LPC, Aubière

Le 6 mars, 2009

Facteur de forme du proton sous le seuil par annihilation d'antiprotons sur noyaux

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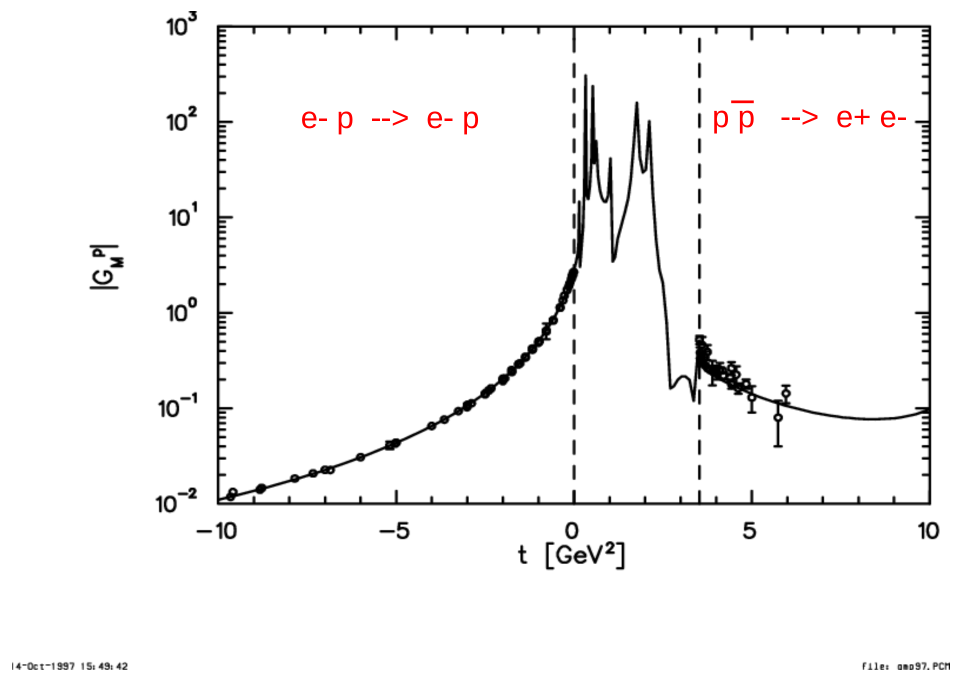
• Why is it interesting?

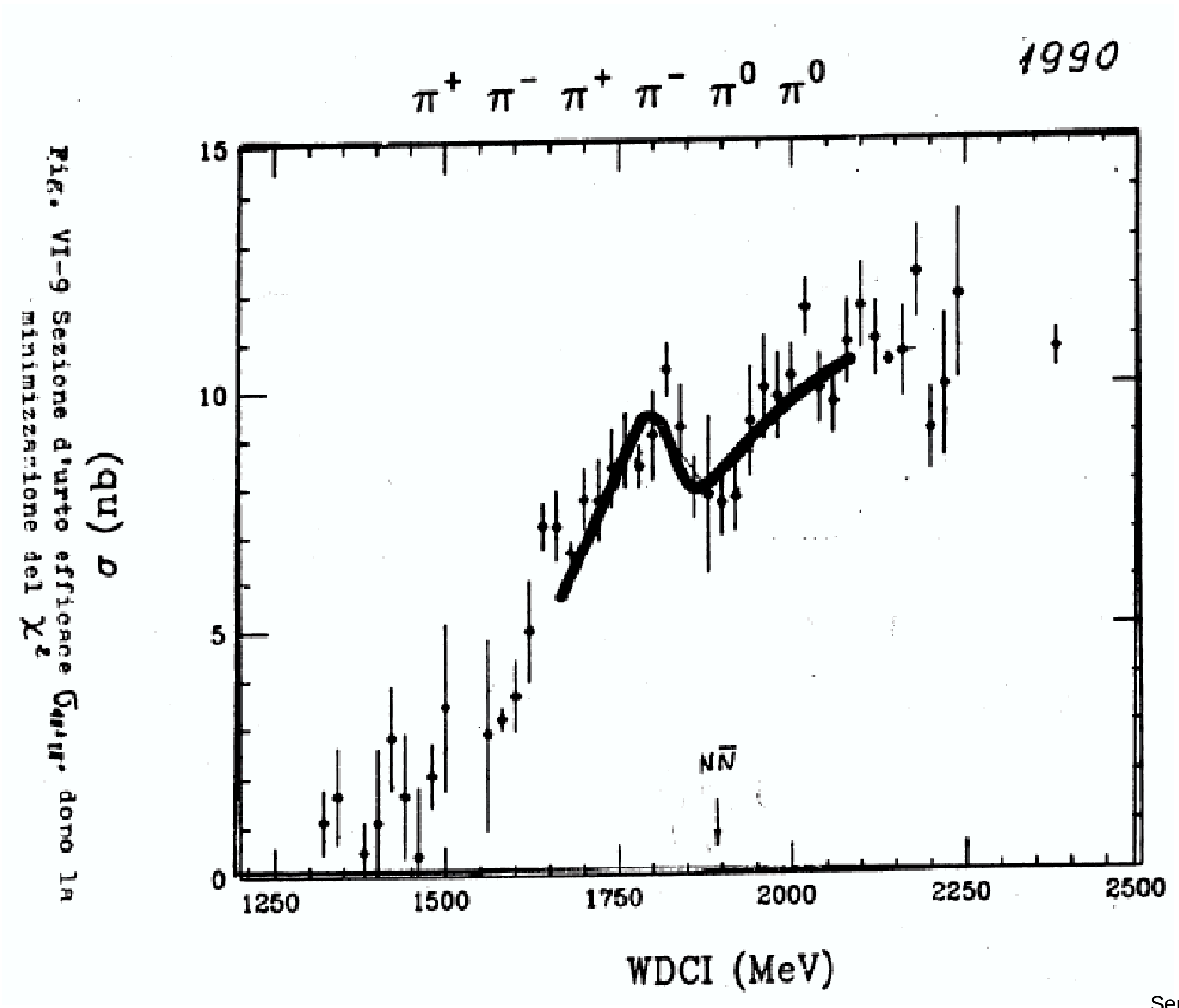
Since the near-threshold $\bar{N}N$ domain is not a desert, but the structures are expected.
Proton form factor at $-10\ GeV^2 \leq q^2 \leq 10\ GeV^2$:



U. Meissner et al., Nucl. Phys. **A 666** (2000) 51.

• Why is it interesting?

Observed structure in $e^+e^- \to \pi^+\pi^-\pi^+\pi^-\pi^0\pi^0$ below $\bar{N}N$ threshold.



• $\bar{N}N$ interaction v.s. NN

e^+e^- interaction v.s. e^-e^-

$$V_{e^+e^-}(r) = -V_{e^-e^-}(r)$$

since the charges of e^- and e^+ are opposite.

$$V_{NN}(r) = V_\pi(r) + V_\rho(r) + V_\omega(r) + V_\eta(r) + V_{\sigma_1}(r) + V_{\sigma_2}(r)$$

$$V_{\bar{N}N}(r) = -V_\pi(r) + V_\rho(r) - V_\omega(r) + V_\eta(r) - V_{\sigma_1}(r) + V_{\sigma_2}(r)$$

Since: $G_\pi = G_\omega = G_{\sigma_1} = -1, \quad G_\eta = G_\rho = G_{\sigma_0} = +1$

$G = C\ \exp(iI_2)$ is the G-parity (C is the charge parity).

• $\bar{N}N$ resonances

I.S. Shapiro, Physics Reports, **35** (1978) 129.

Bound states and resonances in the $\bar{N}N$ system are possible.

If they exist, they manifest themselves as structures in the time-like nucleon near-threshold form factor (below and above threshold).

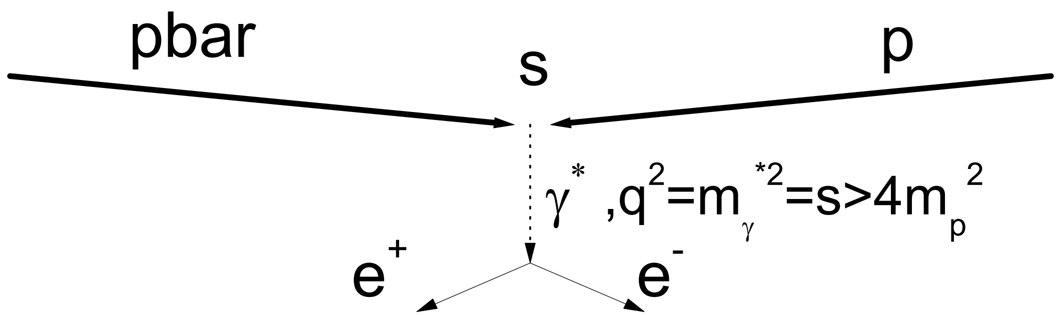
What to do if $\bar{p}$'s are fast?

Panda will provide $\bar{p}$ with $p_{\bar{p},lab} \geq 1500$ MeV/c.
In $\bar{p}p$ collision at $p_{\bar{p},lab} \geq 1500$ MeV/c., c.m. mass energy:
$\sqrt{s} = 2257$ MeV.
– Far from $\bar{p}p$ threshold $2m = 1880$ MeV
(but close to $\Lambda\bar{\Lambda}$ threshold).

Possibilities:

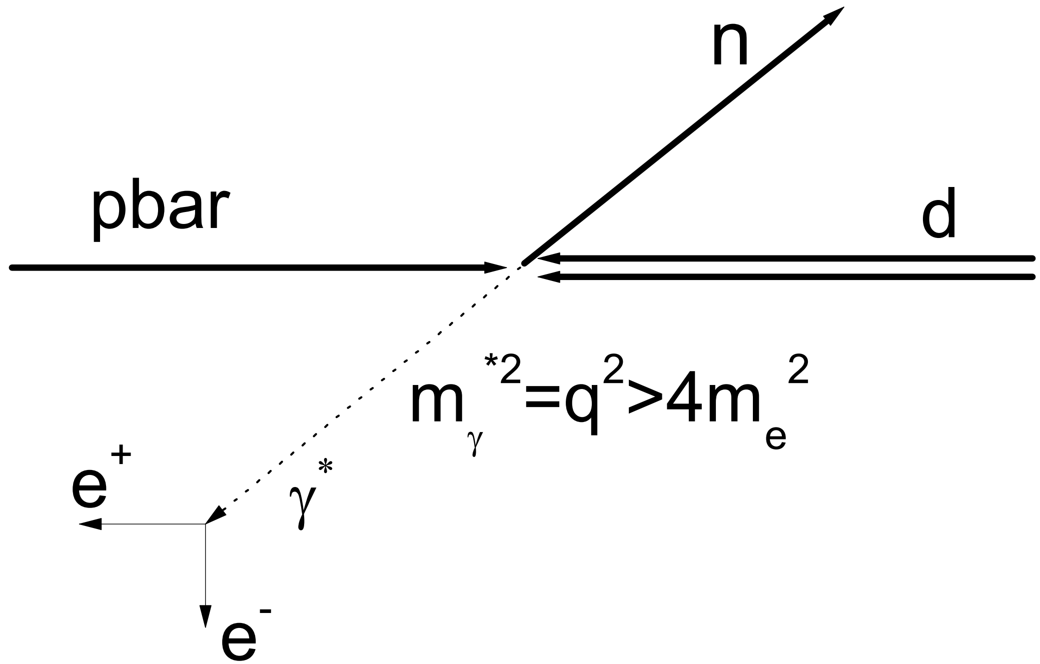
- Deaccelerate $\bar{p}$ (impossible at PANDA).
- Take a nuclear target.

• Annihilation on proton



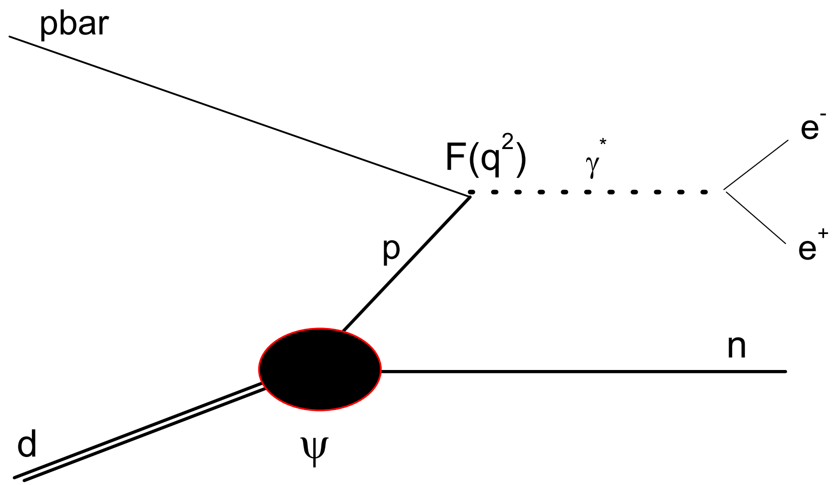
In annihilation on proton: $q^2 > 4m_p^2$, since all the $\bar{p}p$ energy is transfered to the e^+e^- pair.

• Annihilation on deuteron



In annihilation on deuteron: $q^2 > 4m_e^2$ since the $\bar{p}p$ energy can be transfered to the neutron. Virtual γ^* (mass of e^+e^-) can be very light.

• Impulse approximation



For fixed e^+e^- mass $\mathcal{M}_{e^+e^-}$, we have two-body kinematics. When mass $\mathcal{M}_{e^+e^-}$ varies, we can calculate mass distribution $\frac{d\sigma}{d\mathcal{M}_{e^+e^-}}$.

Amplitude:

$$M_{\bar{p}d \to ne^+e^-} \sim \psi_d \; M_{\bar{p}p \to e^+e^-} \; F(q^2)$$

• Cross section

Cross section:

$$d\sigma_{\bar{p}d\to ne^+e^-} \sim \int \psi_d^2 \; M^2_{\bar{p}p\to e^+e^-} \; F^2(q^2) \; dV_3$$

V_3 is three-body phase volume.

• Differential cross section

$$\frac{d\sigma}{d\mathcal{M}_{e^+e^-}} = \sigma_0(\mathcal{M}_{e^+e^-})\,\eta(\mathcal{M}_{e^+e^-})$$

where

$$\sigma_0(\mathcal{M}_{e^+e^-}) \;=\; \frac{\overline{\left|M_{\bar{p}p\to e^-e^+}\right|^2}}{32\pi m p_{\bar{p},lab}} = \frac{2\alpha^2\pi(2m^2+\mathcal{M}^2_{e^+e^-})}{3\mathcal{M}^2_{e^+e^-} m p_{\bar{p},lab}},$$

– cross section on free proton

$$\eta(\mathcal{M}_{e^+e^-}) \;=\; \frac{m p_n^* \mathcal{M}_{e^+e^-}}{(2\pi)^2\sqrt{s}} \int_{-1}^{1} |\psi(k)|^2 dz.$$

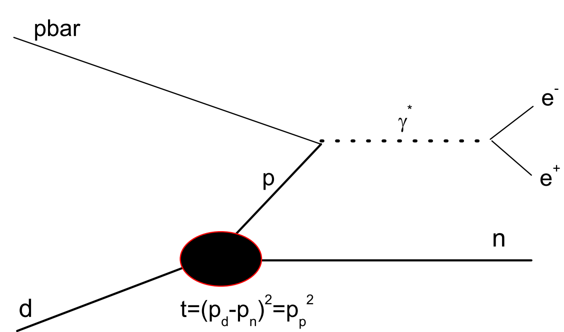
– e^+e^- mass distribution.

• Argument of wave function

$$\eta(\mathcal{M}_{e^+e^-}) \;=\; \frac{m p^*_{\mathcal{M}_{e^+e^-}n} \mathcal{M}_{e^+e^-}}{(2\pi)^2\sqrt{s}} \int_{-1}^{1} |\psi(k)|^2 dz.$$

$z = \cos\theta$, θ is the angle between $\vec{p}_d$ and $\vec{p}_n$ momenta in c.m. frame of reaction $\bar{p}d$.

$$\begin{aligned} t &= (p_d - p_n)^2 = M_d^2 - 2(E_d^* E_n^* - z\, p_d^* p_n^*) + m^2 \\ k^2 &= \frac{1}{4M_d^2}(M_d^2 + m^2 - t)^2 - m^2 \end{aligned}$$



• Annihilation on free proton

On proton:

$$\frac{d\sigma_{\bar{p}p\to e^-e^+}}{d\mathcal{M}_{e^+e^-}} = \sigma_0(\mathcal{M}_{e^+e^-})\,\delta(\mathcal{M}_{e^+e^-} - \sqrt{s_{p\bar{p}}})$$

On deuteron:

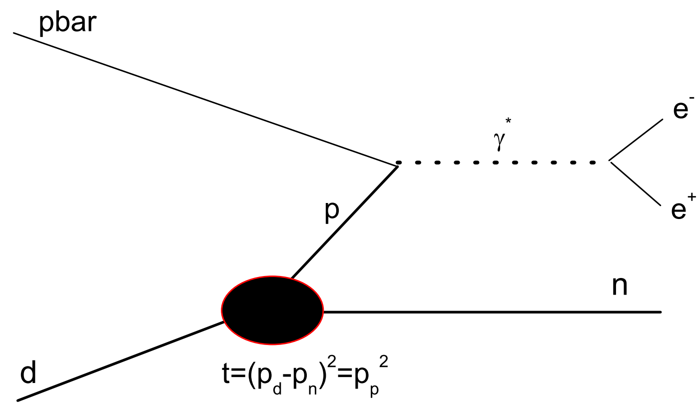
$$\frac{d\sigma}{d\mathcal{M}_{e^+e^-}} = \sigma_0(\mathcal{M}_{e^+e^-})\,\eta(\mathcal{M}_{e^+e^-})$$

Normalization of $\eta(\mathcal{M}_{e^+e^-})$:

$$\int \eta(\mathcal{M}_{e^+e^-})d\mathcal{M}_{e^+e^-} = 1 \quad \Leftarrow \text{automatic, not imposed}$$

Effect of nucleus: dilation
$\delta(\mathcal{M}_{e^+e^-} - \sqrt{s_{p\bar{p}}}) \quad \Rightarrow \quad \eta(\mathcal{M}_{e^+e^-})$.

• Momentum of proton in deuteron



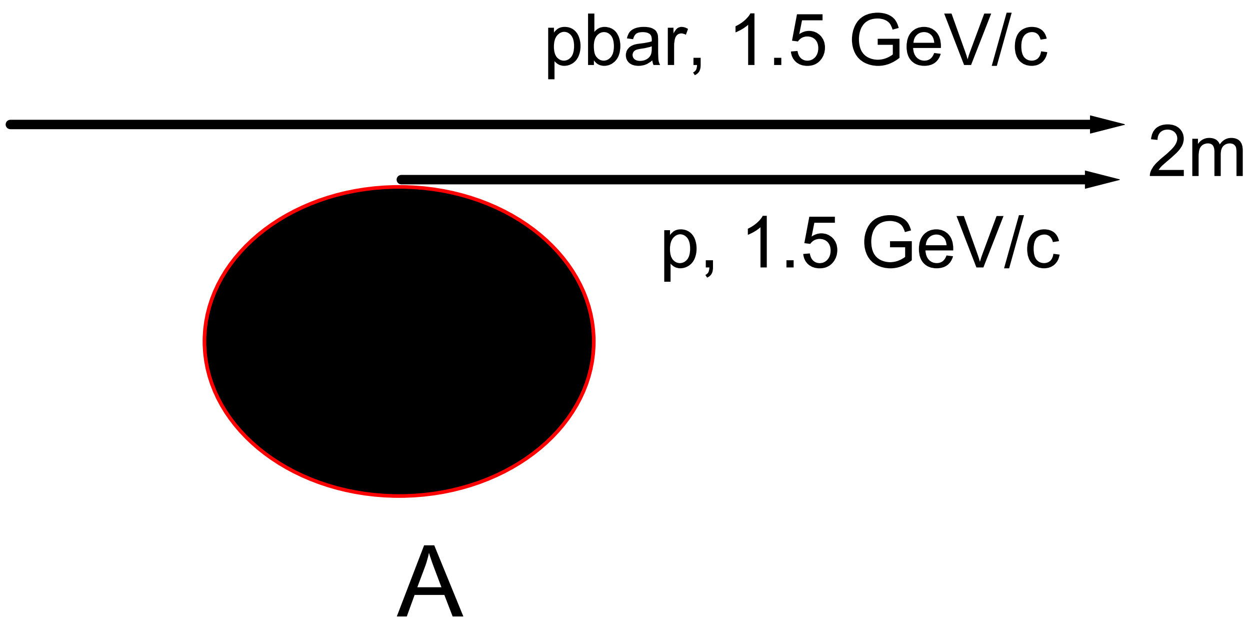
The proton momentum $\vec{p}_p$ and argument k depend on the scattering angle θ.
For given $\mathcal{M}^2_{e^+e^-} = (p_{\bar{p}} + p_p)^2 = (p_{e^+} + p_{e^-})^2$,
when $-1 \leq z \leq 1$, then k varies in the limits:

$$350\ MeV/c \leq k \leq 1500\ MeV/c$$

How is it possible to have so small $k \approx 350\ MeV/c$?
(instead of 1500 MeV/c)

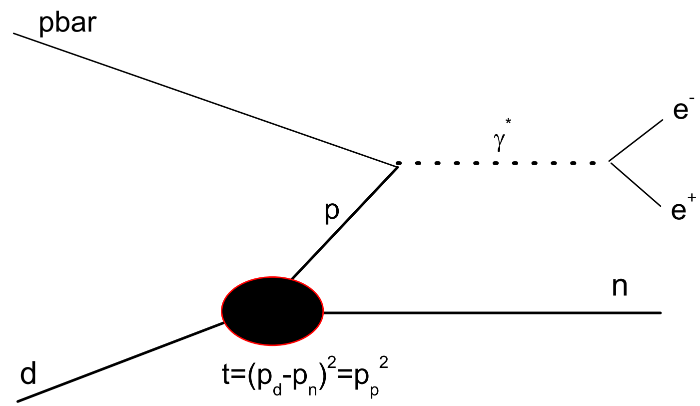
• Nucleus as a source of fast protons

Naively, we need very fast protons in nucleus:



But protons with $p = 1.5$ GeV/c are very seldom.

• Virtuality of proton in deuteron



Effective e^+e^- mass squared:

$$\mathcal{M}^2_{e^+e^-} = (p_{\bar{p}} + p_p)^2 = (E_{\bar{p}} + E_p)^2 - (\vec{p}_{\bar{p}} + \vec{p}_p)^2$$

Proton is off mass shell: $m_p^{*2} = p_p^2 = t < m_p^2$

• Proton momentum and virtuality

Effective e^+e^- mass squared:

$$\mathcal{M}^2_{e^+e^-} = (p_{\bar{p}} + p_p)^2 = (E_{\bar{p}} + E_p)^2 - (\vec{p}_{\bar{p}} + \vec{p}_p)^2$$

where $E_p = E_d - E_n$, $\vec{p}_p = \vec{p}_d - \vec{p}_n$.
The proton mass does not appear here.
It is replaced by the virtual mass $m^{*2} = t < m^2$

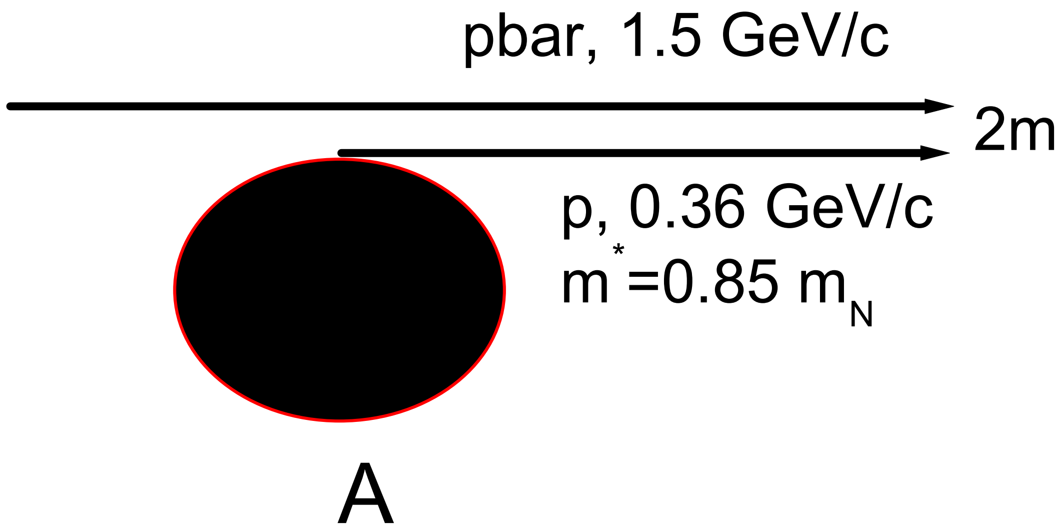
For $p_{\bar{p}} = 1.5$ GeV/c and $z = 1$, $m^{*2} = (0.85m)^2$.
To get $\mathcal{M}_{e^+e^-} = 2m$, we need:

If	$m^{*2} = m^2$,	then	$p_p = 1500$ MeV/c
Since	$m^{*2} = (0.85m)^2$,	then	$p_p = 360$ MeV/c

We need not so high momenta in deuteron, for which momentum distribution is well known.

• Nucleus as a source of not so fast

but off-mass shell protons



Little decrease of the proton mass $m \to m^* = 0.85m$ *due to the off-shellness allows to reduce the proton momentum* $p = 1.5 \to 0.36\ GeV/c$ *and still to get the* e^+e^- *mass near* $\bar{p}p$ *threshold.*

Protons with $p = 0.36$ *GeV/c are not seldom!*

This was a physical explanation of what automatically happens in $\bar{p}$ annihilation on deuteron.

Virtuality of proton in deuteron is **not** an assumption.

Remark: virtuality of proton $m^* = 0.85m$ does not mean change of the proton structure in nucleus.

• Numerical calculations

Cross section on free proton

$$\sigma_0(\mathcal{M}_{e^+e^-}) = \frac{\overline{\left|M_{\bar{p}p\to e^-e^+}\right|^2}}{32\pi m p_{\bar{p},lab}} = \frac{2\alpha^2\pi(2m^2+\mathcal{M}^2_{e^+e^-})}{3\mathcal{M}^2_{e^+e^-} m p_{\bar{p},lab}}$$

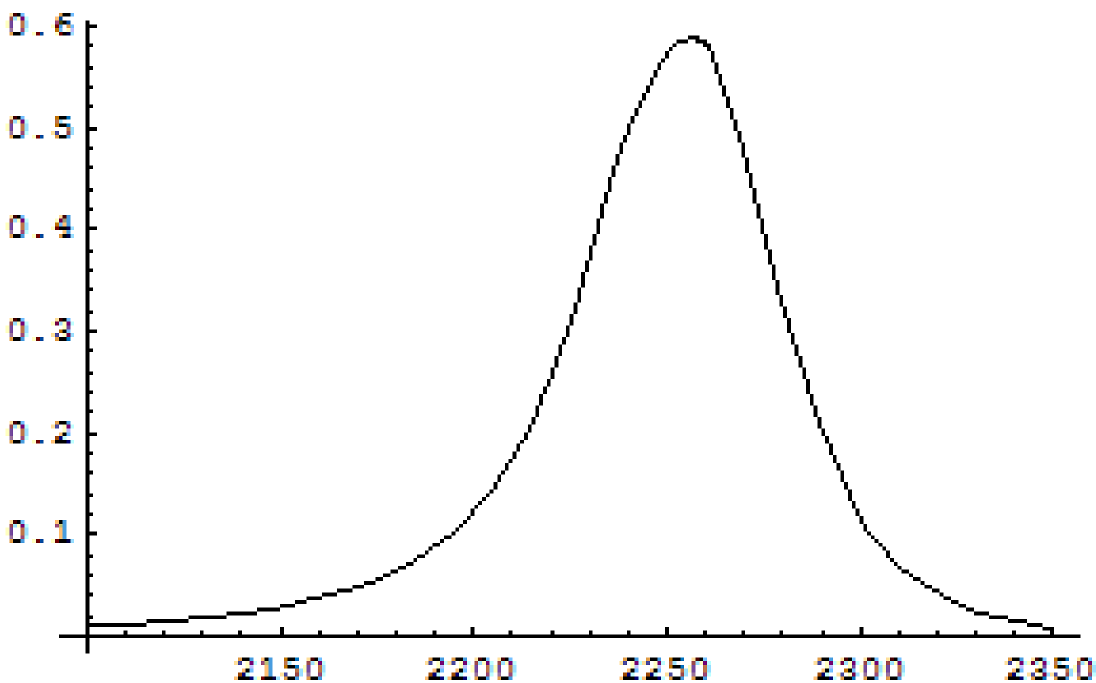
$$p_{\bar{p},lab} = 1.5\ \text{GeV/c} \quad \Rightarrow \quad \mathcal{M}_{e^+e^-} = 2257\ \text{MeV/c}^2$$

$$\sigma_0 \approx 40\ nb$$

$\mathcal{M}_{e^+e^-}$ distribution

$$\frac{d\sigma}{d\mathcal{M}_{e^+e^-}} = \sigma_0(\mathcal{M}_{e^+e^-})\,\eta(\mathcal{M}_{e^+e^-})$$

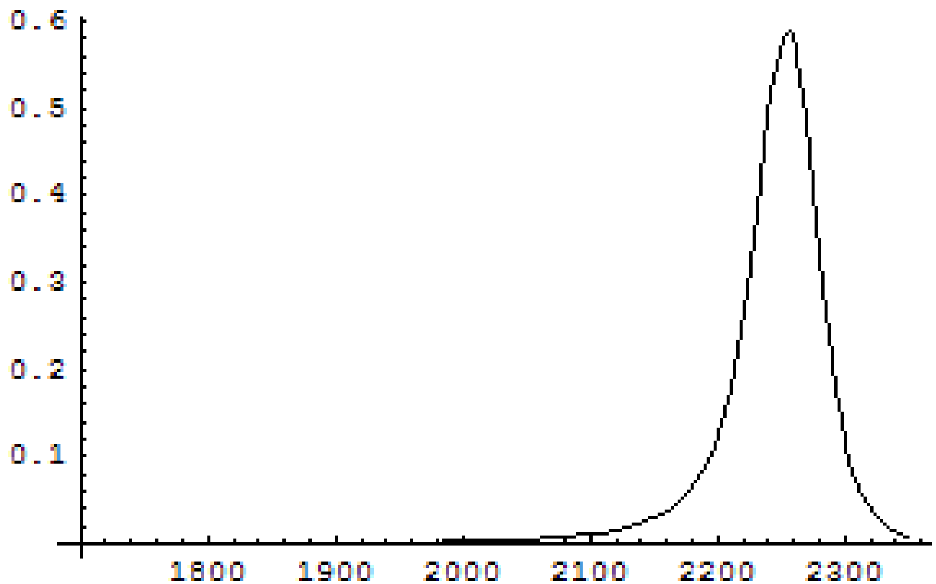
Near peak interval: 2100 MeV $\leq \mathcal{M}_{e^+e^-} \leq$ 2350 MeV



Peak at $\mathcal{M}_{e^+e^-} = 2257$ Mev – just the $\bar{\Lambda}\Lambda$ threshold.

$\mathcal{M}_{e^+e^-}$ distribution

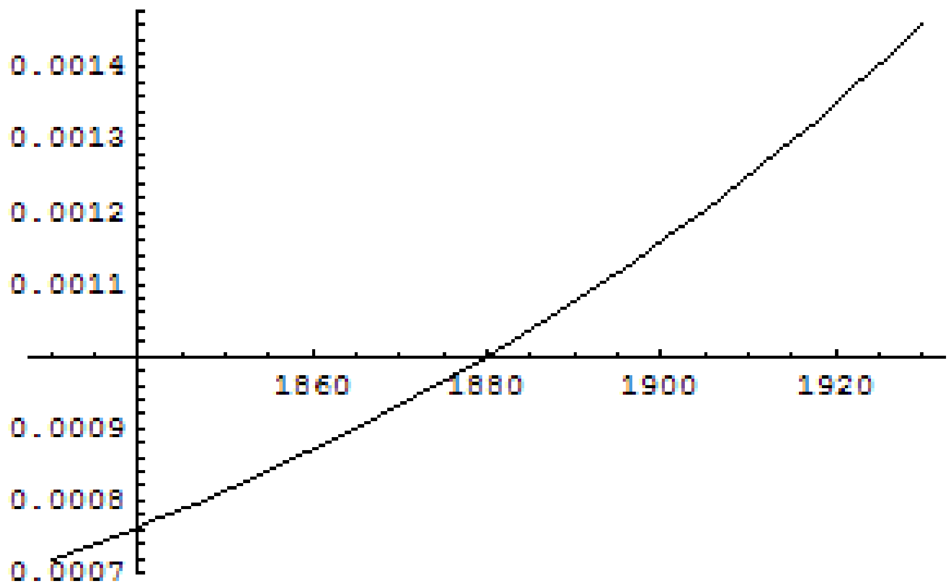
More wide interval: 1750 MeV $\leq \mathcal{M}_{e^+e^-} \leq$ 2350 MeV



$$\left.\frac{d\sigma}{d\mathcal{M}_{e^+e^-}}\right|_{\mathcal{M}_{e^+e^-}=1880\ MeV} \approx \frac{1}{600}\left.\frac{d\sigma}{d\mathcal{M}_{e^+e^-}}\right|_{\mathcal{M}_{e^+e^-}=2257\ MeV}$$

$\mathcal{M}_{e^+e^-}$ distribution

Near $p\bar{p}$ threshold: 1830 MeV $\leq \mathcal{M}_{e^+e^-} \leq$ 1930 MeV
$\frac{d\sigma}{d\mathcal{M}_{e^+e^-}}$ is in nb/MeV:



$$\left.\frac{d\sigma}{d\mathcal{M}_{e^+e^-}}\right|_{\mathcal{M}_{e^+e^-}=2m} \approx 1\frac{pb}{MeV}$$ –for "pointlike" proton

Annihilation on heavy nuclei

$$\bar{p}A \rightarrow e^+e^-(A-1)^*$$

Residual nucleus $(A-1)$ may be excited (discrete and continuous spectrum).

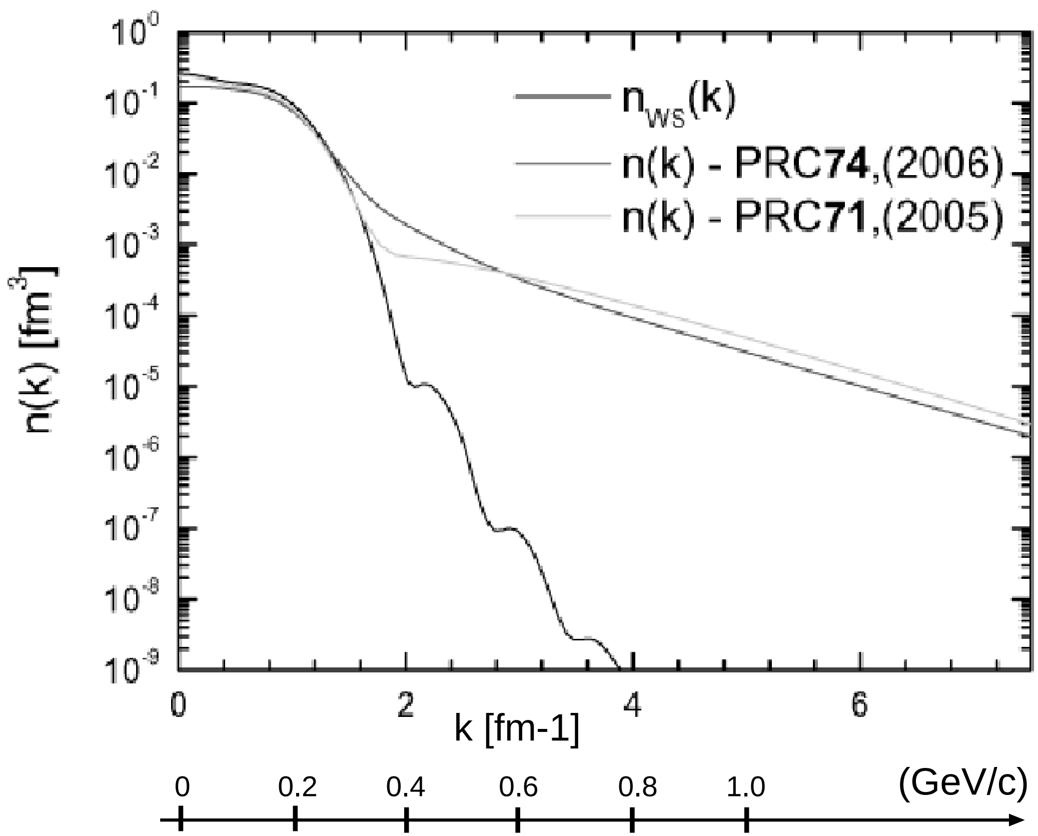
$$\frac{d\sigma}{d\mathcal{M}_{e^+e^-}} = \sigma_0(\mathcal{M}_{e^+e^-})\,\eta(\mathcal{M}_{e^+e^-}), \quad \eta(\mathcal{M}_{e^+e^-}) \sim \int_{-1}^{1} dz \int_{E_{min}}^{E_{max}} dE\, S$$

$$\int_{E_{min}}^{\infty} S(E,q)\, dE = n(q)$$

$S(E,q)$ is nuclear spectral function
$n(q)$ is momentum distribution in nucleus.

Nuclear momentum distributions

A. Antonov et al., Phys. Rev. **C71**, 014317 (2005)
A. Antonov et al., Phys. Rev. **C74**, 024603 (2006)



The tails of distributions are almost the same for all nuclei.

Mass distributions

$$\frac{d\sigma_{\bar{p}A\to e^+e^-X}}{d\mathcal{M}_{e^+e^-}} = Z\sigma_0(\mathcal{M})_{e^+e^-}\ \eta_A(\mathcal{M}_{e^+e^-}), \quad \eta_A(\mathcal{M}_{e^+e^-}) \sim \int_{-1}^{1} dz n_A$$

$$\frac{d\sigma_{\bar{p}A\to e^+e^-X}}{d\mathcal{M}_{e^+e^-}} \approx 6.5Z\ \frac{pb}{MeV}$$

$$Z(C^{12}) = 6,\ Z(Fe^{56}) = 26,\ Z(Au^{197}) = 79$$

$$\sigma(C^{12}) = 39\ pb, \quad \sigma(Fe^{56}) = 169\ pb, \quad \sigma(Au^{197}) = 513\ pb$$

– Seems quite measurable. However, luminosity for heavy nuclei is much smaller than for deuteron (Helene Fonvieille).

Measuring form factor

$$\frac{d\sigma}{d\mathcal{M}_{e^+e^-}} = \sigma_0(\mathcal{M}_{e^+e^-})\,\eta(\mathcal{M}^2_{e^+e^-})\,F^2(\mathcal{M}_{e^+e^-}), \quad \mathcal{M}^2_{e^+e^-} \equiv q^2$$

In general, form factor depends on three off-shell variables:

$$F = F(q^2, p_p^2, p_{\bar{p}}^2)$$

In annihilation on nucleus it is off-shell in two variables:

$$F = F(q^2, p_p^2 \leq (0.85m)^2, m^2)$$

Whereas, the structures are predicted for:

$$F = F(q^2, m^2, m^2)$$

Proton form factor

Do the structures in

$$F = F(q^2, m^2, m^2)$$

survive in the off-shell form factor

$$F = F(q^2, p_p^2 \leq (0.85m)^2, m^2) \quad ?$$

– Most probably, yes, but, strictly speaking, unknown.
One can expect that the dependence of $F(q^2, p_p^2, m^2)$ on p_p^2 is smooth. Therefore, the structures, most probably, survive.

Conclusion

- In annihilation of fast anti-protons on deuteron (~ 1.5 GeV/c), the production of the e^+e^- pair near $\bar{N}N$ threshold does not require extremely high proton momenta, since the proton is off-mass shell.
- Because of that, the cross section is not negligibly small:

$$\left.\frac{d\sigma}{d\mathcal{M}_{e^+e^-}}\right|_{\mathcal{M}_{e^+e^-}=2m} \approx 1\,\frac{pb}{MeV}$$

Can it be measured at Panda?

To be continued by Helene Fonvieille.