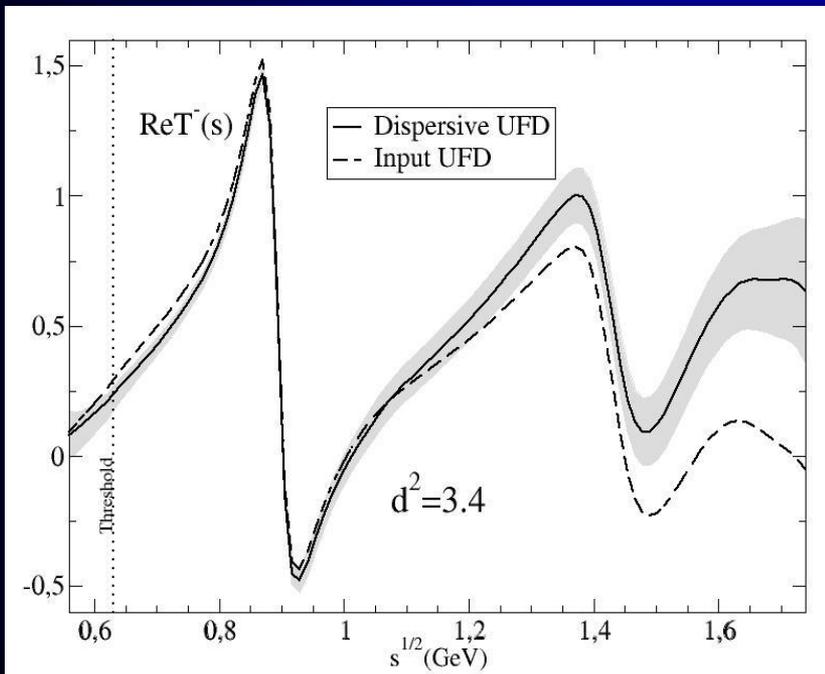
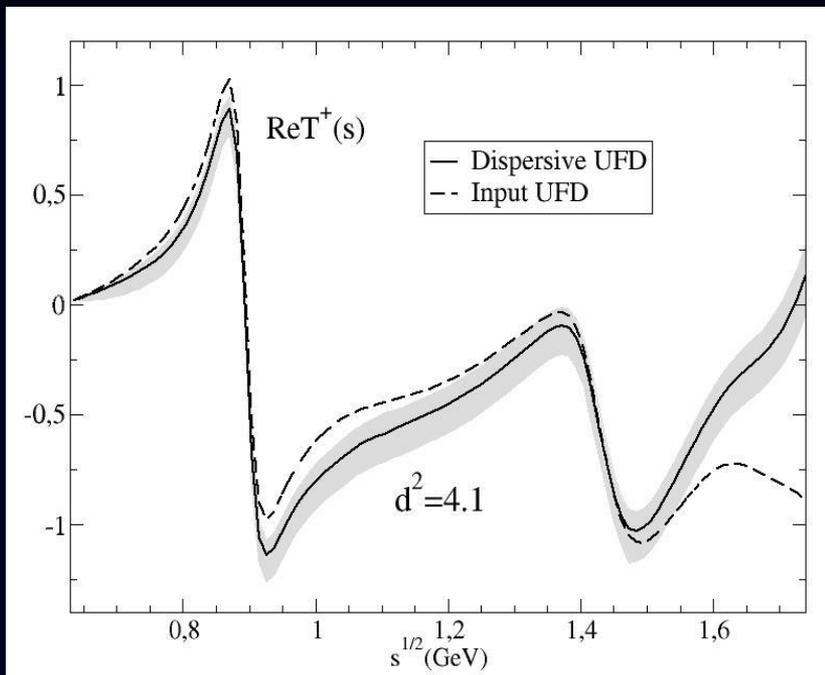
Forward Dispersion relations

Not well satisfied by data

Particularly at high energies

So we use

Forward Dispersion Relations as CONSTRAINTS on fits



How well Dispersion Relations are satisfied by unconstrained fits

Every 22 MeV calculate the difference between both sides of the DR /uncertainty

Define an averaged χ^2 over these points, that we call d^2

d^2 close to 1 means that the relation is well satisfied

$d^2 \gg 1$ means the data set is inconsistent with the relation.

This can be used to check DR

To obtain CONSTRAINED FITS TO DATA (CFD) we minimize:

$$\chi^2 = W \left\{ \overline{d_{T+}}^2 + \overline{d_{T-}}^2 \right\} + \overline{d_{1/2}}^2 + \overline{d_{3/2}}^2 + \sum_k \frac{(p_k - p_k^{exp})^2}{\delta p_k^2}$$

2 FDR's

Sum Rules
threshold

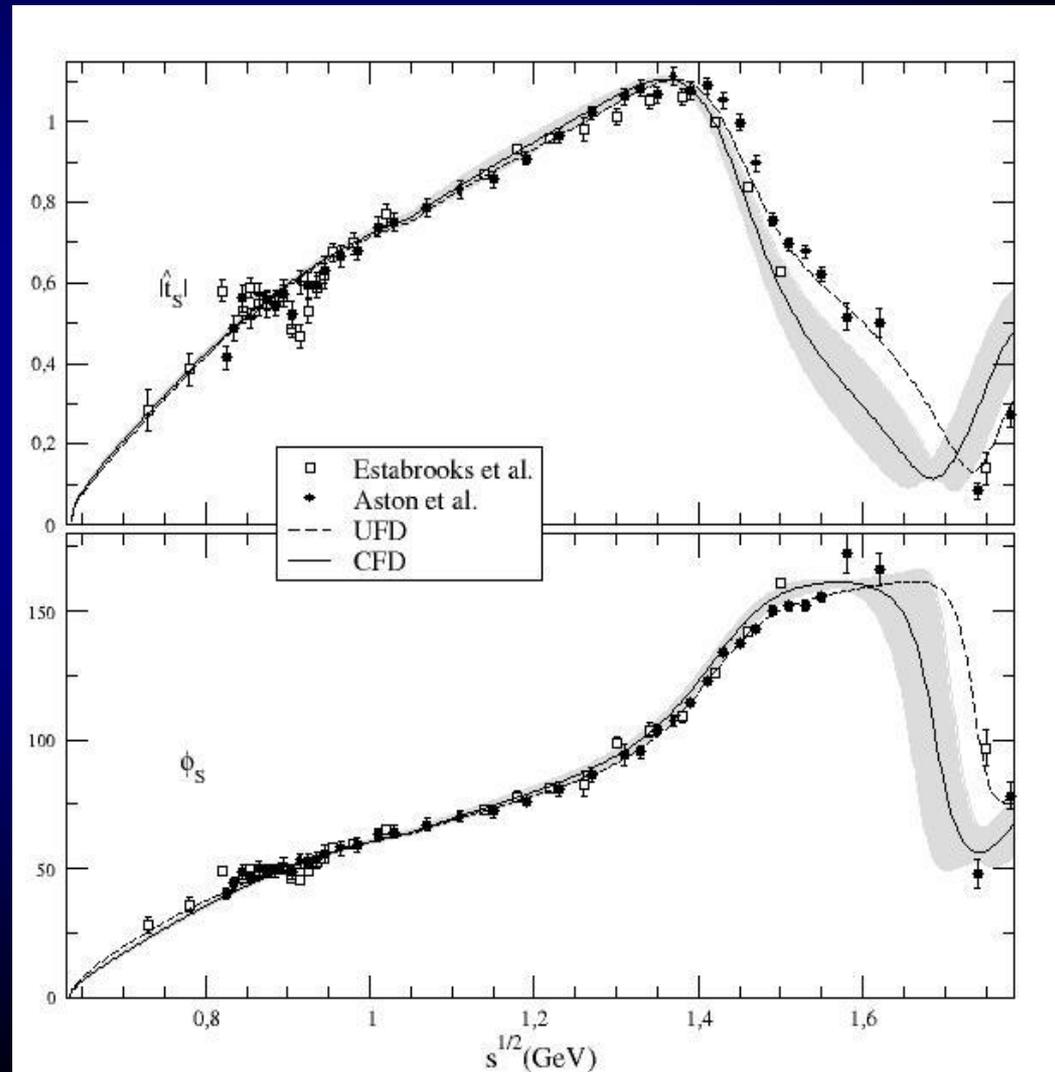
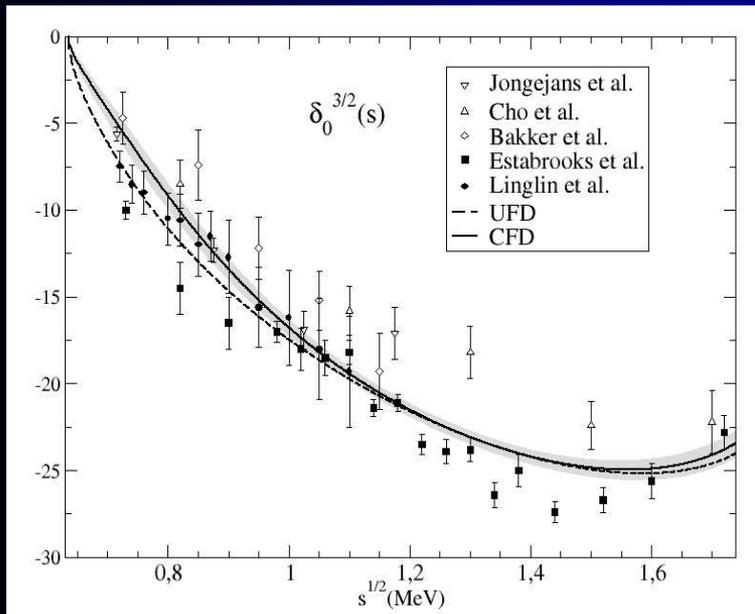
Parameters of the
unconstrained data fits

W roughly counts the number
of effective degrees of freedom
(sometimes we add weight on certain energy regions)

From Unconstrained (UFD) to Constrained Fits to data (CFD)

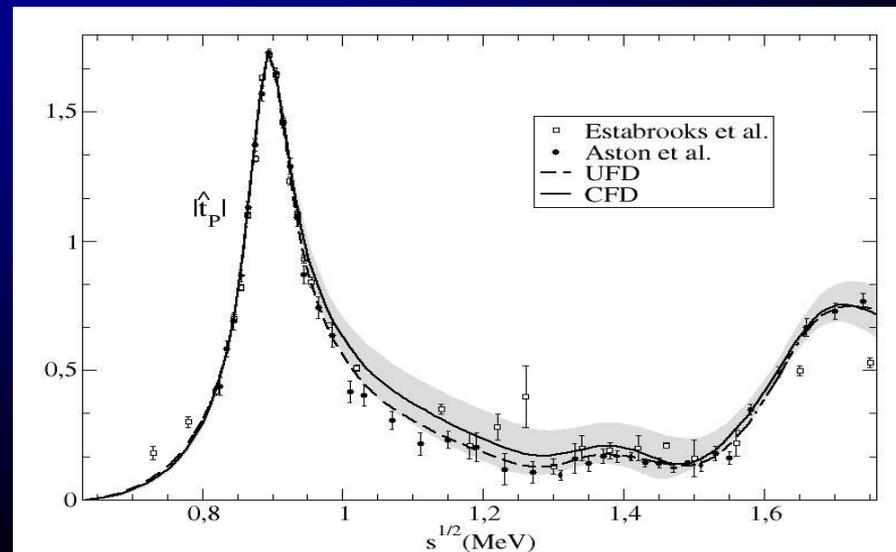
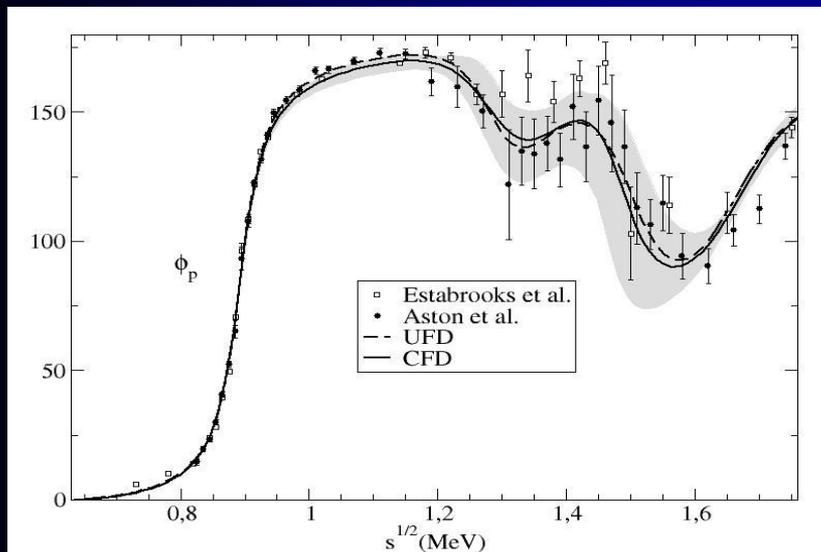
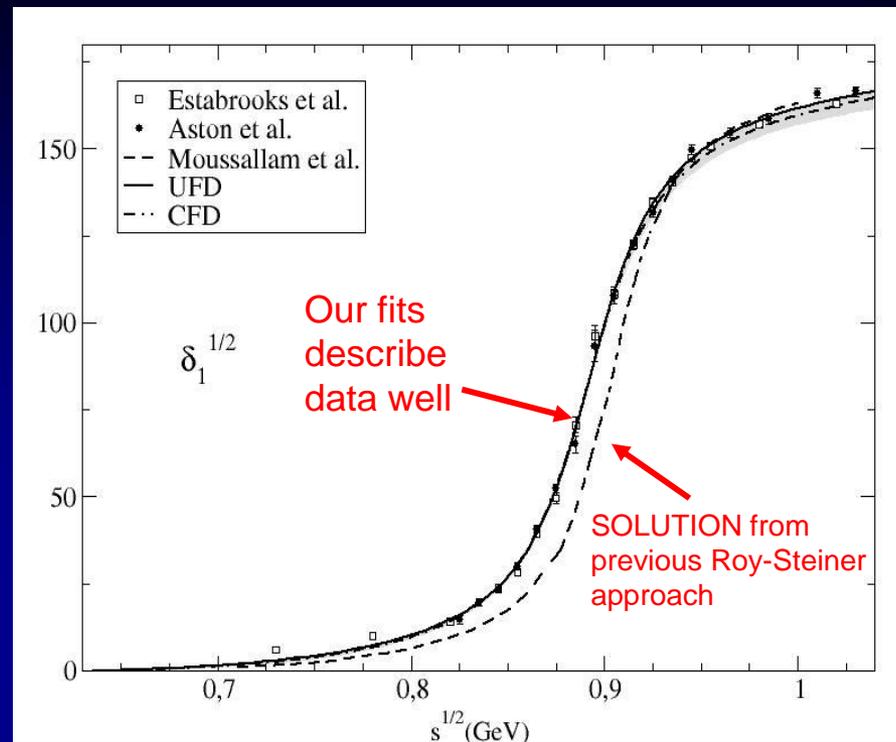
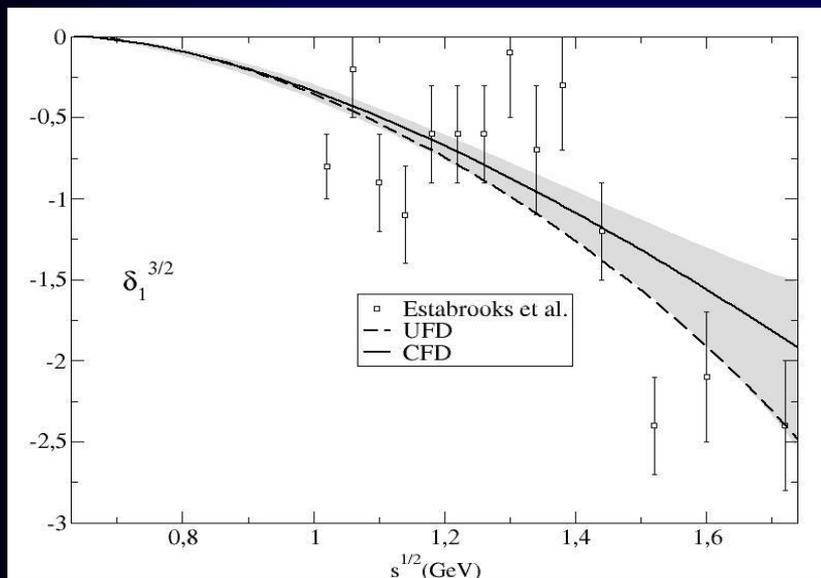
S-waves. The most interesting for the K_0^* resonances

Largest changes from UFD to CFD
at higher energies



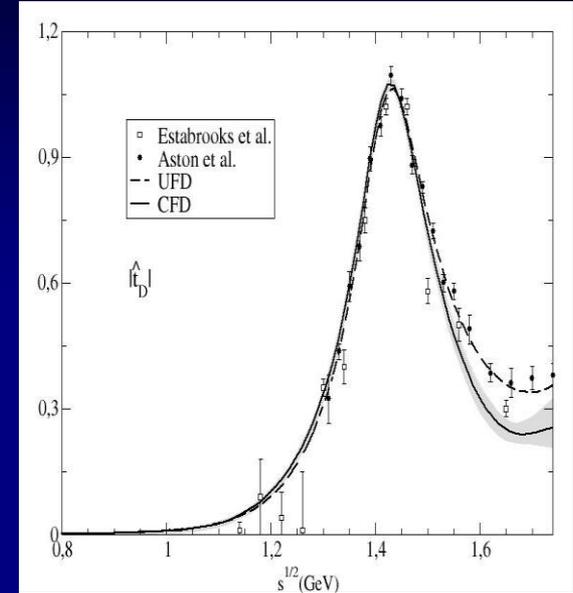
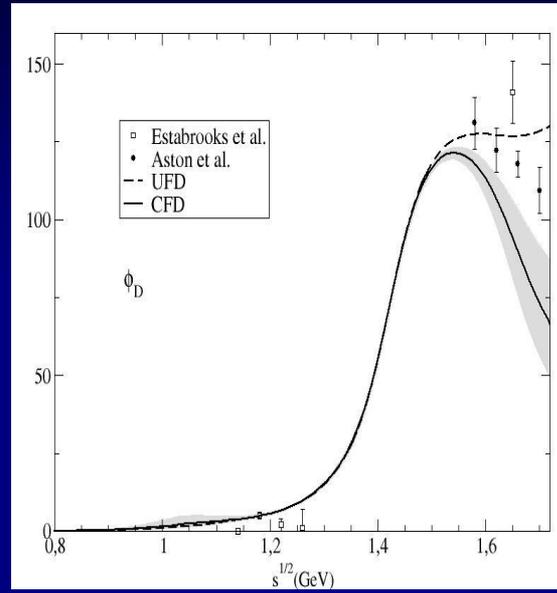
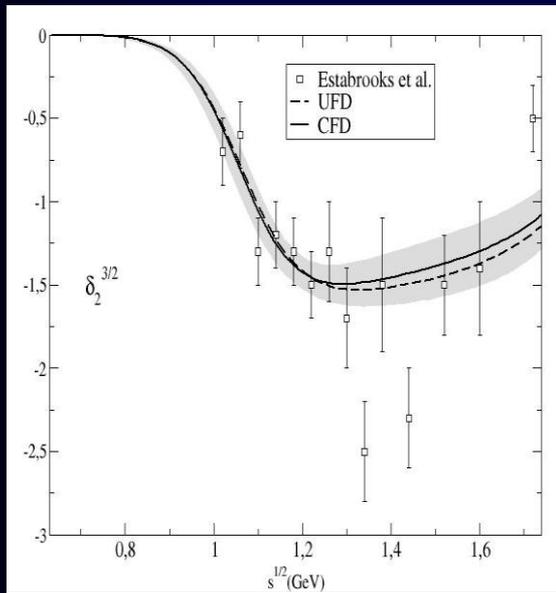
From Unconstrained (UFD) to Constrained Fits to data (CFD)

P-waves: Small changes



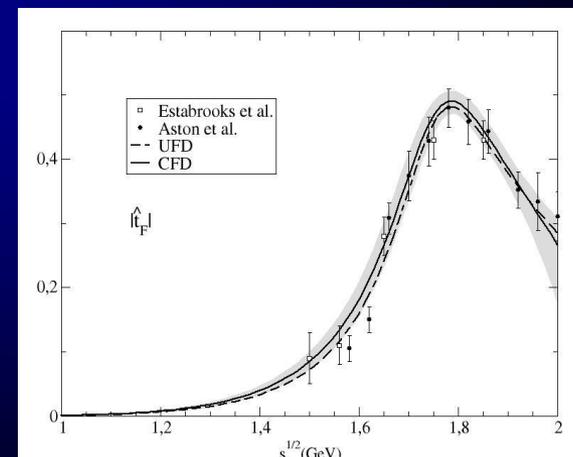
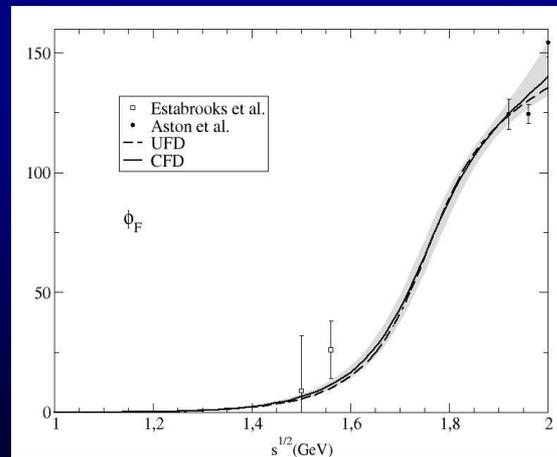
From Unconstrained (UFD) to Constrained Fits to data (CFD)

D-waves: Largest changes of all, but at very high energies

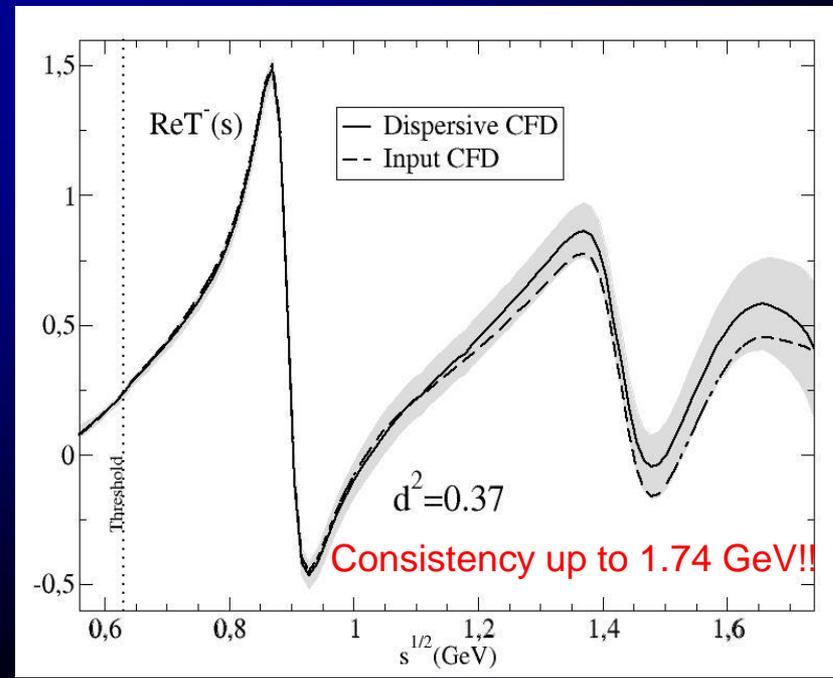
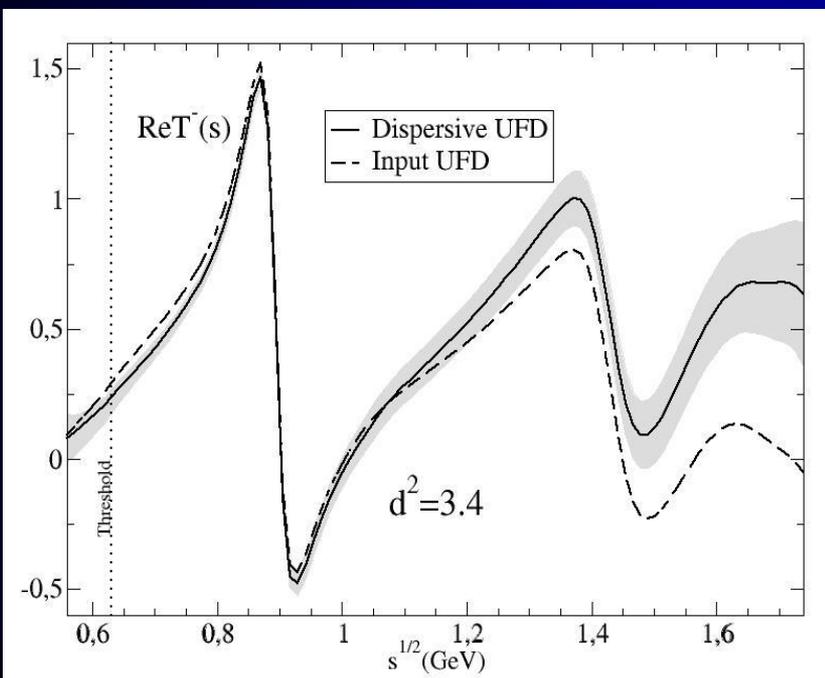
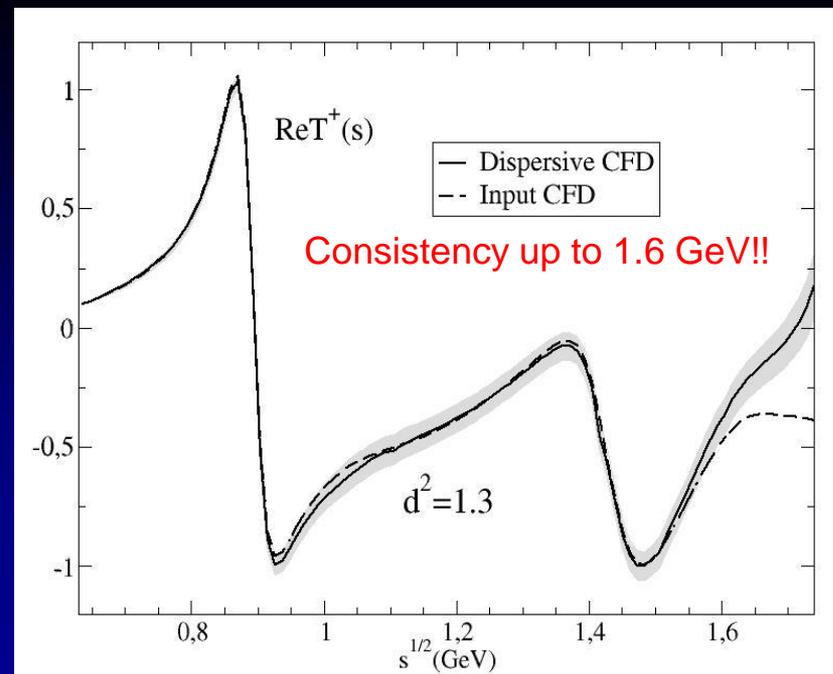
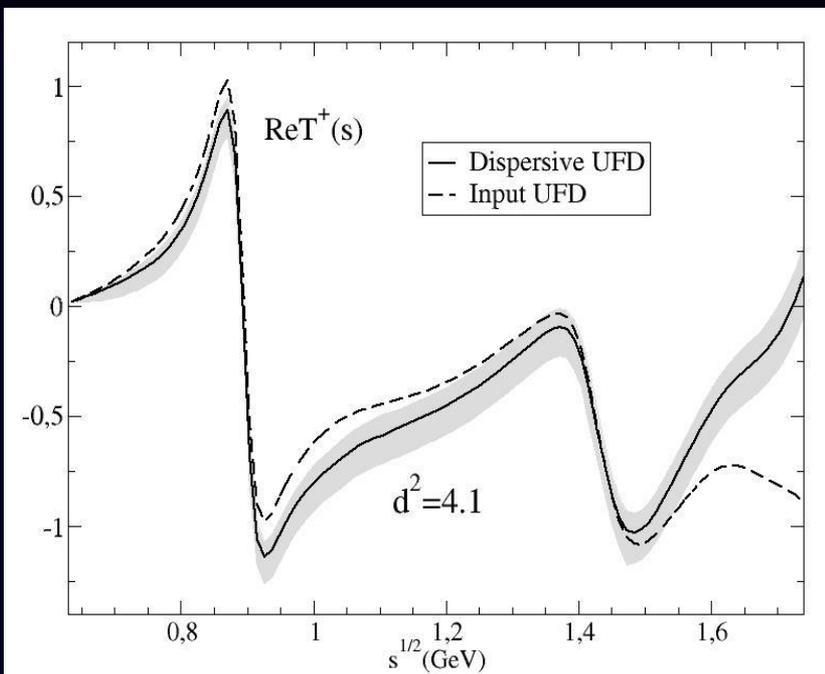


F-waves:

Imperceptible changes



Regge parameterizations allowed to vary: Only πK - ρ residue changes by 1.4 deviations



- We have used FORWARD DISPERSION RELATIONS to constraint πK scattering amplitudes **up to 1.6 GeV**:
 - Simple parameterizations. Easy to use
 - Still describe data
 - Consistent with unitarity, ANALYTICITY and crossing

In progress:

We are about to finish the $\pi\pi\rightarrow KK$ Roy-Steiner analysis up to 1.5 GeV

Working on the Roy-Steiner analysis **for** $\pi K\rightarrow\pi K$. See final slides

Strange scalar resonances from dispersive analysis and analyticity

J. R. Peláez, A. Rodas, J. Ruiz de Elvira

Eur.Phys.J. C77 (2017) no.2, 91

JRP, A. Rodas in preparation

Kappa pole from CFD

We have amplitudes that describe data and satisfy dispersion relations up to 1.6 GeV

There is also a κ POLE in the elastic piece of our CFD parameterizations

Unconstrained Fits (UFD):Elastic region

- We use the unitary functional form for the partial waves

$$t_l^I(s) = \frac{1}{\sigma(s)} \frac{1}{\cot\delta_l^I(s) - i} \quad (5)$$

- Where

$$\cot\delta_l^I(s) = \frac{\sqrt{s}}{2q^{2l+1}} \sum B_n \omega(s)^n \quad (6)$$

- with $\omega(s) = \frac{\sqrt{y(s)} - \alpha\sqrt{y(s_0) - y(s)}}{\sqrt{y(s)} + \alpha\sqrt{y(s_0) - y(s)}}$ as our new variable (conformal mapping).
- Here $y(s) = \left(\frac{s-su}{s+su}\right)^2$ defines the circular cut on the next figure.
- ω used to maximize the analyticity domain.

Unconstrained Fits (UFD): Elastic region

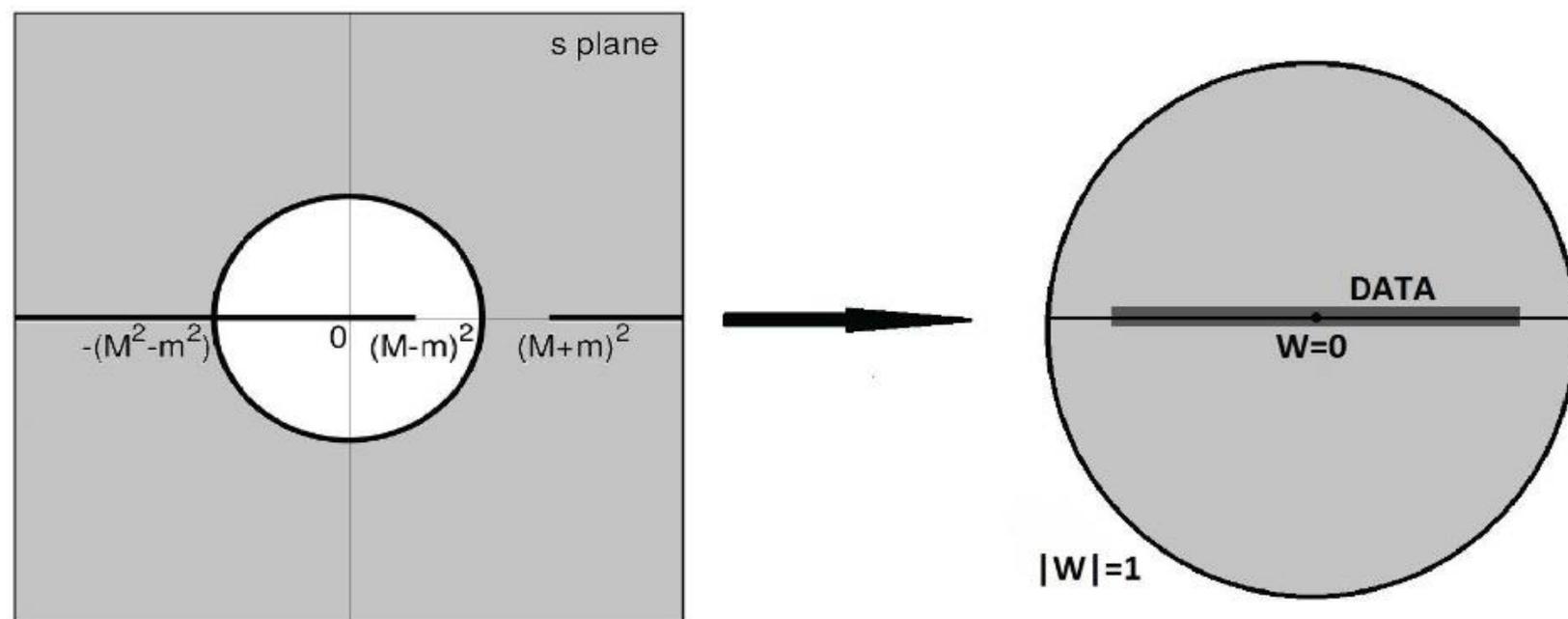


Figure: Structure of the PW.

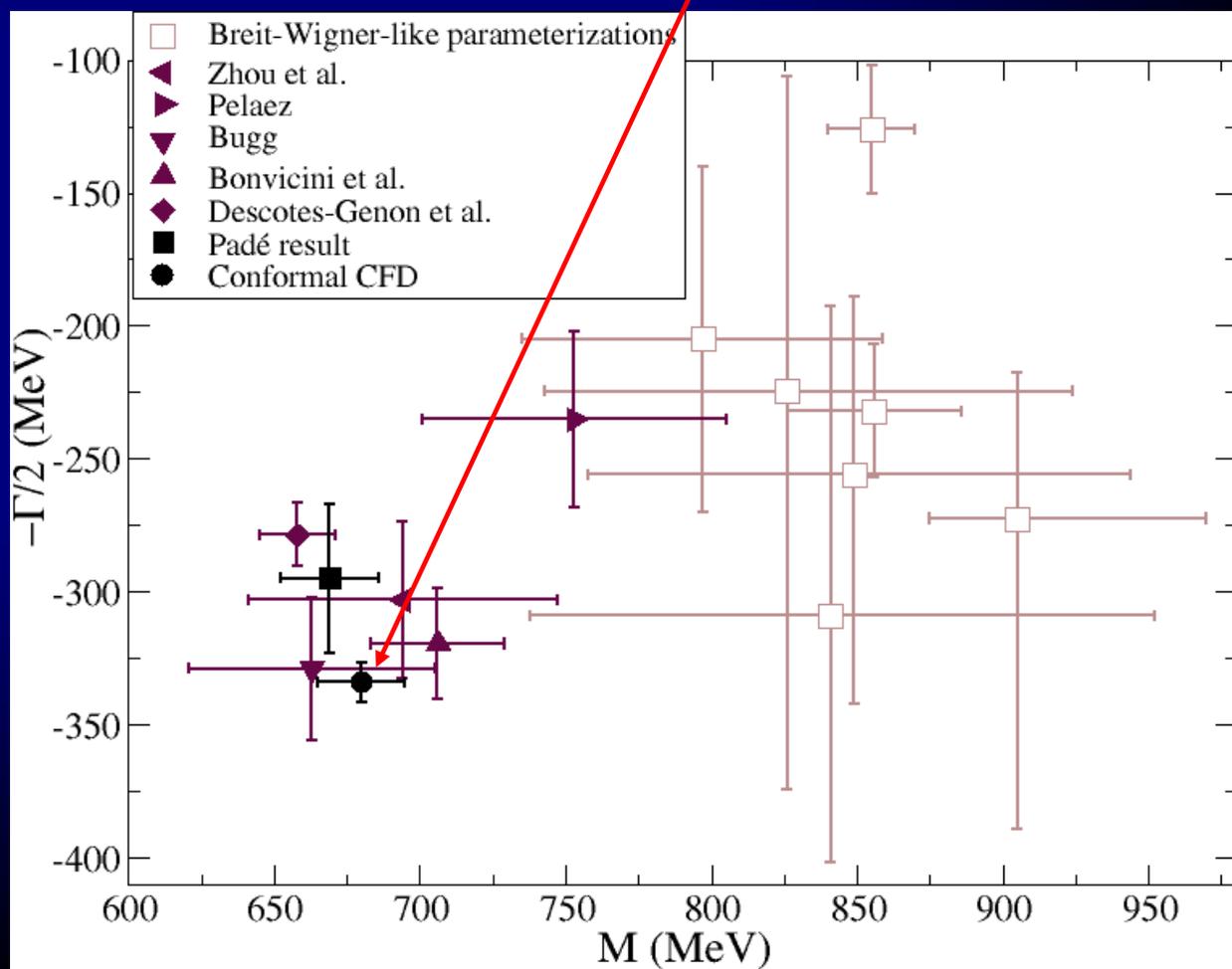
- α is used to center the point of energy s_c for the expansion.

Kappa pole from CFD

1) Extracted from our conformal CFD parameterization [A.Rodas & JRP, PRD93,074025 \(2016\)](#)

Fantastic analyticity properties,
but still not completely model independent

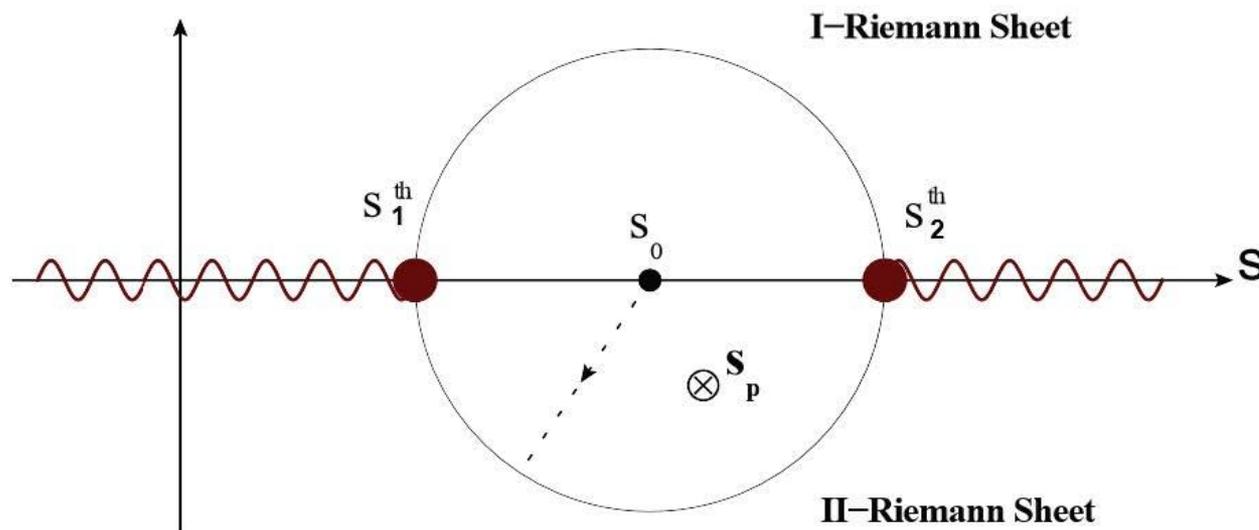
$(680 \pm 15) - i(334 \pm 7.5)$ MeV



Almost model independent: Does not assume any particular functional form (but local determination)

Based on previous works by P.Masjuan, J.J. Sanz Cillero, I. Caprini, J.Ruiz de Elvira

- The method is suitable for the calculation of both elastic and inelastic resonances.
- The Padé sequence gives us the continuation to the continuous Riemann Sheet.
- We take care of the calculation of the errors. Apart from the experimental and systematic errors of each parameterization we also include different fits.



Kappa pole from CFD

1) Extracted from our conformal CFD parameterization [A.Rodas & JRP, PRD93,074025 \(2016\)](#)

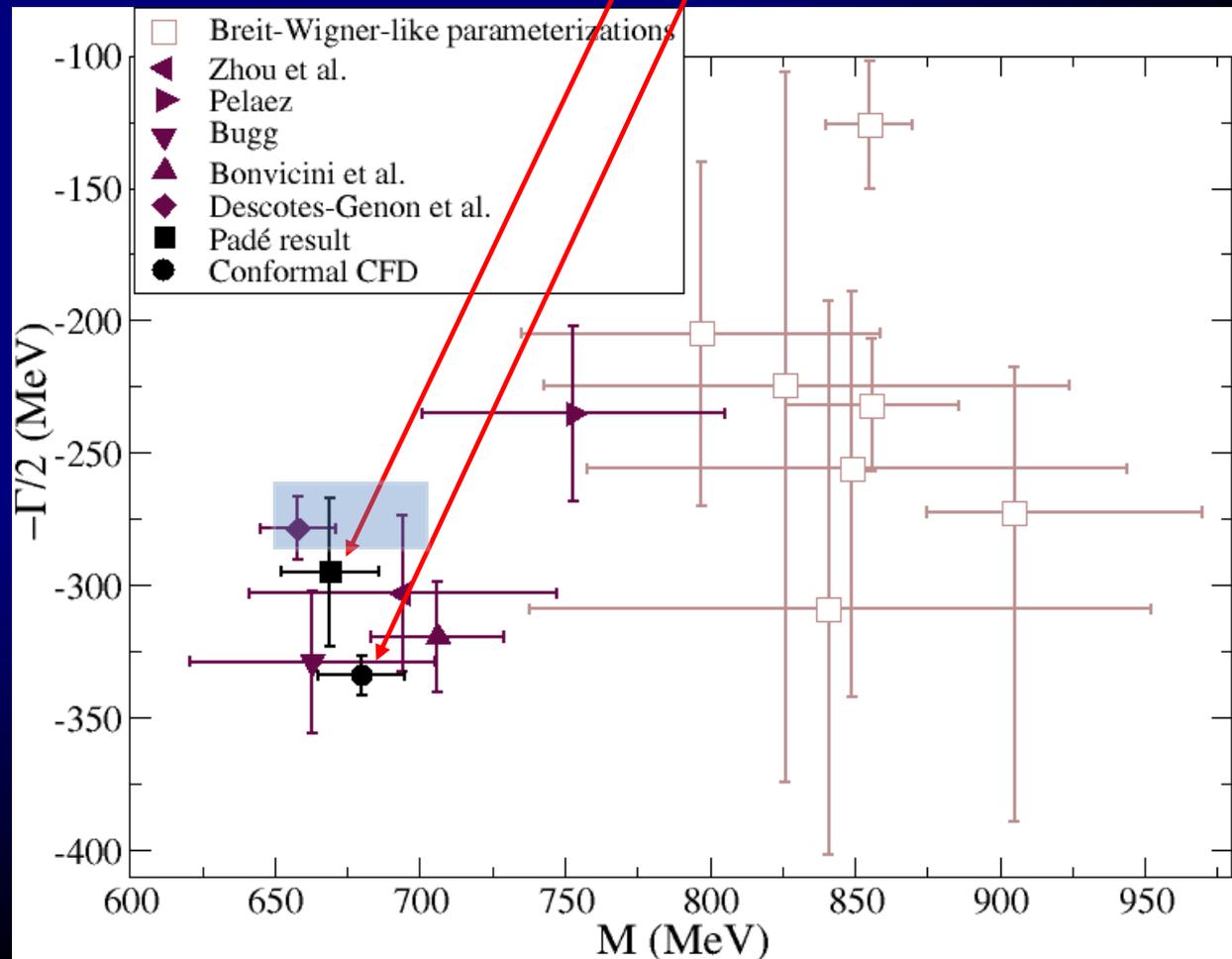
Fantastic analyticity properties,
but still not completely model independent

$$(680 \pm 15) - i(334 \pm 7.5) \text{ MeV}$$

2) Using Padé Sequences...

[A.Rodas & JRP & J. Ruiz de Elvira. Eur. Phys. J. C \(2017\) 77:91](#)

$$(670 \pm 18) - i(295 \pm 28) \text{ MeV}$$



Compare to PDG:
 $(682 \pm 29) - i(273 \pm 12) \text{ MeV}$

Model independent analysis

Roy-Steiner SOLUTION from Paris group

Descotes-Genon-Moussallam 2006

Our Roy-Steiner analysis of FIT to data

JRP, A. Rodas, in preparation

NEW!!

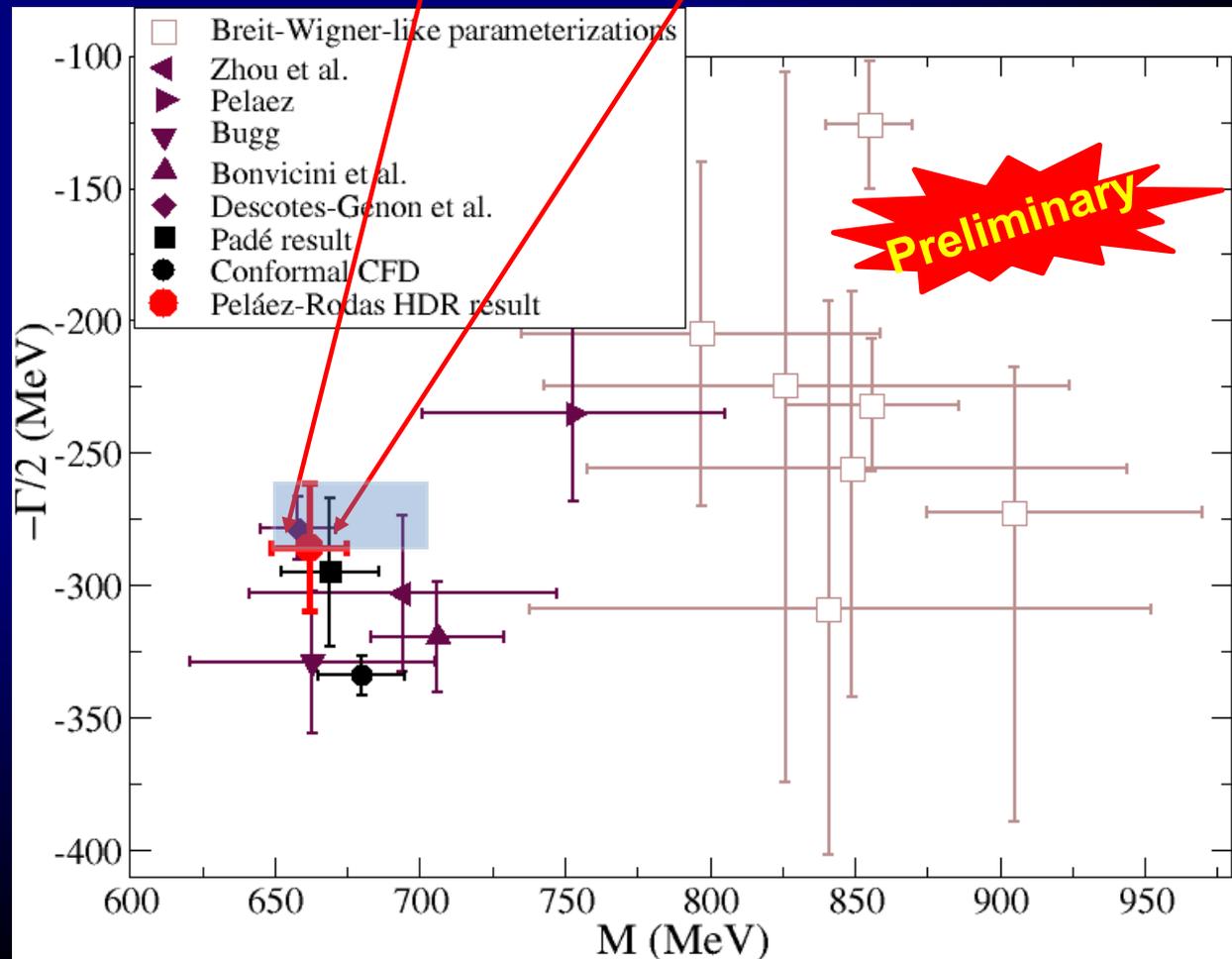
We have:

- Constrained Fit to data (not solved)
- Improved P-wave (data OK)
- Used Hyperbolic DR both in real axis and complex plane.
- Improved Pomeron
- Constrained $\pi\pi \rightarrow KK$ input with DR
- Other technicalities

Independent dispersive determination of the $K_0^*(800)$

$(658 \pm 13) - i(278.5 \pm 12)$ MeV

$(662 \pm 13) - i(289 \pm 25)$ MeV



Summary

- Dispersion relations have been useful for establishing the existence of resonances and for rigorous determinations of their parameters
- For light scalars, they have settled the longstanding σ -meson controversy and are on the way to settle that of the κ -meson.
- We have provided first preliminary results for $K_0^*(800)$

Still in progress:

A second dispersive determination with Roy-Steiner and FDRs will finally settle the $\kappa/K_0^*(800)$ issue at the PDG. Our group has been asked to do it.

First preliminary results available. Nice agreement with other dispersive approach

We are about to finish the $\pi\pi \rightarrow KK$ analysis needed as input for $\pi K \rightarrow \pi K$