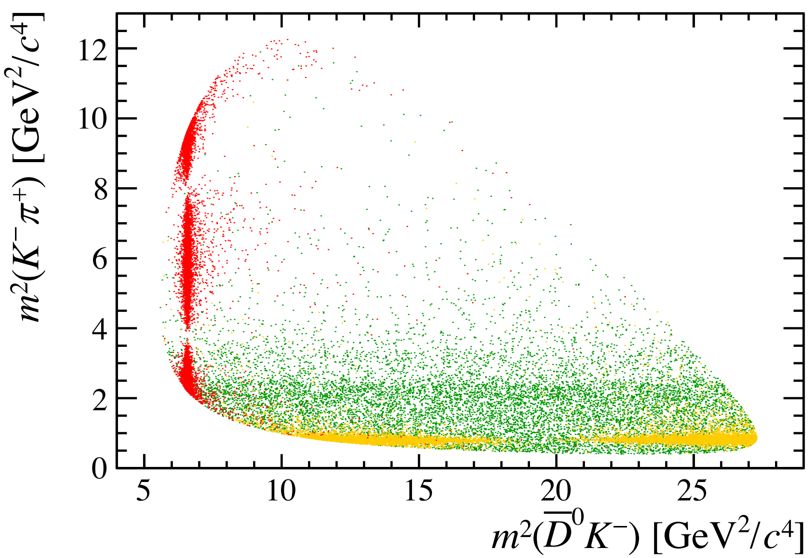
Laura++ :
a Dalitz plot fitter

Thomas Latham

Workshop on multibody charmless B-hadron decays

LPNHE, Paris

7th June 2018



History

- Package originally developed within BaBar experiment by members of the Warwick group
  - The LAURA (Fortran) package was written in 2002 by Paul Harrison
  - Converted into C++ by John Back in 2003
  - Developed subsequently by John Back and TL, adding many extra features over the years
- TL now main developer (with several contributors)
- First public version (v1r0) released on HepForge in September 2013

  http://laura.hepforge.org
- Latest version (v3r4) released to accompany submission of journal paper draft to Computer Physics Communications
- Paper has very recently been published (open access):

  https://doi.org/10.1016/j.cpc.2018.04.017

  arXiv:1711.09854

LAURA^{++} : a Dalitz plot fitter

John Back, Tim Gershon, Paul Harrison, Thomas Latham,†
Daniel O'Hanlon,‡ Wenbin Qian
Department of Physics, University of Warwick, Coventry, United Kingdom

Pablo del Amo Sanchez
LAPP, Université Savoie Mont-Blanc, CNRS/IN2P3, Annecy-Le-Vieux, France

Daniel Craik
Massachusetts Institute of Technology, Cambridge, United States of America

Jelena Ilic
STFC Rutherford Appleton Laboratory, Didcot, United Kingdom

Juan Otalora Goicochea
Universidade Federal do Rio de Janeiro (UFRJ), Rio de Janeiro, Brazil

Eugenia Puccio
Stanford University, Stanford, United States of America

Rafael Silva Coutinho
Physik-Institut, Universität Zürich, Zürich, Switzerland

Mark Whitehead
European Organization for Nuclear Research (CERN), Geneva, Switzerland

November 27, 2017

Abstract

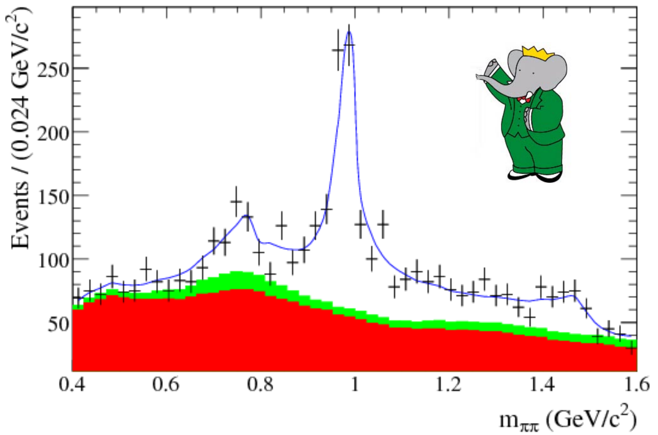
The Dalitz plot analysis technique has become an increasingly important method in heavy flavour physics. The LAURA^{++} fitter has been developed as a flexible tool that can be used for Dalitz plot analyses in different experimental environments. Explicitly designed for three-body decays of heavy-flavoured mesons to spinless final state particles, it is optimised in order to describe all possible resonant or nonresonant contributions, and to accommodate possible *CP* violation effects.

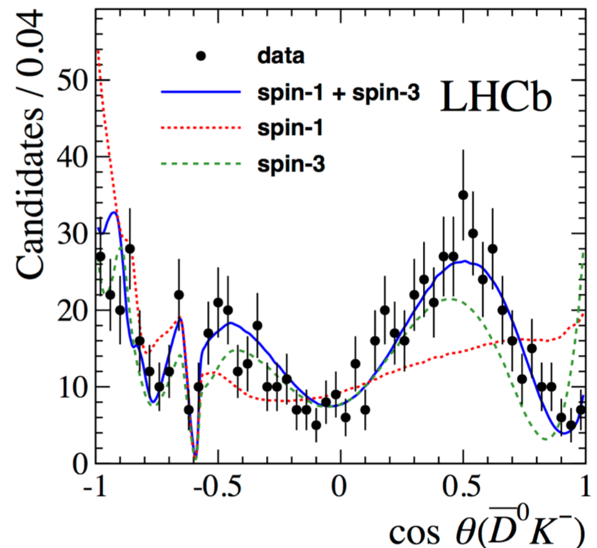
†Corresponding author, email T.Latham@warwick.ac.uk
‡Now at Sezione INFN di Bologna, Bologna, Italy

Publications

- Used in many Dalitz-plot (and other) analyses published by BaBar collaboration
  - Mainly charmless 3-body B decays
- Already used by several publications from LHCb, e.g.
  - Observation of overlapping spin-1 and spin-3 D^0K^- resonances at mass 2.86 GeV/c^2
  - Search for *CP* violation in $D^0 \to \pi^+\pi^-\pi^0$ decays with the energy test
  - First observation and amplitude analysis of the $B^- \to D^+K^-\pi^-$ decay
  - Constraints on the unitarity triangle angle γ from Dalitz plot analysis of $B^0 \to DK^+\pi^-$ decays
- Numerous others in progress within LHCb, with a variety of final states, e.g. $B_s^0 \to K_S^0K^\pm\pi^\mp$, $B^+ \to h^+h^+h^-$, $B^0 \to \eta_c K^+\pi^-$, $B^- \to D^0\overline{D}^0K^-$
- Package also has several users in Belle, Belle II and BES III collaborations

[Phys. Rev. D 78, 012004 (2008)]





[Phys. Rev. Lett. 113, 162001 (2014)]

Technical introduction

- A set of C++ classes – built into (shared) library
- Only external dependency is ROOT (mainly for reading and writing data files, histogram representations and interface to Minuit)
- Several example programs provided that can be built into executables to generate and fit toy MC data
  - Latest versions of package (from v3r3) also include an example in python
- Code uses Doxygen system for documentation

Main features (1)

- Form amplitude model (e.g. using isobar formalism) for the decay of a spin 0 parent into 3x spin 0 children
- Can then use in one of three modes:
  - Toy MC generator – generate pseudoexperiments from the model
  - (Extended) maximum likelihood fitter – fit the model to existing data (from experiment, MC, etc.) to determine parameters of interest
  - Weight data sample (e.g. full simulation generated uniformly in phase space or square DP) based on the defined model

Isobar model

- We want to describe the contributing amplitudes in the decay and the interference between them
- Very common technique known as isobar model
- Total amplitude is sum of contributing amplitudes, each with a complex coefficient:

$$\mathcal{A}\left(m_{13}^2, m_{23}^2\right) = \sum_{j=1}^{N} c_j F_j\left(m_{13}^2, m_{23}^2\right)$$

Complex coefficient

Decay dynamics
(function of DP position)

Resonance dynamics

- The F_j functions are set by choosing:
  - The mass, width and spin (various pre-set values available but can also be modified)
  - A particular lineshape model (many are available – full list in backup)
    - Several parameterisations available for modelling S-wave contributions, including K-matrix
  - The children to which the resonance decays

```
// Create the isobar model
LauIsobarDynamics* sigModel = new LauIsobarDynamics(daughters, effModel);
LauAbsResonance* reson(0);
reson = sigModel->addResonance("rho0(770)",       1, LauAbsResonance::GS);
reson = sigModel->addResonance("rho0(1450)",      1, LauAbsResonance::RelBW);
reson = sigModel->addResonance("f_0(980)",        1, LauAbsResonance::Flatte);
reson->setResonanceParameter("g1",0.2);
reson->setResonanceParameter("g2",1.0);
reson = sigModel->addResonance("f_2(1270)",       1, LauAbsResonance::RelBW);
reson = sigModel->addResonance("NonReson",        0, LauAbsResonance::FlatNR);
```

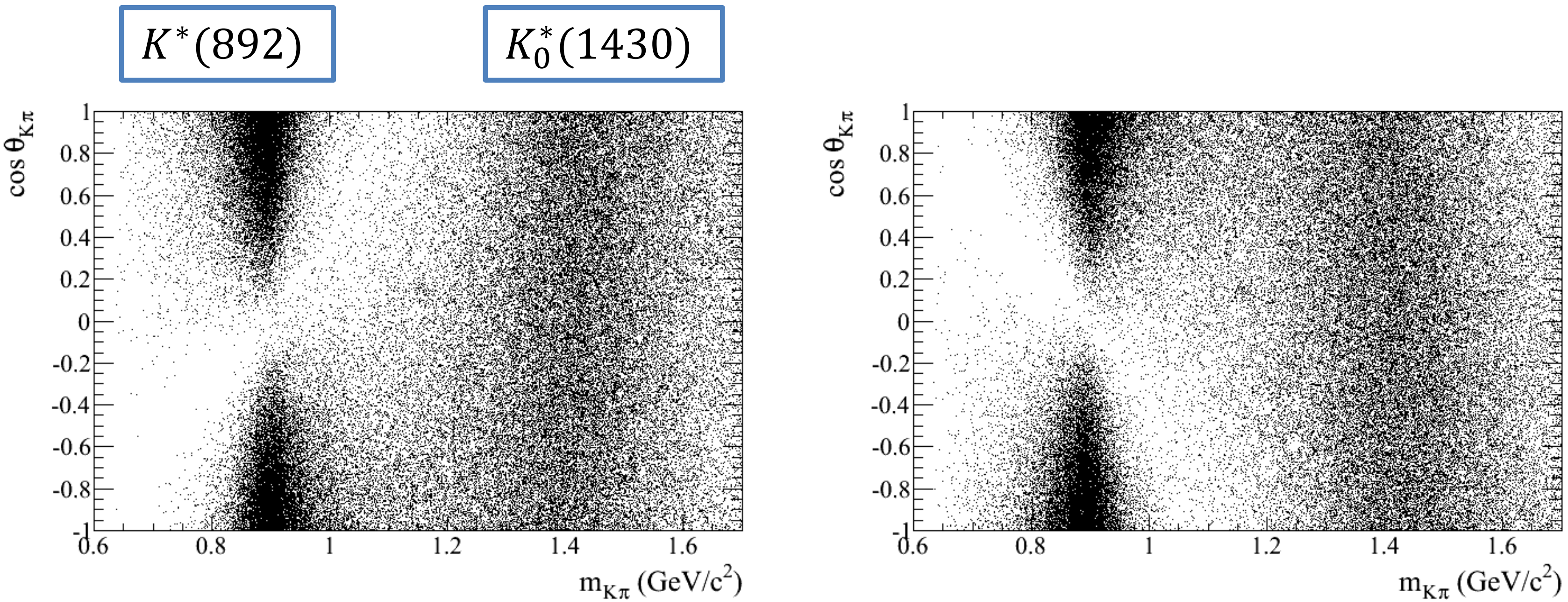
Isobar coefficients

- Values of c_j are the main parameters of interest in fits
- Need to be cast into a pair real numbers, e.g.:
  - $c_j = x_j + i\,y_j$ or $c_j = a_j\,e^{i\theta_j}$
- One component must be fixed as reference

```
// Create the fit model
LauSimpleFitModel* fitModel = new LauSimpleFitModel(sigModel);

// Create the complex coefficients for the isobar model
// Here we're using the magnitude and phase form:
// c_j = a_j exp(i*delta_j)
std::vector<LauAbsCoeffSet*> coeffset;
coeffset.push_back( new LauMagPhaseCoeffSet("rho0(770)",  1.00,  0.00,  kTRUE,  kTRUE) );
coeffset.push_back( new LauMagPhaseCoeffSet("rho0(1450)", 0.37,  1.99, kFALSE, kFALSE) );
coeffset.push_back( new LauMagPhaseCoeffSet("f_0(980)",   0.27, -1.59, kFALSE, kFALSE) );
coeffset.push_back( new LauMagPhaseCoeffSet("f_2(1270)",  0.53,  1.39, kFALSE, kFALSE) );
coeffset.push_back( new LauMagPhaseCoeffSet("NonReson",   0.54, -0.84, kFALSE, kFALSE) );
for (std::vector<LauAbsCoeffSet*>::iterator iter=coeffset.begin(); iter!=coeffset.end(); ++iter) {
        fitModel->setAmpCoeffSet(*iter);
}
```

Interference



- Both resonances modelled as relativistic Breit-Wigner in this toy example
- Different relative phases lead to different interference patterns

Integration cache

- When calculating the normalisation of the amplitude, the c_j's factorise:

$$\int |A(x,y)|^2 \, dx\, dy = |c_1|^2 \int |F_1(x,y)|^2 \, dx\, dy + c_1 c_2^* \int F_1(x,y) F_2^*(x,y) \, dx\, dy + \ldots$$

- Calculate all the integrals once and cache the values
  - Also cache values of F_j's for each event in data sample
- Advantage: gives massive speed benefits!
- Disadvantage: resonance parameters must be fixed
- Since v3r0 it is possible to float resonance parameters
  - Caching & bookkeeping system ensures minimum amount of recalculation at each fit iteration

Main features (2)

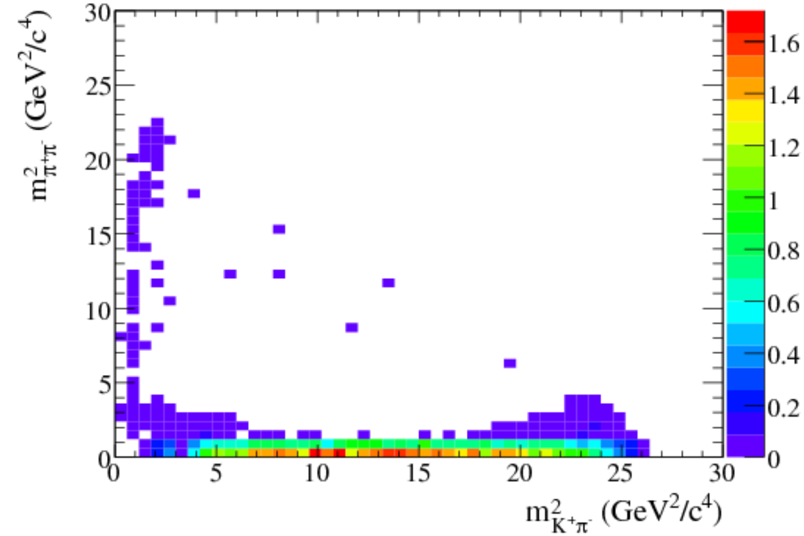
- Possible to model variation of reconstruction efficiency over the DP
  - Also resolution and migration of mis-reconstructed events (although this is usually negligible in LHCb analyses of all-track final states)

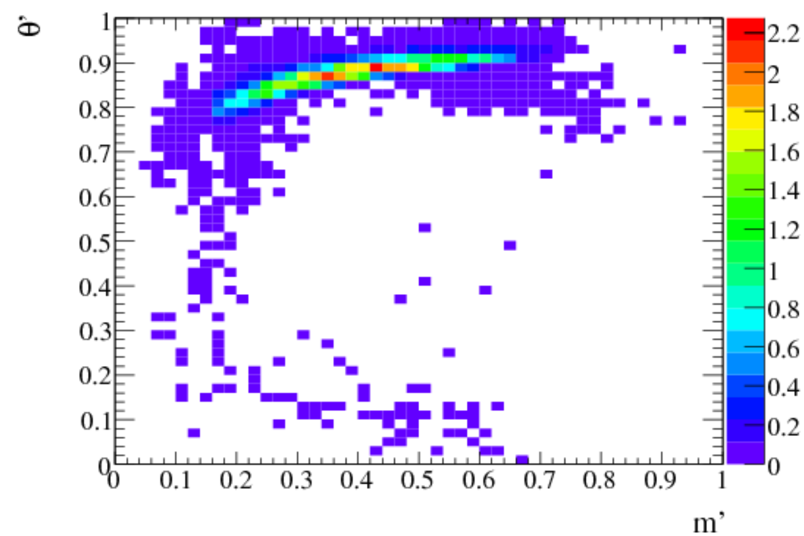
- Can add background component(s) as needed

- Can include additional discriminating variables to improve signal and background separation
  - Many PDF shapes for non-DP variables, including forms that allow for correlation with the DP position
  - Feature used extensively in recent DP analysis by BaBar studying *CP* violation in $B^+ \to K_S^0 \pi^+ \pi^0$ decays [Phys. Rev. D 96 (2017) 072001]

Backgrounds

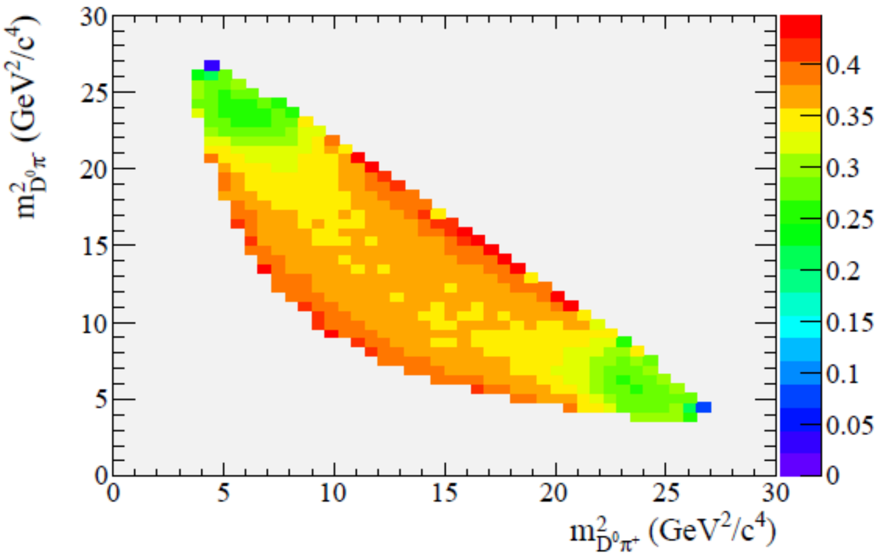
- Background model included as 2D histogram
- Either in conventional DP co-ordinates or in the "square DP"
- Can optionally use a simple linear or cubic spline interpolation to smooth shape

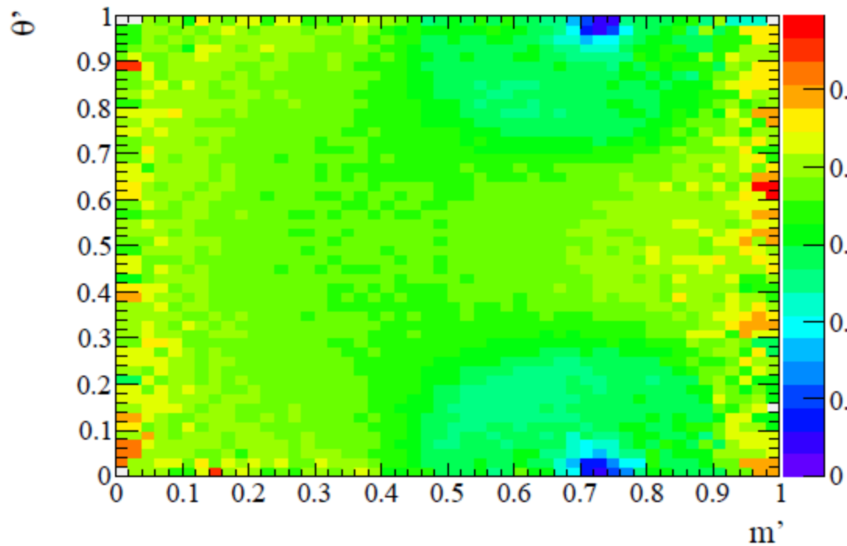




Efficiency variation

- Implemented in similar way to backgrounds (including smoothing using linear or cubic spline interpolation)
- Can also add veto regions if needed (e.g. if removing a peaking background)

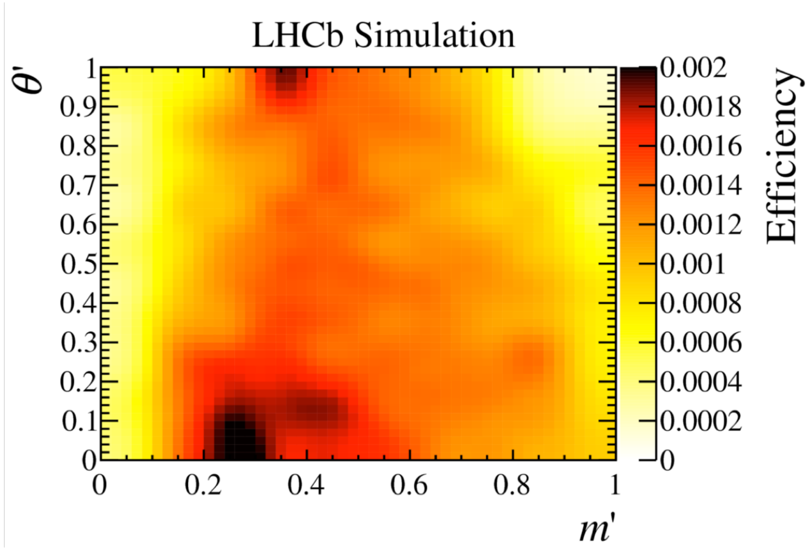


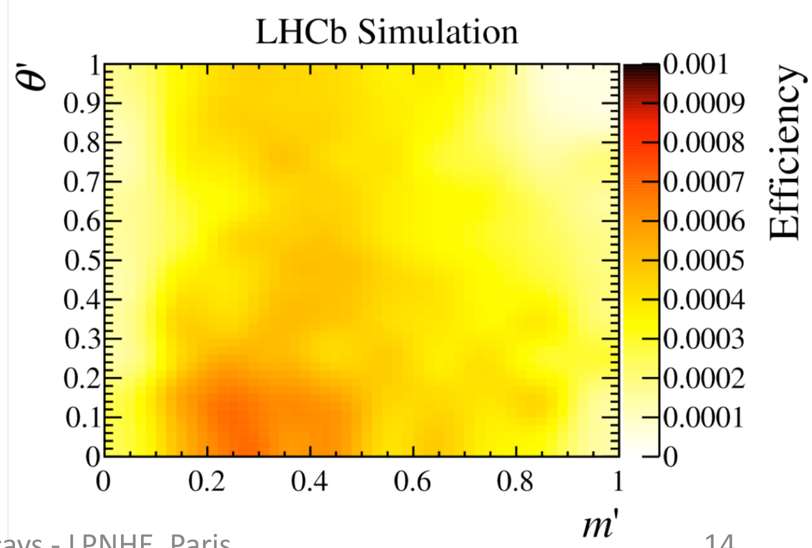


Main features (3)

- Can (since v2r0) perform simultaneous fits to different data categories
- Implementation of the JFit architecture [arXiv:1409.5080]
- Originally devised for performing joint fits of data from different experiments
- Also useful for dealing with multiple data categories, e.g. different triggers can lead to different efficiency variation
- Used for this purpose in LHCb analysis of $B^- \to D^+ K^- \pi^-$ decays (plots show efficiency behaviour in two trigger categories)

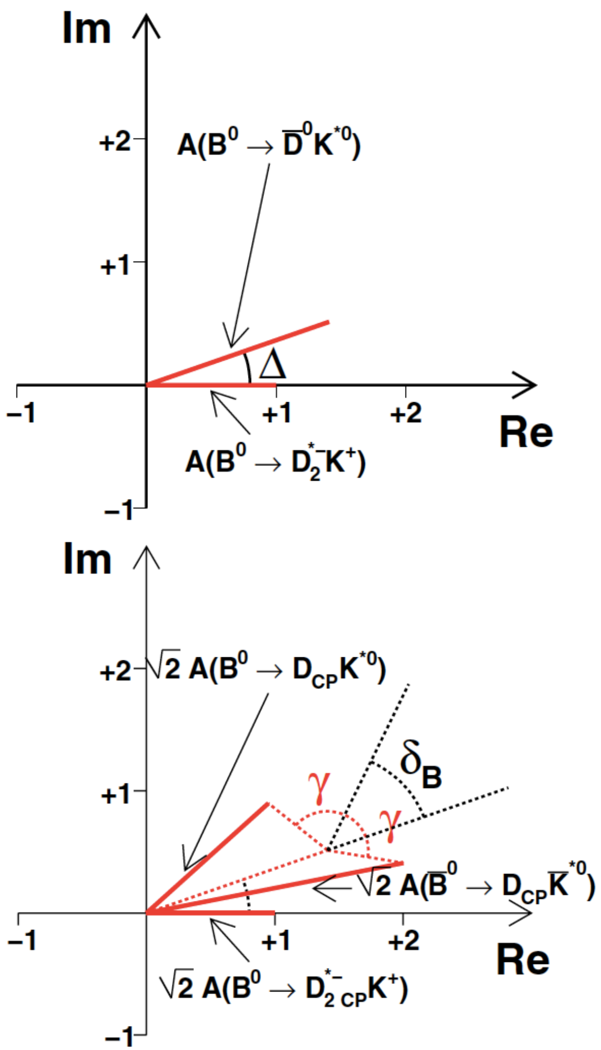
  [Phys. Rev. D91, 092002 (2015)]





Main features (3)

- Another use-case for simultaneous fitting is coupled-channel analysis, for extraction of hadronic parameters and/or *CP* violation information
- For example, used by LHCb analysis to constrain the CKM angle gamma from $B^0 \to DK^+\pi^-$ decays
  [Phys. Rev. D 93, 112018 (2016),
  Erratum: Phys. Rev. D 94, 079902 (2016)]
- Perform simultaneous fits to different B-decay Dalitz plots where the D decays either to (top) flavour-specific final state or to (bottom) *CP* eigenstates
- Method from:
  [Phys. Rev. D79, 051301 (2009)] &
  [Phys. Rev. D80, 092002 (2009)]



Main features (4)

- Can easily fit for *CP* violation in decay using LauCPFitModel class
  - Used in several BaBar and LHCb analyses of charmless B decays:
  - e.g. $B^+ \to K^+\pi^+\pi^-$, $B^+ \to \pi^+\pi^+\pi^-$, $B^+ \to K_S^0\pi^+\pi^0$
- The ability to perform a blind analysis is also available (since v3r1)

- Also possible to model mixing-induced CPV
  - Used in BaBar analysis of $B^0 \to K_S^0\pi^+\pi^-$
  - Code currently being updated to be more general
    - Cover decays of D^0, B^0 and B_s^0 mesons (allowing for $\Delta\Gamma \neq 0$)
    - Work for both correlated (e^+e^-) and uncorrelated ($pp/p\bar{p}$) production
    - Include in more general manner the effects of mis-tagging and decay time resolution and acceptance, production asymmetries, etc.
  - Not yet available in any public release – main development target for this year
  - Expected to be used in future LHCb analyses of $B^0 \to Dh^+h^-$, $B_{(s)}^0 \to K_S^0 h^+h^-$ and $B_{(s)}^0 \to h^+h^-\pi^0$
  - See next talk by Daniel and hackathon on Friday!

Summary

- Laura++ is a mature package
  - Used for many published analyses (mainly from BaBar and LHCb)
  - Specialised fitter for Dalitz-plot analysis – very fast!
  - Well tested, lots of features, e.g. K-matrix, sPlot, etc.

- Well documented: Doxygen, code examples, and journal paper

- Under active development
  - Several major new features added in last few years
  - e.g. JFit implementation, floating resonance parameters, model-independent partial wave components

- We aim to respond quickly to questions and feedback from users!

Conclusion

- Please take a look:

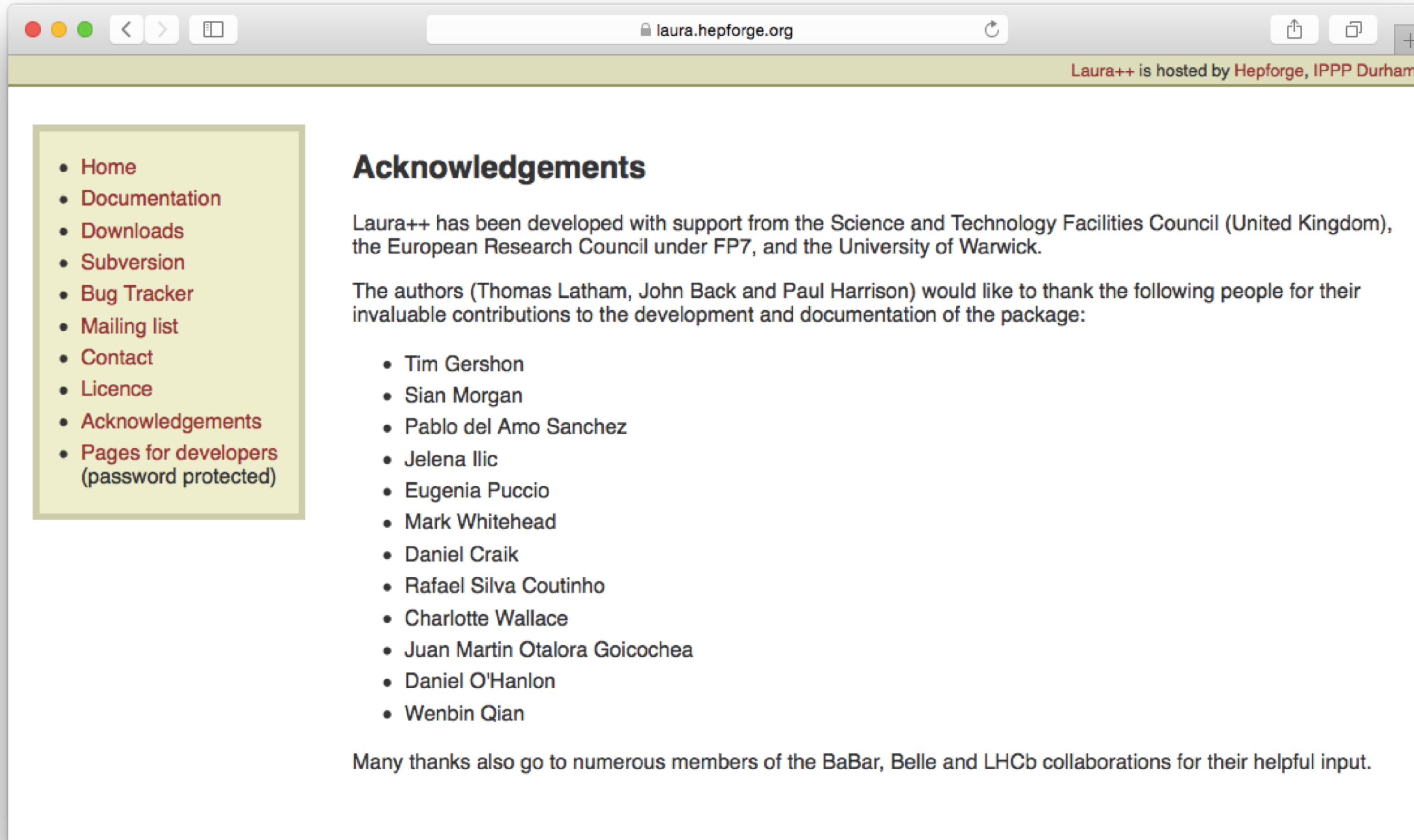
  http://laura.hepforge.org

- There is documentation on the website to help you get started, including a quick-start tutorial
- Several detailed examples included in the package
- The journal paper documents the conventions and formulae used as well as work flows and some hints on best practice:

  https://doi.org/10.1016/j.cpc.2018.04.017

- There are also two mailing lists:
  - laura-announce@projects.hepforge.org – sign-up to receive announcements of new releases etc.
  - laura@projects.hepforge.org – send an email to ask questions or to provide feedback or suggestions for improvements to the developers.

Thanks!



- Home
- Documentation
- Downloads
- Subversion
- Bug Tracker
- Mailing list
- Contact
- Licence
- Acknowledgements
- Pages for developers (password protected)

Acknowledgements

Laura++ has been developed with support from the Science and Technology Facilities Council (United Kingdom), the European Research Council under FP7, and the University of Warwick.

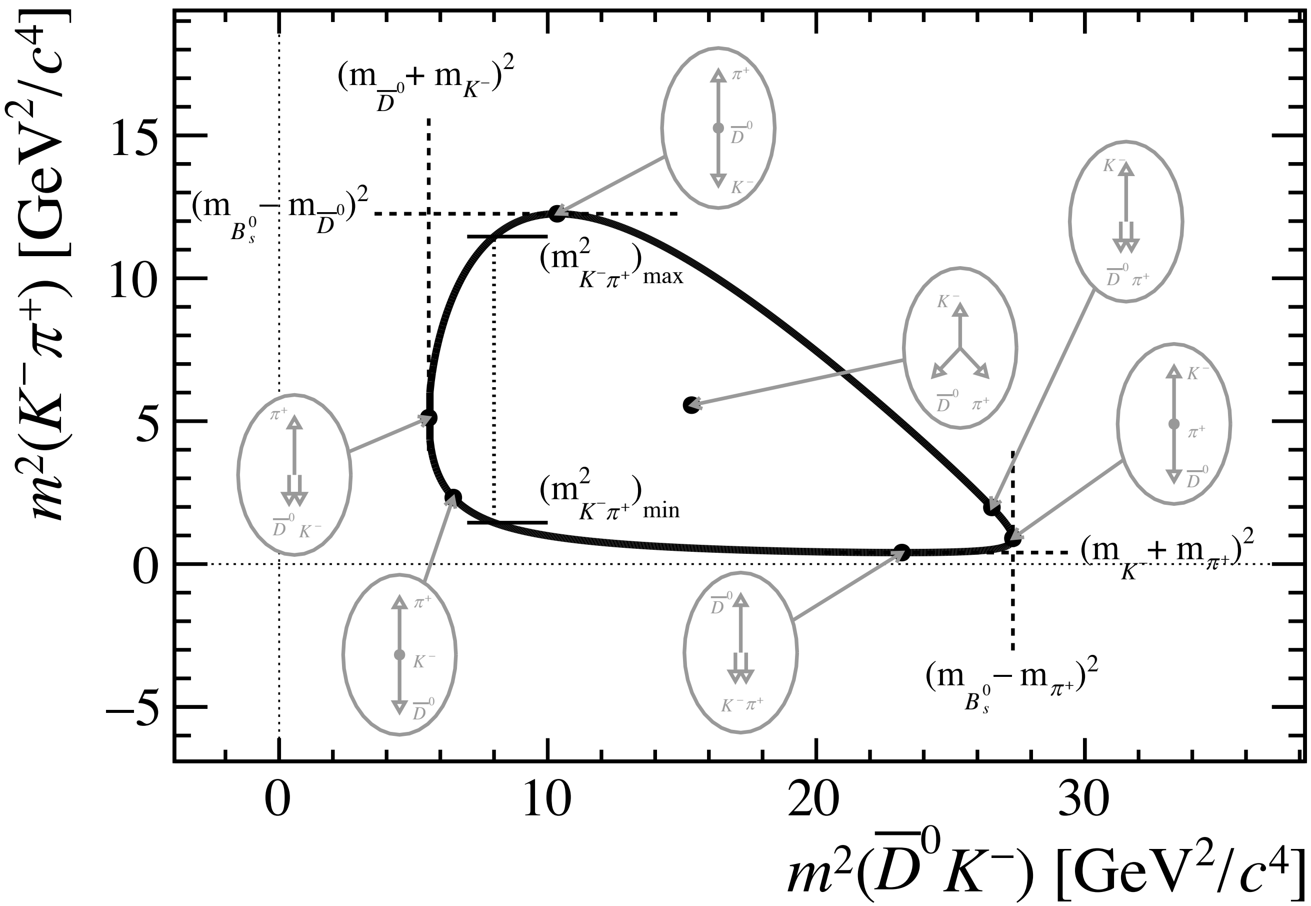
The authors (Thomas Latham, John Back and Paul Harrison) would like to thank the following people for their invaluable contributions to the development and documentation of the package:

- Tim Gershon
- Sian Morgan
- Pablo del Amo Sanchez
- Jelena Ilic
- Eugenia Puccio
- Mark Whitehead
- Daniel Craik
- Rafael Silva Coutinho
- Charlotte Wallace
- Juan Martin Otalora Goicochea
- Daniel O'Hanlon
- Wenbin Qian

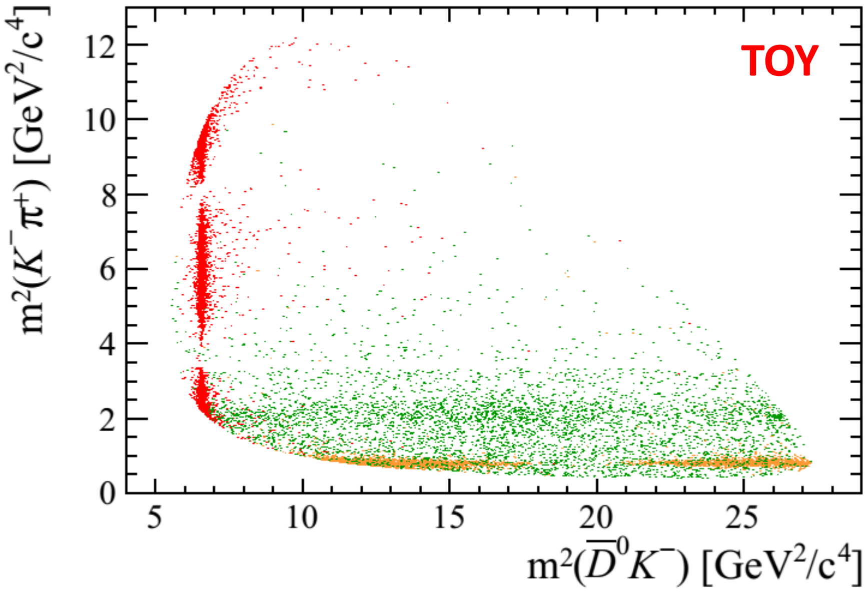
Many thanks also go to numerous members of the BaBar, Belle and LHCb collaborations for their helpful input.

BACKUP

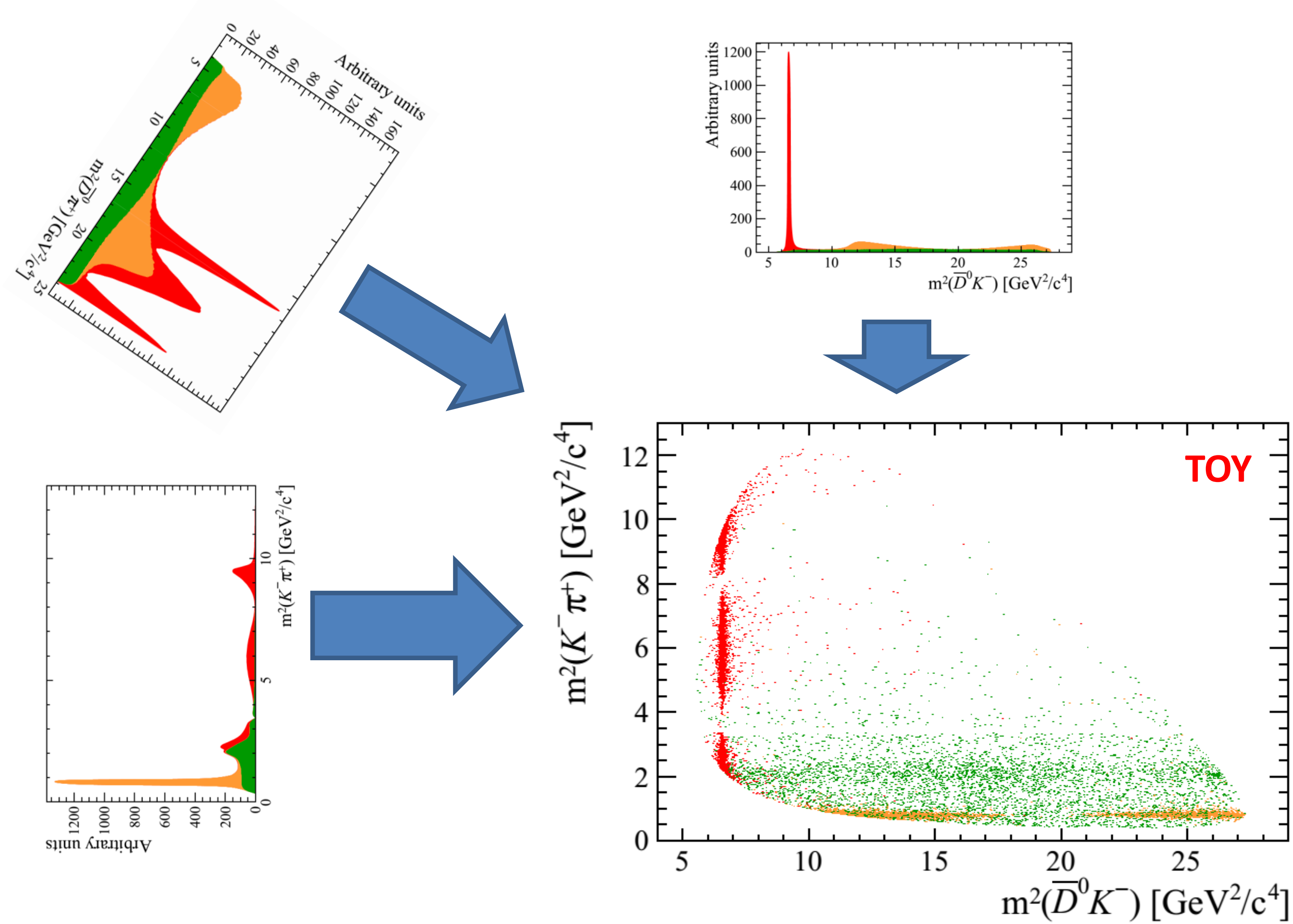
The Dalitz plot



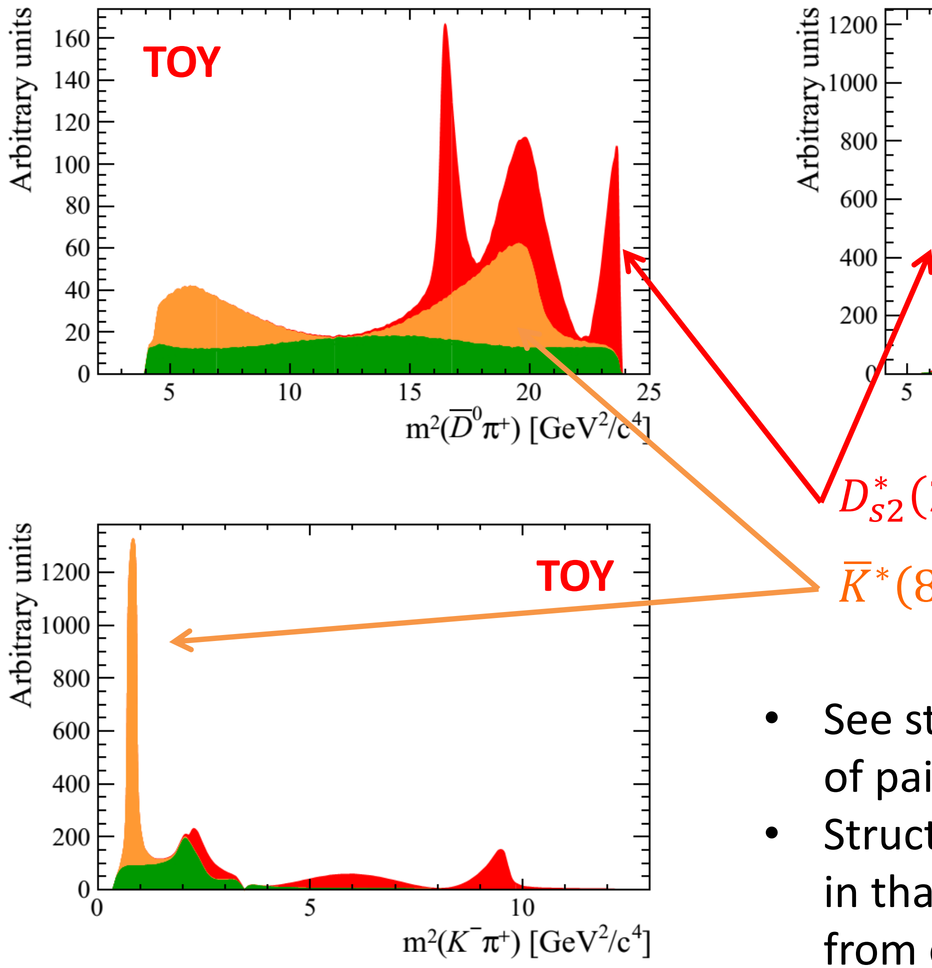
The Dalitz plot



The Dalitz plot



The Dalitz plot



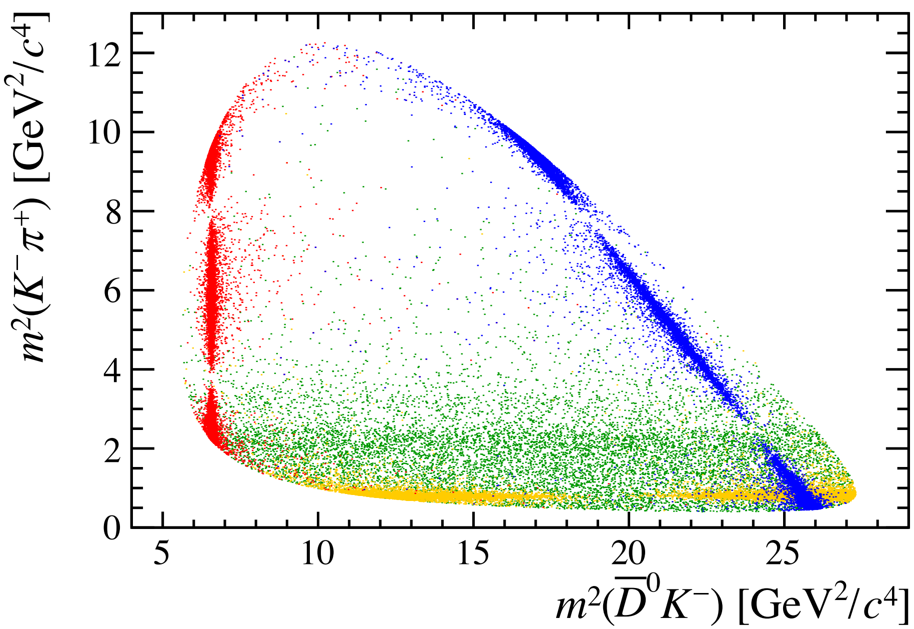
$D_{s2}^{*}(2573)^{-}$ resonance in $m_{\overline{D}^0 K^-}$

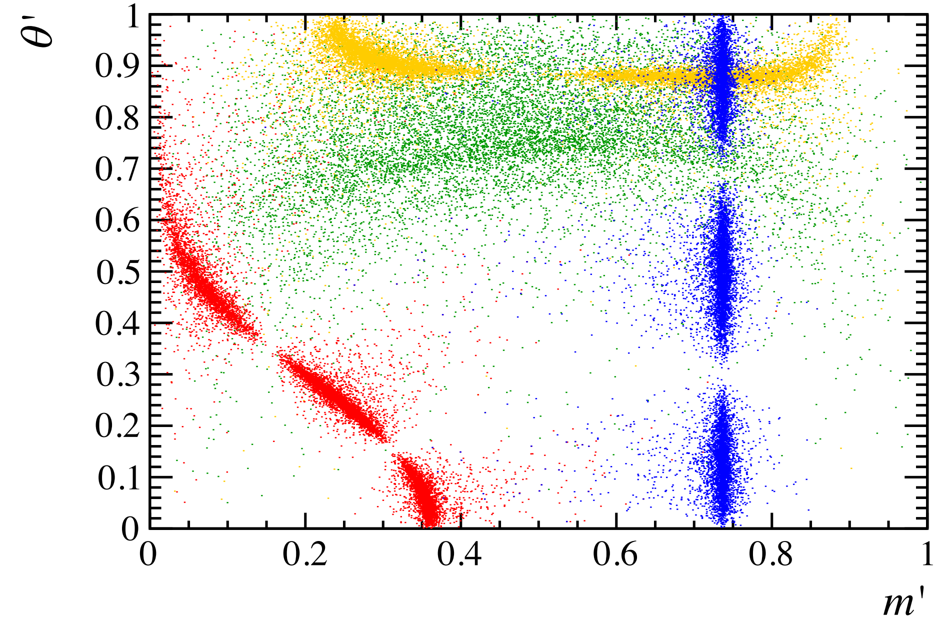
$\overline{K}^{*}(892)^0$ resonance in $m_{K^-\pi^+}$

- See structures in the invariant masses of pairs of daughters
- Structures correspond to resonances in that pair of daughters or reflections from other resonances

The square Dalitz plot

- For greater convenience for binned quantities (such as efficiency), we transform such that the Dalitz plot is mapped to a square

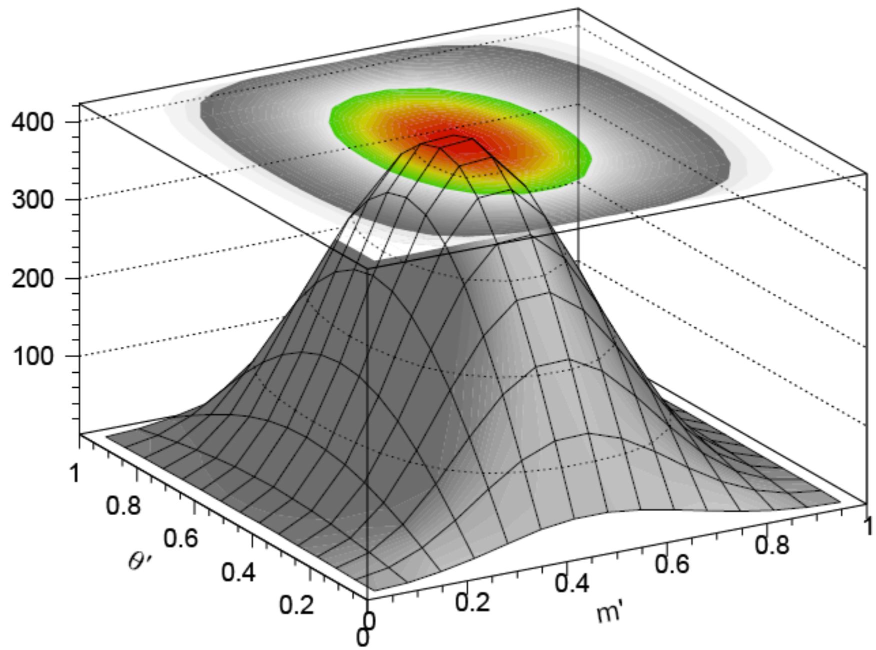




The square Dalitz plot

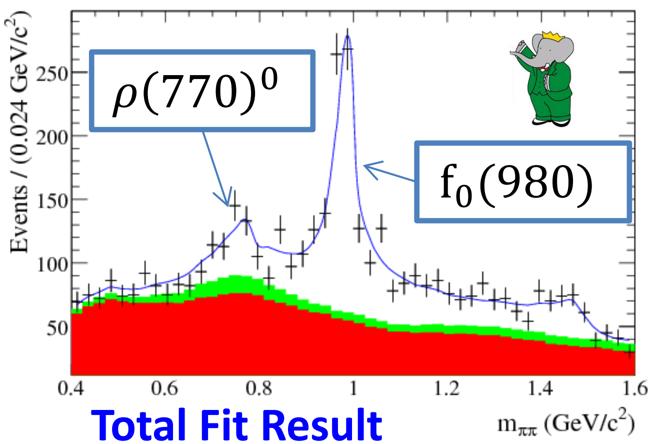
$$m' \equiv \frac{1}{\pi}\cos^{-1}\left(2\frac{m_{12}-m_{12}^{\min}}{m_{12}^{\max}-m_{12}^{\min}}-1\right) \qquad \text{and} \qquad \theta' \equiv \frac{1}{\pi}\theta_{12}$$

- Transformation of coordinates
- "Zooms" into the areas around the boundary of the conventional Dalitz plot
- Increases resolution in those areas of interest
- Also avoids effect from kinematic boundary not being aligned with bin edges



Some example fits

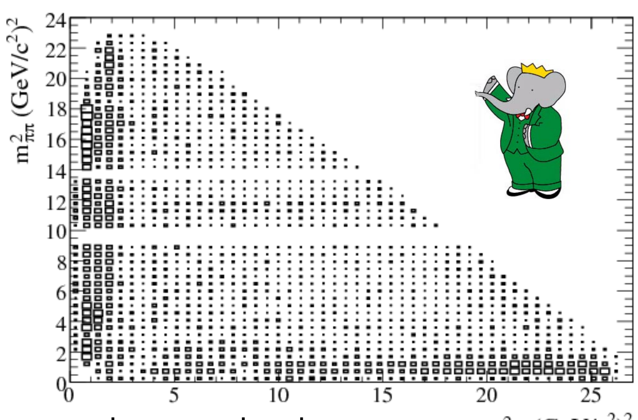
[Phys. Rev. D 78 (2008) 012004]



Total Fit Result

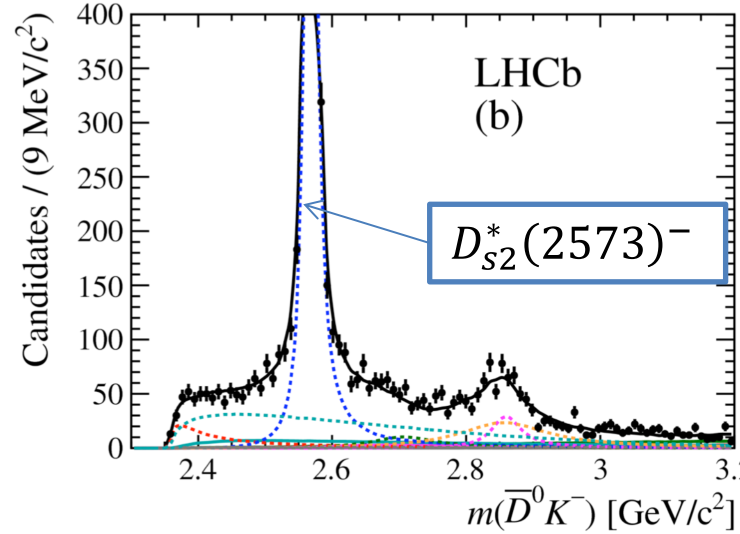
Continuum background

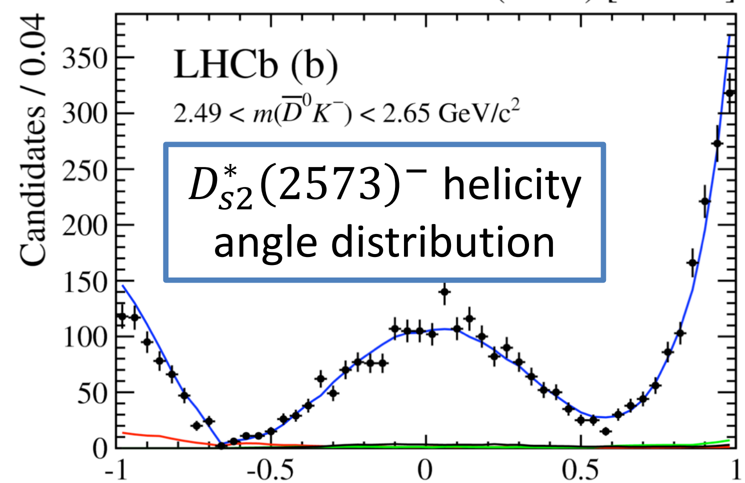
***$B\overline{B}$* background**



$B^+ \to K^+\pi^+\pi^-$

[Phys. Rev. D 90 (2014) 072003]





$B^0_s \to \overline{D}^0K^-\pi^+$

Available lineshapes

Shape name	Enumeration
Simple Breit–Wigner	BW
Relativistic Breit–Wigner (RBW)	RelBW
Modified Breit–Wigner from Gounaris–Sakurai (GS)	GS
Flatté or coupled–channel Breit–Wigner	Flatte
σ or $f_0(500)$	Sigma
κ or low–mass $K\pi$ scalar	Kappa
Low–mass $D\pi$ scalar	Dabba
LASS $K\pi$ S–wave	LASS
Resonant part of $K\pi$ LASS	LASS_BW
Non–resonant part of $K\pi$ LASS	LASS_NR
Form–factor description of the $K\pi$ S–wave	EFKLLM
S–wave using K–matrix and P–vector	KMatrix
Uniform non–resonant (NR)	FlatNR
Theoretical NR model	NRModel
Empirical NR exponential	BelleNR
Empirical NR power–law	PowerLawNR
Empirical NR exponential for symmetrised DPs	BelleSymNR
Empirical NR Taylor expansion for symmetrised DPs	TaylorNR
Empirical NR polynomial	PolNR
Model-independent partial wave (magnitude & phase)	MIPW_MagPhase
Model-independent partial wave (real & imaginary)	MIPW_RealImag
Incoherent Gaussian shape	GaussIncoh
$\rho-\omega$ mixing: GS for ρ, RBW for ω	RhoOmegaMix_GS
neglecting Δ^2 denominator term	RhoOmegaMix_GS_1
$\rho-\omega$ mixing: RBW for both ρ and ω	RhoOmegaMix_RBW
neglecting Δ^2 denominator term	RhoOmegaMix_RBW_1

List of BaBar DP analysis papers

- B. Aubert et al. (BABAR Collaboration), Dalitz-plot analysis of the decays $B^+ \to K^+\pi^+\pi^-$,
  - Phys. Rev. D 72, 072003 (2005), Erratum: Phys. Rev. D 74, 099903 (2006), arXiv:hep-ex/0507004
- B. Aubert et al. (BABAR Collaboration), An amplitude analysis of the decay $B^+ \to \pi^+\pi^+\pi^-$,
  - Phys. Rev. D 72, 052002 (2005), arXiv:hep-ex/0507025
- B. Aubert et al. (BABAR Collaboration), Evidence for direct *CP* violation from Dalitz-plot analysis of $B^+ \to K^+\pi^+\pi^-$,
  - Phys. Rev. D 78, 012004 (2008), arXiv:0803.4451 [hep-ex]
- B. Aubert et al. (BABAR Collaboration), Dalitz plot analysis of $B^+ \to \pi^+\pi^+\pi^-$ decays,
  - Phys. Rev. D 79, 072006 (2009), arXiv:0902.2051 [hep-ex]
- B. Aubert et al. (BABAR Collaboration), Time-dependent amplitude analysis of $B^0 \to K_S^0\pi^+\pi^-$,
  - Phys. Rev. D 80, 112001 (2009), arXiv:0905.3615 [hep-ex]
  - (Laura++ used as a cross-check, not for the main results.)
- P. del Amo Sanchez et al. (BABAR Collaboration), Dalitz-plot analysis of $B^0 \to \overline{D}^0\pi^+\pi^-$,
  - Presented at ICHEP 2010, arXiv:1007.4464 [hep-ex]
- J. P. Lees et al. (BABAR Collaboration), Evidence for *CP* violation in $B^+ \to K^*(892)^+\pi^0$ from a Dalitz-plot analysis of $B^+ \to K_S^0\pi^+\pi^0$,
  - Phys. Rev. D 96, 072001 (2017), arXiv:1501.00705 [hep-ex]

List of other BaBar papers

- B. Aubert et al. (BABAR Collaboration), Observation of the decay $B^+ \to K^+K^-\pi^+$,
  - Phys. Rev. Lett. 99, 221801 (2007), arXiv:0708.0376 [hep-ex]
- B. Aubert et al. (BABAR Collaboration), Search for the highly suppressed decays $B^- \to K^+\pi^-\pi^-$ and $B^- \to K^-K^-\pi^+$,
  - Phys. Rev. D 78, 091102(R) (2008), arXiv:0808.0900 [hepex]
- B. Aubert et al. (BABAR Collaboration), Search for the decay $B^+ \to K_S^0 K_S^0 \pi^+$,
  - Phys. Rev. D 79, 051101(R) (2009), arXiv:0811.1979 [hep-ex]
- P. del Amo Sanchez et al. (BABAR Collaboration), Observation of the rare decay $B^0 \to K_S^0 K^+\pi^-$,
  - Phys. Rev. D 82, 031101(R) (2010), arXiv:1003.0640 [hep-ex]
- J. P. Lees et al. (BABAR Collaboration), Observation of the rare decay $B^+ \to K^+\pi^0\pi^0$ and measurement of the quasi-two body contributions $B^+ \to K^*(892)^+\pi^0$, $B^+ \to \mathrm{f}_0(980)K^+$ and $B^+ \to \chi_{c0}K^+$,
  - Phys. Rev. D 84, 092007 (2011), arXiv:1109.0143 [hep-ex]

List of LHCb DP analysis papers

- R. Aaij et al. (LHCb Collaboration), Observation of overlapping spin-1 and spin-3 $\overline{D}^0 K^-$ resonances at mass 2.86 GeV/c2,
  - Phys. Rev. Lett. 113, 162001 (2014), arXiv:1407.7574 [hep-ex]
- R. Aaij et al. (LHCb Collaboration), Dalitz-plot analysis of $B_s^0 \to \overline{D}^0 K^- \pi^+$ decays,
  - Phys. Rev. D 90, 072003 (2014), arXiv:1407.7712 [hep-ex]
- R. Aaij et al. (LHCb Collaboration), First observation and amplitude analysis of the $B^- \to D^+ K^- \pi^-$ decay,
  - Phys. Rev. D 91, 092002 (2015), arXiv:1503.02995 [hep-ex]
- R. Aaij et al. (LHCb Collaboration), Amplitude analysis of $B^0 \to \overline{D}^0 K^+ \pi^-$ decays,
  - Phys. Rev. D 92, 012012 (2015), arXiv:1505.01505 [hep-ex]
- R. Aaij et al. (LHCb Collaboration), Constraints on the unitarity triangle angle γ from Dalitz plot analysis of $B^0 \to D K^+ \pi^-$ decays,
  - Phys. Rev. D 93, 112018 (2016), Erratum: Phys. Rev. D 94, 079902 (2016), arXiv: 1602.03455 [hep-ex]
- R. Aaij et al. (LHCb Collaboration), Amplitude analysis of $B^- \to D^+ \pi^- \pi^-$ decays,
  - Phys. Rev. D 94, 072001 (2016), arXiv:1608.01289 [hep-ex]

List of other LHCb papers

- R. Aaij et al. (LHCb Collaboration), Search for *CP* violation in $D^0 \to \pi^+ \pi^- \pi^0$ decays with the energy test,
  - Phys. Lett. B 740, 158 (2015), arXiv:1410.4170 [hep-ex]

List of other papers

- T. Latham and T. Gershon, A method to measure $\cos 2\beta$ using time-dependent Dalitz plot analysis of $B^0 \to D_{CP}\pi^+\pi^-$,
  - J. Phys. G 36, 025006 (2009), arXiv:0809.0872 [hep-ph]

- E. Ben-Haim, R. Brun, B. Echenard and T.Latham, JFIT: a framework to obtain combined experimental results through joint fits,
  - arXiv:1409.5080 [physics.data-an]

- J. Back, et al., Laura++ : a Dalitz plot fitter,
  - arXiv:1711.09854 [hep-ex]