QCDF in hadronic multibody B decays

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Workshop on multibody charmless B-hadron decays - Paris June 2018

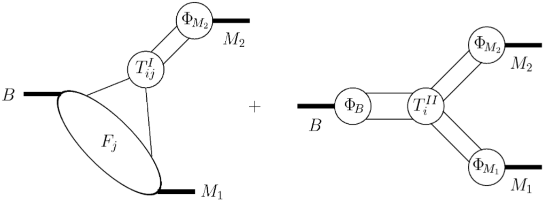
Theor. Physik 1

DFG FOR 1873

QCD Factorization in two-body decays

Beneke, Buchalla, Neubert, Sachrajda [1999]; Beneke, Neubert [2003]

At leading order in the heavy-quark expansion (Λ/m_b)



Hard scattering kernels $T^{I,II}$ perturbatively calculable

$$\langle M_1 M_2|\mathcal{O}_i|\bar{B}\rangle = F^{B\to M_1}\int du T_i^I(u)\Phi_{M_2}(u) + \int d\omega\, du\, dv T_i^{II}\phi_B(\omega)\Phi_{M_1}(u)\Phi_{M_2}(v)$$

Form factors and LCDAs universal non-perturbative objects

- Vertex corrections $T^I = 1 + \mathcal{O}(\alpha_s/\pi)$
- Spectator scattering $T^{II} = \mathcal{O}(\alpha_s)$ and real

NNLO penguin contributions are being calculated Beneke, Bell, Huber [in progress]

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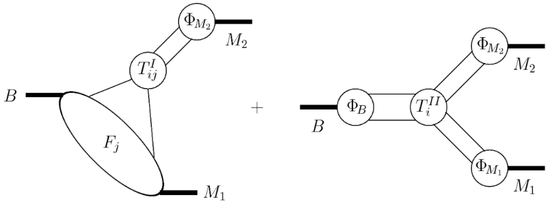

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Form factors and LCDAs universal non-perturbative objects

$$A_{\mathrm{CP}} = \mathcal{O}(\alpha_s/\pi) + \mathcal{O}(\Lambda/m_b)$$

Indications for the presence of subleading terms

Factorization in three-body B decays

Kraenkl, Mannel, Virto [2015]; Klein, Mannel, Virto, KKV [2018]

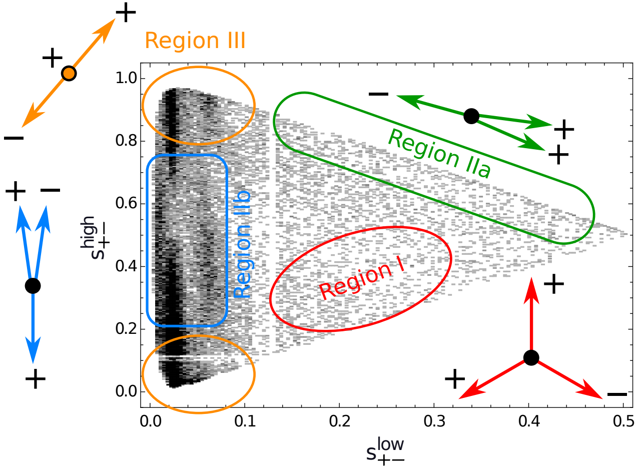
Data-driven model-independent factorization approach

- Improvement over quasi-two body interpretation
- Introduces new non-perturbative strong phases
- Focuss here on $B \to \pi\pi\pi$ but similar for $B \to hhh$

(First) Challenge: Reach the same level as two-body QCDF

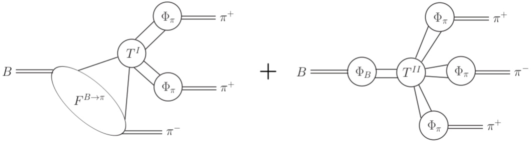
Dalitz distribution - Kinematics

- $B^+ \to \pi^+(k_1)\pi^-(k_2)\pi^+(k_3)$ Symmetric Dalitz plot
- Kinematic variables $s_{+-}^{\text{low}} = \frac{(k_1+k_2)^2}{m_B^2}$ and $s_{+-}^{\text{high}} = \frac{(k_2+k_3)^2}{m_B^2}$



Factorization in three-body decays - Central Region

Kraenkl, Mannel, Virto [2015]

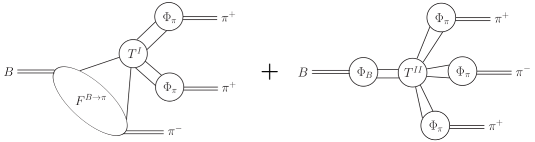


$$\langle \pi^+\pi^+\pi^-|\mathcal{O}_i|\bar{B}\rangle = F^{B\to\pi}\int du\, dv T_i^I(u,v)\Phi_\pi(u)\Phi_\pi(v)$$
$$+\int du\, dv\, dz\, d\omega\, T_i^{II}(u,v,z,\omega)\Phi_B(\omega)\Phi_\pi(u)\Phi_\pi(v)\Phi_\pi(z)$$

- Hard kernels depend on momentum fractions
- At leading order all convolutions are finite
- $1/m_b^2$ and α_s suppressed compared to the edge
- $A_{\rm CP} = \mathcal{O}(\alpha_s/\pi) + \mathcal{O}(\Lambda/m_b)$

Factorization in three-body decays - Central Region

Kraenkl, Mannel, Virto [2015]



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$$+\int du\, dv\, dz\, d\omega\, T_i^{II}(u,v,z,\omega)\Phi_B(\omega)\Phi_\pi(u)\Phi_\pi(v)\Phi_\pi(z)$$

Perturbatively calculable region might not exist for $m_B = 5$ GeV

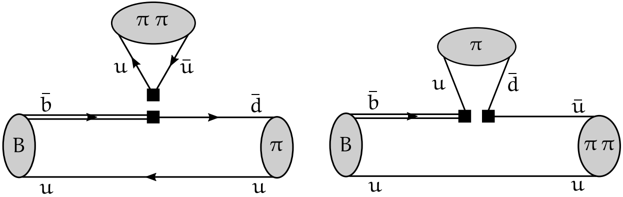
- Interesting to study QCD factorization properties
- Study power-corrections/weak annihilation? Bediaga, Frederico, Magalhaes [2017]

Factorization in three-body decays - Edges

Kraenkl, Mannel, Virto [2015]; Klein, Mannel, Virto, KKV [2018]

Breakdown of factorization at edges requires new input

- Resonances only close to the edges
- Three-body decays resemble two-body



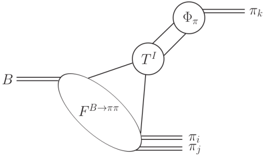
Leading contributions to hard kernels

Same operators as in two-body case, different final states

- Always an improvement over quasi-two-body decays
- Reduces to $B \to \rho\pi$ for ρ dominance and zero-width approximation

Factorization in three-body decays - Edges

Kraenkl, Mannel, Virto [2015]



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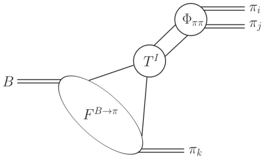


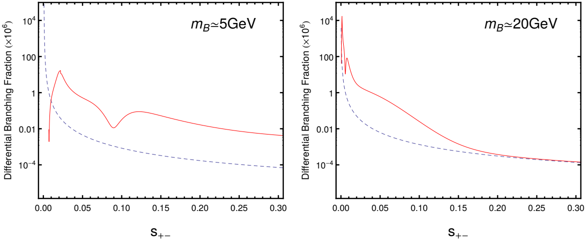
$$\langle \pi^+\pi^+\pi^- | \mathcal{Q}_i | B \rangle_{s_{+-}\ll 1} = T_i^I \otimes F^{B\to\pi^+} \otimes \Phi_{\pi^+\pi^-} + T_i^I \otimes F^{B\to\pi^+\pi^-} \otimes \Phi_{\pi^+}$$

New non-perturbative input $\rightarrow$ new strong phases

- Two-pion light-cone distribution amplitude Polyakov, Diehl, Gousset, Pire, Gozin, ...
- Generalized Form Factor Feldmann, Khodjamirian, Faller, Mannel, van Dyk, ...

Matching of the two approaches

Kraenkl, Mannel, Virto [2015]



Full 2πLCDA (red) and perturbative contribution (dashed)

Two approaches do not merge for realistic B meson masss

- Power-corrections not suppressed enough
- No part of the Dalitz plot is really center-like

New non-perturbative inputs

Focus on $B \to \pi\pi\pi$ but can be adapted for $B \to hhh$ decays

2π **LCDA**

Polyakov, Diehl, Gousset, Pire, Teryaev

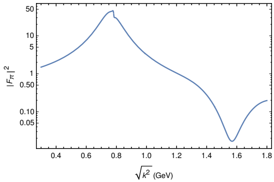
Reduces at leading order to the normalization

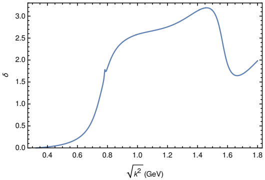
- Both isoscalar ($I = 0$) and isovector ($I = 1$) contribute

$$\int du\, \phi_{\pi\pi}^{I=1}(u,\zeta,s) = (2\zeta - 1) F_\pi(s) \qquad \int du\, \phi_{\pi\pi}^{I=0}(u,\zeta,s) = 0$$

Time-like pion formfactor $F_\pi(s)$: Babar data on $e^+e^- \to \pi\pi(\gamma)$

Hanhart, Kubis, Shekhovtsova, Roig, Was, Predzinski





$B \to \pi\pi$ form factor

Only vector form factor relevant [Faller, Feldmann, Khodjamirian, Mannel, van Dyk '14]

- Partial wave expansion: P wave always $I = 1$ and S wave has $I = 0$

$$k_{3\mu} \langle \pi^+(k_1)\pi^-(k_2)|\bar{b}\gamma^\mu\gamma^5 u|B^+(p)\rangle = -\sqrt{k_3^2} F_t(s, \zeta)$$

Theory efforts:

- $B \to \pi\pi$ form factors factorize at large k^2 [Boër, Feldmann, van Dyk '17, Feldmann, van Dyk, KKV [in progress]]
- Relevant kinematics in regime of Light-Cone Sum Rules [Khodjamirian, Virto, Cheng '17, Descotes-Genon, Khodjamirian, Virto, KKV [in progress]]

$B \to \pi\pi$ form factor from B-meson LCSRs

Correlation function with pseudoscalar heavy-light current

$$F_\mu(k,q) = i\int d^4x e^{ik\cdot x}\langle 0|T\bar{d}(x)\gamma_\mu u(x), im_b\bar{u}(0)\gamma_5 b(0)|\bar{B}^0(q+k)\rangle$$

Light-cone OPE in terms of B-meson LCDA and dispersive relation:

$$F_\mu^{OPE}(k^2,q^2) = \frac{1}{\pi}\int_{s_{th}}^{\infty} ds \frac{2\mathrm{Im}F_\mu}{s-q^2}$$

Unitarity Relation

$$2\mathrm{Im}F_\mu = m_b\int d\tau \langle 0|\bar{d}\gamma_\mu u|\pi(k_1)\pi(k_2)\rangle\langle\pi\pi|\bar{u}\gamma_5 b|\bar{B}^0(q+k)\rangle + \cdots$$

$$= q_\mu \frac{s\sqrt{q^2}\beta_\pi^2}{4\sqrt{6}\pi\sqrt{\lambda}} F_\pi^*(s) F_t^{l=1}(s,q^2) + \cdots$$

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$$= q_\mu \frac{s\sqrt{q^2}\beta_\pi^2}{4\sqrt{6}\pi\sqrt{\lambda}} F_\pi^*(s) F_t^{I=1}(s,q^2) + \cdots$$

- $F_\pi F_t^{I=1}$ real for $s < 16 m_\pi^2$ [Kang, Kubis, Hanhart, Meissner '14, Khodjamirian, Virto, Cheng [2017]]

Phase F_π = Phase $F_t^{I=1}$ Important for CP violation

$B \to \pi\pi$ form factor: LCSR

LCSR with B meson DAs (P-wave only)

[Khodjamirian, Virto, Cheng '17]

$$-\int_{4m_\pi^2}^{s_0^{2\pi}} ds\, e^{-s/M^2} \frac{s\sqrt{q^2}\beta_\pi^2}{4\sqrt{6}\pi\sqrt{\lambda}} F_\pi^*(s) F_t^{l=1}(s,q^2) = F_{\rm OPE}(M^2, q^2)$$

- Borel transformation and quark-hadron duality to obtain LCSR
- OPE depends on B-meson DA
- Usefull to constrain resonance models
- Reduces to $B \to \rho$ form factor in ρ-dominance, zero-width approx.
  - Finite-width effects in $B \to \rho$ estimated at $\mathcal{O}(10\%)$

LCSR for S-wave for $B \to hh$

Khodjamirian, Virto, Descotes-Genon, KKV [in progress]

Study of CP violation in $B^+ \to \pi^+\pi^-\pi^+$

R. Klein, Th. Mannel, J. Virto, KKV

JHEP 1710(1017) 117 [arXiv:1708.020407]

$B \to \pi\pi\pi$ decay amplitude

At leading order, leading twist

$$\mathcal{A}_{s_\pm^{\rm low}<<1} = \frac{G_F}{\sqrt{2}} m_B^2 \left[f_\pi \frac{m_\pi}{m_B^2} (\lambda_u (a_1 + a_4^u) + \lambda_c a_4^c) F_t(s_\pm^{\rm low}, \zeta) \right.$$
$$\left. + (\lambda_u (a_2 - a_4^u) - \lambda_c a_4^c)(2\zeta - 1) F_\pi(s_\pm^{\rm low}) f_0(s_\pm^{\rm low}) \right] ,$$

- a_i as in two-body decay, contain perturbative strong phases $\mathcal{O}(\alpha_s)$
- $\lambda_u = |\lambda_u| e^{i\gamma}$ weak phase

Only 4 inputs that can be obtained from data

- $B \to \pi$ form factor f_0
- Single pion DA gives the pion decay constant f_π
- $B \to \pi\pi$ form factor F_t
- 2π LCDA gives F_π

$B \to \pi\pi\pi$ decay amplitude

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- a_i as in two-body decay, contain perturbative strong phases $\mathcal{O}(\alpha_s)$
- $\lambda_u = |\lambda_u| e^{i\gamma}$ weak phase

CP violation requires two strong phases $F_t \neq F_\pi$

- Both isoscalar (S-wave) and isovector (P-wave) contribute

$$F_t = F_t^{I=0} + F_t^{I=1}$$

$B \to \pi\pi$ form factor: LCSR

Different approach using dipion DAs [Khodjamirian, Virto, Cheng '17]

$$F_5(k^2, q^2) = i \int d^4x e^{iq\cdot x} \langle \pi^+(k_1)\pi^-(k_2)| T\left\{\bar{u}(x) i m_b \gamma_5 b(x), i m_b \bar{b}(0)\gamma_5 d(0)\right\}|0\rangle$$

- Light-cone OPE now depends on the 2πLCDA
  - At leading order again only normalization F_π
  - Gegenbauer and partial-wave expansion required!

$B \to \pi\pi$ form factor: LCSR

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- Light-cone OPE now depends on the 2πLCDA
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  - Gegenbauer and partial-wave expansion required!
- Dispersion relation + Borel + duality

$$F_t(q^2,\zeta)^{I=1} = \frac{6 m_b^2 (2\zeta - 1) F_\pi(q^2)}{m_\pi f_B m_B^2} \int_{u_0}^1 \frac{du}{u}\, \bar{u}\, e^{\frac{m_B^2 - s(u)}{M^2}} (m_b^2 - m_\pi^2 + u^2 q^2)$$

$$s(u) \equiv \frac{m_b^2 - \bar{u} m_\pi^2 + u\bar{u} q^2}{u}$$

$B \to \pi\pi$ form factor: LCSR

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$$s(u) \equiv \frac{m_b^2 - \bar{u} m_\pi^2 + u\bar{u} q^2}{u}$$

- Reduces to $B \to \rho$ form factor (apply ρ-dominace, zero-width approximation)

$$F_t^{I=1} \propto (2\zeta - 1) A_0^{B\rho} \frac{g_{\rho\pi\pi} m_\rho}{\sqrt{2}(m_\rho^2 - s - i m_\rho \Gamma_\rho)} \propto (2\zeta - 1) F_\pi(s) A_0^{B\rho}$$

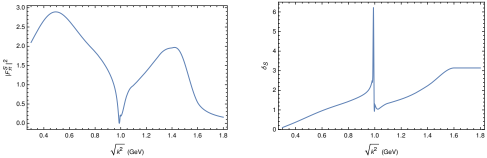
$B \to \pi\pi$ form factor: Isoscalar contribution

Daub, Hanhart, Kubis, Passemar, Cirigliano

F_π^S scalar pion form factor (analogous to F_π)

$$\langle \pi^-(k_1)\pi^+(k_2)| m_u \bar{u}u + m_d \bar{d}d |0\rangle = m_\pi^2 F_\pi^S(k^2) \ .$$

- Dispersion theory, coupled Omnes-equations (only non-strange)
- Only reliable* up to about 1.3 GeV



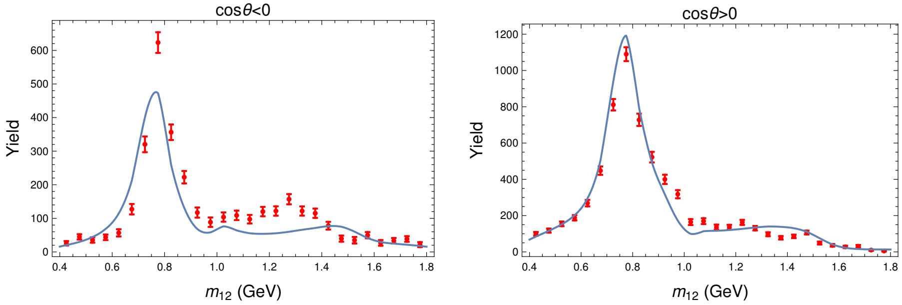
LCSR inspired model similar to $F_t^{I=1}$: not necessary in future!

β, ϕ constant fit parameters

$$F_t^{I=0}(q^2) = \frac{m_B^2}{m_\pi f_\pi} \beta e^{i\phi} F_\pi^S(q^2)$$

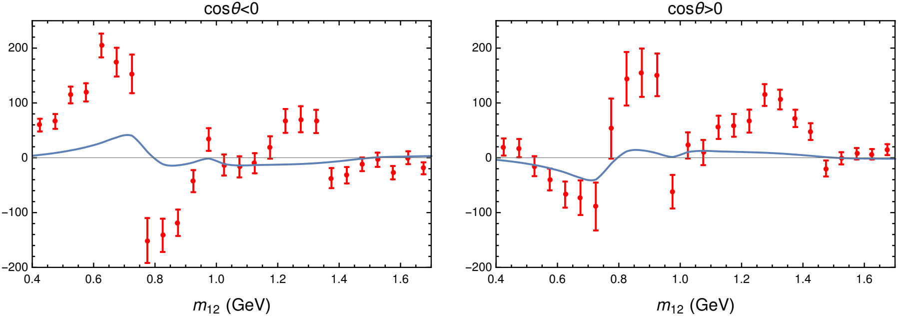
Dalitz Distribution

KKV, Virto, Mannel, Klein



- Difficult to reproduce with our current inputs
- Full Dalitz distribution preferred over projections

Dalitz and CP Distributions



$$A_{\mathrm{CP}} \propto \beta \sin\gamma \sin\phi \cos\theta_\pi$$

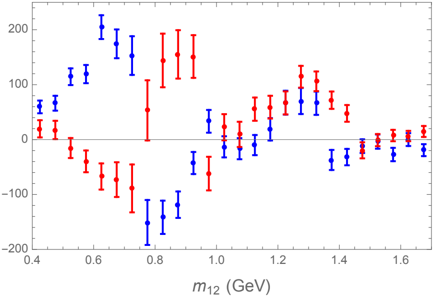
Only vector-scalar interferences (at this order!)

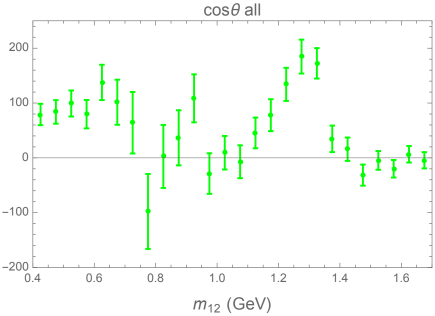
CP Distributions

Data from LHCb

$$A_{\mathrm{CP}} \propto \beta \sin\gamma \sin\phi \cos\theta + \beta' \sin\phi' \cos^2\theta + \beta'' \sin\phi'' \cos^4\theta$$

- Distinguish between region above and below $m_{12} = 1.0$ GeV
- Include higher-twist and $\mathcal{O}(\alpha)$ corrections





Outlook

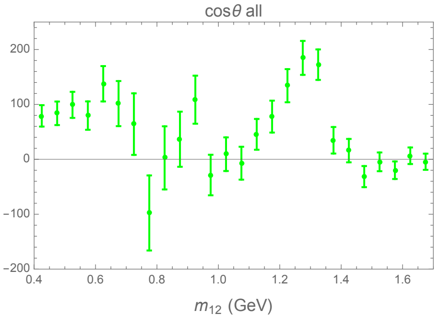
First challenge: [in progress]

- QCD sum rules for scalar form factors
- Prove QCDF using SCET and $B^0 \to D^-(\pi^+\pi^0)$
- Apply to all $B \to hhh$; $SU(3)$ analysis
- Include $\mathcal{O}(\alpha)$ corrections and higher-twist corrections
- Extend to B_s decays (?)
- Devise "optimal" observables

Experimental inputs required:

- Dalitz distributions with background and efficiency corrected
- Data in different kinematic regions
- Connection with $B \to \pi\pi\ell\nu$, $B \to \pi\pi\ell\ell$ and $B \to K^*\ell\ell$!
- Updated $B \to \rho\pi$ measurements

Optimal Observables: D waves?



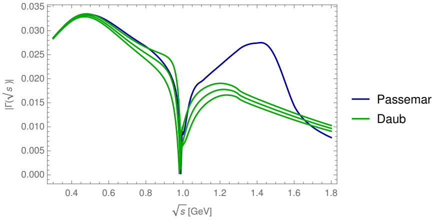
Data from LHCb

$$\int_{-1}^{1} P_l^{(0)}(\cos\theta_\pi) P_k^{(0)}(\cos\theta_\pi) \propto \delta_{lk}$$

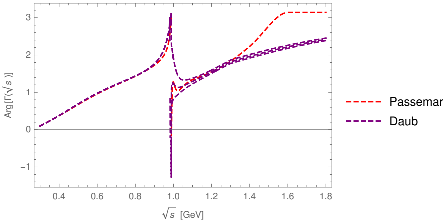
- Define angular moments to distinguish between different interferences
- Pattern arises from D-D or S-D interference ?

$B \to \pi\pi$ form factor: Isoscalar comparison

Daub, Hanhart, Kubis, Passemar, Cirigliano



Scalar form factor



Phase

$$\left\langle \pi^-(k_1)\pi^+(k_2)\middle| m_u\bar{u}u + m_d\bar{d}d \middle|0\right\rangle = m_\pi^2 F_\pi^S(k^2) \ .$$

$B \to \pi\pi$ form factor: Isoscalar contribution

- Quasi-two body approach with scalar S dominance

$$F_\pi^S \propto \left\langle \pi^-(k_1)\pi^+(k_2)|\bar{u}u|0\right\rangle = \left\langle \pi^-(k_1)\pi^+(k_2)|S^0\right\rangle \mathrm{BW}_S \left\langle S^0|\bar{u}u|0\right\rangle$$

$$\left\langle S^0|\bar{u}u|0\right\rangle = f_S m_S \ , \quad \left\langle \pi^-(k_1)\pi^+(k_2)|S^0\right\rangle = g_{S\pi^-\pi^+} m_S$$

$$F_\pi^S(q^2) = \frac{2m_u}{m_\pi^2}\frac{f_S m_S^2 g_{S\pi^-\pi^+}}{m_S^2 - q^2 - i\sqrt{q^2}\Gamma_S}$$

- For the $B \to \pi\pi$ form factor:

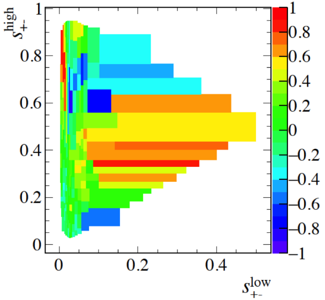
$$\begin{aligned} F_t^{I=0} &\propto \left\langle \pi^-(k_1)\pi^+(k_2)|J_\nu|B^-(p)\right\rangle \\ &= \left\langle \pi^-(k_1)\pi^+(k_2)|S^0\right\rangle \mathrm{BW}_S \left\langle S^0(q)|J_\nu|B^-(p)\right\rangle \end{aligned}$$

- Similar to $F_t^{I=1}$:

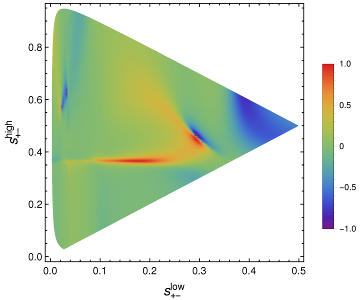
$$F_t^{I=0}(q^2) = \frac{m_B^2}{m_\pi f_\pi}\beta e^{i\phi} F_\pi^S(q^2)$$

- First study: β and ϕ free constant parameters

Charm model scenario



Scenario



Experimental Data

$$\mathcal{A}_c = \mathcal{A}_c^{(0)} + g\,\frac{4m_c^2}{m_B^2 s_{+-}^{\text{low}} - 4m_c^2 + i m_c \Gamma}$$

Inspired by strong phases generated above the charm threshold: Final-state-interactions

Edge versus Center