

G4BetheHeitler5DModel : un générateur d'événements

$$\gamma \rightarrow e^+e^-$$

dans Geant4

D. Bernard

LLR, Ecole Polytechnique and CNRS/IN2P3, France

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[links page](#)



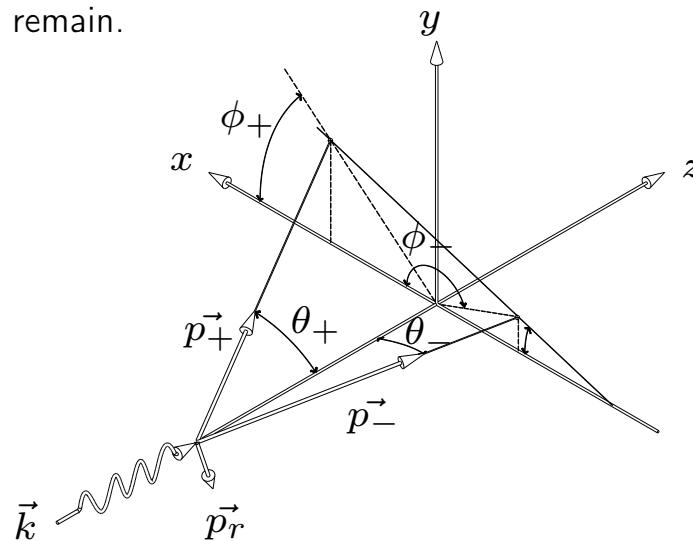
γ conversions to e^+e^- pairs

- Target, final state
 - nucleus (nuclear conversion) $\gamma Z \rightarrow e^+e^-Z$ threshold $2mc^2$
 - electron (triplet conversion) $\gamma e^- \rightarrow e^+e^-e^-$ threshold $4mc^2$

A three-particle final state, even when the trajectory of the recoiling target cannot be detected.

- A 5D phase space

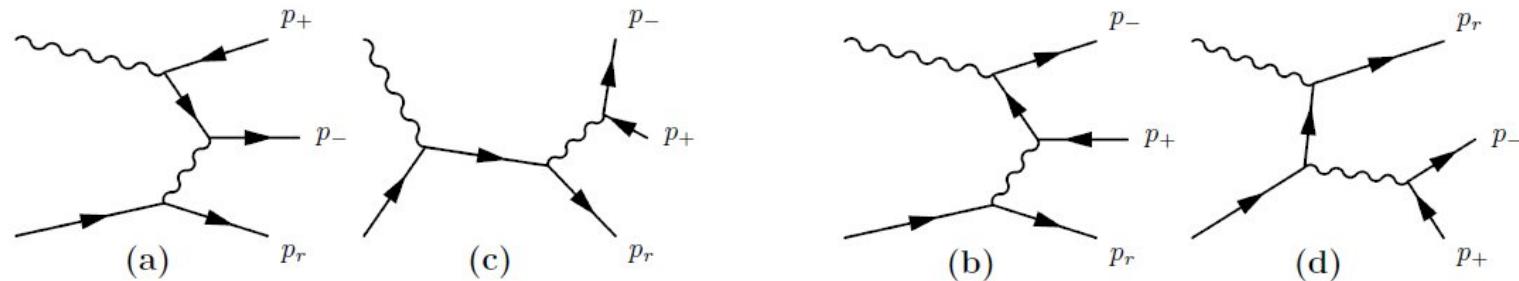
- 3 particle final state, $4 - 1 = 3$ free parameters for each one,
- energy-momentum conservation fixes 4 of them.
- $3 \times 3 - 4 = 5$ variables remain.



- $+, -, r$ = positron, electron, recoil. ϕ azimuthal, θ polar angles.
- $\Omega \equiv (\phi_+, \phi_-, \theta_+, \theta_-, x_+ \equiv E_+/E_\gamma)$

The Bethe-Heitler differential cross section

- Analytical 5D differential cross section



- First-order Born approximation: (a) and (b) diagrams. Excellent approximation:
 - for nuclear conversion ($M_{\text{nucleus}} \gg m$), m electron mass.
 - for high-energy triplet conversion ($E \gg mc^2$) Mork, Phys. Rev. 160 (1967) 1065.
- Non-polarised (Bethe and Heitler, Proc. R. Soc. Lond. A 146 (1934) 83), Polarised (Berlin and Madansky, Phys. Rev. 78 (1950) 623; May, Phys. Rev. 84 (1951) 265.)
 - Linear photon polarisation (not circular), no polarisation transferred to the final leptons
 - Point-like target (extended-nucleus very-high- q^2 suppression neglected)
 - Very-high-energy LPM (Landau-Pomeranchuk-Migdal) suppression neglected
 - Coulomb (final state electrostatic interactions between leptons) corrections neglected
 - At face value, assumes that the recoil energy is negligible $\Rightarrow E_- = E - E_+$

Bethe-Heitler differential cross section: divergences

- Here the non-polarised expression

Bethe and Heitler, Proc. R. Soc. Lond. A 146 (1934) 83

$$\begin{aligned} d\sigma = & \frac{-\alpha Z^2 r_0^2 m^2}{(2\pi)^2 \omega^3} dE_+ d\Omega_+ d\Omega_- \frac{|p_-||p_+|}{|\vec{q}|^4} \\ & \left[\left(\frac{p_+ \sin \theta_+}{E_+ - p_+ \cos \theta_+} \right)^2 (4E_-^2 - q^2) + \left(\frac{p_- \sin \theta_-}{E_- - p_- \cos \theta_-} \right)^2 (4E_+^2 - q^2) + \right. \\ & \left. \frac{2p_+ p_- \sin \theta_+ \sin \theta_- \cos \phi}{(E_- - p_- \cos \theta_-)(E_+ - p_+ \cos \theta_+)} (4E_+ E_- + q^2 - 2\omega^2) - 2\omega^2 \frac{(p_+ \sin \theta_+)^2 + (p_- \sin \theta_-)^2}{(E_+ - p_+ \cos \theta_+)(E_- - p_- \cos \theta_-)} \right] \end{aligned}$$

with: $|\vec{q}|^2 = |\vec{p}_+ + \vec{p}_- - \vec{k}|^2$.

- Divergences:

- For e^+ and e^- $\frac{1}{(E - p \cos \theta)}$ forward divergence at high energies, the easy ones
- $\frac{1}{q^4}$ small recoil divergence, involves several kinematic variables in a correlated way

Correlated divergences: Consequences for pre-existing physics models

- No attempt to sample the 5D differential cross-section
- No attempt to generate a target recoil momentum
 - The electron and the positron are generated in a plane that contains the photon
 - Azimuthal angles back to back, $\phi_- = \phi_+ \pm \pi$
- Single-particle polar angle θ distribution, decent.
- “Energy share”, x_+ distribution, exact. $x_+ \equiv E_+/E_\gamma$
- Electron and positron polar angles θ_- and θ_+ generated independently
 - energy-momentum not conserved in the reaction !
 - some artificial transverse recoil apparently generated, but distribution wrong and direction in the “conversion plane”
- Polarised model wrong. P. Gros et al., Astropart. Phys. 88 (2017) 60

[polarisation **phase** bug corrected, though, in the 10.4beta release V. Ivantchenko & Ph. Gros, Sept. 2017.]

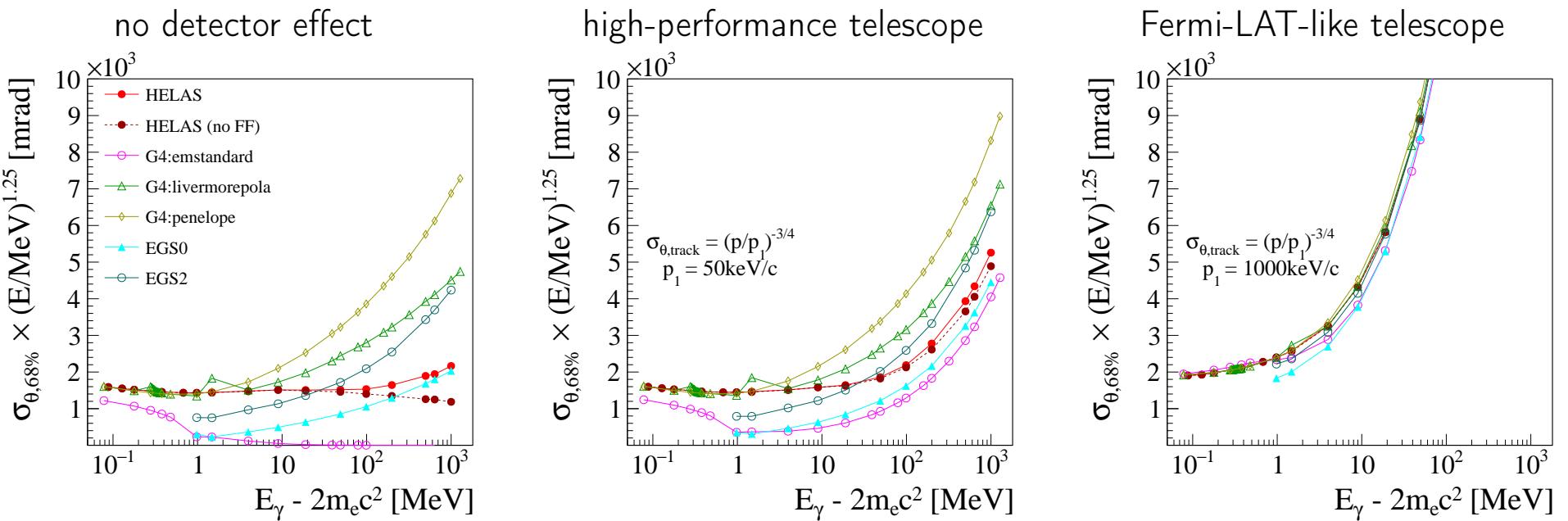
$(E < 1 \text{ GeV})$ high-performance $\gamma \rightarrow e^+e^-$ telescopes: ongoing projects

Three active targets (i.e. combined converter + tracker) techniques being developed:

- Homogeneous low-density active targets (gas detectors): HARPO, Adept
- Tungsten-less, silicon-wafer stacks (e-ASTROGAM, AMEGO) and (so) many past other ones
- Homogeneous ultra-high-spatial-precision, high-density, active targets: emulsions (GRAINE)

Angular resolution due to unmeasured nucleus recoil:

(normalized to $E^{-1.25}$)



Need an exact simulation of the conversion !

("HELAS" is one of G4BetheHeitler5DModel's ancestors)

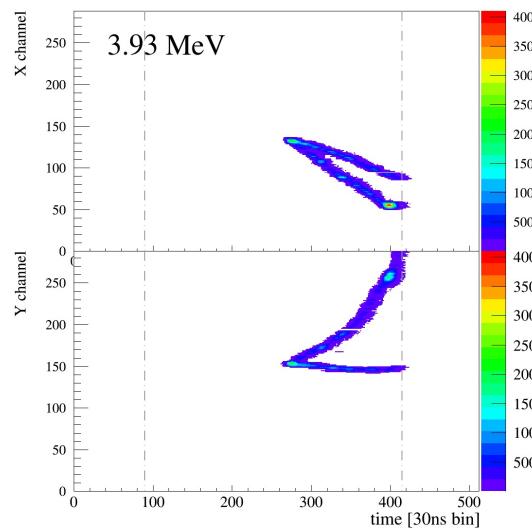
FF: atom electrons screening form-factor

P. Gros et al., Astropart. Phys. 88 (2017) 60

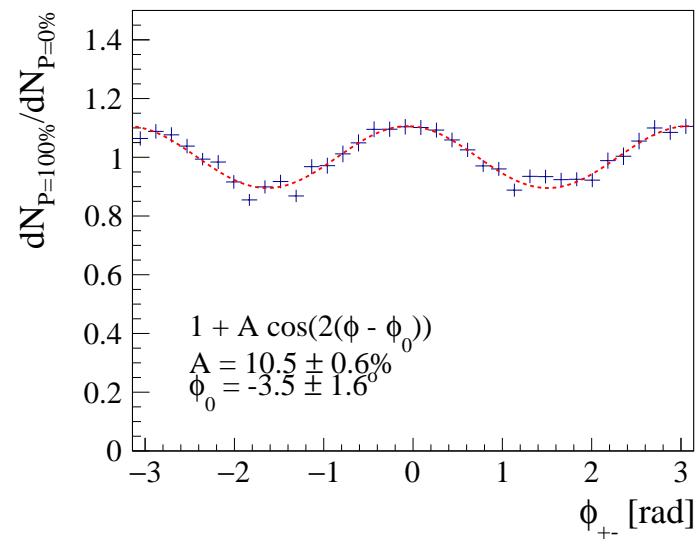
G4BetheHeitler5DModel development: Context: HARPO

- Works on paper
 - Telescope performance D. Bernard, Nucl. Instrum. Meth. A 701 (2013) 225
 - Polarimeter performance D. Bernard, Nucl. Instrum. Meth. A 729 (2013) 765
- Experimental development
 - Gas (argon-isobutane) Time projection chamber (TPC) SPIE 9144 (2014) 91441M
 - Validated in (polarised) gamma-ray beam, NewSUBARU, U. of Hyôgo, Japan

(x, t) and (y, t) signal maps
 $\gamma \rightarrow e^+e^-$ in 2.1 bar Ar-iC4H10 95-5%



Azimuthal angle distribution
11.8 MeV γ in 2.1 bar Ar-iC4H10 95-5%



Ph. Gros et al., Astropart. Phys. 97 (2018) 10

G4BetheHeitler5DModel: Sampling Method: 1

- Mitigation of main, correlated, divergence: perform each step in appropriate Lorentz frame
 - Center-of-mass system (CMS) boost determined from photon energy E and target mass M .
 - Five variables are taken at random, $(\ell: \text{lepton, that is, } e^+ \text{ or } e^-)$

variable	name	Lorentz frame
θ	target and pair polar angle	CMS
μ	e^+e^- invariant mass	
θ_ℓ	electron and positron polar angle	pair frame
ϕ_ℓ	electron and positron azimuthal angle	pair frame
ϕ	target and pair azimuthal angle	CMS

- In the CMS, target (mass M) and pair (mass μ) are back-to-back with opposite momenta.
 - “Decay” of the pair to an electron and a positron performed in the pair Lorentz frame.
 - The lepton 4-vectors are boosted “back” to the CMS.
 - The three final particle 4-vectors are boosted “back” to the laboratory Lorentz frame.
 - The Bethe-Heitler variables are obtained from the 4-vectors.
 - The probability density function (pdf) is computed.
- Final-state phase space normalization for this set of cascade decays:
[Review of Particle Physics \(Particle Data Group\)](#) See eqs. (1)-(3) in Nucl. Instrum. Meth. A 899 (2018) 85
 - Note that in contrast to Bethe-Heitler taken at face value, we **do conserve energy momentum**
 $E = E_- + E_+ + E_r$

G4BetheHeitler5DModel: Sampling Method: 2

Mitigation of residual, mostly uncorrelated, divergences:

- [2012 – 2017]: SPRING/BASES implementation of VEGAS method.

(S. Kawabata, Comput. Phys. Commun. 88 (1995) 309.)

Several seconds CPU overhead for each (photon-energy, target Z)

Sytematic verification against known results (1D distributions, total cross sections ..)

Nucl. Instrum. Meth. A 729 (2013) 765

Astropart. Phys. 88 (2017) 60

- [2018]: change of variable

Nucl. Instrum. Meth. A 899 (2018) 85

I. Semeniouk & D. Bernard, 14th Pisa Meeting on advanced detectors, 27 May - 02 June 2018, Isola d'Elba

- Geant4 presentations:

Aug. 2018	Lund	23rd Geant4 Collaboration Meeting	“Deployed version”
Jan. 2018	CERN	Geant4 EM meeting	“stand-alone C++ version verified”
Sept. 2017	Wollongong	Geant4 User Workshop	“VEGAS-free, but still fortran”
April 2017	Guildford	Geant4 Space Users’ Workshop	“de-VEGAS-ification in progress”

Implementation

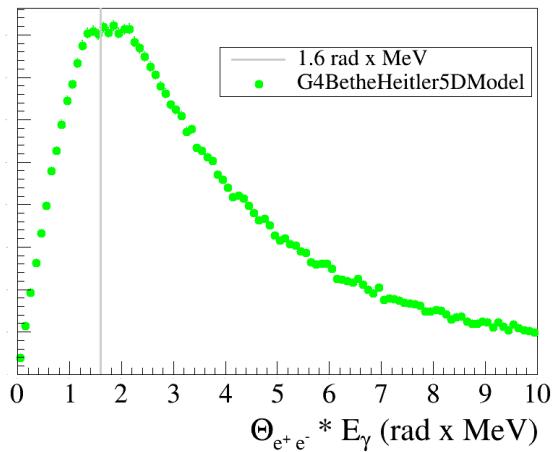
- G4EmLowEPPhysics physics list ($E < 80 \text{ GeV}$)
- Gamma Conversion model G4BetheHeitler5DModel
- Inherit total cross section from G4BetheHeitlerModel
- Provide final state: SampleSecondaries
- Models flags provided via G4EmParameters flags and UI commands
 - Recoil particle (ion or electron): Nuclear / triplet / ($Z / 1$) natural mixture
 - Isolated targets (QED checks), charged targets in atoms (detector simulation)
- Linearly polarised / non polarised photons (from norm of photon polarisation vector)
- SampleSecondaries CPU time ($\mu\text{s}/\text{event}$), argon, 1 GeV

G4BetheHeitler5DModel	G4BetheHeitlerModel
74.	0.9

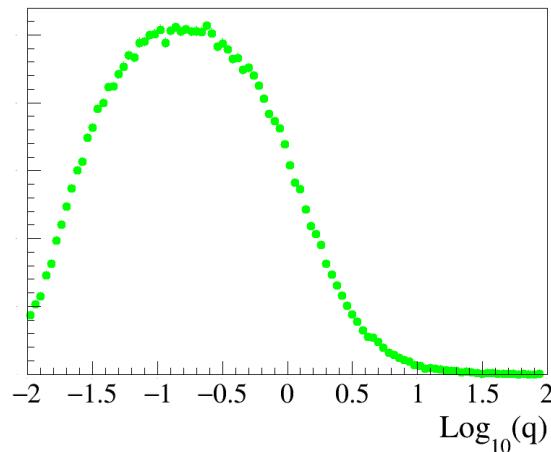
See also Sec. 5.4 and Fig. 8 of Nucl. Instrum. Meth., A 899 (2018) 85.

- Does not dominate EM showers simulation CPU time.
- Region-dependent model choice considered, to be implemented.

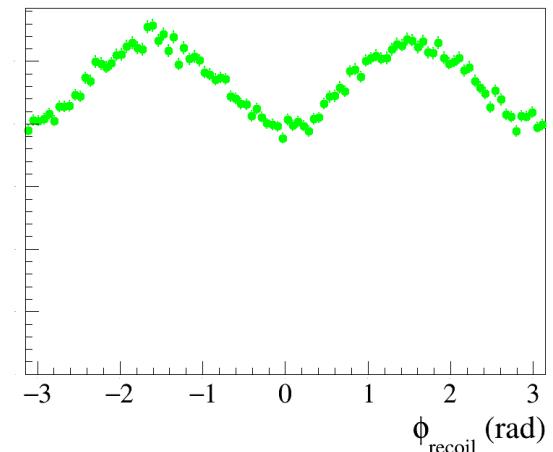
TestEm15



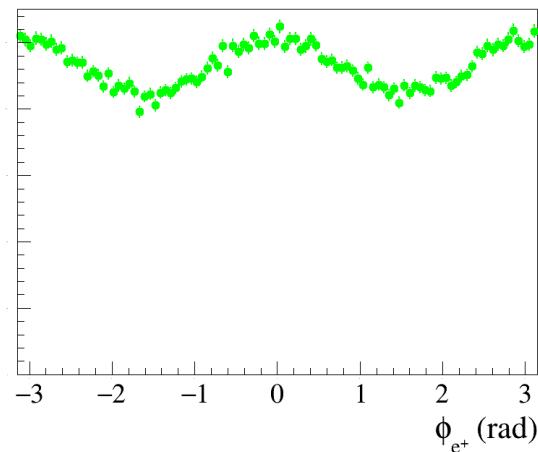
Olsen, Phys. Rev. 131 (1963) 406



q (MeV/c) is recoil momentum

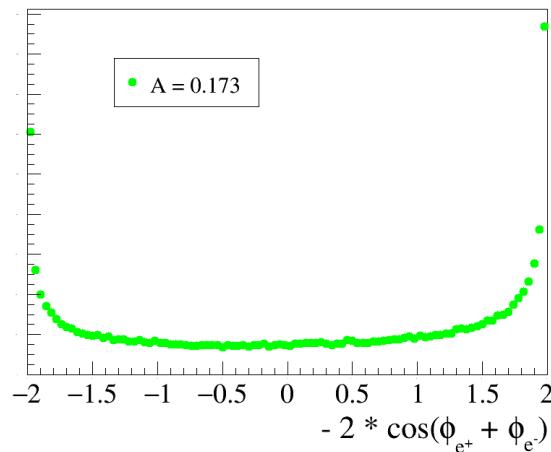


recoil preferentially \perp pol



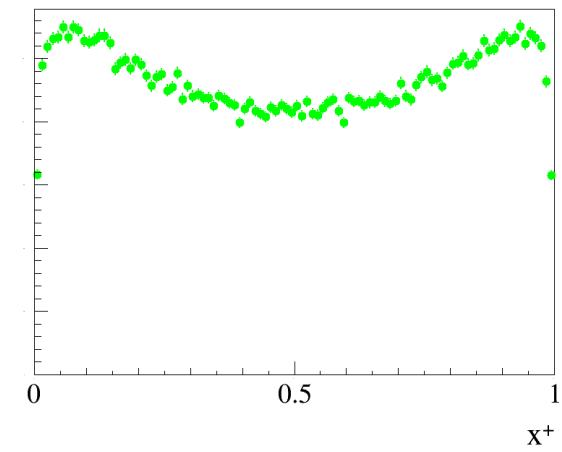
e^+ and e^- preferentially along pol

1 GeV, argon



$A = \langle -2 \cos 2\phi \rangle, \quad \phi = (\phi_+ + \phi_-)/2$
minus sign due to $\pi e^+/e^-$ shift

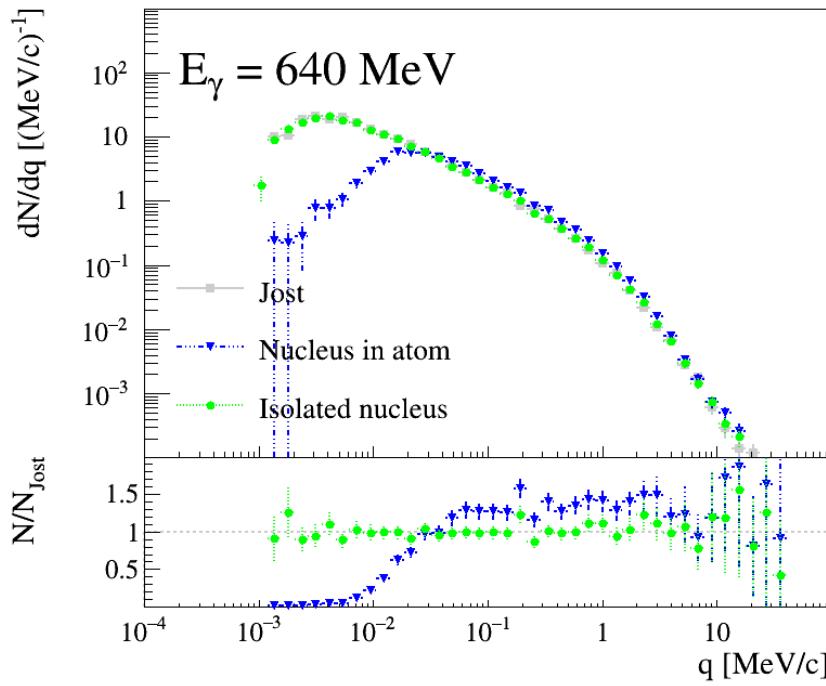
Astropart. Phys. 88 (2017) 30



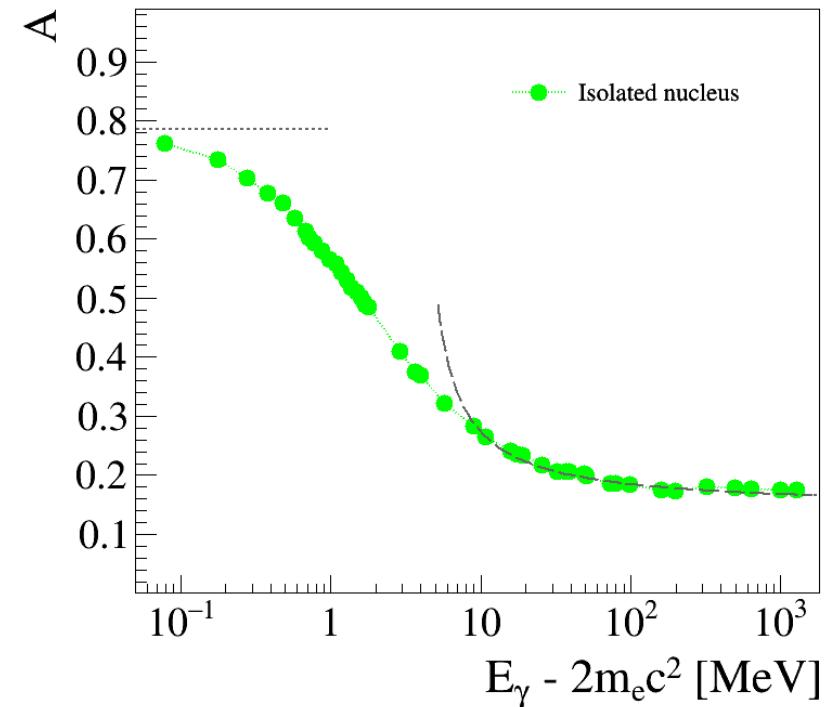
energy share $x_+ \equiv E_+/E_\gamma$

TestEm15/README.gamma

G4BetheHeitler5DModel: Verifications again



Recoil momentum distribution compared to the analytical high-energy expression from
[Jost, Phys. Rev. 80, 189 \(1950\)](#).
The ratio plot is relative to Jost.



Polarisation asymmetry as a function of available energy, compared to published asymptotic expressions

- I. Semeniouk & D. Bernard, 14th Pisa Meeting on advanced detectors, 27 May - 02 June 2018, Isola d'Elba
- P. Gros et al., Astropart. Phys. 88 (2017) 60

Documentation

- Edition of Physics Reference Manual, Release 10.4 (April 2018)
 - 6.5.4 Five-dimensional (5D) Bethe-Heitler gamma Conversion to e+e-
 - 13.10 Pair production by Linearly Polarised Gamma Rays - Five-dimensional (5D) Bethe-Heitler Model

Reference numbers to be checked when 10.5 Physics Reference Manual is out.

- Geant4 Links
 - [Geant4 10.5-beta-01 Release Notes](#), 29 June 2018
 - [Beta Development Source Code Download Geant4 10.5-beta](#), 29 June 2018

Conclusion

- Exact Bethe-Heitler evt generator in Geant4.
 - target recoil momentum distribution
 - linearly polarised γ conversion
 - triplet conversion.

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- Geant4 Collaboration:
 - Vladimir Ivantchenko
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- HARPO Collaboration:
 - Igor Semeniouk
 - Philippe Gros

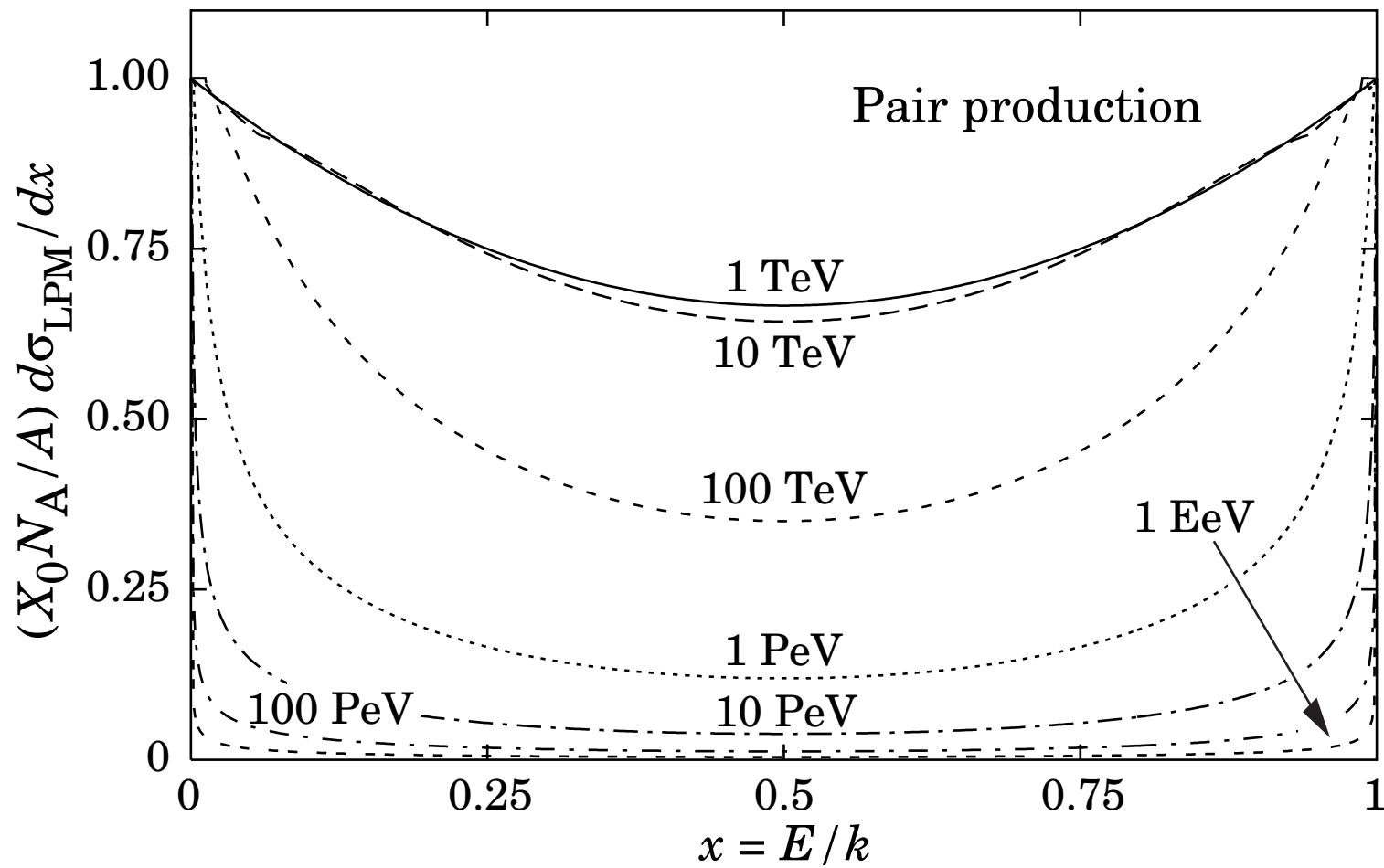
Back-up slides

Screening of target electric field by atomic electrons

small- q^2 screening suppression: $F(q^2)$ form factor

- coherent (nuclear conversion)
- incoherent (triplet conversion)

LPM



Landau-Pomeranchuk-Migdal [Review of Particle Physics \(Particle Data Group\)](#)