Microscopic model for e^+e^- production in πN collisions

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Introduction

- A lot of info is encoded in angular distributions of particle scattering
  - disentangling overlapping resonances
  - finding quantum numbers of particles (J^P)

- Angular distribution is related to orbital motion of particles
  → relation to spin/polarization

- Importance of polarized experiments

- Nontrivial polarization can exist in unpolarized experiments

e.g. $\pi + N \to N^*$: orbital angular momentum component parallel to the beam is 0
→ N^* polarization = N polarization
→ only $\lambda = \pm\frac{1}{2}$ even for higher resonances

Density matrix

- Representation of a general QM state including mixed states
- unpolarized beam/target: mixed state (incoherent mixture of different polarizations)
- The system is in state $|\psi_i\rangle$ with probability P_i : $\rho = \sum_i P_i \, |\psi_i\rangle \, \langle\psi_i|$
- Expectation value of operator O: $\langle O\rangle = \mathrm{Tr}\,(O\rho)$
- two-step process:

$$|\psi_i\rangle \xrightarrow{T_1} T_1\,|\psi_i\rangle = \sum_k |\phi_k\rangle \, \langle\phi_k\,|T_1\,|\,\psi_i\rangle \xrightarrow{T_2} \sum_k T_2\,|\phi_k\rangle \, \langle\phi_k\,|T_1\,|\,\psi_i\rangle$$

- probability of $|\,\psi_f\rangle$ in final state: $\mathcal{M}_{fi} = \sum_k \langle\psi_f\,|\,T_2\,|\,\phi_k\rangle \, \langle\phi_k\,|\,T_1\,|\,\psi_i\rangle$

$$\sum_i P_i |\mathcal{M}_{fi}|^2 = \sum_k \rho^{\mathrm{dec}}_{k'k} \rho^{\mathrm{prod}}_{kk'} = \mathrm{Tr}\left(\rho^{\mathrm{dec}} \rho^{\mathrm{prod}}\right)$$

production and
decay density matrices:

$$\rho^{\mathrm{prod}}_{kk'} = \sum_i P_i \, \langle\phi_k\,|\,T_1\,|\,\psi_i\rangle \, \langle\psi_i\,|\,T_1\,|\,\phi_{k'}\rangle$$

$$\rho^{\mathrm{dec}}_{k'k} = \langle\phi_{k'}\,|\,T_2\,|\,\psi_f\rangle \, \langle\psi_f\,|\,T_2\,|\,\phi_k\rangle$$

Decay of a resonance - polarization

A resonance of spin-J decays into two particles with spins s_1 and s_2:

$$\mathcal{M}_{J\to s_1+s_2} = \langle \mathbf{k} s_1 \lambda_1, -\mathbf{k} s_2 \lambda_2 \,|\, T_{J\to s_1+s_2} \,|\, J\lambda \rangle$$

Final state in terms of eigenstates of total angular momentum:

$$|\mathbf{k} s_1 \lambda_1, -\mathbf{k} s_2 \lambda_2\rangle = 4\pi \sqrt{\frac{\sqrt{s}}{|\mathbf{k}|}} \sum_{J\lambda} \sqrt{\frac{2J+1}{4\pi}} D^J_{\lambda,\lambda_1-\lambda_2}(\Omega) |J\lambda; \lambda_1\lambda_2\rangle$$

Wigner matrix

$$\mathcal{M}_{J\to s_1+s_2} = \sqrt{\frac{2J+1}{4\pi}} D^J_{\lambda,\lambda_1-\lambda_2}(\Omega)^* F^J_{\lambda_1\lambda_2}$$

helicity amplitude

Decay density matrix:

$$\rho^{\text{dec}}_{\lambda',\lambda} = \sum_{\lambda_1\lambda_2} \frac{2J+1}{4\pi} |F^J_{\lambda_1\lambda_2}|^2 D^J_{\lambda',\lambda_1-\lambda_2}(\Omega) D^J_{\lambda,\lambda_1-\lambda_2}(\Omega)^*$$

$\rho \to \pi^+\pi^-$

$\rho \to \pi\pi\colon\ \lambda_1 = \lambda_2 = 0,\ J = 1$

$$\rho_{\lambda'\lambda}^{\rho\to\pi\pi} = \frac{3}{4\pi}|F|^2 D^1_{\lambda',0}(\Omega) D^1_{\lambda,0}(\Omega)^*$$

$$D^1_{1,0}(\Omega) = -\frac{1}{\sqrt{2}}\sin\theta e^{i\phi}$$
$$D^1_{1,0}(\Omega) = \cos\theta$$
$$D^1_{1,0}(\Omega) = \frac{1}{\sqrt{2}}\sin\theta e^{-i\phi}$$

$$\rho^{\text{dec}}_{\lambda',\lambda} = 2|\mathbf{p}|^2 \begin{pmatrix} 1-\cos^2\theta_\pi & \frac{\sqrt{2}}{2}\sin 2\theta_\pi e^{i\phi_\pi} & -\sin^2\theta_\pi e^{2i\phi_\pi} \\ \frac{\sqrt{2}}{2}\sin 2\theta_\pi e^{-i\phi_\pi} & 2\cos^2\theta_\pi & -\frac{\sqrt{2}}{2}\sin 2\theta_\pi e^{i\phi_\pi} \\ -\sin^2\theta_\pi e^{-2i\phi_\pi} & -\frac{\sqrt{2}}{2}\sin 2\theta_\pi e^{-i\phi_\pi} & 1-\cos^2\theta_\pi \end{pmatrix}$$

$\rho \to e^+ e^-$

$$\rho^{\rm dec}_{\lambda',\lambda} = 4|\mathbf{p}|^2 \begin{pmatrix} 1+\cos^2\theta_e+\alpha & -\frac{\sqrt{2}}{2}\sin 2\theta_e e^{i\phi_e} & \sin^2\theta_e e^{2i\phi_e} \\ -\frac{\sqrt{2}}{2}\sin 2\theta_e e^{-i\phi_e} & 2(1-\cos^2\theta_e)+\alpha & \frac{\sqrt{2}}{2}\sin 2\theta_e e^{i\phi_e} \\ \sin^2\theta_e e^{-2i\phi_e} & \frac{\sqrt{2}}{2}\sin 2\theta_e e^{-i\phi_e} & 1+\cos^2\theta_e+\alpha \end{pmatrix}$$

angular distribution:

$$\sum_{\rm pol}|\mathcal{M}|^2 \propto (1+\cos^2\theta_e)(\rho^{\rm had}_{-1,-1}+\rho^{\rm had}_{1,1}) + 2(1-\cos^2\theta_e)\rho^{\rm had}_{0,0}$$
$$+\sin^2\theta_e(e^{2i\phi_e}\rho^{\rm had}_{-1,1}+e^{-2i\phi_e}\rho^{\rm had}_{1,-1})$$
$$+\sqrt{2}\cos\theta_e\sin\theta_e\left[e^{i\phi_e}(\rho^{\rm had}_{-1,0}+\rho^{\rm had}_{0,1})+e^{-i\phi_e}(\rho^{\rm had}_{1,0}+\rho^{\rm had}_{0,-1})\right]$$

$\rho \to e^+e^-$

angular distribution:

$$\frac{d\sigma}{d\Omega_e} \propto 4|\mathbf{p}|^2 \mathcal{N}_e(1 + \lambda_\theta \cos^2\theta_e + \lambda_e \sin^2\theta_e \cos 2\phi_e + \lambda_{\theta\phi} \sin 2\theta_e \cos\phi_e$$
$$+ \lambda_\phi^\perp \sin^2\theta_e \sin 2\phi_e + \lambda_{\theta\phi}^\perp \sin 2\theta_e \sin\phi_e).$$

anisotropy coefficients:

$$\lambda_\theta = \frac{1}{\mathcal{N}_e}(\rho_{-1,-1}^{\text{prod}} + \rho_{+1,+1}^{\text{prod}} - 2\rho_{0,0}^{\text{prod}}),$$

$$\lambda_\phi = \frac{1}{\mathcal{N}_e} 2\text{Re}\,(\rho_{-1,+1}^{\text{prod}}),$$

$$\lambda_{\theta\phi} = \frac{1}{\mathcal{N}_e}\sqrt{2}\text{Re}\,(\rho_{0,+1}^{\text{prod}} - \rho_{-1,0}^{\text{prod}}),$$

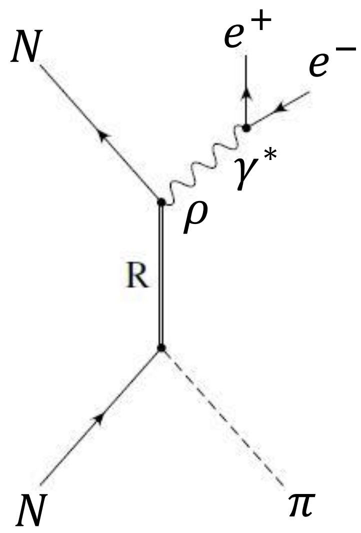
$$\lambda_\phi^\perp = \frac{1}{\mathcal{N}_e} 2\text{Im}\,(\rho_{-1,+1}^{\text{prod}}),$$

$$\lambda_{\theta\phi}^\perp = \frac{1}{\mathcal{N}_e}\sqrt{2}\text{Im}\,(\rho_{0,+1}^{\text{prod}} - \rho_{-1,0}^{\text{prod}}),$$

normalization:

$$\mathcal{N}_e = \rho_{-1,-1}^{\text{prod}} + \rho_{+1,+1}^{\text{prod}} + 2\rho_{0,0}^{\text{prod}} + \alpha(\rho_{-1,-1}^{\text{prod}} + \rho_{+1,+1}^{\text{prod}} + \rho_{0,0}^{\text{prod}})$$

$\pi N \to N e^+ e^-$



$\rho^{\rm prod}$ calculated from an effective lagrangian

Consistent interactions for higher spin resonances:

invariance under

$$\psi_\mu \to \psi_\mu + i\partial_\mu \chi \qquad \text{spin-3/2}$$

$$\psi_{\mu\nu} \to \psi_{\mu\nu} + \frac{i}{2}(\partial_\mu \chi_\nu - \partial_\nu \chi_\mu) \qquad \text{spin-5/2}$$

Gauge invariant operators:

$$G_{\mu,\nu} = i(\partial_\mu \psi_\nu - \partial_\nu \psi_\mu)$$

$$G_{\mu\nu,\lambda\rho} = -\partial_\mu \partial_\nu \psi_{\lambda\rho} - \partial_\lambda \partial_\rho \psi_{\mu\nu} + \frac{1}{2}(\partial_\mu \partial_\lambda \psi_{\nu\rho} + \partial_\mu \partial_\rho \psi_{\nu\lambda} + \partial_\nu \partial_\lambda \psi_{\mu\rho} + \partial_\nu \partial_\rho \psi_{\mu\lambda})$$

new fields:

$$\Psi_\mu = \gamma^\nu G_{\mu,\nu} \qquad \Psi_{\mu\nu} = \gamma^\lambda \gamma^\rho G_{\mu\nu,\lambda\rho}$$

$\pi N \to N e^+ e^-$

$$\mathcal{L}^1_{R_{3/2}N\rho} = \frac{ig_1}{4m_N^2}\bar{\Psi}^\mu_R \vec{\tau}\gamma^\nu\tilde{\Gamma}\psi_N \cdot \vec{\rho}_{\nu\mu}$$

$$\mathcal{L}^2_{R_{3/2}N\rho} = \frac{ig_2}{8m_N^3}\bar{\Psi}^\mu_R \vec{\tau}\tilde{\Gamma}\partial^\nu\psi_N \cdot \vec{\rho}_{\nu\mu}$$

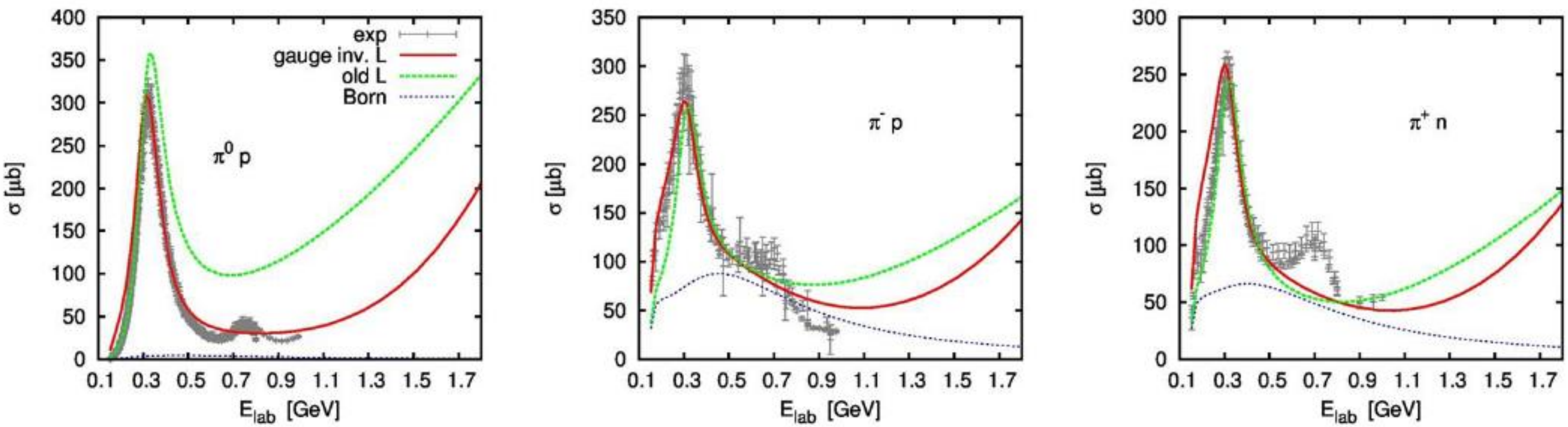
$$\mathcal{L}^3_{R_{3/2}N\rho} = \frac{ig_3}{8m_N^3}\bar{\Psi}^\mu_R \vec{\tau}\tilde{\Gamma}\psi_N \cdot \partial^\nu\vec{\rho}_{\nu\mu}$$

$$\mathcal{L}^1_{R_{5/2}N\rho} = -\frac{g_1}{(2m_N)^4}\bar{\Psi}^{\mu\nu}_R \vec{\tau}\tilde{\Gamma}\gamma^\rho(\partial_\mu\psi_N) \cdot \vec{\rho}_{\rho\nu}$$

$$\mathcal{L}^2_{R_{5/2}N\rho} = -\frac{g_2}{(2m_N)^5}\bar{\Psi}^{\mu\nu}_R \vec{\tau}\tilde{\Gamma}\partial^\rho(\partial_\mu\psi_N) \cdot \vec{\rho}_{\rho\nu}$$

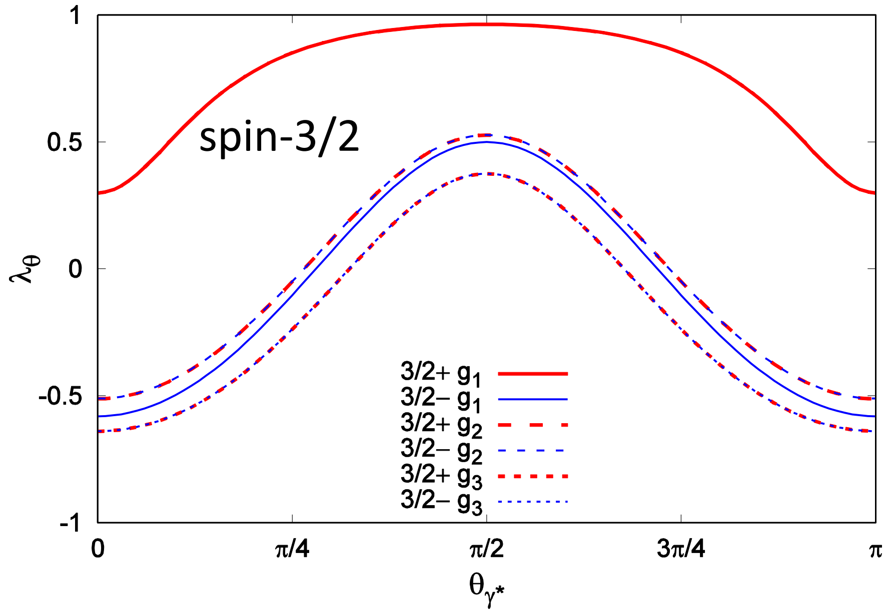
$$\mathcal{L}^3_{R_{5/2}N\rho} = -\frac{g_3}{(2m_N)^5}\bar{\Psi}^{\mu\nu}_R \vec{\tau}\tilde{\Gamma}(\partial_\mu\psi_N) \cdot \partial^\rho\vec{\rho}_{\rho\nu}$$

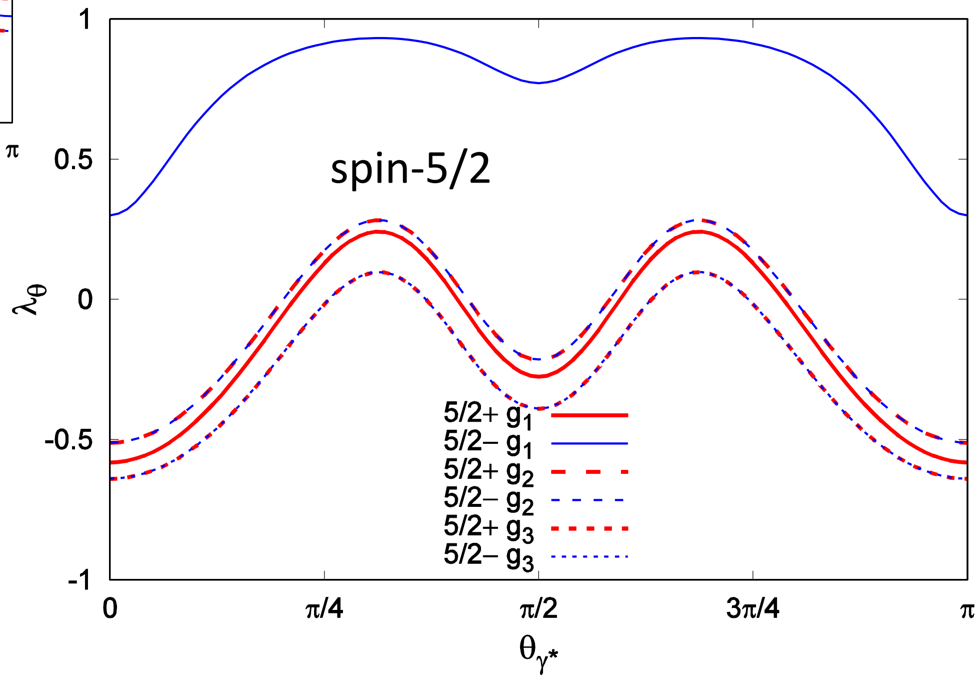
Consistent interactions in pion photoproduction



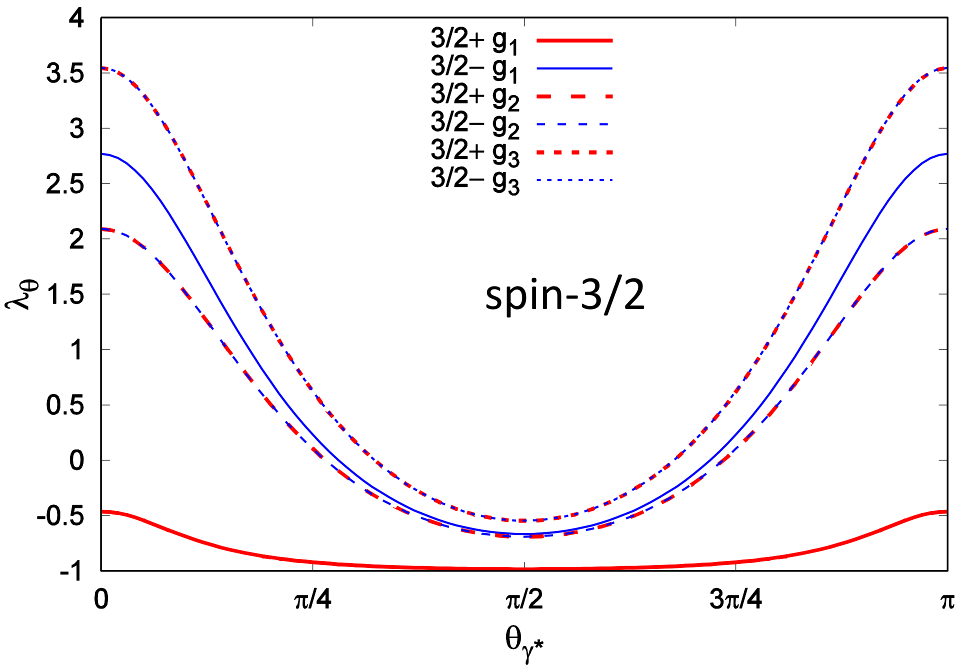
- Born + s- and u-channel Δ contribution
- u-channel violates unitarity → unphysical rise at high E_{lab}
- "gauge invariant" Lagrangian is better

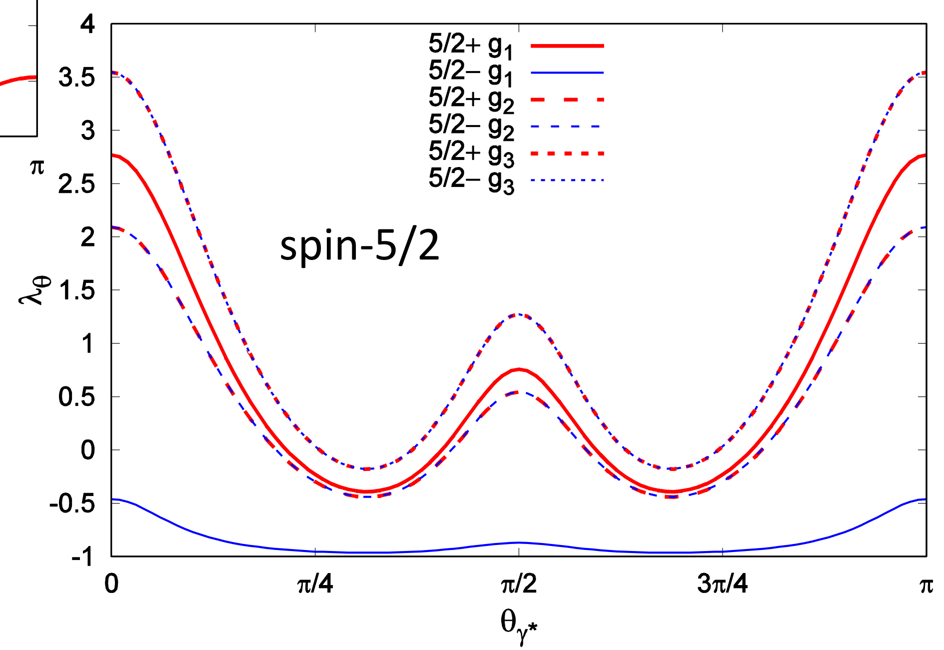
Anisotropy coefficient for e^+e^-





Anisotropy coefficient for $\pi^+\pi^-$

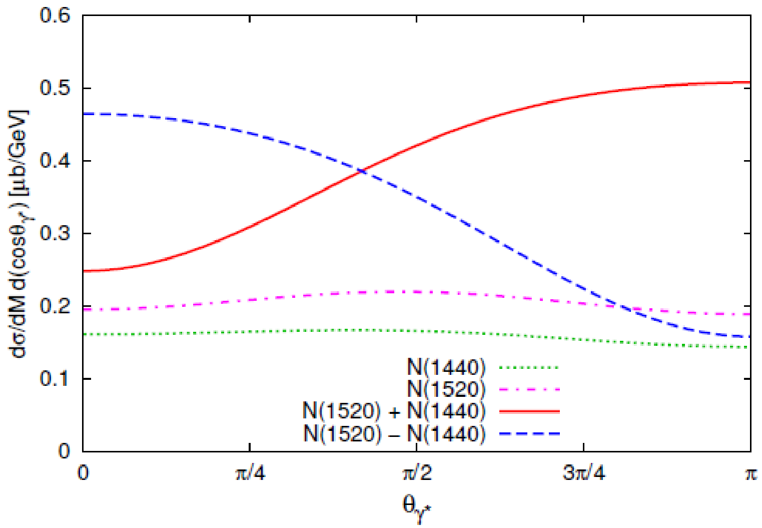




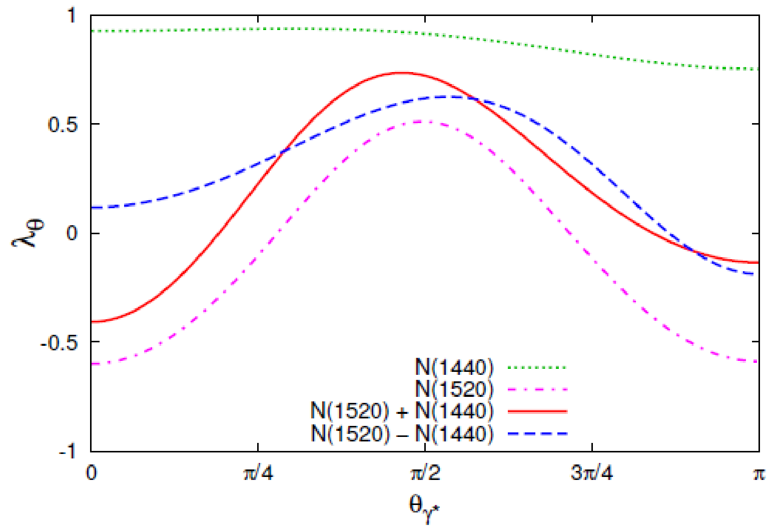
N(1440) and N(1520)

$\sqrt{s} = 1.49\ \text{GeV} \qquad M = 0.5\ \text{GeV}$

$|A_s^i + A_u^i|^2$



$|A_s^i + A_u^i|^2$



- Unknown relative phase between contributions of the two resonances
- λ_θ does not depend strongly on the relative phase

Conclusions

- The anisotropy coefficient λ_θ reflects the polarization state of the virtual photon
- Shape of $\lambda_\theta(\theta_{\gamma^*})$ as a function of θ_{γ^*} is characteristic of the spin-polarization of intermediate resonances
- Triple differential cross section is needed – attempt by HADES
- ρ contribution to $\pi^+\pi^-$ is related to e^+e^-

Still to do:

- Add nonresonant contributions
- Relate to helicity amplitudes
- Add sequential decays to $\pi^+\pi^-$ $(R \to R'\pi \to N\pi\pi)$