Workshop on multibody charmless B-hadron decays
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Extraction of the CKM phase γ using charmless 3-body decays of *B* mesons

Emilie Bertholet, Eli Ben-Haïm, Matthew Charles

Outline

- **Introduction**
- **Method overview**
  - Fit model
  - Choice of points on the DP
- **Results and systematic uncertainties**
- **Tests of flavour SU(3) breaking**
- **Conclusion and perspectives**

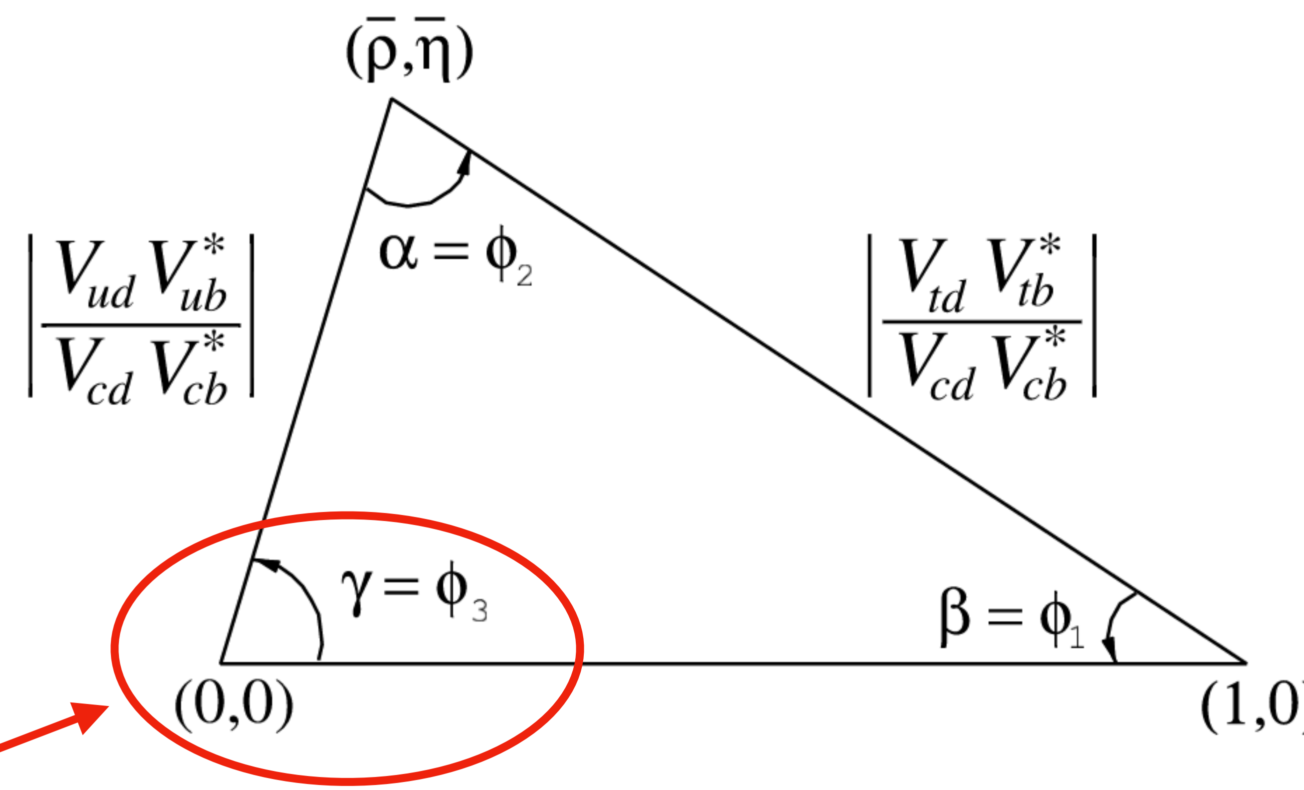
Unitarity Triangle

CKM matrix is unitary $\begin{cases} \sum_i V_{ij} V^*_{ik} & = \delta_{jk} \\ \sum_j V_{ij} V^*_{kj} & = \delta_{jk} \end{cases}$

Unitarity Triangle defined by:

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0$$

$$\overline{\rho} + i\overline{\eta} = -\frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}}$$



$$\gamma = \arg\left[-\frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}}\right]$$

b→u highly suppressed:
great precision on γ is hard to achieve.

CKM parameters and charmless B meson decyays

Measure CKM parameters:

- SM: V_{CKM} is unitary.
- SM + NP: V_{CKM} may not be unitary.
- Need to test unitarity and self-consistency.

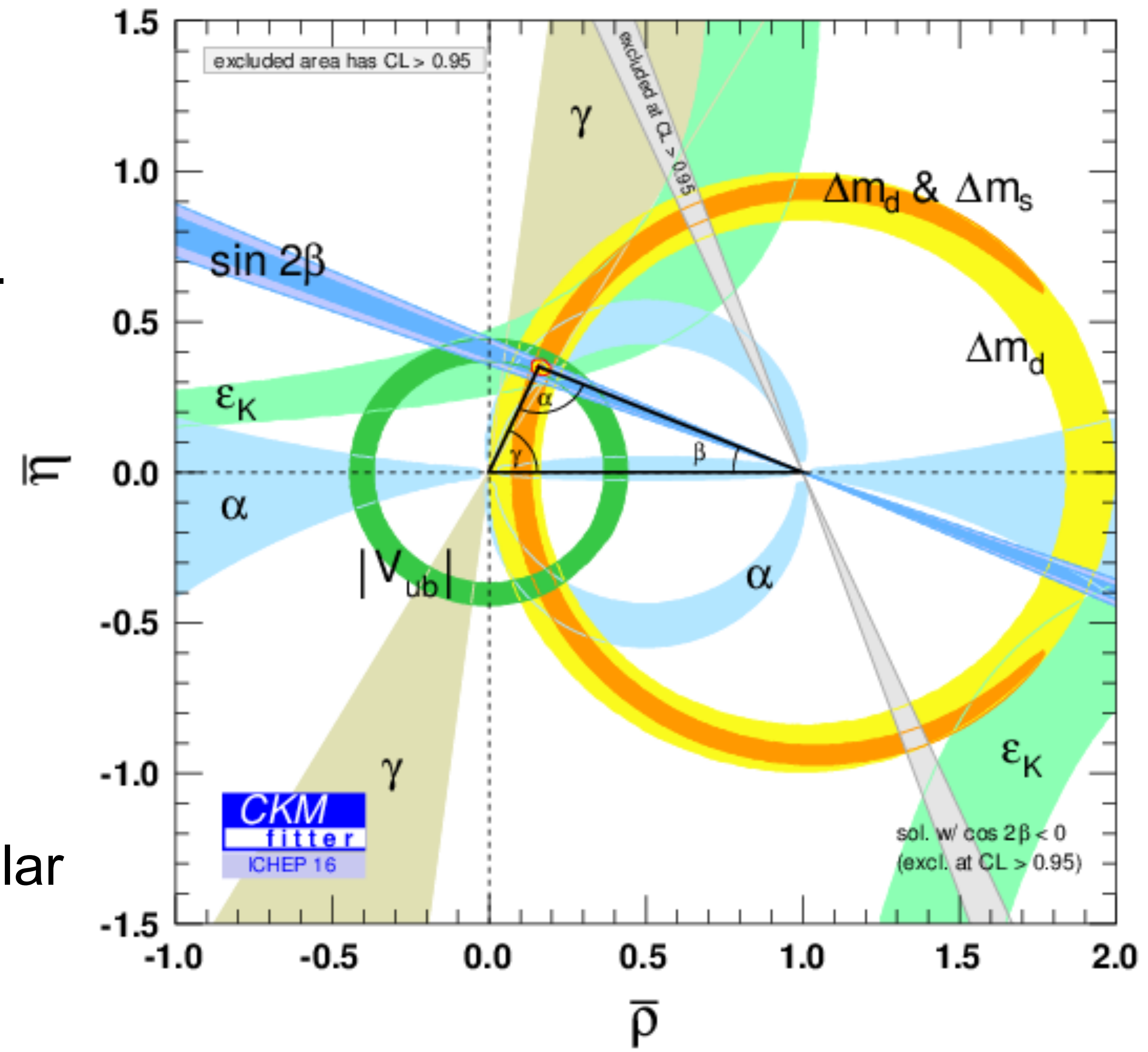
→ over-constrain the Unitarity Triangle.

Measure γ:

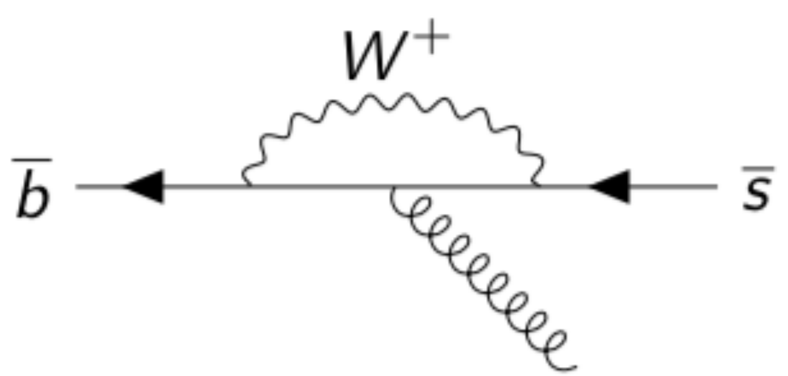
- from tree decays (eg. *B*→*DK*).
- from loop decays [charmless].
- least known CKM parameter to date.

Charmless *B* meson decays:

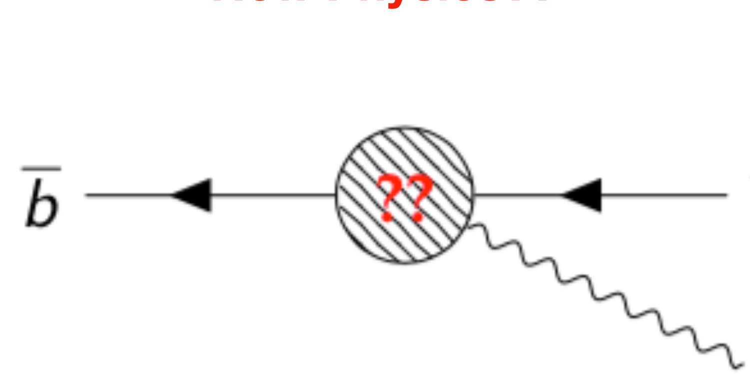
- Tree and penguin diagrams can have similar size.
- CPV
- NP searches



Standard Model



New Physics??



$$\alpha = 88.8^{\circ}{}^{+2.3^{\circ}}_{-2.3^{\circ}}$$

$$\beta = 21.85^{\circ}{}^{+0.68^{\circ}}_{-0.67^{\circ}}$$

$$\gamma = 72.1^{\circ}{}^{+5.4^{\circ}}_{-5.8^{\circ}}$$

Method overview

Phys. Lett. B728 (2014) 206-209

- Method to extract the CKM angle γ from charmless loop processes (NP sensitive) developed by David London, Bhubanjyoti Bhattacharya and Maxime Imbeault.

- Combine information from 5 charmless 3-body decays of *B* mesons under an assumption of flavour SU(3) symmetry.

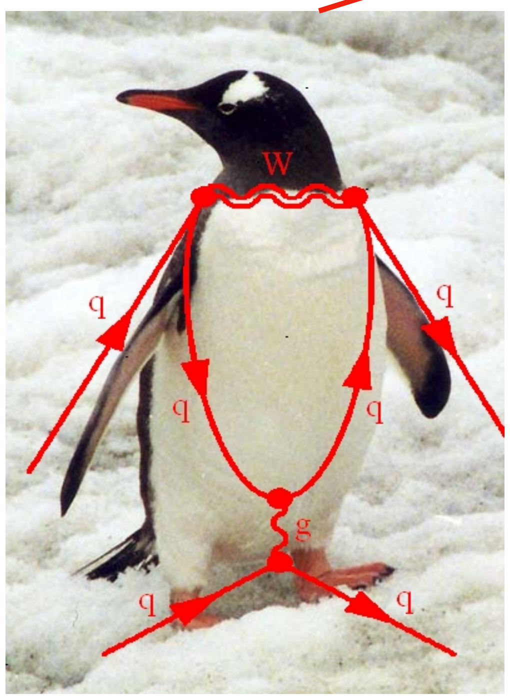
$$B^0 \to K_S K_S K_S \quad B^0 \to K^+\pi^0\pi^- \quad B^+ \to K^+\pi^+\pi^-$$
$$B^0 \to K_S K^+ K^- \quad B^0 \to K_S\pi^+\pi^-$$

- We extracted γ and its uncertainty using BABAR's results.

Flavour SU(3) symmetry I

Under flavour SU(3) symmetry assumption, tree and penguin diagrams are proportional for *b*→*s* transitions:

$$P_{EW}^{(C)} = \kappa T^{(C)}$$



$$\kappa \equiv -\frac{3}{2}\frac{|\lambda_t^{(s)}|c_9 + c_{10}}{|\lambda_u^{(s)}|c_1 + c_2}$$

with

$$\lambda_p^{(s)} = V_{pb}^* V_{ps}$$

c_i :Wilson coefficients

This relation holds only for fully symmetric amplitudes:

$$A_{\rm fs}(s_{12}, s_{13}) = \frac{1}{\sqrt{6}}\left(A(s_{12}, s_{13}) + A(s_{12}, s_{23}) + A(s_{13}, s_{23}) + A(s_{13}, s_{12}) + A(s_{23}, s_{12}) + A(s_{23}, s_{13})\right)$$

Theoretical expressions for the amplitudes

Theoretical amplitudes for each mode can be expressed in terms of:

- 5 effective diagrams
- 1 weak phase
- 1 parameter related to flavour SU(3) breaking

$$2A_{\rm fs}(B^0 \to K^+\pi^0\pi^-) = Be^{i\gamma} - \kappa C$$

$$\sqrt{2}A_{\rm fs}(B^0 \to K^0\pi^+\pi^-) = -De^{i\gamma} - \tilde{P}'_{\rm uc}e^{i\gamma} - A + \kappa D$$

$$A_{\rm fs}(B^0 \to K^0K^0\overline{K}^0) = \alpha_{\rm SU(3)}(\tilde{P}'_{\rm uc}e^{i\gamma} + A)$$

$$\sqrt{2}A_{\rm fs}(B^0 \to K^+K^0K^-) = \alpha_{\rm SU(3)}(-Ce^{i\gamma} - \tilde{P}'_{\rm uc}e^{i\gamma} - A + \kappa B)$$

Theoretical expressions for the amplitudes

Theoretical amplitudes for each mode can be expressed in terms of:

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Theoretical expressions for the amplitudes

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Parameter counting for 4 modes (5 modes)

10 (11) theoretical parameters

Observables

From the extracted amplitudes of the 4 (5) modes, we construct observables

$$X(s_{13}, s_{23}) = |A_{fs}(s_{13}, s_{23})|^2 + |\overline{A}_{fs}(s_{13}, s_{23})|^2$$

X: branching ratio [available for 4 (5) modes]

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Parameter counting for 4 modes (5 modes)
11 (13) observables

Fit principle

10 (11) parameters

11 (13) observables

→ **observables as functions of the parameters**

→ **γ extracted with a fit**

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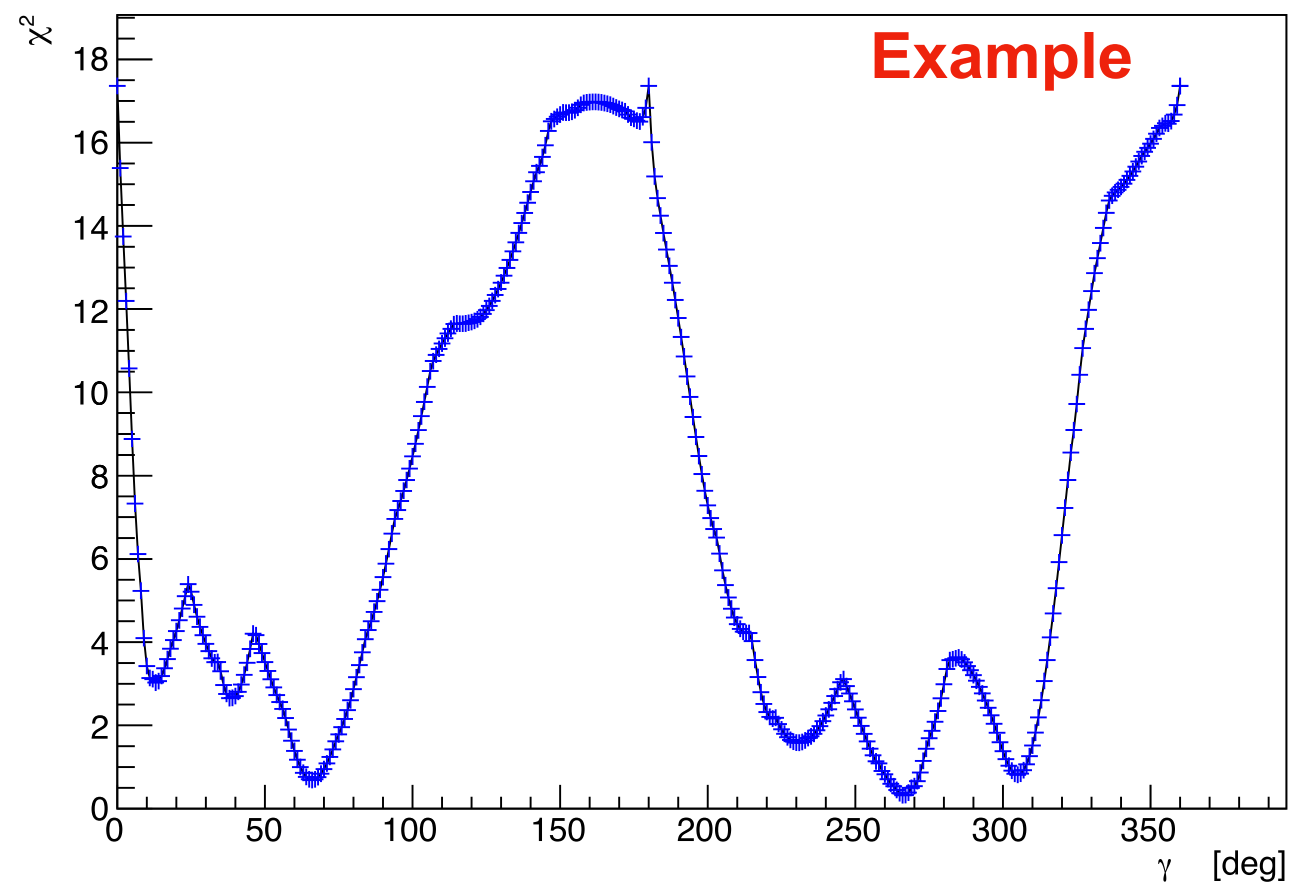
11 (13) observables

→ **observables as functions of the parameters** → **γ extracted with a fit**

Extraction of γ at one point (s_{13}, s_{23}) on the DP:

- Compute observables: $X(s_{13}, s_{23})$, $Y(s_{13}, s_{23})$, $Z(s_{13}, s_{23})$.
- Compute the covariance matrix including the correlations.
- Scan on γ: fix γ to consecutive values and evaluate the other parameters minimising a χ^2 function.

Cov matrix: 11x11 (13x13)



Fit principle

10 (11) parameters → **observables as functions of the parameters** → **γ extracted with a fit**

11 (13) observables → **observables as functions of the parameters**

Extraction of γ at one point (s_{13}, s_{23}) on the DP:

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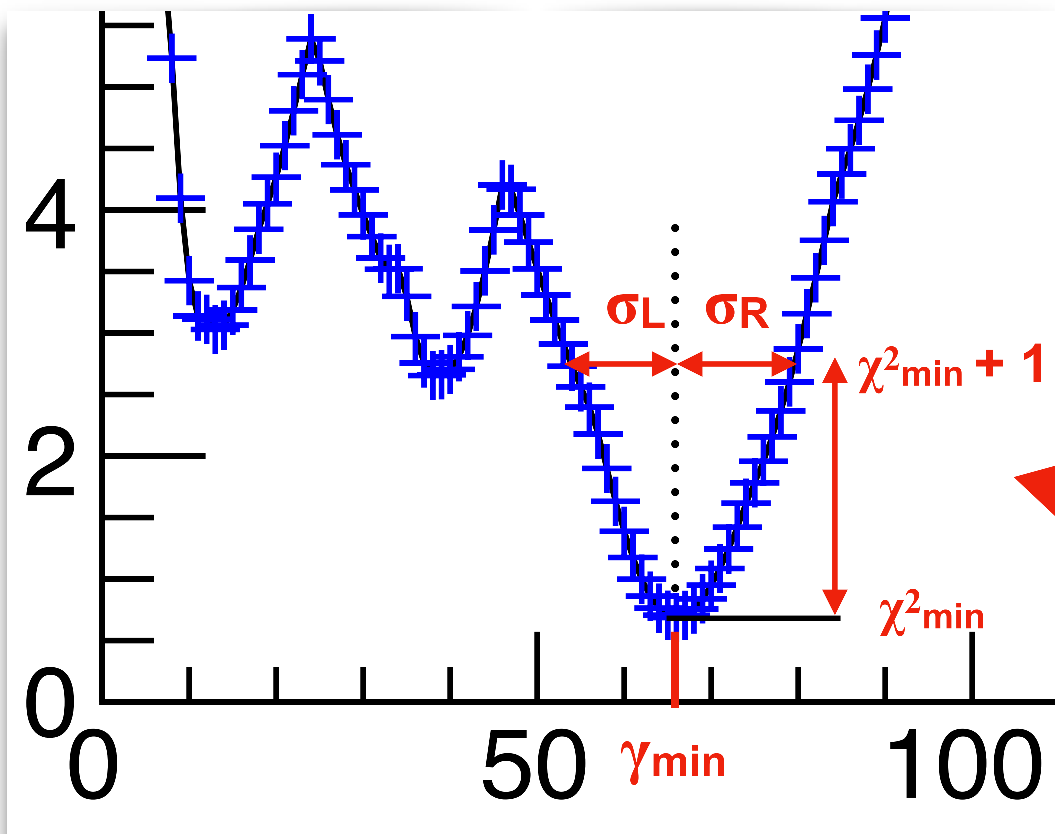
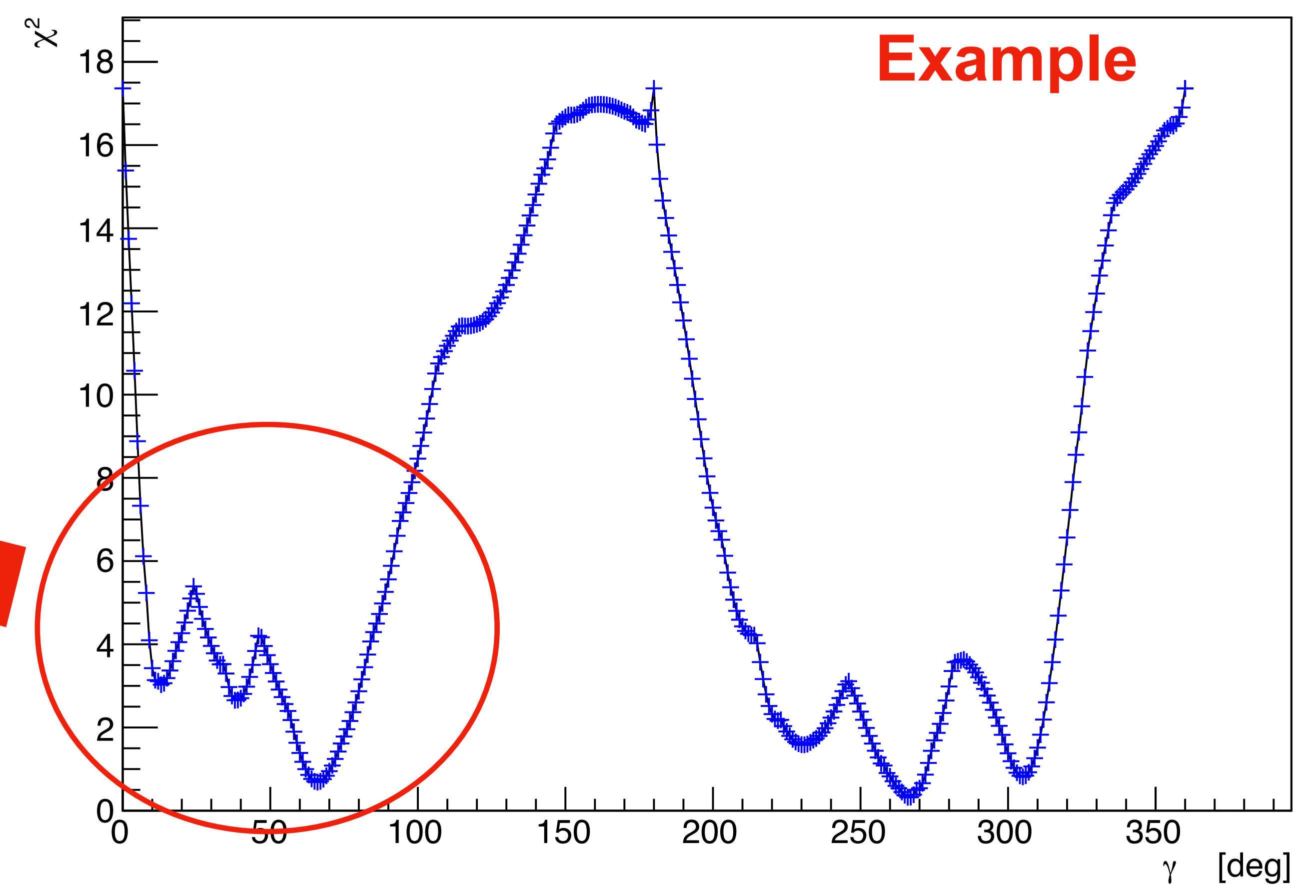
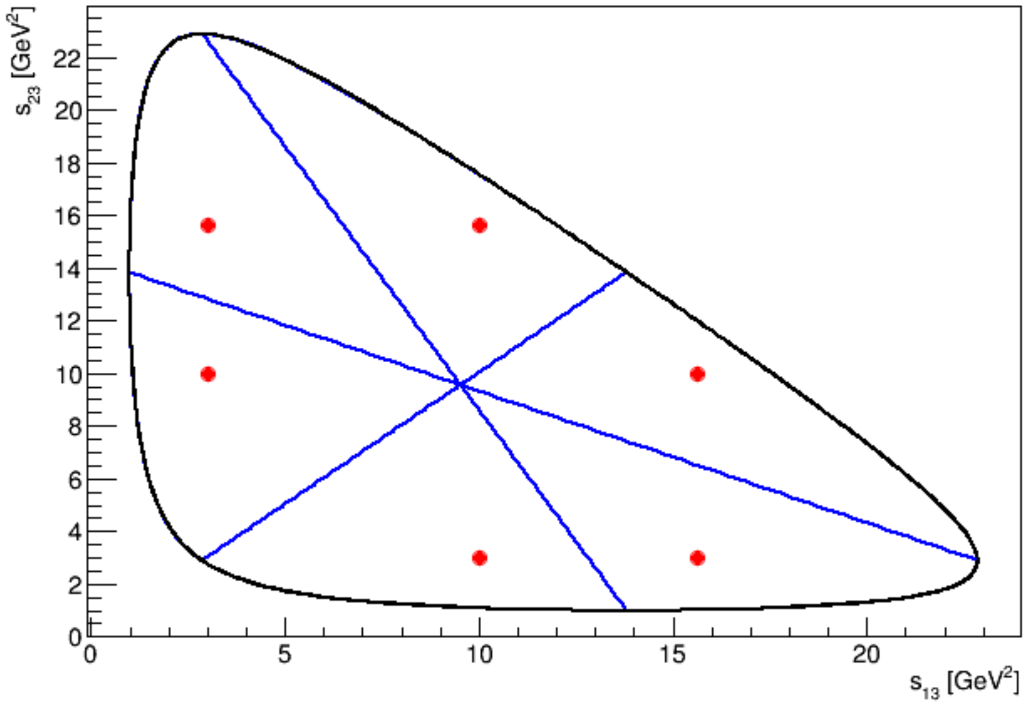




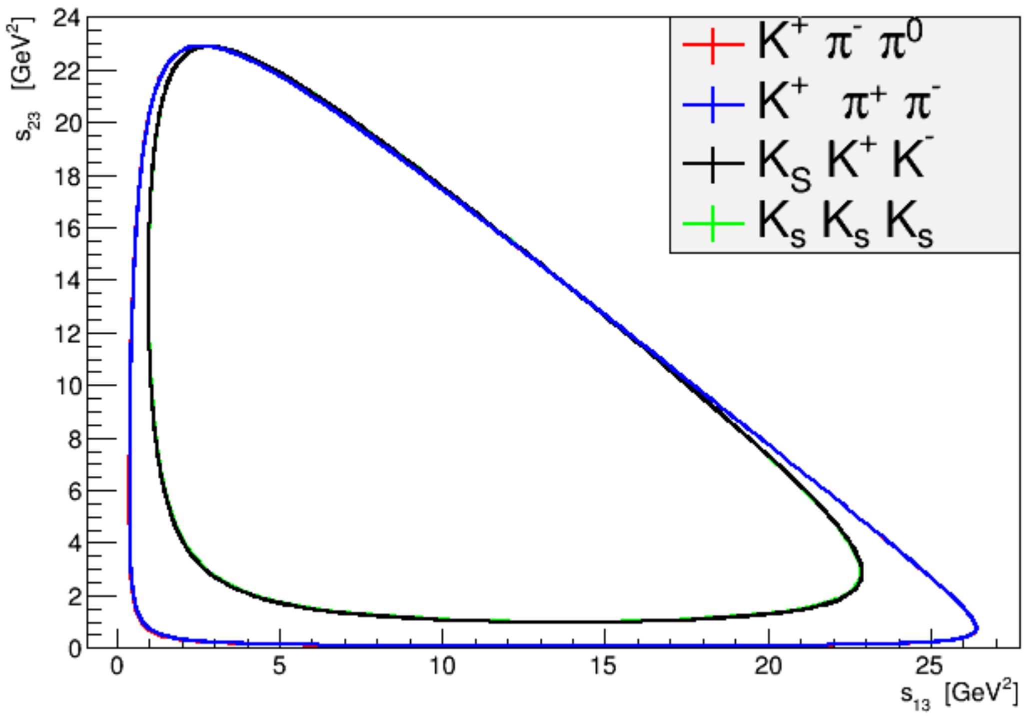
Choice of points on the DP I

Fully symmetrised amplitudes

$$A_{\text{fs}}(s_{12}, s_{13}) = \frac{1}{\sqrt{6}}(A(s_{12}, s_{13}) + A(s_{12}, s_{23}) + A(s_{13}, s_{23}) + A(s_{13}, s_{12}) + A(s_{23}, s_{12}) + A(s_{23}, s_{13}))$$



The fully symmetric DP is divided into 6 regions containing the same information.



Kinematic boundaries of the different modes

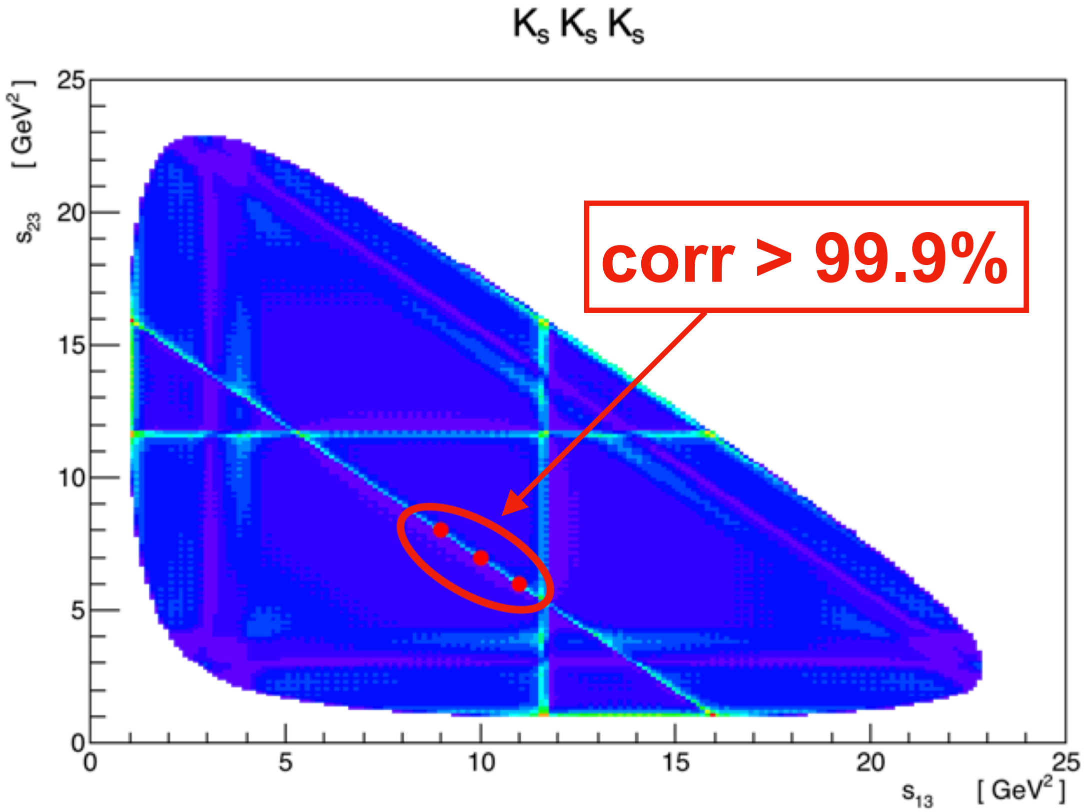
The information we can use is limited by the size of $B^0 \to K_S K_S K_S$ DP (smallest one).

Choice of points on the DP II

The use of several points allows:

- Improving the validity of flavour SU(3) hyp.
- Using the maximum amount of information.
- Improving the statistical uncertainties.

Extract γ using the maximum possible number of points on the DP.



In practice, due to very high correlations between certain points we are limited to the use of 3 simultaneous points.

Cov matrix: 33x33 (39x39)

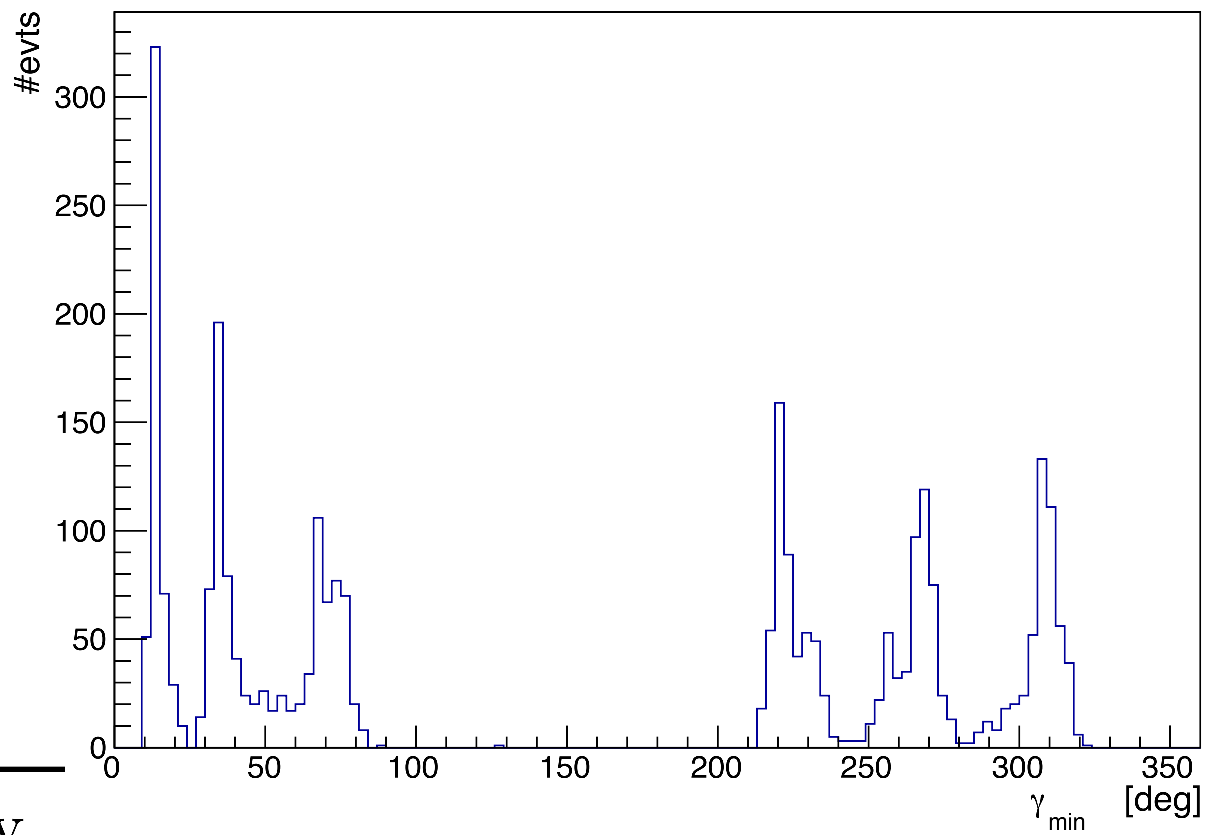
Method for extracting the results

- Combinations of 3 points randomly scattered over the DP
- For each set of points: scan on the value of γ (500 fits with random initial parameters).
- Extract minima and statistical uncertainties for each scan.
- Combine results of all scans.
- Estimate systematic uncertainties.

Baseline results: extraction of γ using 4 modes

- $\alpha_{SU(3)}$ fixed to 1 in the fit.
- 501 sets of random 3-points combinations (correlations < 70%).
- 500 fits randomising the initial values of the parameters per set.

Histogram of the minima extracted from the 501 sets of points.



Preferred values for γ: central values (μ) and statistical uncertainties (σ_L, σ_R).

	μ	σ_L	σ_R	frequency
minimum 1	12.9°	4.3 °	8.4°	484
minimum 2	36.6 °	6.1°	6.6°	474
minimum 3	68.9 °	8.6 °	8.6°	461
minimum 4	223.2°	7.5°	10.9°	499
minimum 5	266.4°	10.8°	9.2°	487
minimum 6	307.5°	8.1°	6.9°	488

$$\gamma_{SM} = 72.1^{\circ}{}^{+5.4^{\circ}}_{-5.8^{\circ}}$$

Results

- 6 possible values for γ.
- 3rd minimum compatible with SM.
- Statistical error of the order of 10°.

Systematic uncertainties

Influence of "poorly resolved" minima

- To combine the results obtained from the different sets of 3 points we average on the central values of the minima.
- Some minima are not deep enough to extract statistical uncertainties. They are labelled as "**poorly resolved minima**" and are **not included** in the average for the baseline result.
- The central value including all the minima, μ^{all}, is used to assing a systematic uncertainty

$$\text{Syst1} = |\mu - \mu^{all}|$$

Influence flavour SU(3) breaking

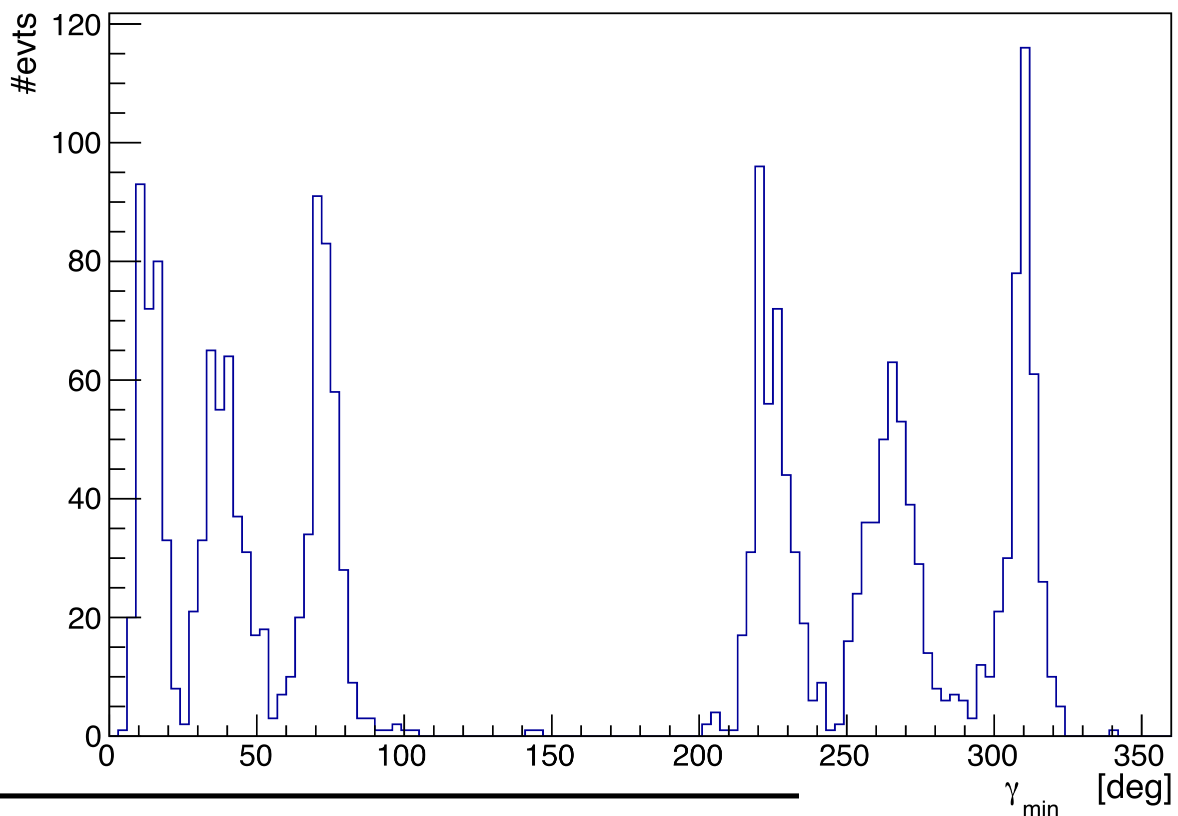
- So far we do not take into account flavour SU(3) breaking.
- γ is re-extracted with 5 modes, letting $\alpha_{SU(3)}$ float in the fit. [next slide]
- Central values found with 5 modes are used to assign a systematic uncertainty

$$\text{Syst2} = |\mu^{4modes} - \mu^{5modes}|$$

Extraction of γ using 5 modes

- $\alpha_{SU(3)}$ free in the fit.
- 401 sets of random 3-points combinations (correlations < 80%).
- 500 fits randomising the initial values of the parameters per set.

Histogram of the minima extracted from the 401 sets of points.



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	μ	σ_L	σ_R	$\|\mu - \mu^{all}\|$	$\|\mu^{4modes} - \mu^{5modes}\|$	frequency
minimum 1	11.9	5.8	9.1	1.3	1.0	306
minimum 2	39.2	6.3	6.7	1.2	2.6	329
minimum 3	71.3	9.5	9.3	0.4	2.4	372
minimum 4	223.9	7.4	9.5	0.1	0.7	383
minimum 5	265.0	11.0	10.0	1.2	1.3	378
minimum 6	308.4	8.8	7.0	0.6	0.9	391

Central values and statistical uncertainties are compatible with the ones obtained extracting γ with 4 modes.

Summary of systematic uncertainties

	minimum 1	minimum 2	minimum 3	minimum 4	minimum 5	minimum 6
syst 1	$0.8°$	$0.3°$	$0.2°$	$0.7°$	$1.4°$	$0.7°$
syst 2	$1.0°$	$2.6°$	$2.4°$	$0.7°$	$1.3°$	$0.9°$

Statistical uncertainties dominate (≈ 10°)

Further tests of flavour SU(3) breaking I

- From the theoretical expressions for the amplitudes:

$$A(B^0 \to K^+K^0K^-)_{\rm fs} = \alpha_{SU(3)} A(B^+ \to K^+\pi^+\pi^-)_{\rm fs}$$

- If flavour SU(3) symmetry is conserved, $\alpha_{SU(3)} = 1$, and thus these amplitudes are equal.

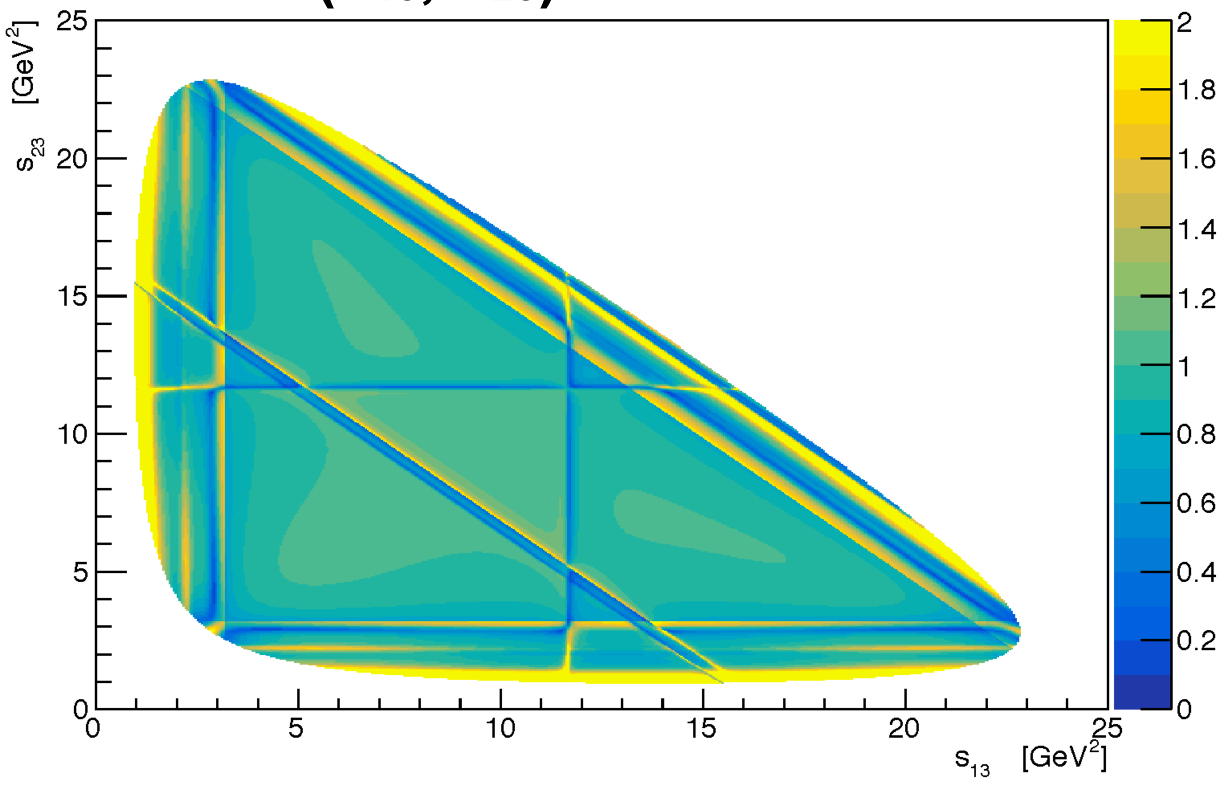
- We define the ratio $R(s_{13}, s_{23})$

$$R(s_{13}, s_{23}) = \frac{A^{K^+\pi^+\pi^-}(s_{13}, s_{23}) + \bar{A}^{K^+\pi^+\pi^-}(s_{13}, s_{23})}{A^{K_S K^+ K^-}(s_{13}, s_{23}) + \bar{A}^{K_S K^+ K^-}(s_{13}, s_{23})}$$

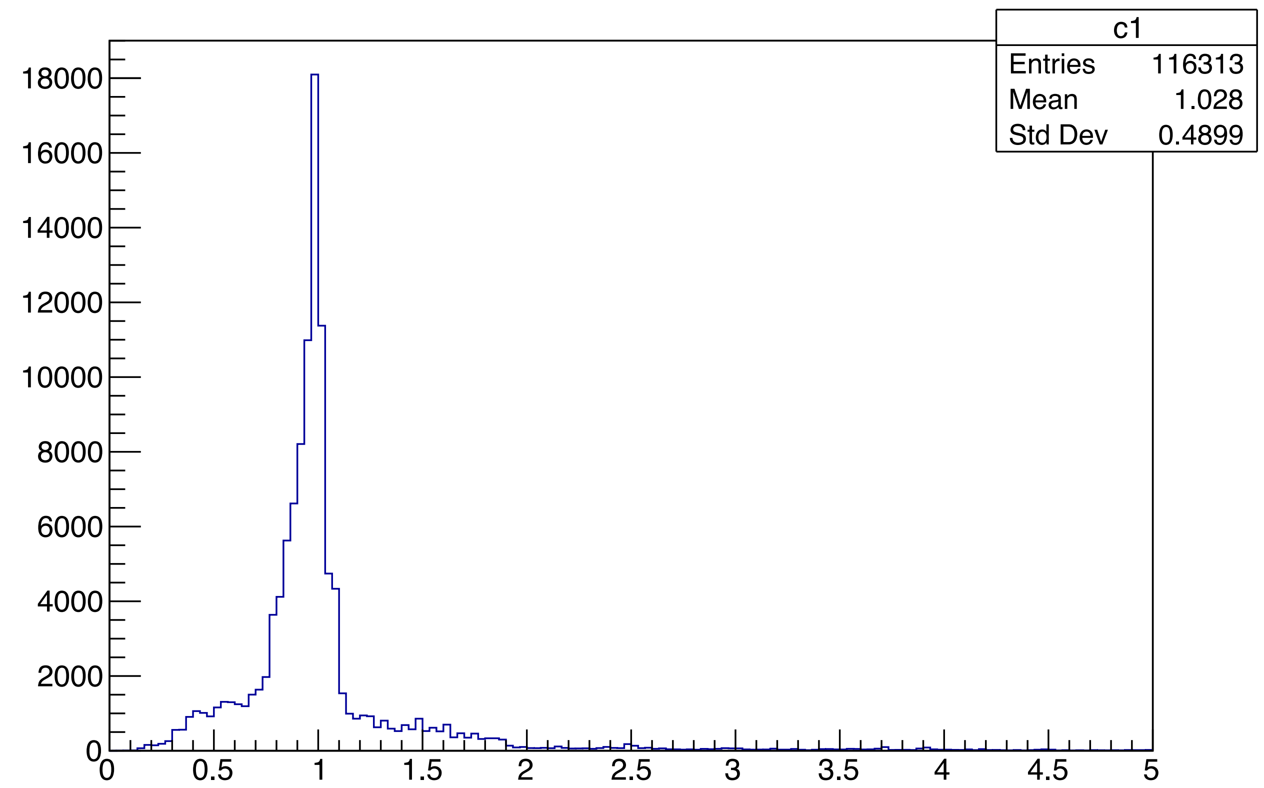
Hypothesis:
- Flavour SU(3) symmetry is conserved when averaging over many points over the DP.

Further tests on flavour SU(3) breaking I

$R(s_{13}, s_{23})$ over the DP



Histogram of the values of $R(s_{13}, s_{23})$



Remarks:

- $R(s_{13}, s_{23})$ varies over the DP, especially near resonances
- $< R(s_{13}, s_{23}) > = 1.03$, close to 1

Further tests on flavour SU(3) breaking I

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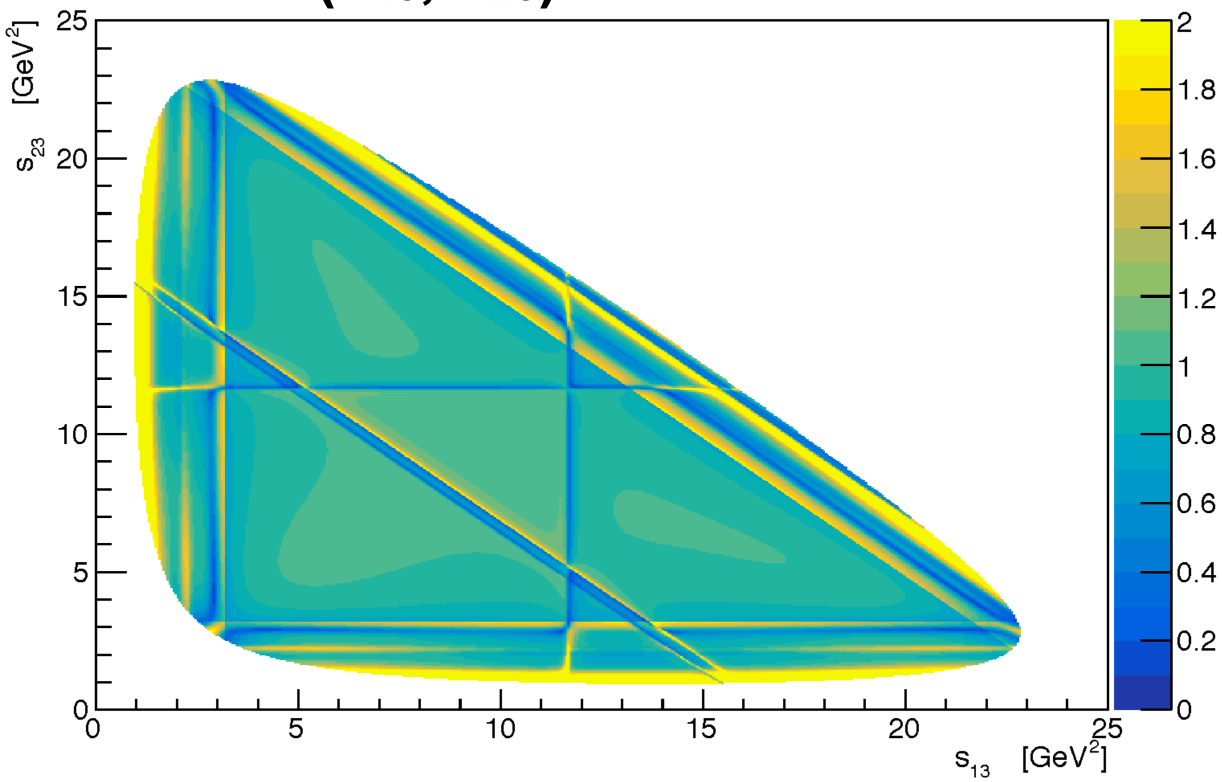

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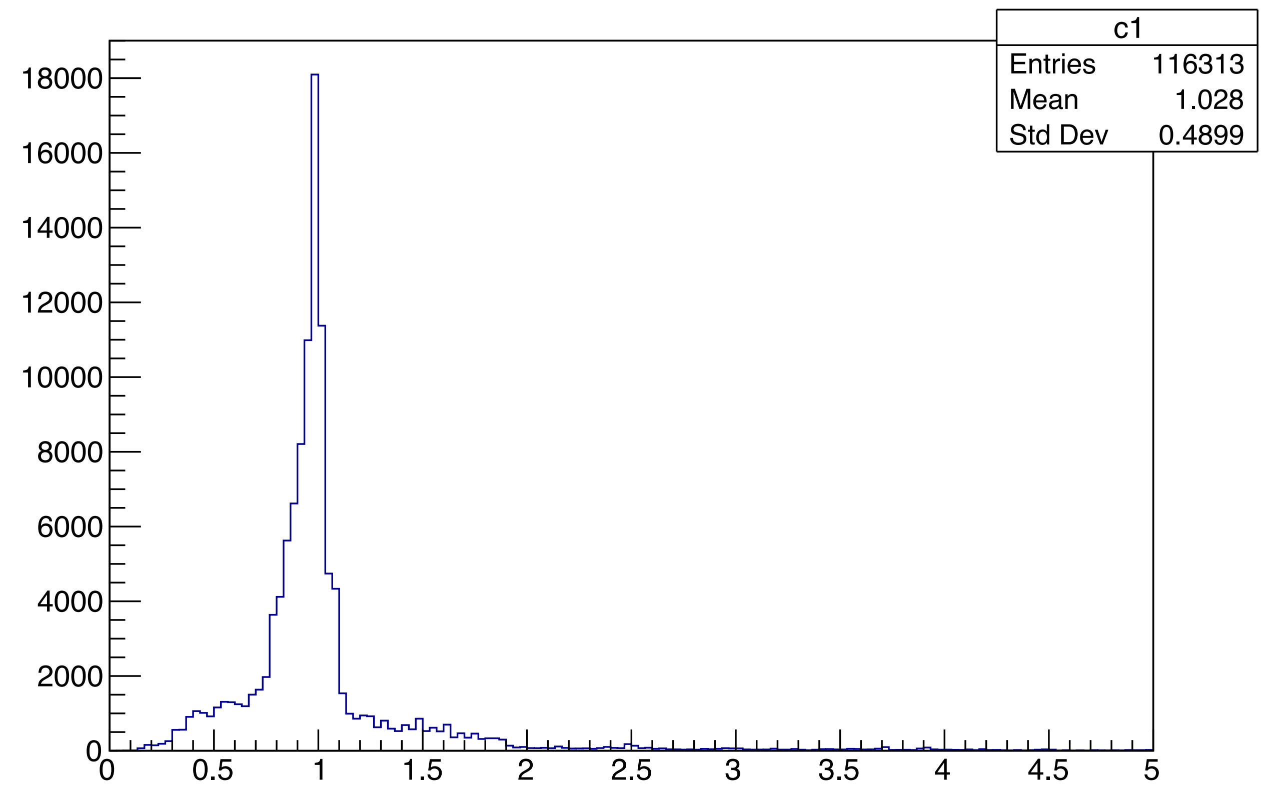


Remarks:

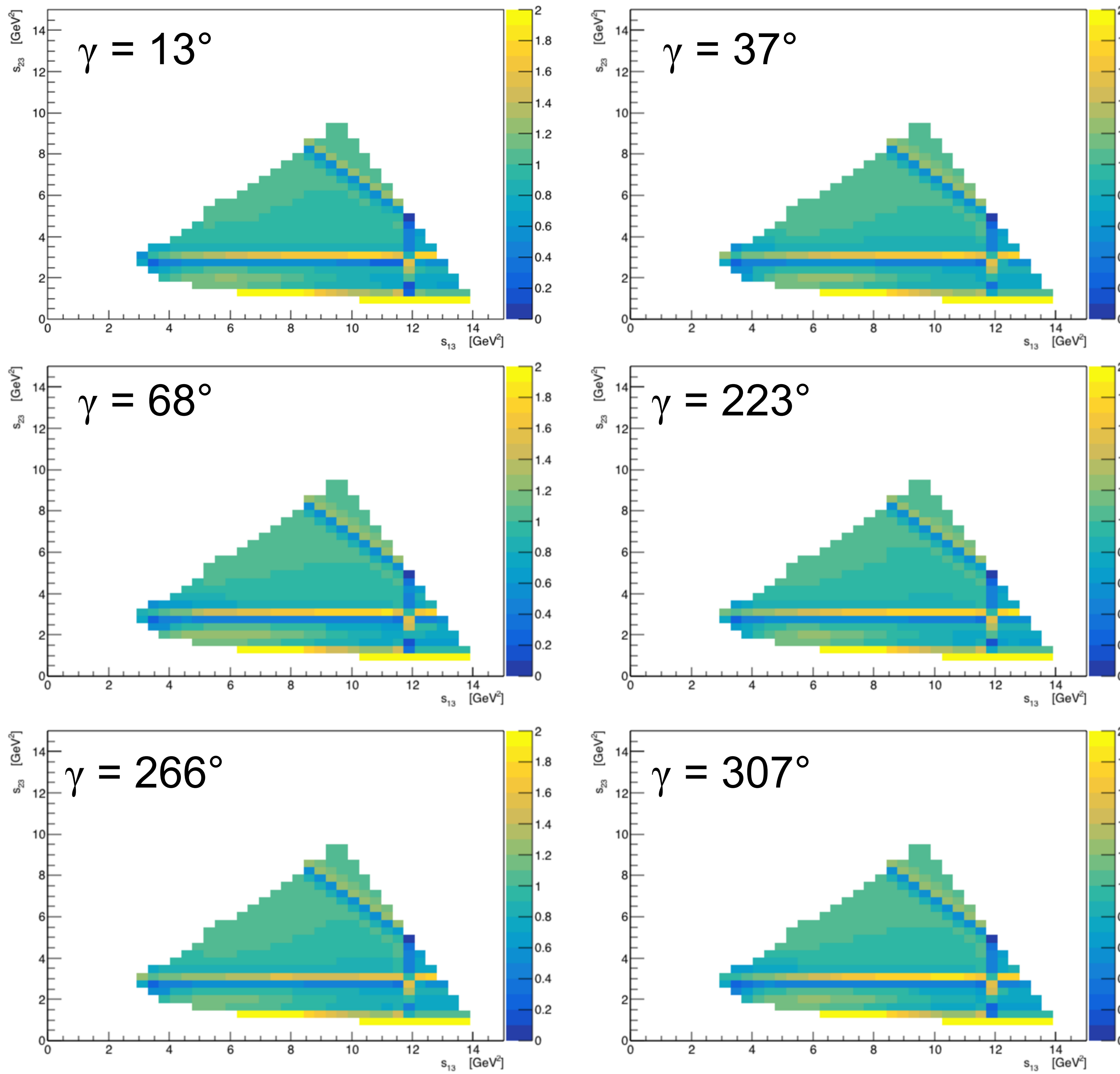
- $R(s_{13}, s_{23})$ varies over the DP, especially near resonances → **as expected.**
- $< R(s_{13}, s_{23}) > = 1.03$, close to 1 → **as expected.**

The hypothesis of flavour SU(3) symmetry conserved "on average" holds.

Further tests on flavour SU(3) breaking II

- Extract $\alpha_{SU(3)}$ value by a fit at different single points over the DP fixing γ to the values of the 6 minima we found previously.

γ	$\langle \alpha_{SU(3)} \rangle$
13°	1.06
37°	1.06
68°	1.05
223°	1.06
266°	1.05
307°	1.05



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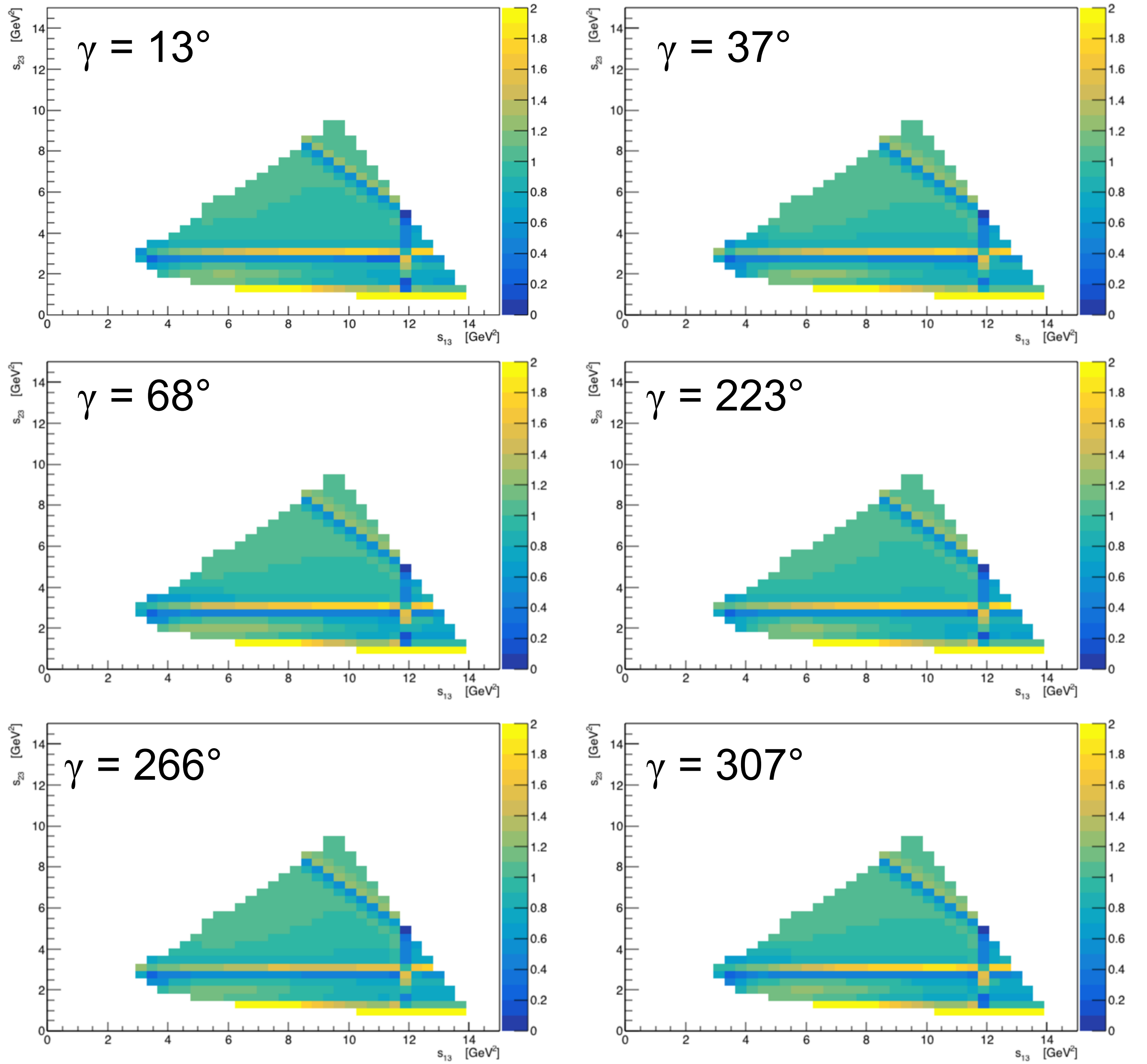


The hypothesis of flavour SU(3) symmetry conserved "on average" holds.

Conclusion

- We studied a method for extracting γ from charmless 3-body decays relying on flavour SU(3) symmetry.

- The values of γ were obtained using BABAR results.
  - 6 minima (1 consistent with SM).
  - Well separated, no overlap.
  - Statistical error about 10° (BABAR results only).
  - Statistical error dominates over Systematics.

$$\gamma_1 = 12.9^{\circ}{}^{+8.4^{\circ}}_{-4.3^{\circ}} \ (\mathrm{stat}) \pm 0.8^{\circ} \ (\mathrm{syst}) \pm 1.0^{\circ} \ (\mathrm{syst})$$

$$\gamma_2 = 36.6^{\circ}{}^{+6.6^{\circ}}_{-6.1^{\circ}} \ (\mathrm{stat}) \pm 0.3^{\circ} \ (\mathrm{syst}) \pm 2.6^{\circ} \ (\mathrm{syst})$$

$$\gamma_3 = 68.9^{\circ}{}^{+8.6^{\circ}}_{-8.6^{\circ}} \ (\mathrm{stat}) \pm 0.2^{\circ} \ (\mathrm{syst}) \pm 2.4^{\circ} \ (\mathrm{syst})$$

$$\gamma_4 = 223.2^{\circ}{}^{+10.9^{\circ}}_{-7.5^{\circ}} \ (\mathrm{stat}) \pm 0.7^{\circ} \ (\mathrm{syst}) \pm 0.7^{\circ} \ (\mathrm{syst})$$

$$\gamma_5 = 266.4^{\circ}{}^{+9.2^{\circ}}_{-10.8^{\circ}} \ (\mathrm{stat}) \pm 1.4^{\circ} \ (\mathrm{syst}) \pm 1.3^{\circ} \ (\mathrm{syst})$$

$$\gamma_6 = 307.5^{\circ}{}^{+6.9^{\circ}}_{-8.1^{\circ}} \ (\mathrm{stat}) \pm 0.7^{\circ} \ (\mathrm{syst}) \pm 0.9^{\circ} \ (\mathrm{syst})$$

- Paper in preparation.

Perspectives

- Dedicated analysis in a single experiment (LHCb, BELLE 2...) or even joint analysis?

- Take into account different symmetry states:
  - totally anti-symmetric states
  - mixed states

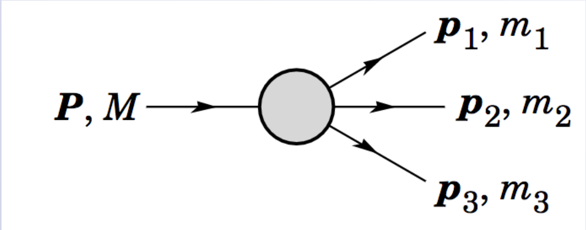
  } may help to decrease the statistical uncertainties and reduce the number of solutions.

Thomas Grammatico's Masters thesis

$$B^+ \to K^+\pi^+\pi^-$$
$$B^+ \to K^0\pi^+\pi^0$$
$$B^0 \to K^0\pi^+\pi^-$$

$$B^0 \to K^+\pi^0\pi^-$$
$$B^0 \to K^0 K^+ K^-$$

Fully anti-symmetric amplitudes:



$$|A\rangle \equiv \frac{1}{6}(|123\rangle - |132\rangle + |312\rangle - |321\rangle + |231\rangle - |213\rangle)$$