

# Transition form factors and radiative decays of hyperons

Stefan Leupold

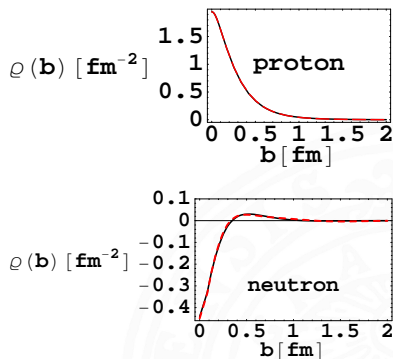
Uppsala University

Orsay, May 2018



# Femtoscience

- Want to understand structure of matter at femtometer scale
  - (electromagnetic) form factors contain structure information
  - spatial distributions related to Fourier transform of space-like form factors

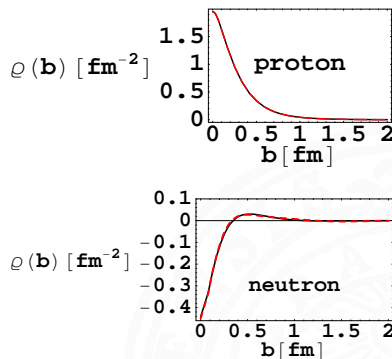


G.A. Miller,

Phys.Rev.Lett. 99, 112001 (2007)

# Femtoscience

- Want to understand structure of matter at femtometer scale
  - (electromagnetic) form factors contain structure information
  - spatial distributions related to Fourier transform of space-like form factors
- How does hadronic structure change when replacing up/down by **strange** quarks?
  - Would like to get information about **form factors of hyperons**



G.A. Miller,

Phys.Rev.Lett. 99, 112001 (2007)

# Electromagnetic form factors of hyperons

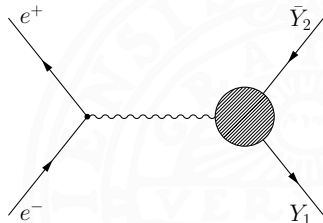
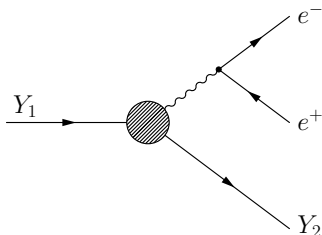
to large extent  
*terra incognita*



- electron-hyperon scattering complicated  
⇒ instead:
  - reactions  $e^+ e^- \rightarrow \text{hyperon anti-hyperon } (Y_1 \bar{Y}_2) \rightsquigarrow \text{BESIII}$
  - ⇒ form factors and transition form factors for **large** time-like  $q^2 > (m_{Y_1} + m_{Y_2})^2$
  - decays  $Y_1 \rightarrow Y_2 e^+ e^- \rightsquigarrow \text{HADES+PANDA}$
  - ⇒ transition form factors for **small** time-like  $q^2 < (m_{Y_1} - m_{Y_2})^2$

# Structure information about hyperons

- need space-like form factors to obtain structure information
  - ↪ hard to obtain for hyperons
  - ↪ alternative: explore time-like region and use dispersion theory  
(time-like means  $q^2 > 0$ , i.e. energy transfer > momentum transfer)



# Unitarity and analyticity

- constraints from local quantum field theory:  
partial-wave amplitudes for reactions/decays **must be**
  - **unitary** (optical theorem):

$$S S^\dagger = 1, \quad S = 1 + iT \quad \Rightarrow \quad 2 \operatorname{Im} T = T T^\dagger$$

↪ note that this is a matrix equation:

$$\operatorname{Im} T_{A \rightarrow B} = \sum_X T_{A \rightarrow X} T_{X \rightarrow B}^\dagger$$

↪ in practice: use most relevant intermediate states  $X$

- **analytical** (**dispersion relation** for form factors):

$$T(q^2) = T(0) + \frac{q^2}{\pi} \int_{\text{threshold}}^{\infty} ds \frac{\operatorname{Im} T(s)}{s(s - q^2 - i\epsilon)}$$

↪ can be used to calculate whole amplitude from imaginary part

# Dispersion relation for form factor

example:  $e^+e^- \rightarrow \gamma^* \rightarrow Y_1 \bar{Y}_2 (= B)$

$$T_{\gamma^* \rightarrow B}(q^2) = T_{\gamma^* \rightarrow B}(0) + \frac{q^2}{\pi} \int_{\text{threshold}}^{\infty} ds \frac{\sum_X T_{\gamma^* \rightarrow X}(s) T_{X \rightarrow B}^\dagger(s)}{s(s - q^2 - i\epsilon)}$$

- physical threshold is at  $m_{Y_1} + m_{Y_2}$ , but in dispersion relation the **threshold** is given by the lowest-mass states that can couple to virtual photons (and hyperons)
- ↪  $X = 2$  pions, ...
- ↪ **threshold**  $= (2m_\pi)^2$
- ↪ need information in unphysical region starting at **2-pion threshold**
- most important contribution for low values of  $q^2$  (time-like or space-like)

# Pion contribution

$$T_{\gamma^* \rightarrow Y_1 \bar{Y}_2}(q^2) \approx T_{\gamma^* \rightarrow Y_1 \bar{Y}_2}(0) + \frac{q^2}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{T_{\gamma^* \rightarrow 2\pi}(s) T_{2\pi \rightarrow Y_1 \bar{Y}_2}^\dagger(s)}{s(s - q^2 - i\epsilon)}$$

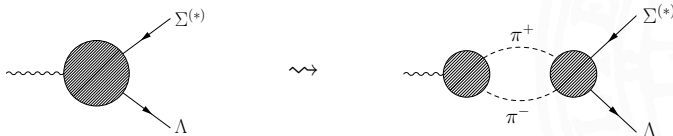
- ↪ need  $T_{2\pi \rightarrow Y_1 \bar{Y}_2}$  ( $T_{\gamma^* \rightarrow 2\pi}$  very well known)
- ↪ use again analyticity, dispersion theory and crossing symmetry
- ↪ need pion-hyperon scattering amplitudes  $\pi Y_1 \rightarrow \pi Y_2$
- ↪ ideally from data, but not available
- ↪ use instead chiral perturbation theory ( $\chi$ PT)
- ↪ contains in part unknown low-energy constants (LECs)
- ↪ determine  $\text{LEC}(s)$  from data



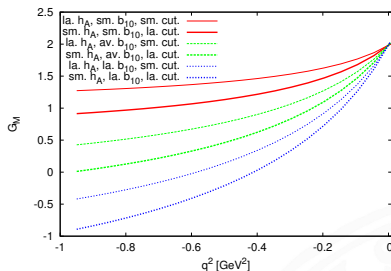
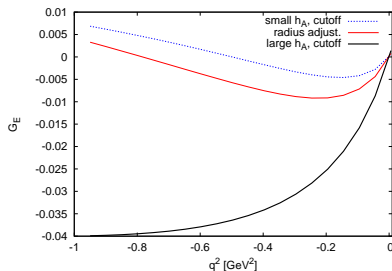
# Strategy

$$T_{\gamma^* \rightarrow \Upsilon_1 \bar{\Upsilon}_2}(q^2) \approx T_{\gamma^* \rightarrow \Upsilon_1 \bar{\Upsilon}_2}(0) + \frac{q^2}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{T_{\gamma^* \rightarrow 2\pi}(s) T_{2\pi \rightarrow \Upsilon_1 \bar{\Upsilon}_2}^\dagger(s)}{s(s - q^2 - i\epsilon)}$$

- measure **left-hand side** for small time-like  $q^2 > 0$ , e.g. slope
- determine **LEC(s)** for  $T_{2\pi \rightarrow \Upsilon_1 \bar{\Upsilon}_2}$  on right-hand side
- predict **left-hand side** for small space-like  $q^2 < 0$

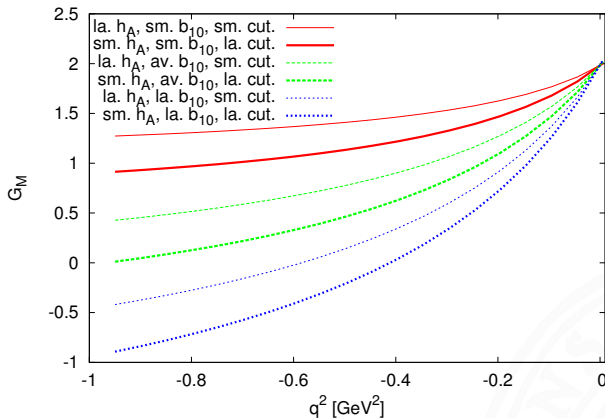


# Example: Transition form factors $\Sigma\text{-}\Lambda$



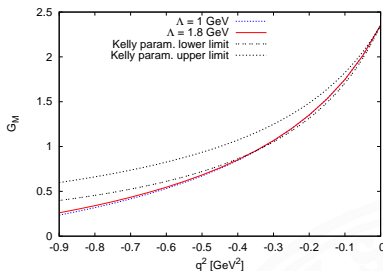
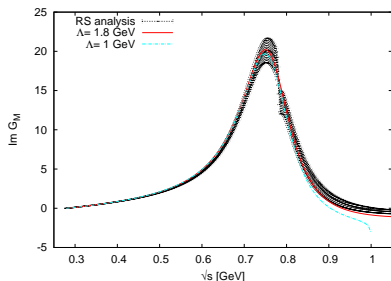
- electric transition form factor very small over large range
- ↪ what one might measure at low energies is magnetic transition form factor
- ↪ Dalitz-decay data  $\Sigma^0 \rightarrow \Lambda e^+ e^-$  integrated over  $\Lambda\text{-}e^-$  angle, but differential in  $q^2$  might be sufficient
- note: Dalitz decay region  $4m_e^2 < q^2 < (m_\Sigma - m_\Lambda)^2$  hardly visible here

# Magnetic transition form factor $\Sigma \rightarrow \Lambda$



- large uncertainty
- ↪ directly related to uncertainty in NLO low-energy constant  $b_{10}$
- ↪ can be determined from measuring magnetic transition radius

# Magnetic isovector form factor of nucleon



↪ low-energy framework works up to  $|q^2| \approx 0.4 \text{ GeV}^2$

- “RS” (Roy-Steiner equation): fully dispersive analysis
- “Kelly param.”: parametrization of form factor data  
J.J. Kelly, Phys.Rev.C 70, 068202 (2004)

↪ scheme (disp. th. &  $\chi$ PT) works very well for nucleon

# Extension to spin-3/2 hyperons has started

- aiming at transition form factors, Dalitz decays  
 $Y^*(J = 3/2) \rightarrow Y e^+ e^-$
- ↪ hyperon-pion scattering amplitudes in  
chiral perturbation theory ( $\chi$ PT)
- ↪ master thesis Olov Junker, work in progress
- $\chi$ PT prediction for radiative decays  $Y^*(J = 3/2) \rightarrow Y \gamma$
- ↪ next page
- axial-vector transition form factors  
and relation to  $Y^*(J = 3/2) \rightarrow Y \gamma \pi$
- ↪ master thesis Måns Holmberg, work in progress

# Radiative decays

Constructing and using chiral perturbation theory  
at next-to-leading order for decuplet states:

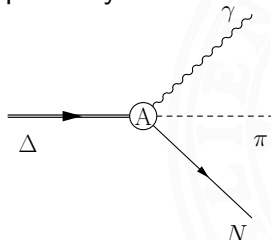
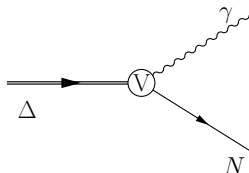
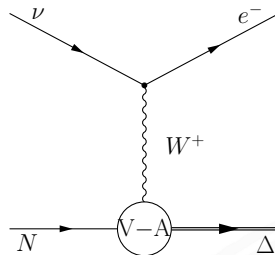
| Decay                                     | $c/(c_M e)$   | BR [%]                            | $ c_M $ [GeV <sup>-1</sup> ] |
|---|---------------|-----------------------------------|------------------------------|
| $\Delta \rightarrow N\gamma$              | $2/\sqrt{3}$  | $0.60 \pm 0.05$                   | $2.00 \pm 0.03$              |
| $\Sigma^{*+} \rightarrow \Sigma^+ \gamma$ | $-2/\sqrt{3}$ | $0.70 \pm 0.17$                   | $1.89 \pm 0.08$              |
| $\Sigma^{*-} \rightarrow \Sigma^- \gamma$ | 0             | $< 0.024$                         | —                            |
| $\Sigma^{*0} \rightarrow \Sigma^0 \gamma$ | $1/\sqrt{3}$  | <b><math>0.18 \pm 0.01</math></b> | —                            |
| $\Sigma^{*0} \rightarrow \Lambda \gamma$  | -1            | $1.25 \pm 0.13$                   | $1.89 \pm 0.05$              |
| $\Xi^{*0} \rightarrow \Xi^0 \gamma$       | $-2/\sqrt{3}$ | <b><math>4.0 \pm 0.3</math></b>   | —                            |
| $\Xi^{*-} \rightarrow \Xi^- \gamma$       | 0             | $< 4$                             | —                            |

(predictions in boldface)

Måns Holmberg, SL, arXiv:1802.05168 [hep-ph], to appear in EPJ A

# Axial-vector transition form factors

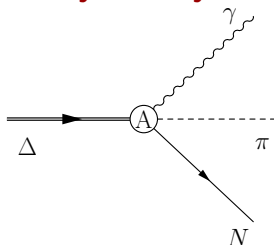
- interesting for scattering neutrino-nucleon to electron-Delta
- low energies: want to know deviation from current-algebra result  $\rightsquigarrow$  LEC  $C_E$
- vector and axial-vector transition form factors contribute also to  $\Delta \rightarrow N\gamma$  and  $\Delta \rightarrow N\pi\gamma$ , respectively



# Axial-vector TFFs and three-body decays

problems:

- needs to be disentangled from bremsstrahlung
- hard to measure for broad Delta

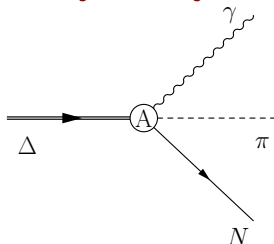




# Axial-vector TFFs and three-body decays

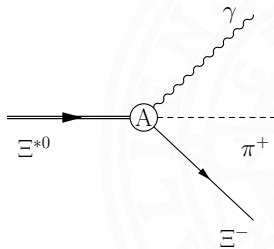
problems:

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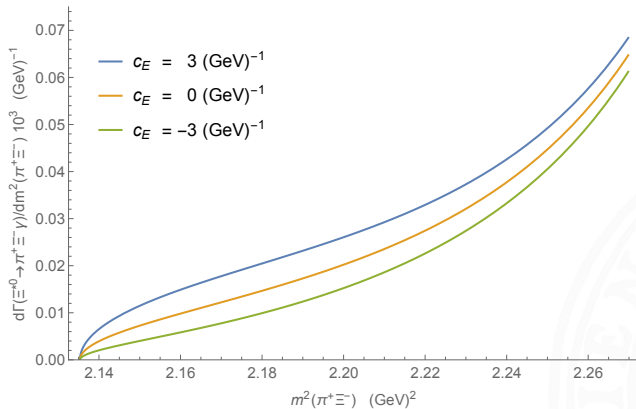
↪ in chiral perturbation theory at next-to-leading order:  
only one LEC  $c_E$  for whole multiplet

↪ get some clue from radiate  
three-body decays of hyperons,  
e.g. [cascades](#)



# Radiative three-body decay — preliminary results

- consider  $\Xi^{*0}(1530) \rightarrow \Xi^- \pi^+ \gamma$



- branching ratio  $(3.7 \pm 0.7) \cdot 10^{-4}$   
(cut on photon energy at 50 MeV)

from Måns Holmberg

# Collaborators

- Carlos Granados (postdoc, Washington D.C.)
- Elisabetta Perotti (PhD student, Uppsala)
- Måns Holmberg (master student, Uppsala)
- Olov Junker (master student, Uppsala)

C. Granados, SL, E. Perotti, Eur.Phys.J. A53 (2017) 117

SL, Eur.Phys.J. A54 (2018) 1

M. Holmberg, SL, Eur.Phys.J.A, in print, arXiv:1802.05168 [hep-ph]

# Summary

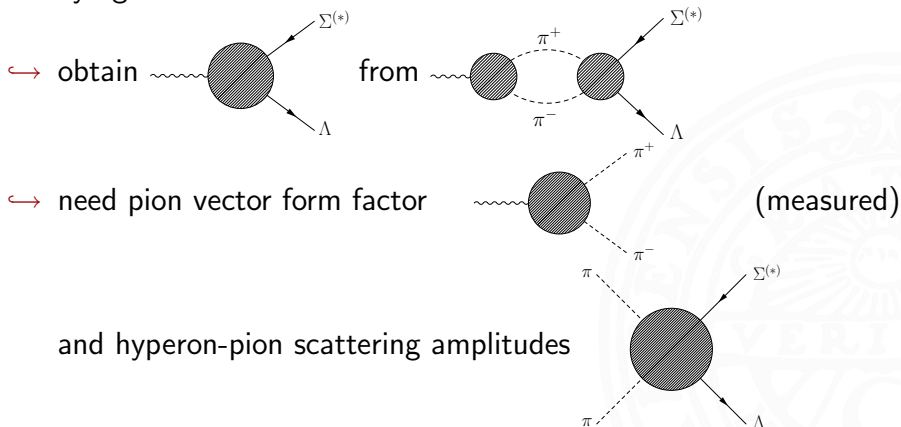
- excellent prospects to explore structure of hyperons
- ↪ experiments: BES-III, HADES, PANDA, ...
- ↪ Uppsala theory group: develop and utilize dispersion theory and chiral perturbation theory
  - can predict magnetic transition form factor  $\Sigma^0\text{-}\Lambda$  in low-energy space-like region if slope is measured in Dalitz decay
  - work in progress for transition form factors  $\Sigma^{*0}\text{-}\Lambda$  etc.
  - radiative decays  $\Sigma^{*0} \rightarrow \Sigma^0\gamma$ ,  $\Xi^{*0} \rightarrow \Xi^0\gamma$  predicted
  - decays  $Y^*(J = 3/2) \rightarrow Y\pi\gamma$  related to axial-vector transition form factors, in turn related to  $\nu N \rightarrow e^- \Delta$

backup slides

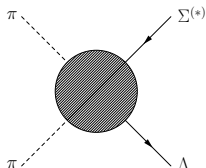


# Hyperon transition form factors

- for  $\Sigma/\Sigma^* \rightarrow \Lambda e^+ e^-$  need transition form factors
- dispersive framework: at low energies  $q^2$  dependence is governed by lightest intermediate states

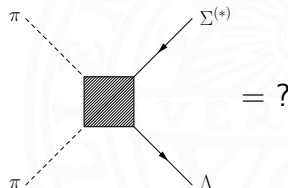
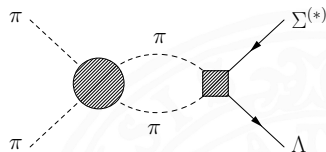


# Hyperon-pion scattering amplitudes



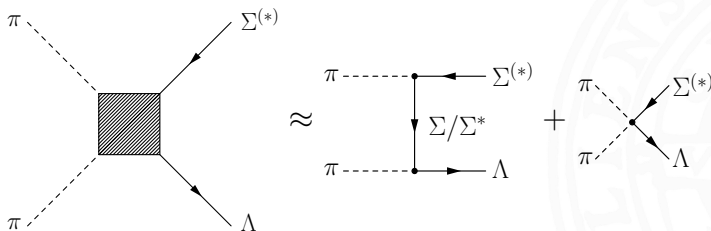
contain

- “right-hand cuts” (pion rescattering)
  - straightforward from unitarity and analyticity (and experimental pion phase shift)
- and rest:
  - left-hand cuts, polynomial terms
  - not straightforward
  - use three-flavor baryon chiral perturbation theory



# Input for hyperon-pion scattering amplitudes

- ideally use data
  - ↪ available for pion-nucleon, but not for pion-hyperon
  - ↪ instead: three-flavor baryon chiral perturbation theory ( $\chi$ PT) at leading and next-to-leading order (NLO)
    - including decuplet states
    - (optional for  $\Sigma^0 \rightarrow \Lambda$  transition, but turns out to be important!)





# $\chi$ PT input for hyperon-pion scattering amplitudes

three-point couplings fairly well known,  
but how to determine NLO four-point coupling constants?

- only one parameter ( $b_{10}$ ) for  $\Sigma$ - $\Lambda$  transition

↪ but not very well known

- “resonance saturation” estimates

Meißner/Steininger/Kubis, Nucl.Phys. B499, 349 (1997);

Eur.Phys.J. C18, 747 (2001)

or from fit to  $\pi N$  and  $KN$  scattering data with  
coupled-channel Bethe-Salpeter approach

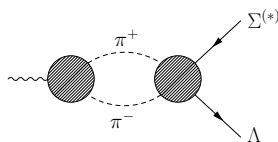
Lutz/Kolomeitsev, Nucl.Phys. A700, 193 (2002)

- maybe in the future:  
cross-check from lattice QCD

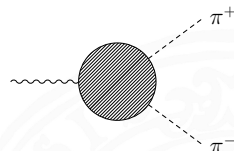
parameter is directly related to magnetic transition radius of  $\Sigma$ - $\Lambda$



# Where are the vector mesons?



- contained in measured pion vector form factor



- and contained in four-point coupling



- interactions  $\sim \bar{\Lambda} \sigma_{\mu\nu} \Sigma^0 V^{\mu\nu}$  and  $\sim V_{\mu\nu} \partial^\mu \pi^+ \partial^\nu \pi^-$   
(with  $V$  vector-meson field)  
contribute to  $\chi$ PT four-point interaction  $\sim \bar{\Lambda} \sigma_{\mu\nu} \Sigma^0 \partial^\mu \pi^+ \partial^\nu \pi^-$
- dispersion theory takes care of proper interplay