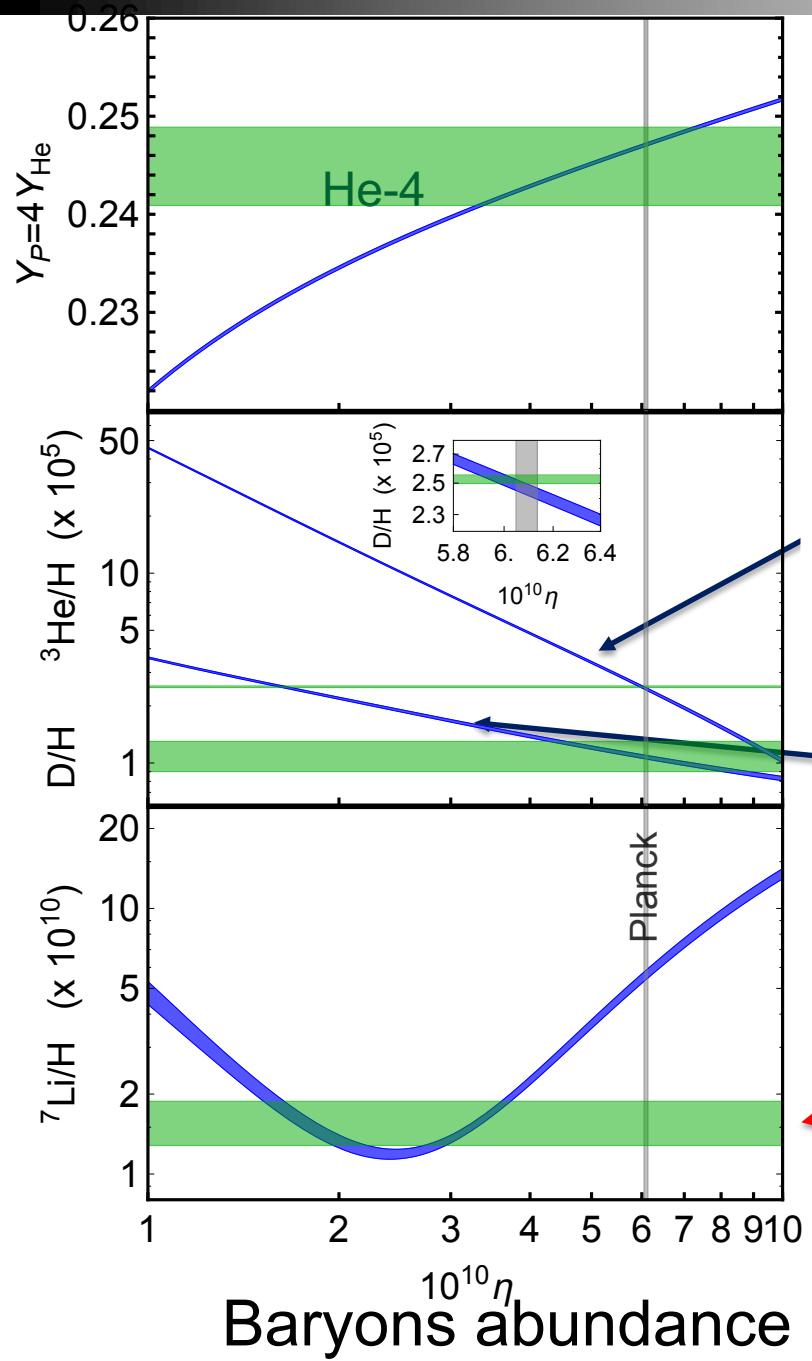


# Precision Big-Bang Nucleosynthesis with improved He-4 predictions

Cyril Pitrou (collaboration with A. Coc, J.-P. Uzan, E. Vangioni)

PRIMAT <http://www2.iap.fr/users/pitrou/primat.htm> (1801.08023, Physics Reports)

# Why precision for BBN ?



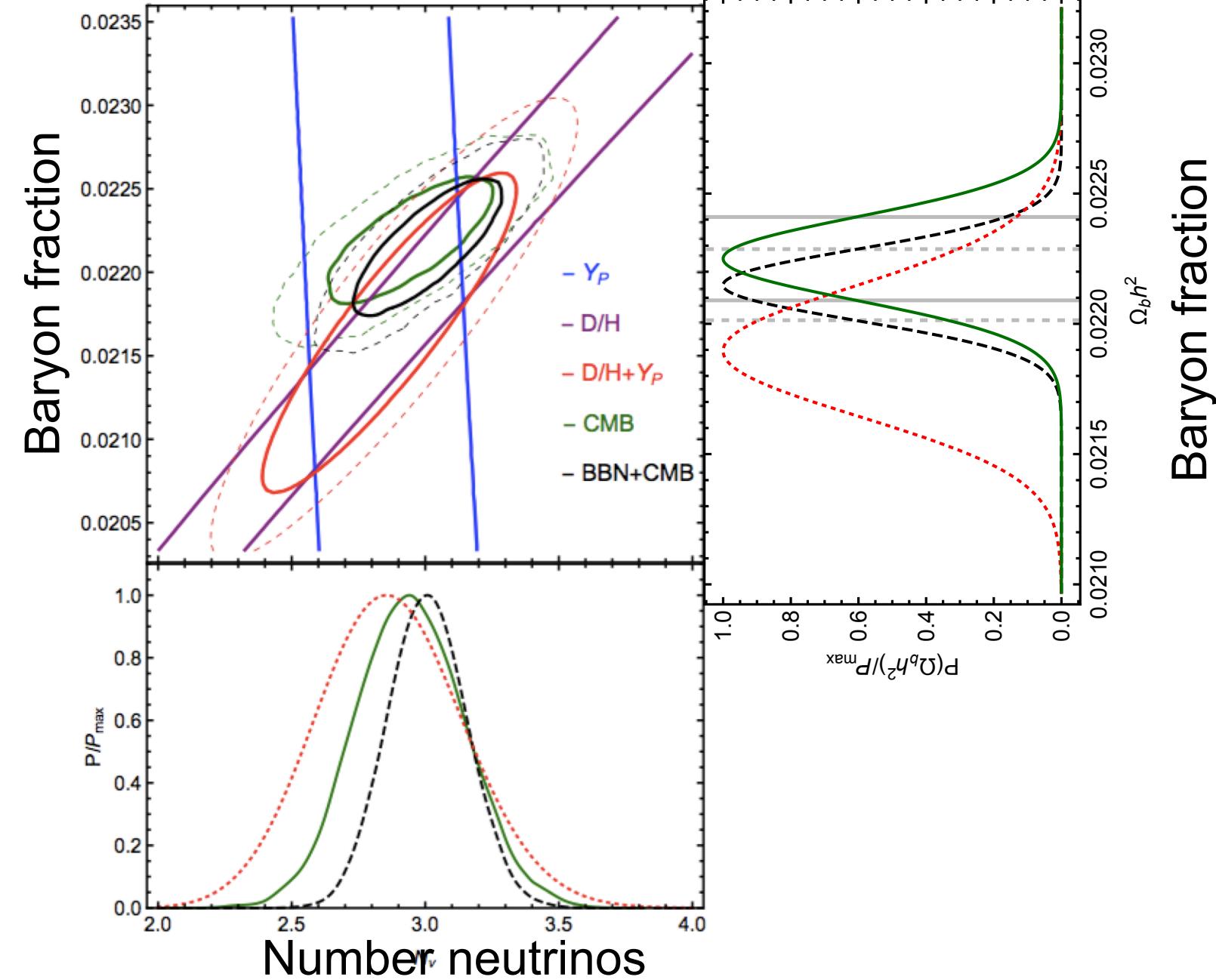
Aver et al. 2015

$$Y_P = 0.2449 \pm 0.0040,$$

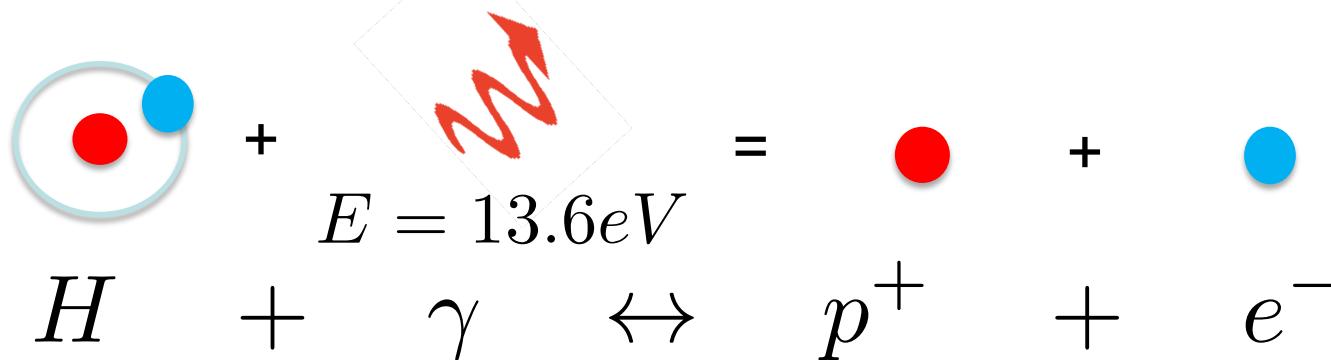
1.6 %

$$D/H = (2.527 \pm 0.030) \times 10^{-5}$$

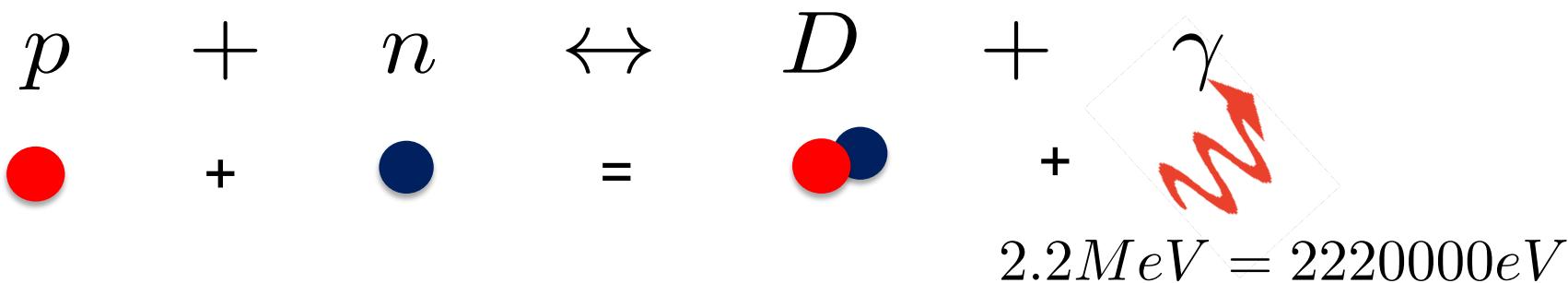
1.2 %



## Chemical reaction



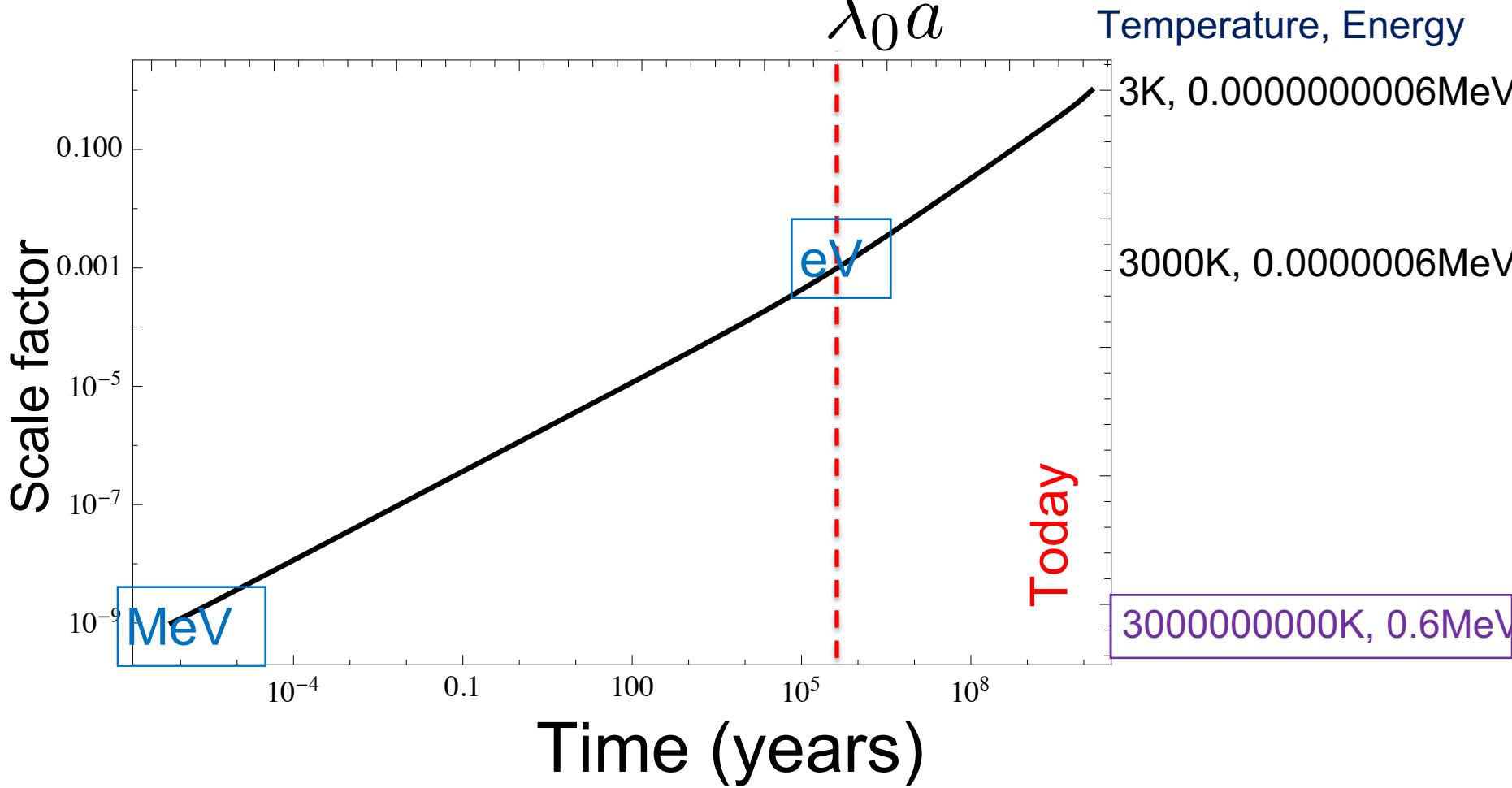
## Nuclear reaction



$$938.2 + 939.5 = 1875.5 + 2.2$$

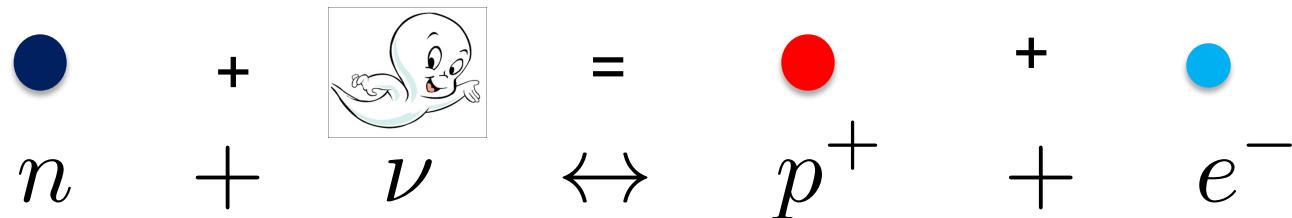
# Cosmic scaling

$$E = 2.70 k_B T = \frac{hc}{\lambda_0 a}$$



# Neutron abundances

## Weak interactions



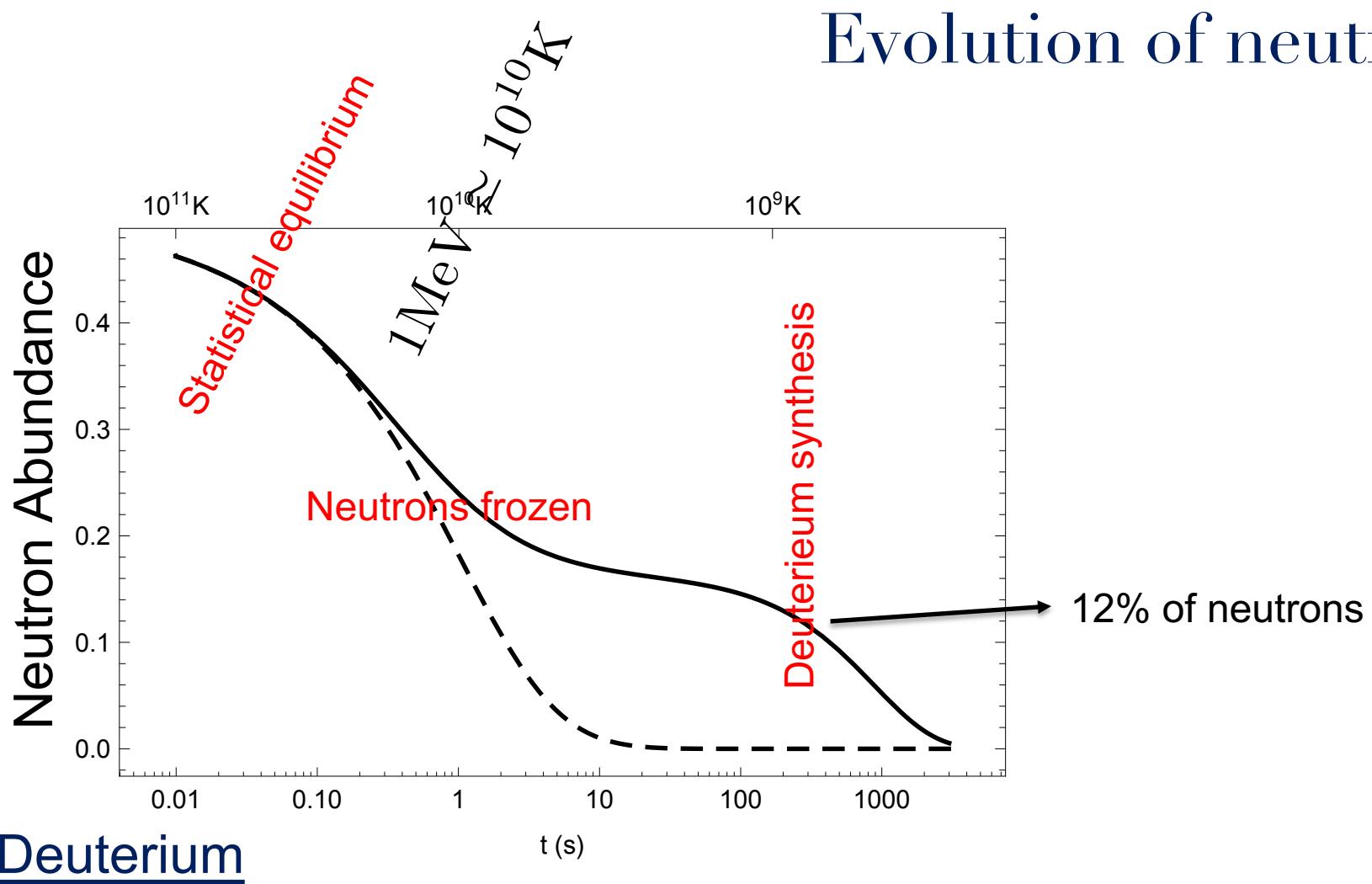
If enough interactions, then statistical equilibrium

$$n = e^{-\frac{E}{k_B T}}$$

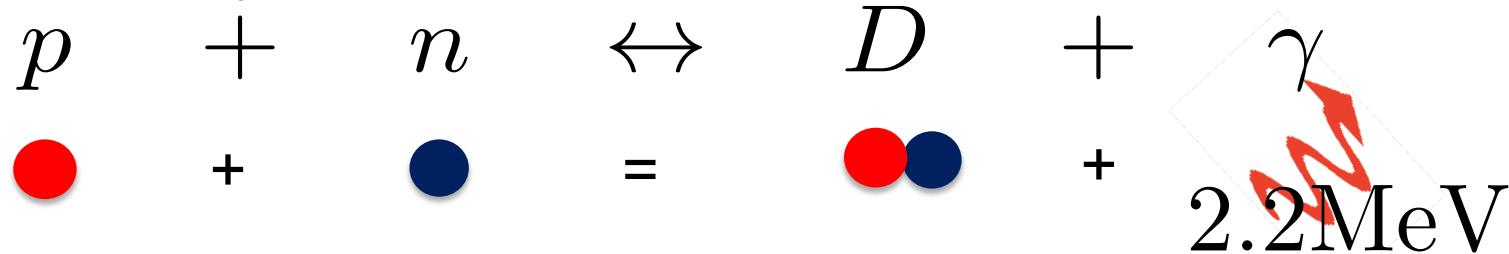
Protons       $n = e^{-\frac{938.2}{k_B T}}$

Neutrons       $n = e^{-\frac{939.5}{k_B T}} = e^{-\frac{938.2}{k_B T}} e^{-\frac{1.3}{k_B T}}$

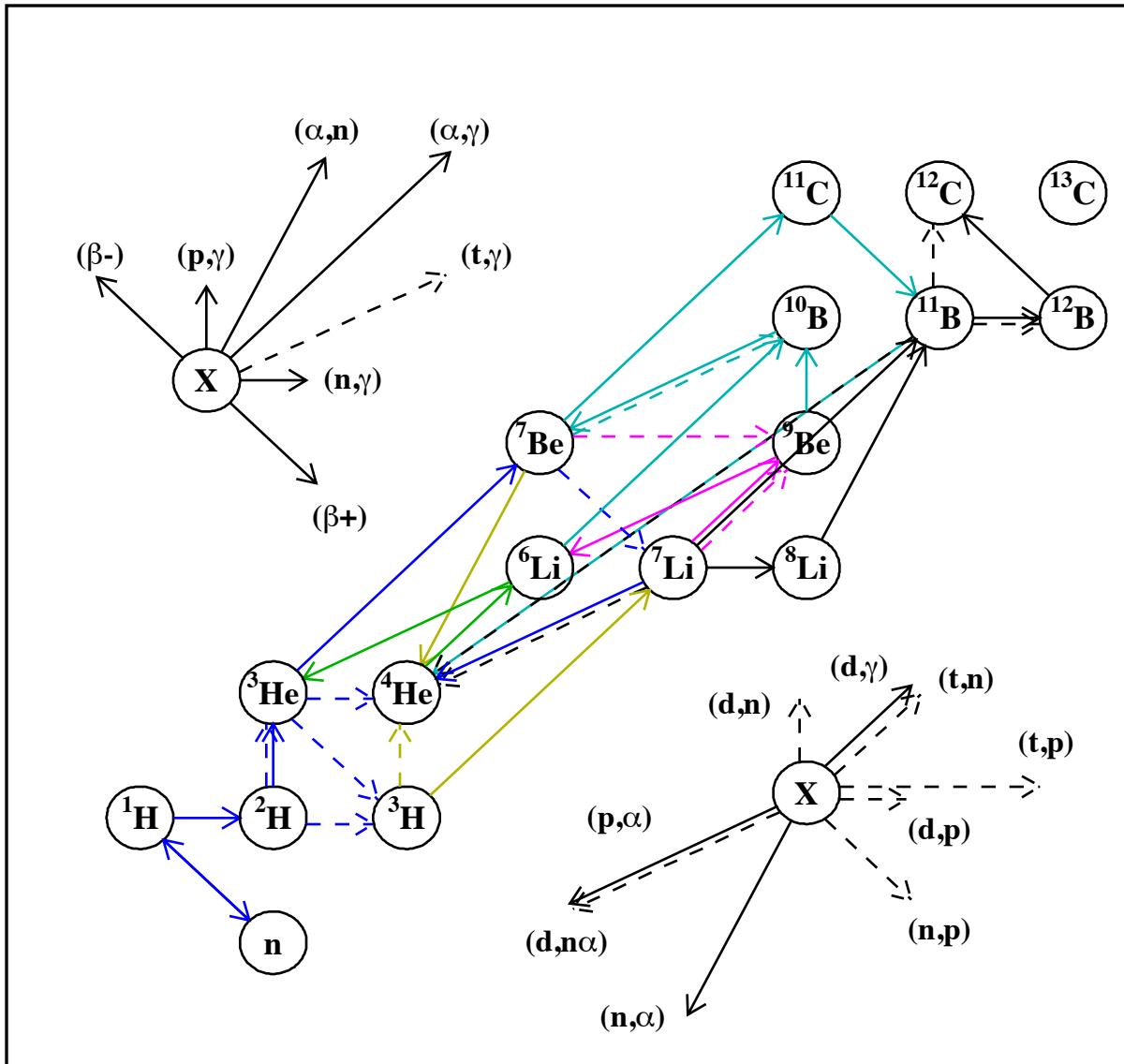
# Evolution of neutrons



Baryons :  $2.4 \cdot 10^{-7} \text{ cm}^{-3}$



# Other reactions



# Nuclear Reactions

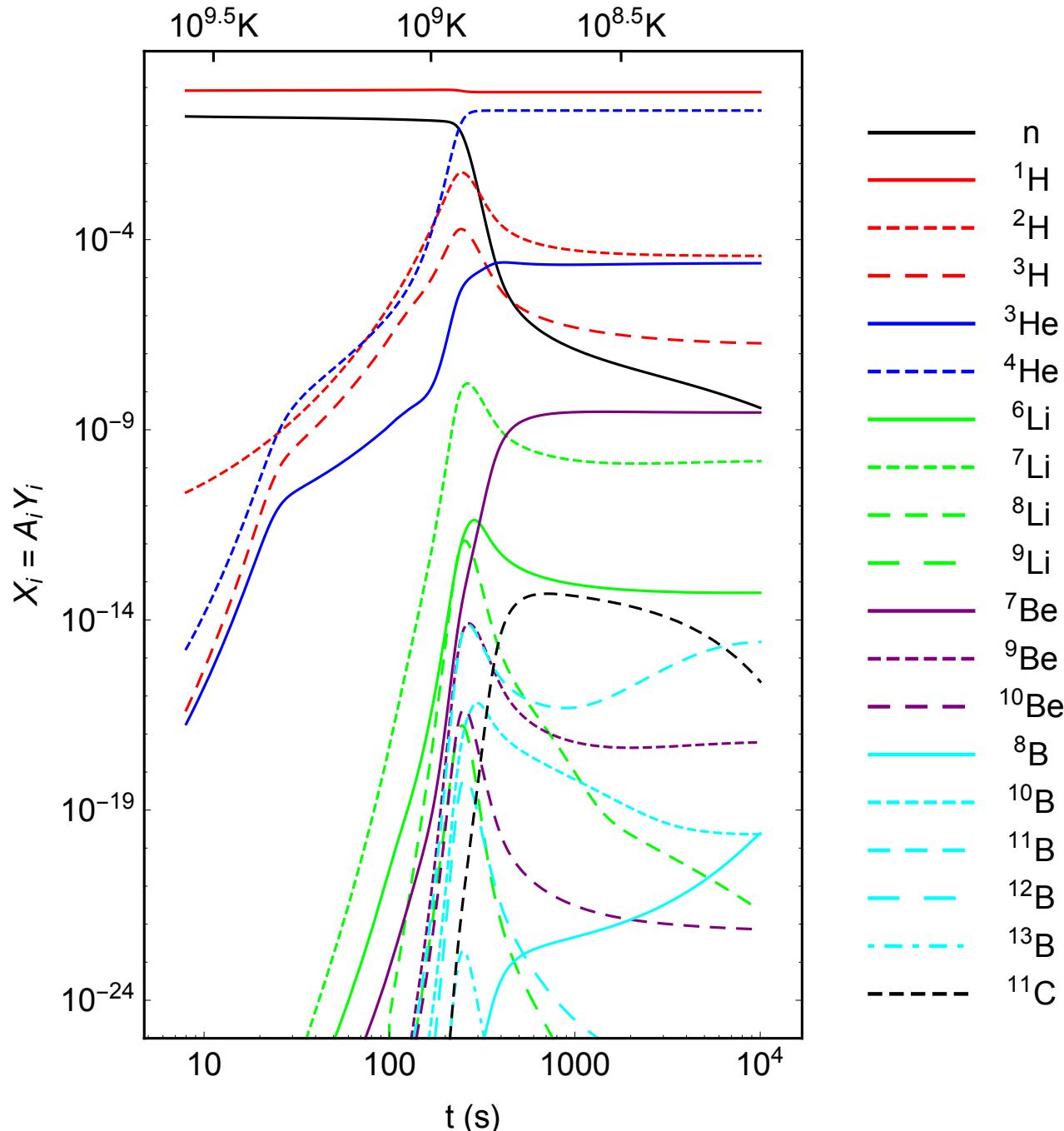
$$Y_i \equiv \frac{n_i}{n_{\text{tot}}}$$

$$\dot{Y}_{i_1} = \sum_{i_2 \dots i_p, j_1 \dots j_q} N_{i_1} \left( \Gamma_{j_1 \dots j_q \rightarrow i_1 \dots i_p} \frac{Y_{j_1}^{N_{j_1}} \dots Y_{j_q}^{N_{j_q}}}{N_{j_1}! \dots N_{j_q}!} - \Gamma_{i_1 \dots i_p \rightarrow j_1 \dots j_q} \frac{Y_{i_1}^{N_{i_1}} \dots Y_{i_p}^{N_{i_p}}}{N_{i_1}! \dots N_{i_p}!} \right)$$

Tabulated nuclear rates (Alain Coc, Elisabeth Vangioni)  
433 reactions + weak rates.

Reverse rates from detailed balance :

$$\frac{\gamma_{j_1 \dots j_q \rightarrow i_1 \dots i_p}}{\gamma_{i_1 \dots i_p \rightarrow j_1 \dots j_q}} = \frac{\prod_{i=i_1 \dots i_p} \frac{1}{N_i!} \left[ g_i \left( \frac{m_i T}{2\pi} \right)^{3/2} \right]^{N_i}}{\prod_{j=j_1 \dots j_q} \frac{1}{N_j!} \left[ g_j \left( \frac{m_j T}{2\pi} \right)^{3/2} \right]^{N_j}} \exp \left( \frac{\sum_{j=1}^q m_j - \sum_{i=1}^p m_i}{T} \right)$$



# Precision BBN

## Numerical Method

- 1) Solve for plasma (and cosmology)  $t, a, T$
- 2) Compute weak rates *with all small corrections*
- 3) Solve nuclear network (uncertainty on nuclear rates)

*This is valid because*

- 1) Baryons are subdominant.
- 2) Energy release by weak reactions is negligible.
- 3) Energy release by nuclear reactions is also negligible

## Plasma thermodynamics

$$n = g \int f(p) \frac{4\pi p^2 dp}{(2\pi)^3}$$

$$\rho = g \int f(p) E \frac{4\pi p^2 dp}{(2\pi)^3}$$

$$P = g \int f(p) \frac{p^2}{3E} \frac{4\pi p^2 dp}{(2\pi)^3}$$

Conservation of Entropy  $s = \frac{\rho + P}{T}$

$$sa^3 = \text{Cte}$$

$$a(T) \leftrightarrow T(a)$$

## Solve for cosmological evolution

$$\rho_{\text{plasma}} = \rho_{e^+} + \rho_{e^-} + \rho_\gamma$$

$$\rho_{\text{rad}} = \rho_{\text{neutrinos}} + \rho_{\text{plasma}}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho_{\text{tot}}(T(a))$$

Allows to obtain  $a(t)$  and  $t(a)$

# QED Plasma effects

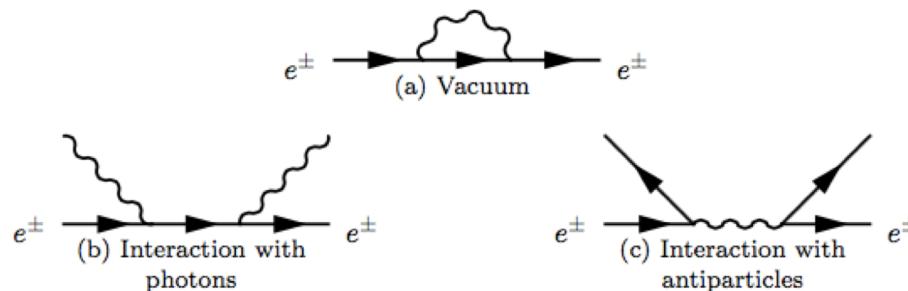


FIG. 5 *Top* : electron/positron self-energy. *Bottom* : electron/positron mass shift from interaction with plasma.



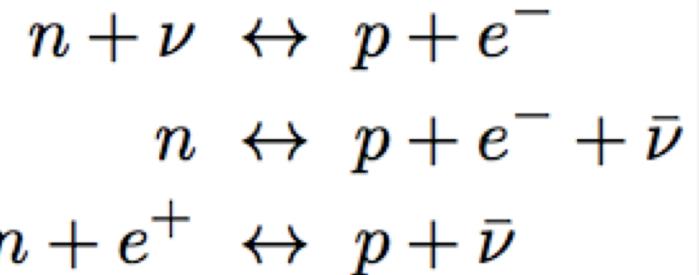
FIG. 6 *Left* : photon self-energy. *Right* : photon mass shift from interaction with electron/positron plasma.

# Incomplete decoupling of neutrinos

Around 0.511 MeV  $e^+ + e^- \rightarrow 2\gamma$

However there are some residual  $e^+ + e^- \rightarrow \nu + \bar{\nu}$

$$\begin{aligned}\dot{n}_n + 3Hn_n &= -n_n\Gamma_{n \rightarrow p} + n_p\Gamma_{p \rightarrow n} \\ \dot{n}_p + 3Hn_p &= -n_p\Gamma_{p \rightarrow n} + n_n\Gamma_{n \rightarrow p}\end{aligned}$$



- 1) Born approximation
- 2) Finite nucleon mass effects
- 3) Radiative corrections at T=0 (virtual photons)
- 4) True photons corrections (bremsstrahlung)
- 5) Finite temperature radiative corrections

$$E_n - E_p = \Delta + \delta Q$$

$$\Delta = m_n - m_p.$$

Born order

Finite nucleon mass corrections

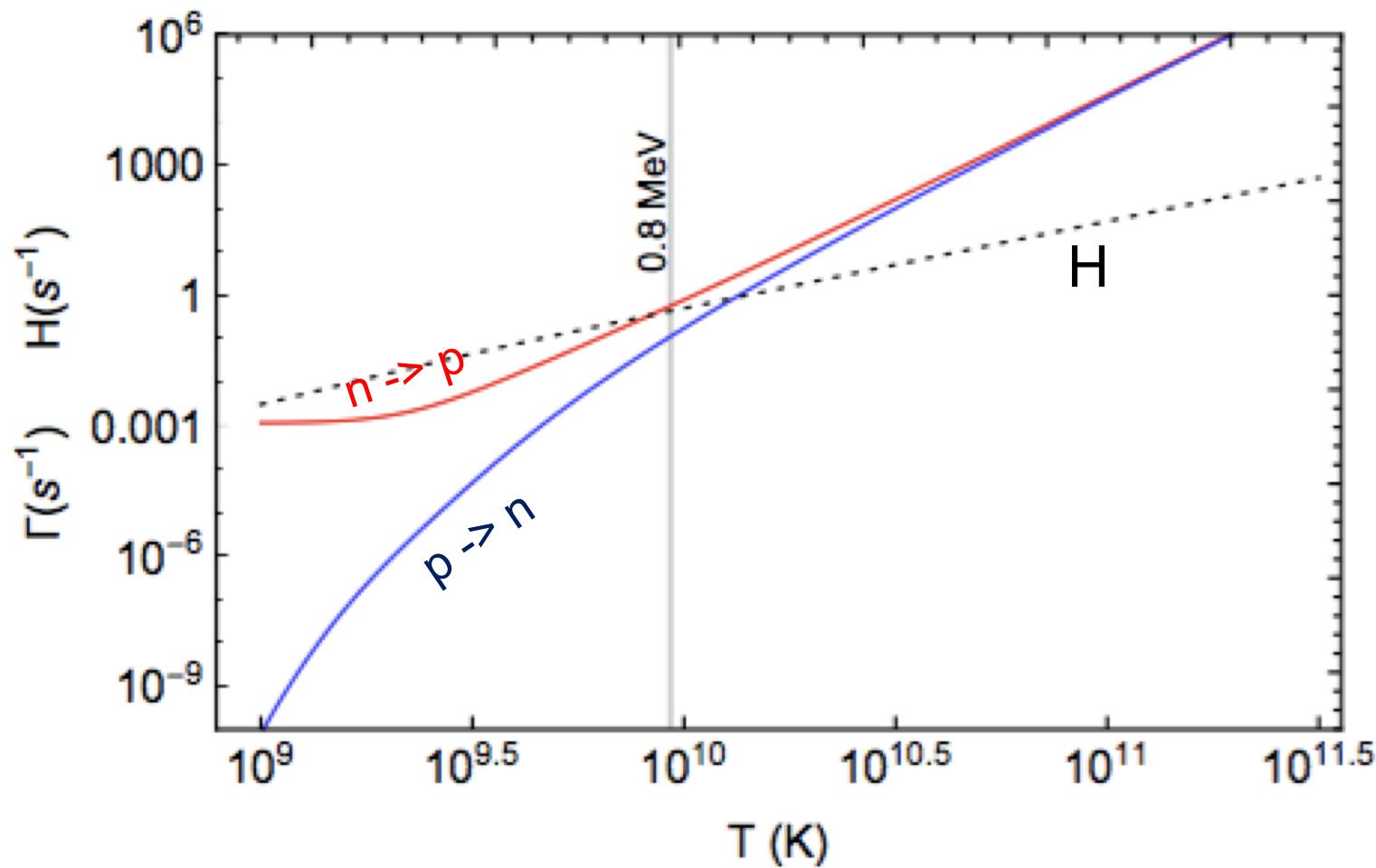
$$\delta(E_n - E_p + E_\nu - E_e) = \boxed{\delta(\Sigma)} + \delta'(\Sigma)\delta Q + \frac{1}{2}\delta''(\Sigma)(\delta Q)^2 + \dots$$

$$\Sigma \equiv \Delta + E_\nu - E_e$$

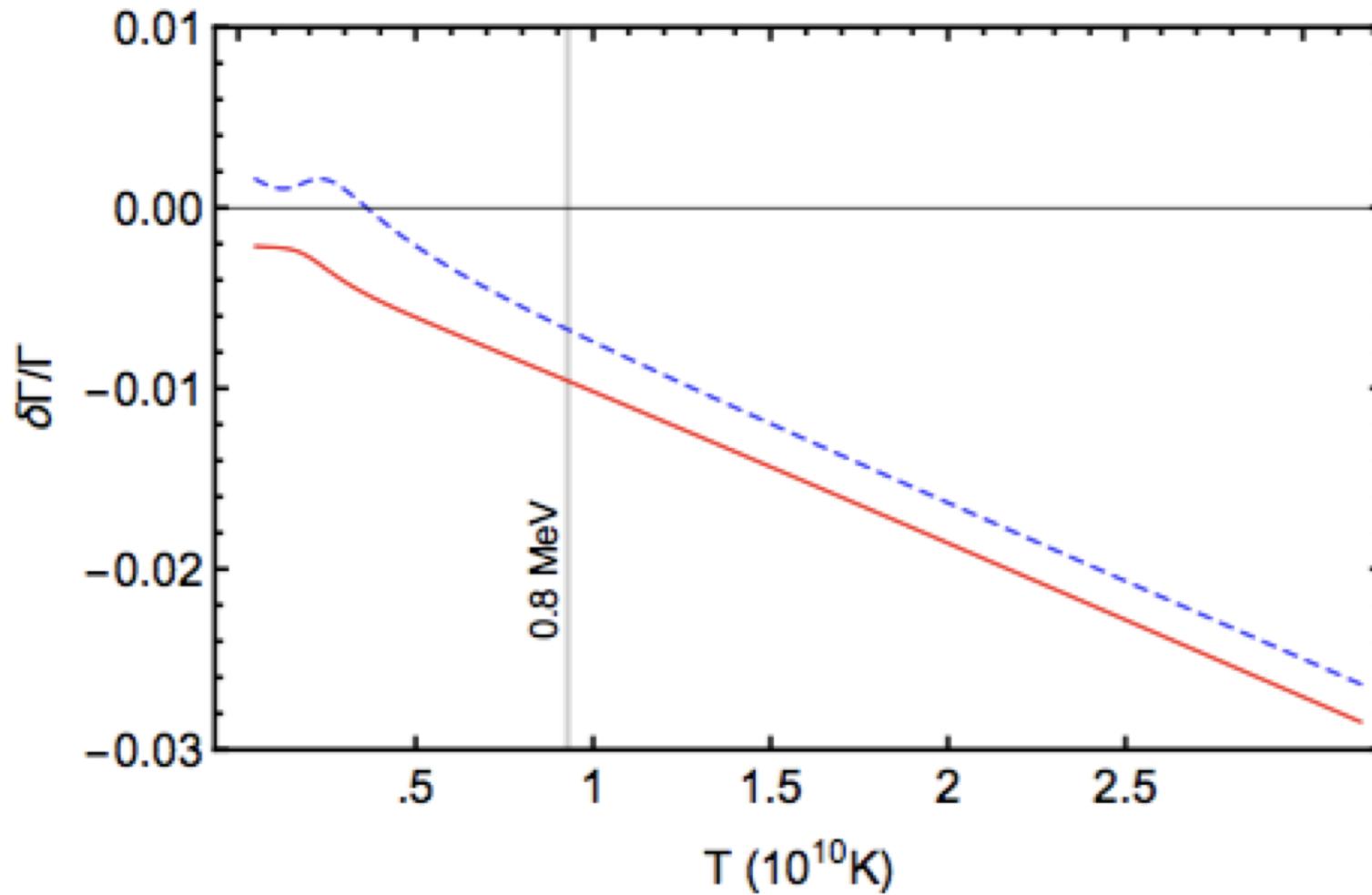
Same kind of expansion for the matrix element  $|M|^2$

$$\Gamma_{n \rightarrow p} = \boxed{\bar{\Gamma}_{n \rightarrow p}} + \boxed{\delta \Gamma_{n \rightarrow p}}$$

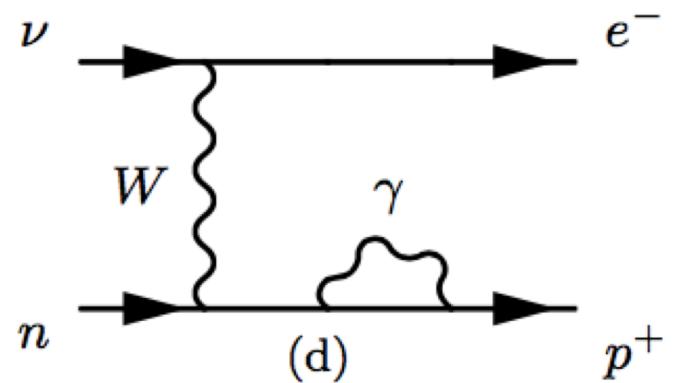
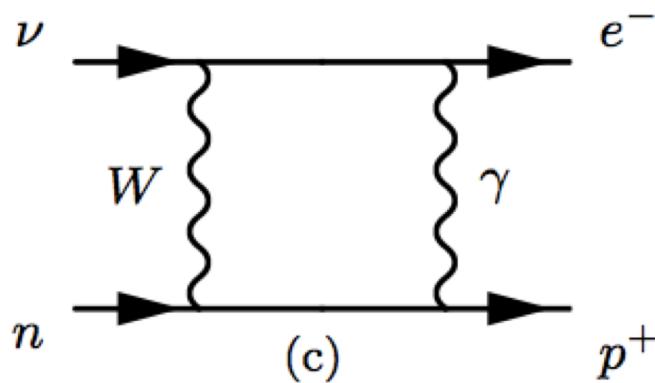
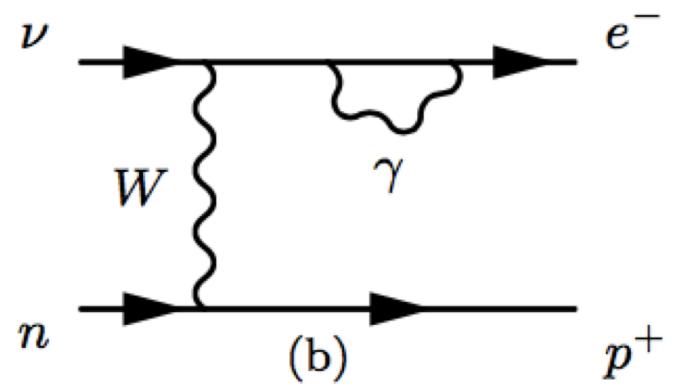
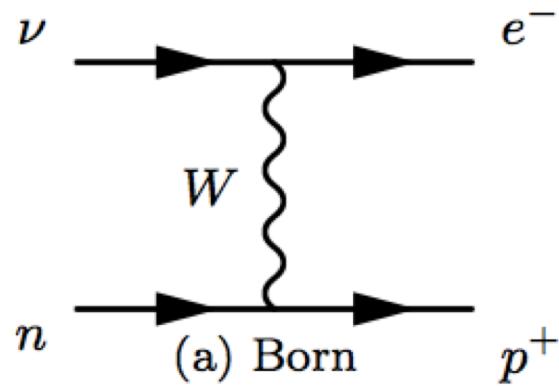
# BORN approximation rates



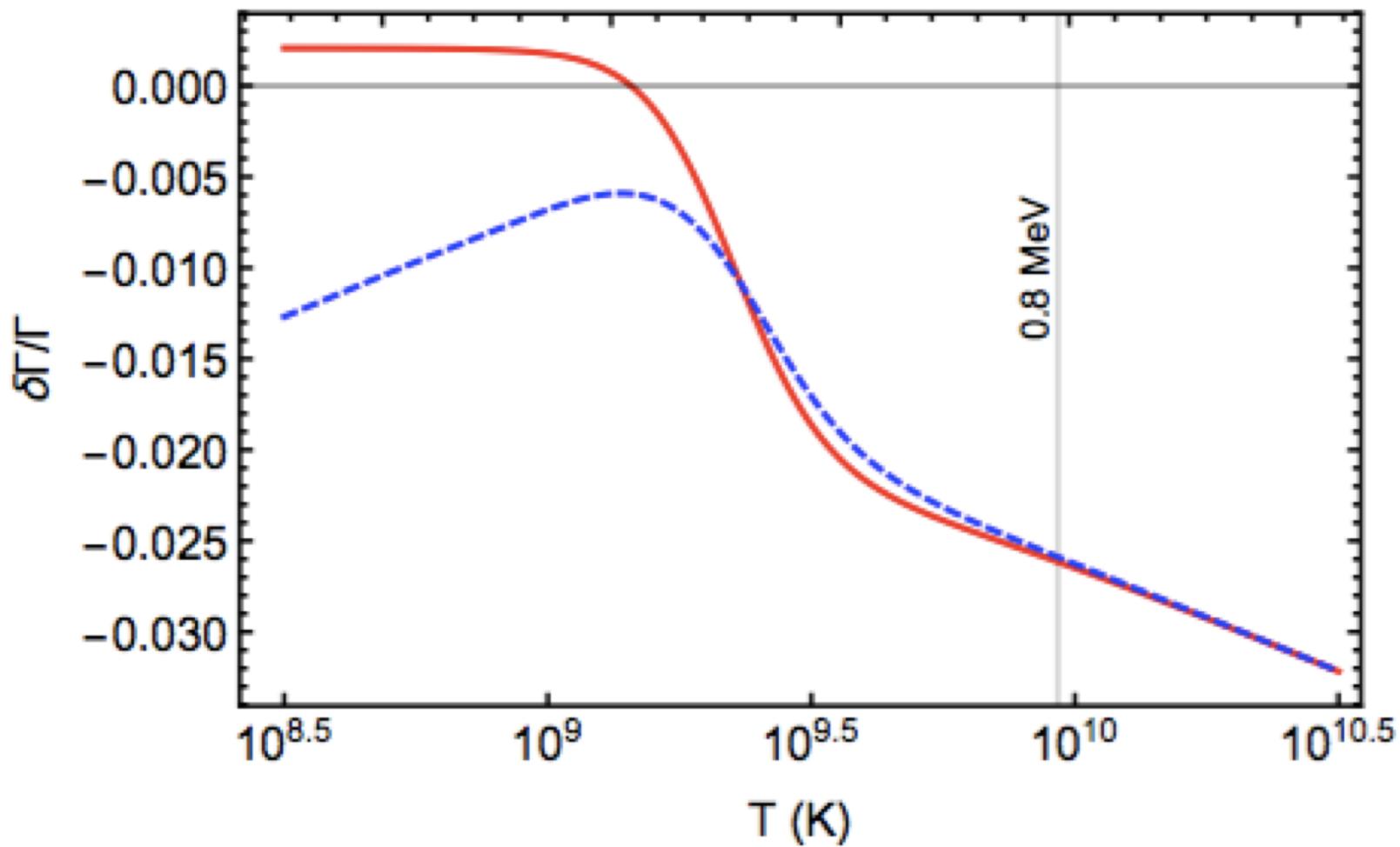
# Finite nucleon mass corrections



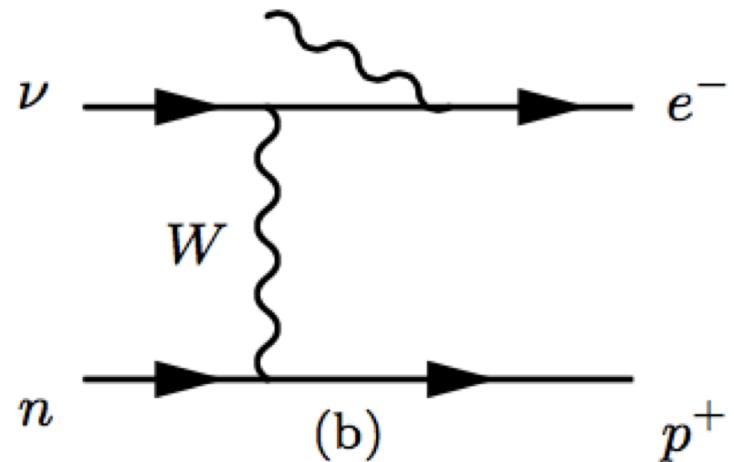
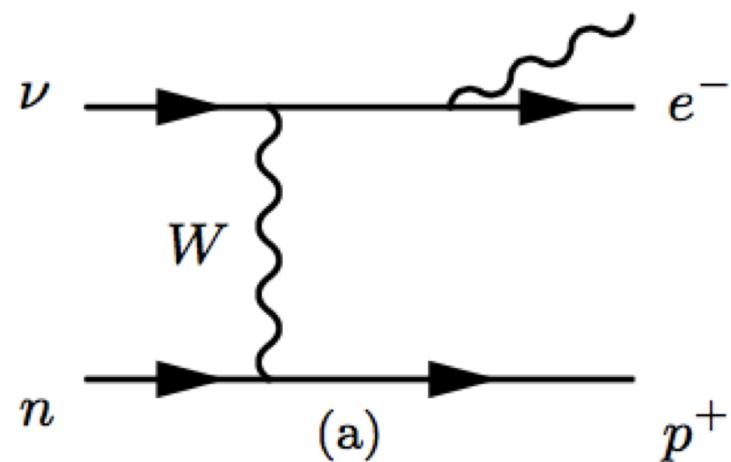
# Radiative corrections



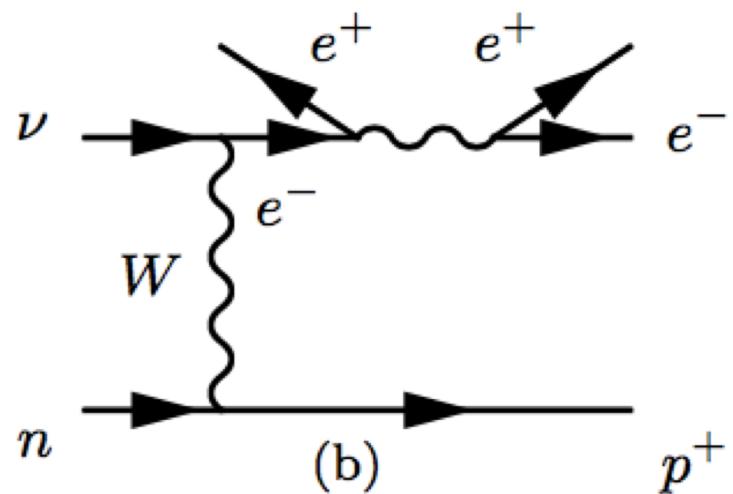
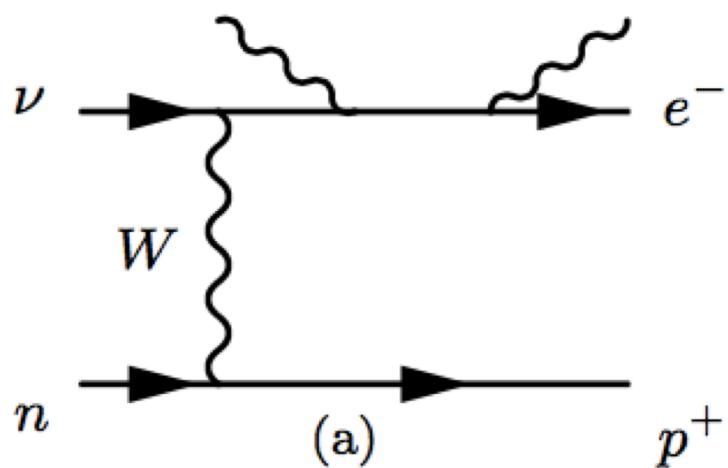
## Radiative corrections



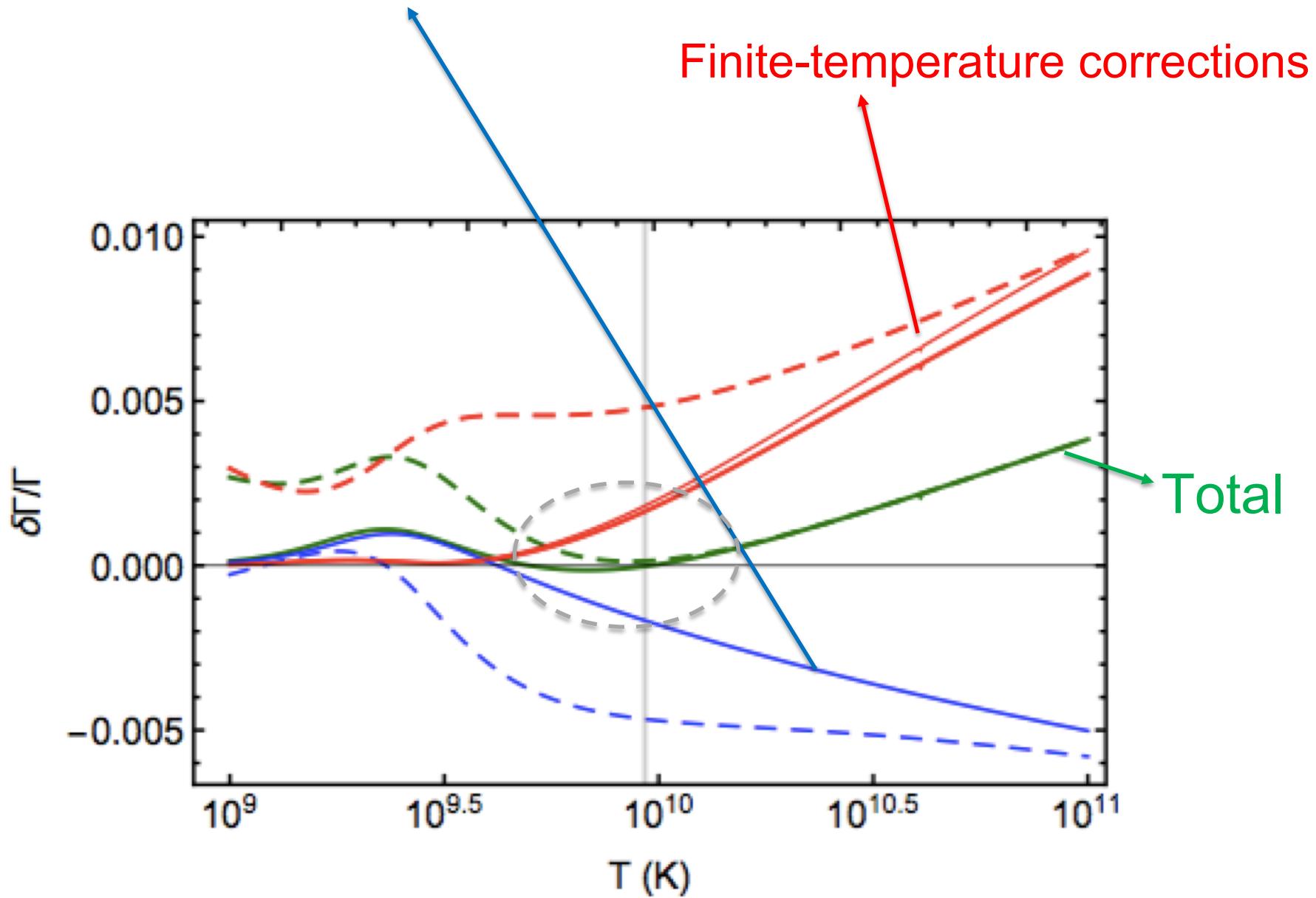
# True photons



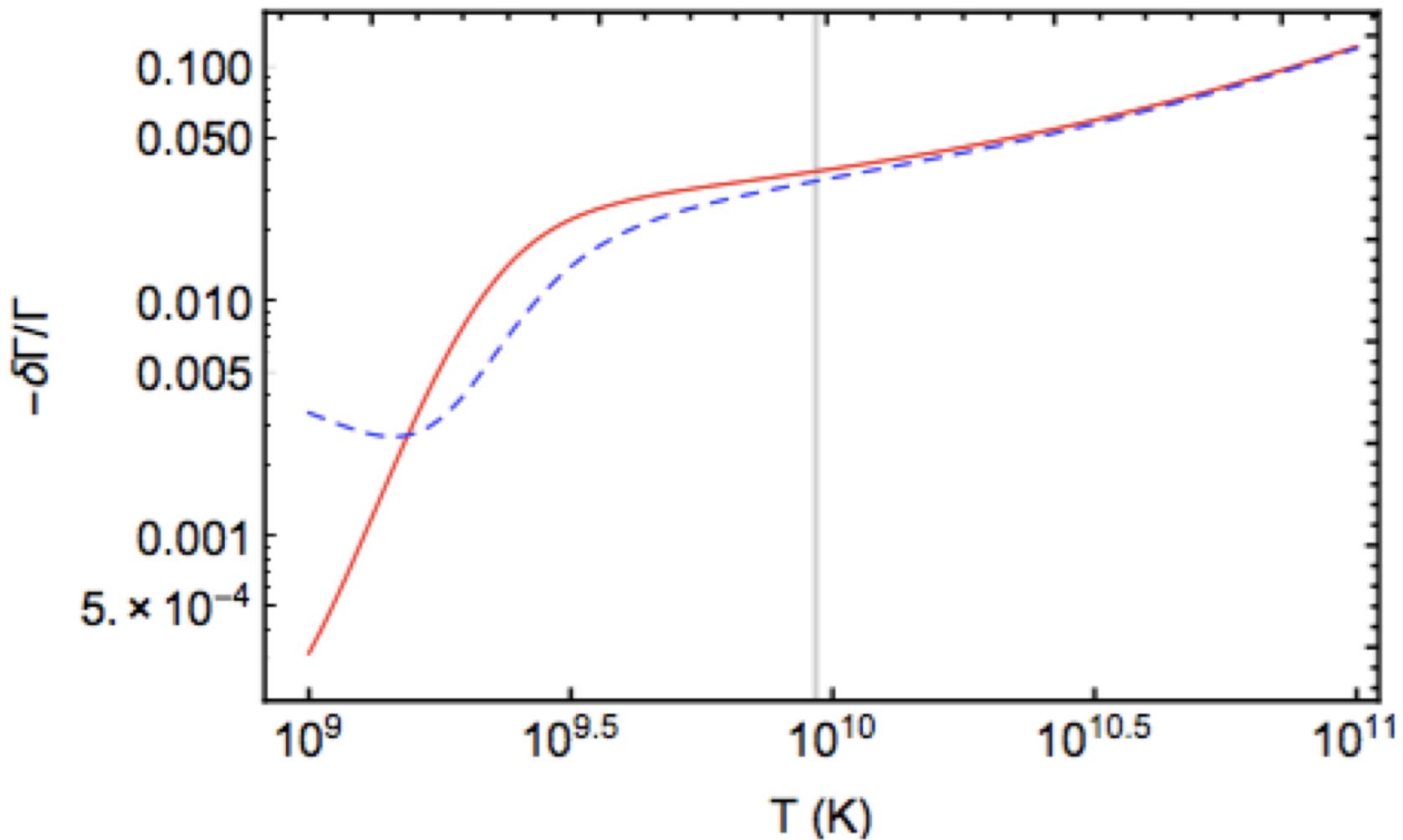
# Finite Temperature corrections



## True photons (bremsstrahlung)



## Total corrections



# Size of corrections

PRIMAT <http://www2.iap.fr/users/pitrou/primat.htm>

Corrections	$Y_P$	$\delta Y_P \times 10^4$	$\delta Y_P / Y_P (\%)$	$D/H \times 10^5$	$\Delta (D/H) (\%)$	${}^3\text{He}/\text{H} \times 10^5$	${}^7\text{Li}/\text{H} \times 10^{10}$
Born	0.24276	0	0	2.424	0	1.069	5.637
Born+ID	0.24289	1.2	0.05	2.433	0.37	1.070	5.615
Born+FM	0.24388	11.2	0.46	2.430	0.25	1.070	5.654
Born+FM+WM	0.24404	12.5	0.53	2.431	0.29	1.070	5.657
RCa [Eq. (B30), Non. Rel. Fermi]	0.24586	31.0	1.27	2.441	0.70	1.071	5.684
RCb [Eq. (B35), Non. Rel. Fermi]	0.24589	31.3	1.29	2.441	0.70	1.071	5.685
RC [Eq. (B35), Rel. Fermi]	0.24591	31.5	1.30	2.441	0.70	1.071	5.685
RC+QED-MS	0.24602	32.9	1.36	2.442	0.74	1.071	5.687
RC+QED-PI	0.24591	31.5	1.30	2.444	0.82	1.072	5.677
RC+ID	0.24602	32.6	1.34	2.450	1.07	1.073	5.663
RC+ID+QED-PI	0.24602	32.6	1.34	2.453	1.19	1.073	5.655
RC+FM+WM	0.24720	44.4	1.83	2.448	0.99	1.072	5.704
RC+FM+WM+QED-MS	0.24733	45.7	1.88	2.449	1.03	1.073	5.706
RC+FM+WM+QED-PI	0.24719	44.3	1.82	2.451	1.11	1.073	5.696
RC+FM+WM+ID	0.24725	44.9	1.85	2.457	1.36	1.074	5.681
RC+FM+WM+ThRC (No BS)	0.24751	47.5	1.96	2.450	1.07	1.073	5.709
RC+FM+WM+ThRC+BS	0.24720	44.4	1.83	2.448	0.99	1.072	5.704
RC+FM+WM+ThRC+BS+ID+QED-PI	0.24724	44.8	1.85	2.460	1.49	1.074	5.673

He-4 correction 1.85%

Deuterium correction 1.49%

Thank you

PRIMAT

<http://www2.iap.fr/users/pitrou/primat.htm>

## Boltzmann factor

