

Electromagnetic time like and space like transition form factor models

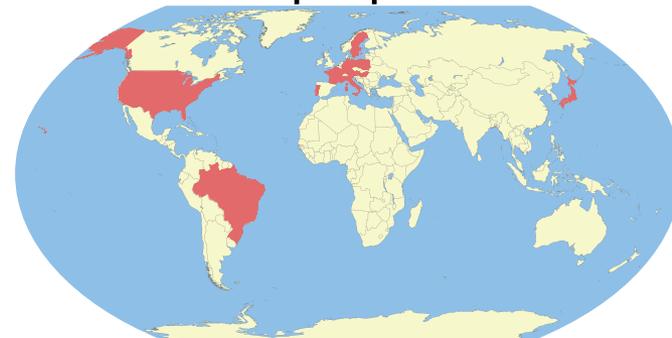
Teresa Peña (IST-ULisboa)
and
Gilberto Ramalho, LFTC,
Universidade Cruzeiro do Sul.



Kandisky "Circles in a circle" (1923)

Electromagnetic time like and space like transition form factor models

Exciting near-future prospects of high-quality data.
Expand our knowledge of the spectrum and structure properties of baryons.



Missing resonances?
Diquark clusters?

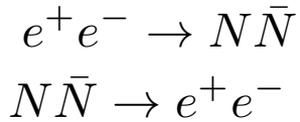
This talk:
Connect Timelike and Spacelike Transition Form Factors (TFF)
Baryon-photon coupling evolution with momentum transfer

Crossing the boundaries to explore baryon resonances

Timelike: $Q^2 < 0$

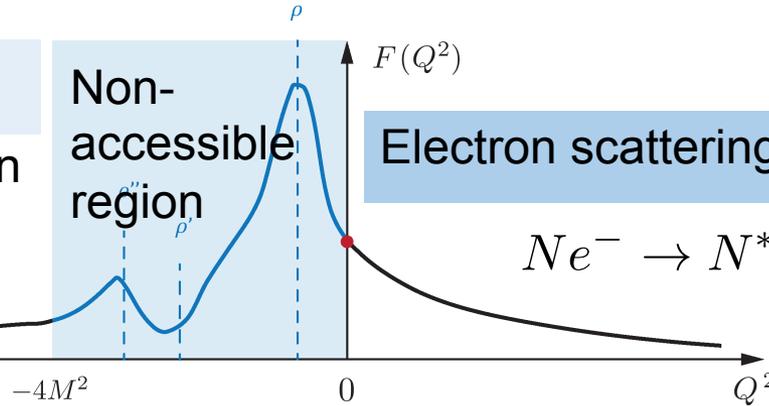
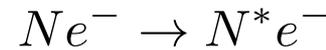
Spacelike: $Q^2 > 0$

Timelike physical region

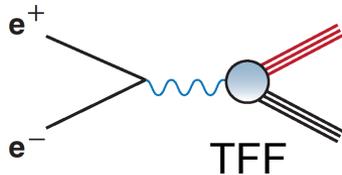


Non-accessible region

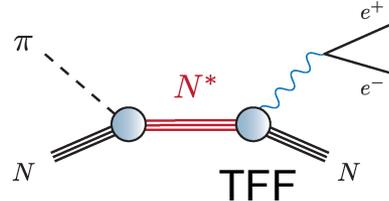
Electron scattering



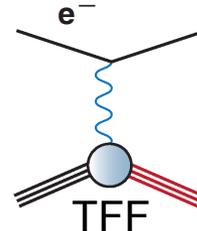
$$Q^2 = -q^2$$



BES III, BELLE II



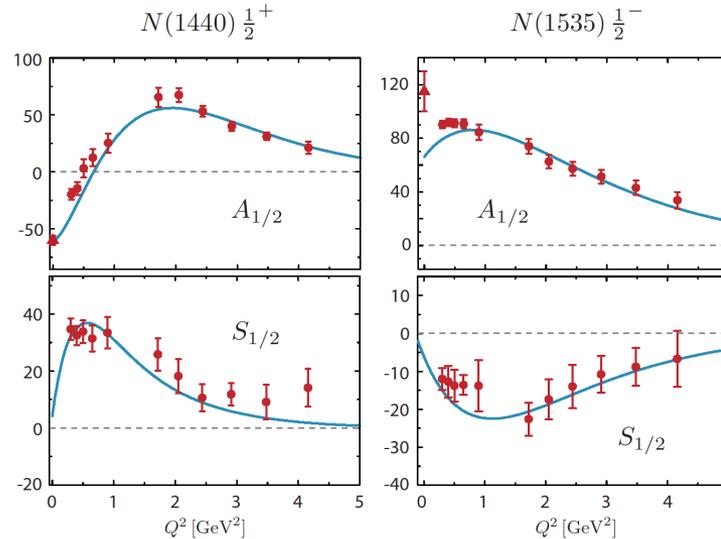
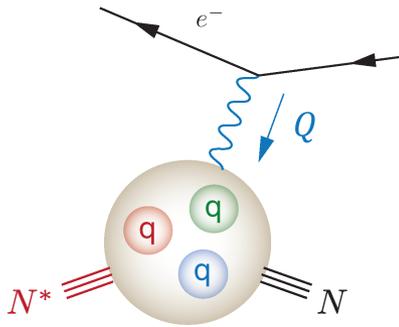
FAIR
HADES



JLab/CLAS:
most world data

CLAS/Jlab electron scattering data constrain interpretation of HADES dilepton production data, results have to match at the photon point

Transition form factors



Baryon resonances transition form factors

CLAS: Aznauryan et al.,
Phys. Rev. C 80 (2009)

MAID: Drechsel, Kamalov,
Tiator, EPJ A 34 (2009)

Spacelike form factors:

- Structure information: shape, qq \bar{q} excitation vs. hybrid, ...
- Evolution of quark-photon coupling with momentum transfer

Timelike form factors:

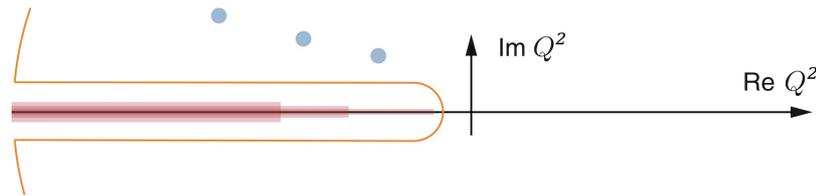
- Particle production channels: vector mesons at small q^2
- Test of vector meson dominance
- Input for the description of in-medium dilepton production

Information on **baryon spectroscopy** needed to extract information in both cases!

Theoretical toolkits

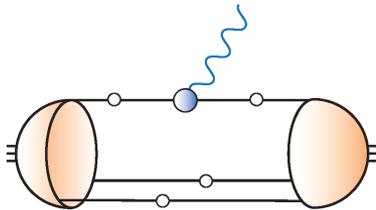
Timelike baryon transition form factors not yet within reach in lattice QCD:
explore alternative methods, estimate theory uncertainty!

- **Analyticity**

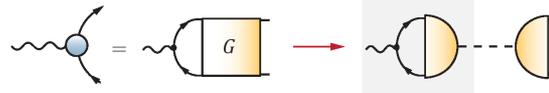


→ Dispersion theory

- **Dynamics**

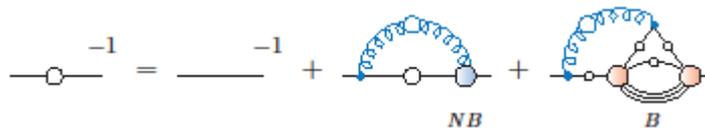


Quark-photon coupling
dynamically generates VM poles!

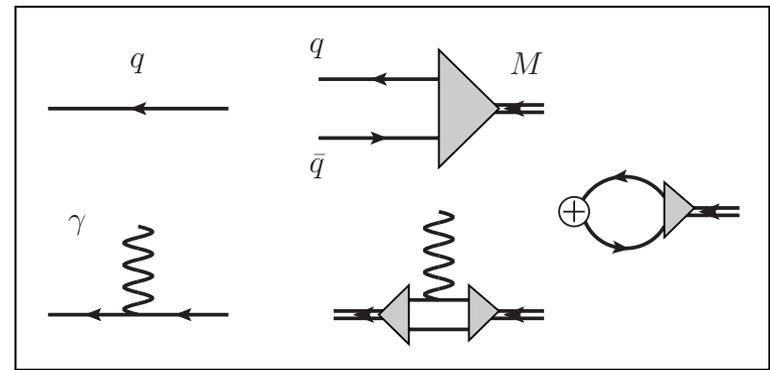


→ Dyson-Schwinger eqs.
→ Effective Lagrangian models
→ Quark models
→ Vector-meson dominance

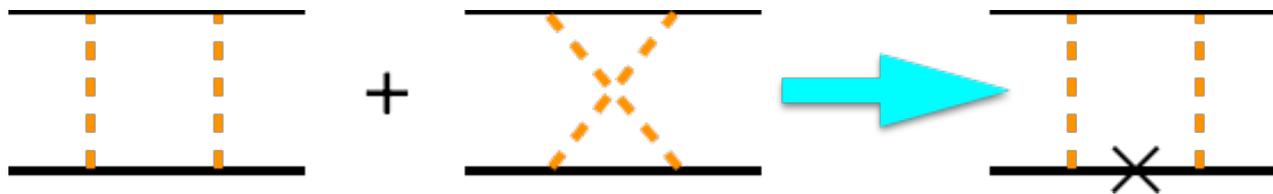
- **Medium effects**



→ In-medium description of
resonances!



- Formulation in Minkowski space.
- Two-body CST equation effectively sums ladder and crossed-ladder exchange diagrams, due to cancelations.

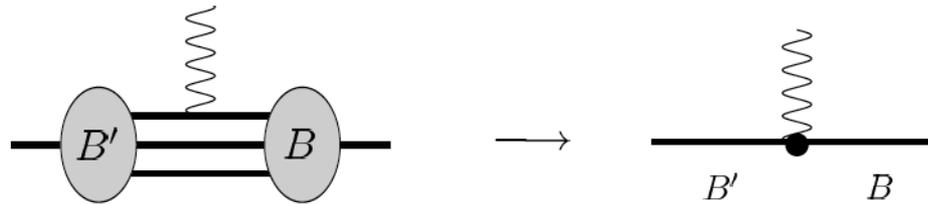


- Provides wave functions from covariant vertex with simple transformation properties under Lorentz boosts, appropriate angular momentum structures and smooth non-relativistic limit.
- Manifestly covariant, but only three-dimensional loop integrations.
- Ensures confinement and dynamical chiral symmetry breaking.

Outline

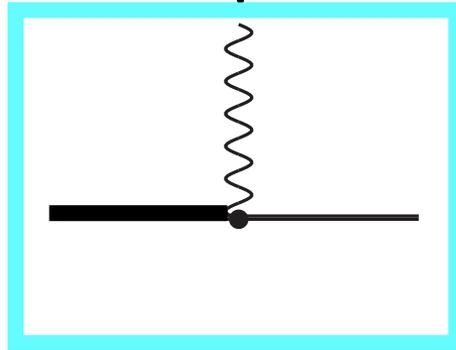
- 1 Evidence of separation of partonic and hadronic (pion cloud) effects?
- 2 Space-like e.m. transitions FFs extension to Time-like e.m. transitions FFs
 $\Delta(1232)$ and $N^*(1520)$ cases.
- 3 Predictions for dilepton mass spectrum and decay widths

Bare quark and pion cloud components

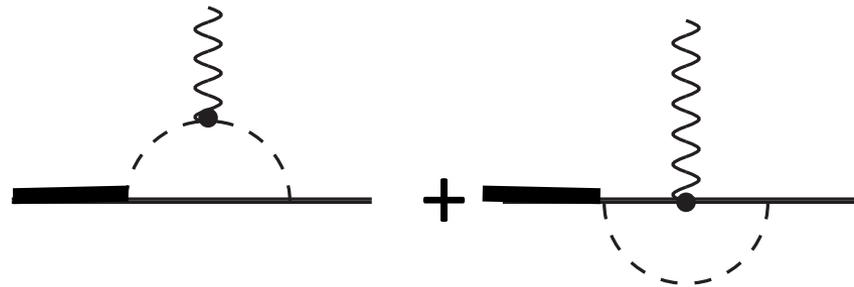


For low Q^2 : add coupling with pion in flight.

Bare quark



Pion cloud



Bare Quark component Pion cloud suppressed for high Q^2 $\frac{1}{Q^8}$

Model dependence?

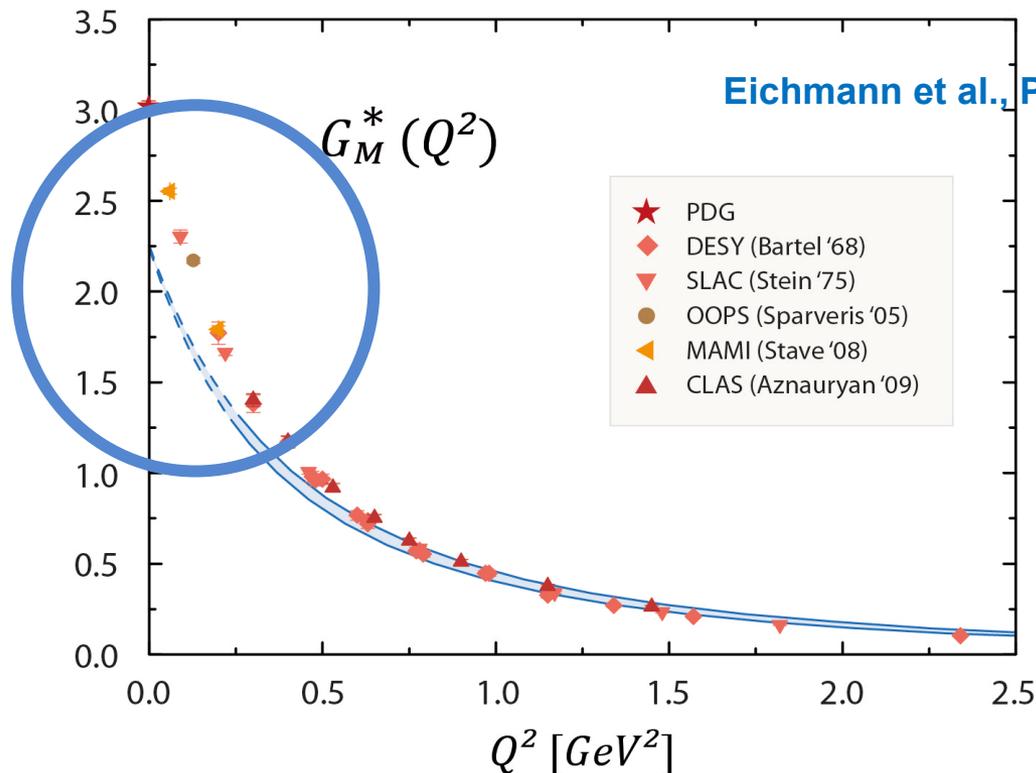
Model independent feature

$$\gamma N \rightarrow \Delta$$

$$|G_M^* = G_M^B + G_M^\pi$$

Separation seems to be supported by experiment.

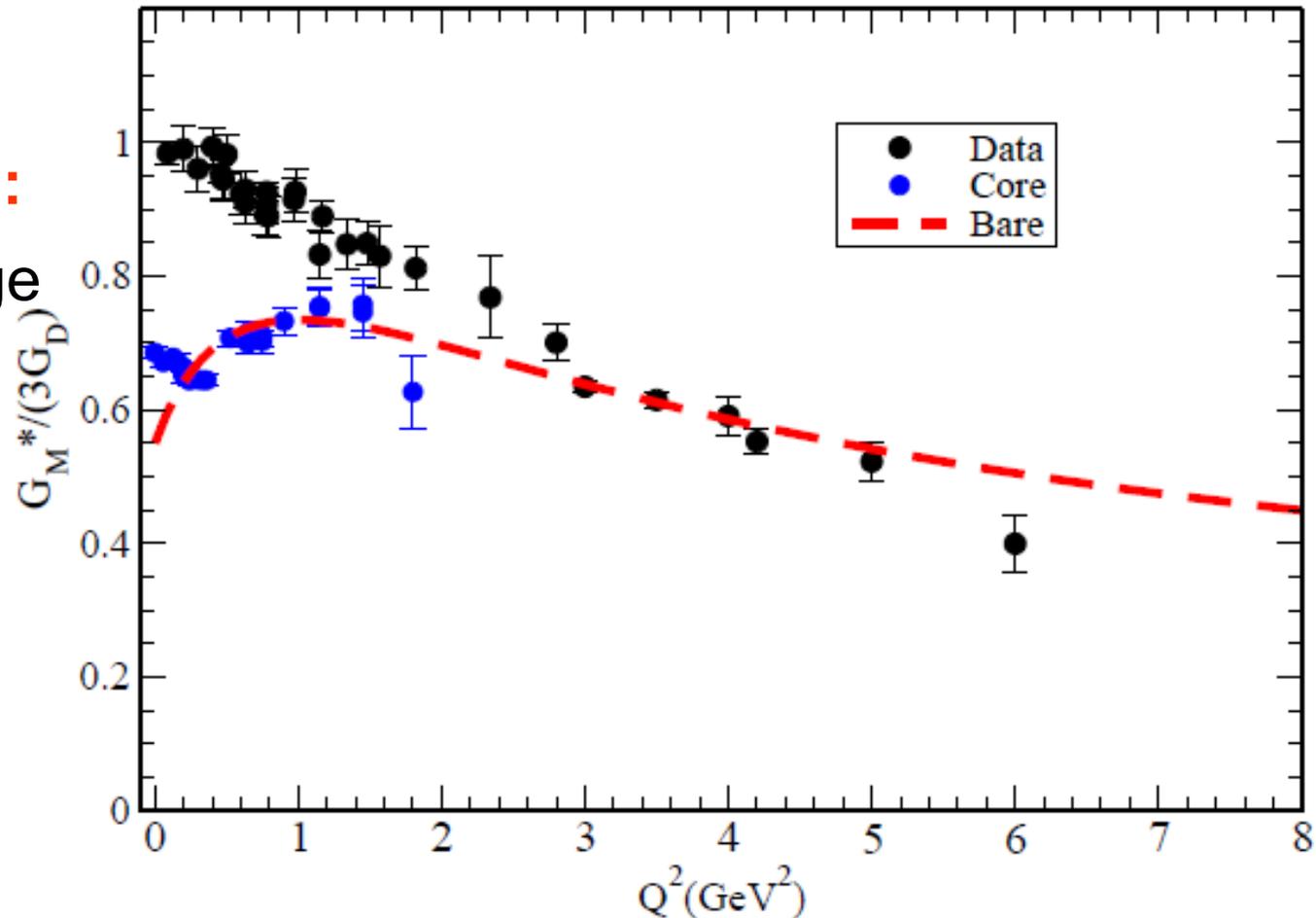
Missing strength of G_M at the origin is an universal feature, even in dynamical quark calculations



Effect of vicinity of the mass pole of the Delta to the pion-nucleon threshold.

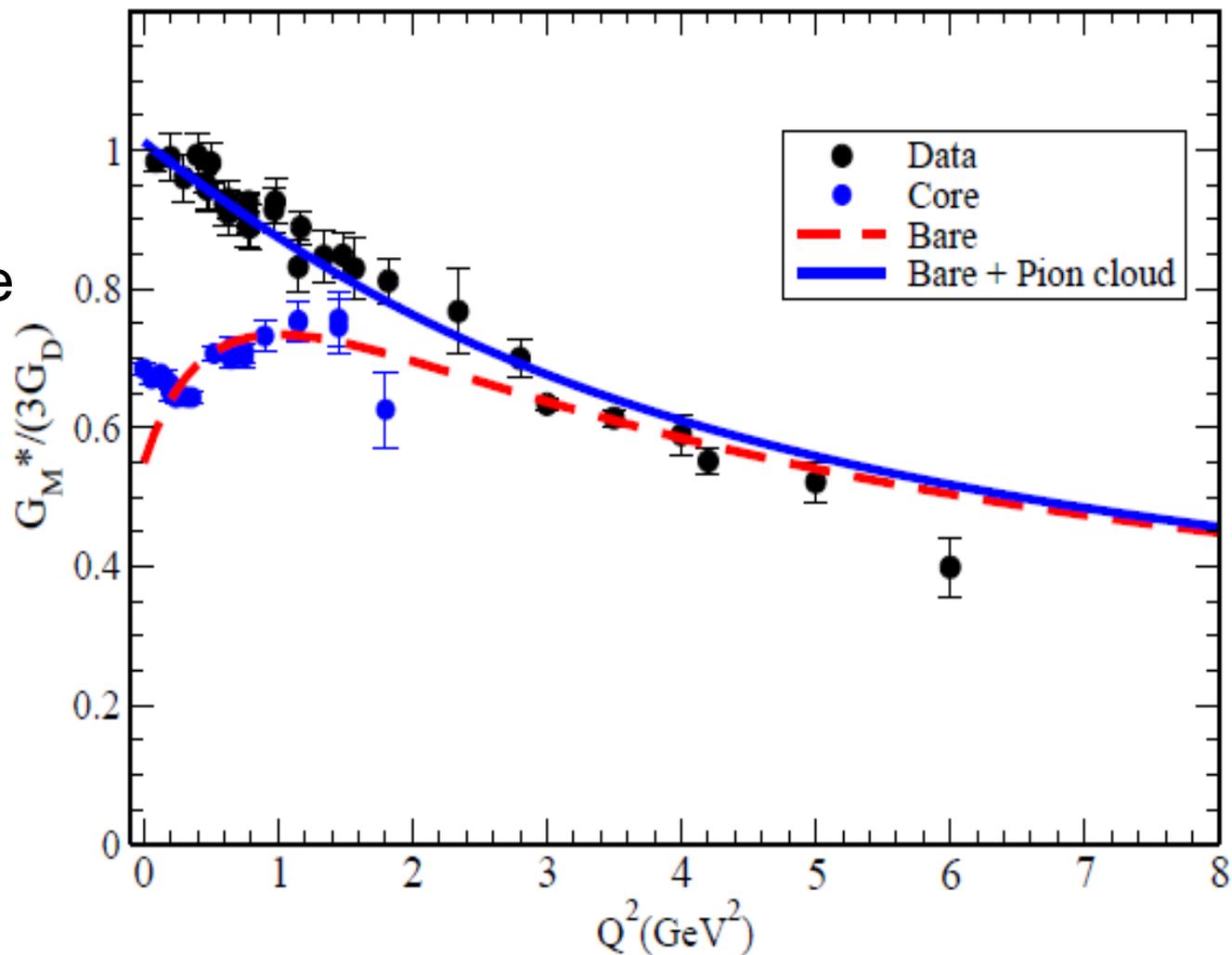
CST[©]**Bare quark core:**

- dominates large Q^2 region.
- agrees with EBAC analysis.

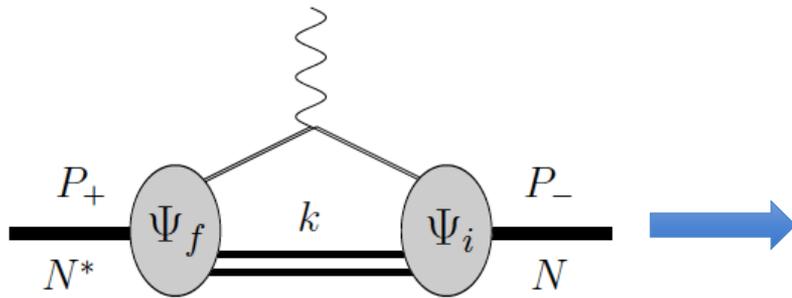


Bare quark core:

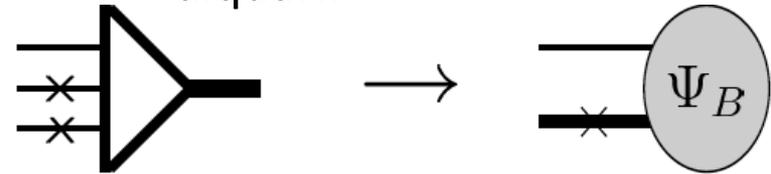
- dominates large Q^2 region.
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E.M. matrix element



Integrate in the relative momentum of the diquark



$$\int_{k_1 k_2} \equiv \int \frac{d^4 k_1 d^4 k_2}{(2\pi)^6} \delta_+(m_1^2 - k_1^2) \delta_+(m_2^2 - k_2^2)$$

$$= \int \frac{d^3 k_1 d^3 k_2}{(2\pi)^6 4E_1 E_2},$$

$$\int_{sk} = \underbrace{\int \frac{d\Omega_{\hat{\mathbf{r}}}}{4(2\pi)^3} \int_{4m_q^2}^{\infty} ds \sqrt{\frac{s - 4m_q^2}{s}}}_{\int_s} \underbrace{\int \frac{d^3 k}{(2\pi)^3 2E_s}}_{\int_k}$$

- **Baryon wavefunction** reduced to an effective quark-diquark structure due to the \$s\$ integration.
- **E.M.** matrix element can be written in terms of an effective baryon composed by an off-mass-shell quark and an on-mass-shell quark pair (diquark) with an average mass.

Wave functions

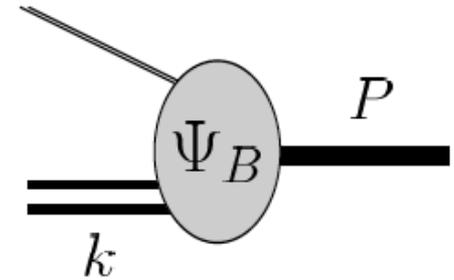
Nucleon wave function

- A quark+ scalar-diquark component
- A quark+ **axial vector**-diquark component

$$\Psi_{N\lambda_n}^S(P, k) = \frac{1}{\sqrt{2}} [\phi_I^0 u_N(P, \lambda_n) - \phi_I^1 \varepsilon_{\lambda P}^{\alpha*} U_\alpha(P, \lambda_n)]$$

$$\times \psi_N^S(P, k).$$

Phenomenological function



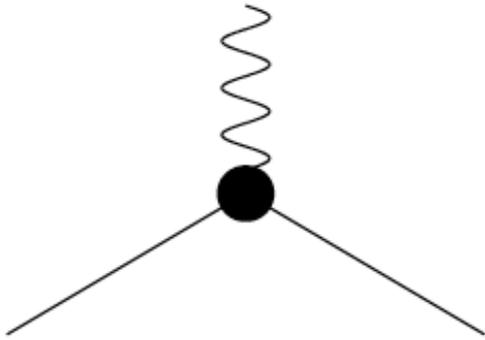
$$U_\alpha(P, \lambda_n) = \frac{1}{\sqrt{3}} \gamma_5 \left(\gamma_\alpha - \frac{P_\alpha}{m_H} \right) u_N(P, \lambda_n),$$

Delta wave function

- A quark+ only **axial vector**-diquark term contributes

$$\Psi_\Delta^S(P, k) = -\psi_\Delta^S(P, k) \tilde{\phi}_I^1 \varepsilon_{\lambda P}^{\beta*} w_\beta(P, \lambda_\Delta)$$

E.M. Current



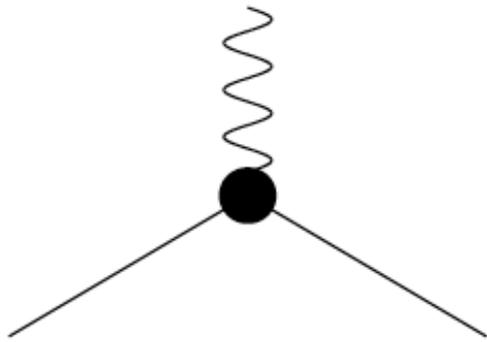
quark-antiquark

⊕ gluon dressing

Constituent quarks (quark form factors)

$$j_I^\mu = \left[\frac{1}{6} f_{1+} + \frac{1}{2} f_{1-\tau_3} \right] \gamma^\mu + \left[\frac{1}{6} f_{2+} + \frac{1}{2} f_{2-\tau_3} \right] \frac{i\sigma^{\mu\nu} q_\nu}{2M_N}$$

E.M. Current



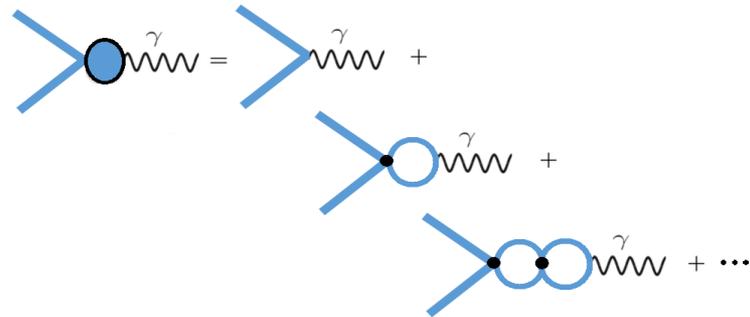
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To parametrize the current use **VMD**, a truncation to the ρ pole of the full meson spectrum contribution to the quark-photon coupling.



$$\Gamma_\mu(p, Q) = \gamma_\mu + \int \frac{d^4 q}{(2\pi)^4} K(p, q, Q) S(q + \eta Q) \Gamma_\mu(q, Q) S(q - \eta Q)$$

4 parameters

VMD as link to LQCD

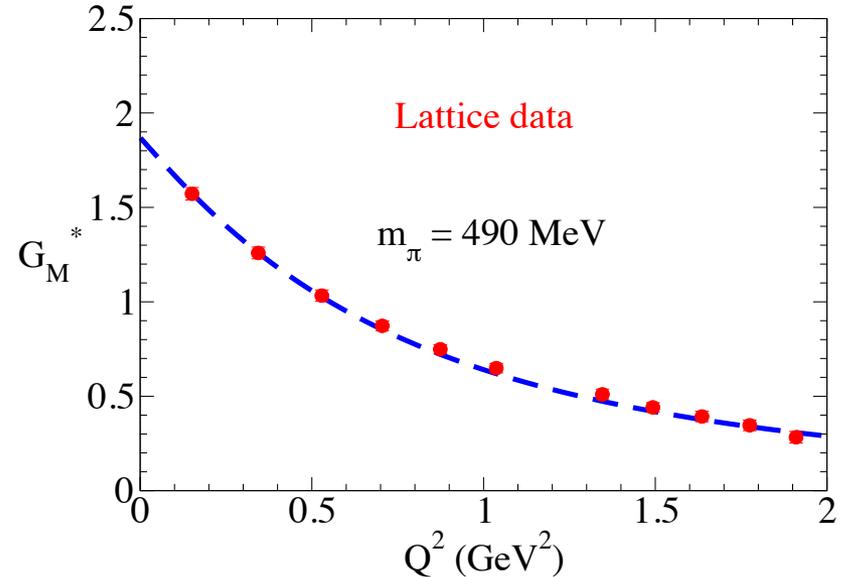
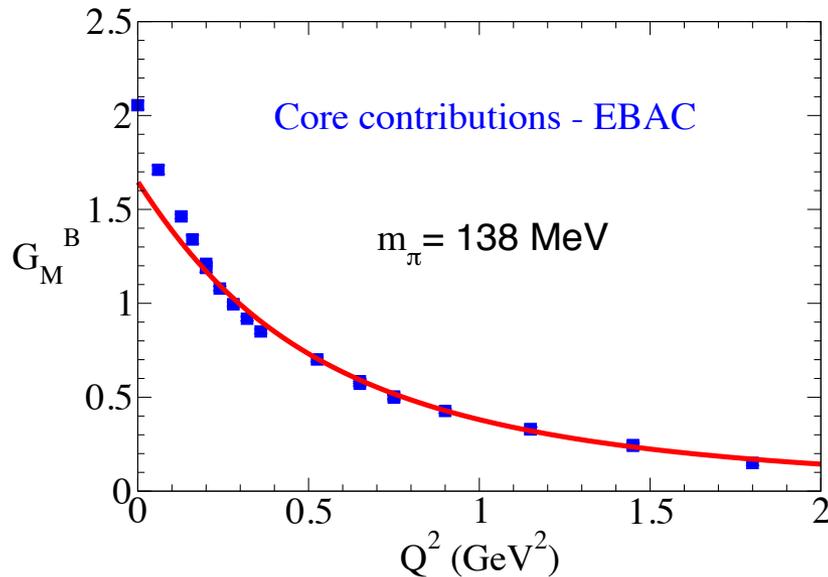
- VMD enables link to LQCD: in the current the vector meson mass is taken a function of the running pion mass.
- Pion cloud contribution negligible for **large pion masses**, and **bare** quark model could be calibrated to the **lattice data**.
- After that, in the limit of the model to the physical pion mass value, the **experimental data** is well described in the large Q^2 region.

Connection to Lattice QCD

To control model dependence:

CST model and LQCD data are made **compatible**.

G. Ramalho and M. T. Peña, Phys. Rev. D 80, 013008 (2009)



Model (no pion cloud) valid for lattice pion mass regime.
No refit of parameters for the physical pion mass limit.

E.M. Current and TFF at the photon point

$$\gamma N \rightarrow \Delta$$

$$\Gamma^{\beta\mu}(P, q) = G_1 q^\beta \gamma^\mu + G_2 q^\beta P^\mu + G_3 q^\beta q^\mu - G_4 g^{\beta\mu}$$

- Only 3 G_i are independent: $q^\mu \Gamma_{\beta\mu} = 0$
E.M. Current has to be conserved



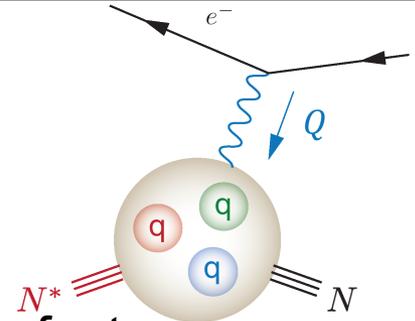
G_M, G_E, G_C Scadron Jones popular choice.

- Only finite G_i are physically acceptable.

Orthogonality between initial and final states necessarily follows from both requirements, giving an important constraint to G_C at $Q^2=0$.

E.M. Current and TFF near the photon point

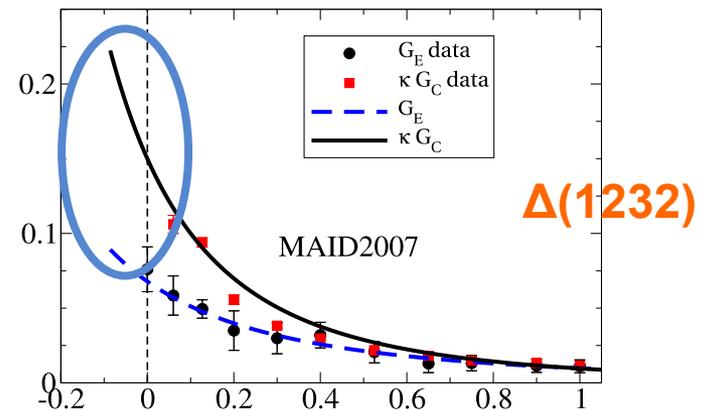
Pseudo Threshold PT $q_0^2 = -(M_R - M_N)^2 ; |q| = 0$



An accident of the definition of the Jones and Scadron form factors:

$$G_E(PT) = \frac{M_R - M}{2M_R} G_C(PT)$$

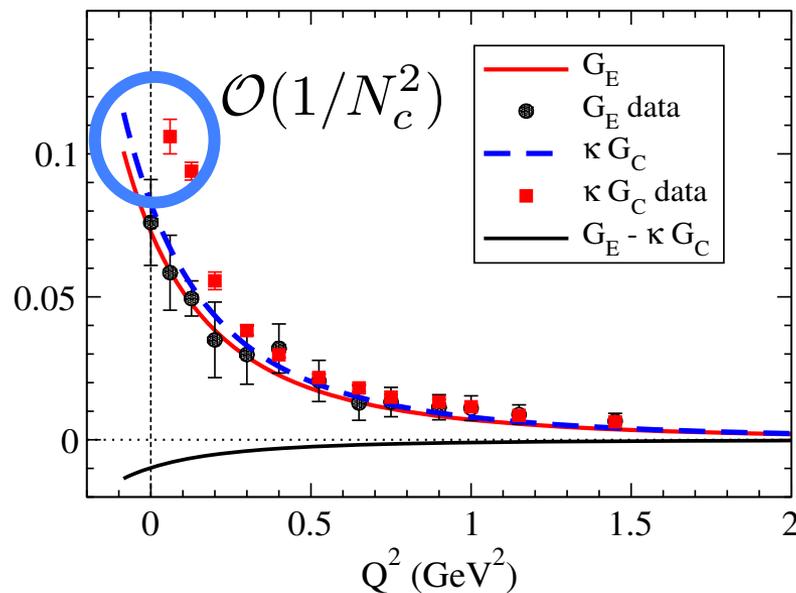
A form of the “Siegert condition”!
 This is implied by orthogonality of states.
 If data analysis proceed through helicity
 Amplitudes this behavior may be missed.



Large N_C limit and SU(6) quark models:

- Suggest that pion cloud effects dominate G_E and G_C

Those effects generate deviations from the Siegert condition of the order $\mathcal{O}(1/N_C^2)$ and do not agree to data at low Q^2 .



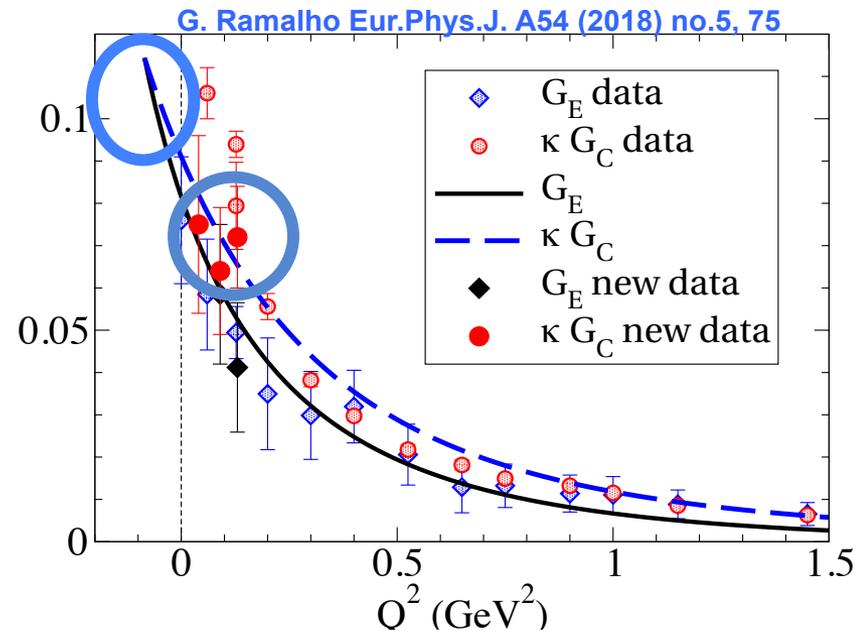
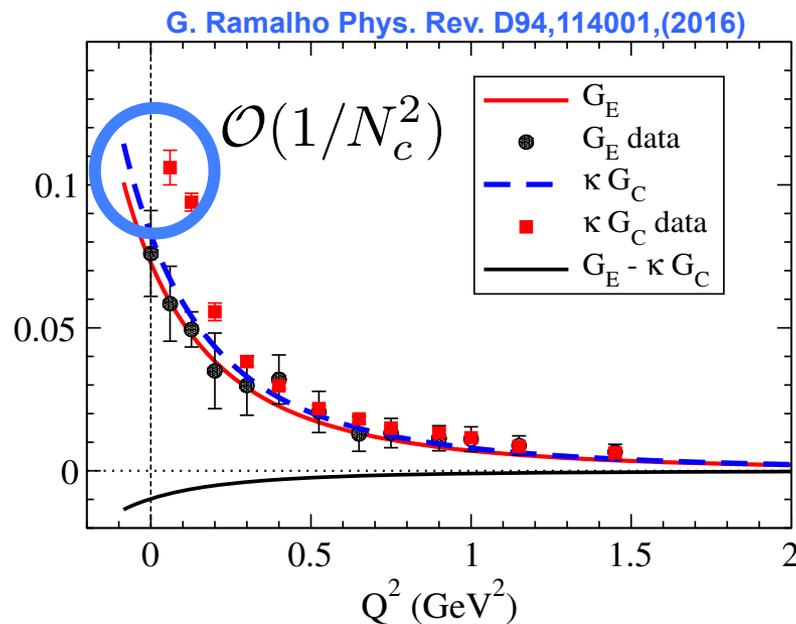
G_E and G_C

Large N_C limit and SU(6) quark models:

- Suggest that pion cloud effects dominate G_E and G_C

Those effects generate deviations from the Siegert condition of the order $\mathcal{O}(1/N_c^2)$

Corrected parametrization with deviations $\mathcal{O}(1/N_c^4)$ generated agreement with 2017 JLAB data

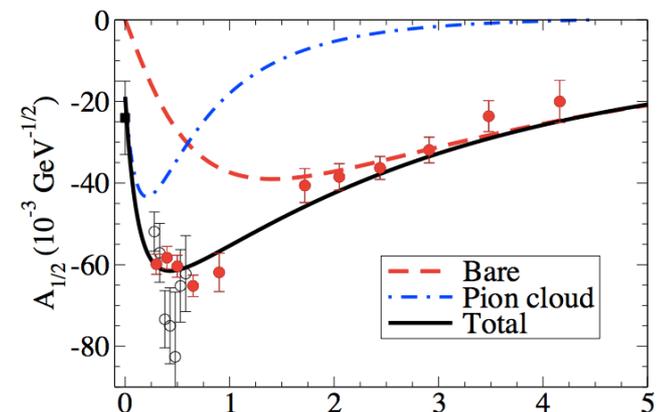
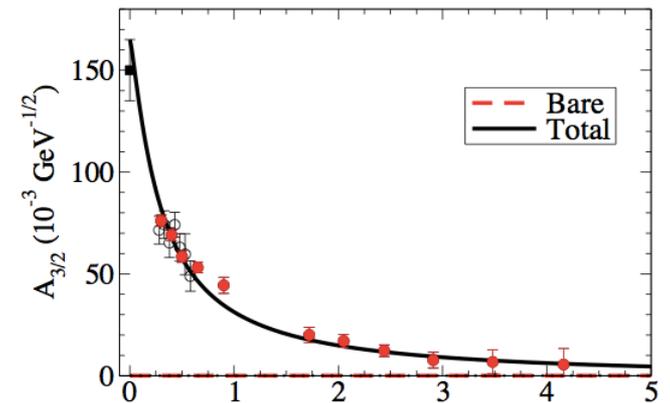
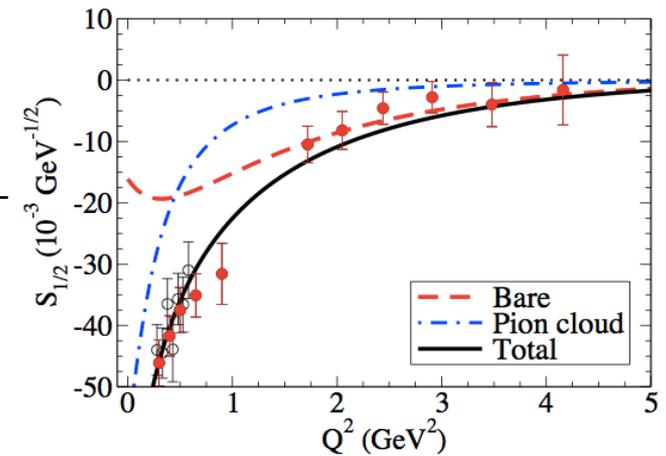


$N \rightarrow N^*(1520)$ Helicity amplitudes

$J^P=3/2^- \quad I=1/2$
60% decay to πN
30% decay to $\pi\pi N$

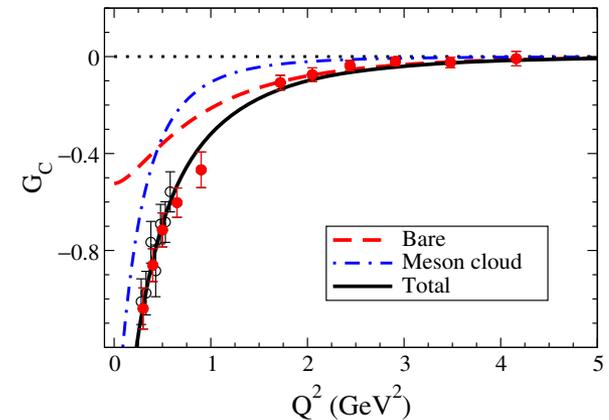
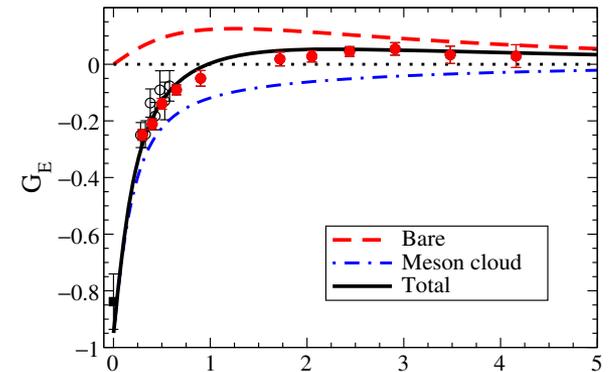
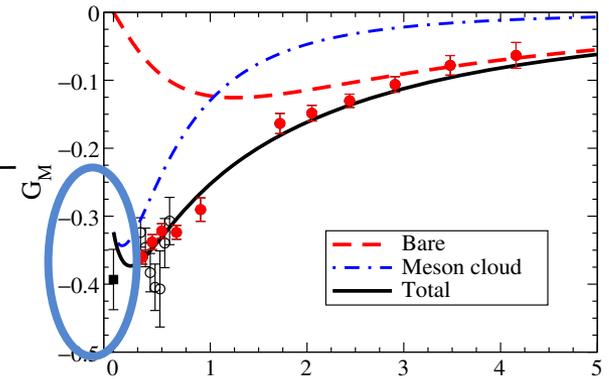
- Bare quark model gives good description of high Q^2 region
- $A_{1/2}$ dominates in the high Q^2 region. (Aznauryan and Burkert, PRC 85 055202 2012)

G. Ramalho, M. T. P., PHYSICAL REVIEW D 85 094016 (2014)



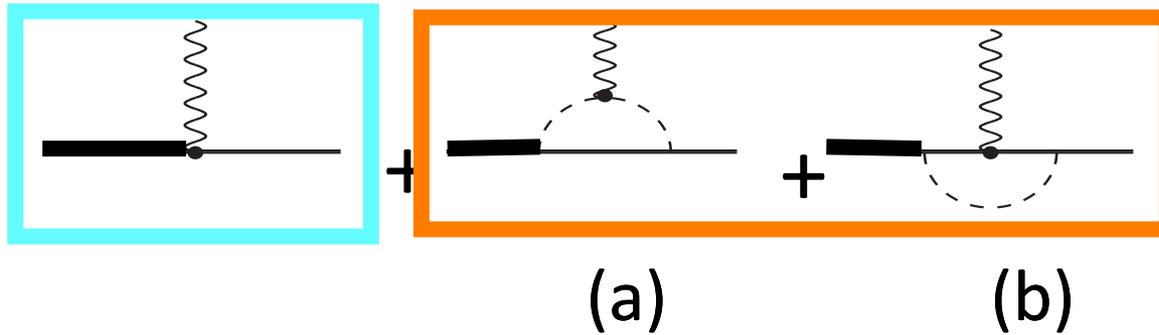
$N \rightarrow N^*(1520)$ TFFs

- Underestimation of G_M close to the photon point due to overall fit
- Important role of meson cloud



G. Ramalho, M. T. P., PHYSICAL REVIEW D 95 014003 (2017)

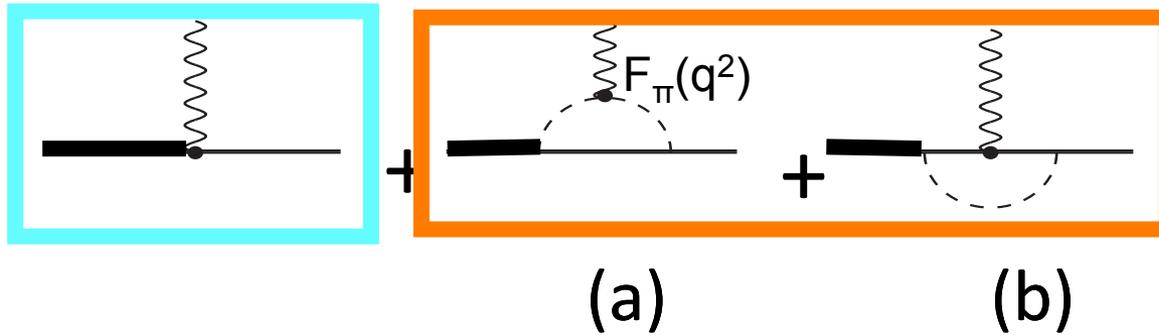
Extension to time like



The residue of the pion form factor $F_\pi(q^2)$ at the timelike ρ pole is proportional to the $\rho \rightarrow \pi\pi$ decay

(a) related with pion electromagnetic form factor $F_\pi(q^2)$

Extension to time like



The residue of the pion from factor $F_\pi(q^2)$ at the timelike ρ pole is proportional to the $\rho \rightarrow \pi\pi$ decay

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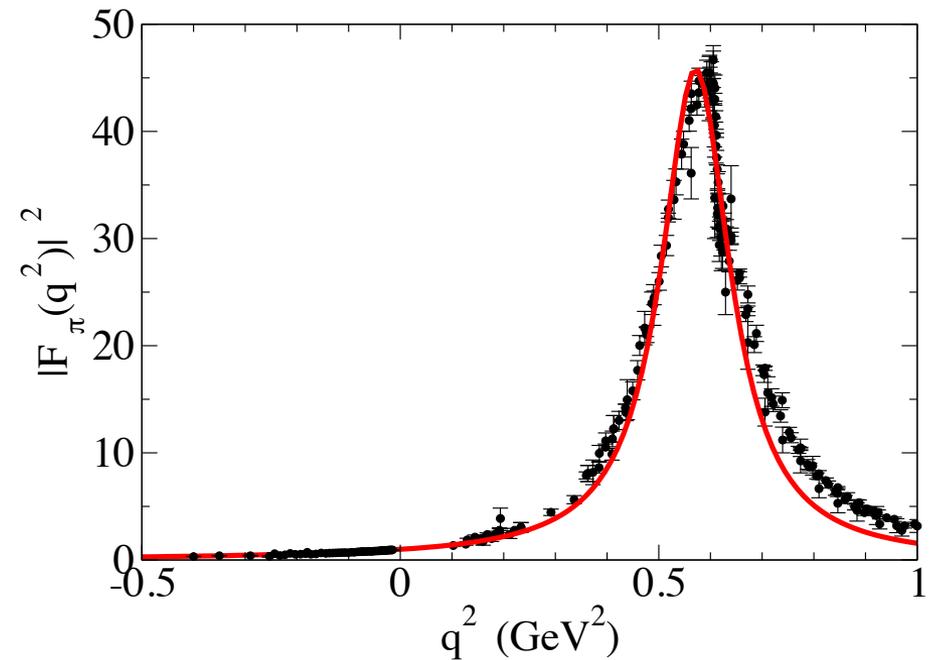
Extension to time like

Parametrization of pion Form Factor

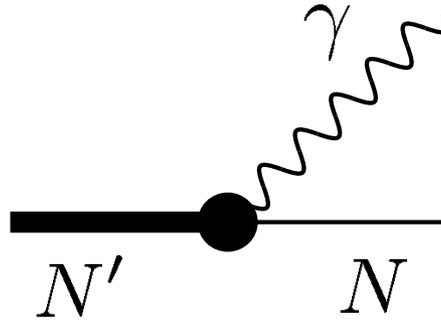
$$F_{\pi}(q^2) = \frac{\alpha}{\alpha - q^2 - \frac{1}{\pi} \beta q^2 \log \frac{q^2}{m_{\pi}^2} + i\beta q^2}$$

$$\alpha = 0.696 \text{ GeV}^2$$

$$\beta = 0.178$$



Extension to time like



R rest frame

$$P_R = (W, 0, 0, 0); \quad P_N = (E_N, 0, 0, -|\mathbf{q}|); \quad q = (\omega, 0, 0, |\mathbf{q}|)$$

Timelike $q^2 > 0$

$$\omega = \frac{W^2 - M^2 + q^2}{2W}$$

$$|\mathbf{q}|^2 = \frac{[(W + M) - q^2][(W - M)^2 - q^2]}{4W^2}$$

$$E_N = \frac{W^2 + M^2 - q^2}{2W}$$

$$\text{TL: } q^2 \leq (W - M)^2$$

Spacelike $-q^2 = Q^2 > 0$

$$\omega = \frac{W^2 - M^2 - Q^2}{2W}$$

$$|\mathbf{q}|^2 = \frac{[(W + M) + Q^2][(W - M)^2 + Q^2]}{4W^2}$$

$$E_N = \frac{W^2 + M^2 + Q^2}{2W}$$

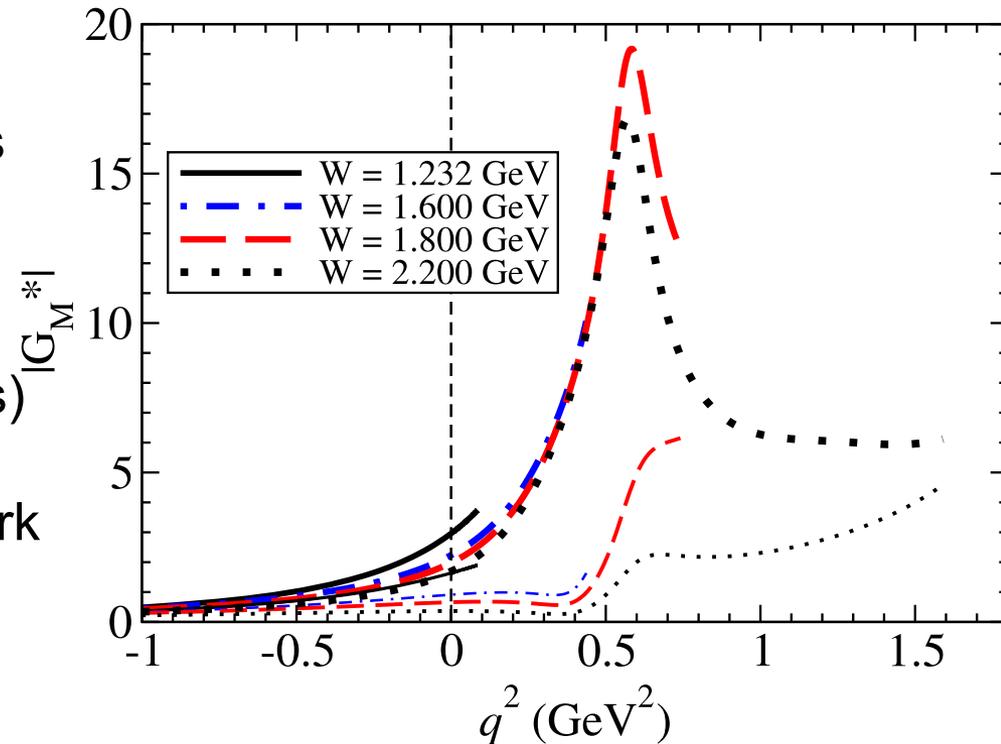
$$W \geq M$$

Transition form factors in the timelike region are restricted to a given kinematic region that depends on the resonance mass.

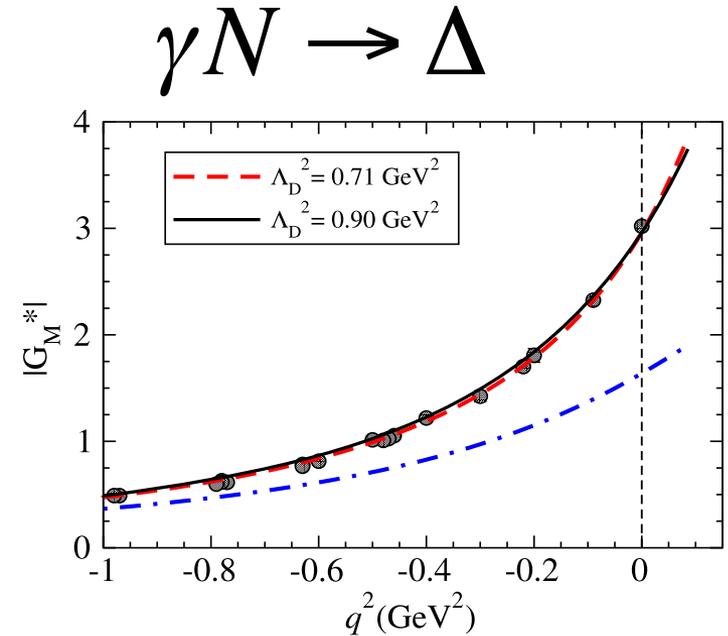
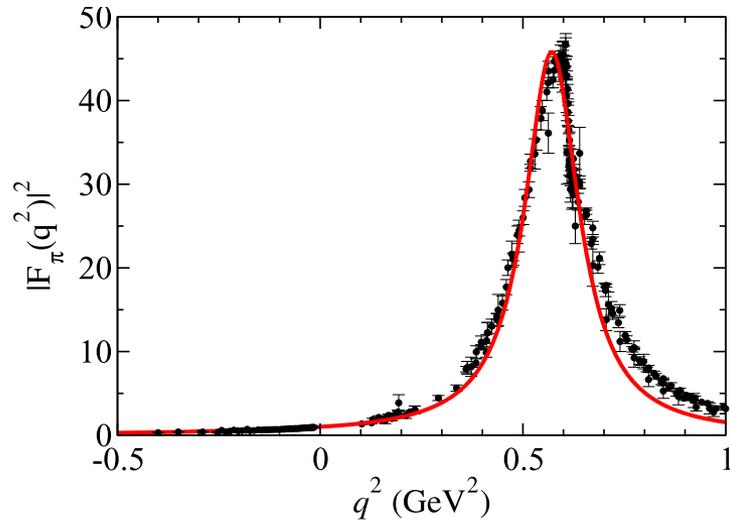
Extension to time like

$$\gamma N \rightarrow \Delta$$

- Extension for higher W shows effect of the rho mass pole
- In that pole region small bare quark contribution (thin lines)
- Away from that pole bare quark component dominates



Crossing the boundaries



Red line : full model
Blue line: Quark core

- Good description of the magnetic dipole physical data ($W = M_\Delta$)

$$\Gamma_{\gamma^* N}(q; W) = \frac{\alpha}{16} \frac{(W + M)^2}{M^2 W^3} \sqrt{y_+ y_-} |G_T(q^2, W)|^2$$

$$|G_T(q^2; M_\Delta)|^2 = |G_M^*(q^2; W)|^2 + 3|G_E^*(q^2; W)|^2 + \frac{q^2}{2W^2} |G_C^*(q^2; W)|^2$$

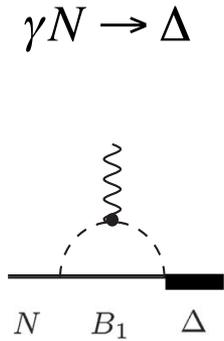
$$y_\pm = (W \pm M)^2 - q^2$$

$$\Gamma_{\gamma N}(W) \equiv \Gamma_{\gamma^* N}(0; W)$$

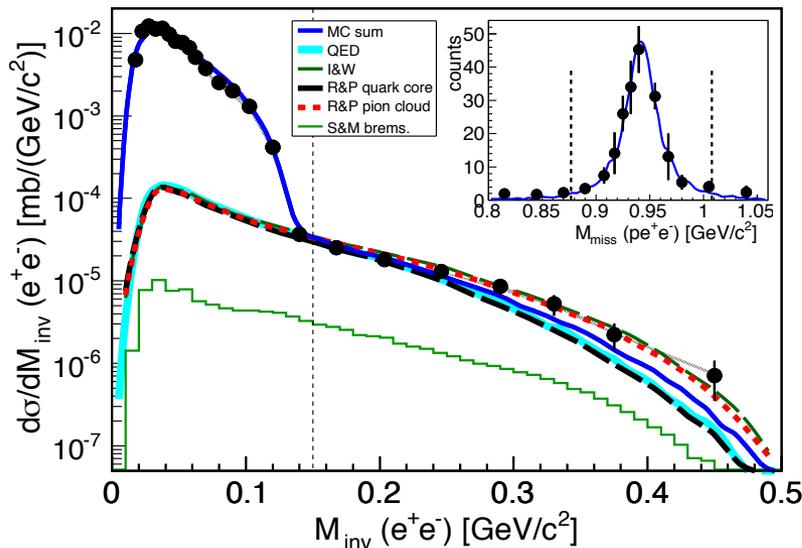
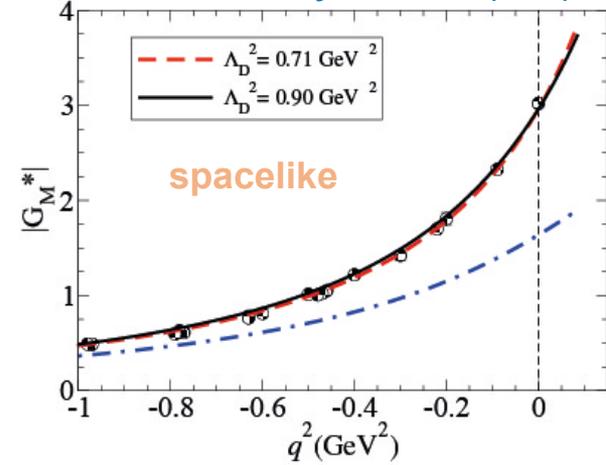
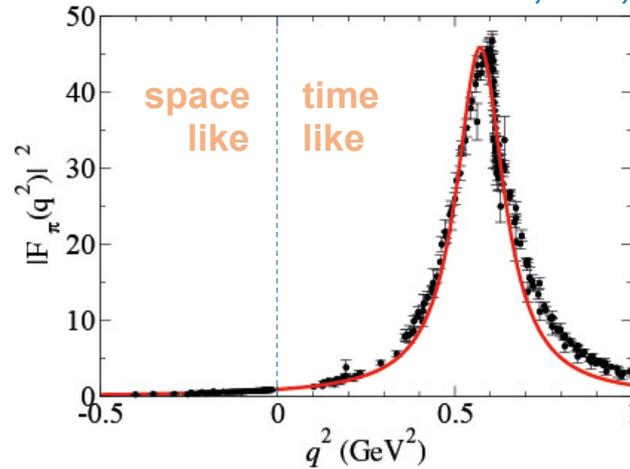
$$\Gamma_{e^+ e^- N}(W) = \frac{2\alpha}{3\pi} \int_{2m_e}^{W-M} \Gamma_{\gamma^* N}(q; W) \frac{dq}{q}$$

Crossing the boundaries

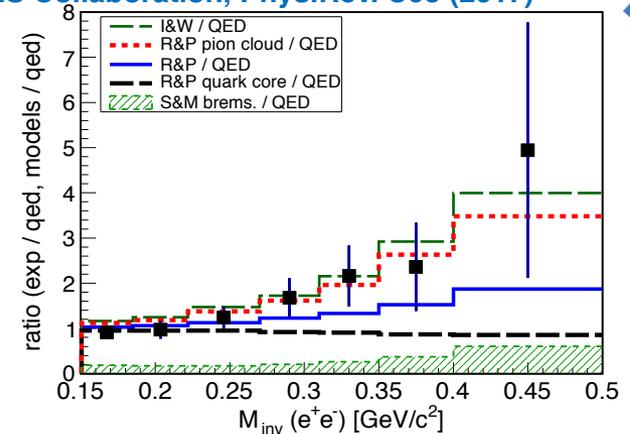
$\Delta(1232)$ Dalitz decay



Ramalho, Pena, Weil, Van Hees, Mosel, Phys.Rev. C93 (2016)



HADES Collaboration, Phys.Rev. C95 (2017)



Δ Dalitz decay branching ratio $4.9 \cdot 10^{-5}$

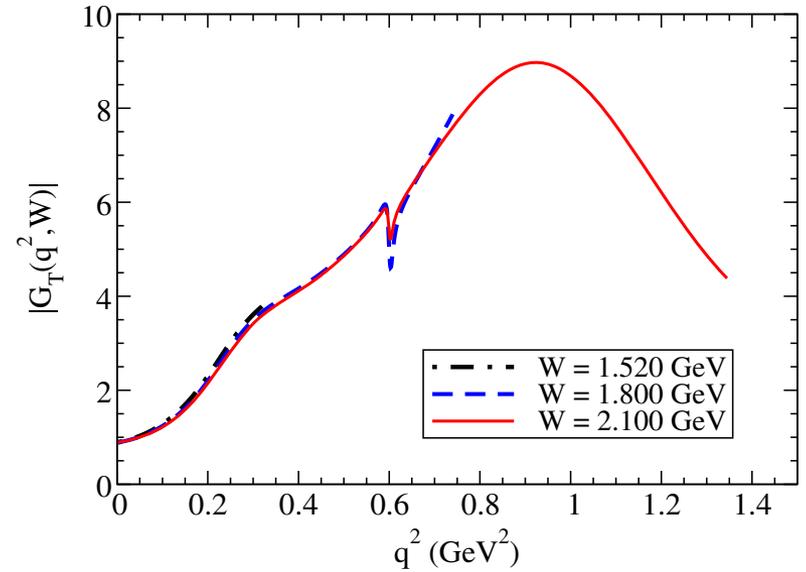
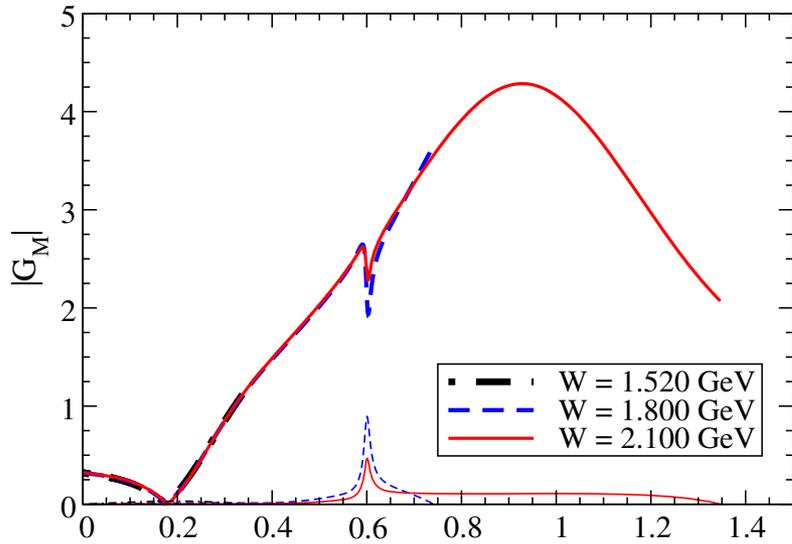
$$\Gamma_{\gamma^*N}(q, W) = \frac{3\alpha(W-M)^2}{16 M^2 W^3} \sqrt{y_+ y_-} y_+ |G_T(q^2, W)|^2$$
$$y_{\pm} = (W \pm M)^2 - q^2$$

$$|G_T(q^2, W)|^2 = 3|G_M(q^2, W)|^2 + |G_E(q^2, W)|^2$$
$$+ \frac{q^2}{2W^2} |G_C(q^2, W)|^2.$$

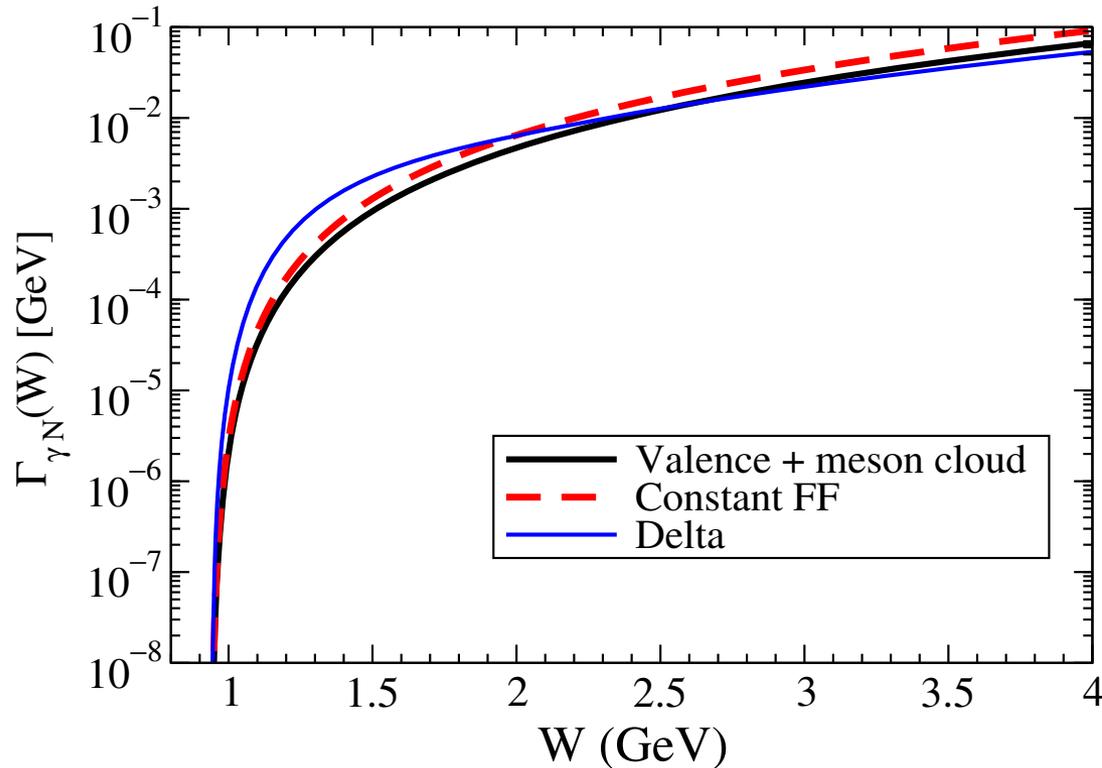
$$\Gamma'_{e^+e^-N}(q, W) \equiv \frac{d\Gamma}{dq}(q, W)$$
$$= \frac{2\alpha}{3\pi q^3} (2\mu^2 + q^2) \sqrt{1 - \frac{4\mu^2}{q^2}} \Gamma_{\gamma^*N}(q, W)$$

Crossing the boundaries

$N^*(1520)$



G. Ramalho, M. T. P., PHYSICAL REVIEW D 95 0104003 (2017)



$$G_T^{CFF} = 1.048$$

We conclude that the N*(1520) and N*(1232) γ decay widths are competitive.

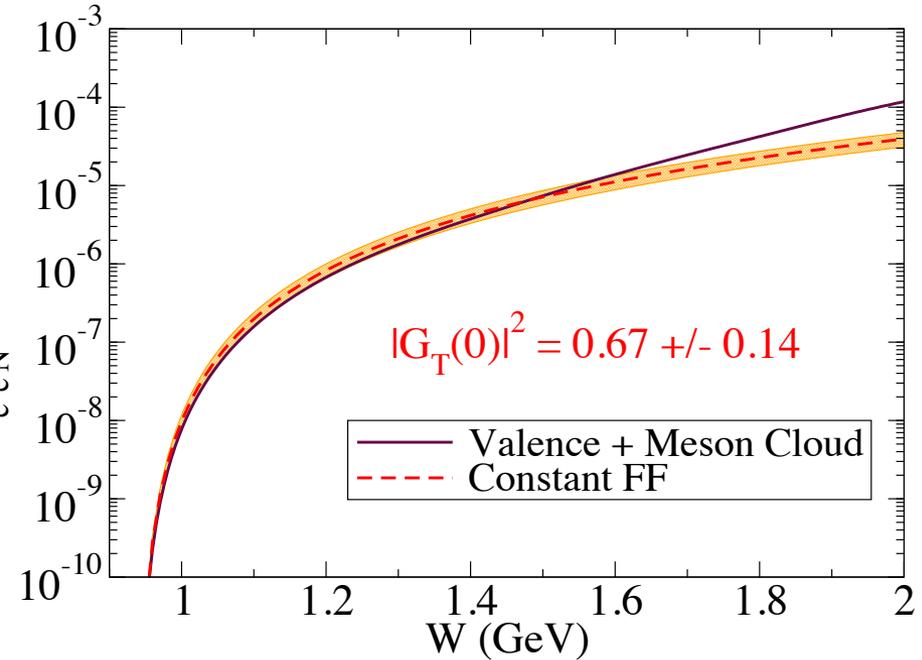
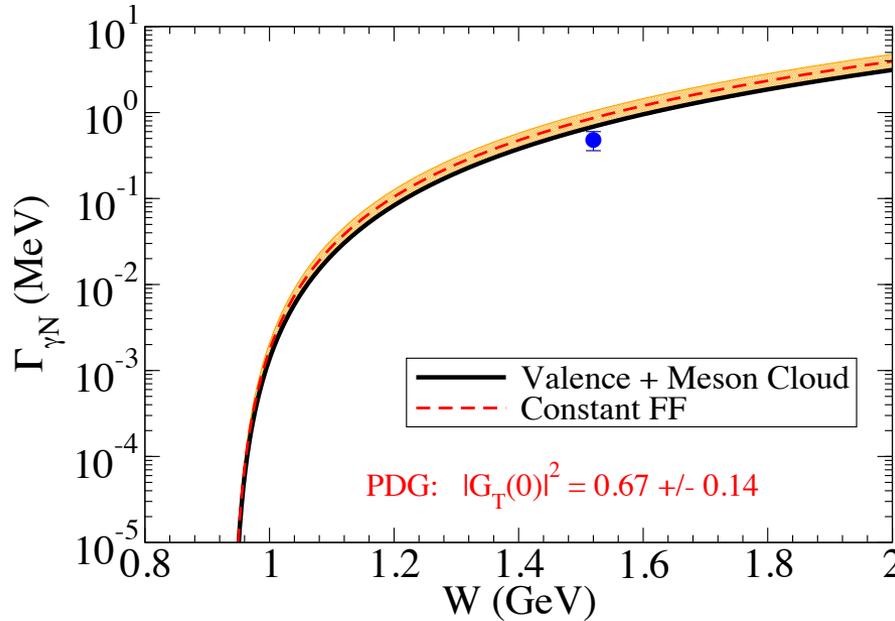
CST underestimates $G^T(0)$

Crossing the boundaries

N*(1520)

Update

Devenish (1976) Normalization



$$G_T^{CLAS} = 0.86 \pm 0.13$$

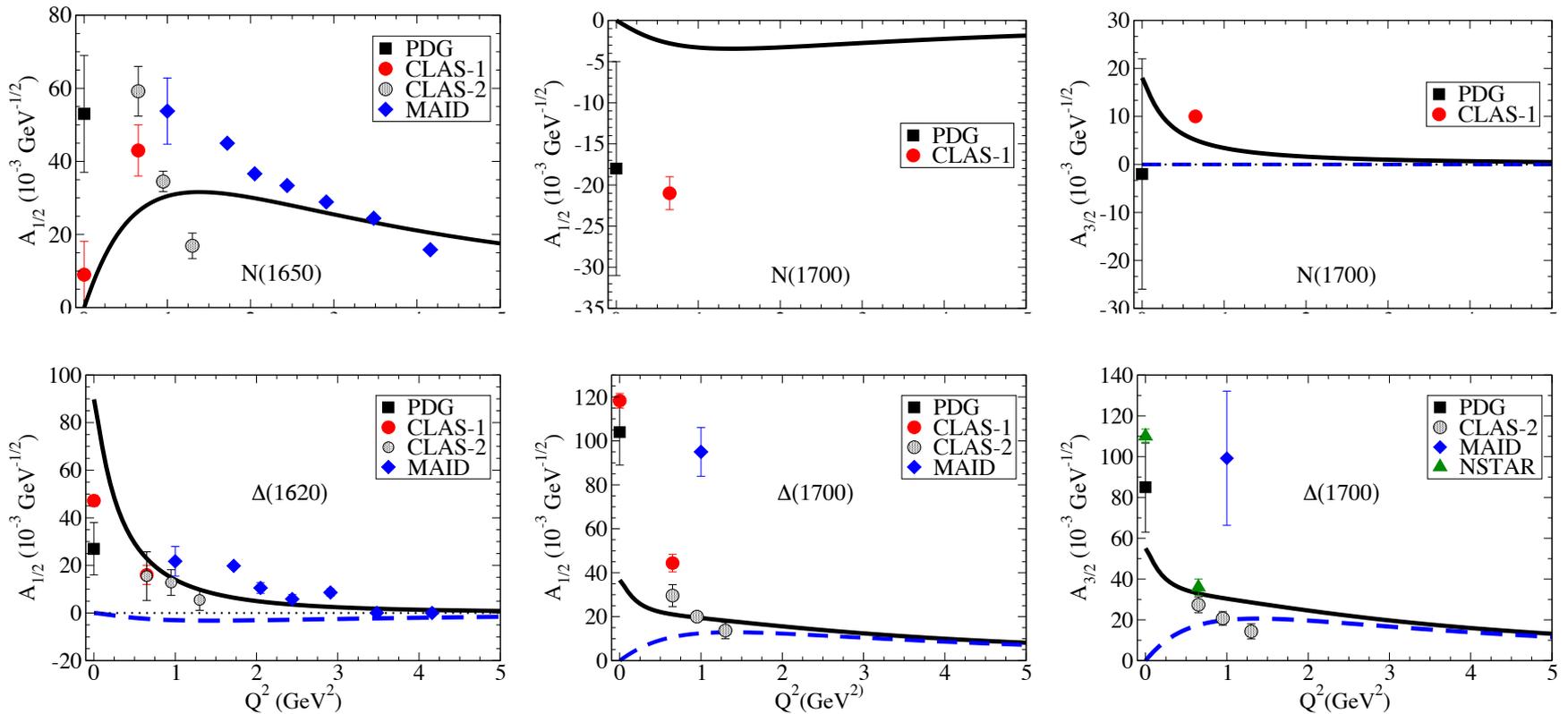
$$G_T^{PDG_{new}} = 0.88 \pm 0.05$$

$$G_T^{CST} = 0.73$$

Consistent with PDG value
based on γN decay

Crossing the boundaries

Input: $N(1520)$, $N(1535)$; **Output:** $N(1650)$, $N(1700)$, $\Delta(1620)$, $\Delta(1700)$



G. Ramalho, PRD 90, 033010 (2014)

Summary

Covariant **S**pectator quark-diquark model enables description of different resonance states (spin/orbital motion).

Several applications: $\Delta(1232)$, $N^*(1440)$, $N^*(1535)$, $N^*(1520)$, ...,
strange sector, DIS, dilepton mass spectrum.

Consistent with experimental data at high Q^2 .

Consistent with LQCD in the large pion mass regime; combined with LQCD **CST** extracts “pion cloud” effects.

VMD and “pion cloud” sustained extension to the time like region of the TFF of the $\Delta(1232)$ and $N^*(1520)$.

Outlook

LQCD simulations below the N^* threshold will help too refine interpretation provided by theoretical quark models.

New data at large Q^2 and even more precise data in all ranges can improve interpretation of empirical results.

Dynamical calculations of diquark vertices within CST to be done, to support quark-diquark picture for baryons, seen within Dyson-Schwinger approach for dynamical quarks.

Thank you!

Gilberto Ramalho, LFTC, Brasil

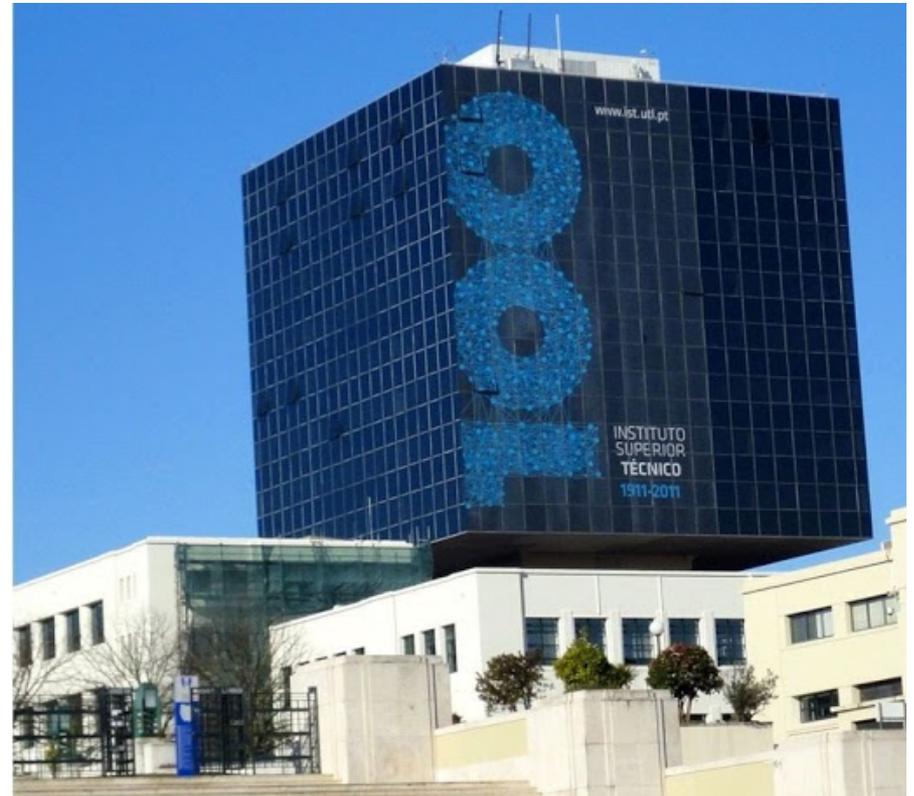
Alfred Stadler (IST-Ulisboa)

Elmar Biernat (IST-Ulisboa)

Gernot Eichmann (IST-Ulisboa)

Sofia Leitão (IST-Ulisboa)

Franz Gross (JLAB)



Crossing the boundaries to explore baryon resonances



Vector meson dominance 2 poles

$$f(Q^2) = e + gB(Q^2)e + gB(Q^2)gB(Q^2)e + \dots = e + \frac{gB(Q^2)e}{1 - gB(Q^2)}$$

$$\text{if } gB(Q^2) = \frac{\lambda^2}{\Lambda^2 + Q^2}, \text{ then } f(Q^2) = e + \frac{\lambda^2 e}{\Lambda^2 - \lambda^2 + Q^2}$$

$$f_{1+} = \lambda + \frac{1 - \lambda}{1 + Q_0^2/m_v^2} + \frac{c_{\pm} Q_0^2/M_h^2}{(1 + Q_0^2/M_h^2)^2}$$

$$f_{2\pm} = \kappa_{\pm} \left(\frac{d_{\pm}}{1 + Q_0^2/m_v^2} + \frac{(1 - d_{\pm})}{1 + Q_0^2/M_h^2} \right)$$

Low-energy behavior encodes high-energy behavior:
Scale enters in the problem! DIS used to fix λ

4 parameters

E.M. Current and TFF at the photon point

$$\Gamma^{\beta\mu}(P, q) = G_1 q^\beta \gamma^\mu + G_2 q^\beta P^\mu + G_3 q^\beta q^\mu - G_4 g^{\beta\mu}$$

- Only finite G_i are physically acceptable
- Only 3 G_i are independent:

E.M. Current has to be conserved $q^\mu \Gamma_{\beta\mu} = 0$

Choice for quark current:

$$j_I^\mu = j_1 \left(\gamma^\mu - \frac{\not{q} q^\mu}{q^2} \right) + j_2 \frac{i \sigma^{\mu\nu} q_\nu}{2m}$$

Not additional contribution to observables from extra term because of contraction with conserved lepton current.

Orthogonality between initial and final states necessarily follows from both requirements, giving an important constraint to G_c at $Q^2=0$.

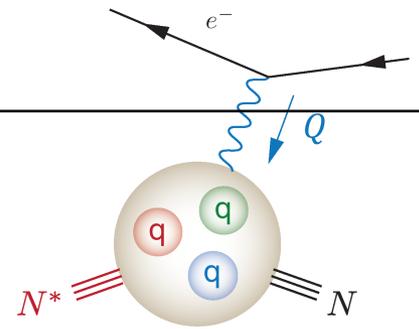
Crossing the boundaries to explore baryon resonances

$$\left[(M + m)G_1 + \frac{M^2 - m^2}{2}G_2 - G_4 \right] = Q^2 G_3,$$
$$G_M^*(Q^2) = \kappa \left\{ [(3M + m)(M + m) + Q^2] \frac{G_1}{M} \right. \\ \left. + (M^2 - m^2)G_2 - 2Q^2 G_3 \right\},$$

$$G_E^*(Q^2) = \kappa \left\{ (M^2 - m^2 - Q^2) \frac{G_1}{M} + (M^2 - m^2)G_2 \right. \\ \left. - 2Q^2 G_3 \right\},$$

$$G_C^*(Q^2) = \kappa \left\{ 4MG_1 + (3M^2 + m^2 + Q^2)G_2 \right. \\ \left. + 2(M^2 - m^2 - Q^2)G_3 \right\},$$

E.M. Current and TFF near the photon point



The two baryons in the transition cannot be both at rest

In the rest frame of the final state:

Photon energy due to baryon mass difference implies 3-momentum transfer

$$E_N = \frac{M_R^2 + M_N^2}{2M_R} \quad |\vec{q}| = \frac{M_R^2 - M_N^2}{2M_R}$$

Pseudo Threshold PT

$$q_0^2 = -(M_R - M_N)^2 ; |q| = 0$$

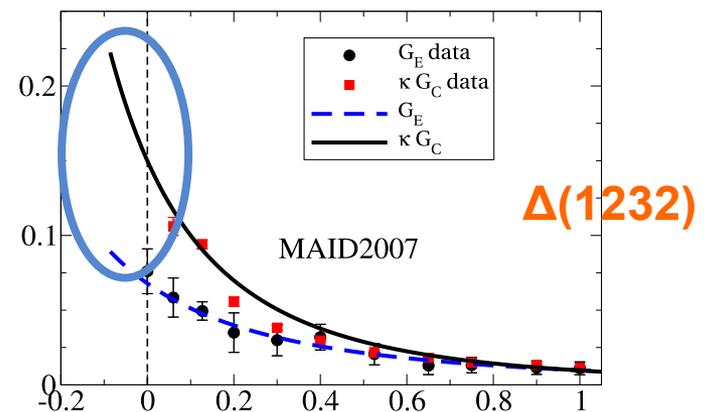
An accident of the definition of the Jones and Scadron form factors:

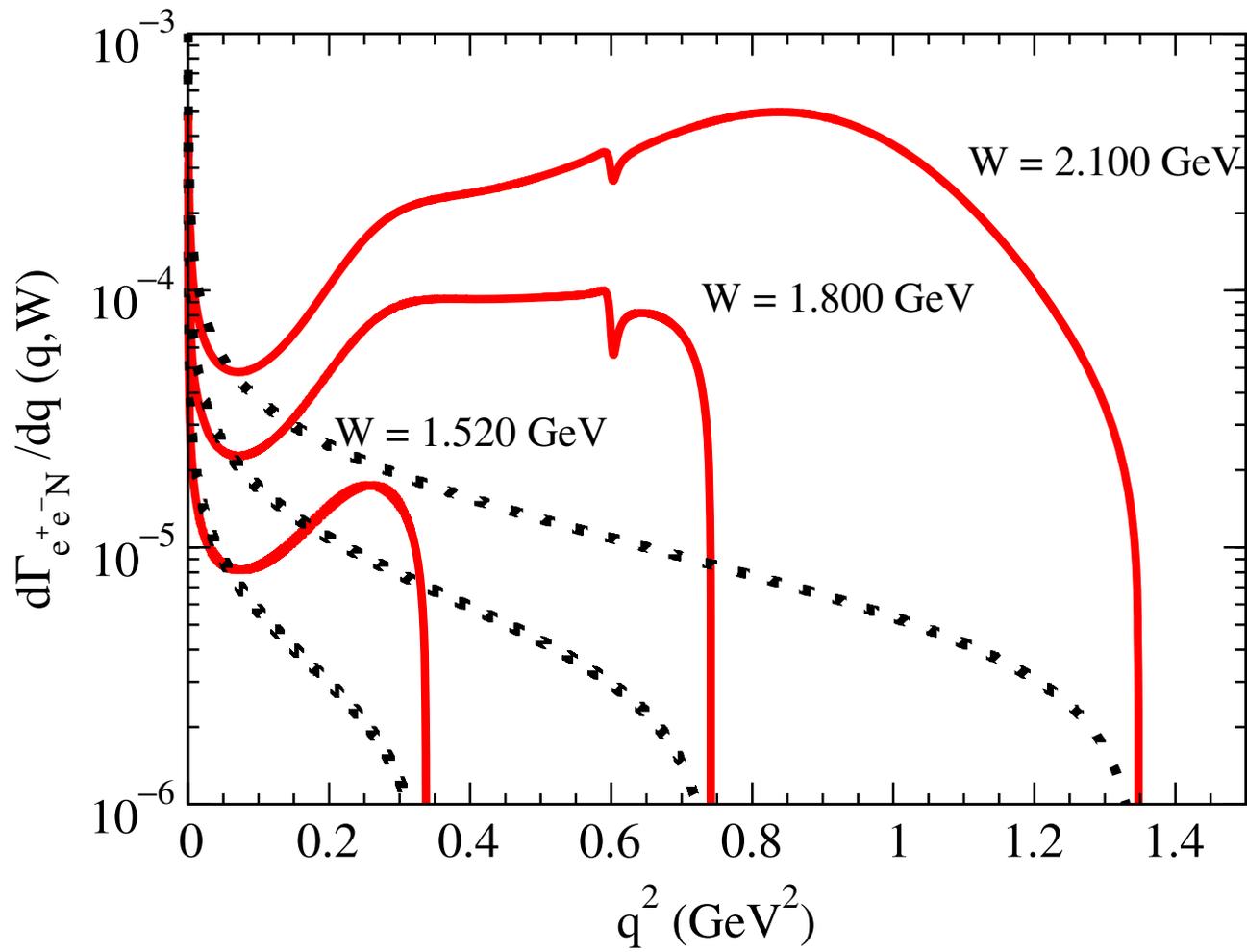
$$G_E(PT) = \frac{M_R - M}{2M_R} G_C(PT)$$

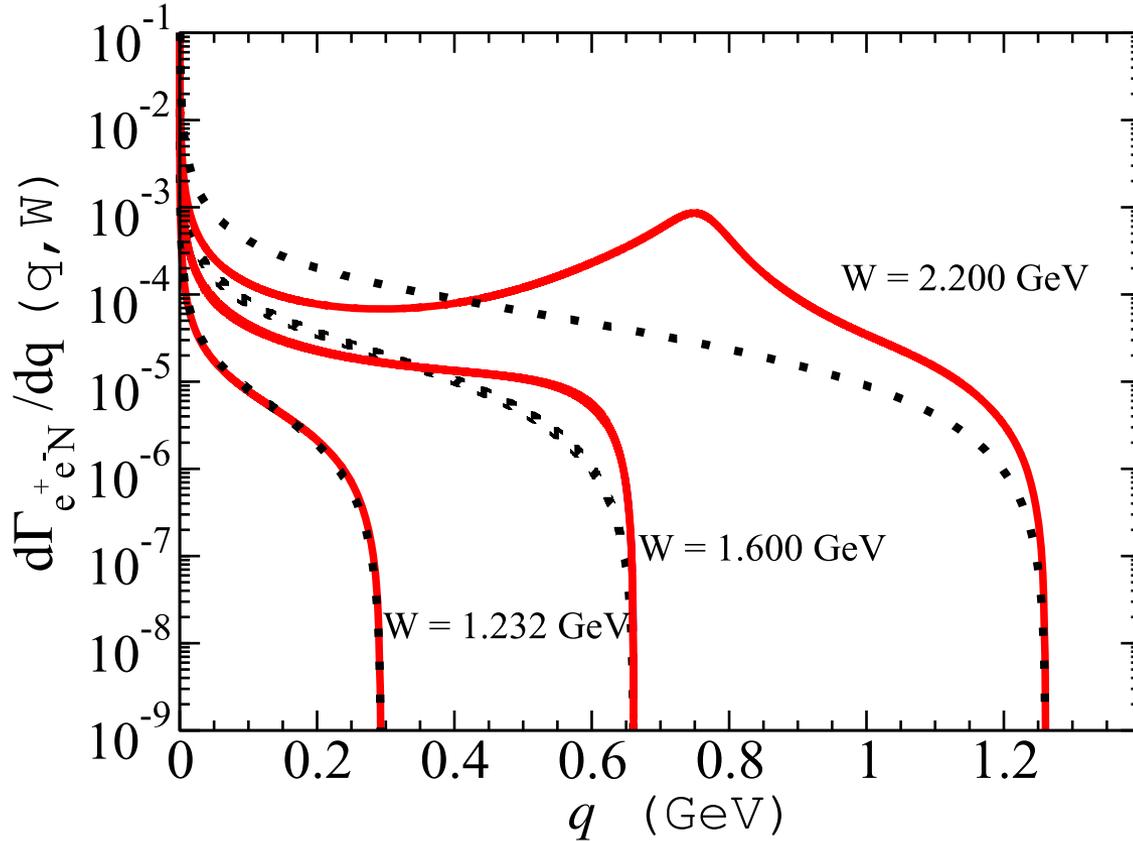
A form of the Siegert condition!

If current conservation is violated this is failed.

If data analysis proceed through helicity amplitudes this behavior may be missed.

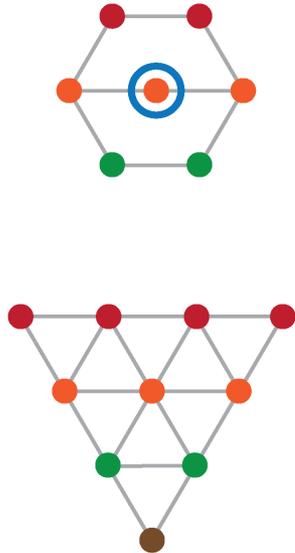




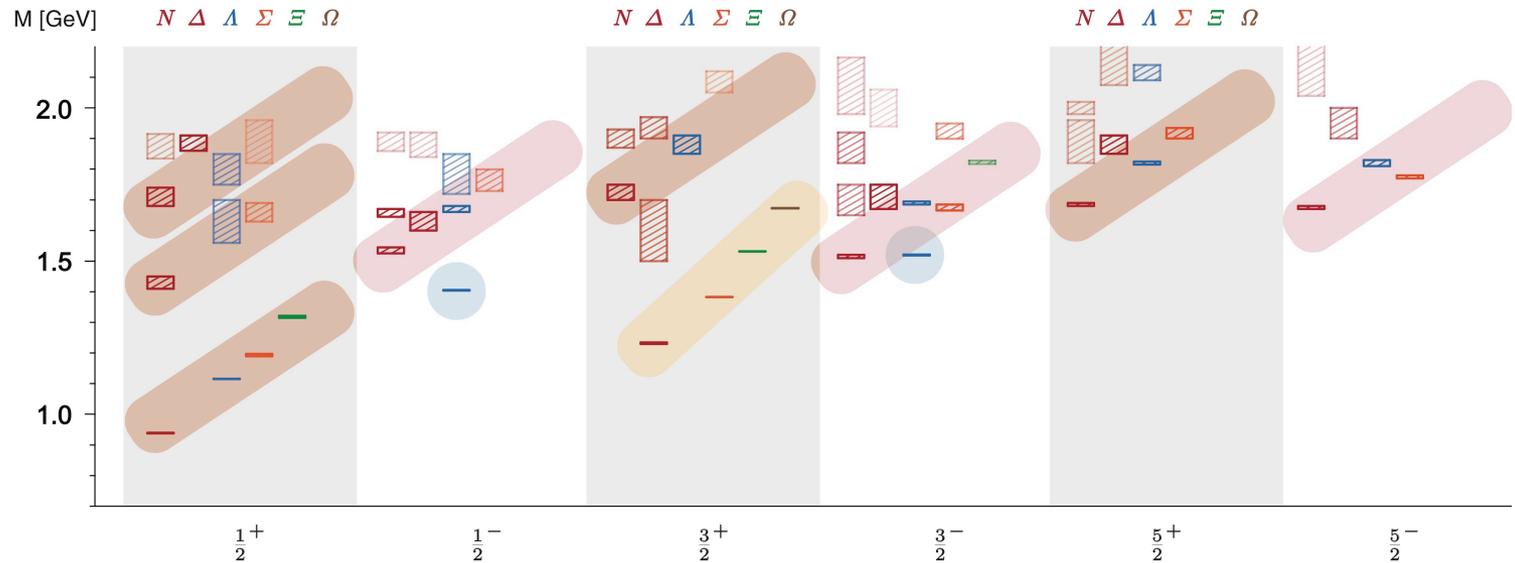


— new model; \cdots const $G_M^*(q^2; W) \equiv G_M^*(0, M_\Delta)$

Strangeness



Hyperons: How little we know! Many missing states...

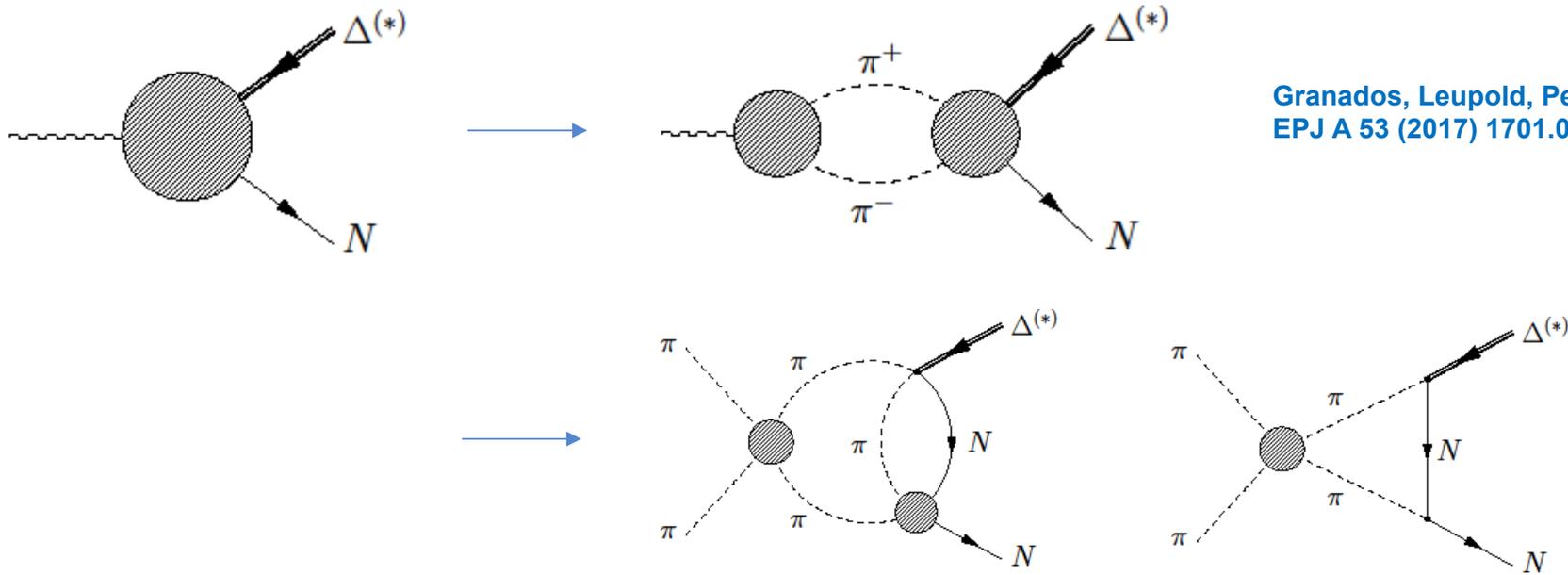


How can we learn more? Hyperon spectroscopy at JLab, PANDA, ...

Electromagnetic probes are ideal tools! [CLEO: Seth et al., PRL 110 \(2013\)](#)

$$\begin{array}{lll}
 e^+ e^- \rightarrow Y_1 \bar{Y}_2 & q^2 > (m_{Y_1} + m_{Y_2})^2 & \text{BESIII} \\
 Y_1 \rightarrow Y_2 e^+ e^- & q^2 < (m_{Y_1} - m_{Y_2})^2 & \text{HADES}
 \end{array}$$

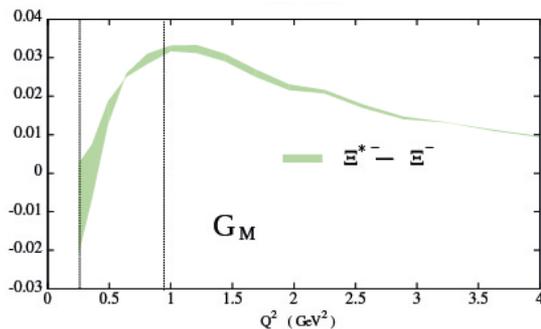
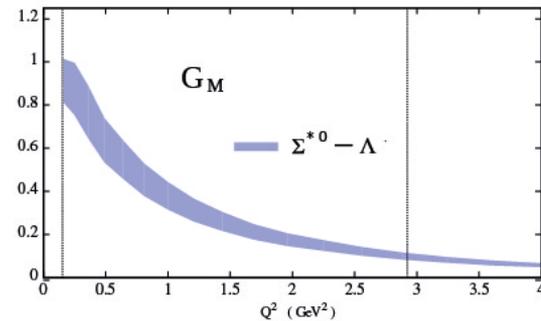
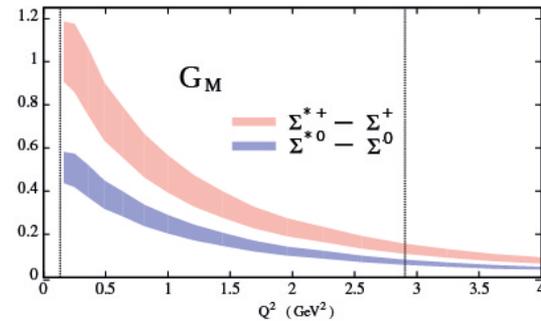
Dispersion theory



Granados, Leupold, Perotti,
EPJ A 53 (2017) 1701.09130

- Allows for **model-independent** calculations of isovector transition form factors at low energies
- Required input (shaded blobs): $\pi\pi$ and πN phase shifts, pion vector form factor \rightarrow all well known!

Dyson-Schwinger approach



- Consistent calculations of meson and baryon spectra, form factors, ...

[Eichmann, Williams, Sanchis-Alepuz, Alkofer, Fischer, Prog. Part. Nucl. Phys. 91 \(2016\)](#)

- Baryons:**

- ✓ qq \bar{q} three-body Faddeev equation
 - ✓ q(q \bar{q}) quark-diquark system
- Similar results!

- Example (qqq): **hyperon transition form factors**

[Sanchis-Alepuz, Alkofer, Fischer, 1707.08463 \[hep-ph\]](#)

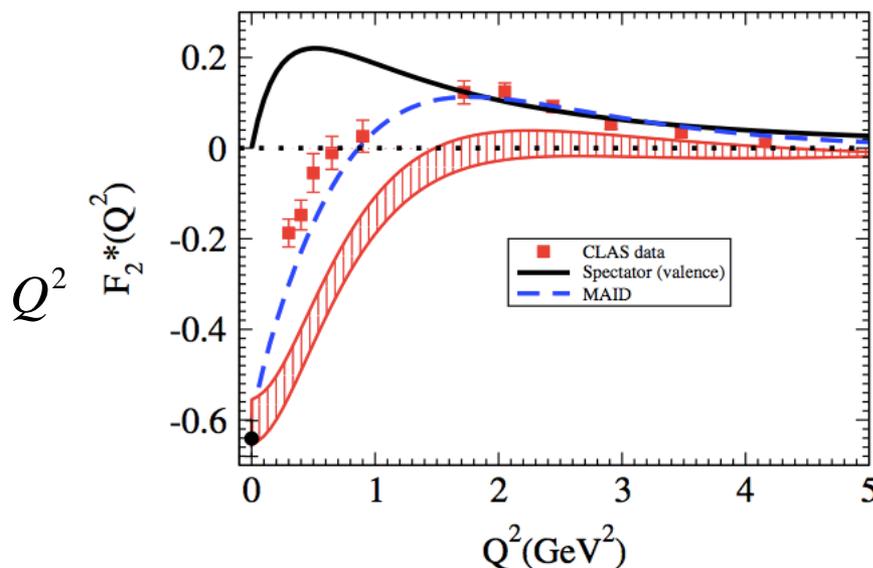
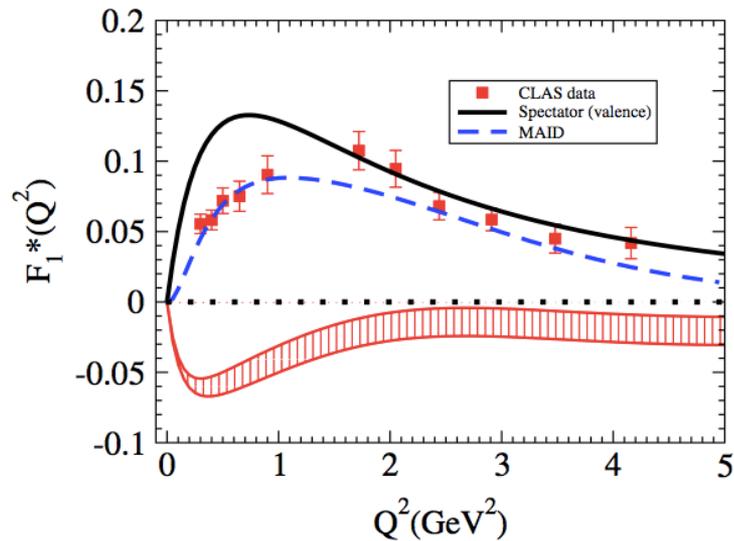
- Timelike form factors:** require treatment of resonances, contour deformation methods, ...

- Non-zero T and μ , **baryon backcoupling effects in medium:** ongoing studies

[Eichmann, Fischer, Welzbacher, PRD 93 \(2016\)](#)

$N \rightarrow N^*(1440)$

- Quark core amplitude describes data at high Q^2 .
- Pion cloud estimated as difference between MAID fit and the quark core.
- Error bands from error bars in the data

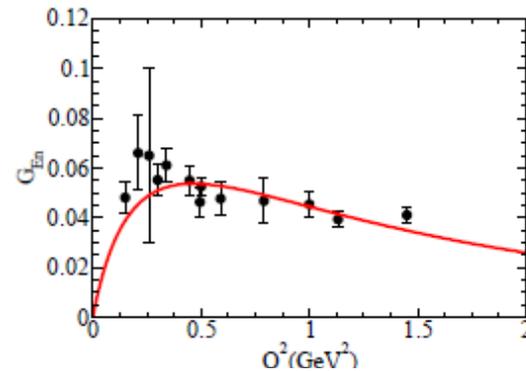
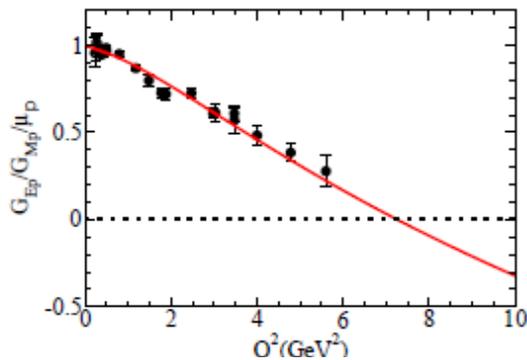
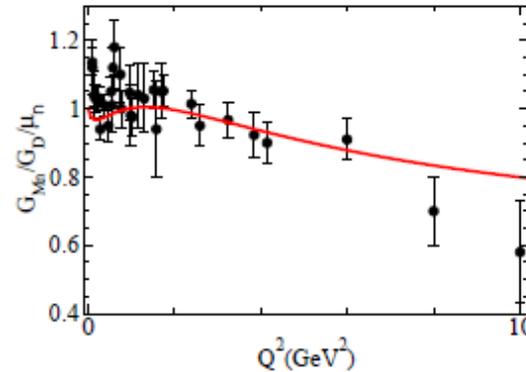
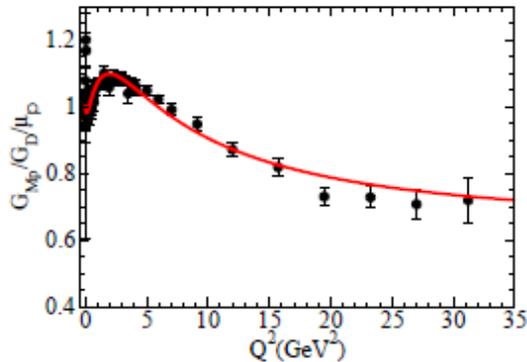


Proton and Neutron form factors

$$G_E = F_1 - \tau F_2, \quad G_M = F_1 + F_2; \quad \tau = \frac{Q^2}{4M_N^2}$$

G. Ramalho, M. T. P. and Franz Gross,

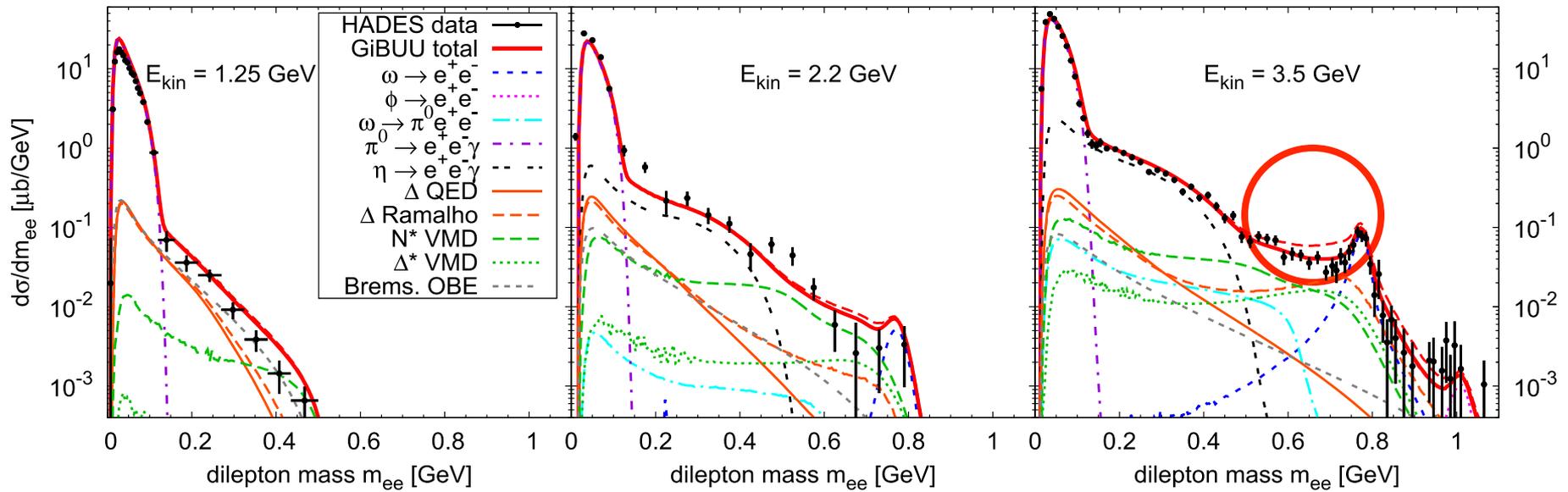
PHYSICAL REVIEW C 77, 015202 (2008)



$$\chi^2 = 1.36$$

Model calibration by Nucleon form factor data:

Fixes parameters of the quark current and radial wave function
(4+2) parameters.



Transport-model calculations of dilepton mass spectra from proton-proton collisions at three different beam energies, with and without form factor.

G. Ramalho, M. T. P, J. Weil, H. van Hees, and U. Mosel

PHYSICAL REVIEW D 93, 033004 (2016)