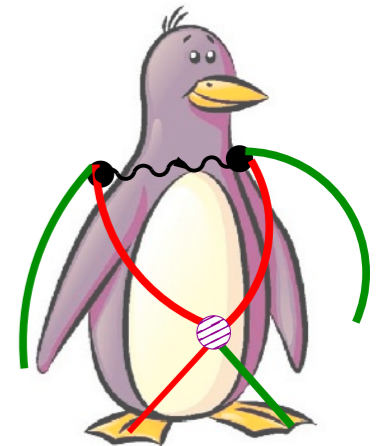


FSI in hadronic B decay

Patricia C. Magalhães

Technical University of Munich - TUM



- 3-body “pure light hadronic” heavy meson decay
 - dynamics, issues,...
- Importance of the FSI in low and high mass
- How to use informations from D meson decay?
ex: $D^+ \rightarrow K^- K^+ K^-$
- How to improve the B amplitude analysis ?
ex: $B^+ \rightarrow K^- K^+ K^-$
- what's next?

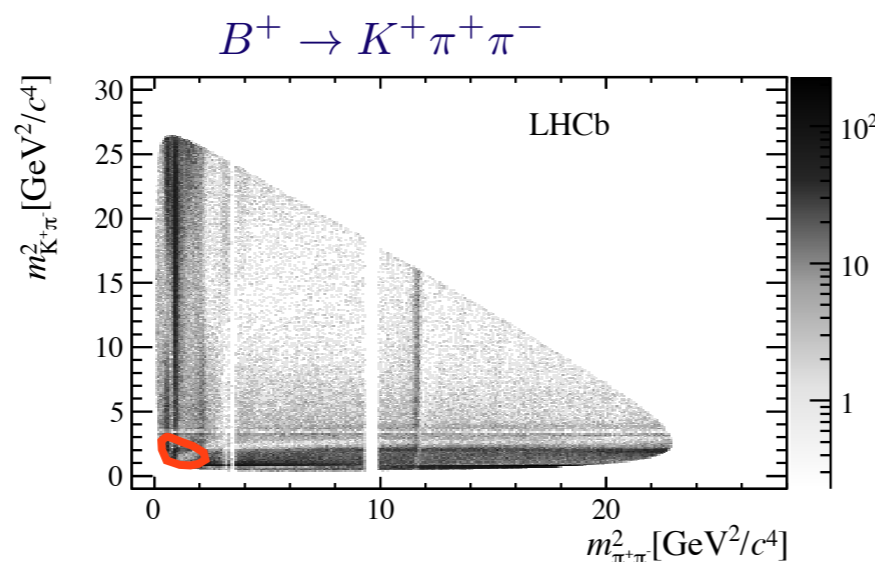
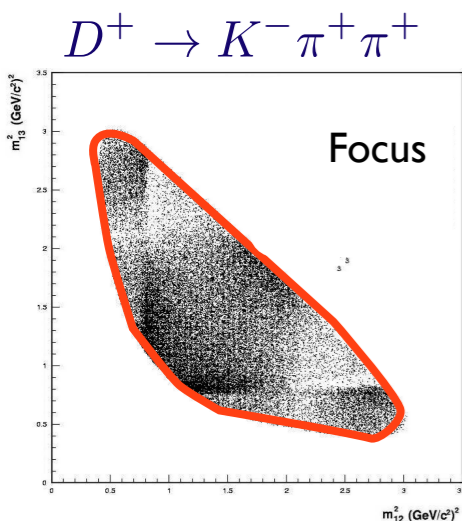
- D and B three-body **HADRONIC** decays are dominated by resonances
 - spectroscopy
 - information of MM interactions \longrightarrow no $K\bar{K}$ data available
 - study of CP-Violation (strong phase needed) \longrightarrow can lead to new physics
- new high data sample from LHCb \longrightarrow more to come from LHCb and Belle II

\longrightarrow deserve better models

isobar model 

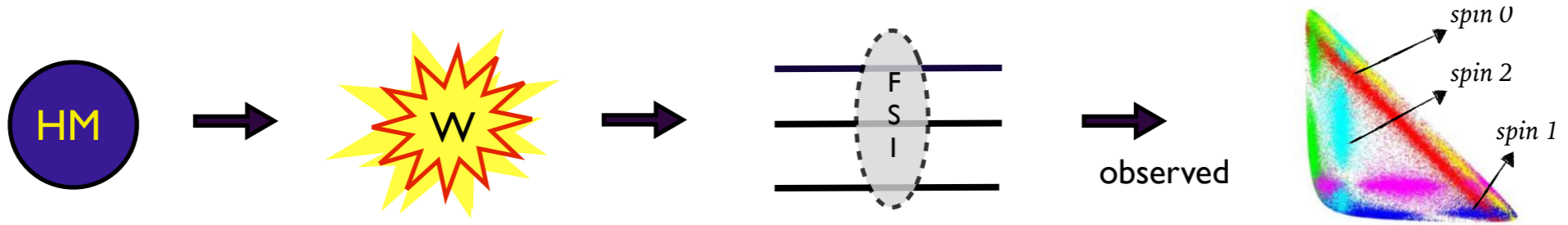
- violates two-body unitarity (2 res in the same channel);
- do NOT include rescattering and coupled-channels;
- free parameters are not connected with theory !

- can we learn something from D decays?



- \neq scales!!! \longrightarrow similar FSI
- 2-body theories “works” up to mD
- B phase-space \longrightarrow + FSI possibilities
charm-penguins,

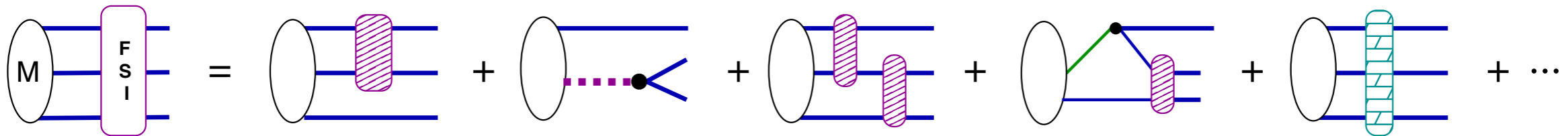
● dynamics



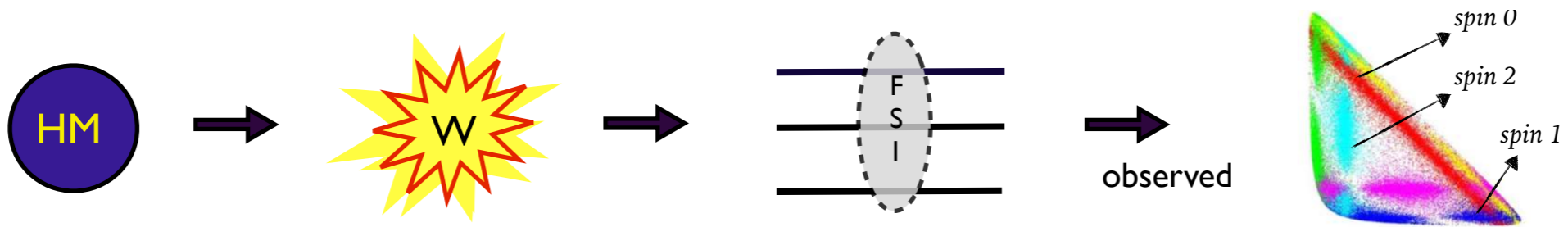
● weak primary vertex (W)



● Final State Interactions (FSI)



- dynamics

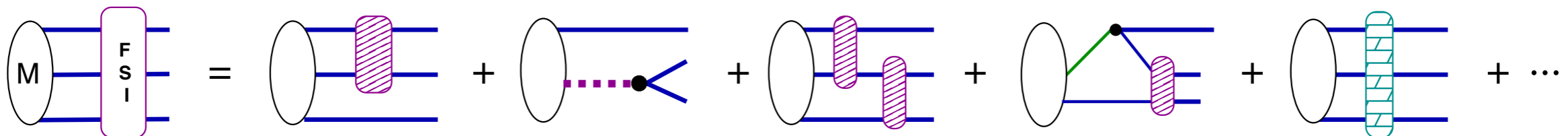


to extract information from data we need an amplitude MODEL

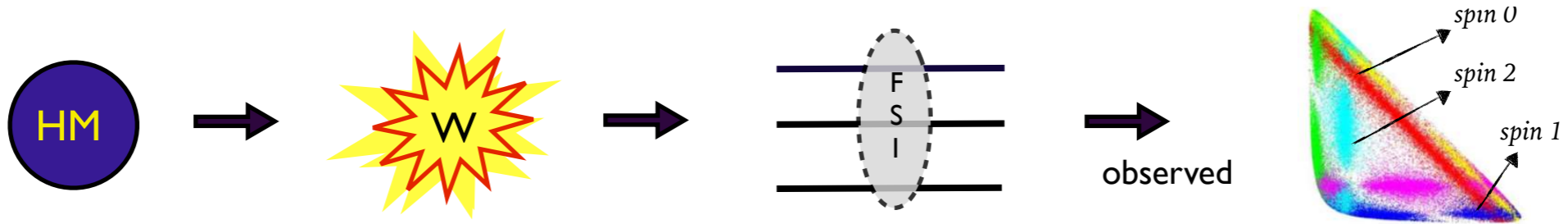
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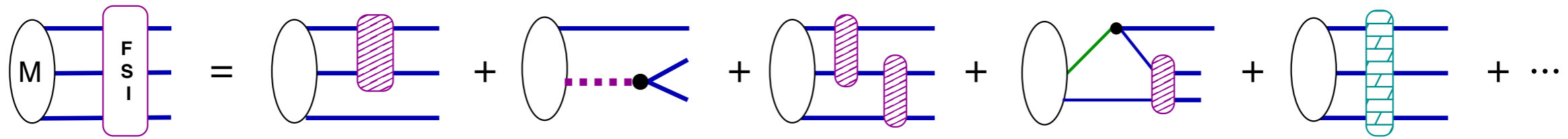
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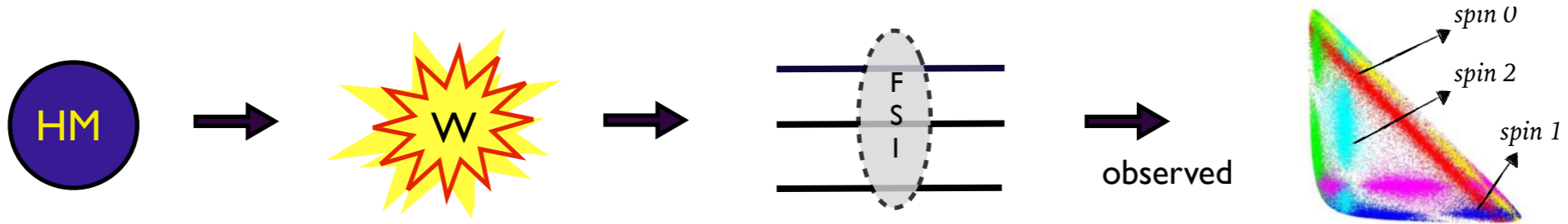


- QCD factorization approach
 - \rightarrow not precise for 3-body
 - not allow all kinds of FSI and 3-body NR

- Final State Interactions (FSI)



- dynamics



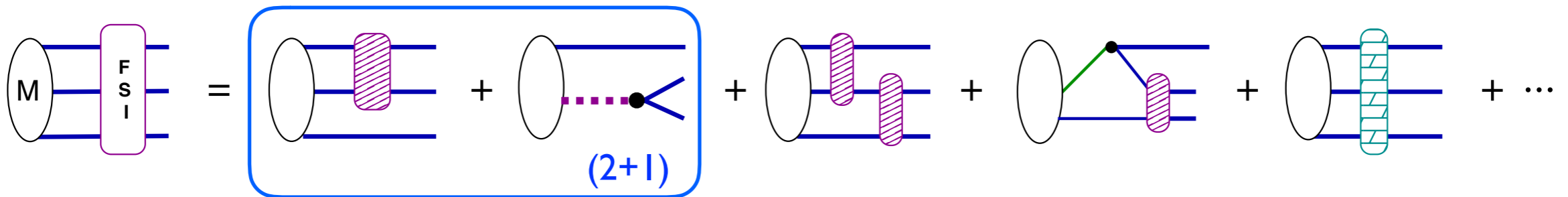
to extract information from data we need an amplitude MODEL

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- QCD factorization approach
 - not precise for 3-body
 - not allow all kinds of FSI and 3-body NR

- Final State Interactions (FSI)



- 2-body is crucial

- full unitarity: Faddeev, Khury-Trieman, triangles

- massive localized Acp

$$B^\pm \rightarrow hhh$$

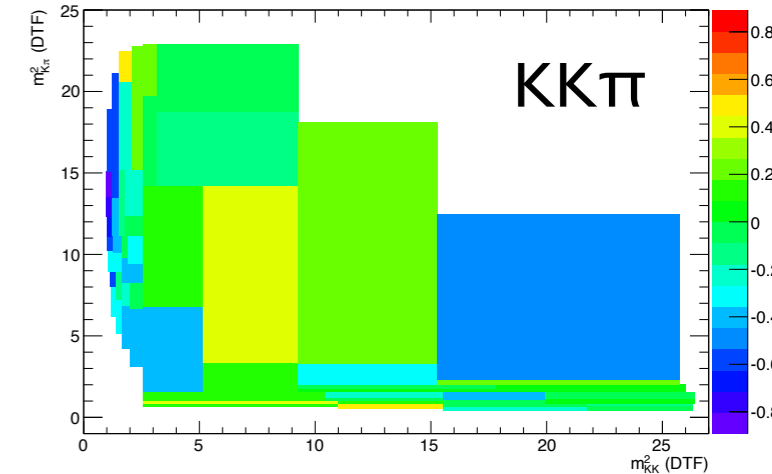
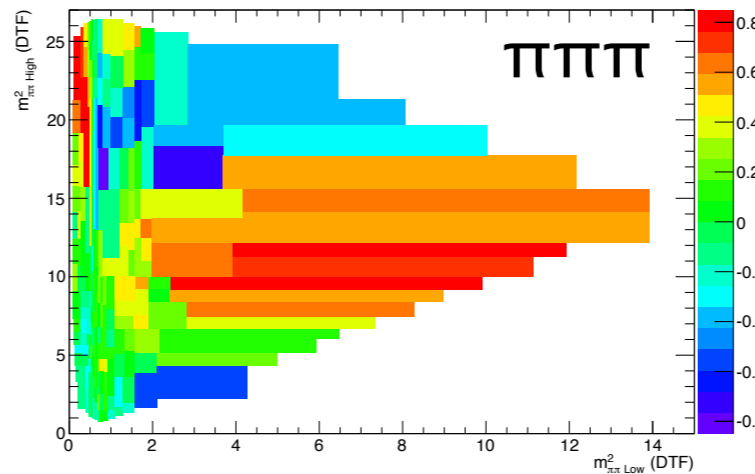
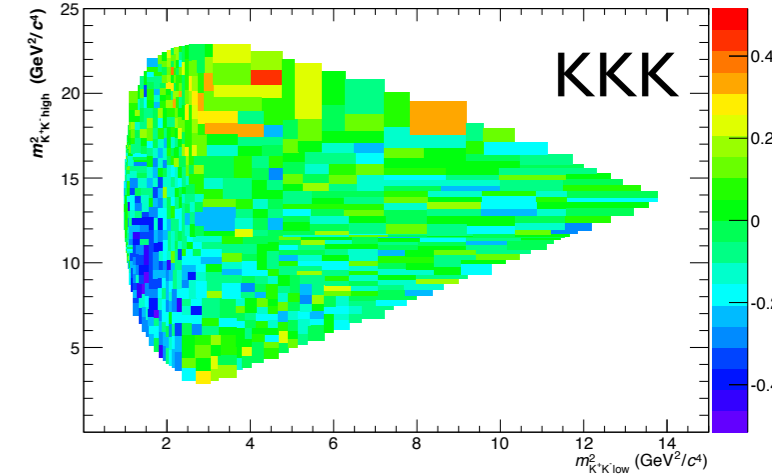
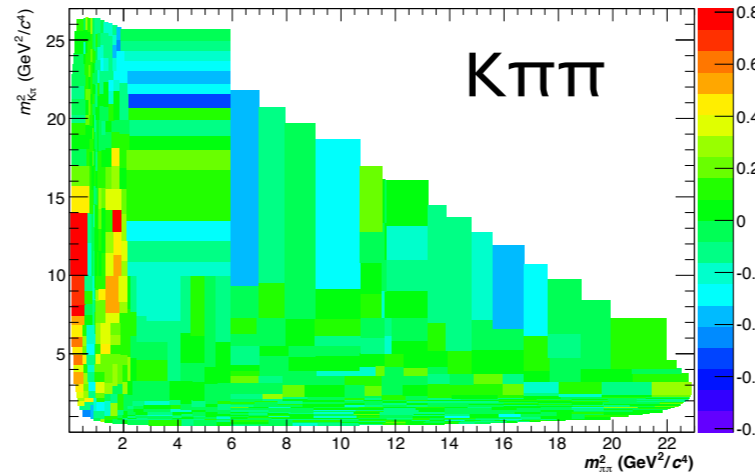


- Charge Parity Violation (CPV):

$$A_{M \rightarrow f} = A_1 e^{i(\delta_1 - \phi_1)} + A_2 e^{i(\delta_2 - \phi_2)}$$

$$|A_{M \rightarrow f}|^2 - |A_{\bar{M} \rightarrow \bar{f}}|^2 = -4A_1 A_2 \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2) \neq 0$$

↳ two \neq weak and strong phases



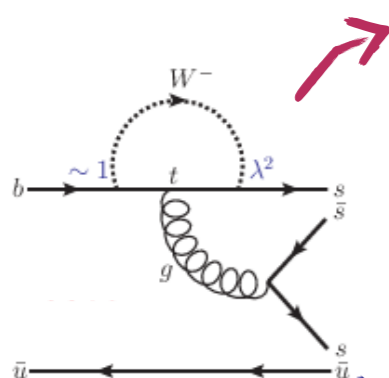
LHCb PRD90 (2014) 112004

- BSS model

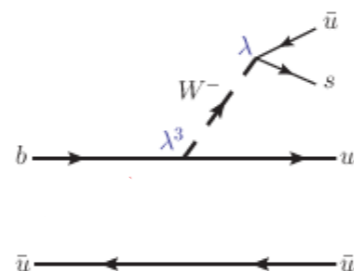


Bander Silverman & Soni PRL 43 (1979) 242

not enough !!!



strong phase



weak phase: CKM

- factorization techniques

- describes well two-body Br;
- do not describe Acp data;

- FSI → strong phase

Wolfenstein PRD43 (1991) 151

- FSI
 - low energy MM rescattering, coupled-channels and resonances
 - 3-body + NR effects

two-body theoretical models (unitary & analyticity) → dispersion relations and ChPT
limited to $\sim 1 - 2 \text{ GeV}$

→ how far we really need 2-body amplitude?! all B phase-space ?

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 - low energy MM rescattering, coupled-channels and resonances
 - 3-body + NR effects

two-body theoretical models (unitary & analyticity) → dispersion relations and ChPT
limited to $\sim 1 - 2 \text{ GeV}$

→ how far we really need 2-body amplitude?! all B phase-space ?

- what do we have in the market ?


L. Benoit talk

- (2+1) parametrization with QCDF + scalar and vector meson-meson FF ; Boito et al [Paris, Krakov, SP]

- 3- FSI at low energy: with triangle loops PCM et. al. [Brazil effort] or Khuri-Treiman Kubis et. al [Bonn] → limited to very low E

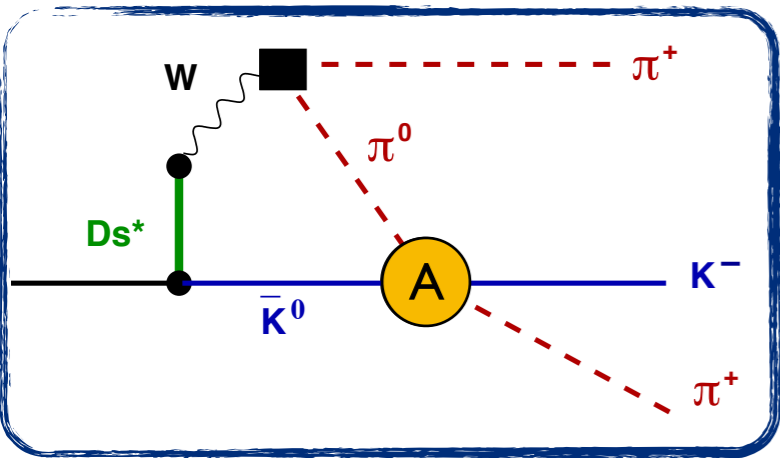
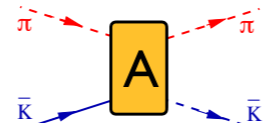
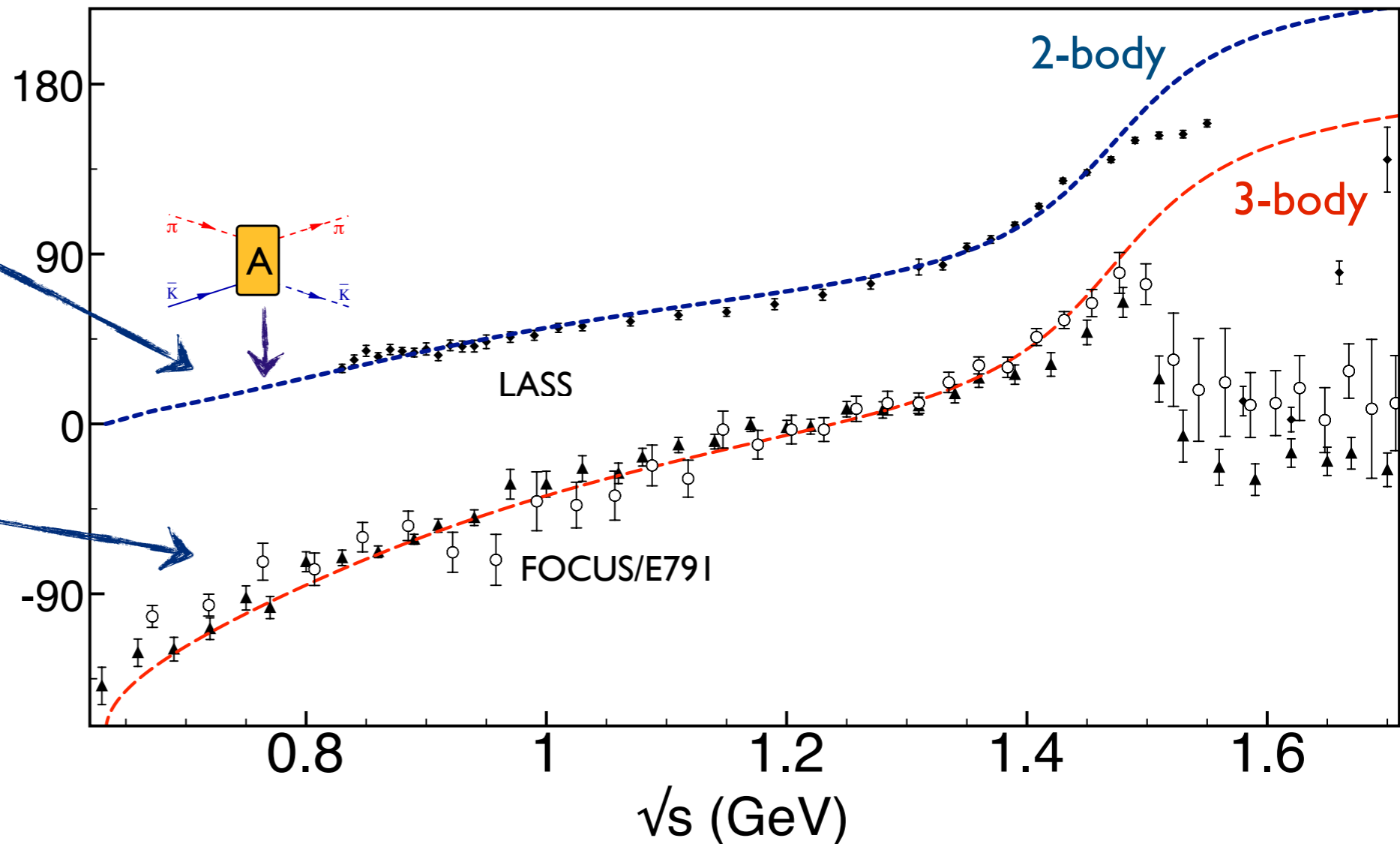
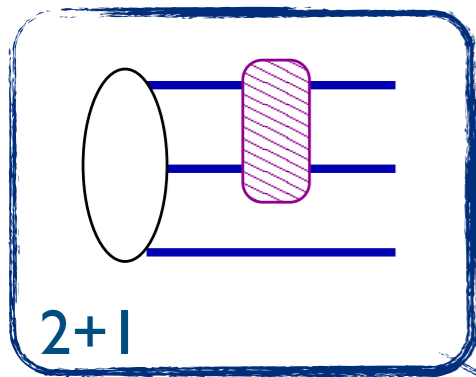
- 3-b FSI high energy: charm penguins  Bediaga, Fredrico & PCM

- K-matrix: 2-b pole + polynomial with free parameter modulated by a production; Anisovich, Babar, LHCb
→ (2+1) difficult to extract informations

- GLASS, LASS use directly 2-body phases for 3-body process 
→ (2+1)

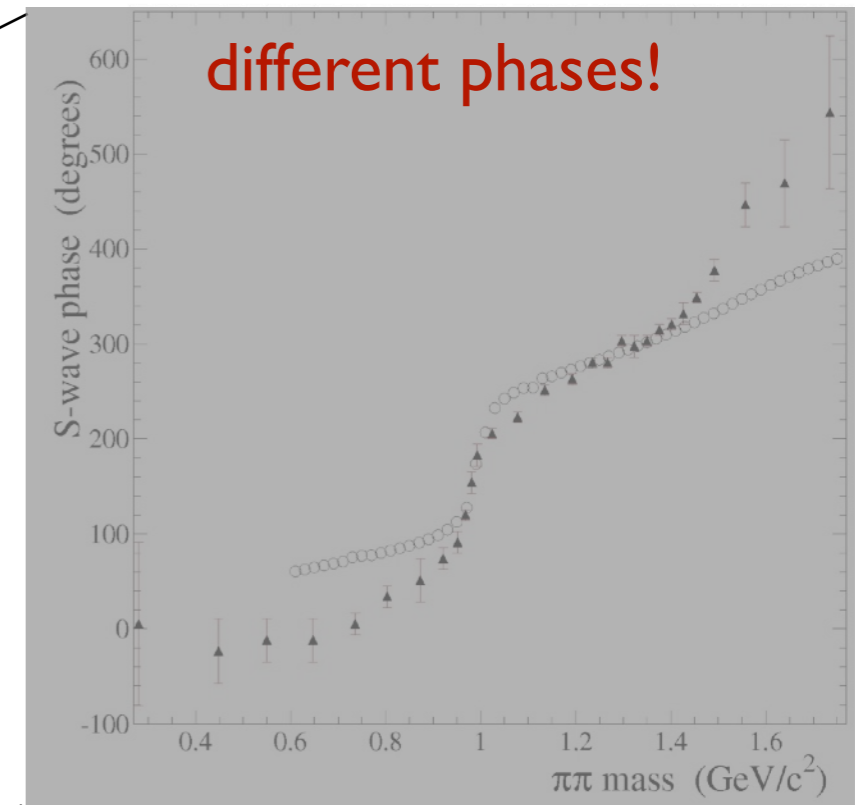
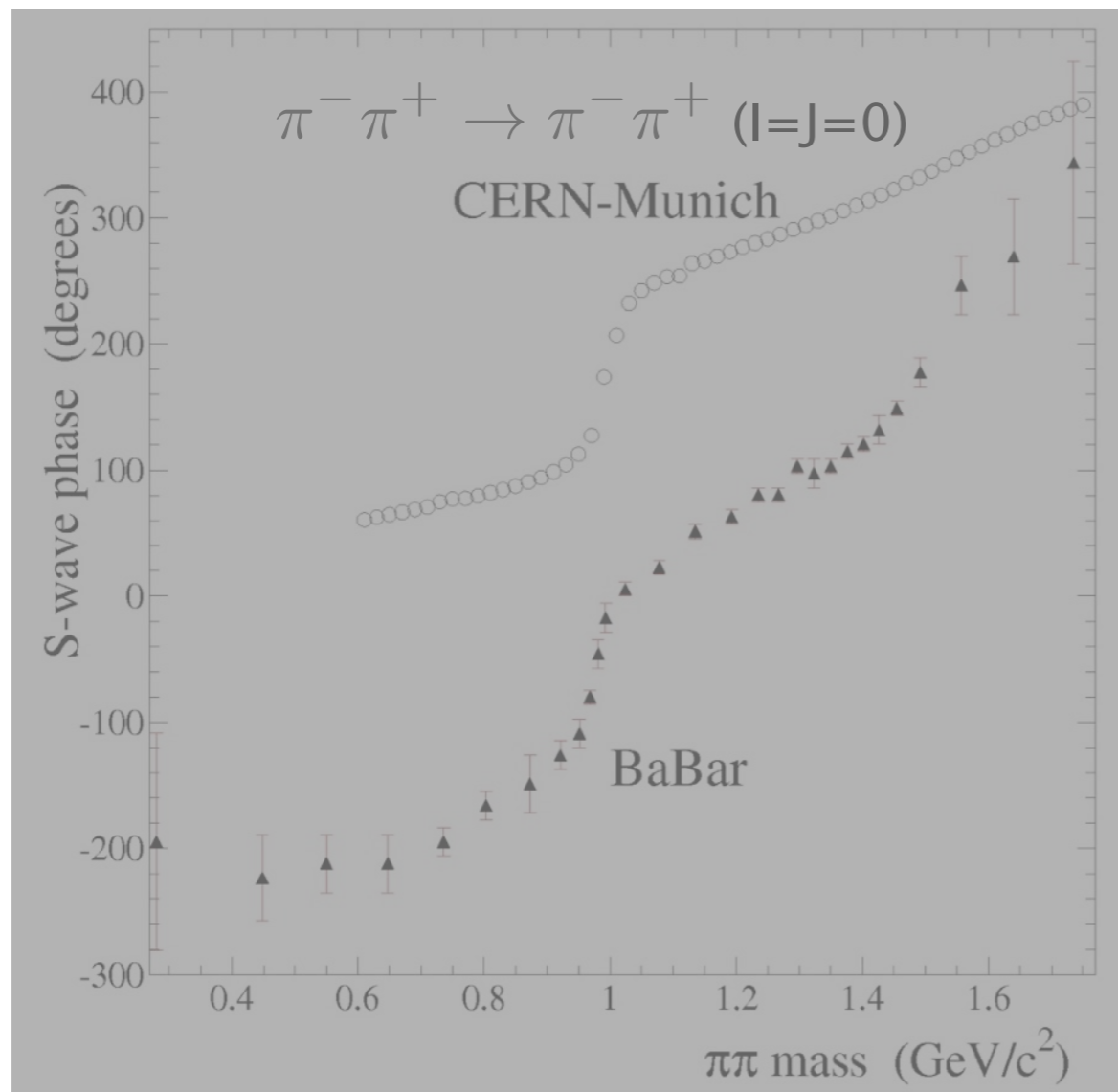
- Can we extract two-body information from 3-body data? **Not directly!**
- $D^+ \rightarrow K^- \pi^+ \pi^+$ \rightarrow different S-wave phase from $K^- \pi^+$

PC Magalhães et.al: PRD84 094001 (2011), PRD92 094005 (2015)



different dynamic!

- $D_s^+ \rightarrow \pi^+ \pi^- \pi^+$



Phys.Rev. D 79 (2009) 032003

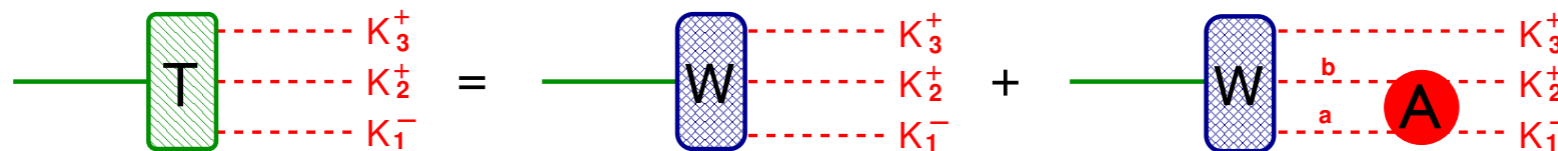
- 2-body amplitude: spin and isospin well defined!
- 3-body data: only spin! $\& \neq$ dynamics (weak vertex, FSI, 3rd particle, ...)

There is no direct connection between phases of the 3-body decay amplitudes and two-body scattering amplitudes

- 2-body is a crucial ingredient $\begin{matrix} \nearrow \text{limited from theory to low E} \\ \searrow \text{limited scattering data} \end{matrix}$
- \rightarrow no KK scattering data and no theoretical model! just extensions

- use 3-body data to obtain information from two-body!

ex: a model for $D^+ \rightarrow K^- K^+ K^-$ that can predict the KK scattering



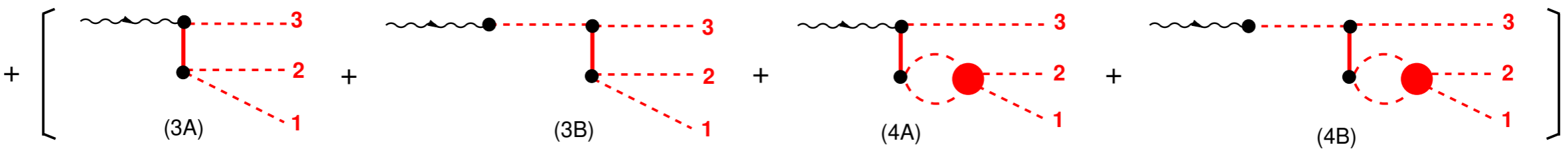
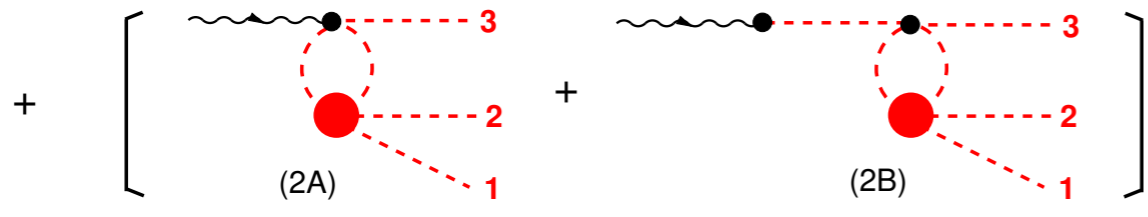
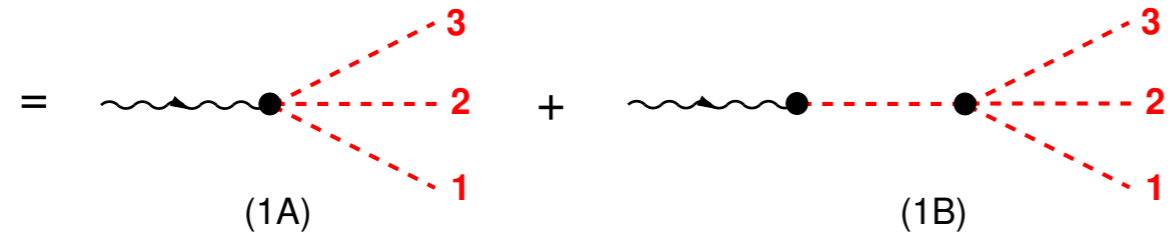
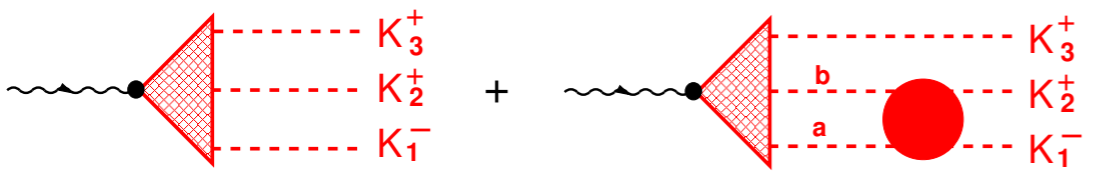
- alternative to isobar model in amplitude analysis [arXiv: 1805.11764](https://arxiv.org/abs/1805.11764)

- hypotheses that annihilation is dominant

- everything can be described by ChPT Lagrangian

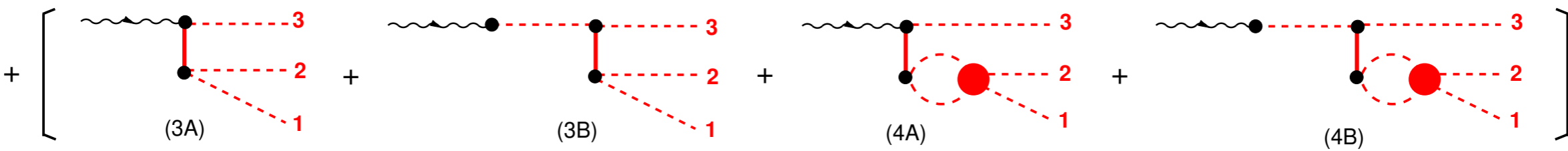
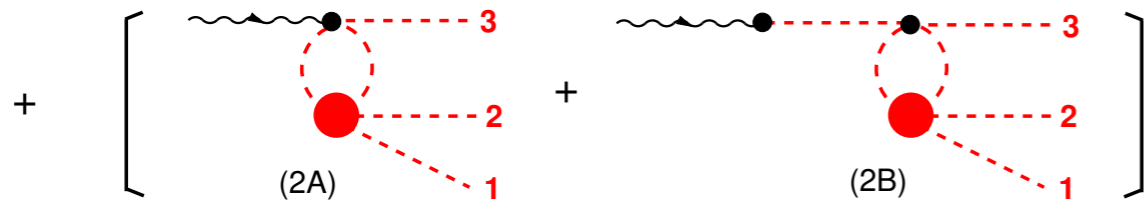
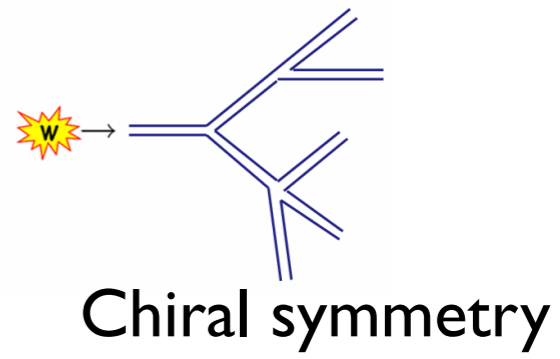
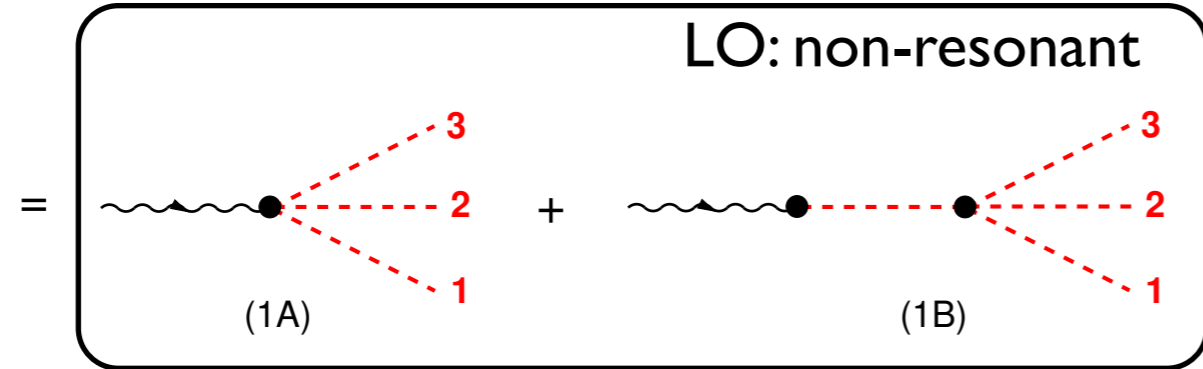
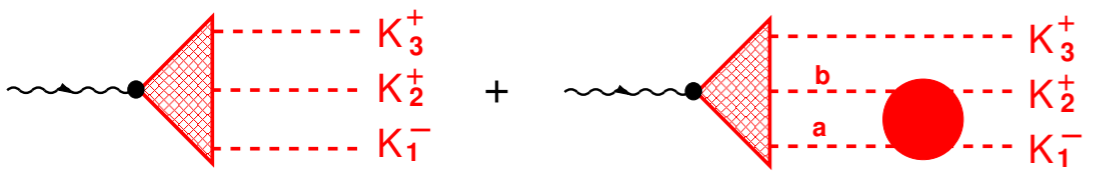
- $A_{ab}^{JI} \rightarrow$ unitary scattering amplitude for $ab \rightarrow K^+ K^-$

\rightarrow parameters have physical meaning: masses and coupling constants



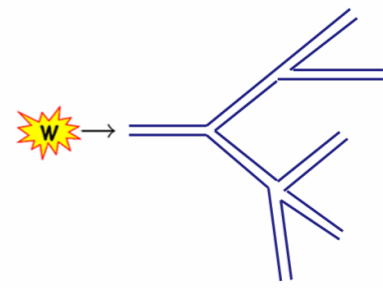
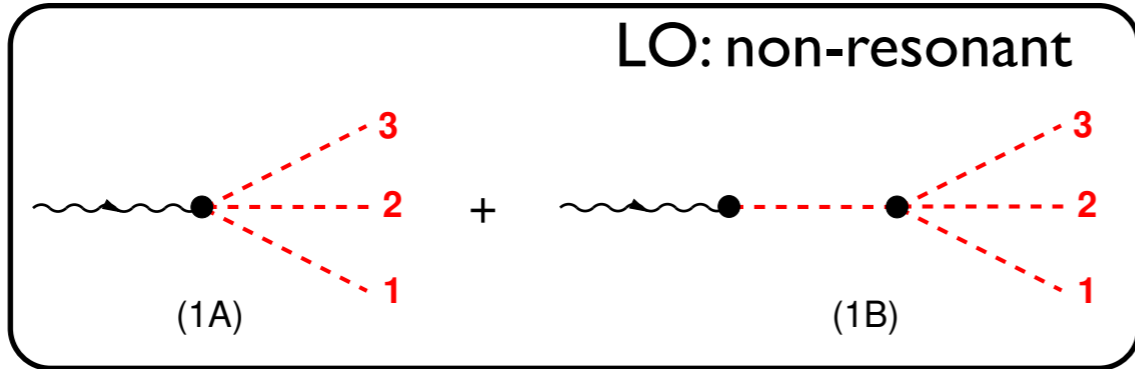
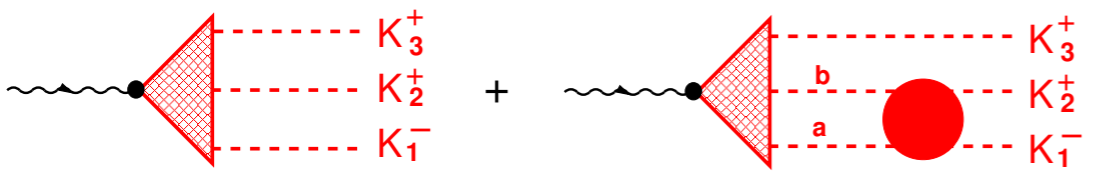
 $K\bar{K}$ scattering amplitude

 isospin decomposition $[J, I = (0, 1), (0, 1)]$

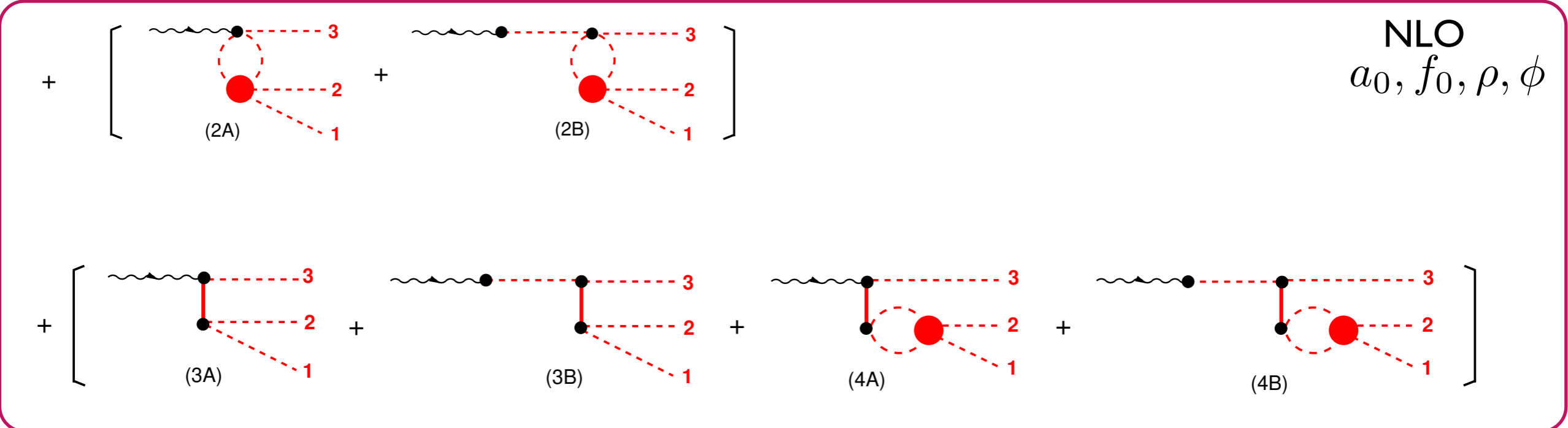


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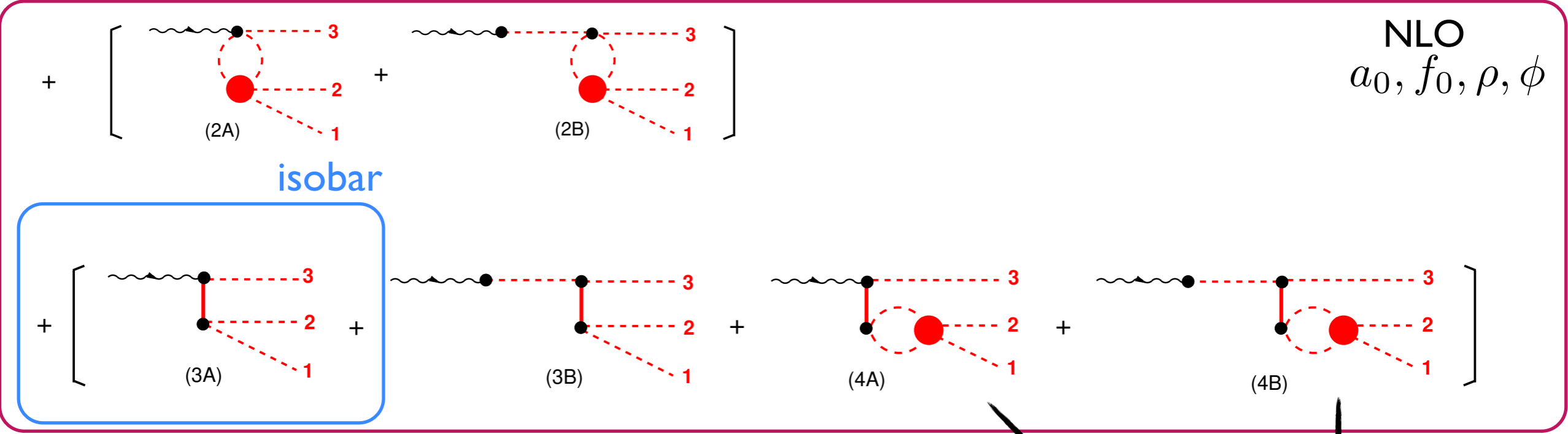
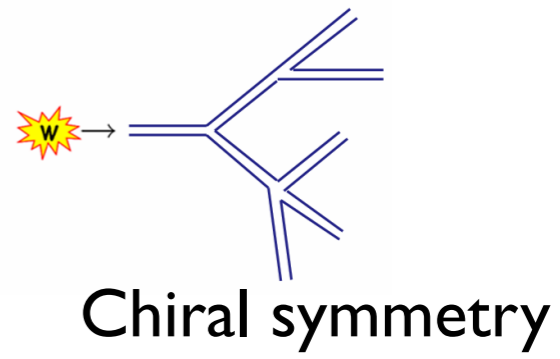
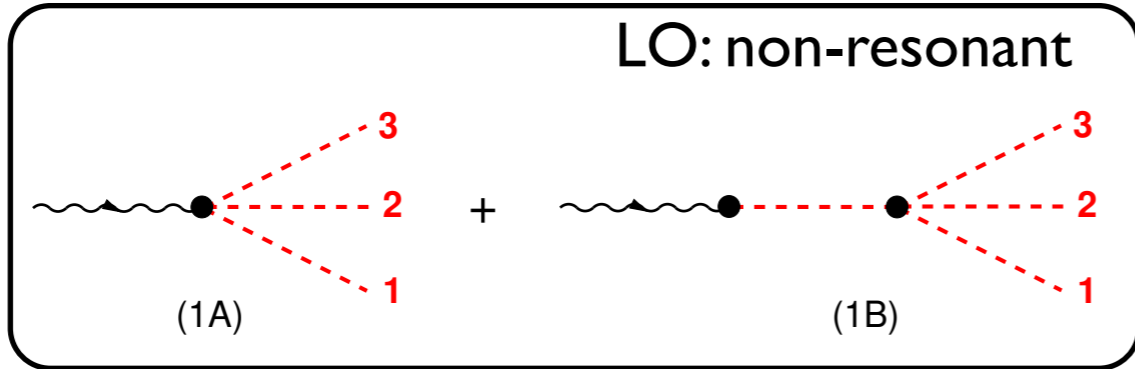
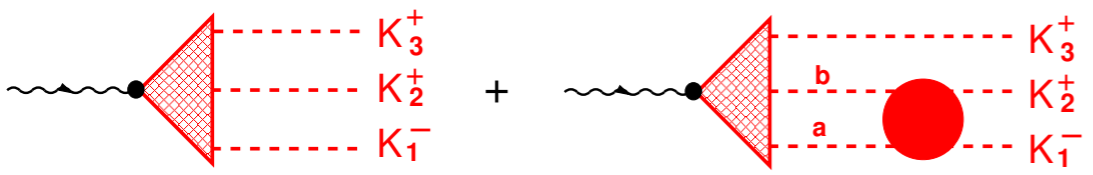


Chiral symmetry



$K\bar{K}$ scattering amplitude

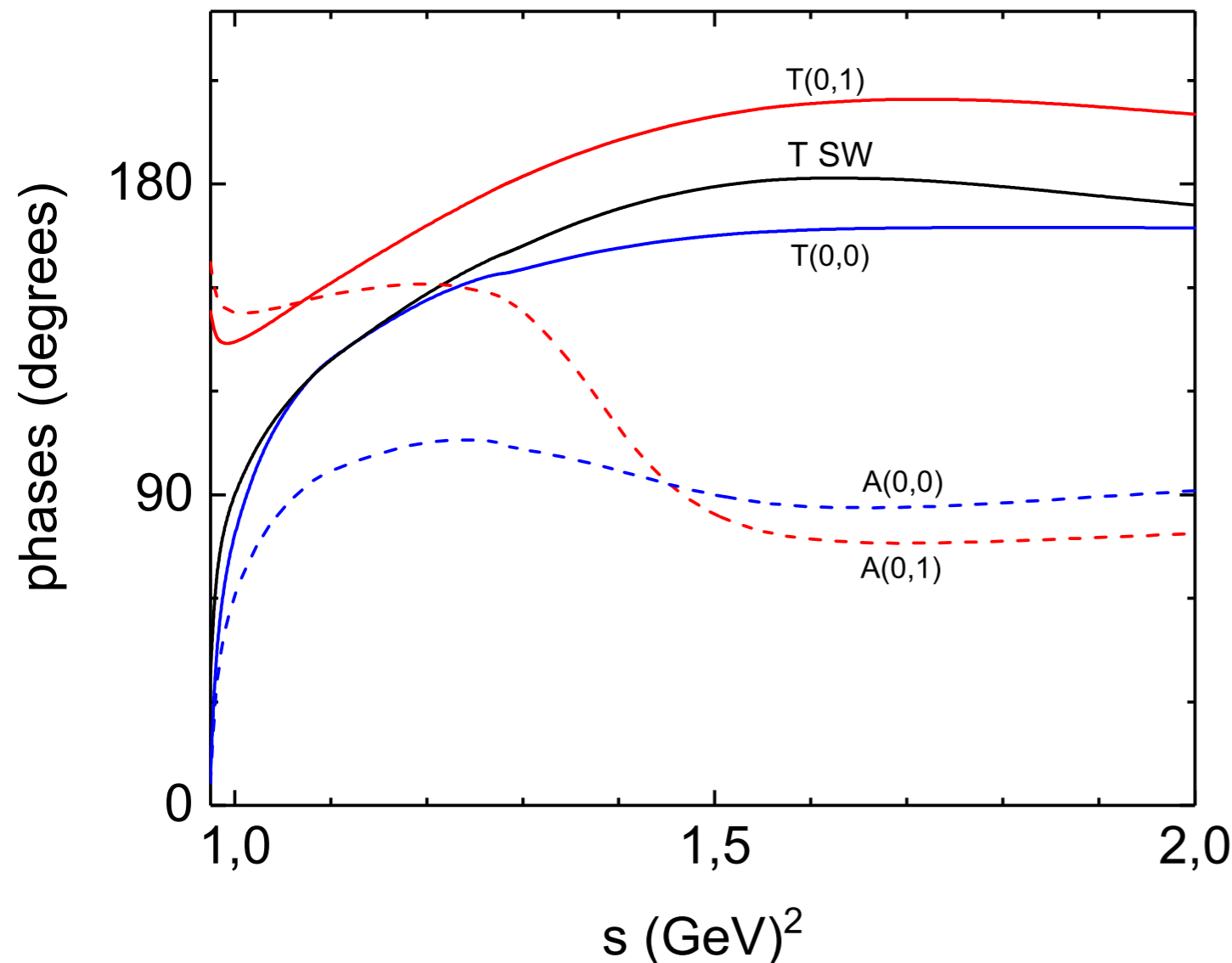
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$K\bar{K}$ scattering amplitude

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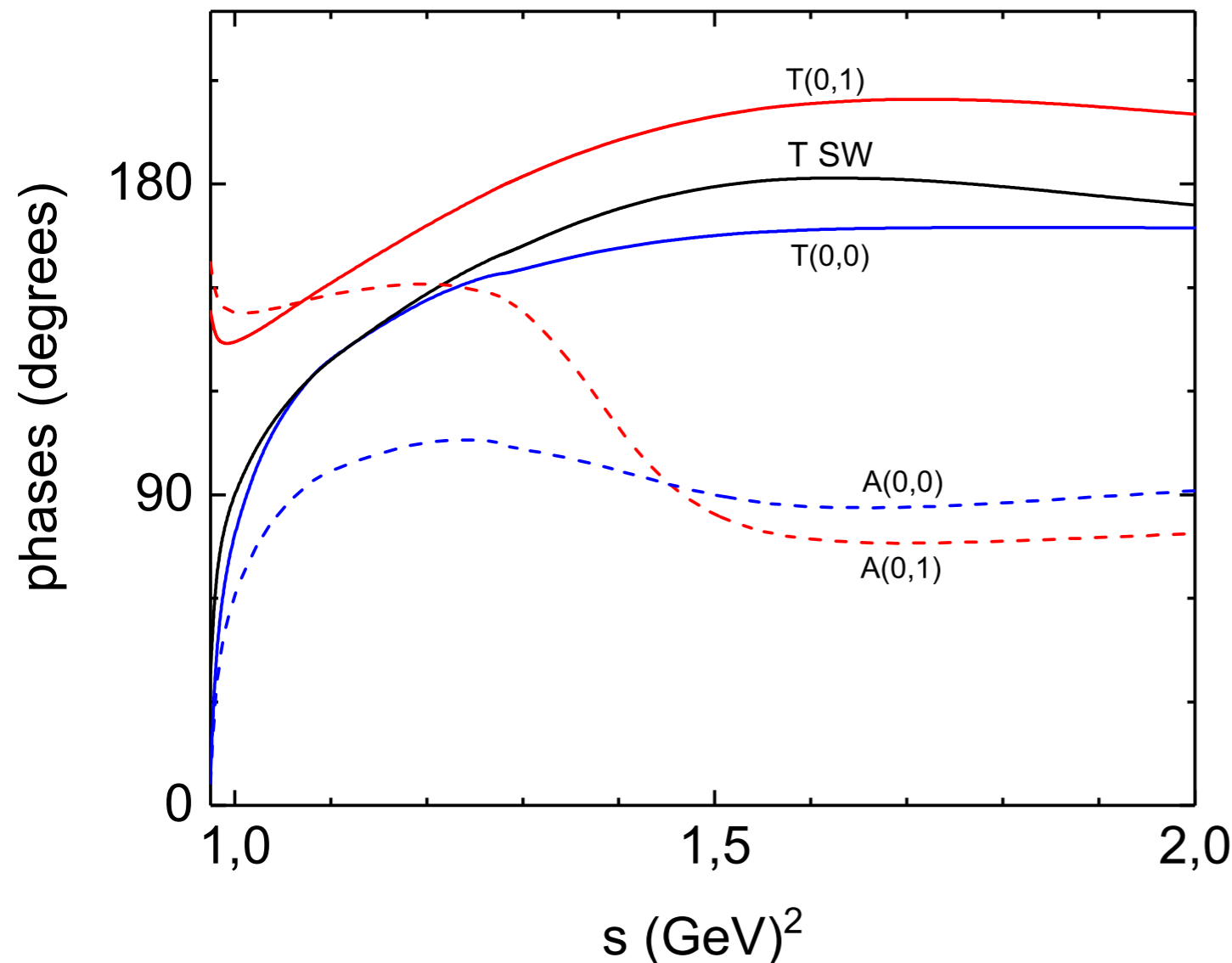
- non-resonant: beyond (2+1) is a 3-body amplitude
- FSI: coupled-channel meson-meson from ChPTR Lagrangian
- intensity of each component is predict by theory $\rightarrow \neq$ isobar model
- Toy studies



- parameters with physical meaning (ChPT)
- can disentangle a_0 and f_0
- couple channel structure: cannot be ignored

arXiv: 1805.11764

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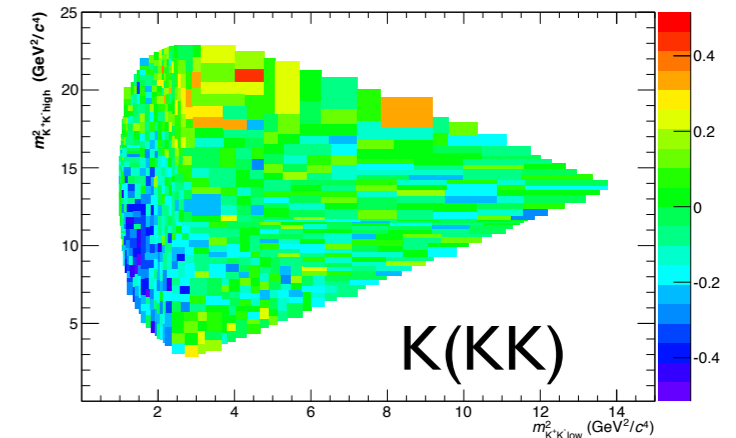
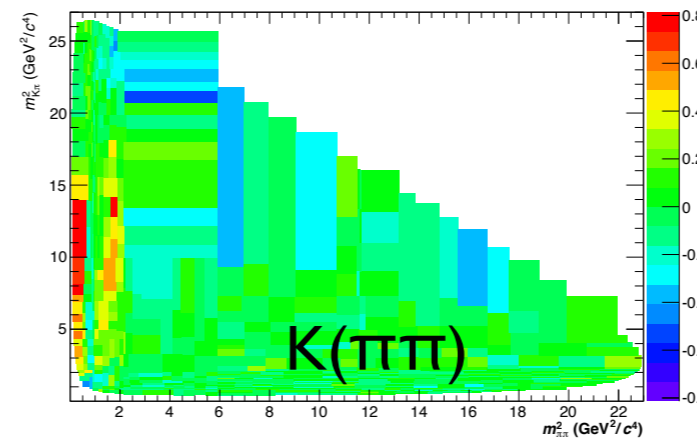
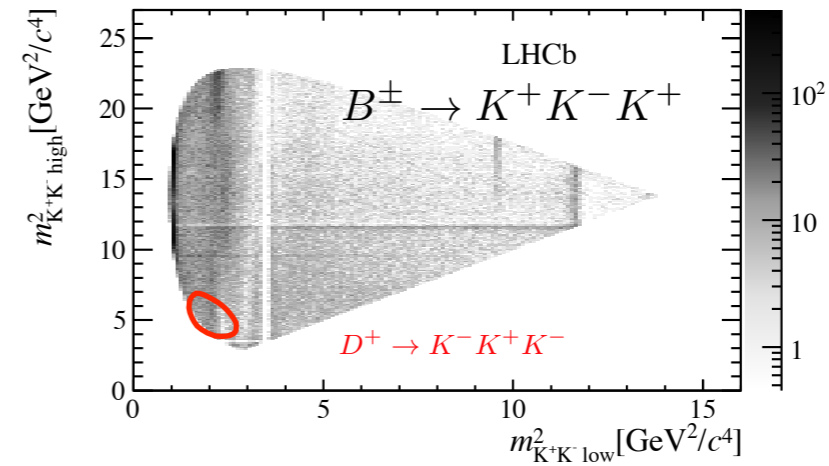
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- \rightarrow couple channel structure: cannot be ignored

Fitting data we can predict
KK scattering phase

arXiv: 1805.11764

B decays

- huge phase-space
- FSI at low and high energy → room for producing all sorts of particles
- localised CPV
- low mass
 - Bediaga, Frederico, & Lourenço
PRD89(2014)094013
- high mass?



B decays

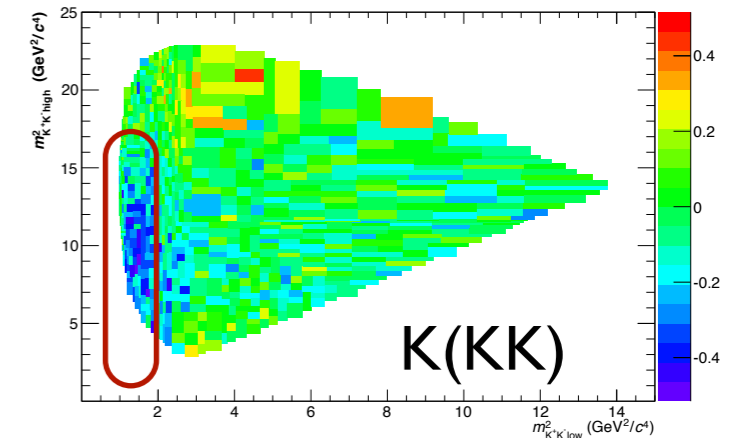
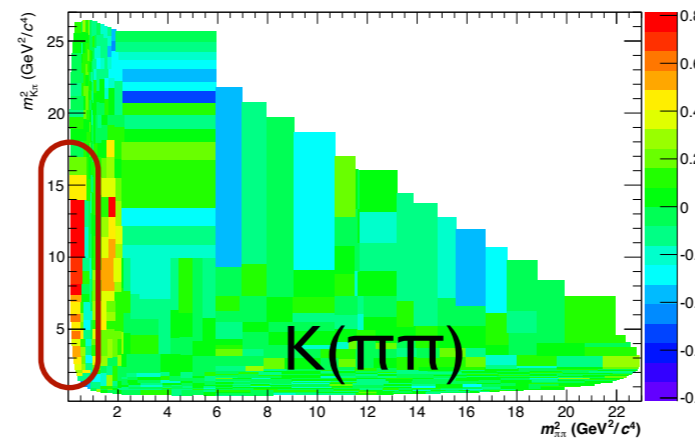
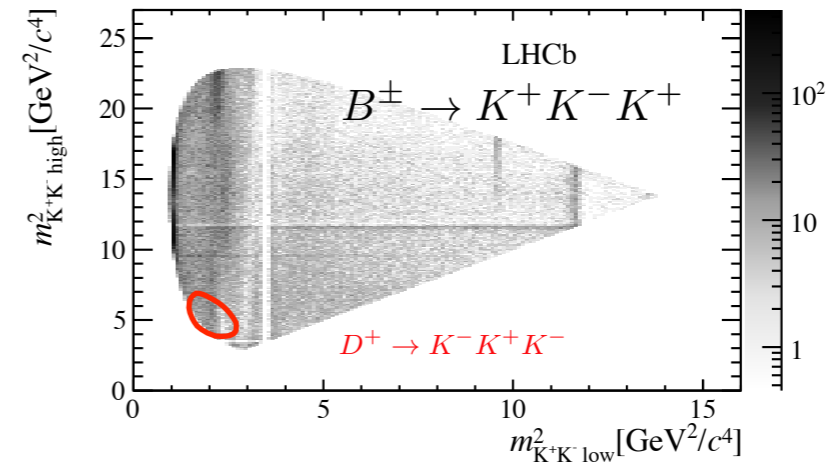
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- localised CPV

- low mass $\pi\pi \rightarrow KK$

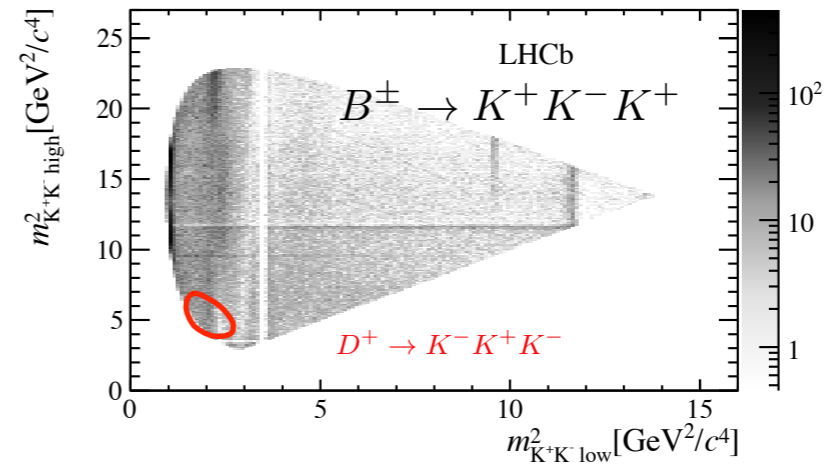
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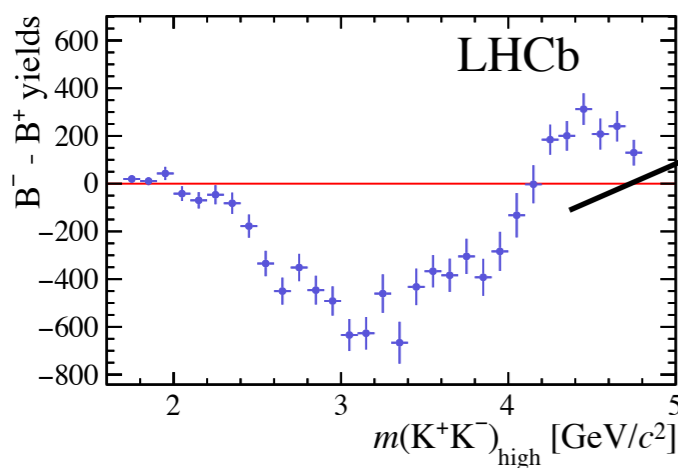
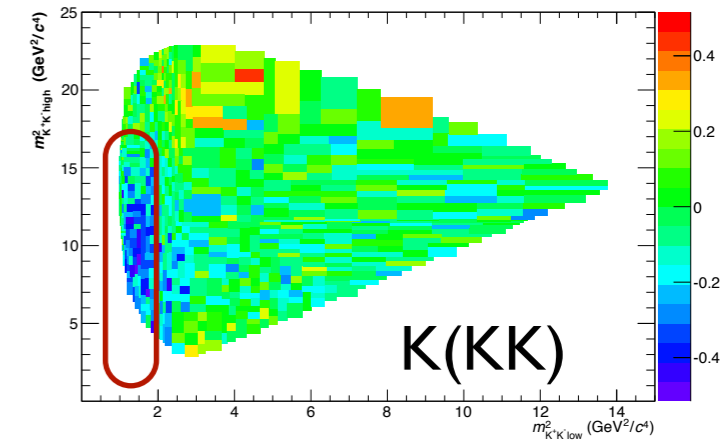
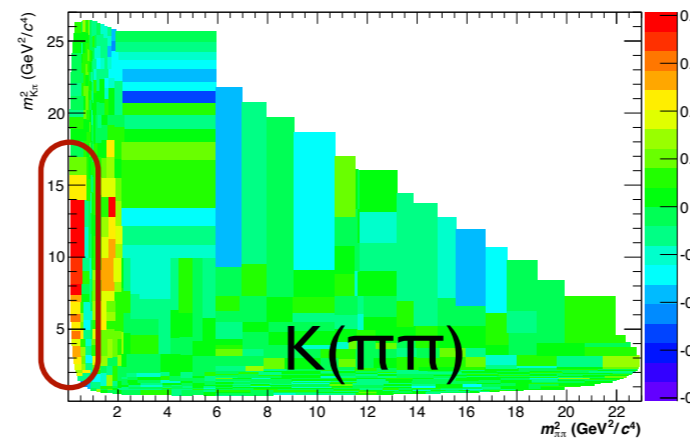


- localised CPV

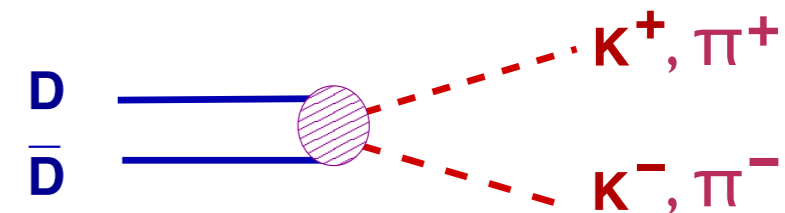
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Bediaga, Frederico, & Lourenço
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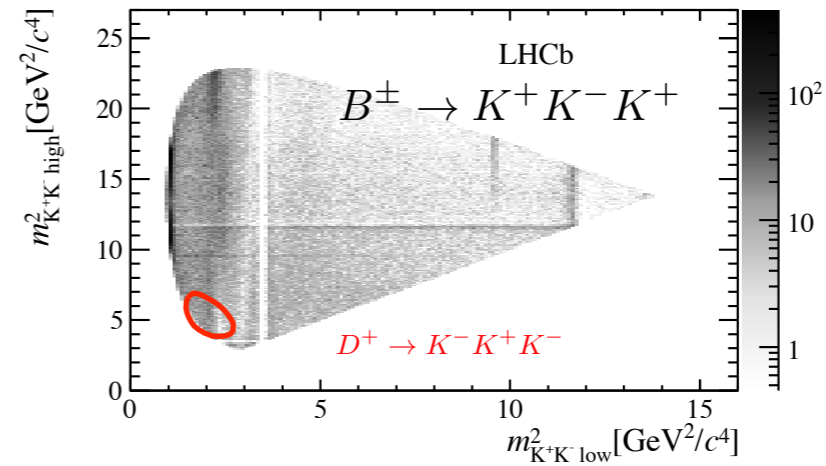


~ $D\bar{D}$ open channel



B decays

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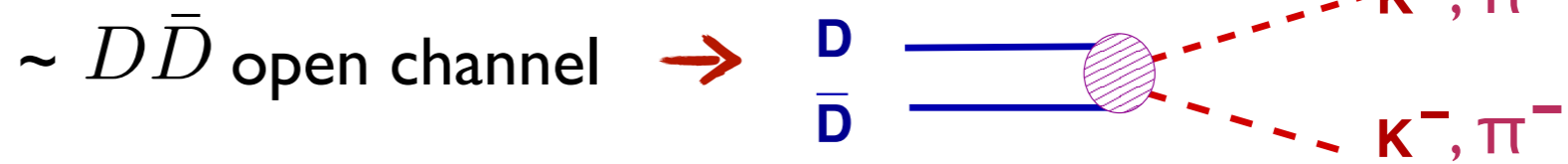
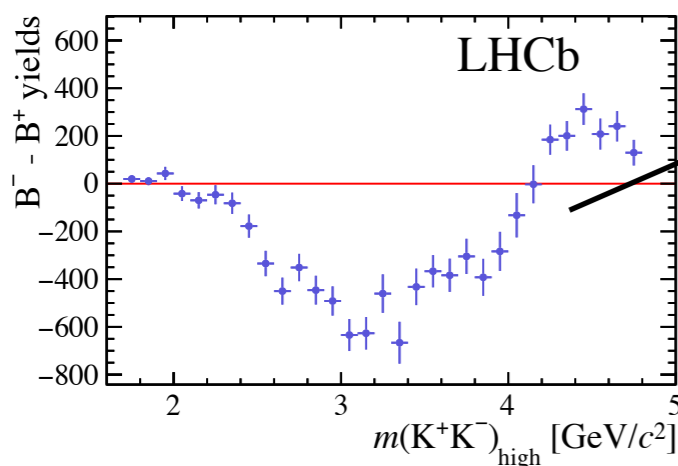
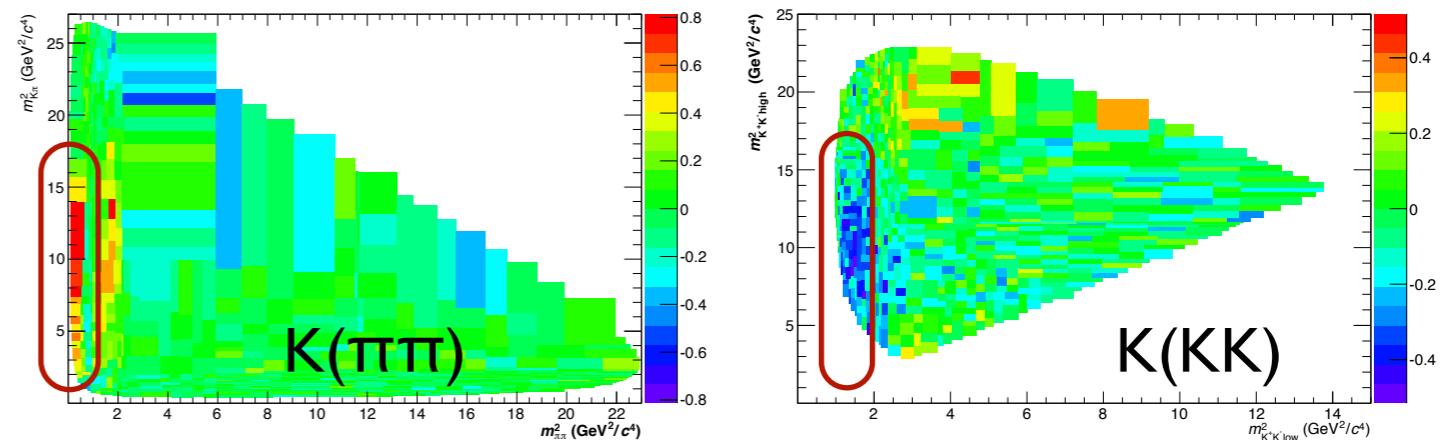


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Bediaga, Frederico, & Lourenço
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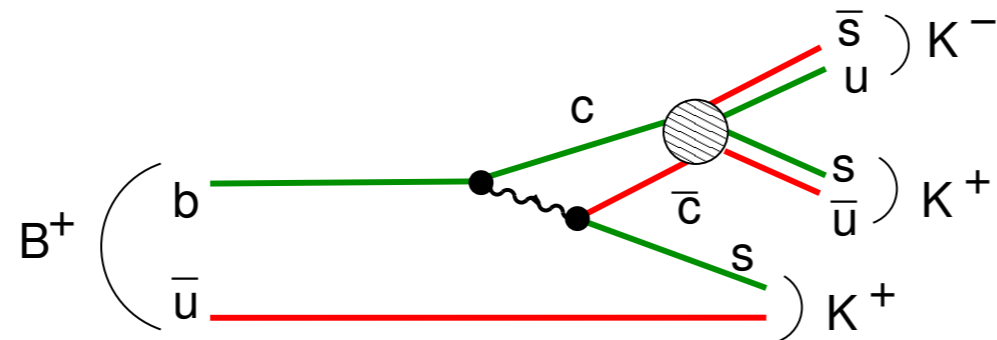
- high mass?



charm loops as source of strong phase

- charm FSI: $B \rightarrow 3h$, $B_c \rightarrow 3h$, $B \rightarrow K^* \mu\mu$, ...

investigate the charm  contribution to $B^\pm \rightarrow K^+ K^- K^+$

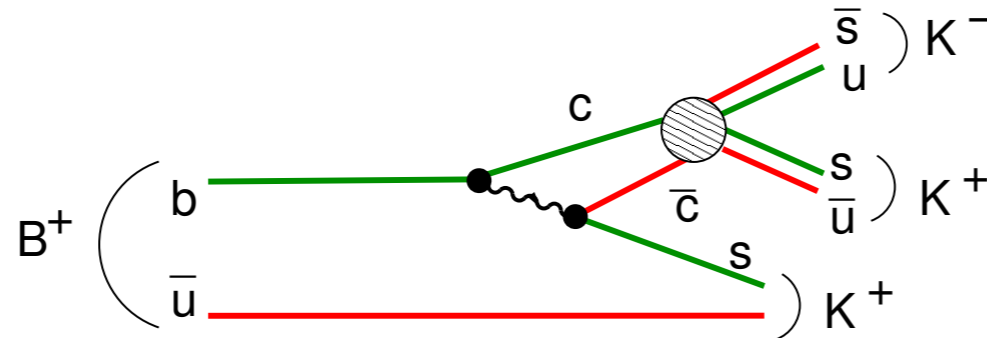


PLB 780 (2018) 357

Cabibbo favoured V_{bc}

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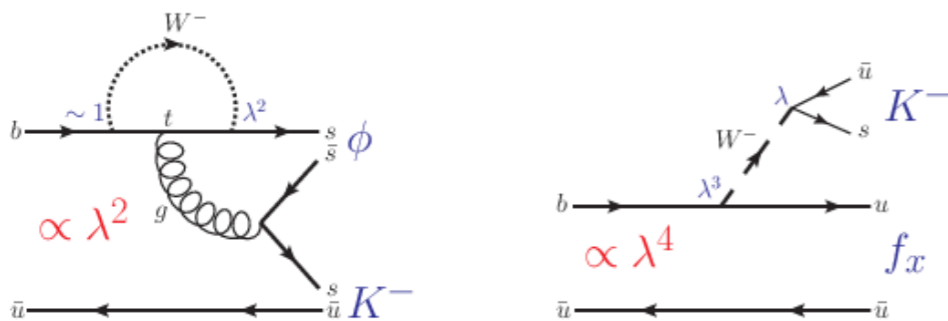
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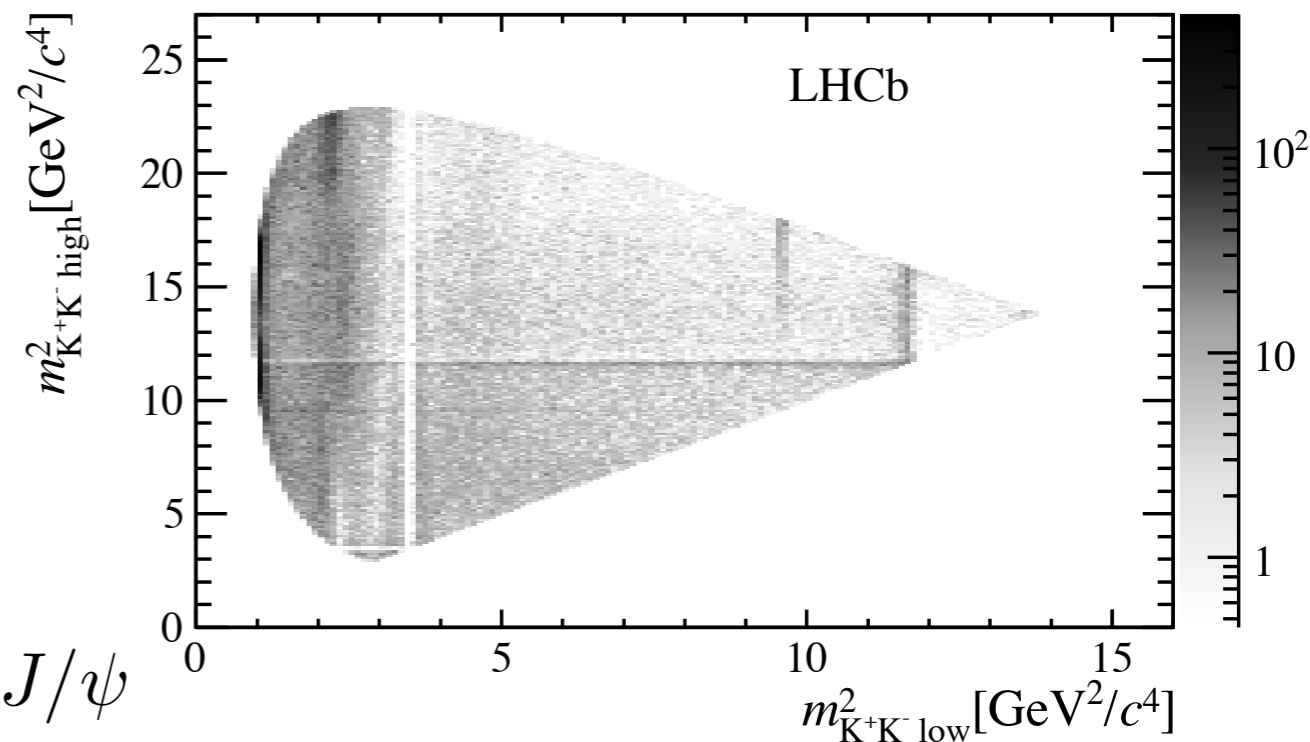
- $B^\pm \rightarrow hhh$ highest statistic 109k 

- nonresonant \rightarrow all phase-space

- dominated by penguin

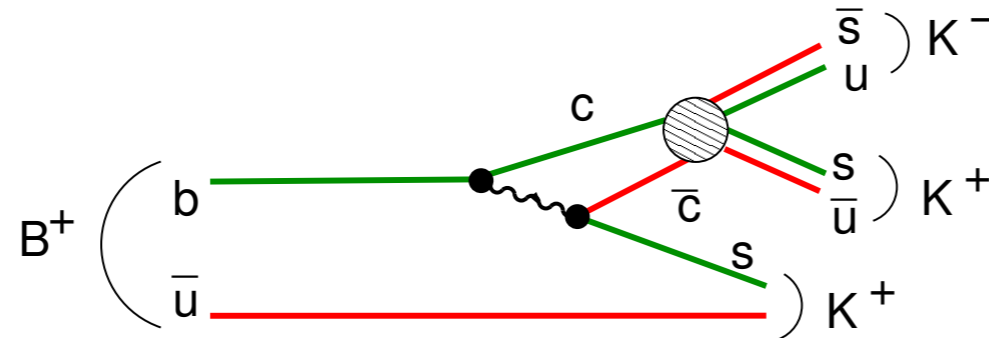


- presence of charm resonances: χ_{c0} J/ψ



- charm FSI: $B \rightarrow 3h$, $B_c \rightarrow 3h$, $B \rightarrow K^* \mu\mu$, ...

investigate the charm  contribution to $B^\pm \rightarrow K^+ K^- K^+$



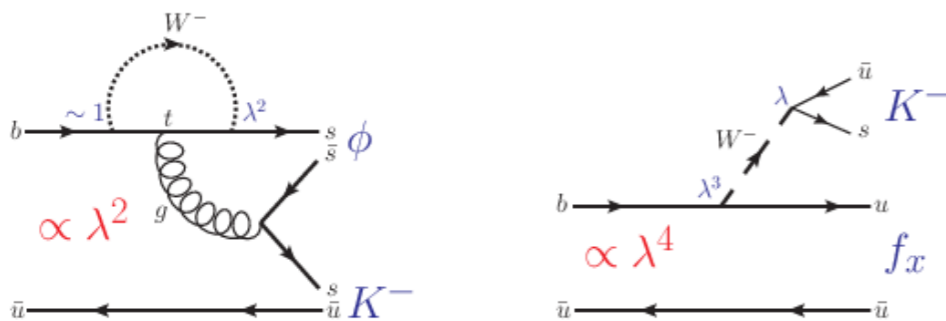
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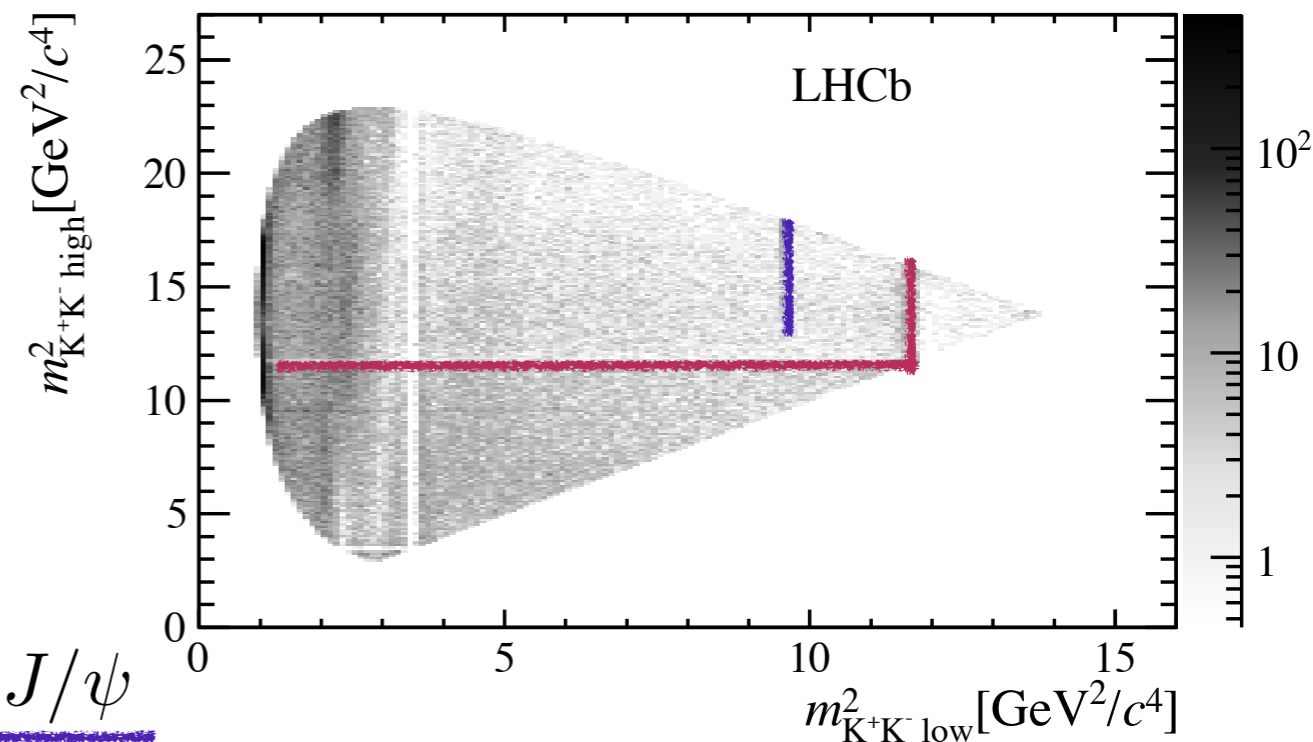
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- dominated by penguin



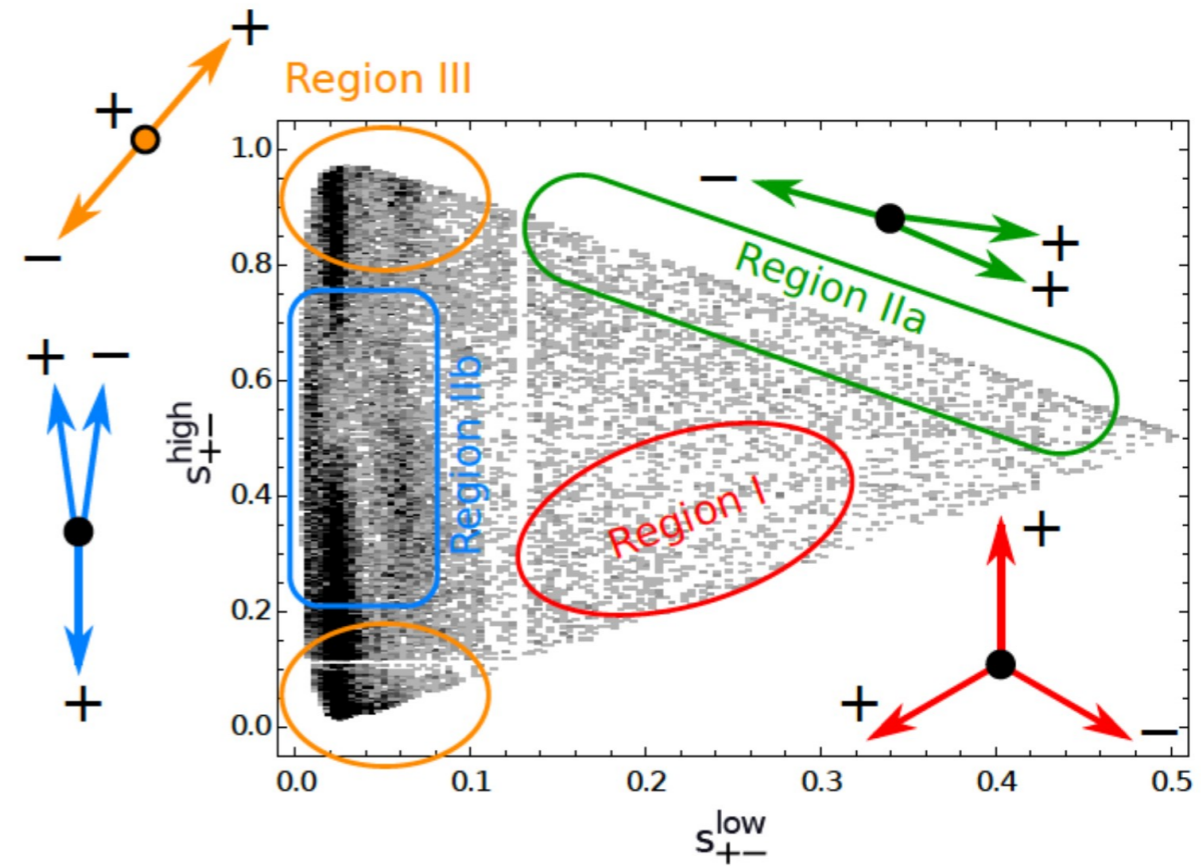
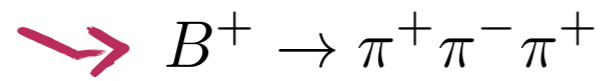
- presence of charm resonances:

χ_{c0} J/ψ



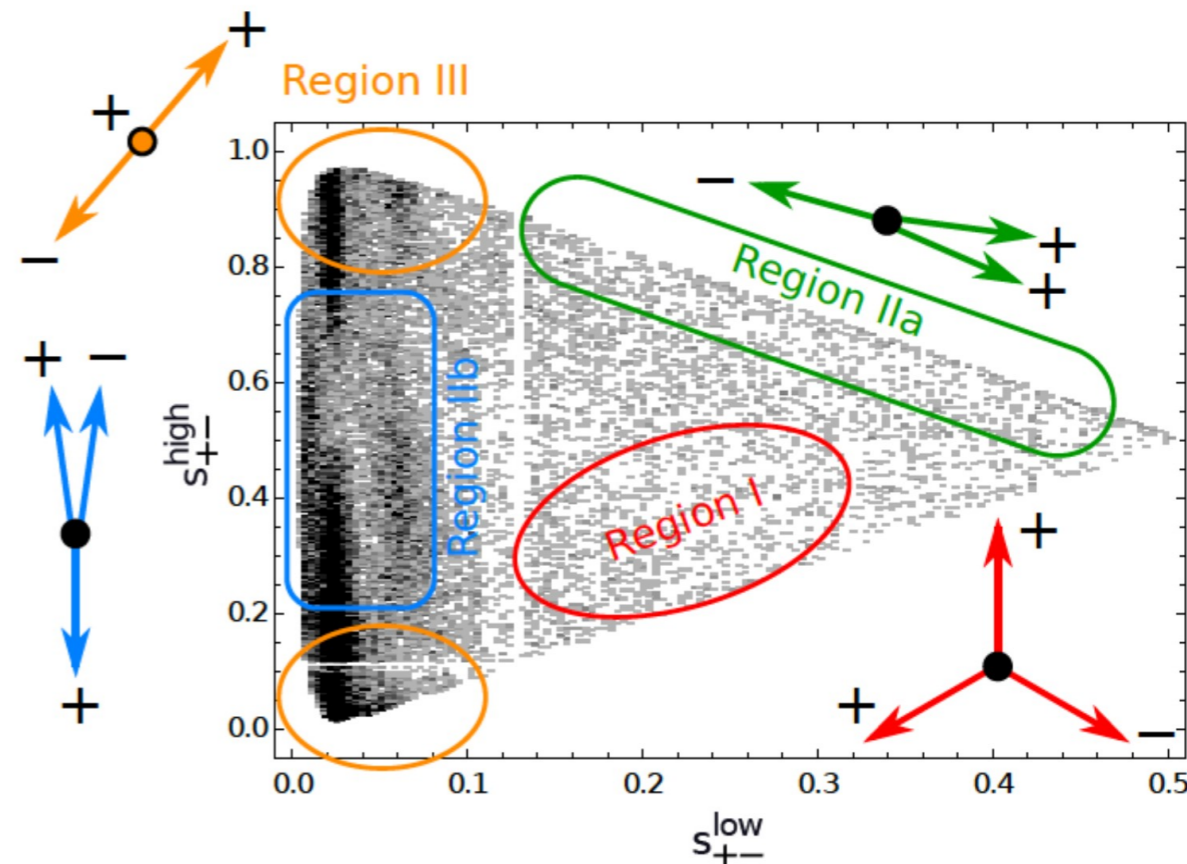
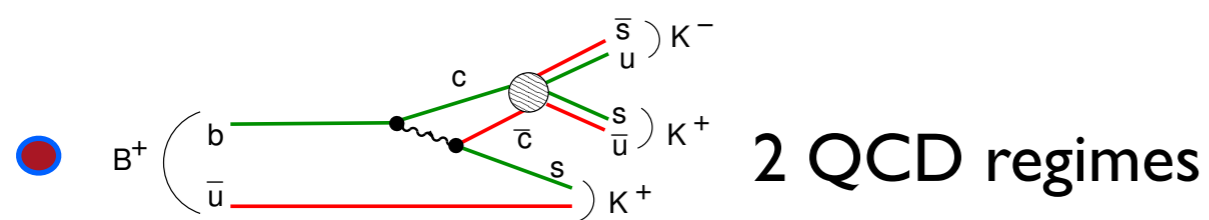
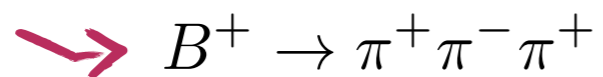
- different QCD regimes in DP

Krüinkl, Mannel & Virto, NPB 899 (2015) 247

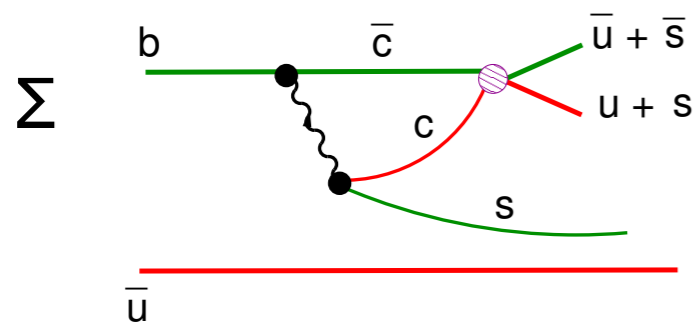


- different QCD regimes in DP

Krönkl, Mannel & Virto, NPB 899 (2015) 247



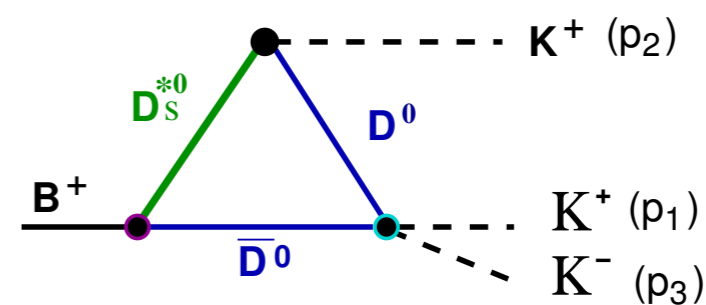
- perturbative: region I



duality principle

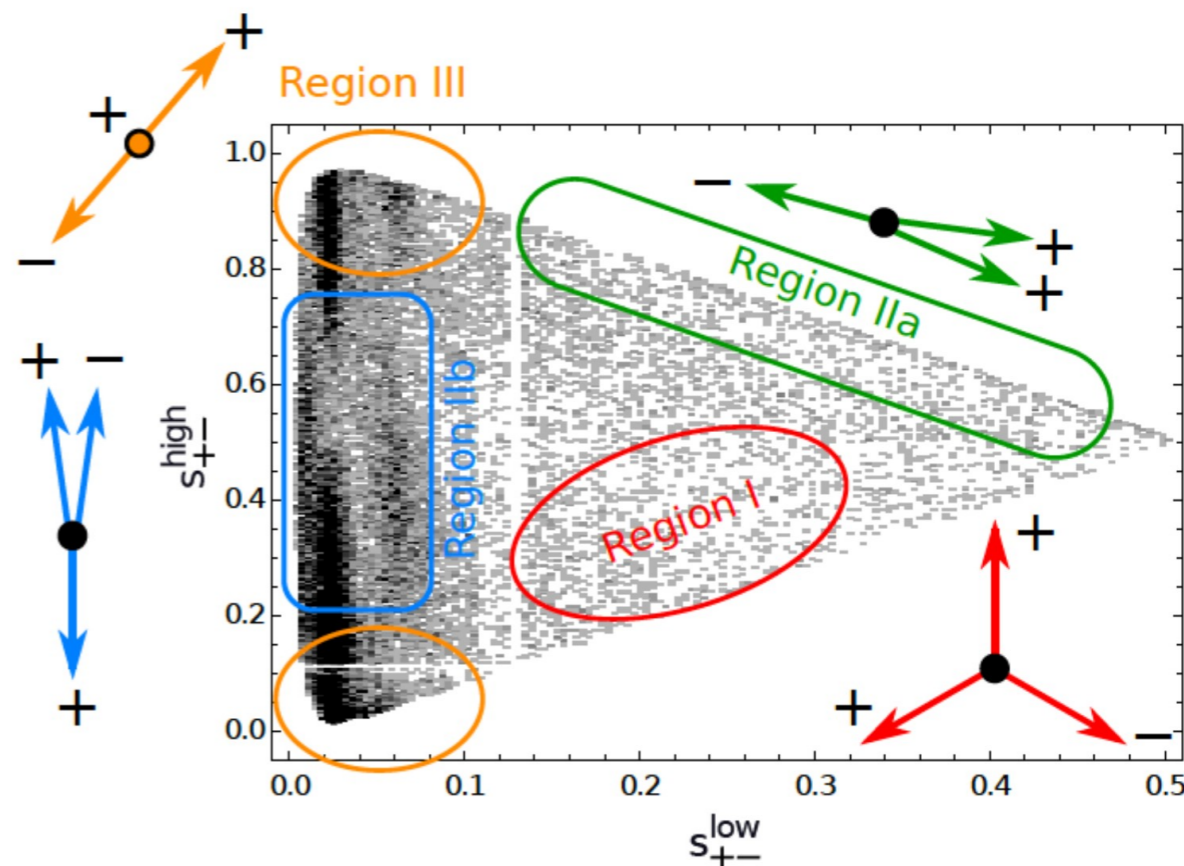
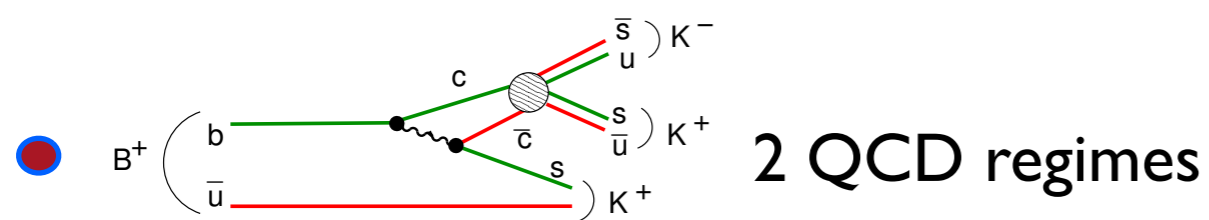
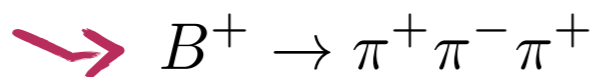


- nonperturbative: region II

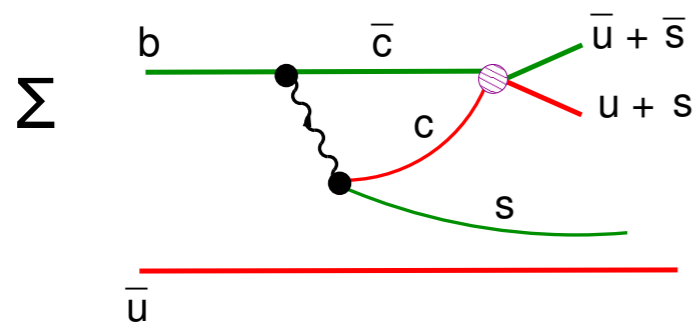


- different QCD regimes in DP

Krönkl, Mannel & Virto, NPB 899 (2015) 247



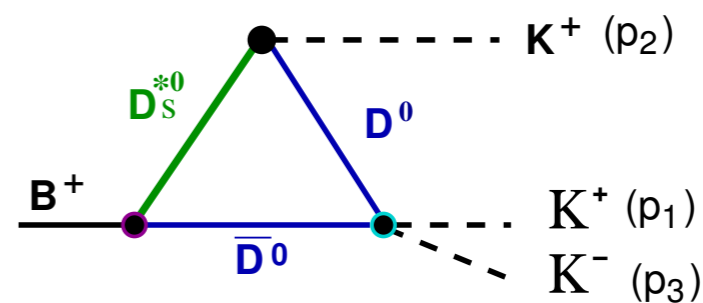
- perturbative: region I



duality principle

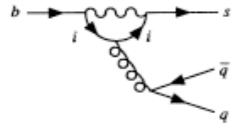


- nonperturbative: region II



identify their signatures to amplitude analyses with LHCb data

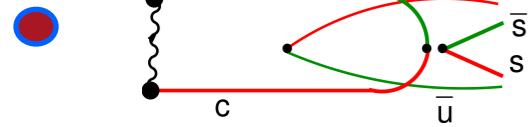
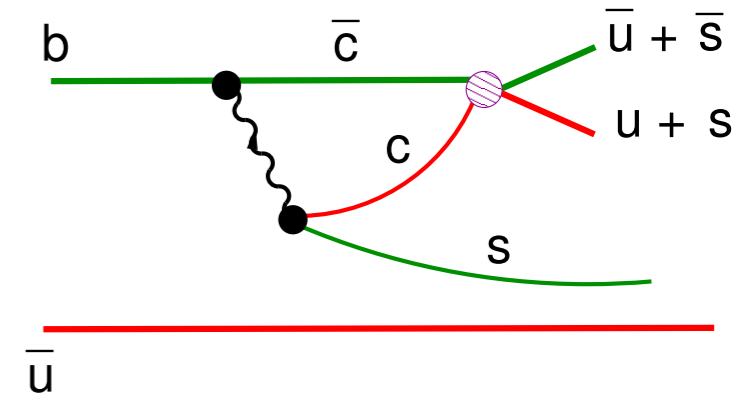
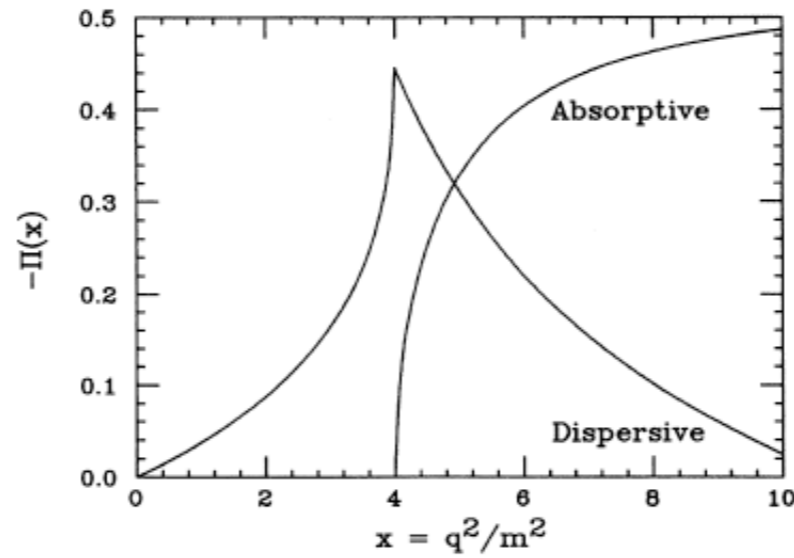
loop



$$Re\Pi(x) = -\frac{1}{6} \left\{ \frac{5}{3} + \frac{4}{x} - \left(1 + \frac{2}{x}\right) \left[\sqrt{1 - \frac{4}{x}} \ln \left(\frac{1 + \sqrt{1 - 4/x}}{1 - \sqrt{1 - 4/x}} \right) \Theta \left[1 - \frac{4}{x} \right] + 2 \sqrt{\frac{4}{x} - 1} \operatorname{arccot} \left[\sqrt{\frac{4}{x} - 1} \right] \Theta \left[\frac{4}{x} - 1 \right] \right] \right\}$$

$$Im\Pi(x) = -\frac{\pi}{6} \left(1 + \frac{2}{x}\right) \sqrt{1 - \frac{4}{x}} \Theta \left[1 - \frac{4}{x} \right]$$

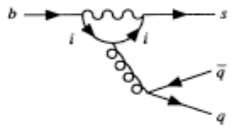
Gerard & Hou PRD43 (1991) 2909



many possible excitations → average of charm mass

$$A_{parton} = \int_{m_{Cmin}}^{m_{Cmax}} \Pi(s) \frac{1}{2\pi\Gamma^2} e^{-\frac{(m-m_{c0})^2}{2\Gamma^2}}$$

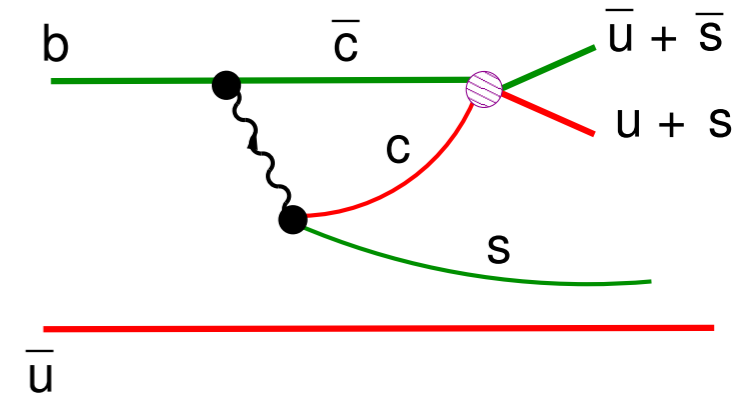
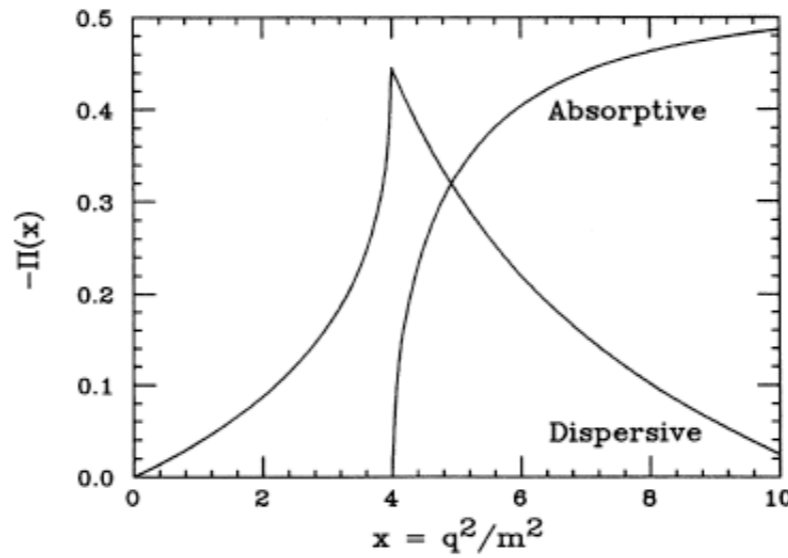
● loop



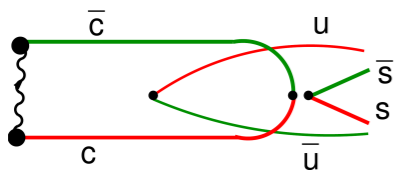
$$Re\Pi(x) = -\frac{1}{6} \left\{ \frac{5}{3} + \frac{4}{x} - \left(1 + \frac{2}{x}\right) \left[\sqrt{1 - \frac{4}{x}} \ln \left(\frac{1 + \sqrt{1 - 4/x}}{1 - \sqrt{1 - 4/x}} \right) \Theta \left[1 - \frac{4}{x} \right] + 2 \sqrt{\frac{4}{x} - 1} \operatorname{arccot} \left[\sqrt{\frac{4}{x} - 1} \right] \Theta \left[\frac{4}{x} - 1 \right] \right] \right\}$$

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Gerard & Hou PRD43 (1991) 2909



●

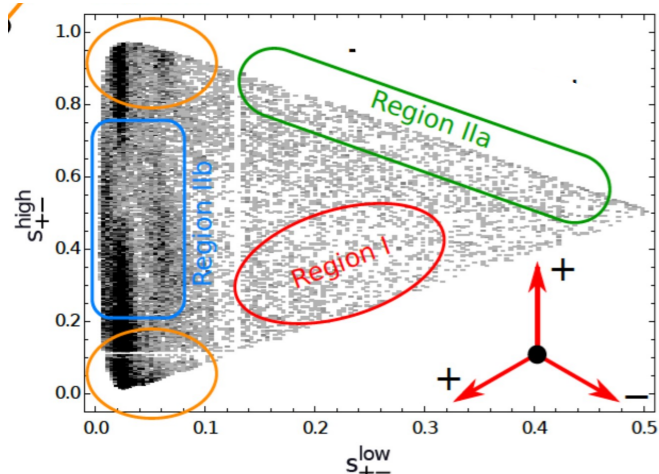


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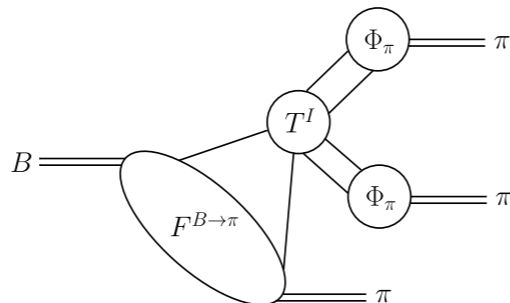
$$A_{parton} = \int_{m_{cmin}}^{m_{cmax}} \Pi(s) \frac{1}{2\pi\Gamma^2} e^{-\frac{(m-m_{c0})^2}{2\Gamma^2}}$$

● Amplitude in region 1 $B \rightarrow 3\pi$

from Mannel et al



$$A_p(s) = T(s)(M_B^2 - s)f_+(s)$$



● Kernel of interaction

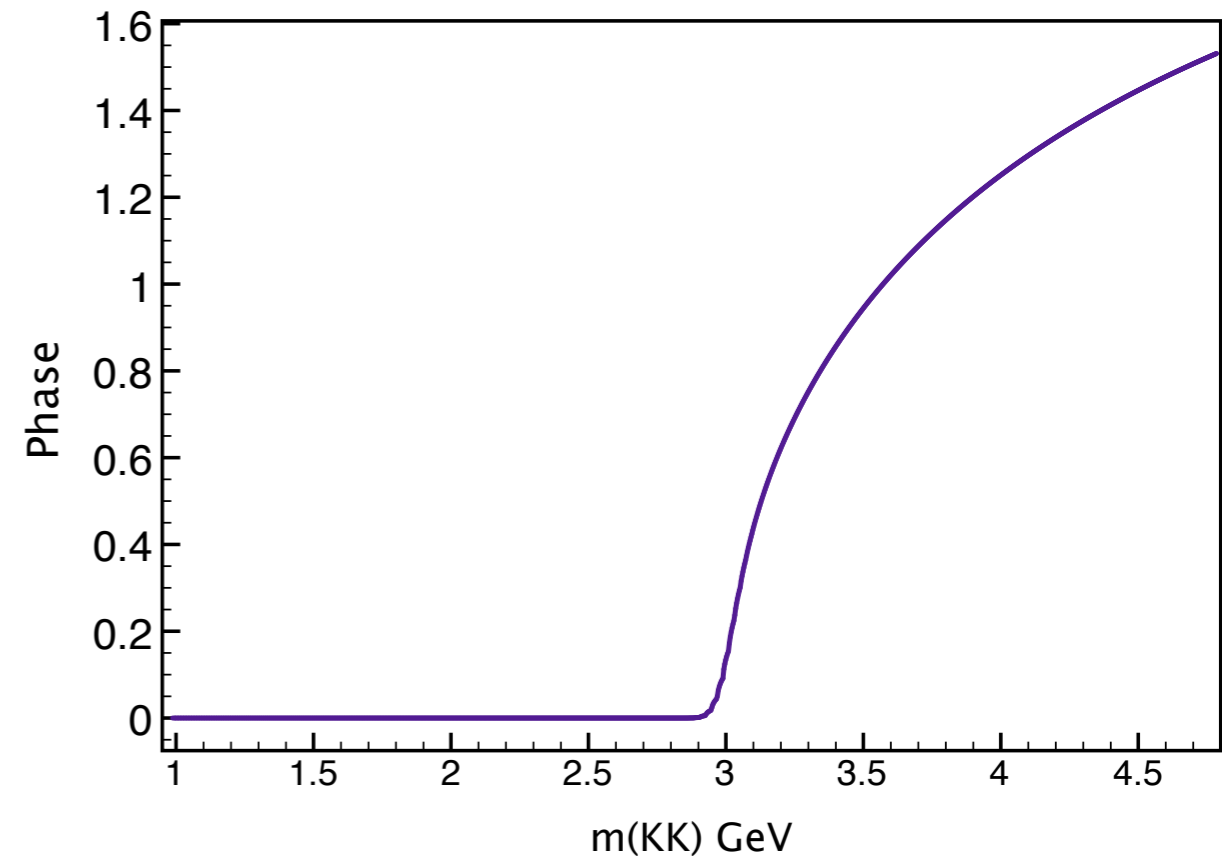
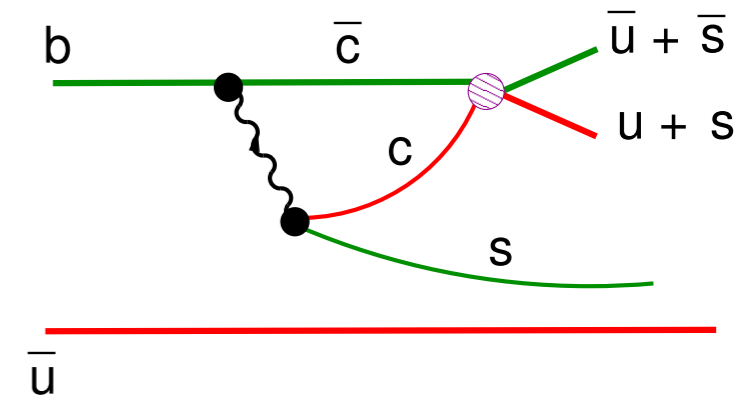
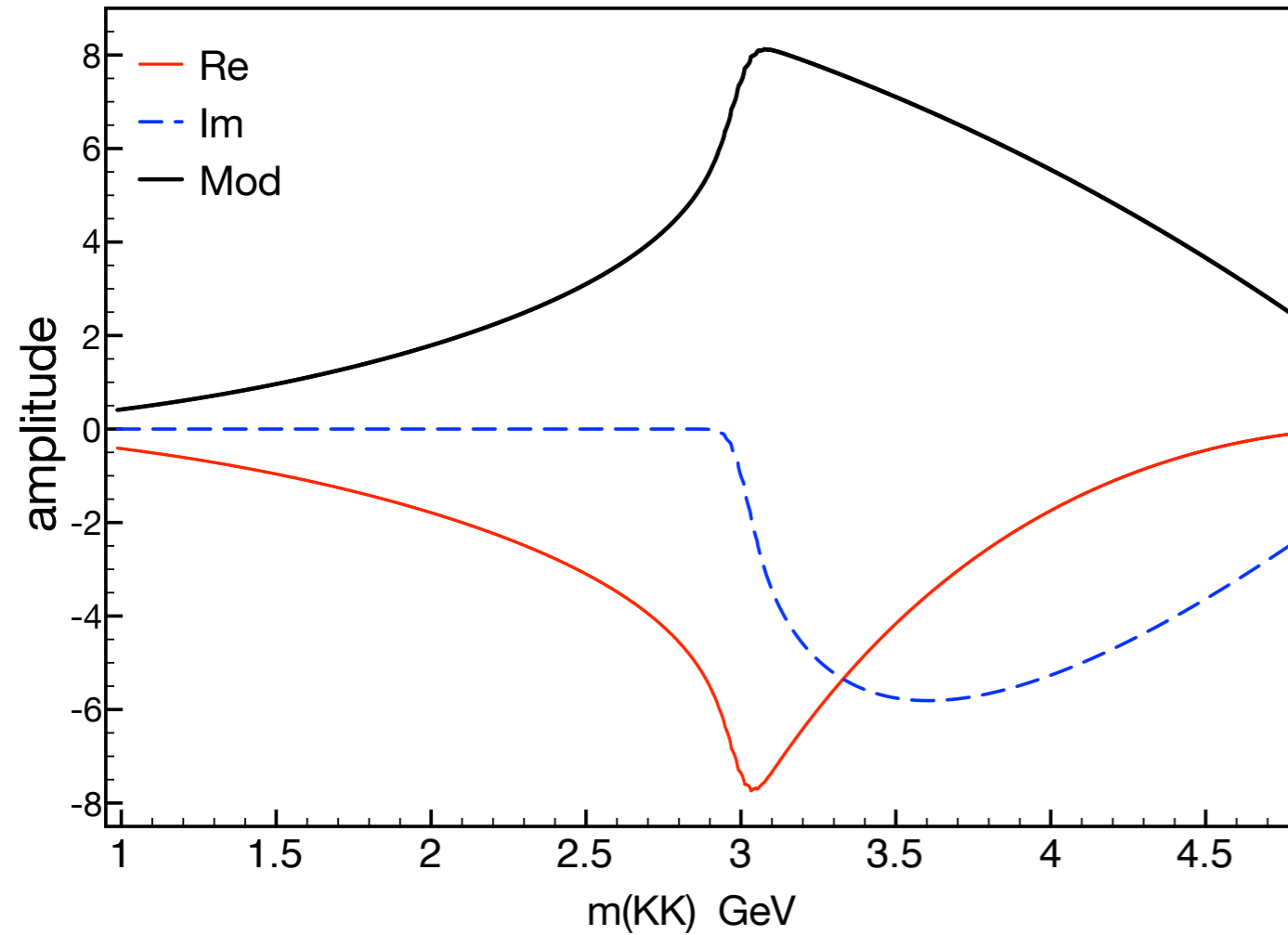
$$T(s) \equiv A_{parton}$$

● $f_+(s) = \frac{1}{1 - s/M_{Bs}^{*2}} \rightarrow B \rightarrow K$

vector form factor

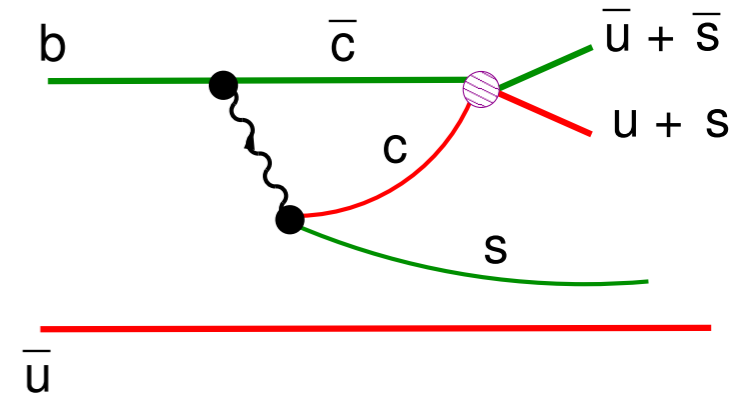
- gaussian parameters should be fitted to data

$$\frac{1}{2\pi\Gamma^2} e^{-\frac{(m-m_{c0})^2}{2\Gamma^2}} \rightarrow \begin{cases} \Gamma = 0.02 \text{ GeV} \\ m_{c0} = 1.5 \text{ GeV} \end{cases}$$

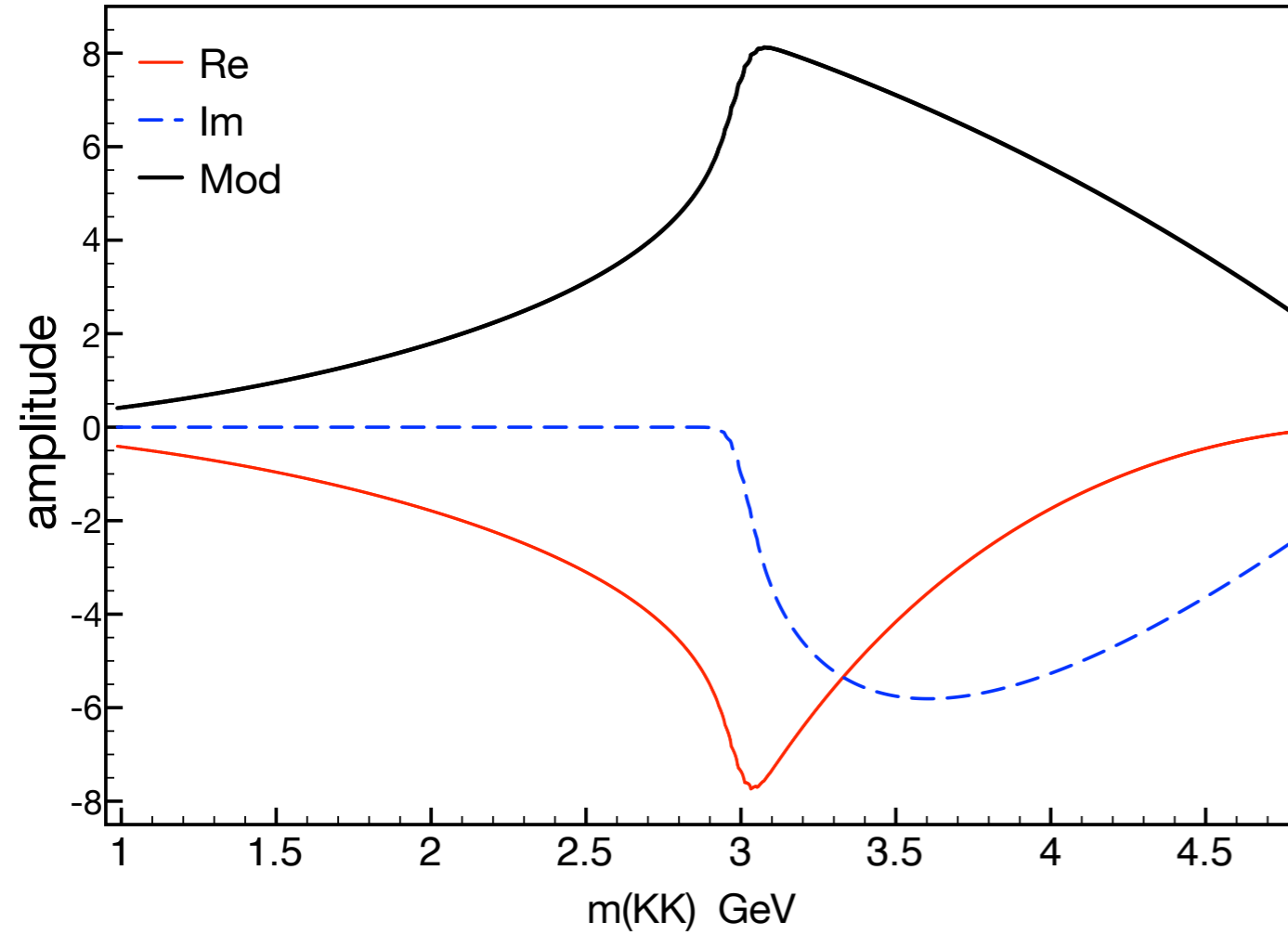


- gaussian parameters should be fitted to data

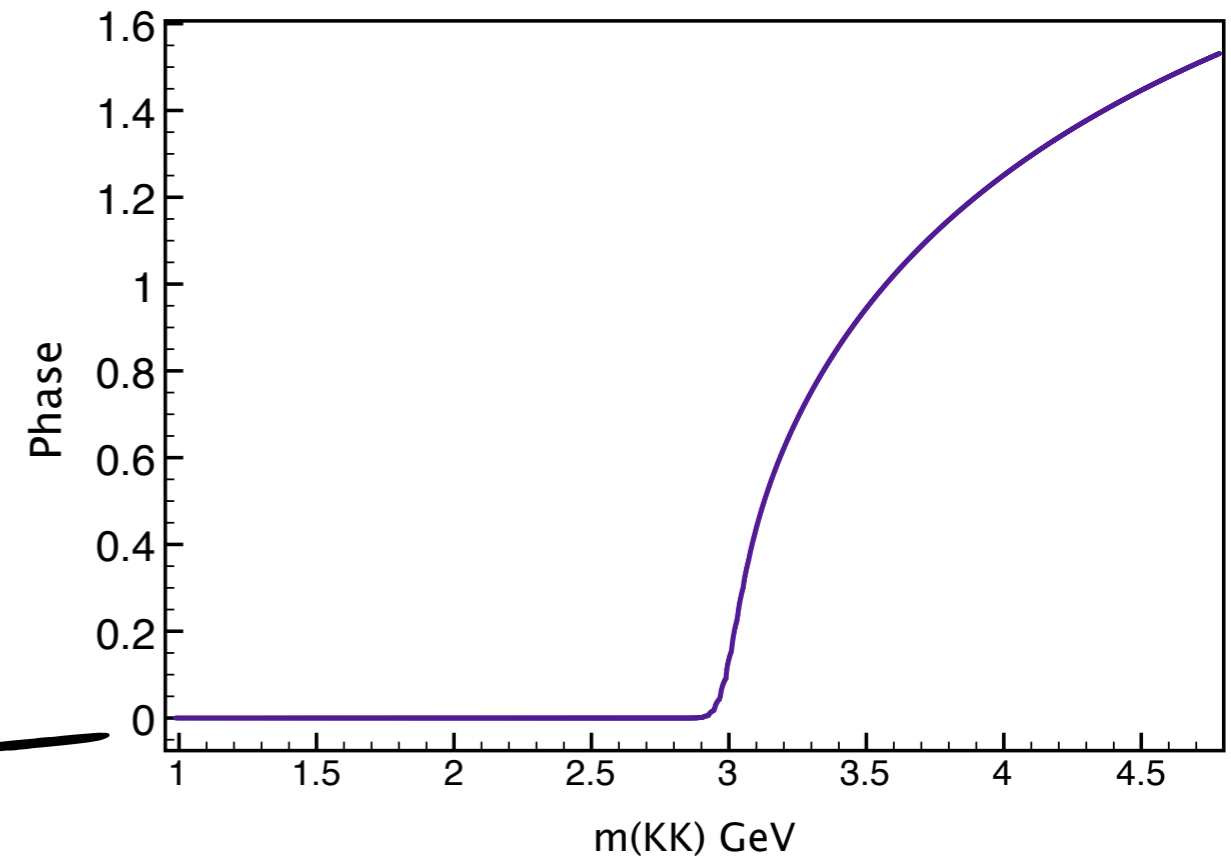
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→ max at ~ 3 GeV

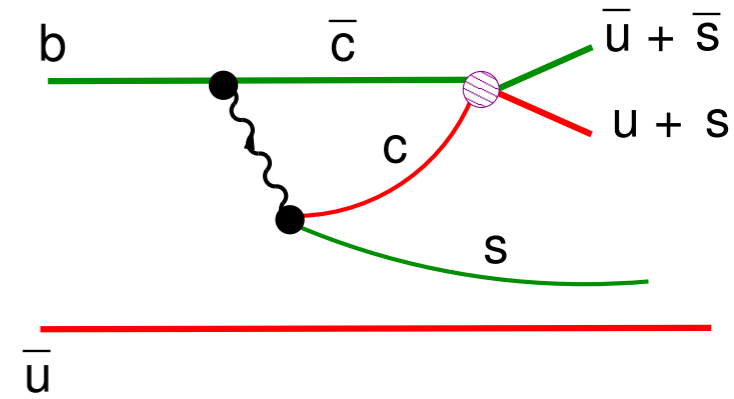


zero below threshold ←



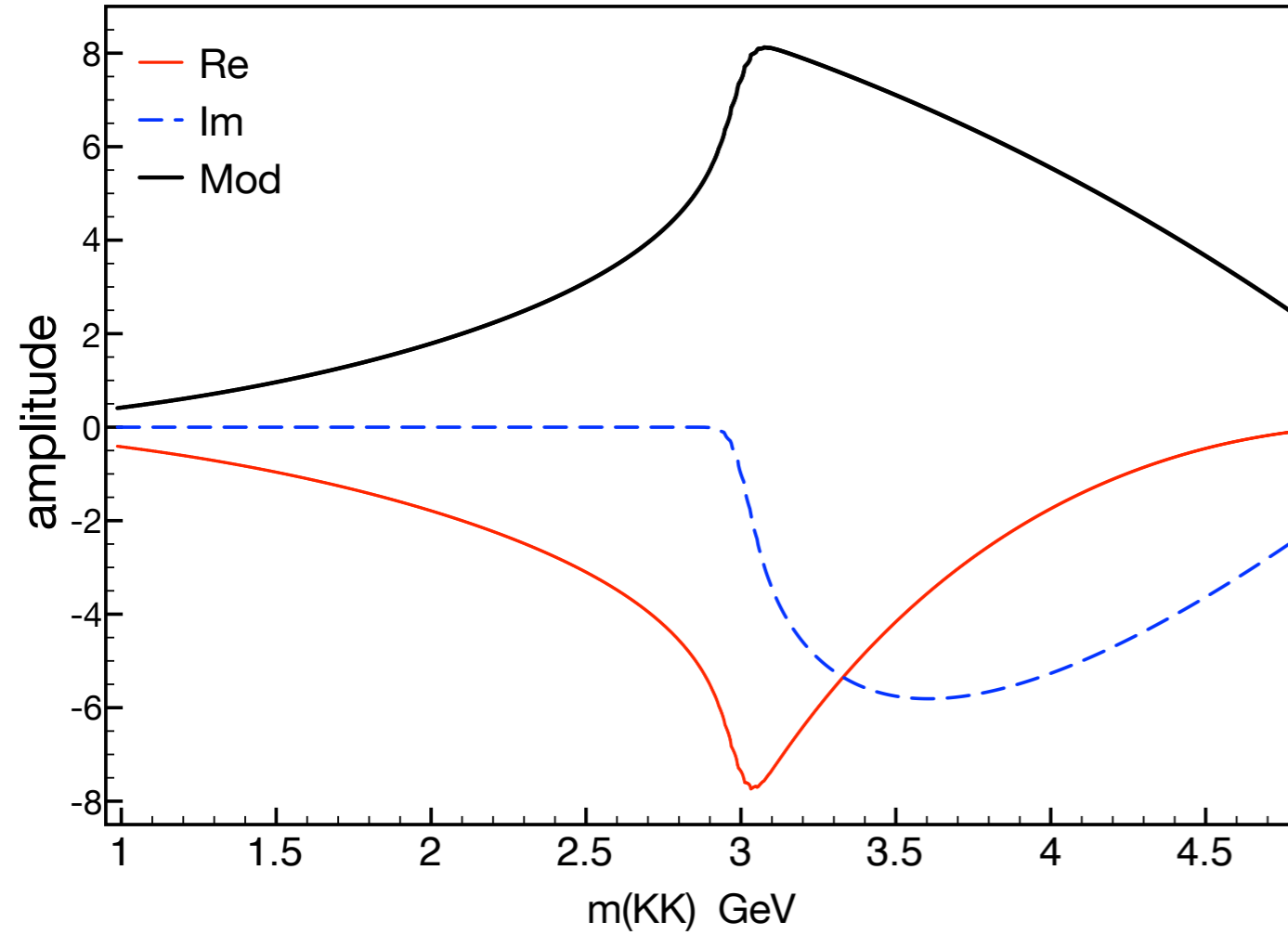
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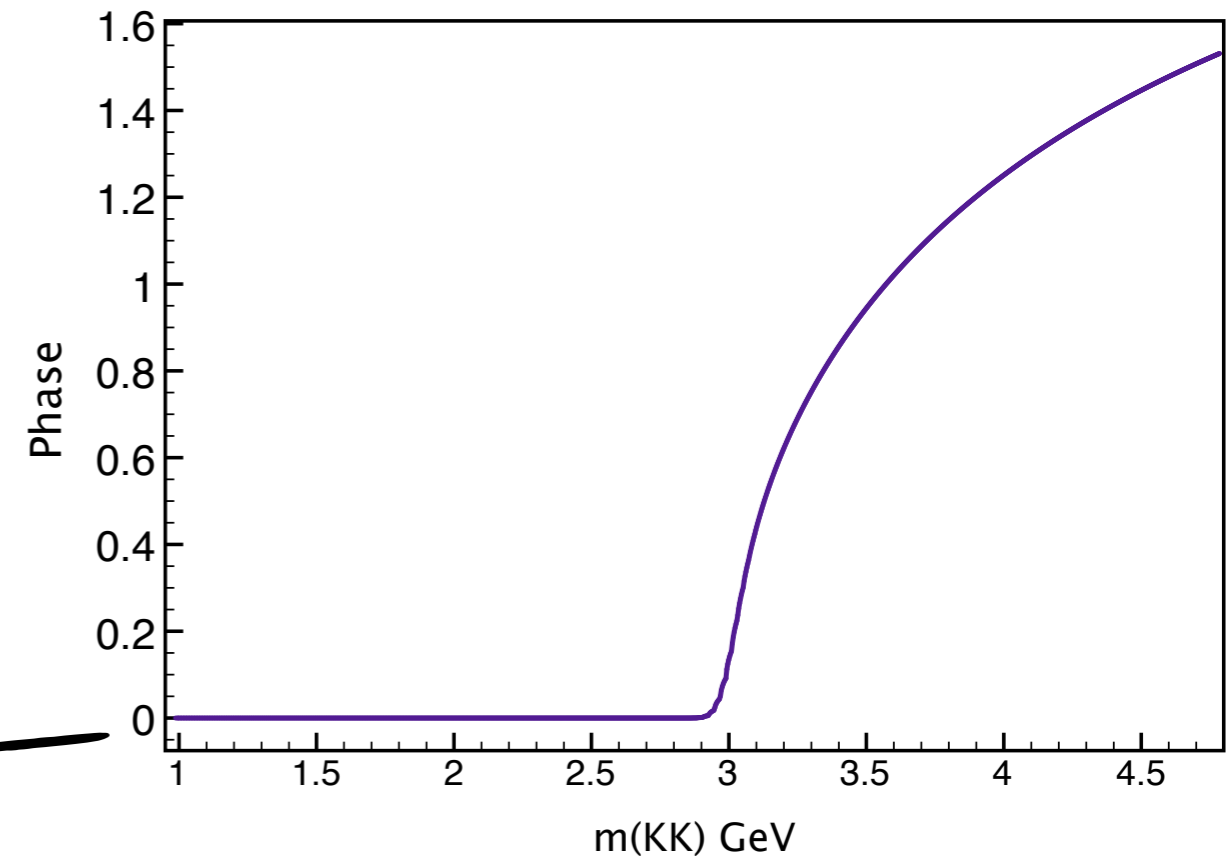


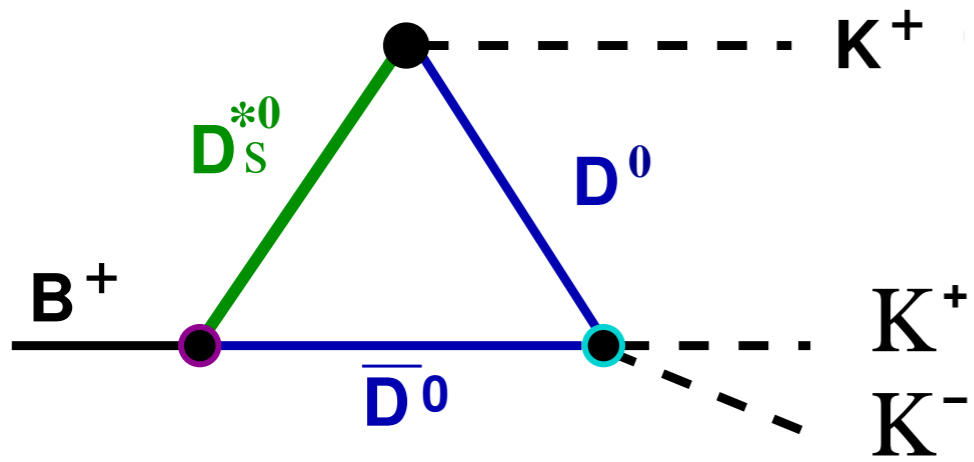
→ max at ~ 3 GeV

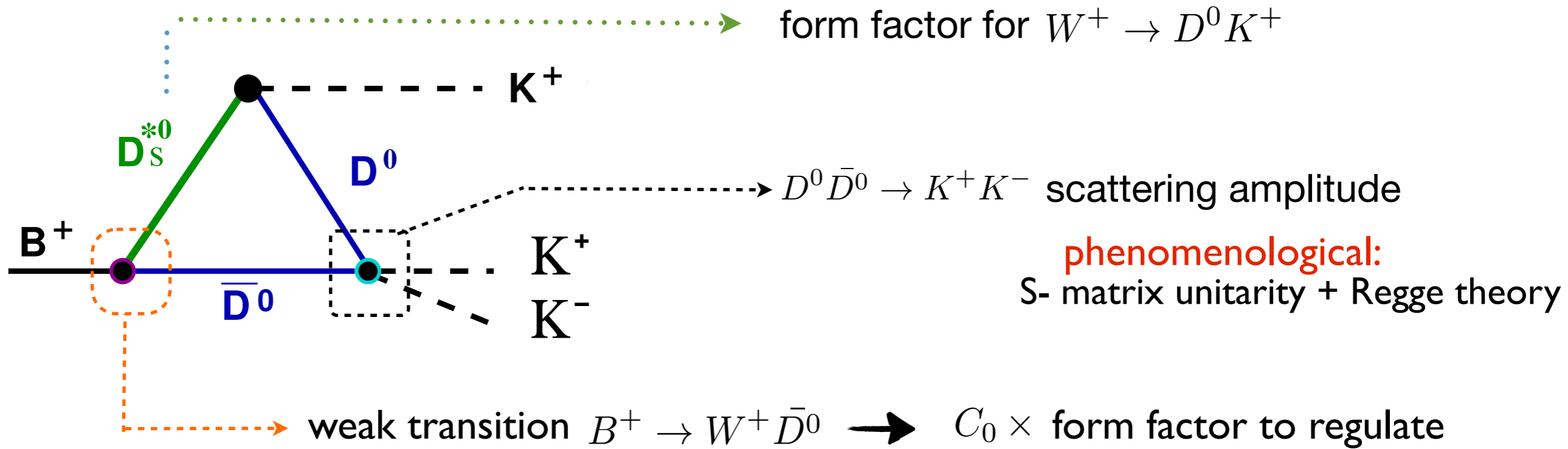
can change the inference pattern



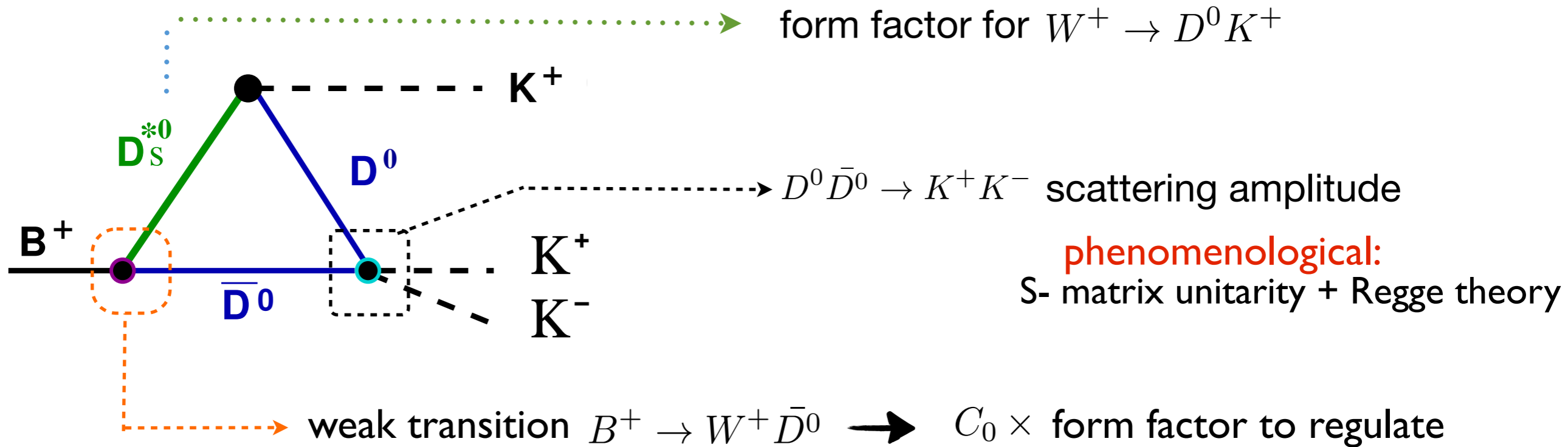
zero below threshold ←







● $Br [B \rightarrow DD_s^*] \sim 1\% \rightarrow 1000 \times Br [B \rightarrow KKK]$



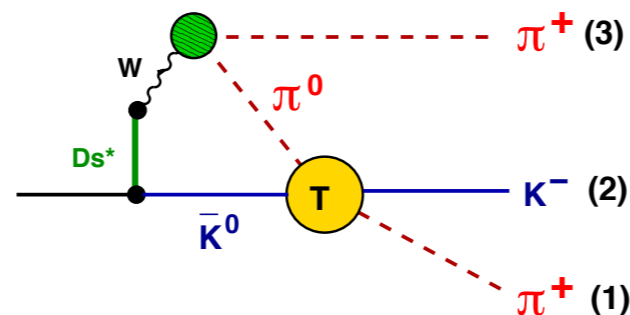
● $Br [B \rightarrow DD_s^*] \sim 1\% \rightarrow 1000 \times Br [B \rightarrow KKK]$

● hadronic loop \rightarrow three-body FSI - introduce new complex structures

● $B^+ \rightarrow \pi^+ \pi^- \pi^+$

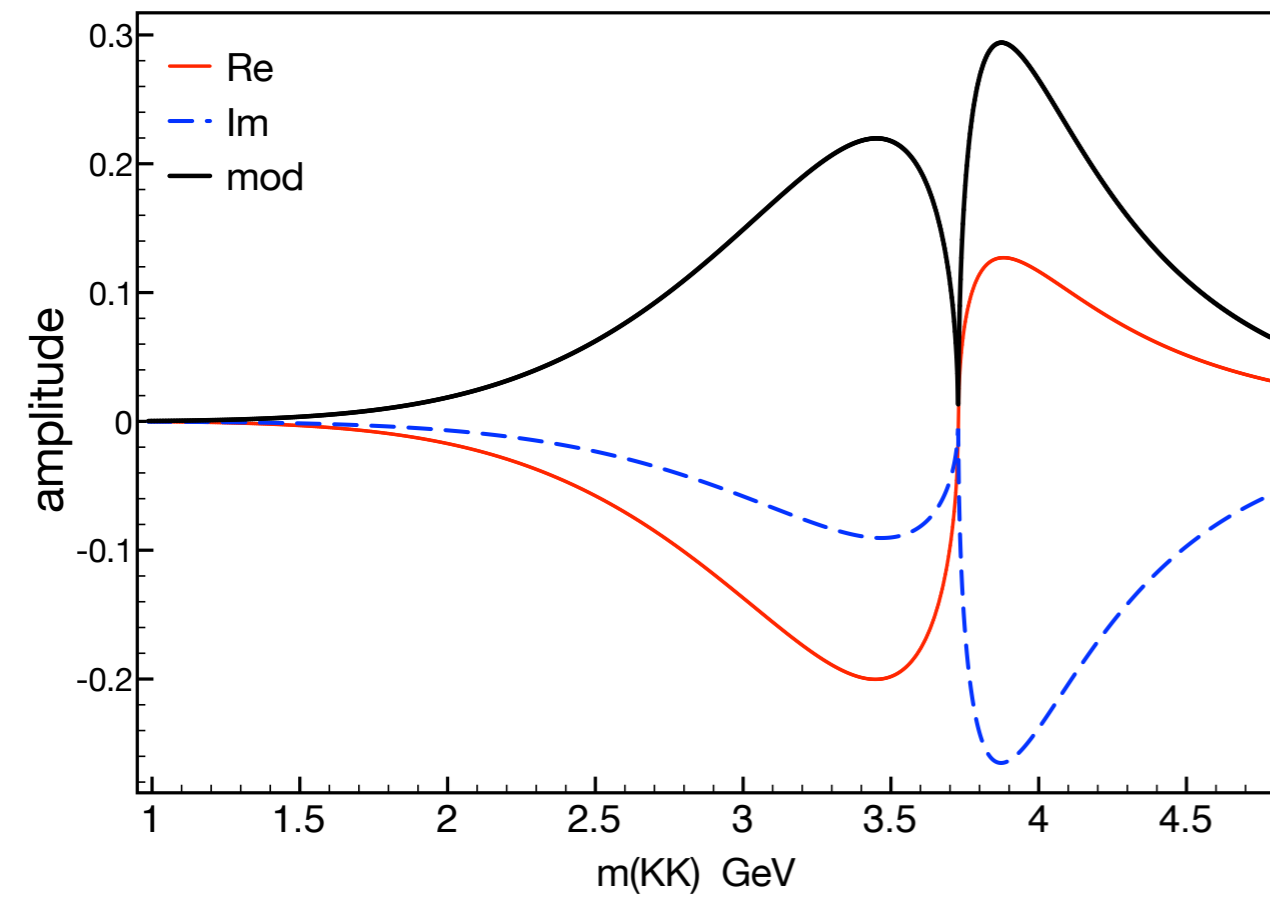
PCM & I Bediaga
arXiv:1512.09284

● $D^+ \rightarrow \pi^+ K^- \pi^+$

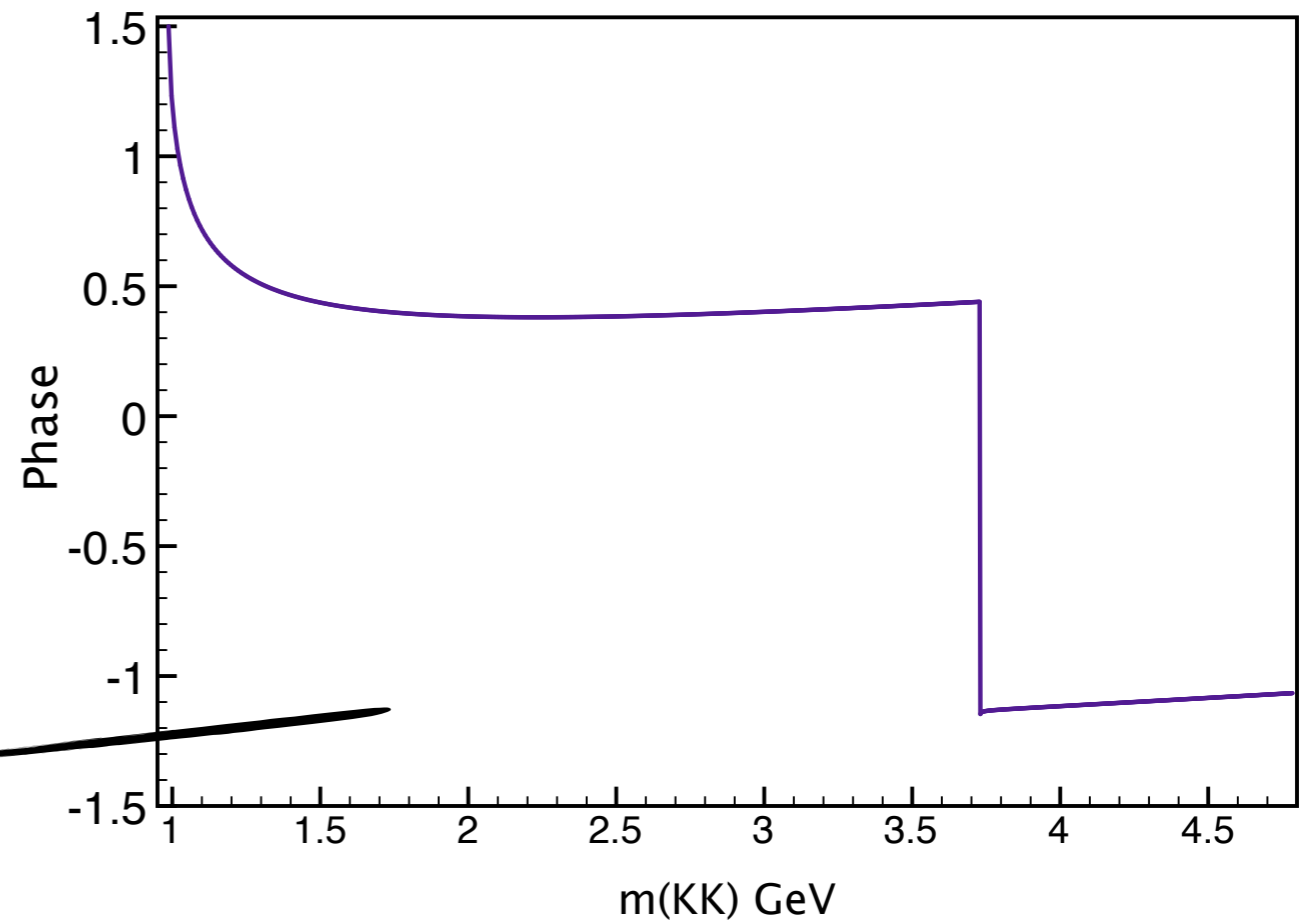


PCM & M Robilotta
PRD 92 094005 (2015) [arXiv:1504.06346]
PCM et al
PRD 84 094001 (2011) [arXiv:1105.5120]

- freedom to chose $D^0 \bar{D}^0 \rightarrow K^+ K^-$ parameters \rightarrow fix by data!

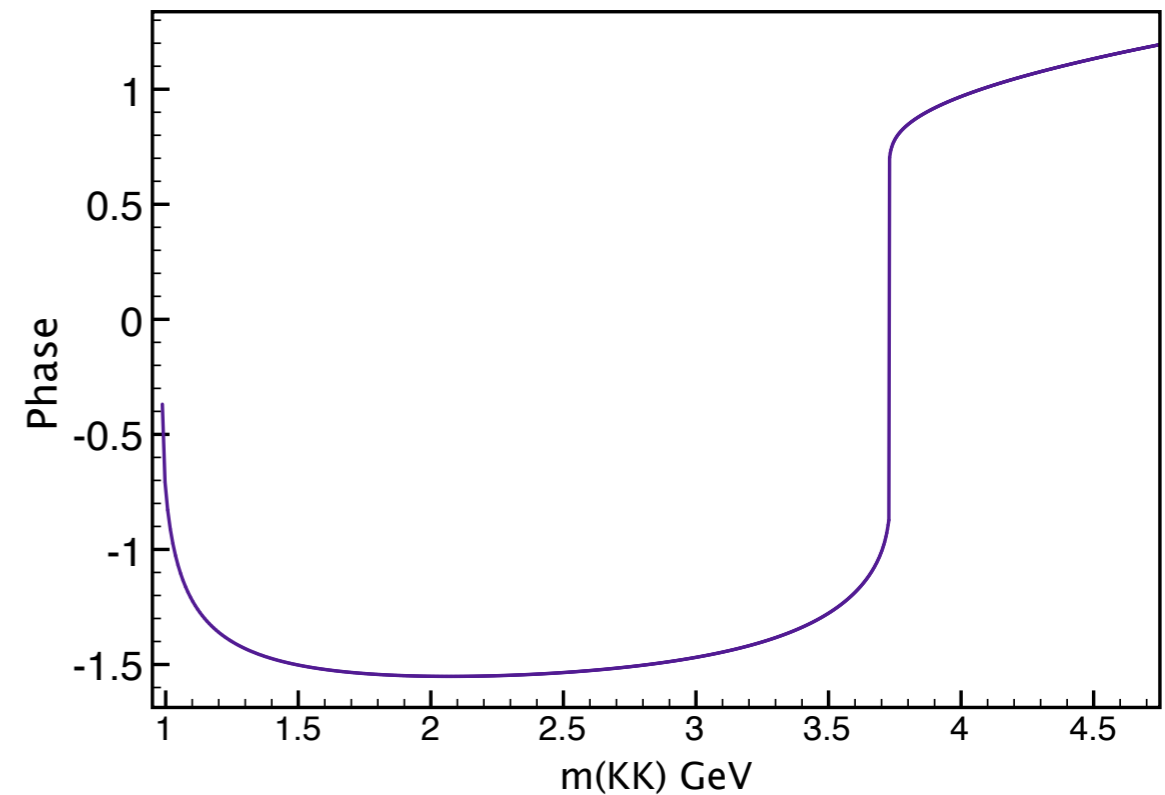
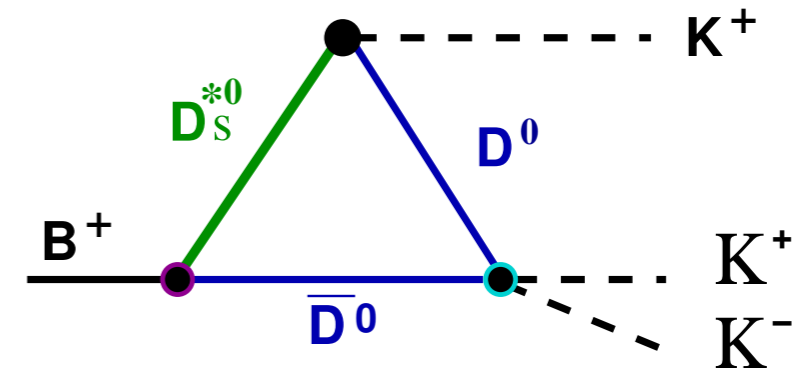
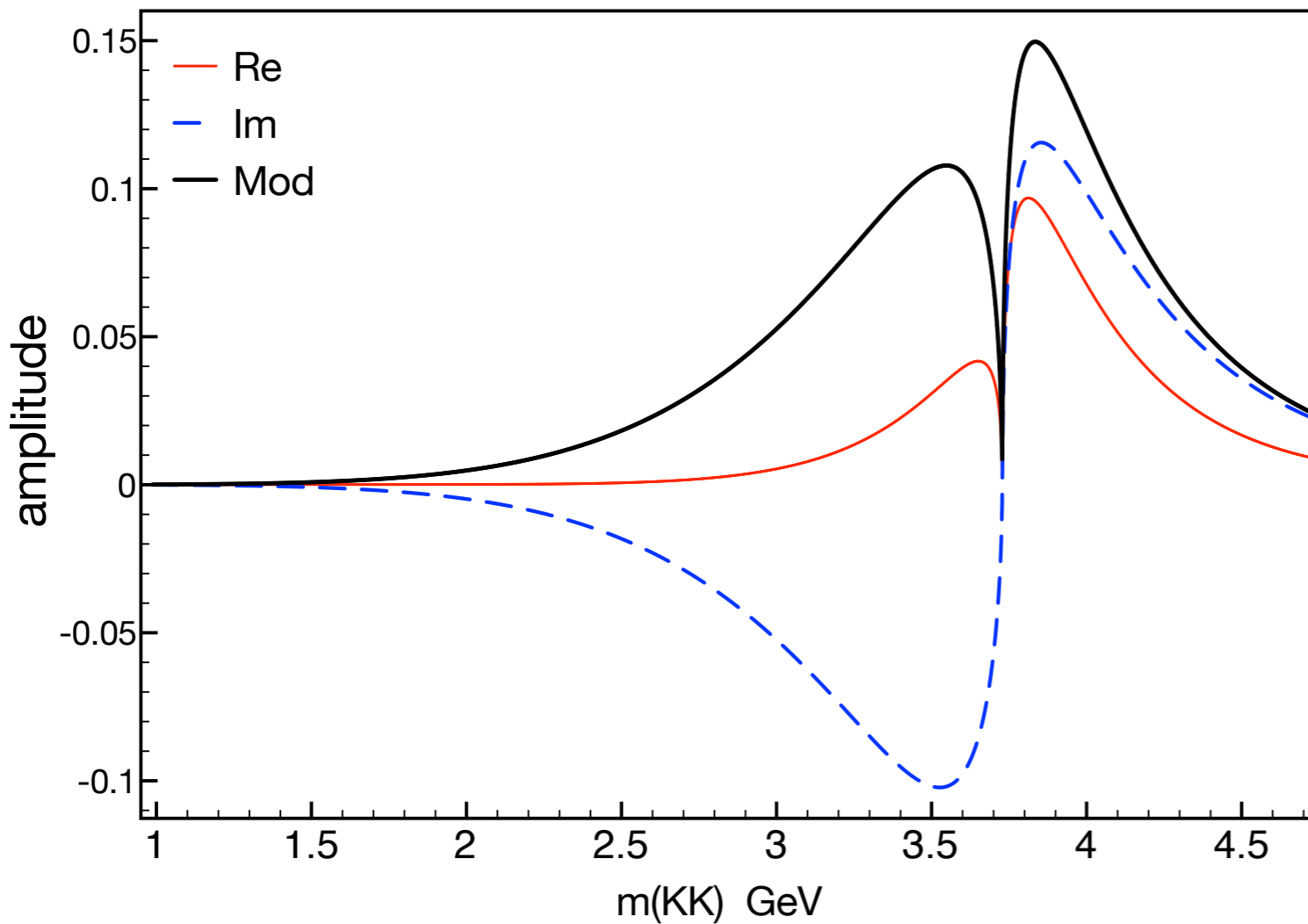


\rightarrow zero at threshold

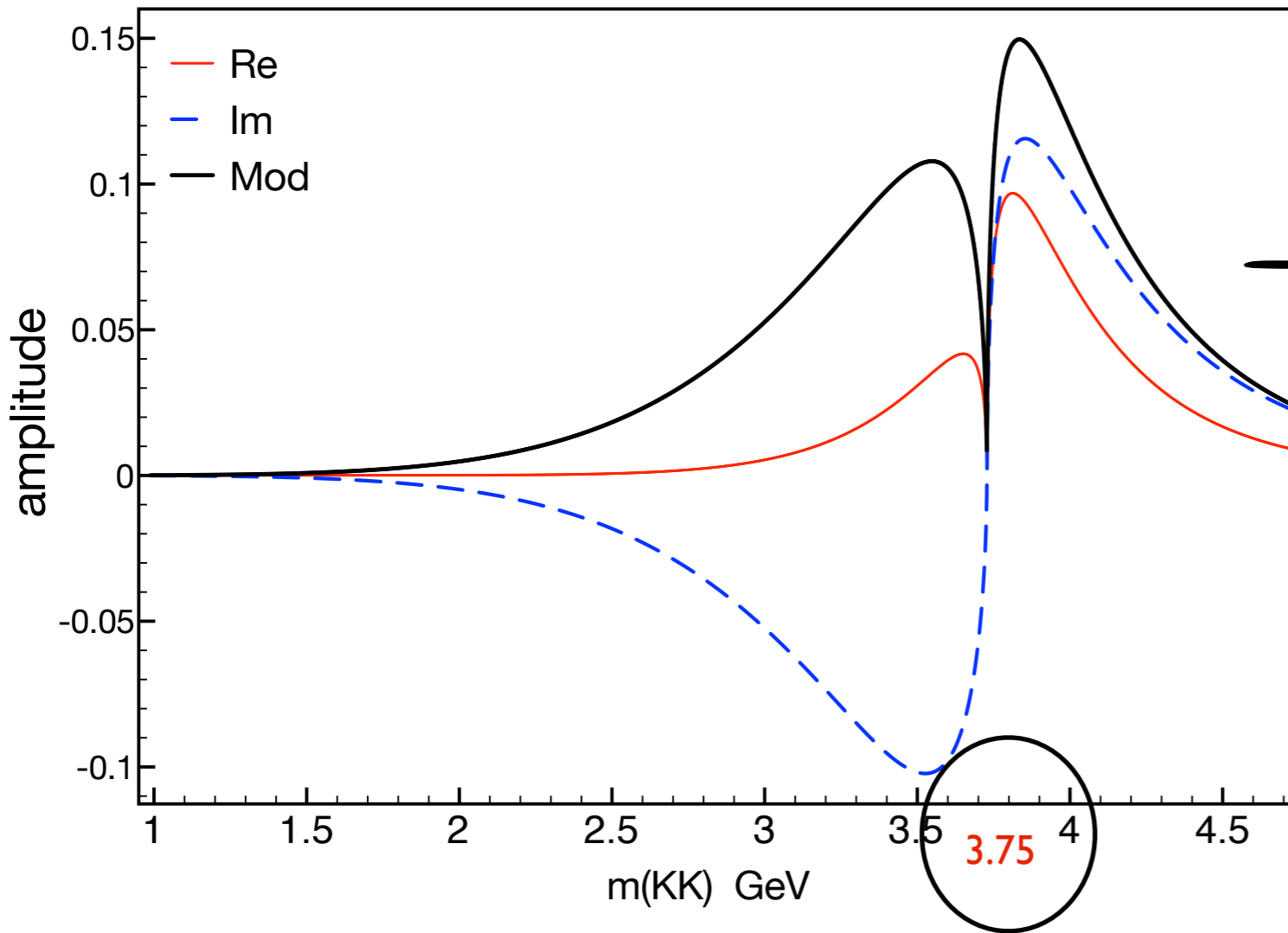


discontinuity at threshold \leftarrow

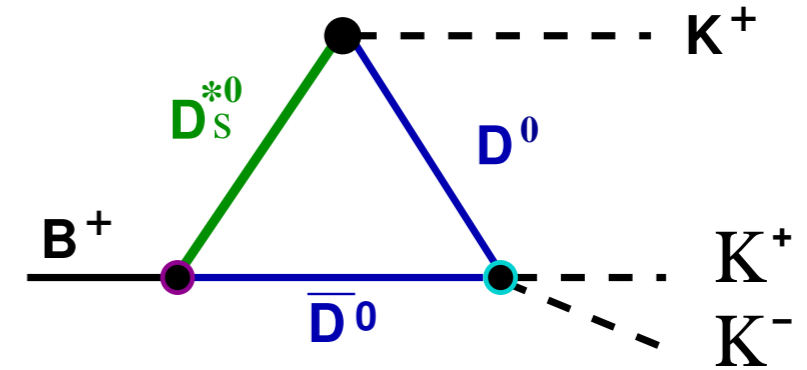
● total amplitude $A = i C_O T_{DD \rightarrow KK} A_P^h$



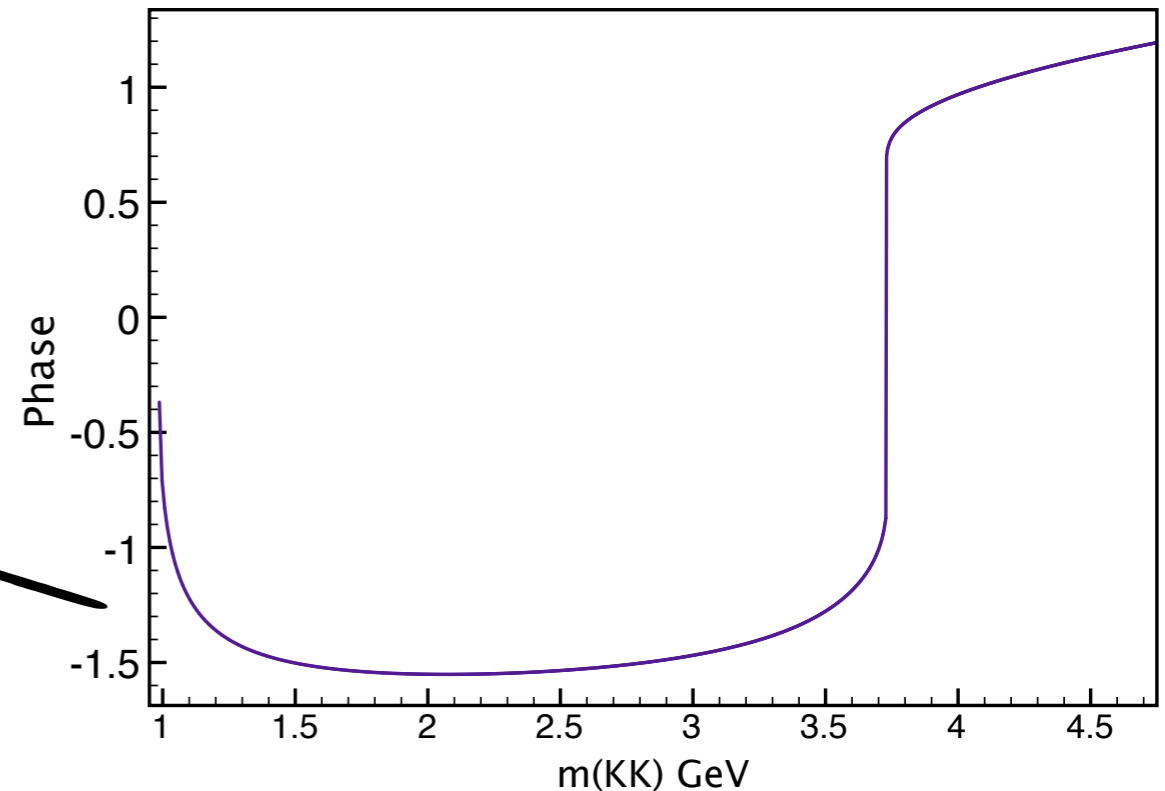
● total amplitude $A = i C_O T_{DD \rightarrow KK} A_P^h$



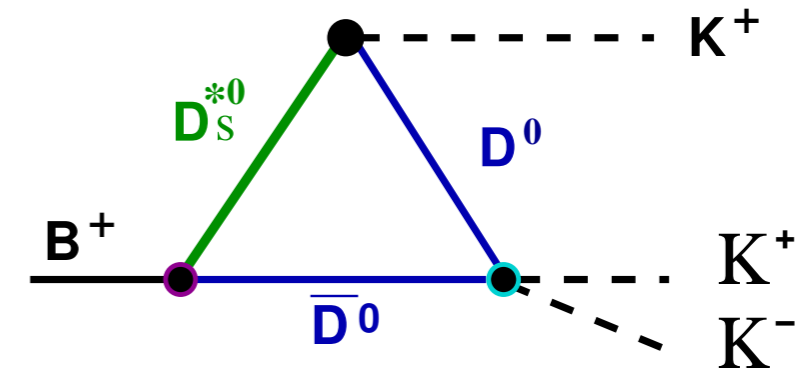
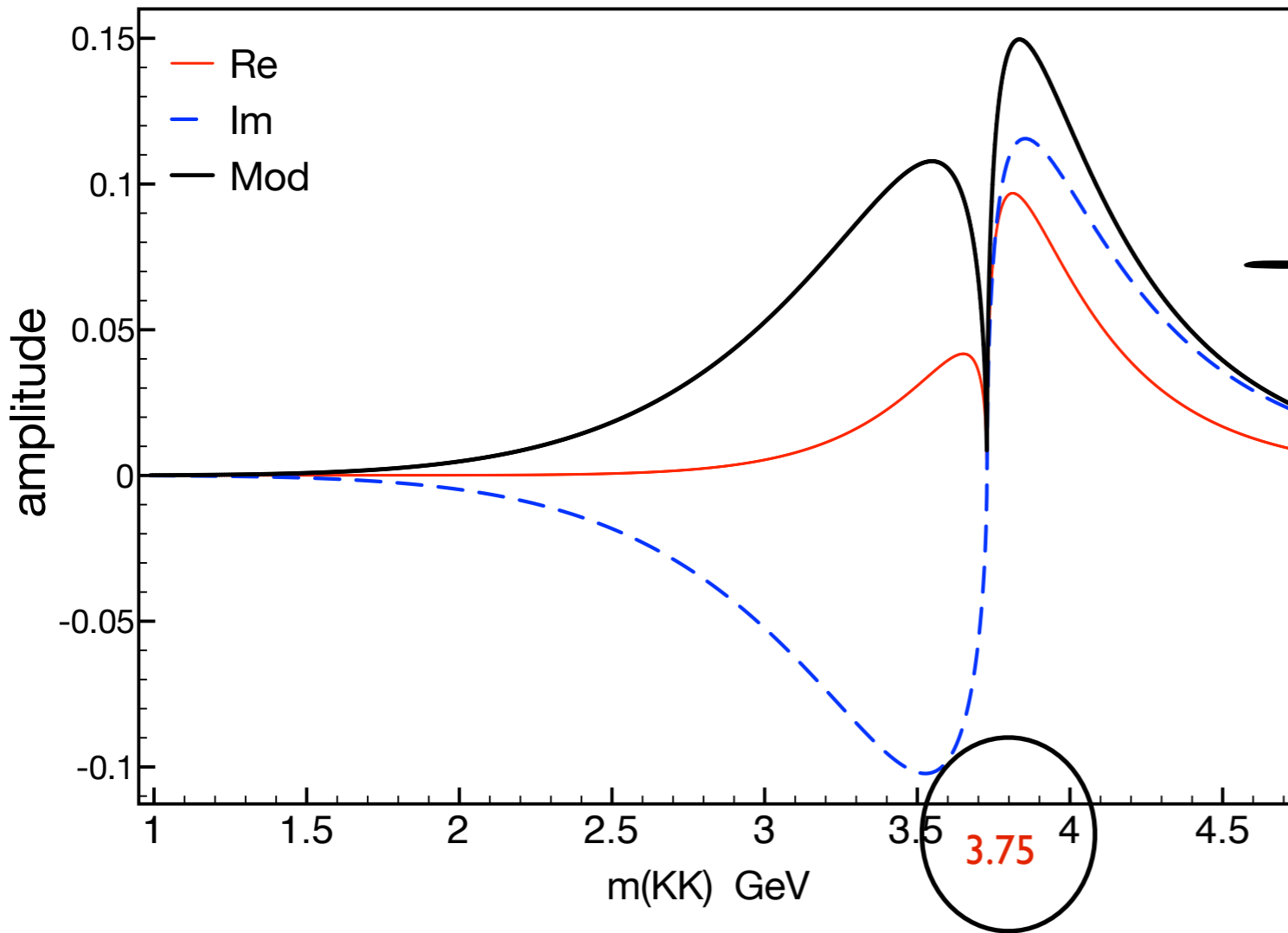
non zero bellow threshold



zero in between two bumps

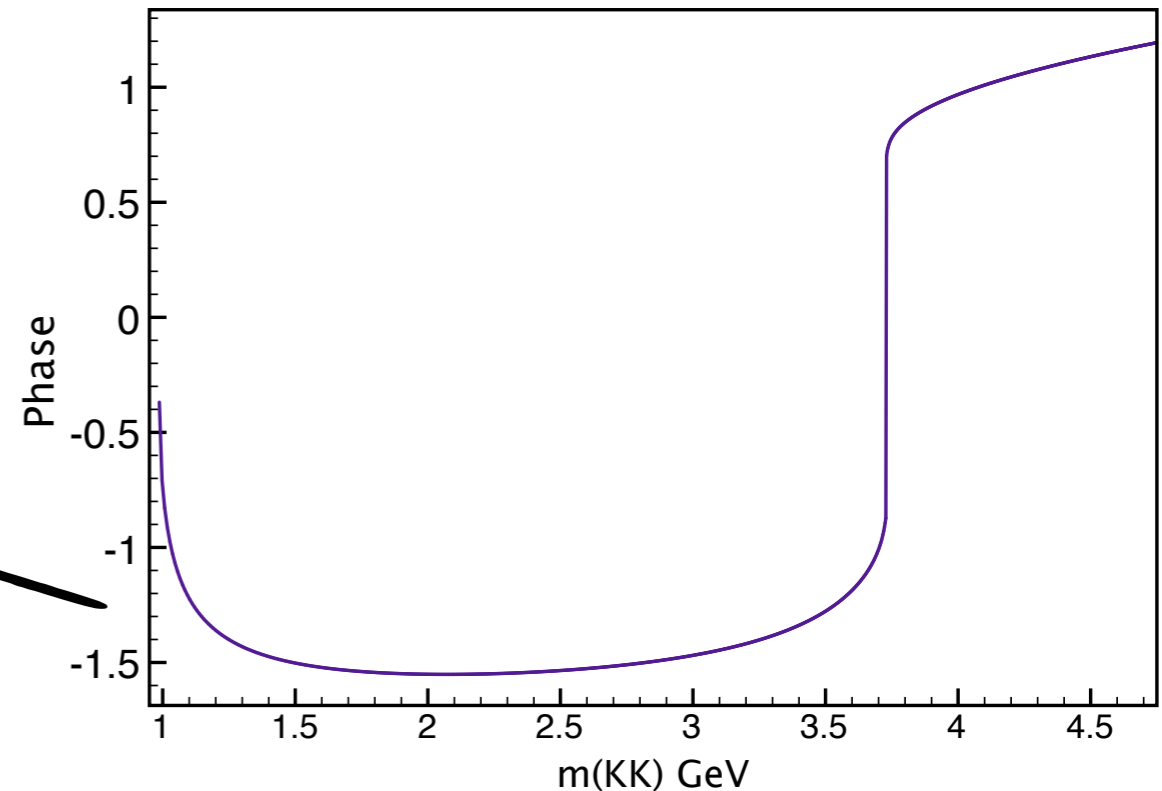


● total amplitude $A = i C_O T_{DD \rightarrow KK} A_P^h$



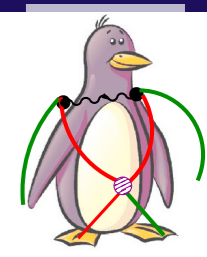
zero in between two bumps

can change CPV signal in DP!!!

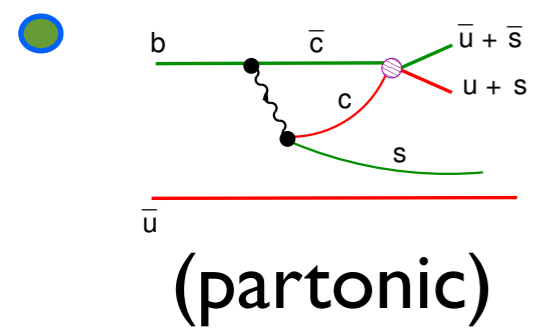


non zero bellow threshold

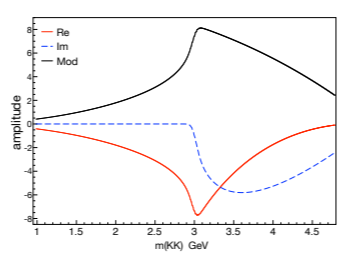
rescattering $D^0 \bar{D}^0 \rightarrow K^+ K^-$
play a major role



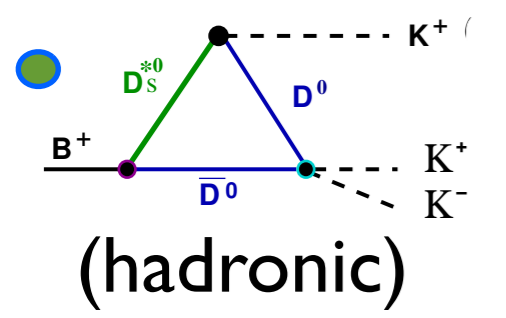
- $B \rightarrow KKK$ charm penguins



- wide amplitude spread in the center of the Dalitz plane



➔ NR population observed!



- amplitude: two narrow peaks in between a zero (threshold)

➔ superposition of triangles

- phase: change sign in a region close where data shows a CP asymmetry change in sign!



FSI mechanism to produce CP asymmetry at high mass

- interference between: triangles & partonic & other NR sources & resonances

➔ can shift the position of the CP asymmetry sign change

➔ should be tested in data ANA!

- FSI \rightarrow superposition of resonant and non-resonant at low and high energy
- different weak vertices topologies for same FS \rightarrow how to add them?
 - need to know about short distance!
- to improve ANA **and** learn from data we need:
 - \rightarrow good analytic and unitary 2-body coupled-channels;
 - with LASS/GLASS and Matrix K we don't learn much...
 - \rightarrow B-decays must include the diff QCD regime dynamics;
 - NR charm penguins!!
 - \rightarrow NR 2 and 3-body effects;
 - at least a way to parametrize this different from 2-body without adding phases

How to do that? ...



TRR110 Workshop - Amplitudes for Three-Body Final States

11-13 July 2018

MIAPP

Europe/Berlin timezone



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Campus of Max-Planck Society
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Germany



Patricia Magalhães
Bastian Kubis
Christoph Hanhart
Norbert Kaiser
Stephan Paul

Scientific Programme

Meson-Meson scattering

Jacobo Ruiz de Elvira (Bern)

Dispersive analysis for πK

Bachir Moussallam (Paris)

Dispersive analysis of $\pi\pi$, πK and $\pi\eta$

Miguel Albaladejo (Murcia)

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visit

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Thank you!!!

Extra slides

FSI in three-body decay :

I. Bediaga, I., T. Frederico, T. and O. Louren Phys. Rev. D89, 094013(2014),[arXiv:1307.8164]

J. H. Alvarenga Nogueira, I. Bediaga, A. B. R. Cavalcante, T. Frederico and O. Louren, Phys. Rev. D92, 054010 (2015) [ArXiv:1506.08332].

PC Magalhães and I Bediaga arXiv:1512.09284;

P. C Magalhães and R.Robilotta, Phys. Rev. D92 094005 (2015) [arXiv:1504.06346] ; P.C.Magalhães et. al. Phys. Rev. D84 094001 (2011) [arXiv:1105.5120]; P.C. Magalhães and Michael C. Birse, PoS QNP2012, 144 (2012).

I. Caprini, Phys. Lett. B 638 468 (2006).

Bochao Liu, M. Buescher, Feng-Kun Guo, C. Hanhart, and Ulf-G. Meissner, Eur. Phys. J. C 63 93 (2009).

F Niecknig and B Kubis - JHEP 10 142 (2015) ArXiv:1509.03188

H. Kamano, S.X. Nakamura, T.-S.H. Lee and T. Sato, Phys. Rev. D 84, 114019 (2011).

S. X. Nakamura, arXiv:1504.02557 (2015).

J. -P. Dedonder, A. Furman, R. Kaminski, L. Lesniak, L. and B. Loiseau, Acta Phys. Polon. B42, 2013 (2011), [Arxiv: 1011.0960]

J.-P. Dedonder, R. Kaminski, L. Lesniak, and B. Loiseau, , Phys. Rev.D89, 094018 (2014).

Donoghue *et al.*, *Phys. Rev Letters* 77(1996) 2178;

Suzuki,Wolfenstein, Phys. Rev. D 60 (1999)074019;

Falk et al. Phys. Rev. D 57,4290(1998);

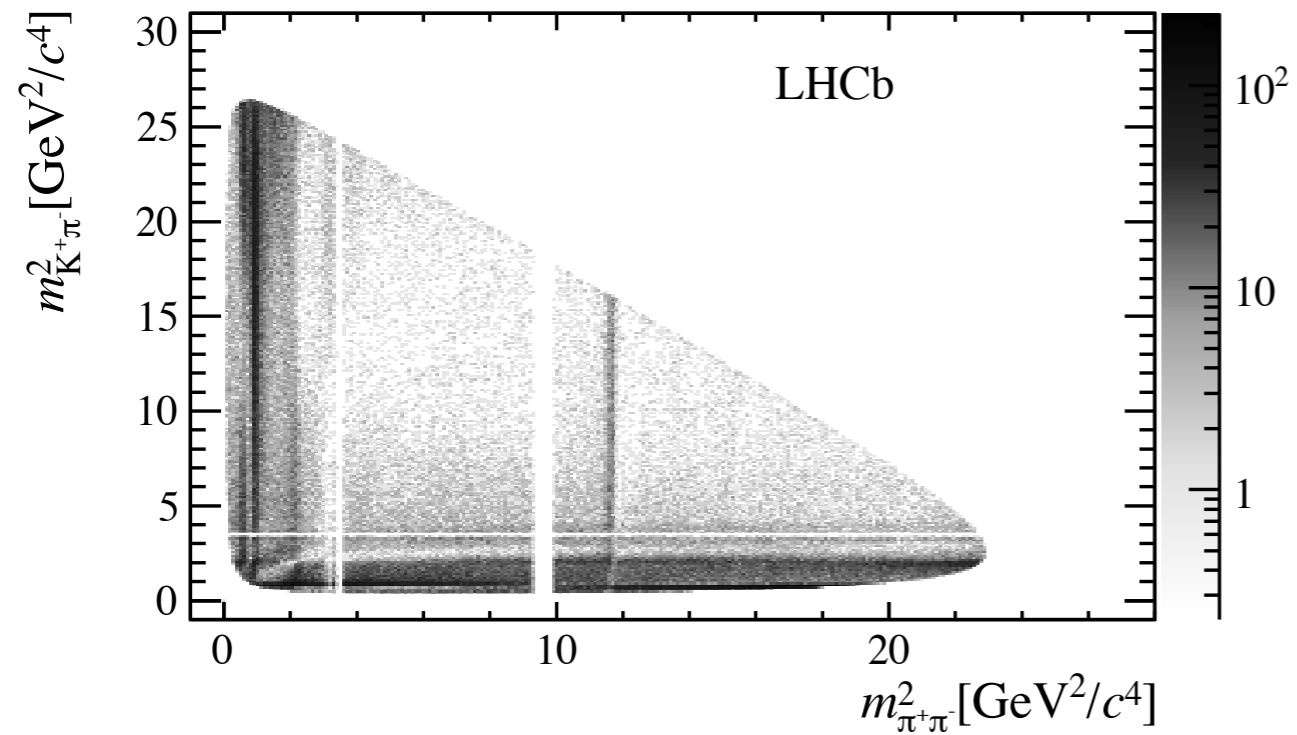
Blok, Gronau, Rosner, *Phys. Rev Letters* 78, 3999 (1997).

many more ...

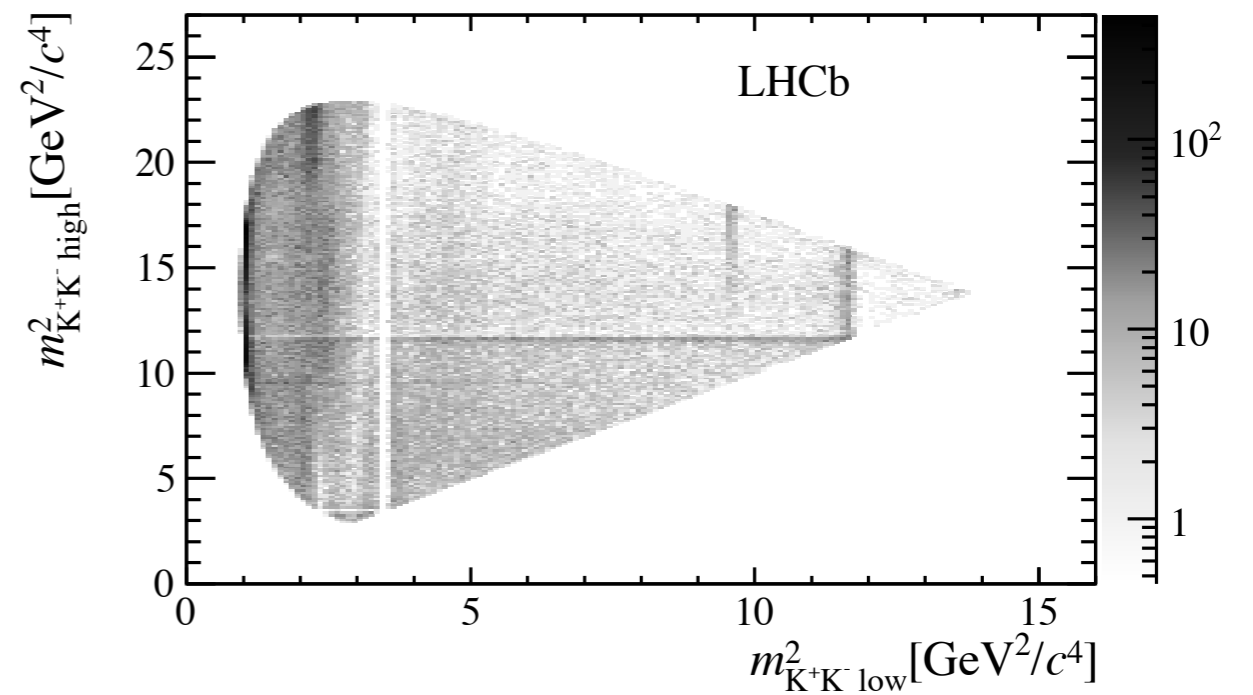
Charm Penguin

if needed

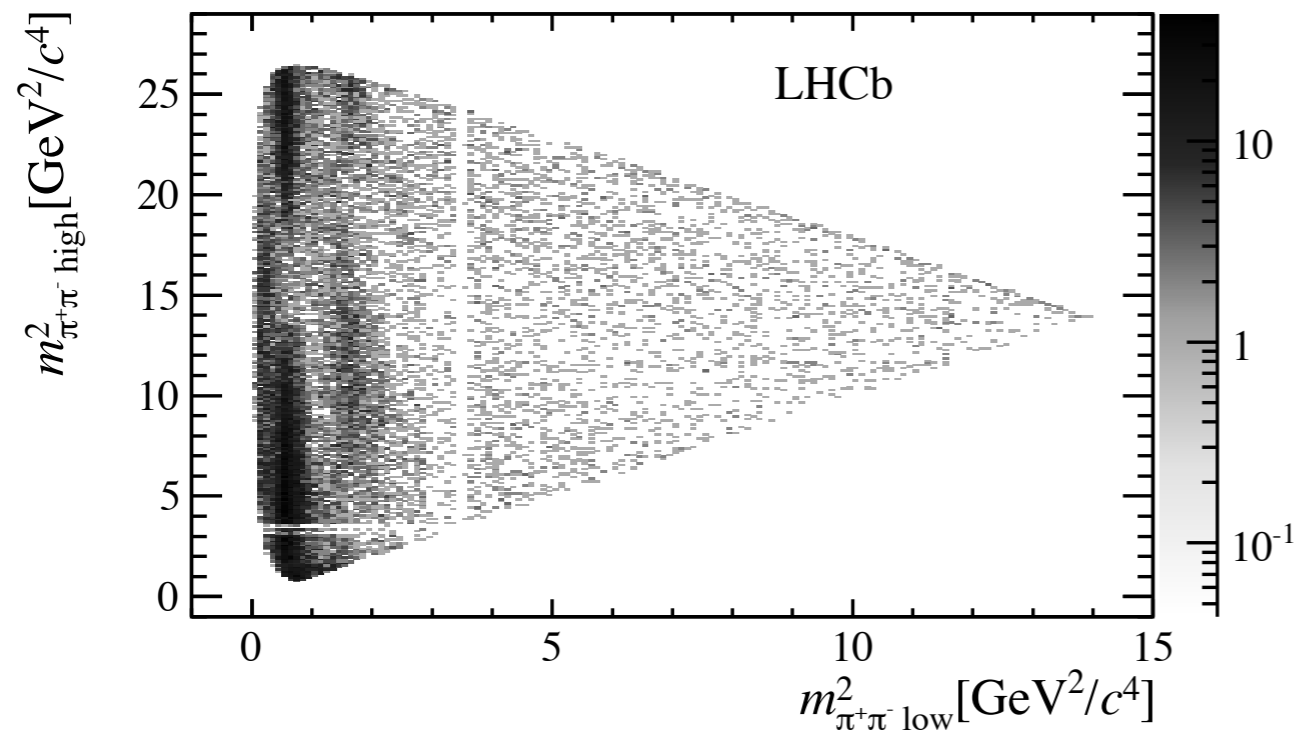
Kpp



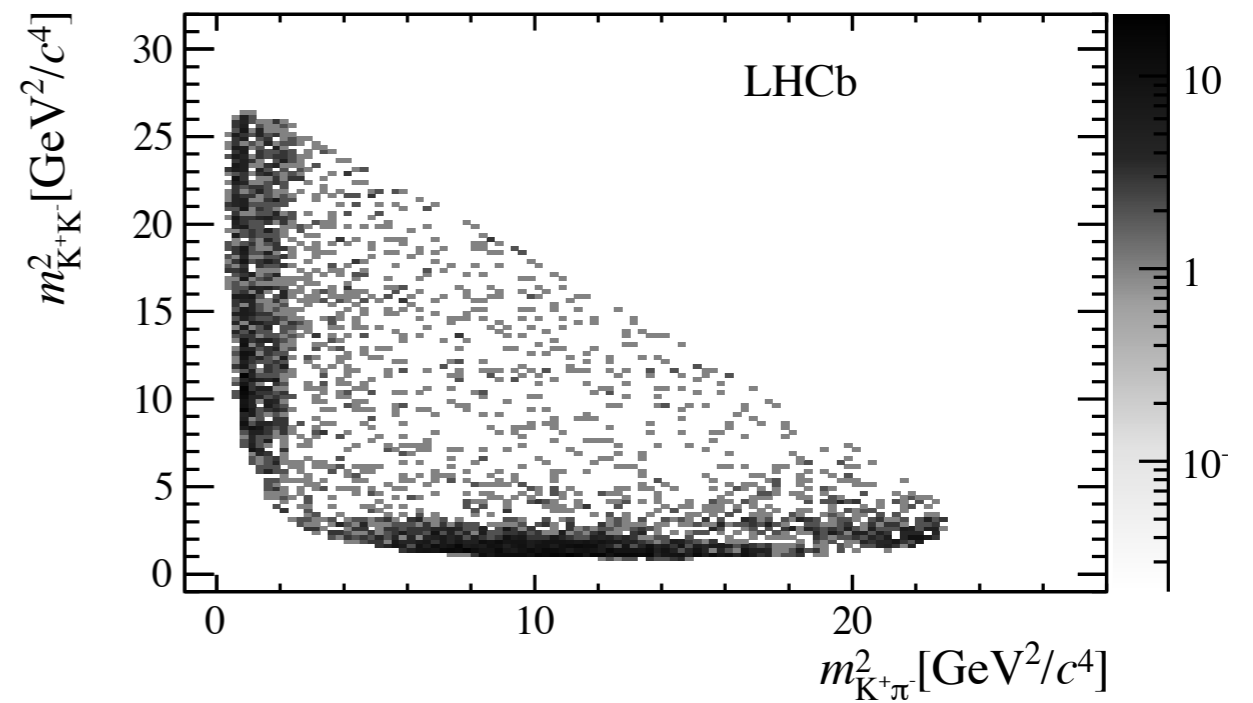
KKK



PPP

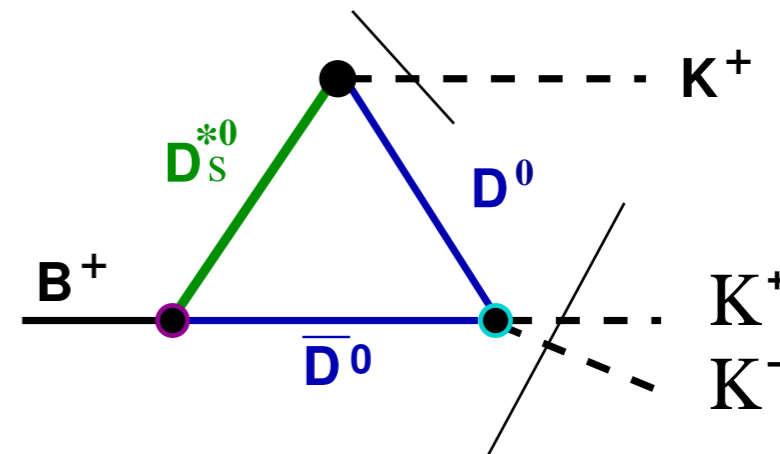


KKp

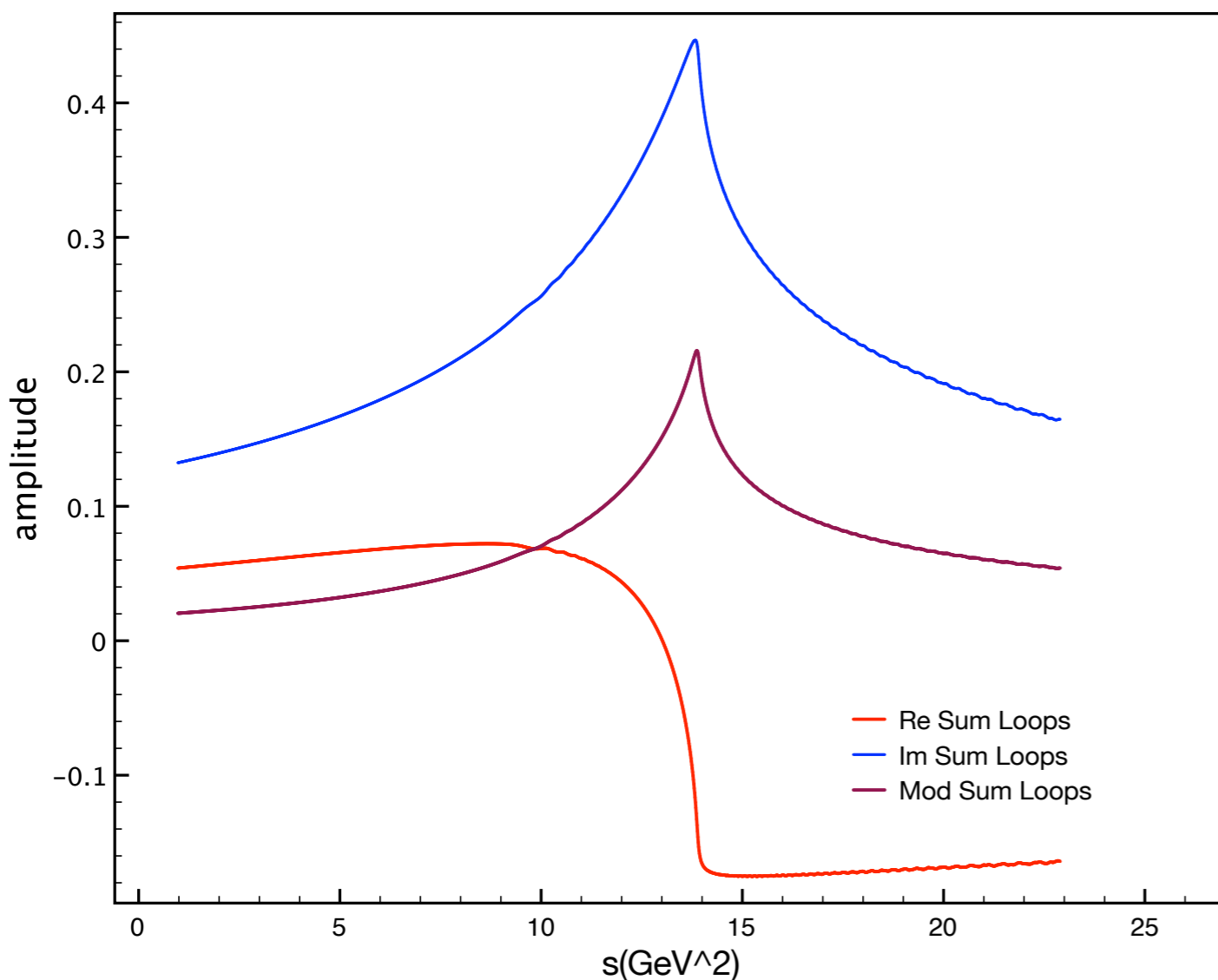


● loop contribution

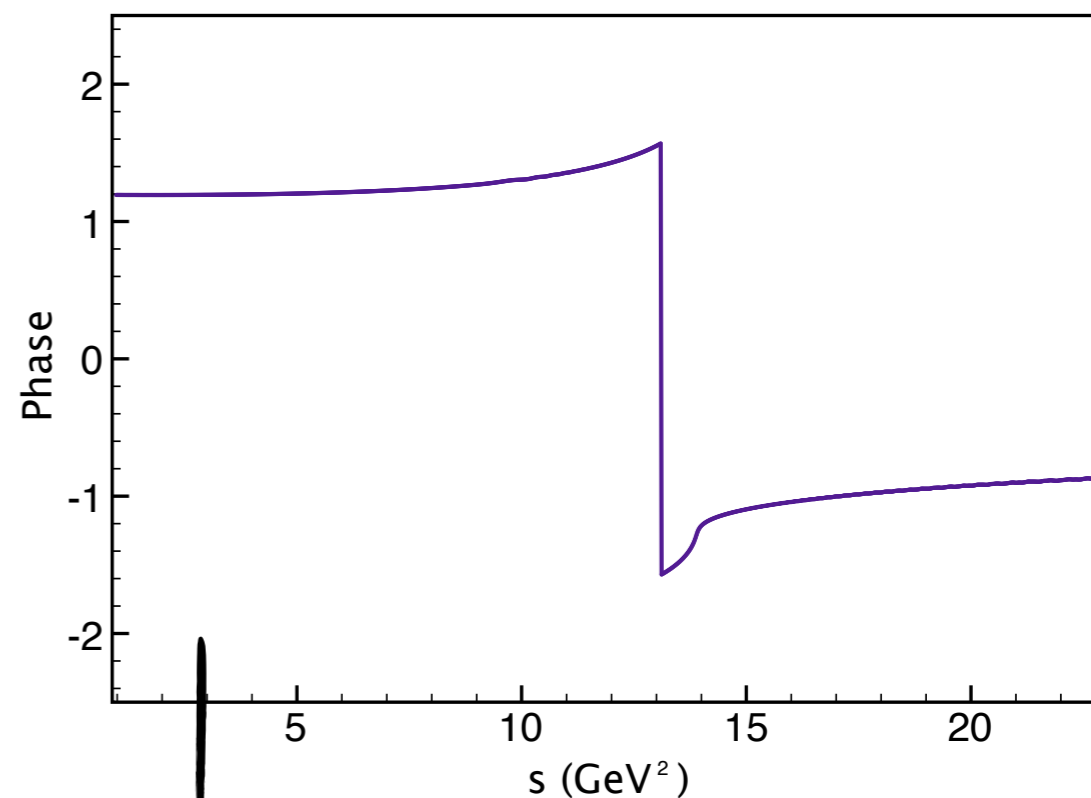
$$A_P^h = i \int \frac{d^4\ell}{(2\pi)^4} \frac{\Delta_{D^0} + 2\Delta_{\bar{D}^0} - 2s_{23} + 3M_K^2 + M_B^2 - l^2}{\Delta_{D^0} \Delta_{\bar{D}^0} \Delta_{D^*} [l^2 - m_{B^*}^2]}$$



● two threshold effect



→ off shell effect



discontinuity at threshold
allow CPV below threshold

$D^0 \bar{D}^0 \rightarrow K^+ K^-$ scattering amplitude

- not well understood on literature
- important as FSI in B two-body decays

Donoghue et al., PRL 77(1996)2178;
 Suzuki, Wolfenstein, PRD 60 (1999)074019;
 Falk et al. PRD 57,4290(1998);
 Blok, Gronau, Rosner, PRL 78, 3999 (1997).

- phenomenological amplitude

Antunes, Bediaga, Frederico, PCM
 ICHEP2016 - proceedings

- unitarity of the S-matrix $S = \begin{pmatrix} \eta e^{2i\alpha} & \sqrt{1-\eta^2} e^{i(\alpha+\beta)} \\ -\sqrt{1-\eta^2} e^{i(\alpha+\beta)} & \eta e^{2i\beta} \end{pmatrix}$

- inspired in the damping factor of the S matrix i.e. $\pi\pi \rightarrow KK$

$$\eta = \mathcal{N} \sqrt{s/s_{th} - 1} / (s/s_{th})^{2.5}$$

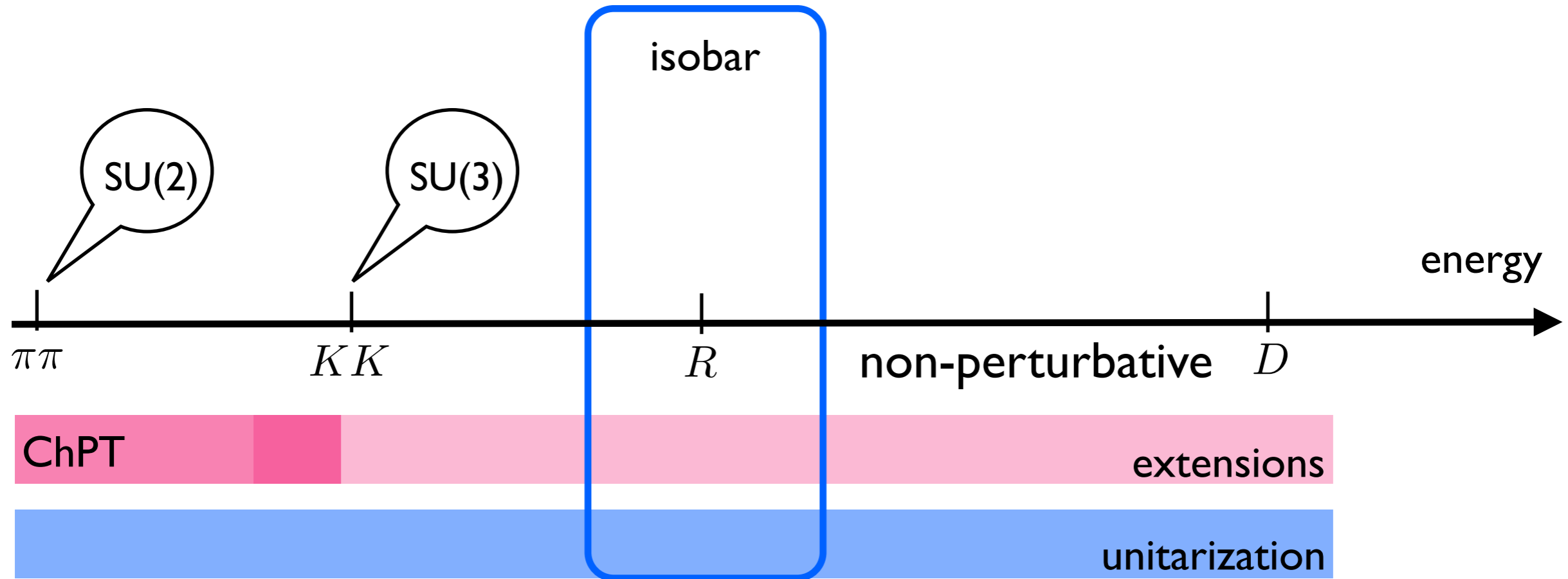
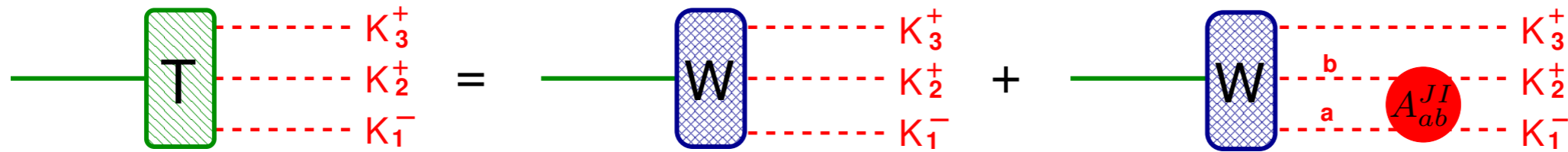
$$\text{KK: } e^{2i\alpha} = 1 - \frac{2ik_1}{\frac{c}{1-k_1/k_0} + ik_1}, \quad \text{DD: } e^{2i\beta} = 1 - \frac{2ik}{\frac{1}{a} + ik}$$

$$k = \sqrt{\frac{s-s_{th}}{4}}, \quad k_1 = \sqrt{\frac{s-s_{th1}}{4}} \quad \text{and} \quad k_0 = \sqrt{\frac{s_0-s_{th}}{4}}$$

$$S_{\beta,\alpha} = \delta_{\beta,\alpha} + it_{\beta,\alpha}$$

$$t_{\beta,\alpha} = \sqrt{1-\eta^2} e^{i(\alpha+\beta)}$$

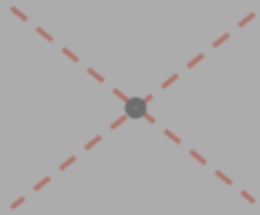
$D \rightarrow KKK$



- A_{ab}^{JI} : unitary coupled-channel amplitude for $[J, I = (0, 1), (0, 1)]$
 $\rightarrow ab = \pi\pi, K\bar{K}, \eta\pi, \rho\pi, \eta\eta$

- solid theory to describe MM interactions at low energy

- LO:



$$\mathcal{L}_M^{(2)} = -\frac{1}{6F^2} f_{ijs} f_{kls} \phi_i \partial_\mu \phi_j \phi_k \partial^\mu \phi_l + \frac{B}{24F^2} \left[\sigma_0 \left(\frac{4}{3} \delta_{ij} \delta_{kl} + 2 d_{ijs} d_{kls} \right) + \sigma_8 \left(\frac{4}{3} \delta_{ij} d_{kl8} + \frac{4}{3} d_{ij8} \delta_{kl} + 2 d_{ijm} d_{klm} d_{8mn} \right) \right] \phi_i \phi_j \phi_k \phi_l$$

Gasser & Leutwyler [Nucl. Phys. B250(1985)]

- NLO: include resonances as a field



Ecker, Gasser, Pich and De Rafael [Nucl. Phys. B321(1989)]

scalars

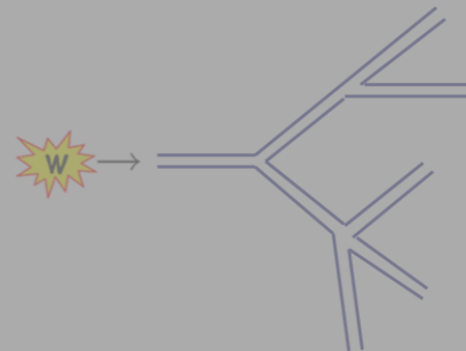
vectors

$$\mathcal{L}_S^{(2)} = \frac{2\tilde{c}_d}{F^2} R_0 \partial_\mu \phi_i \partial^\mu \phi_i - \frac{4\tilde{c}_m}{F^2} B R_0 (\sigma_0 \delta_{ij} + \sigma_8 d_{8ij}) \phi_i \phi_j + \frac{2c_d}{\sqrt{2}F^2} d_{ijk} R_k \partial_\mu \phi_i \partial^\mu \phi_j - \frac{4Bc_m}{\sqrt{2}F^2} \left[\sigma_0 d_{ijk} + \sigma_8 \left(\frac{2}{3} \delta_{ik} \delta_{j8} + d_{i8s} d_{j8s} \right) \right] \phi_i \phi_j R_k$$

$$\mathcal{L}_V^{(2)} = \frac{iG_V}{\sqrt{2}} \langle V_{\mu\nu} u^\mu u^\nu \rangle$$

$$\langle V_{\mu\nu} u^\mu u^\nu \rangle = \frac{1}{F^2} V_a^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_j (i f_{aij} + d_{aij})$$

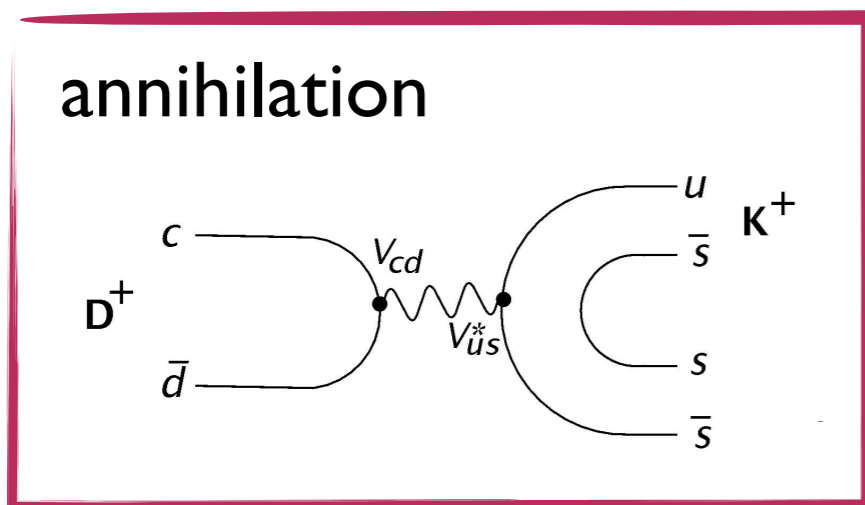
- hadronization of Weak current



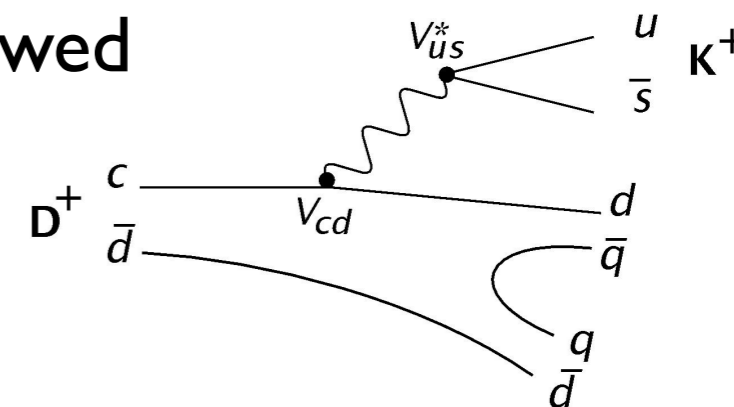
Gasser & Leutwyler [Nucl. Phys. B250(1985)]



- tree level



color allowed



- need a rescattering!

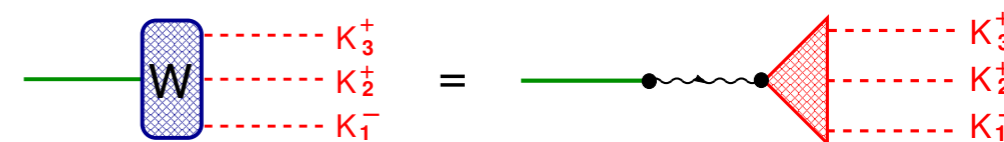
- both are doubly Cabibbo-suppressed
- hypotheses that annihilation is dominant

- separate the different energy scales:

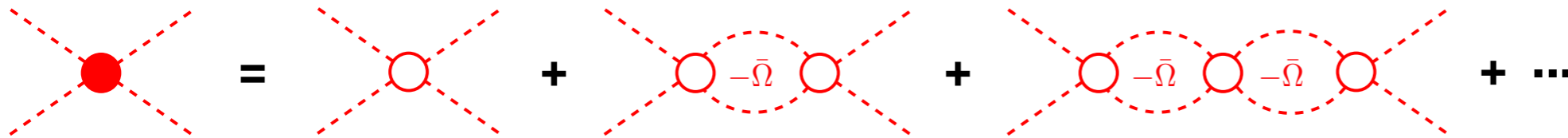
$$\mathcal{T} = \langle (KKK)^+ | T | D^+ \rangle = \underbrace{\langle (KKK)^+ | A_\mu | 0 \rangle}_{\text{ChPT}} \langle 0 | A^\mu | D^+ \rangle.$$

$\hookrightarrow -i G_F \sin^2 \theta_C F_D P^\mu$

\longrightarrow know how to calculate everything

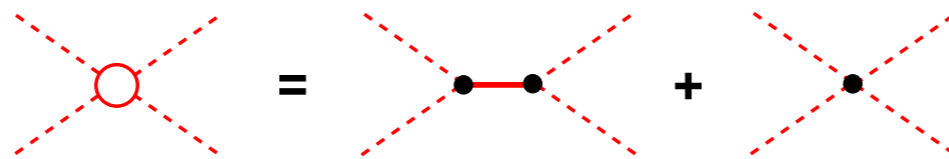


- unitarize amplitude by Bethe-Salpeter eq. [Oller and Oset PRD 60 (1999)]



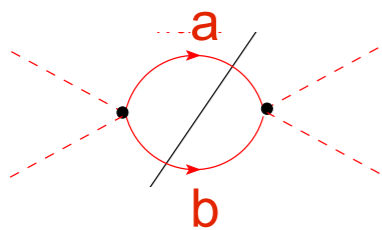
$$\mathcal{A}_{ab}^{JI} = \frac{\mathcal{K}_{ab \rightarrow cd}^{(JI)}}{1 + \bar{\Omega}_{ab} \mathcal{K}_{ab \rightarrow cd}^{(JI)}}$$

- kernel $\mathcal{K}_{ab \rightarrow cd}^{(J,I)}$



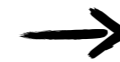
resonance (NLO) + contact (LO)

- loops \rightarrow K-matrix approximation: only on-shell



$$\{I_{ab}; I_{ab}^{\mu\nu}\} = \int \frac{d^4 \ell}{(2\pi)^4} \frac{\{1; \ell^\mu \ell^\nu\}}{D_a D_b}$$

$$D_a = (\ell + p/2)^2 - M_a^2 \quad D_b = (\ell - p/2)^2 - M_b^2$$



$$\bar{\Omega}_{ab}^S = -\frac{i}{8\pi} \frac{Q_{ab}}{\sqrt{s}} \theta(s - (M_a + M_b)^2)$$

$$\bar{\Omega}_{aa}^P = -\frac{i}{6\pi} \frac{Q_{aa}^3}{\sqrt{s}} \theta(s - 4M_a^2)$$

$$Q_{ab} = \frac{1}{2} \sqrt{s - 2(M_a^2 + M_b^2) + (M_a^2 - M_b^2)^2/s}$$

- parameters from ChPT lagrangians $\rightarrow \neq$ meaning!

- masses:

$$m_\rho, m_{a_0}, \boxed{m_{s0}, m_{s1}} \quad SU(3) \text{ singlet and octet}$$

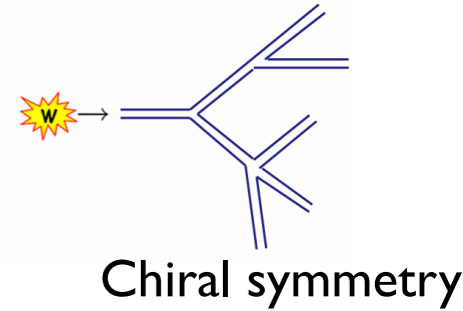
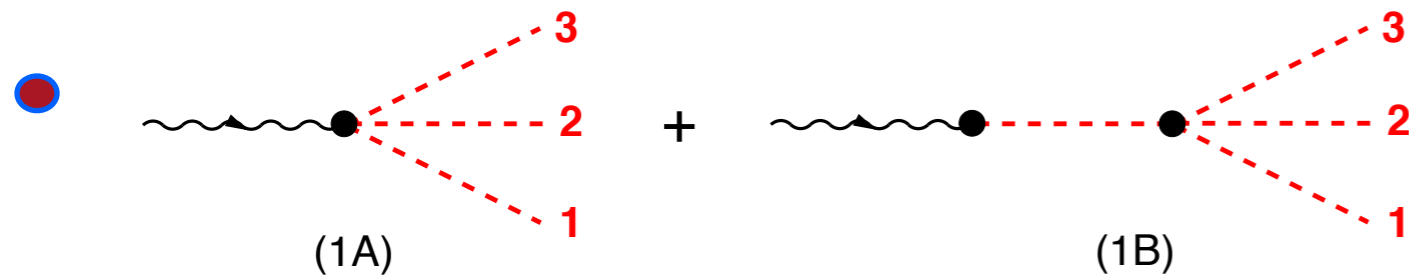
\rightarrow physical f_0 states are linear combination of m_{s0}, m_{s1}

- coupling constants:

$$g_\rho, g_\phi \quad c_d, c_m, \tilde{c}_d, \tilde{c}_m$$

vector

scalar



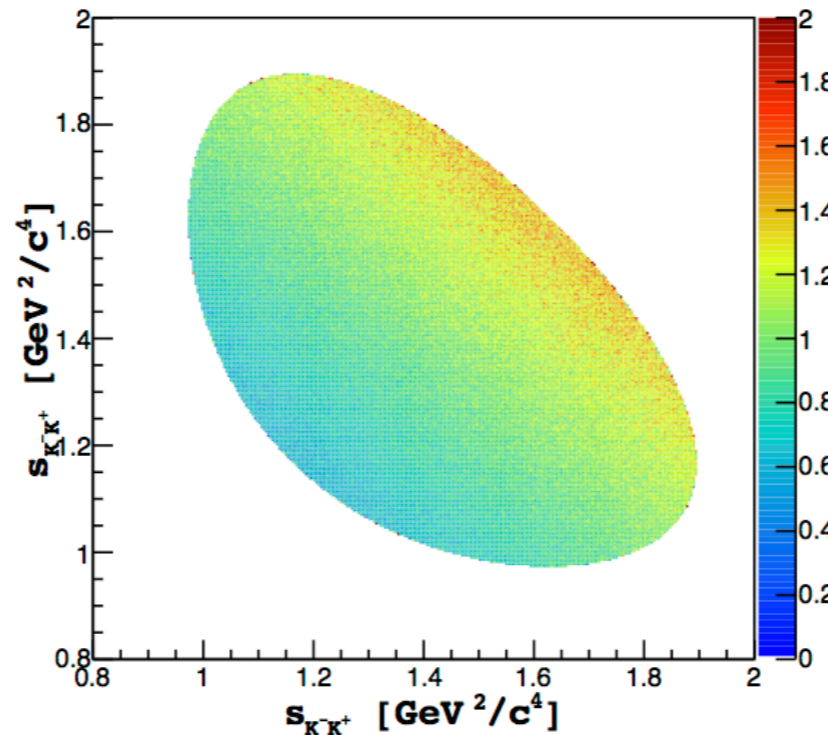
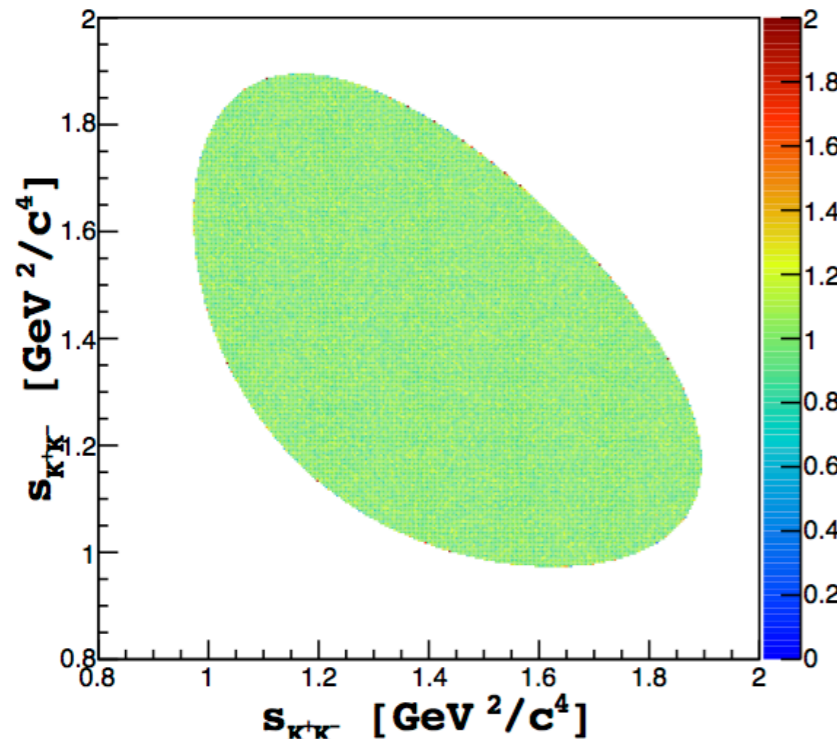
$$T_{NR} = \left[\frac{C}{4} (M^2 - M_K^2 + m_{12}^2) + \frac{C}{4} (m_{13}^2 - m_{23}^2) + (2 \leftrightarrow 3) \right]$$

3-body effect predicted by Chiral symmetry

$$C = \left\{ \left[\frac{G_F}{\sqrt{2}} \sin^2 \theta_C \right] \frac{2F_D}{F} \frac{M_K^2}{M_D^2 - M_K^2} \right\}$$

project into S- and P- wave

● comparing with isobar (constant)



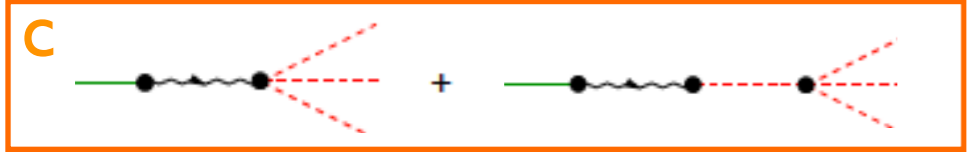
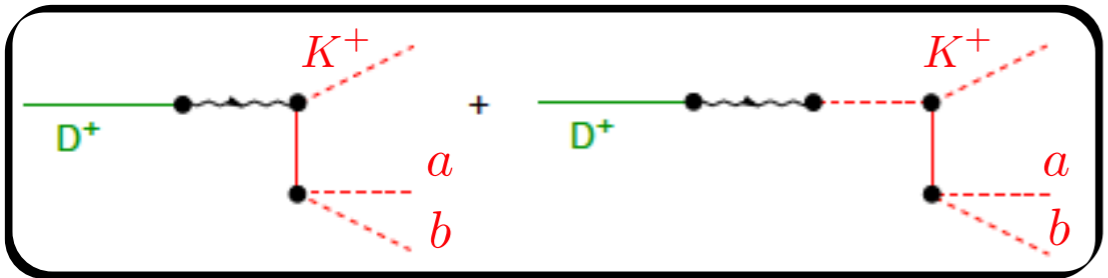
real polynomial

no possible free parameter

● tree $D \rightarrow abK^+$

$$\langle U_3(K^+) | T_{(0)}^{(0,1)} | D \rangle = \left\{ \Gamma_{(0)\pi 8}^{(0,1)} \langle U_3^{\pi 8} | + \Gamma_{(0)KK}^{(0,1)} \langle U_3^{KK} | \right\}$$

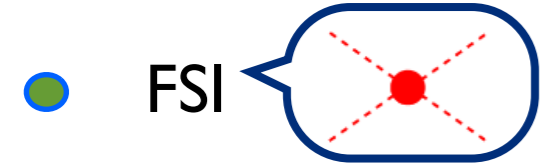
a_0 example
 $[J, I = 0, 1] \rightarrow \eta\pi, KK$



to avoid double counting!

$$\Gamma_{(0)\pi 8}^{(0,1)} = C \left\{ \left[\frac{2\sqrt{2}}{\sqrt{3}F^2} \right] \frac{[-c_d P \cdot p_3 + c_m M_D^2]}{m_{12}^2 - m_{a_0}^2} [c_d (m_{12}^2 - M_\pi^2 - M_8^2) + 2c_m M_\pi^2] + \left[-\frac{\sqrt{3}}{\sqrt{2}} [M_D^2/3 - P \cdot p_3] \right]_c \right\}$$

$$\Gamma_{(0)KK}^{(0,1)} = C \left\{ \left[\frac{2}{F^2} \right] \frac{[-c_d P \cdot p_3 + c_m M_D^2]}{m_{12}^2 - m_{a_0}^2} [c_d (m_{12}^2 - 2M_K^2) + 2c_m M_K^2] + \left[-\frac{1}{2} [M_D^2 - P \cdot p_3] \right]_c \right\}$$



one interaction

$$\Gamma_{(1)\pi 8}^{(0,1)} = -\mathcal{K}_{\pi 8|\pi 8}^{(0,1)} [\bar{\Omega}_{\pi 8}^S] \Gamma_{(0)\pi 8}^{(0,1)} - \mathcal{K}_{\pi 8|KK}^{(0,1)} \left[\frac{1}{2} \bar{\Omega}_{KK}^S \right] \Gamma_{(0)KK}^{(0,1)}$$

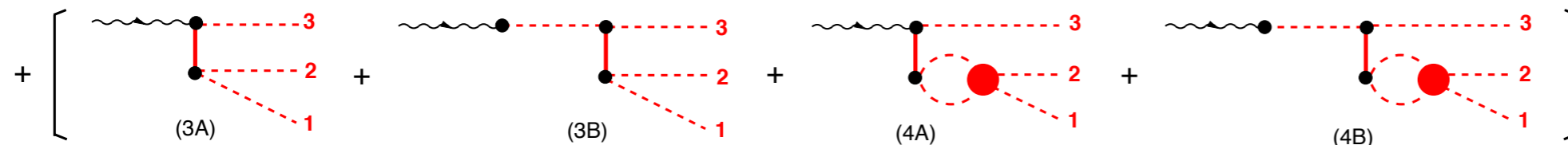
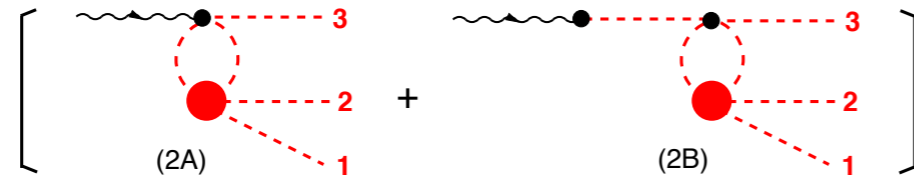
$$\Gamma_{(1)KK}^{(0,1)} = -\mathcal{K}_{\pi 8|KK}^{(0,1)} [\bar{\Omega}_{\pi 8}^S] \Gamma_{(0)\pi 8}^{(0,1)} - \mathcal{K}_{KK|KK}^{(0,1)} \left[\frac{1}{2} \bar{\Omega}_{KK}^S \right] \Gamma_{(0)KK}^{(0,1)}$$

$$\Gamma_{(1)}^{(0,1)} = \begin{bmatrix} \Gamma_{(1)\pi 8}^{(0,1)} \\ \Gamma_{(1)KK}^{(0,1)} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \Gamma_{(0)\pi 8}^{(0,1)} \\ \Gamma_{(0)KK}^{(0,1)} \end{bmatrix} = M^{(0,1)} \Gamma_{(0)}^{(0,1)}$$

infinity interactions

$$\Gamma^{(0,1)} = \{ 1 + M^{(0,1)} + [M^{(0,1)}]^2 + \dots \} \Gamma_{(0)}^{(0,1)} \rightarrow \Gamma^{(0,1)} = [1 - M^{(0,1)}]^{-1} \Gamma_{(0)}^{(0,1)}$$

● full FSI!



a_0 example
 $[J, I = 0, 1] \rightarrow \eta\pi, KK$

●
$$T^{(0,1)} = -\frac{1}{2} \left[\bar{\Gamma}_{KK}^{(0,1)} - \Gamma_{c|KK}^{(0,1)} \right]$$

$$\rightarrow \bar{\Gamma}_{KK}^{(0,1)} = \frac{(m_{12}^2 - m_{a_0}^2)}{D_{a_0}(m_{12}^2)} \left[M_{21} \Gamma_{(0)\pi 8}^{(0,1)} + (1 - M_{11}) \Gamma_{(0)KK}^{(0,1)} \right]$$

$$D_{a_0} = (m_{12}^2 - m_{a_0}^2) [(1 - M_{11})(1 - M_{22}) - M_{12} M_{21}]$$

$$M_{11} = -\mathcal{K}_{\pi 8|\pi 8}^{(0,1)} [\bar{\Omega}_{\pi 8}^S]$$

$$M_{12} = -\mathcal{K}_{\pi 8|KK}^{(0,1)} [(1/2) \bar{\Omega}_{KK}^S]$$

$$M_{21} = -\mathcal{K}_{\pi 8|KK}^{(0,1)} [\bar{\Omega}_{\pi 8}^S]$$

$$M_{22} = -\mathcal{K}_{KK|KK}^{(0,1)} [(1/2) \bar{\Omega}_{KK}^S]$$

● only resonance

$$\bar{\Gamma}_{KK}^{(0,1)} = \frac{(m_{12}^2 - m_{a_0}^2)}{D_{a_0}(m_{12}^2)} \Gamma_{(0)KK}^{(0,1)}$$

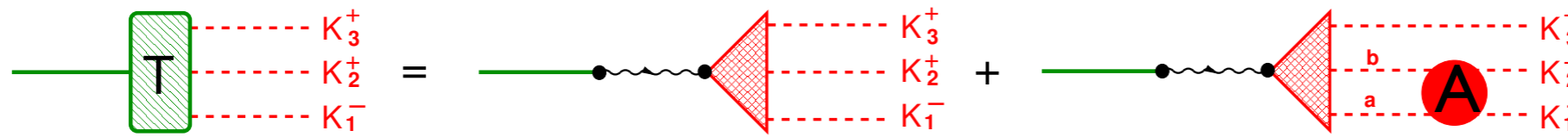
Flatté

$$D_{a_0}(s) = (s - m_{a_0}^2) + i m_{a_0} \Gamma_{a_0}(s)$$

$$m_{a_0} \Gamma_{a_0}(s) = \frac{1}{8\pi \sqrt{s}} \left\{ \left[\frac{4}{3 F^4} \left[c_d (s - M_\pi^2 - M_8^2) + 2 c_m M_\pi^2 \right]^2 Q_{\pi 8} \right. \right. \\ \left. \left. + \left[\frac{1}{F^4} \left[c_d (s - 2 M_K^2) + 2 c_m M_K^2 \right]^2 Q_{KK} \right] \right\}$$

\rightarrow parameter: c_d, c_m, m_{a_0}

access two-body dynamics !



- $T = [T^S + T^P + (2 \leftrightarrow 3)]$

$$T^S = \left[\frac{C}{4} (M_D^2 - M_K^2 + m_{12}^2) + T^{(0,1)} + T^{(0,0)} \right]$$

$$T^P = \left[\frac{C}{4} (m_{13}^2 - m_{23}^2) + T^{(1,1)} + T^{(1,0)} \right].$$

- $A^{IJ} \rightarrow$ prediction by Triple-M

- extend ChPT to non perturbative region \rightarrow parameter have different meaning
 $m_{S1} = m_{S0}$

- parameter for Toy studies :

masses from PDG (GeV)

$$m_\rho = 0.776, m_\phi = 1.019, m_{a0} = 0.960, m_{S0} = 0.980 \rightarrow m_{S1} = 1.370 \text{ GeV}$$

low energy couplings (GeV)

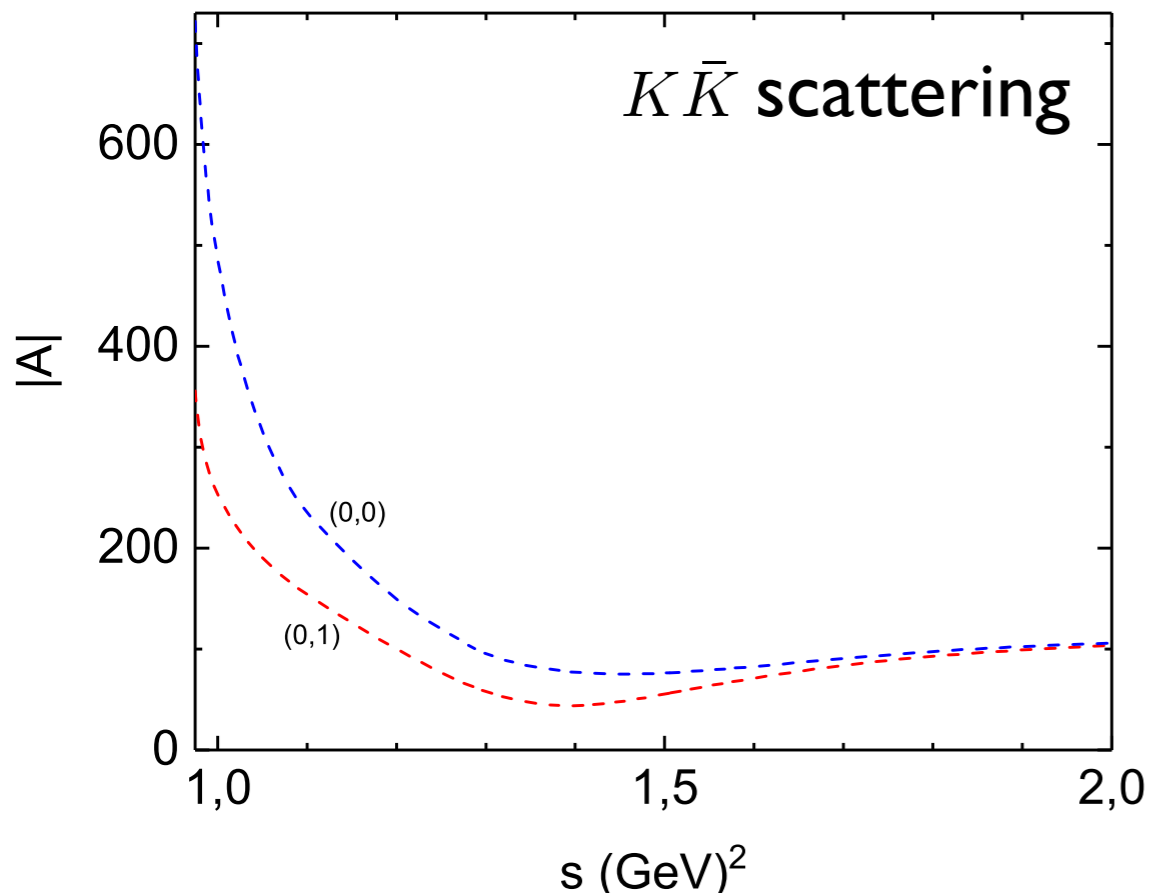
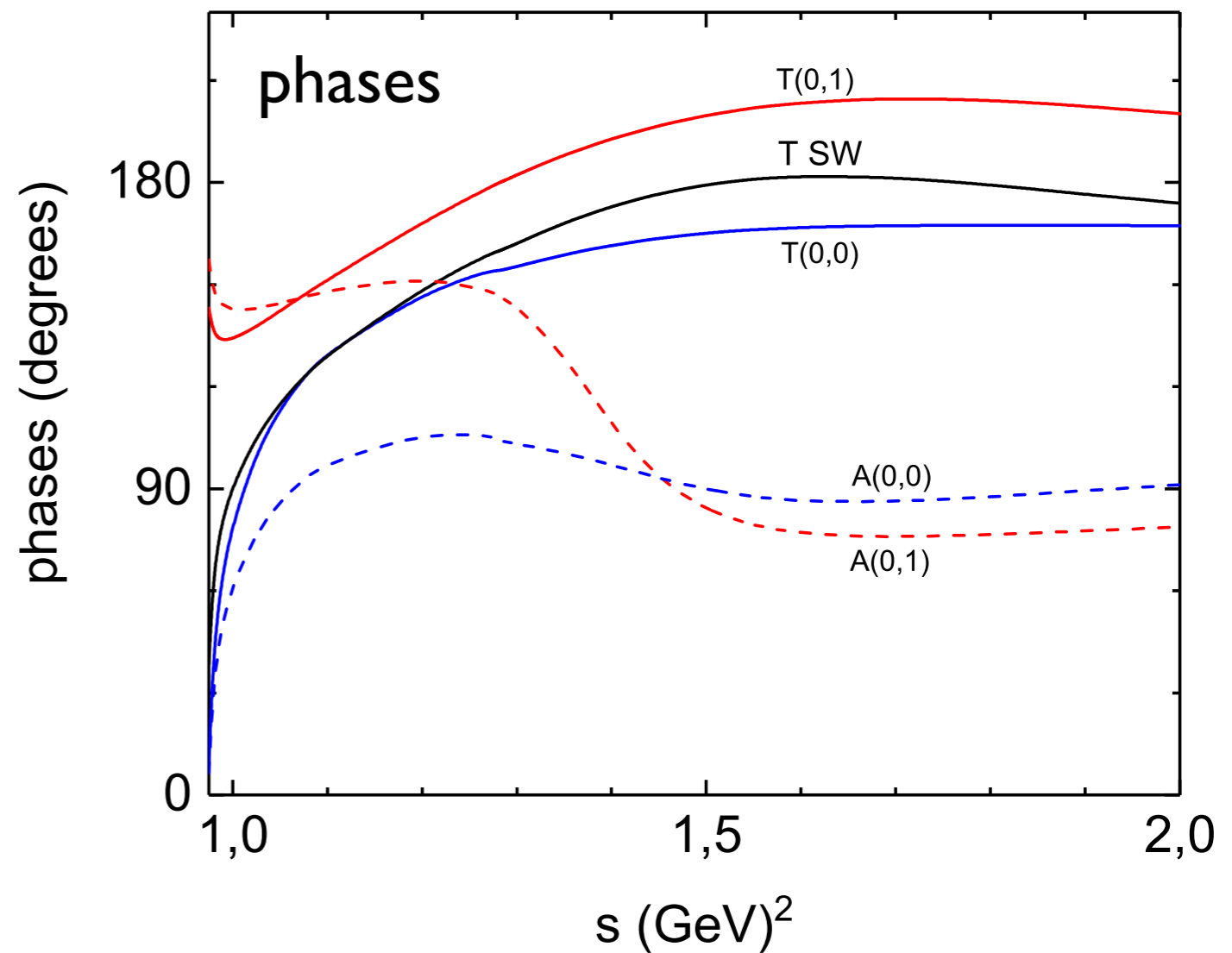
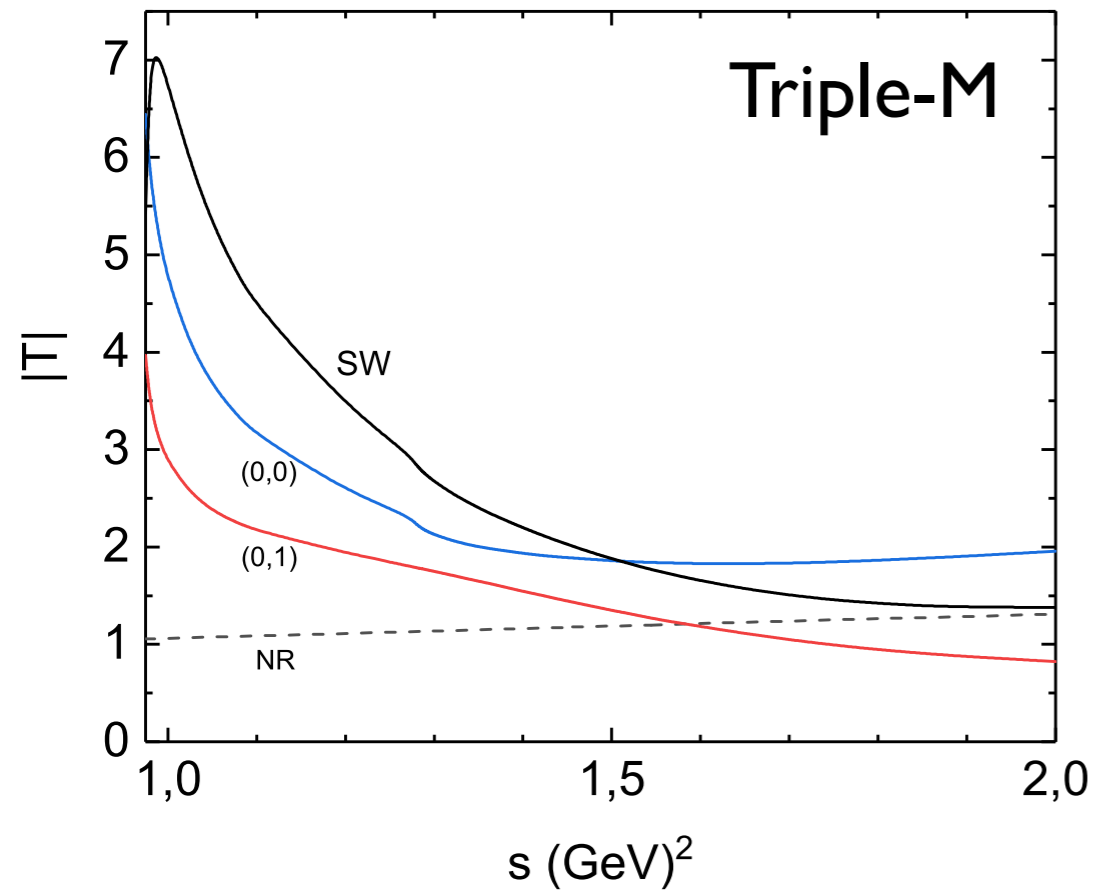
$$[F, G_V] = [0.093, 0.067] \text{ vectors}$$

$$[c_d, c_m] = [0.032, 0.042] \text{ scalar octet}$$

$$[\tilde{c}_d, \tilde{c}_m] = [0.018, 0.025] \text{ scalar singlet}$$

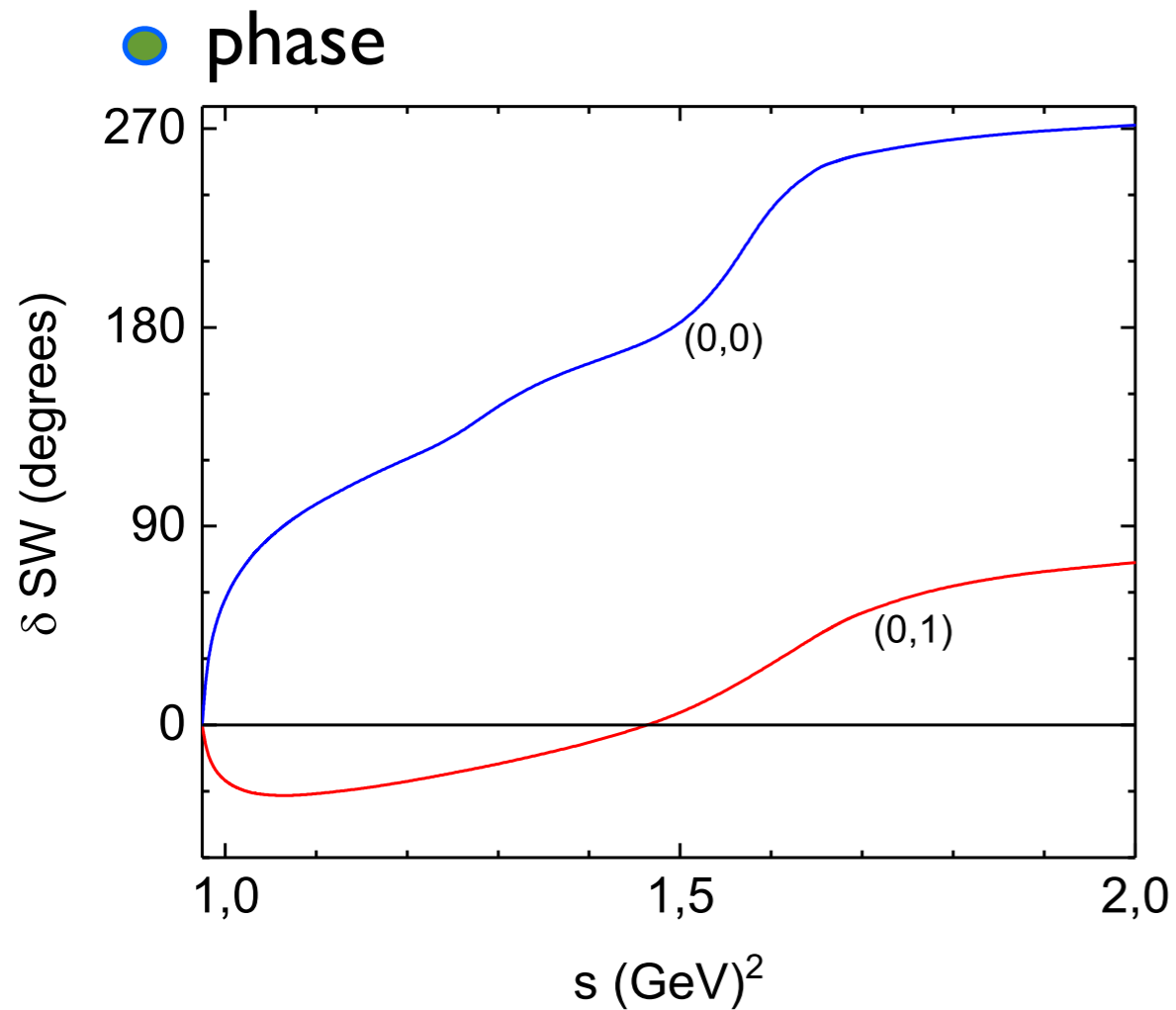
additional PDG
 $\Gamma_{\phi \rightarrow K\bar{K}} \sim 3.54 \text{ MeV}$
 $\sin \theta = 0.605$ ($\phi - \omega$) mixing

\rightarrow all (13) could be free in a fit to data

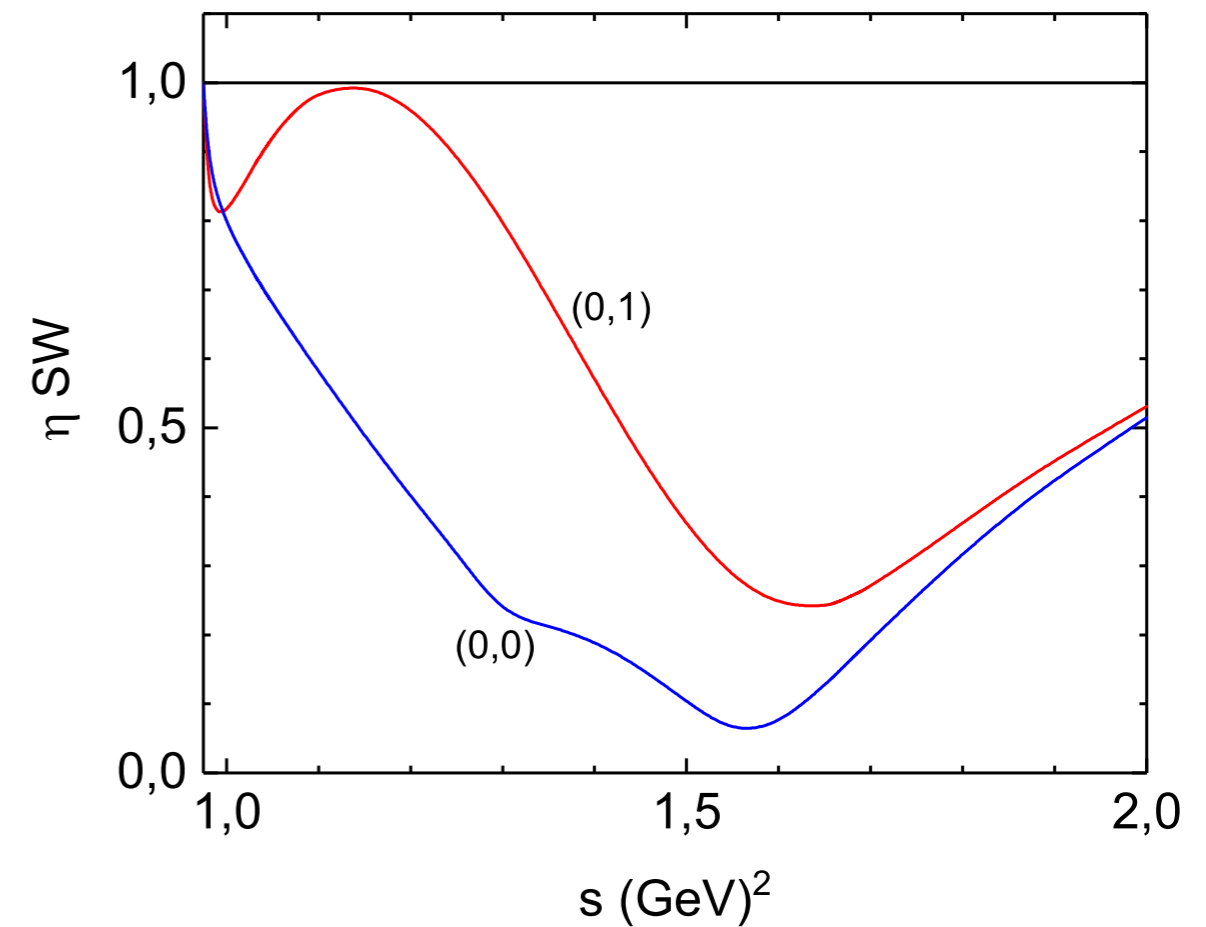


- amplitude with similar behavior \neq phase
- not possible to extract $A(J,I)$ from data
- Triple-M disentangle \neq isospins in data

- Triple-M predictions for A

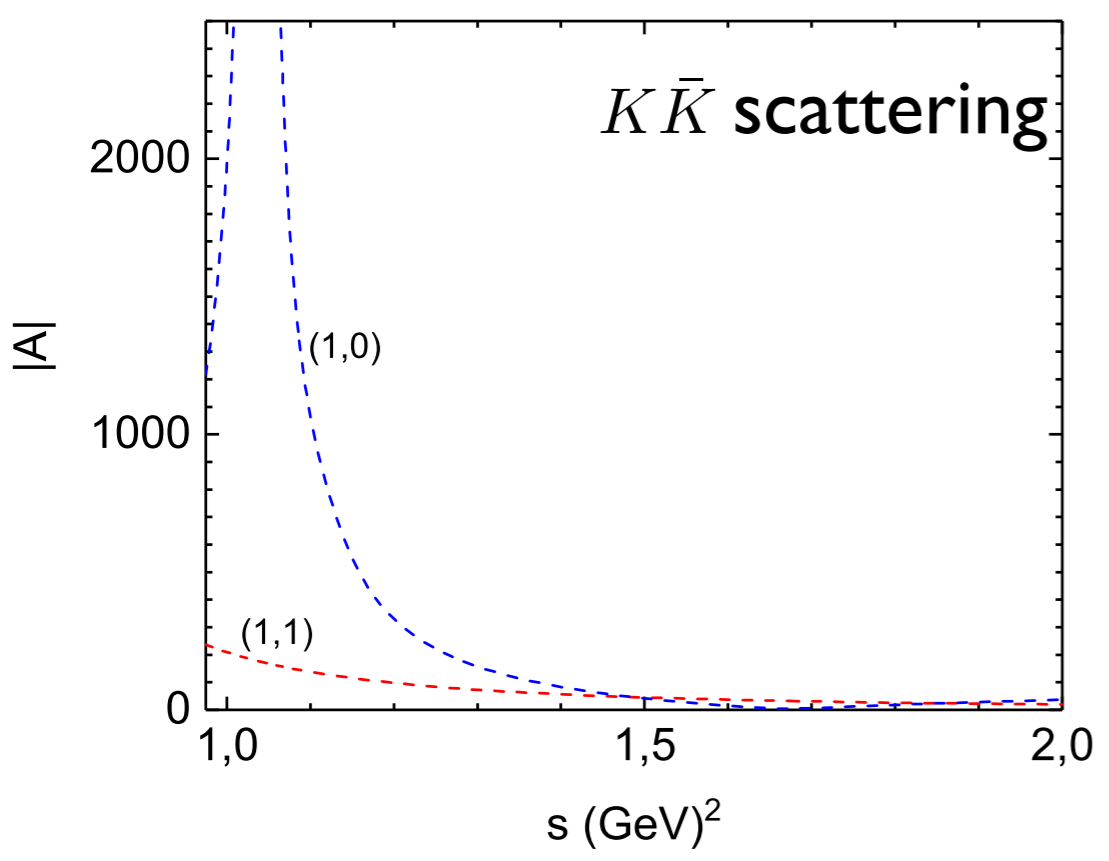
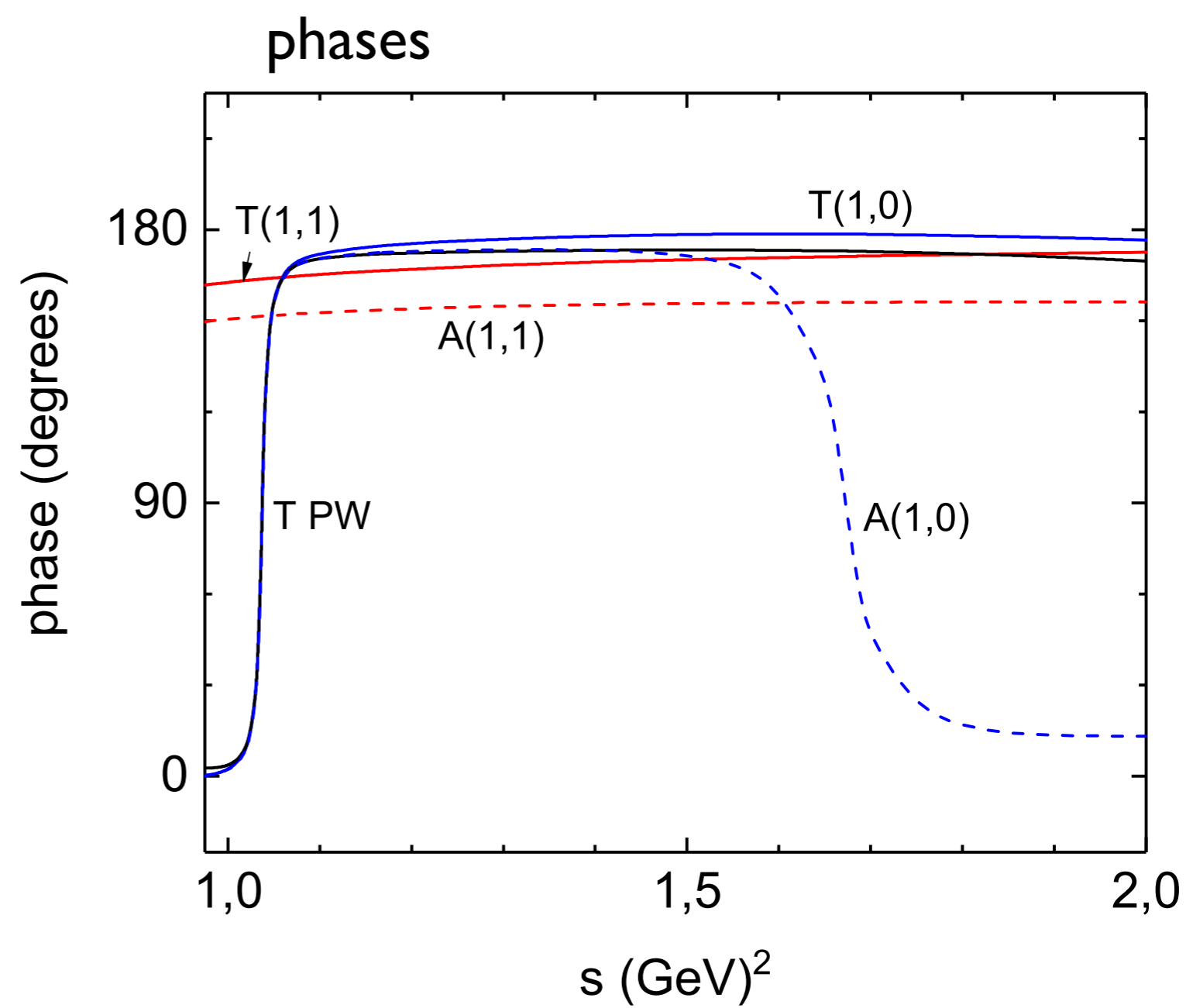
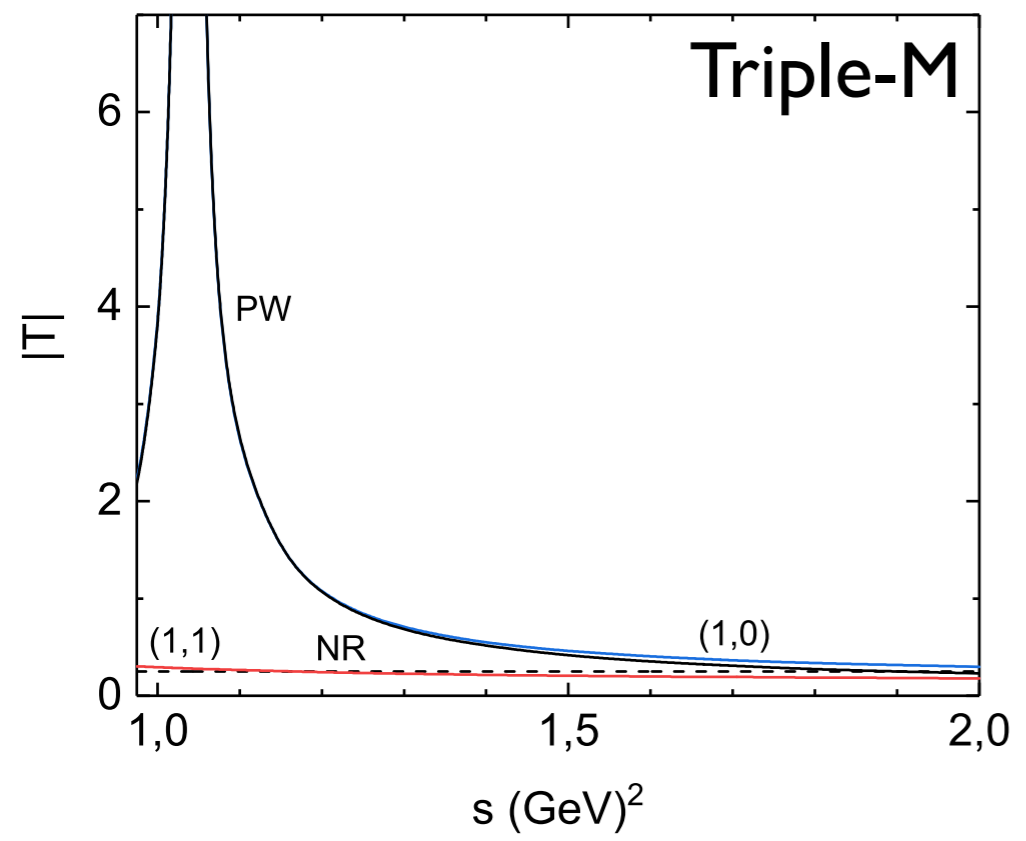


- inelasticity



- importance of coupled-channels

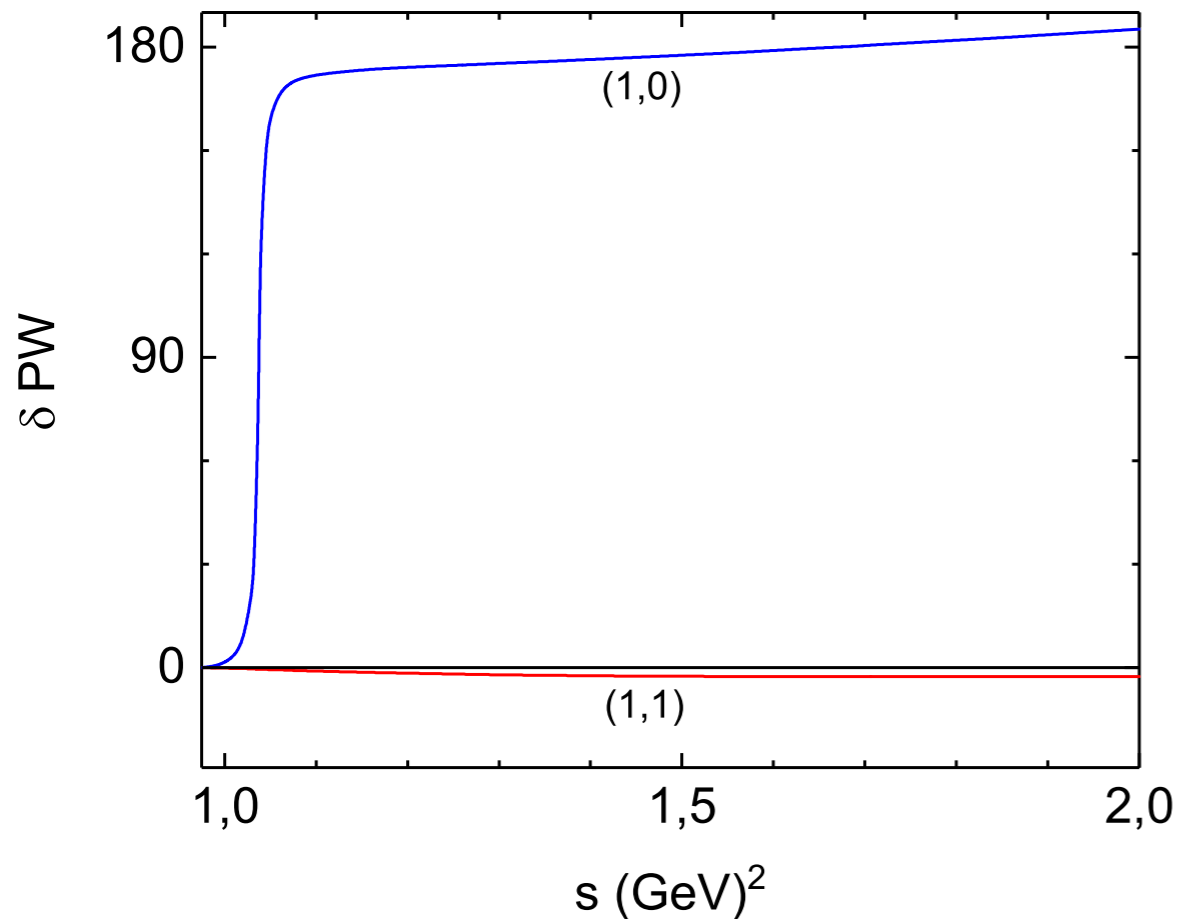
- two resonances in the ($J = 0, l = 0$) channel, preserving unitarity



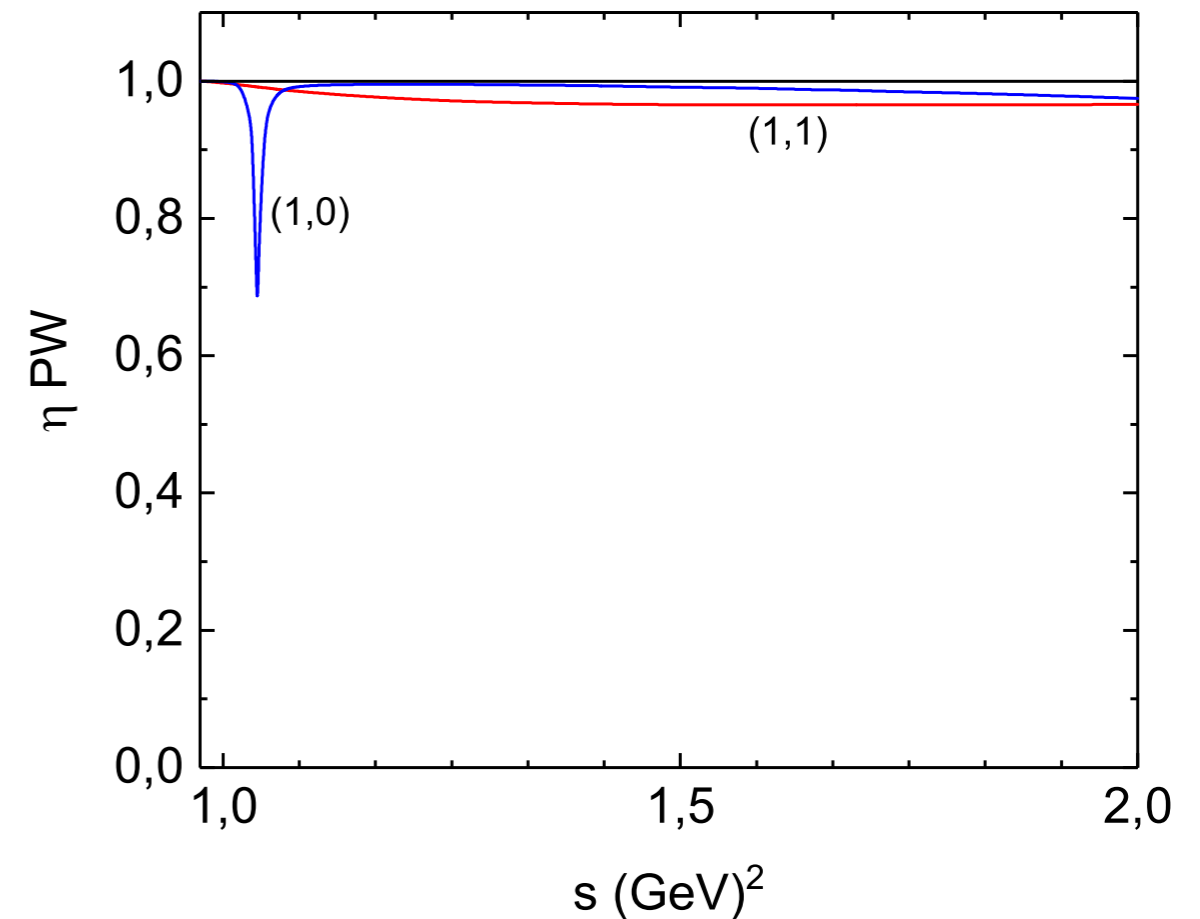
- ρ and NR contributions are tiny in TM
→ small in KK
- ϕ phases are \neq for $s > 1.5$

- Triple-M predictions for A

- phase

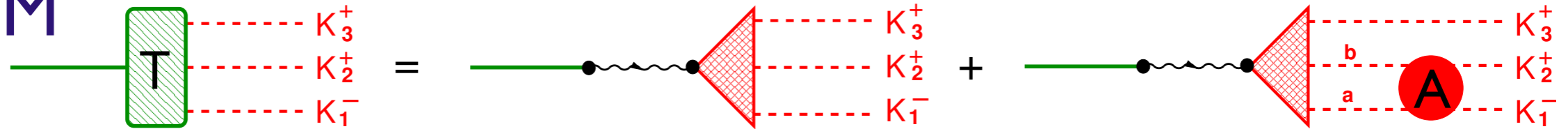


- inelasticity

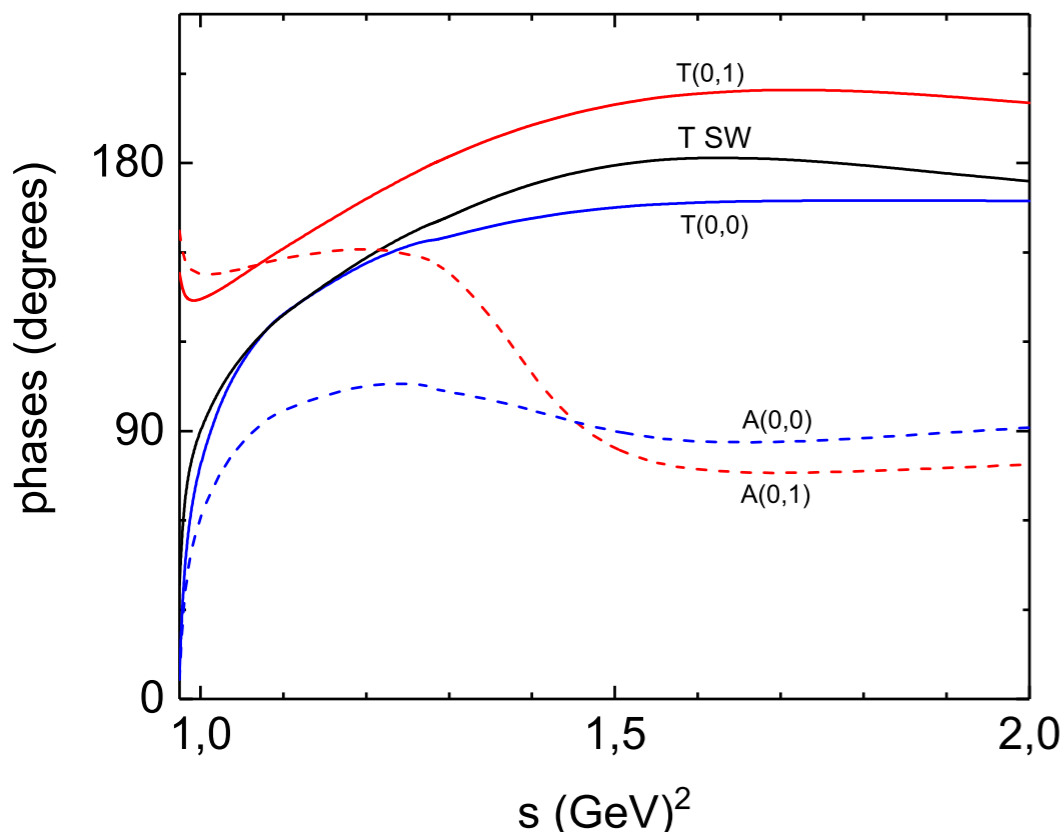


- ϕ is the dominant channel
- $\phi \rightarrow \rho\pi$ inelasticity \rightarrow 15% of the life-time
- $\rho \rightarrow \pi\pi \rightarrow$ constant inelasticity

Triple-M



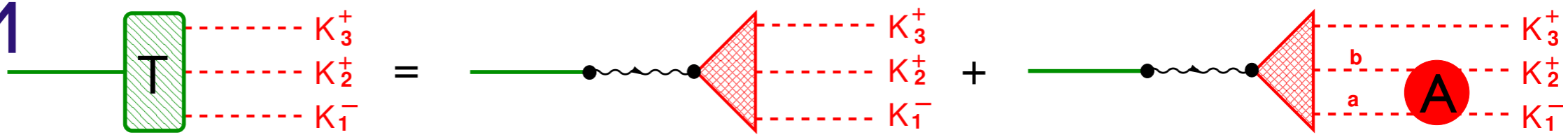
- annihilation weak topology dominance \rightarrow ChPT multi-meson description
 - non-resonant: beyond $(2+1)$ is a 3-body amplitude
- FSI: coupled-channel meson-meson from ChPTR Lagrangian
- intensity of each component is predict by theory $\rightarrow \neq$ isobar model
- Toy studies



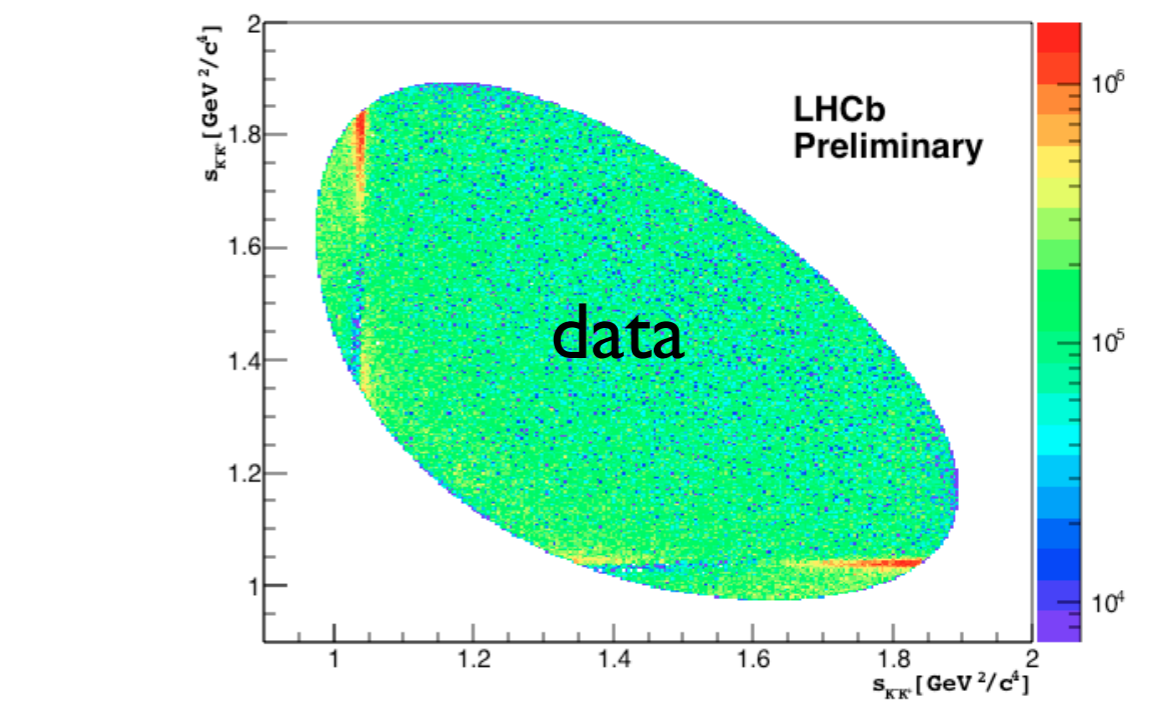
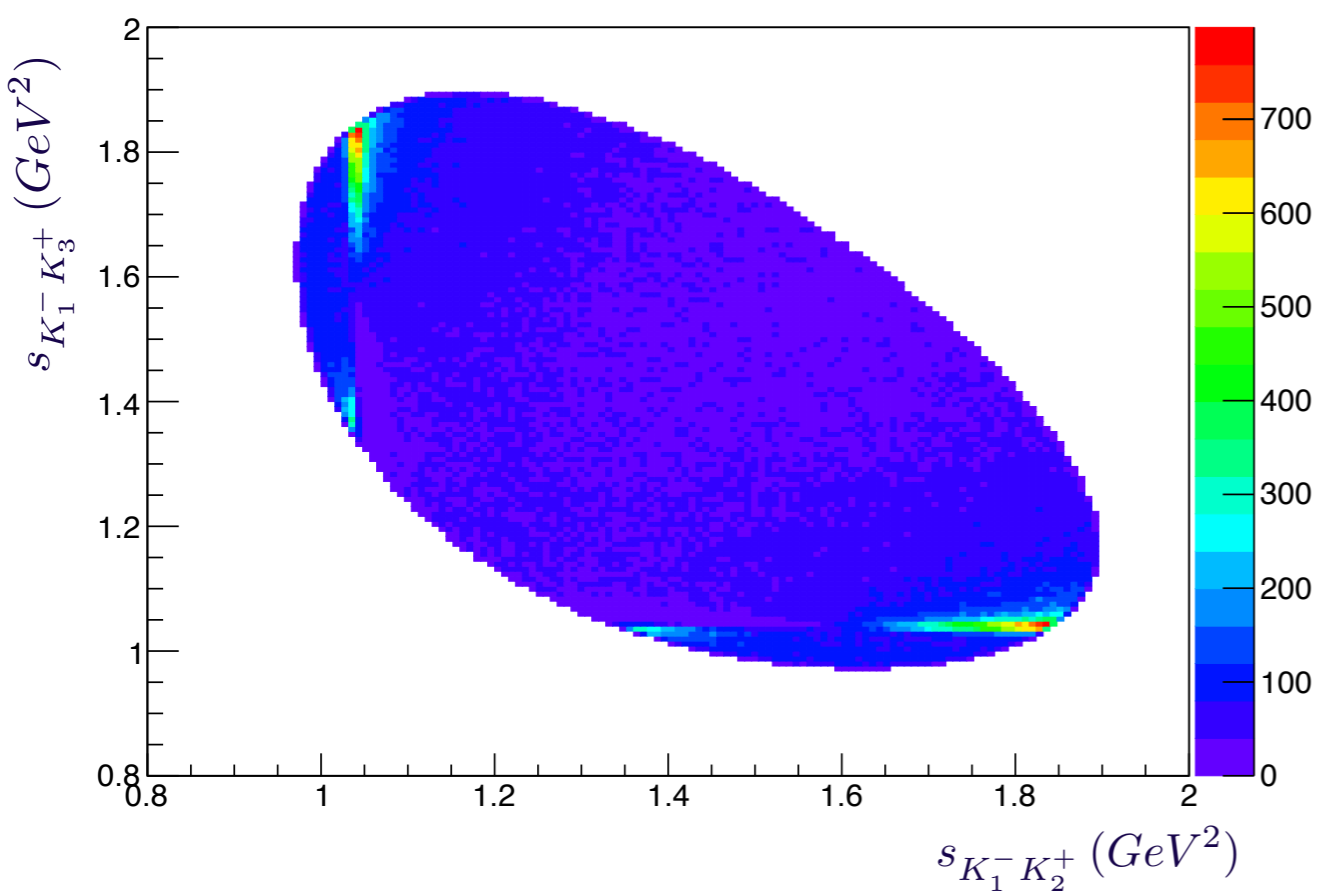
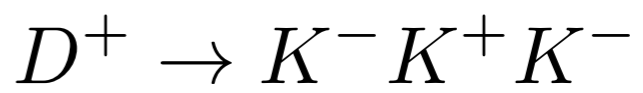
- \rightarrow parameters have physical meaning from ChPT
masses and coupling const. fix by fitting data
- \rightarrow can disentangle a_0 and f_0
- \rightarrow couple channel structure: cannot be ignored

although \neq is possible to extract 2-body phase from 3-body data with TM

• Triple-M



Triple-M Toy Dalitz plot



➔ Fit data with Triple-M

➔ powerful tool to extract KK scattering S-wave

