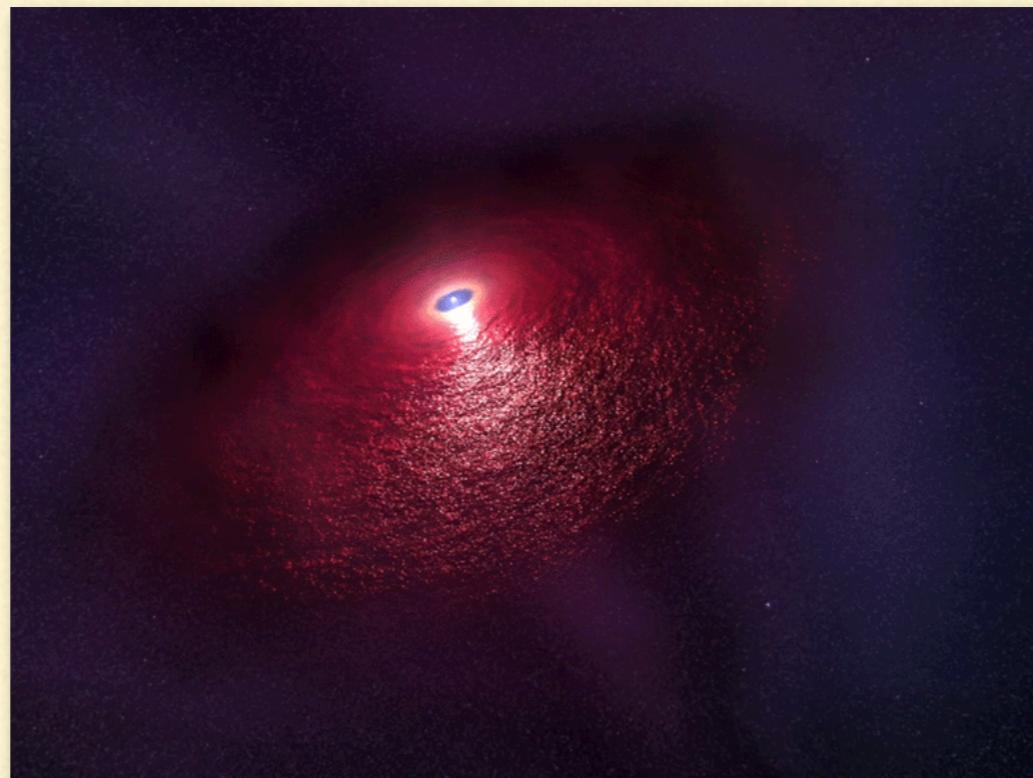


# Quels mécanismes transportent la matière dans les disques d'accrétion ?

Jean-Pierre Lasota

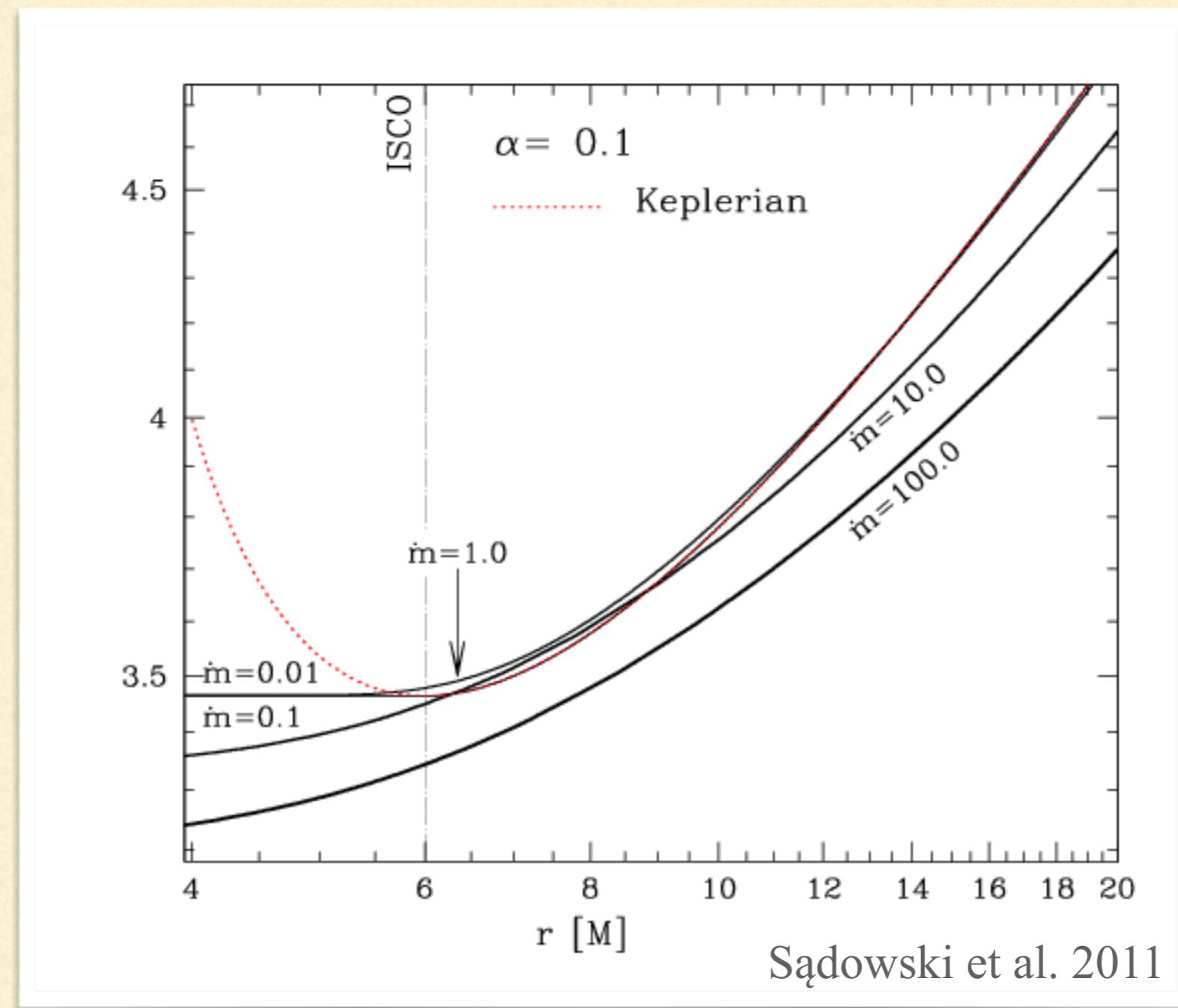
IAP & NCAC



[Journées Théorie PNHE](#)

2 Octobre 2018

# Transport de matière dans les disques = transport du moment angulaire



"anomalous viscosity"=turbulence (Peek 1942, von Weizsäcker 1943)

## Some numbers

$$Re = \frac{\nu_\phi H}{\nu}$$

- Reynolds number

$$Rm = \frac{c_s H}{\eta}$$

- magnetic Reynolds number

$$P_m = \frac{\nu}{\eta} = \frac{Rm}{Re}$$

- Prandtl number

molecular kinematic viscosity:  $\nu = 10^5 \text{ cm}^2 \text{ s}^{-1}$

for  $M \approx 1 \text{ M}_\odot$ ,  $R \approx 10^{10} \text{ cm}$ ,  $H/R \sim 10^{-2}$  :  $Re \sim 10^{11}$

$t_{\text{visc}} \sim \frac{R^2}{\nu} \approx 3 \times 10^7 \text{ yr}$

hence the need for anomalous viscosity

---

Hydrodynamical turbulence cannot drive accretion in astrophysical discs.

Efficiency of turbulent transport

$$\alpha \sim \frac{1}{Re_{\text{crit}}}$$

$Re_{\text{crit}}$  is the critical Reynolds number for transition to turbulence

Lesur & Longaretti (2005)

But beware: in the fluid mechanics community it is considered crackpot to suggest that at Reynolds numbers like our  $10^{14}$  a flow (no matter which or where) can be anything but ‘fiercely turbulent’. If your career depends on being friends with this community, it would be wise to avoid discussions about accretion disks or Couette experiments.

- Henk Spruit

---

In accretion discs angular momentum is transported by "anomalous viscosity" (MHD turbulence) generated by the magneto-rotational instability (MRI)\*

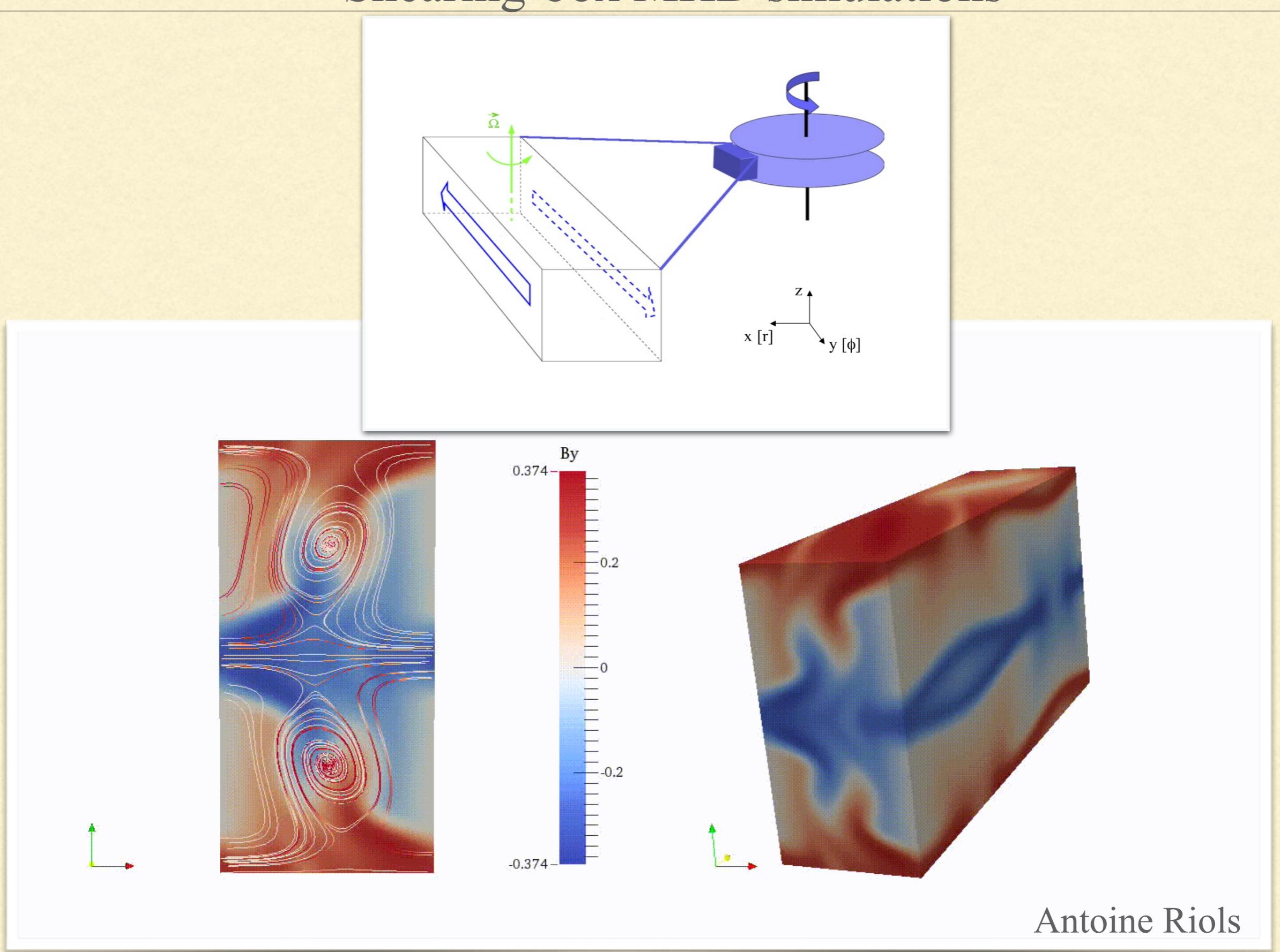
However,

- ➊ Despite enormous progress, MRI is still not well understood, mainly because most numerical simulations are performed in a *shearing box*.
- ➋ Problems: *Non-linear evolution & saturation: how they depend on the size of the box, numerical resolution, dissipation, stratification and the net magnetic field.*

\* Balbus & Hawley 1991 ....

---

# Shearing-box MHD simulations



---

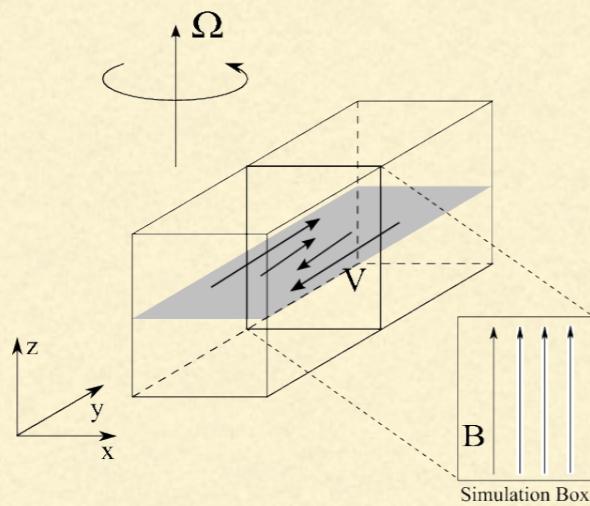
# Problems with MRI

- ➊ can it drive accretion in dwarf-nova discs? (the " $\alpha$ " problem).
- ➋ can it drive accretion in X-ray transient systems (the "high  $\alpha$  problem").
- ➌ can it drive accretion in cold, quiescent dwarf-nova and XRT, and in protostellar discs (the low-ionisation, dissipation problem)?

The role of: *convection, external (strong) magnetic field, disc winds, spiral waves (shocks), radiative pressure, non-ideal MHD effects, general-relativistic effects, hydrodynamical instabilities (?).*

In most cases MRI is, most probably, not acting alone.

# Magneto-rotational-instability (MRI)



$$\Omega \propto R^{-q}$$

$$v_A := \frac{B}{\sqrt{\rho}}$$

$$\kappa^2 := 2(2-q)\Omega^2$$

Perturbations of MHD equations:  $\delta X = \delta \bar{X} \exp(i(\omega t - kz))$

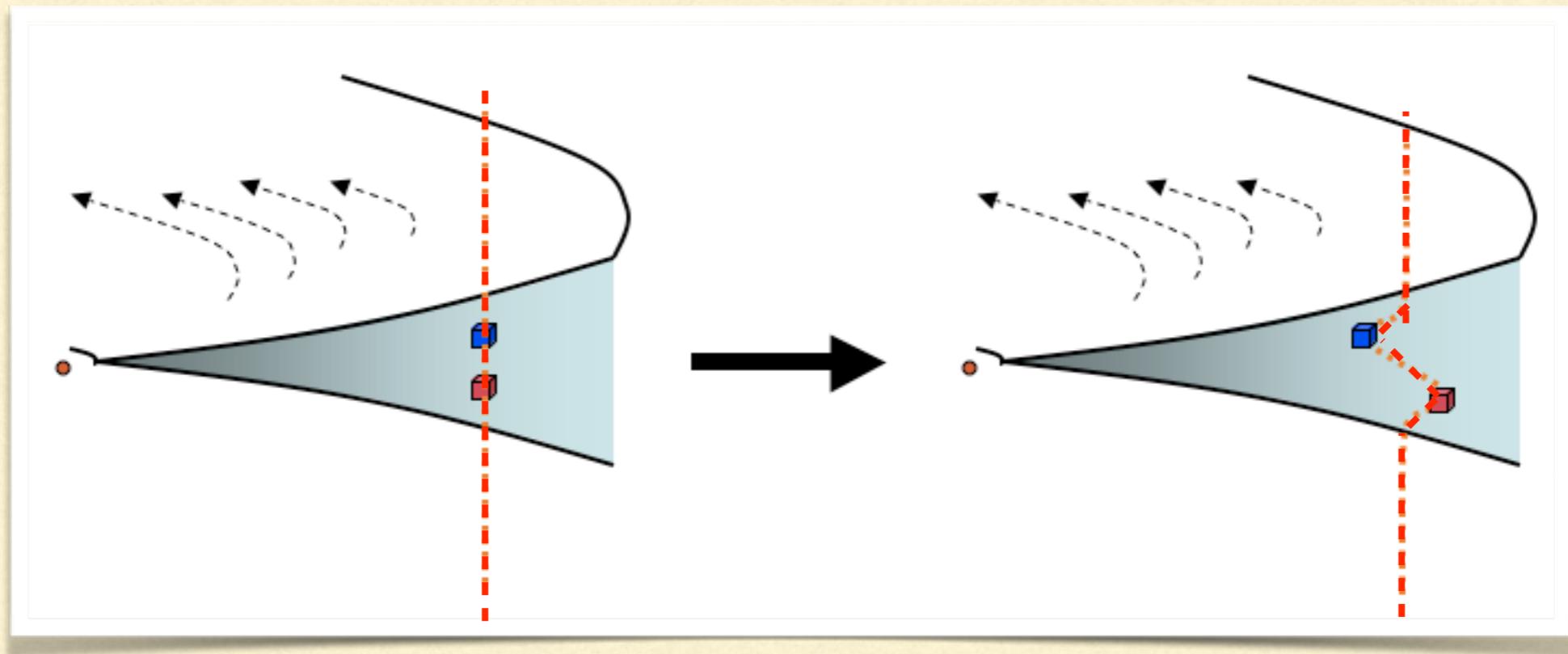
$$\omega^4 - \omega^2 [2k^2 v_A^2 + \kappa^2] + k^2 v_A^2 [k^2 v_A^2 - 2q\Omega^2] = 0$$

$$(v_A = 0, \quad \omega^2 - \kappa^2 = 0; \quad \text{stability : } \kappa^2 \geq 0, \text{ or } q \leq 2)$$

Stability condition:  $k^2 v_A^2 - 2q\Omega^2 \geq 0$  or  $q \leq 0$

For  $q \geq 0$  there is always a  $k_c = \frac{2q\Omega^2}{v_A^2}$  such that for all  $k < k_c$

$$k^2 v_A^2 - 2q\Omega^2 < 0$$



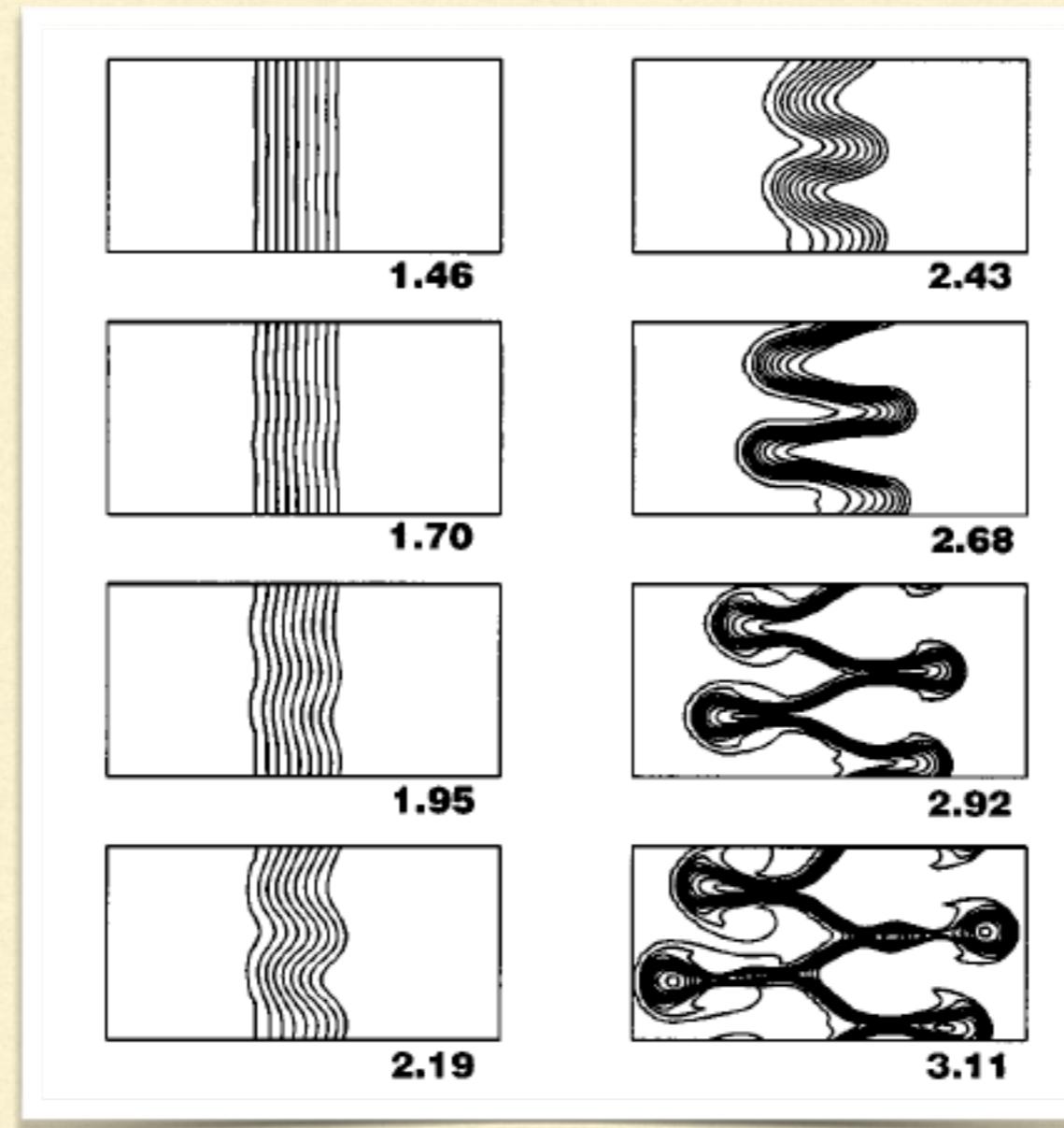
Most unstable mode:  $|\omega_{\max}| = \frac{q\Omega}{2}, \quad k_{\max}^2 v_A^2 = \frac{q}{4} (4 - q) \Omega^2$

Keplerian disc:  $|\omega_{\max}| = 0.75 \Omega, \quad k_{\max} v_A \sim \Omega$

$> 10^4$  energy amplification in one dynamical time

Weak field instability, from  $k_c \geq \frac{2\pi}{H}$  follows  $\frac{p_g}{p_{\text{mag}}} =: \beta \geq \frac{4\pi^2}{q}$

# Saturation ?



Balbus & Hawley 1998

---

## The $\alpha$ - viscosity parameter

Stress tensor:

$$w_{r\varphi} := w_{r\varphi}^{(M)} + w_{r\varphi}^{(R)} := - \frac{B_r B_\varphi}{4\pi} + \rho v_r \delta v_\varphi$$

$$\alpha := \frac{\int w_{r\varphi}(z) dz}{\int [P_g(z) + P_r(z)] dz}$$

Shakura-Sunyaev  $\alpha$ :  $W_{r\varphi} = \rho w_{r\varphi} =: \alpha_{SS} P$

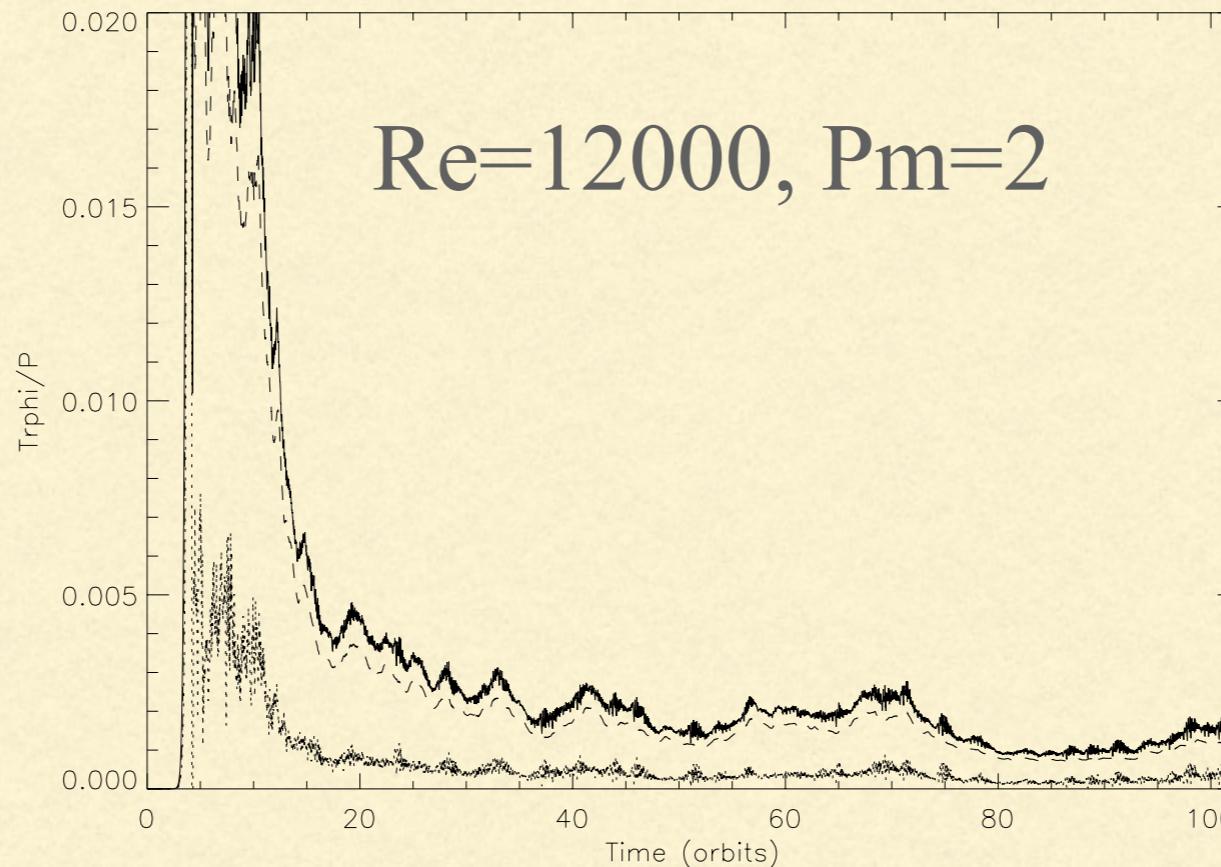
---

# Effect of transport coefficients $\nu$ and $\eta$

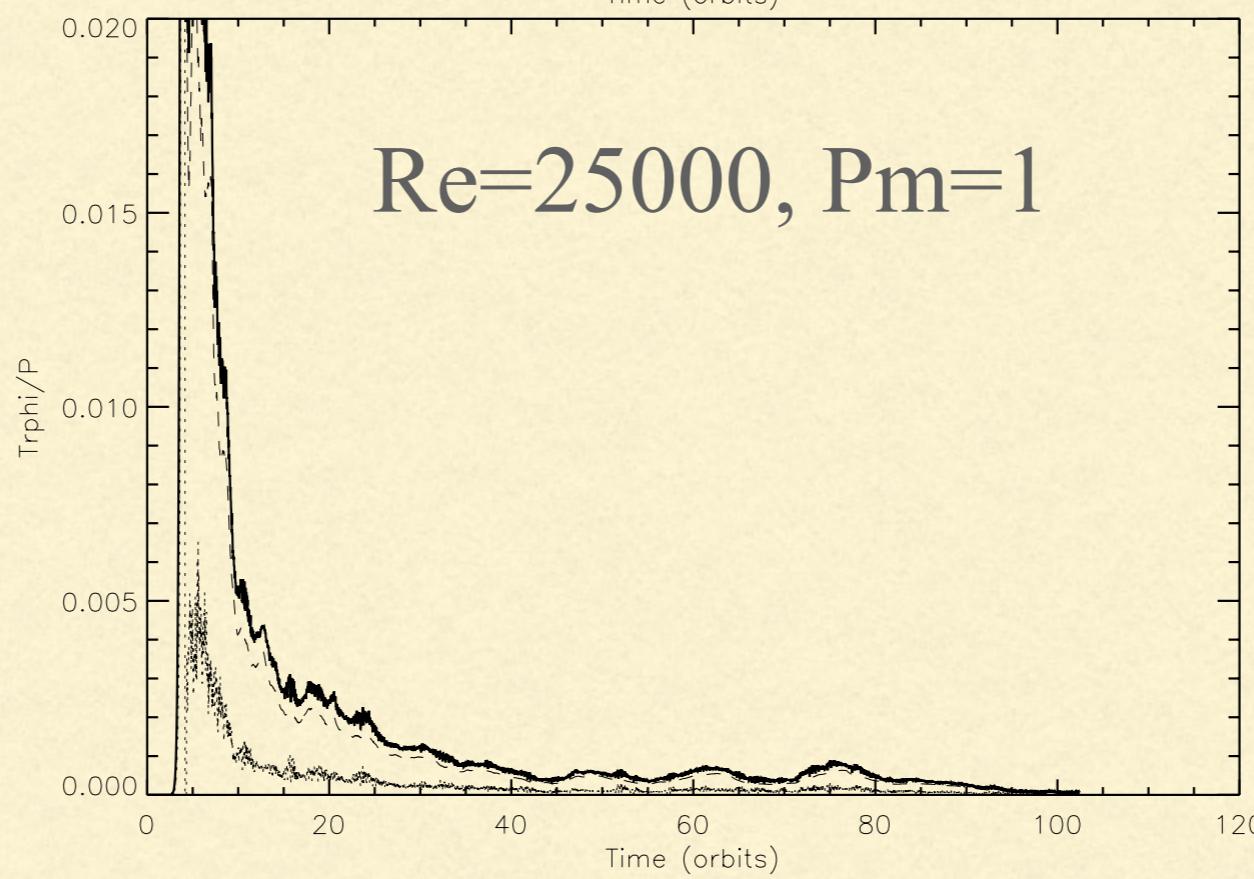
---

No net B flux

$Pm_{crit} \sim 1$



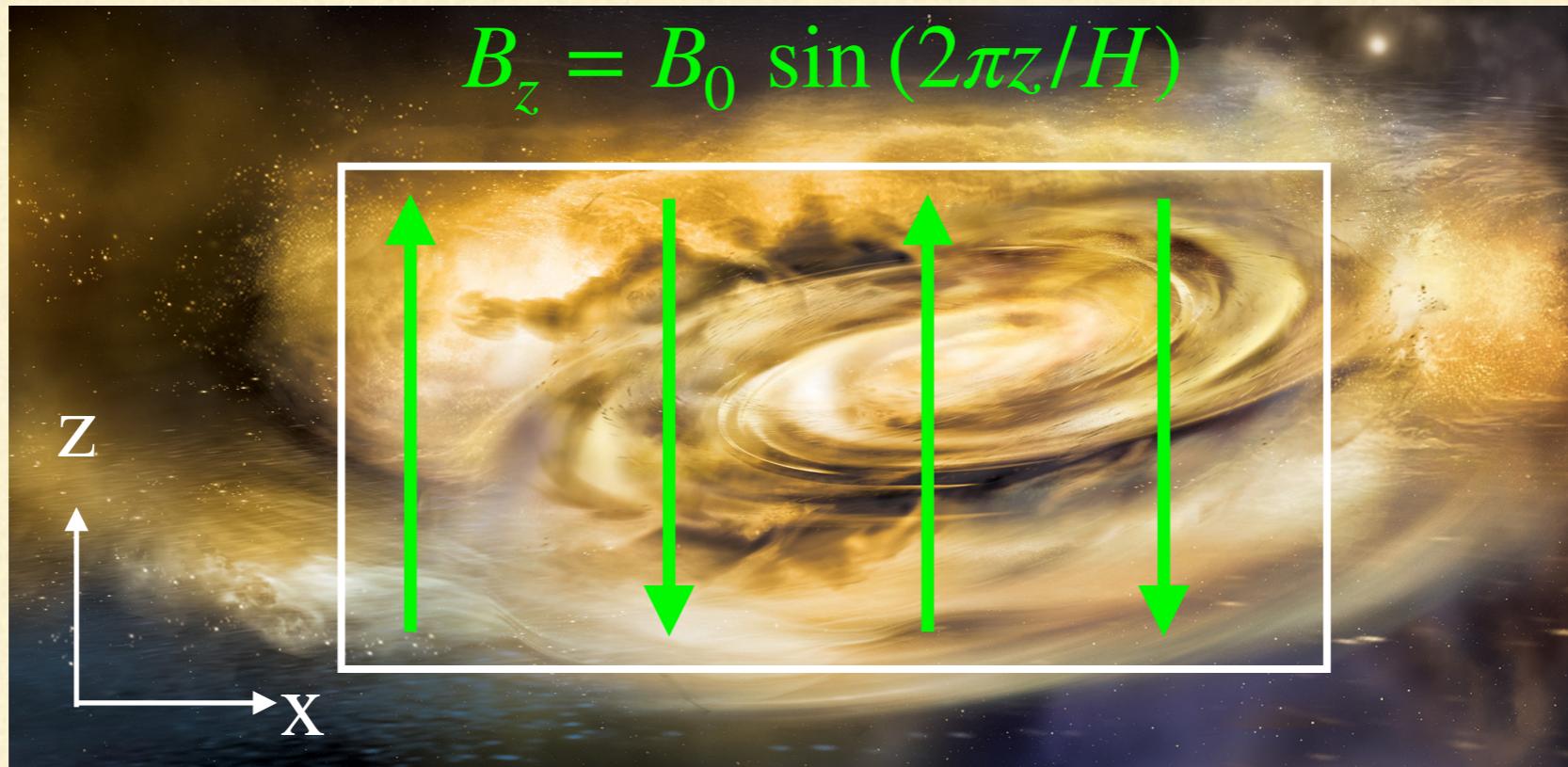
$$\alpha = 1.6 \times 10^{-3}$$



$$\alpha \approx 0$$

Caveat: In non-stratified, zero net-flux shearing boxes simulations with no thermodynamics, MRI turbulence saturation properties depend on numerical resolution and box aspect ratio.

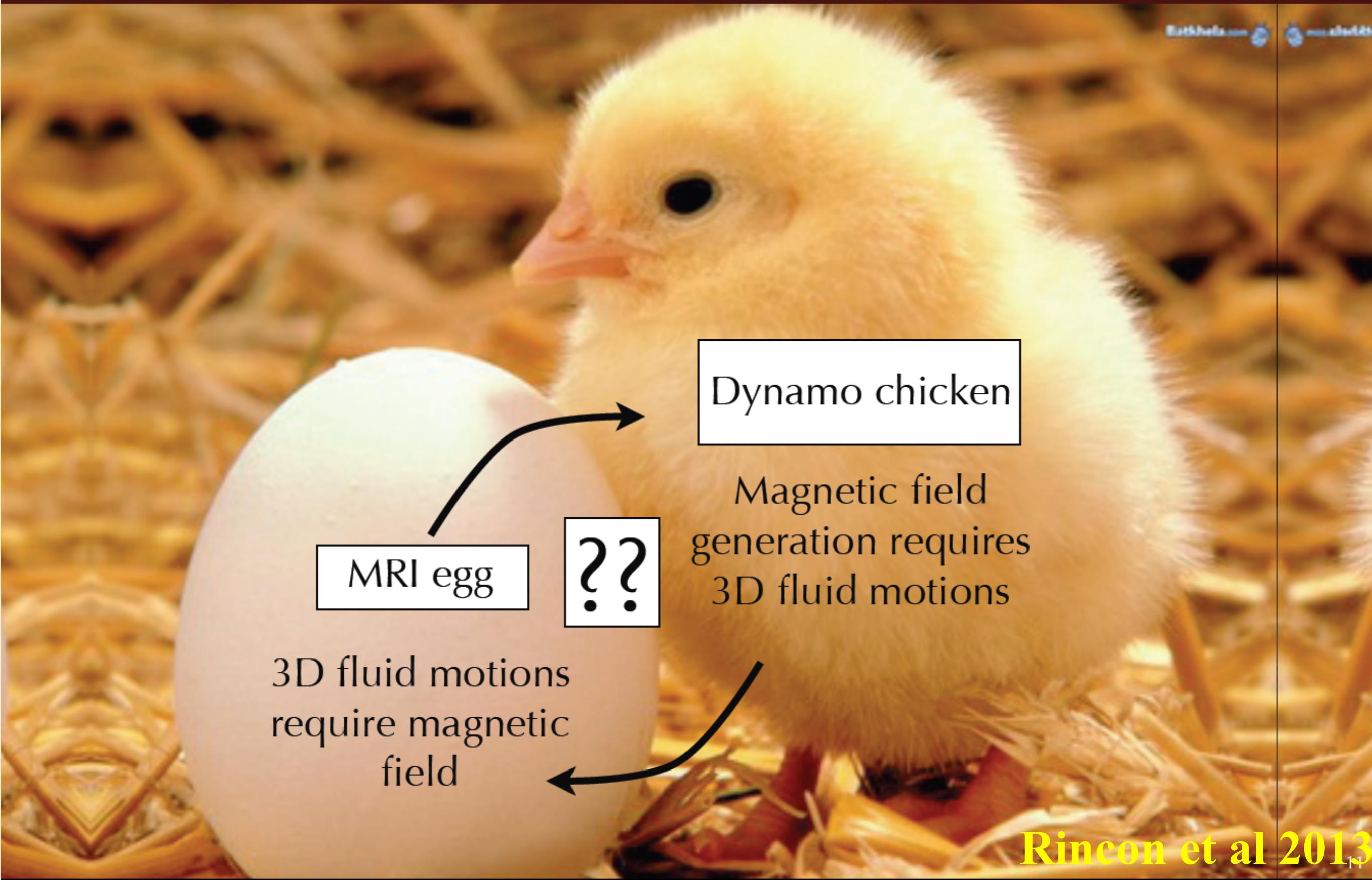
Zero net flux



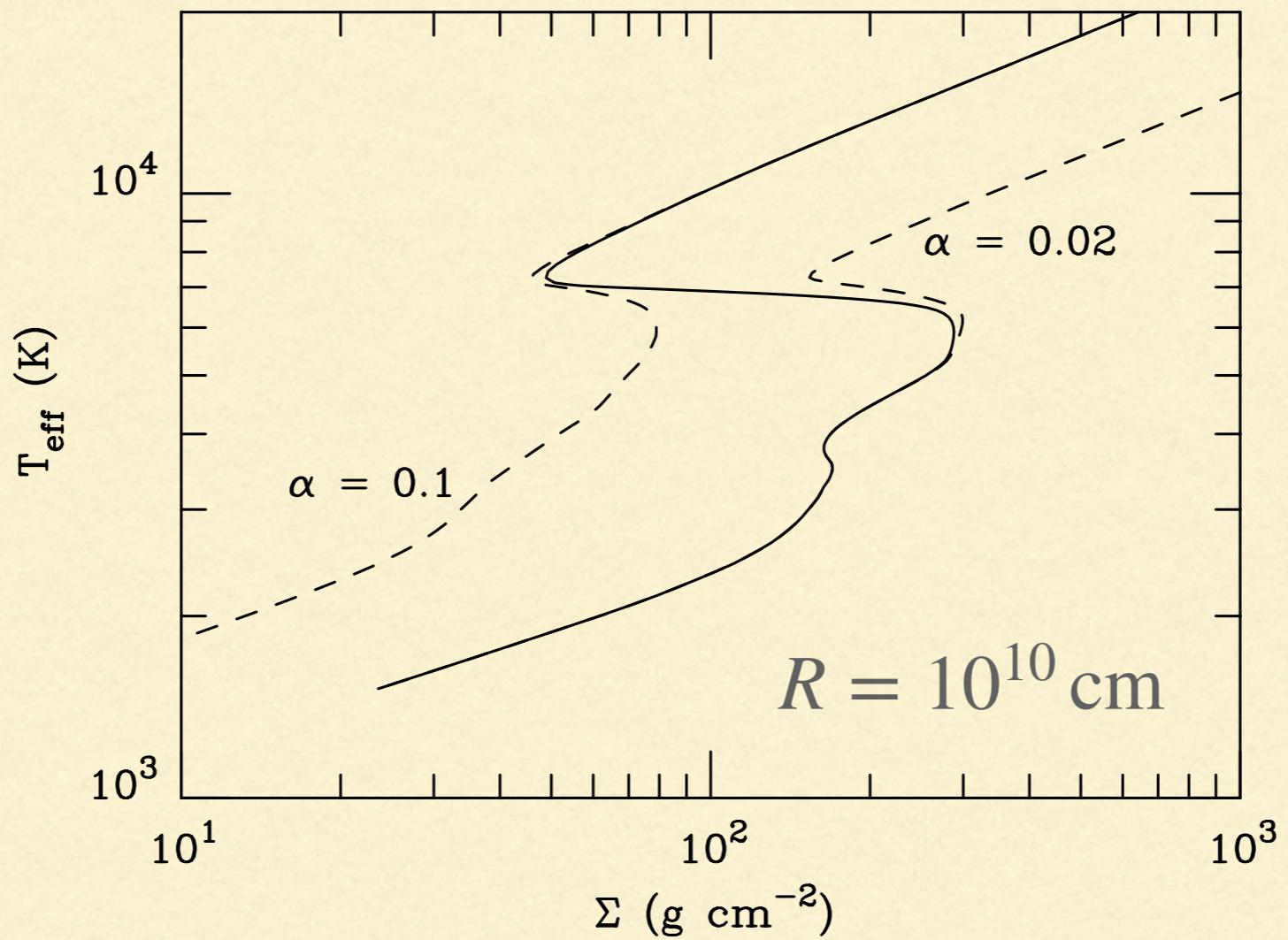
subcritical dynamo

In astrophysical "practice" there is always a non-zero flux (?)

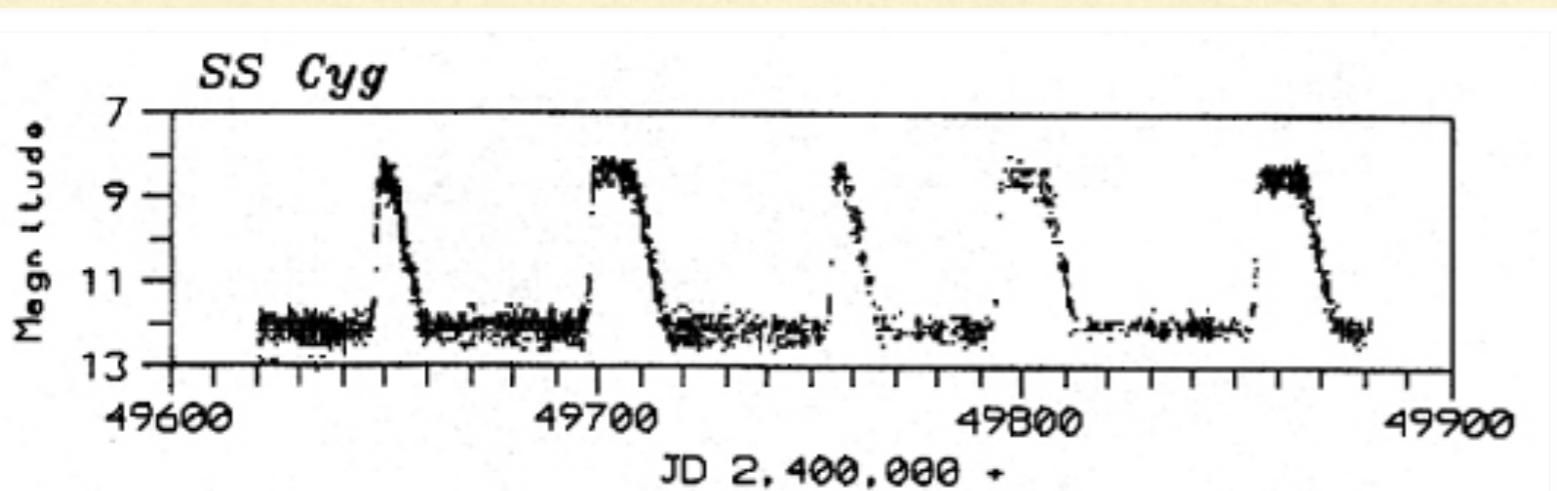
# The MRI dynamo problem



# Dwarf-nova disc instability model (DIM)

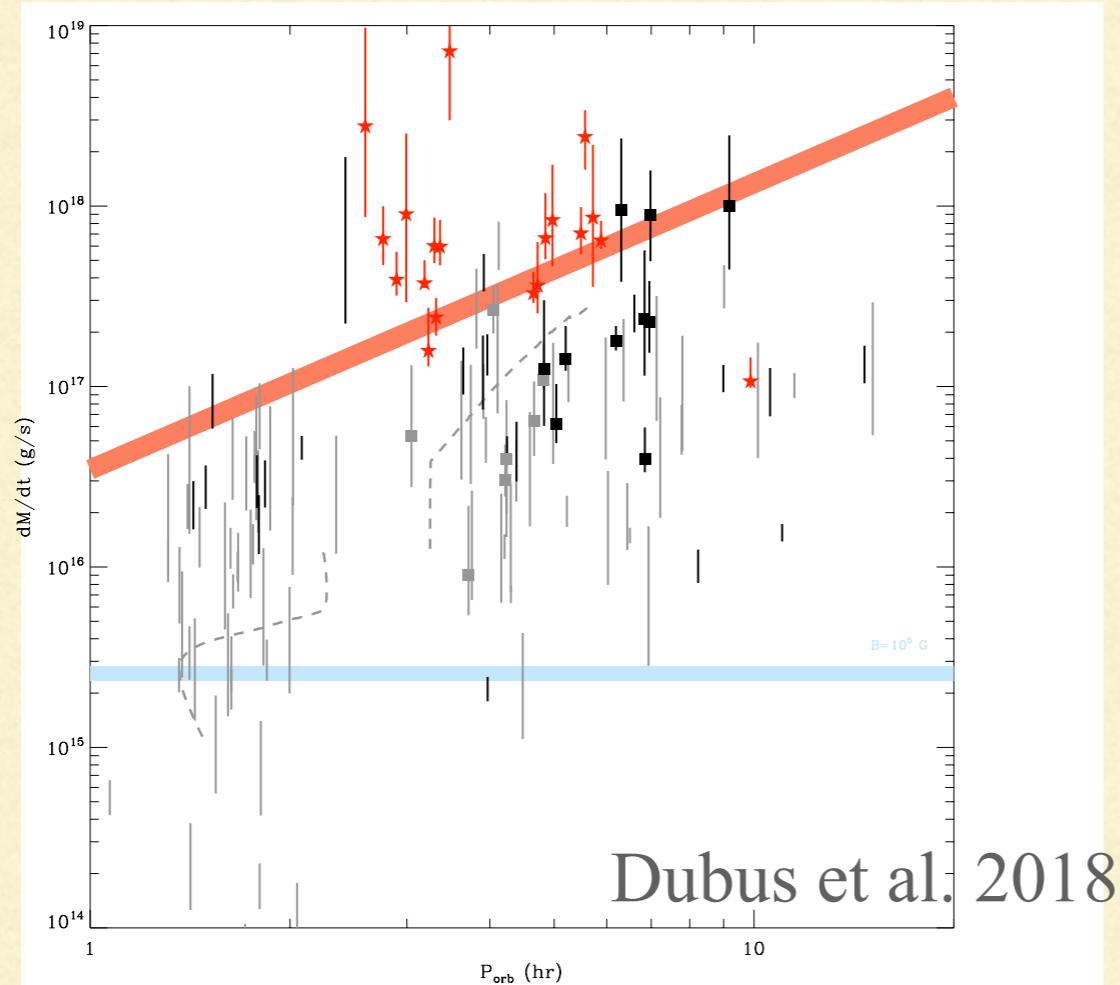


Hameury et al. 1998



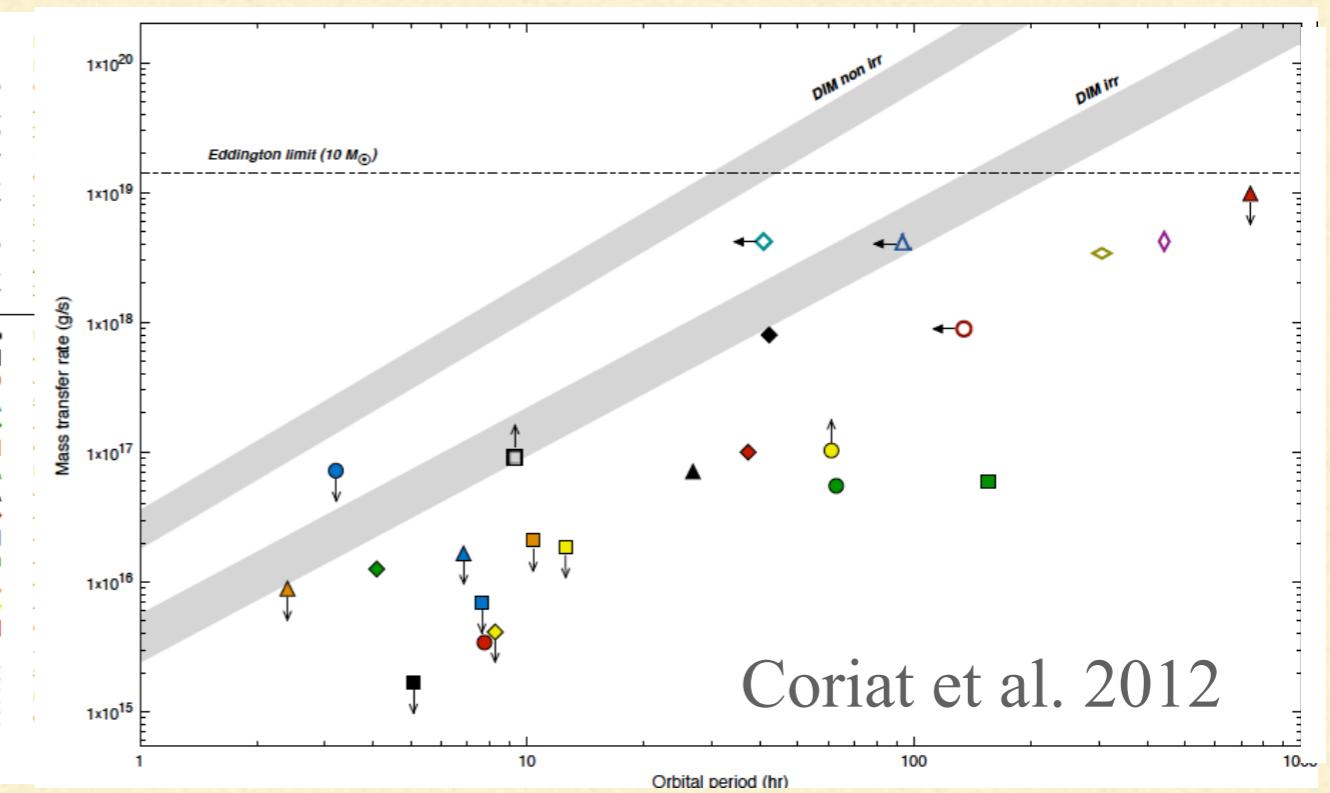
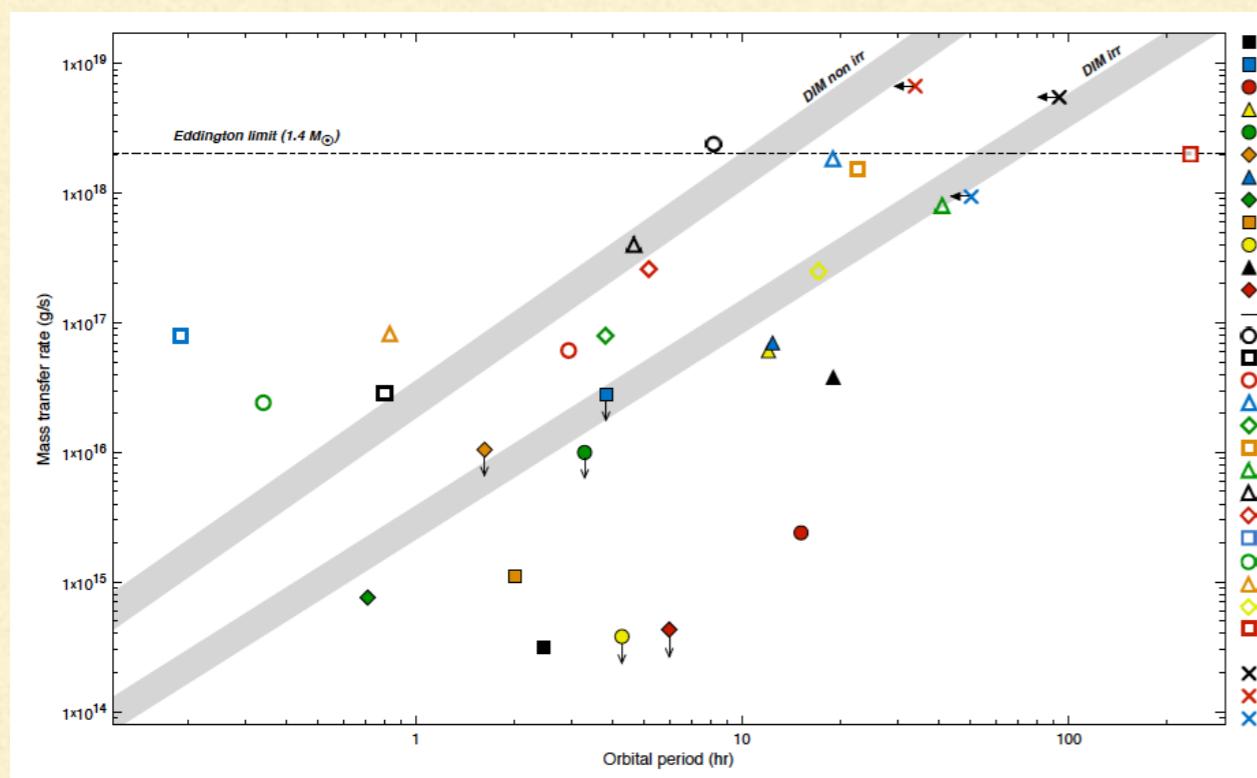
$$\alpha \approx \frac{j_K(R_{\text{disc}})}{c_s^2} t_{\text{decay}}^{-1}$$

The DIM describes DN and XRT outbursts

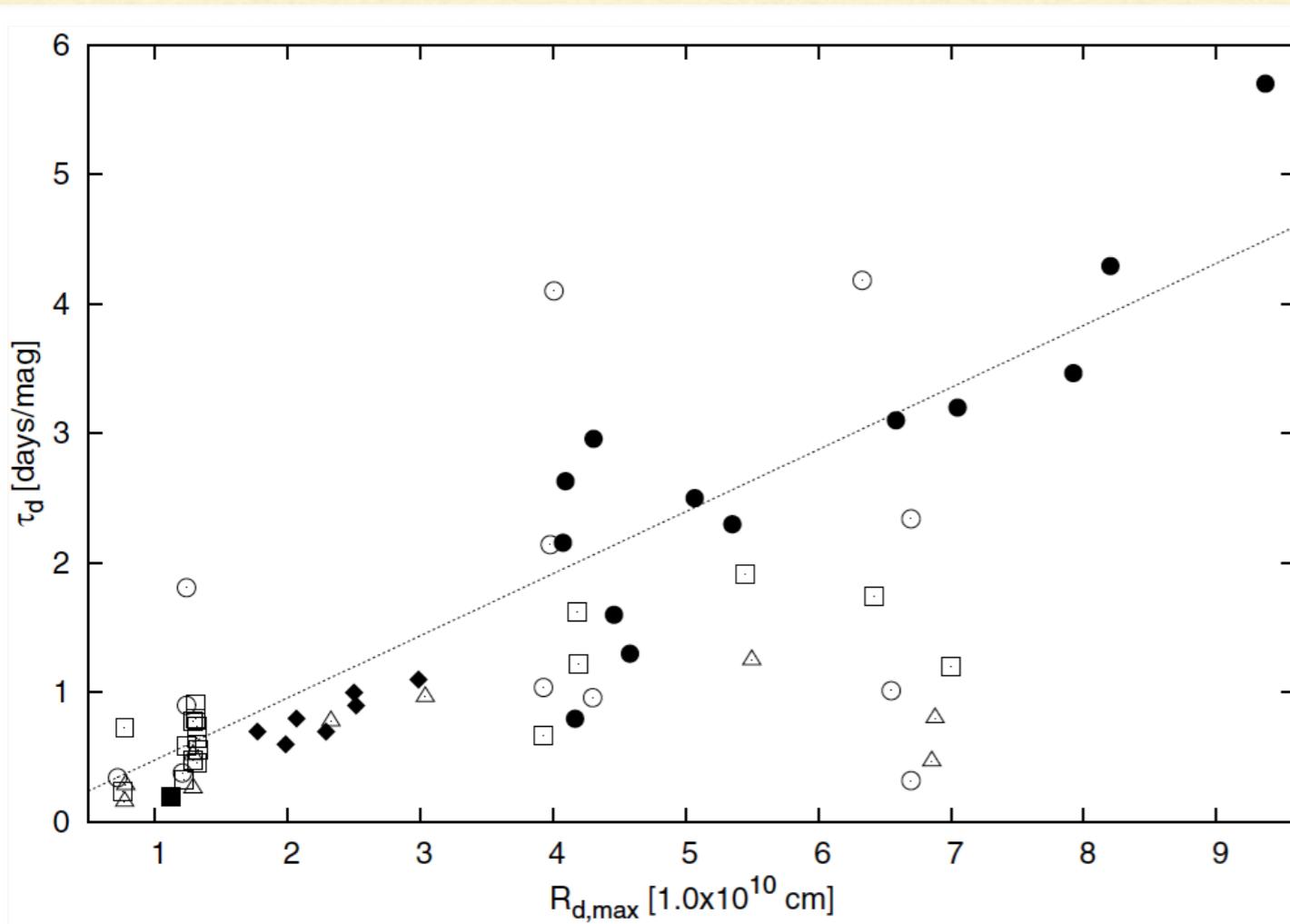


# Stability:

$$\dot{M}_{\text{tr}} > \dot{M}_{\text{crit}}(R_{disc}) \propto R^{2.6}$$

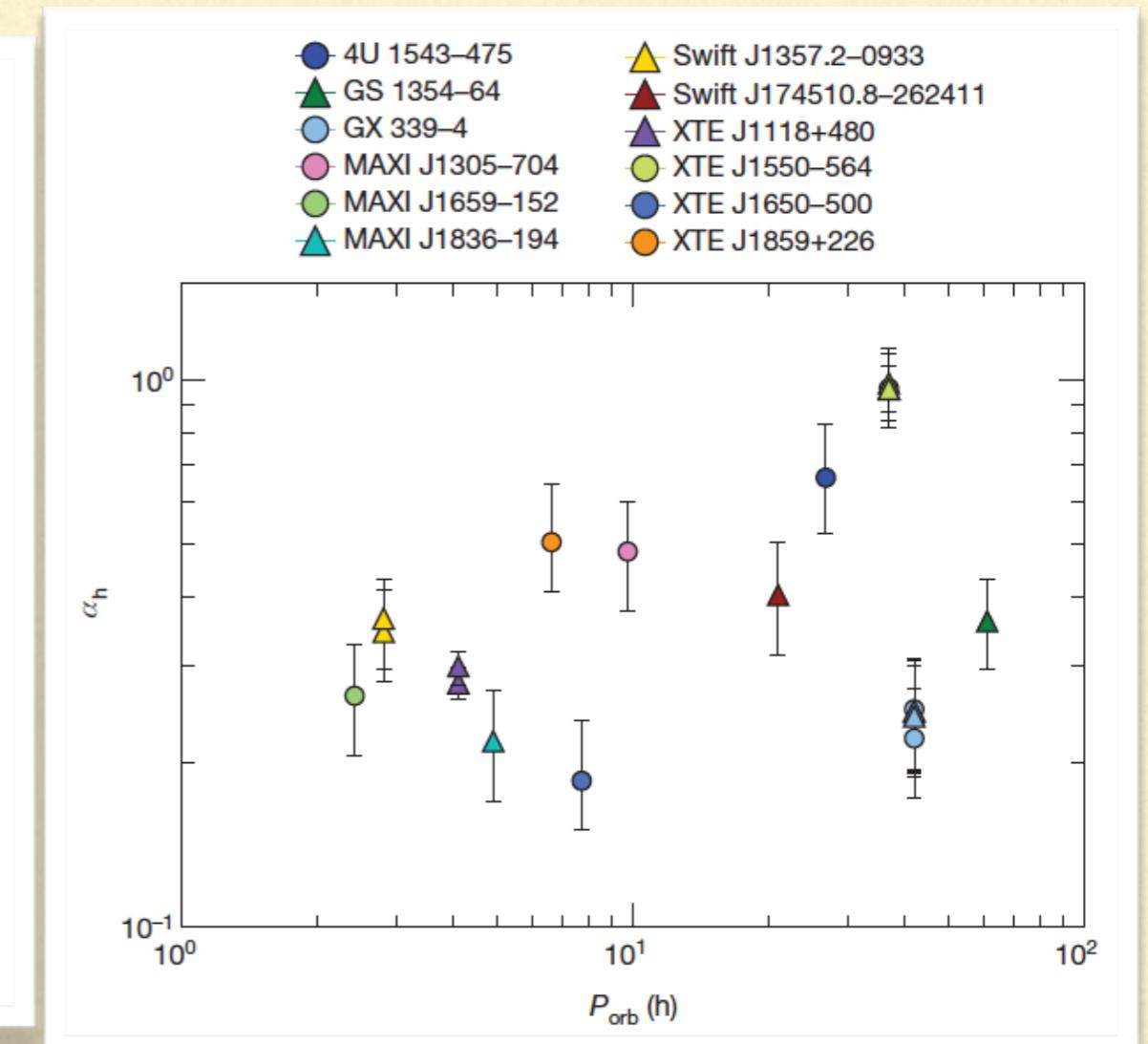


# $\alpha$ - problem I. : MRI with no net $B$ produces $\sim 0.01$



Kotko & Lasota 2012

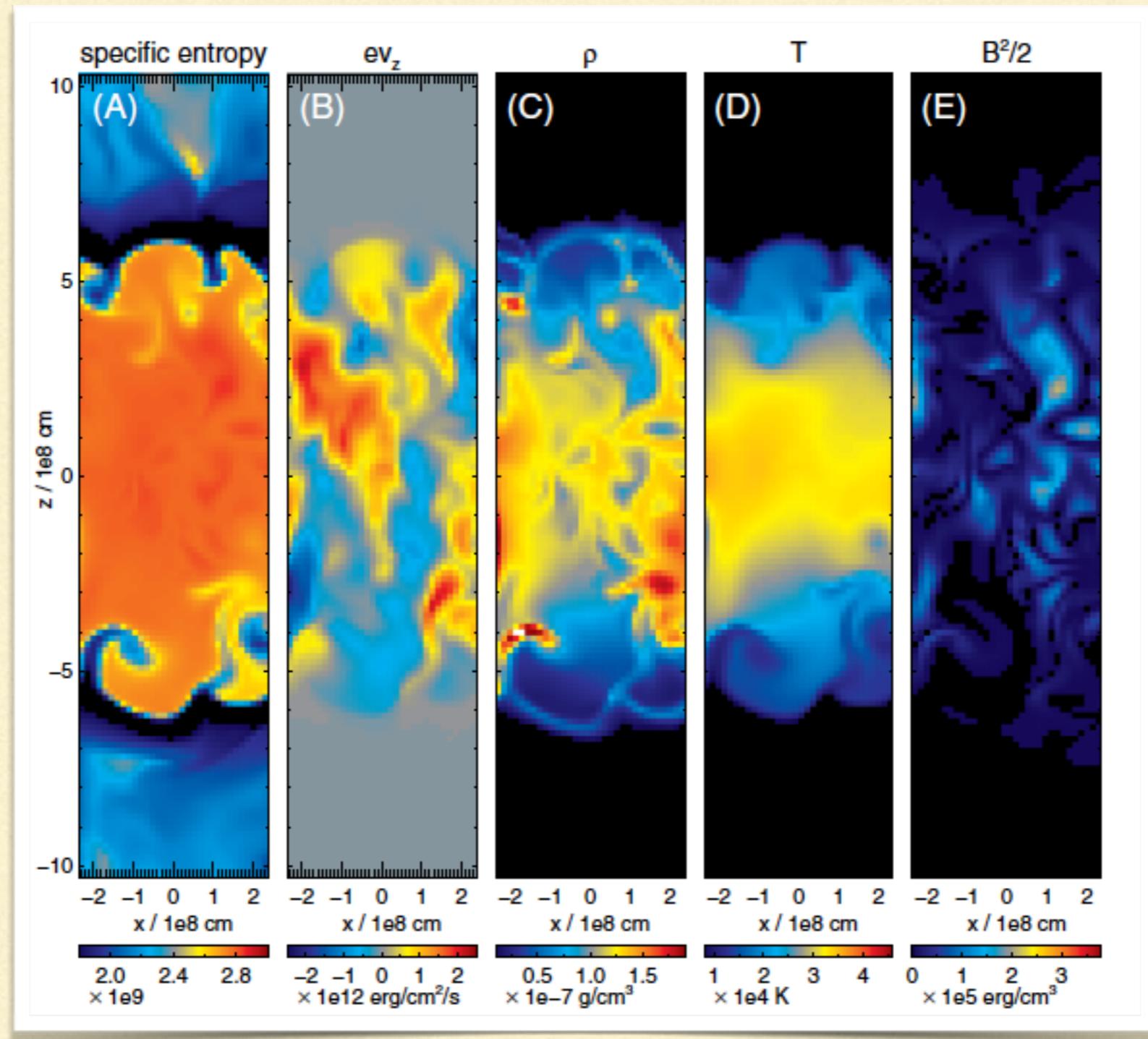
Dwarf-novae:  $\alpha_h \sim 0.1 - 0.2$



Tetarenko et al. 2018

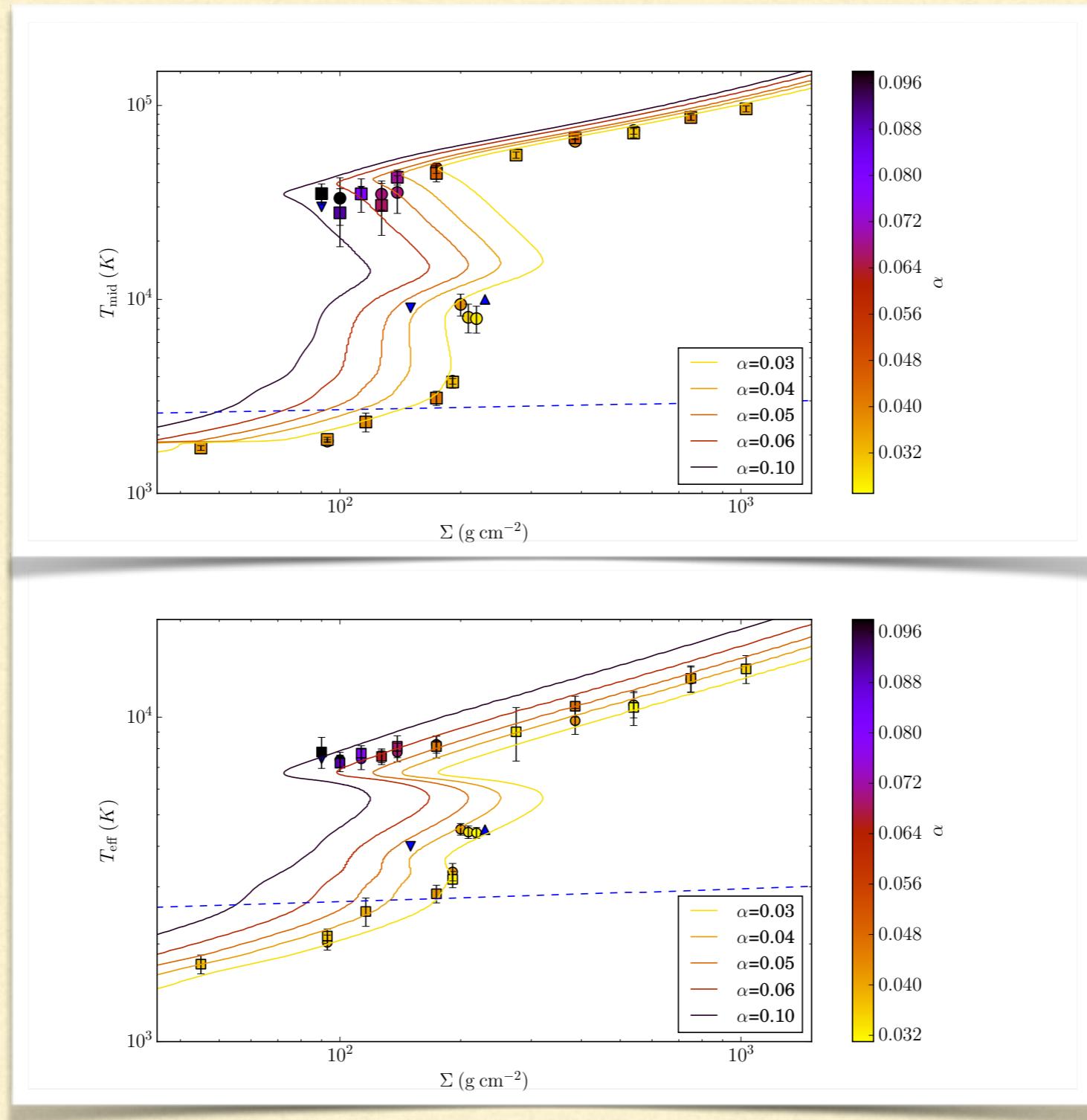
XRTs:  $\alpha_h \geq 0.2$

# $\alpha$ - problem I., solution (?) : convection-enhanced MRI

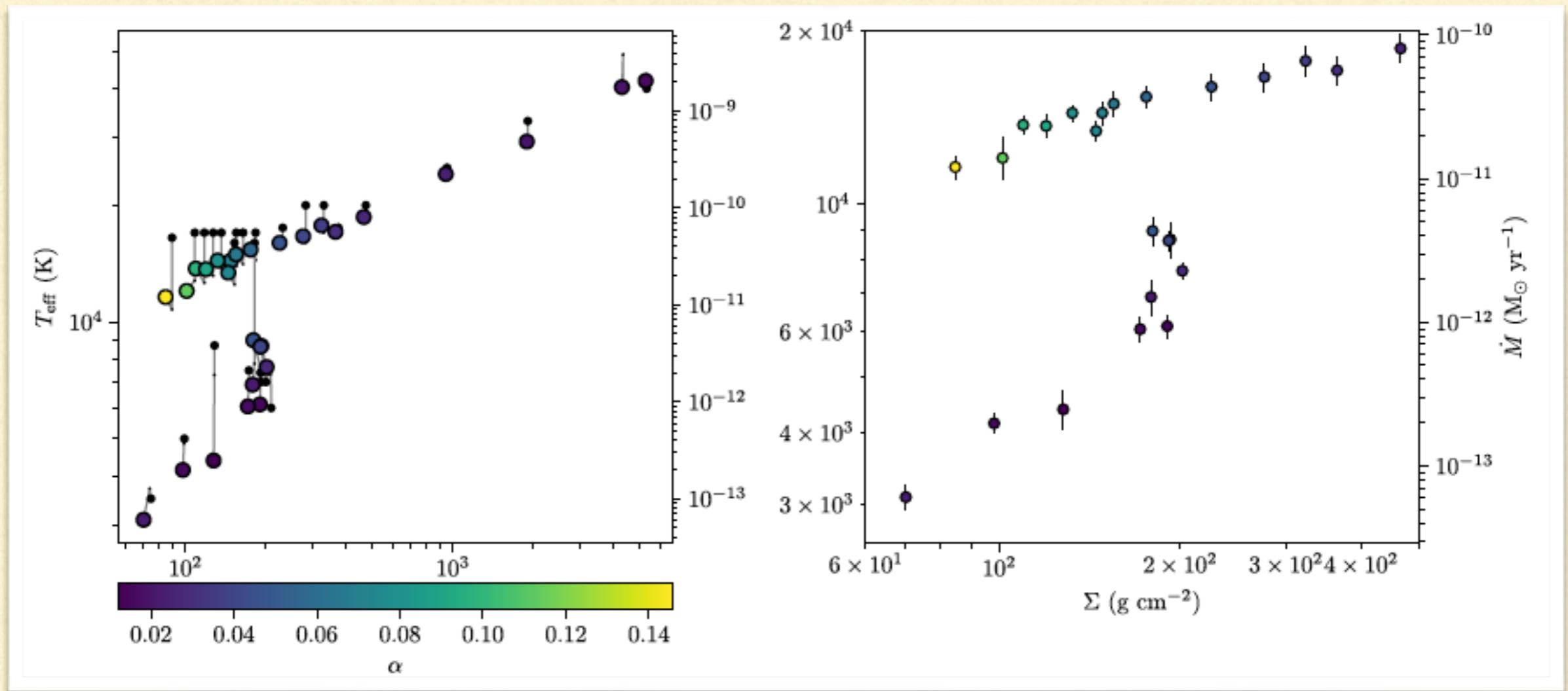


Hirose et al. 2014; Scepi et al. 2018

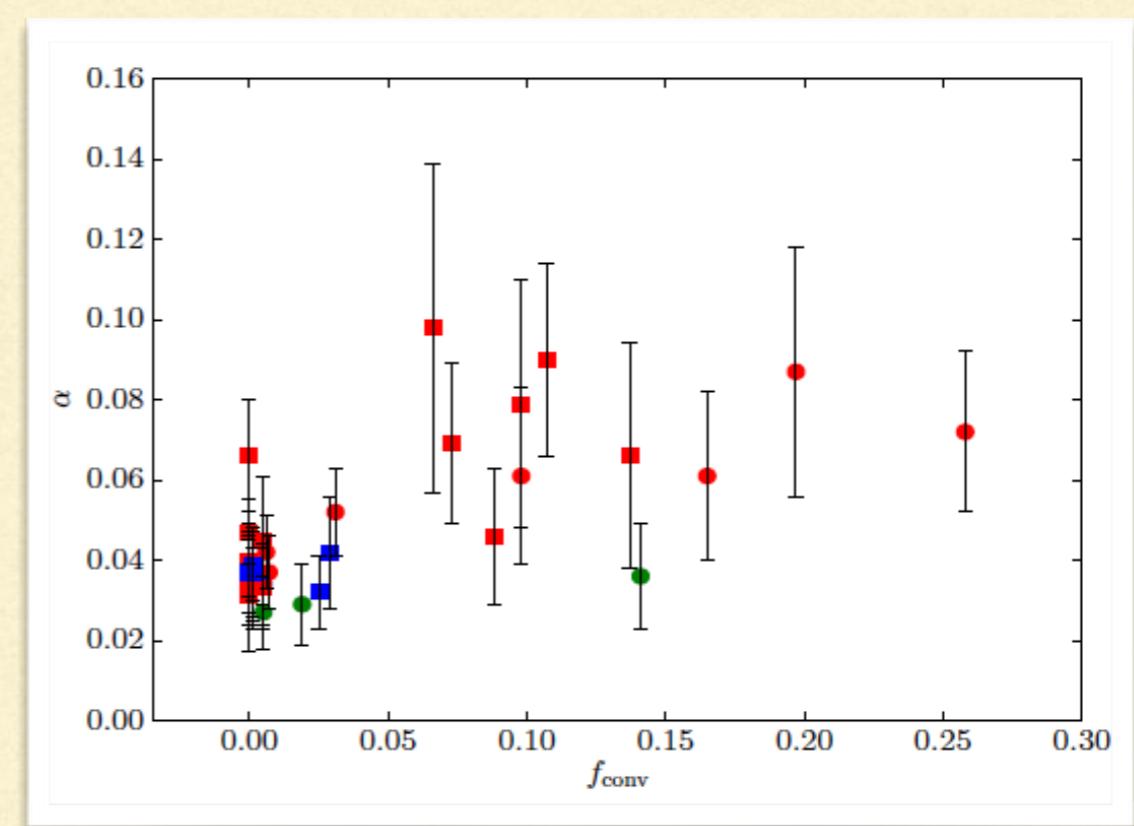
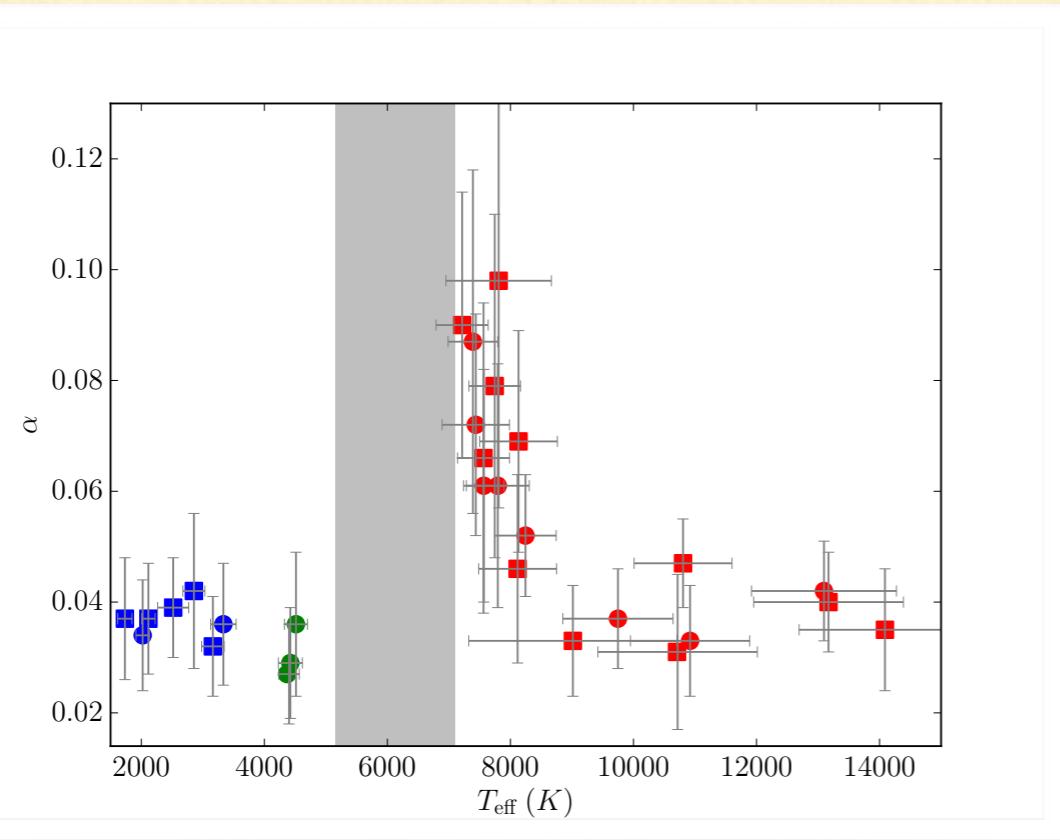
# Effect of convection in MRI discs



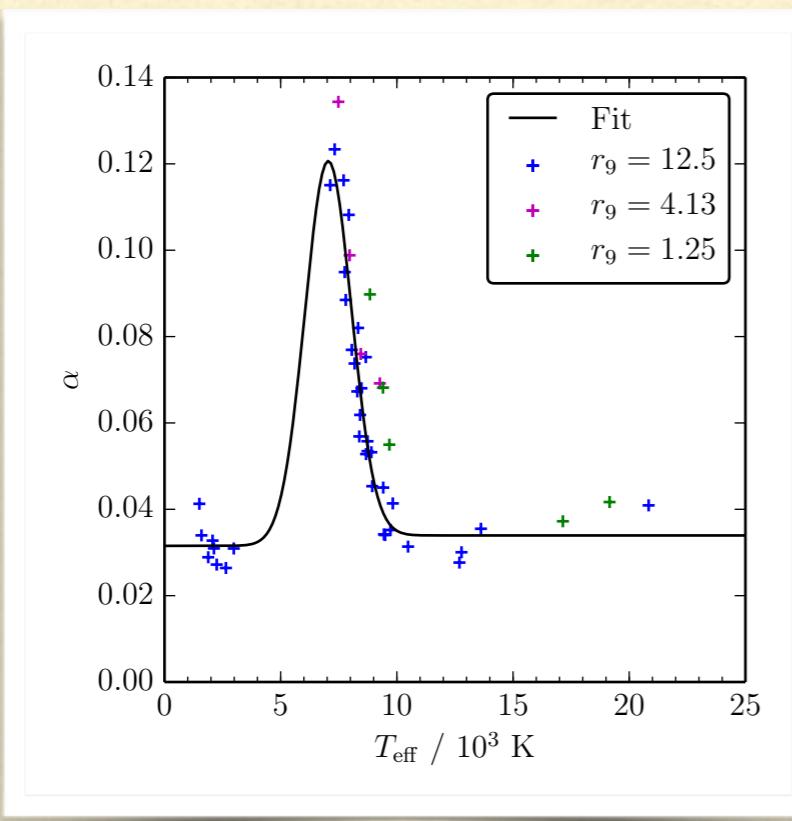
# Effect of convection in helium discs



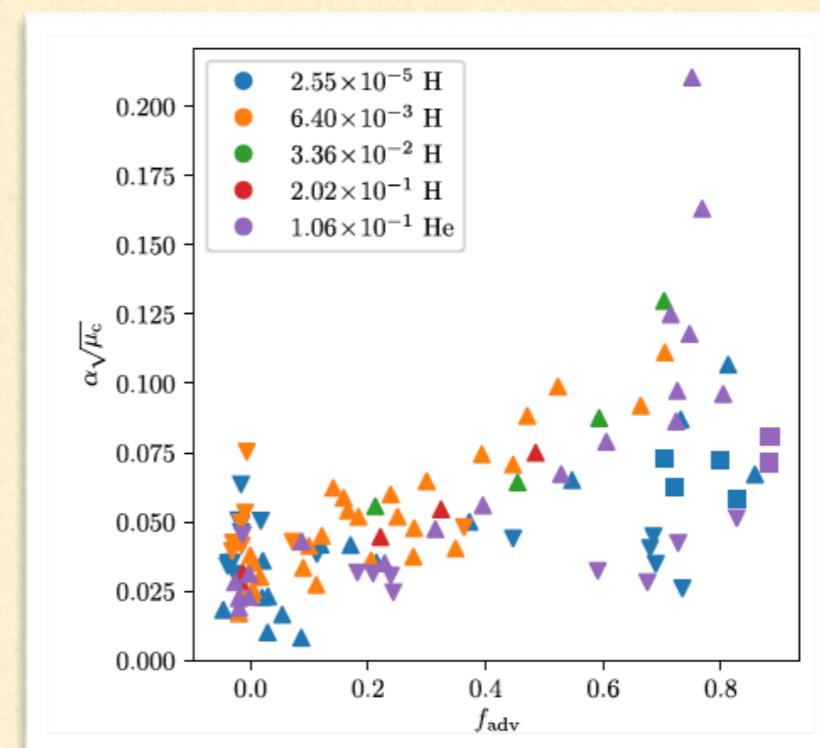
Coleman et al. 2018



Scepi et al. 2017

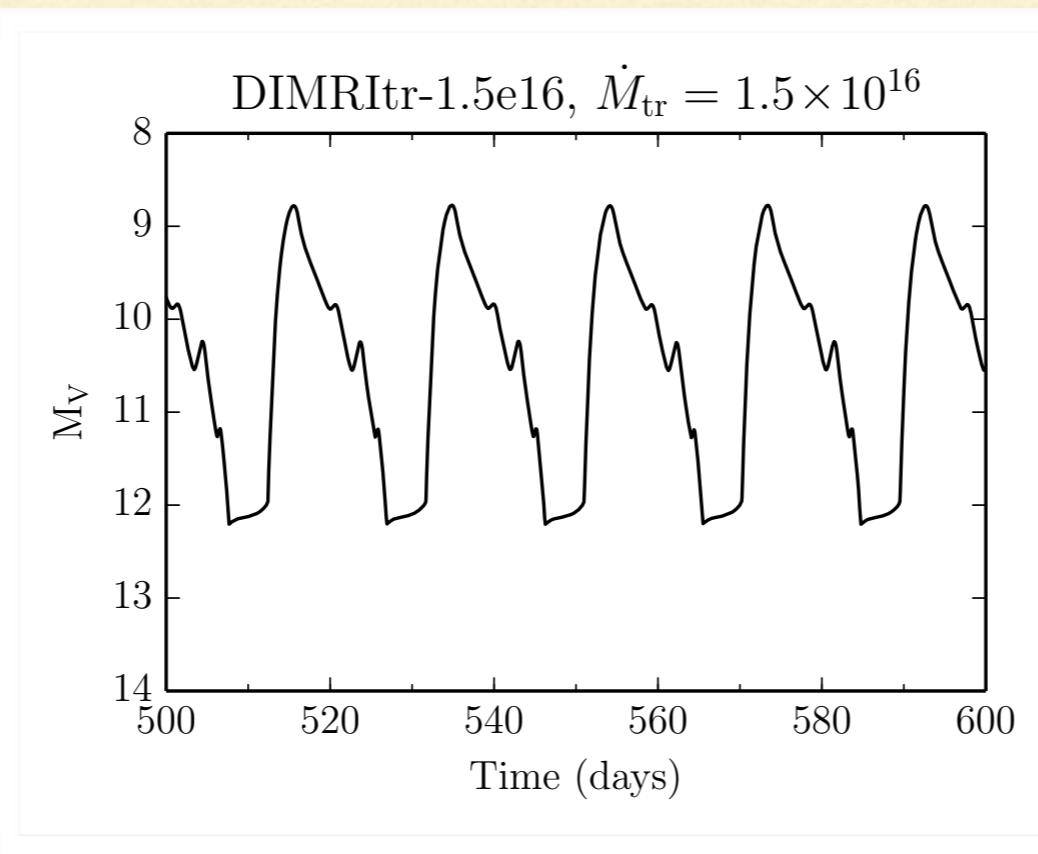


Coleman et al. 2017

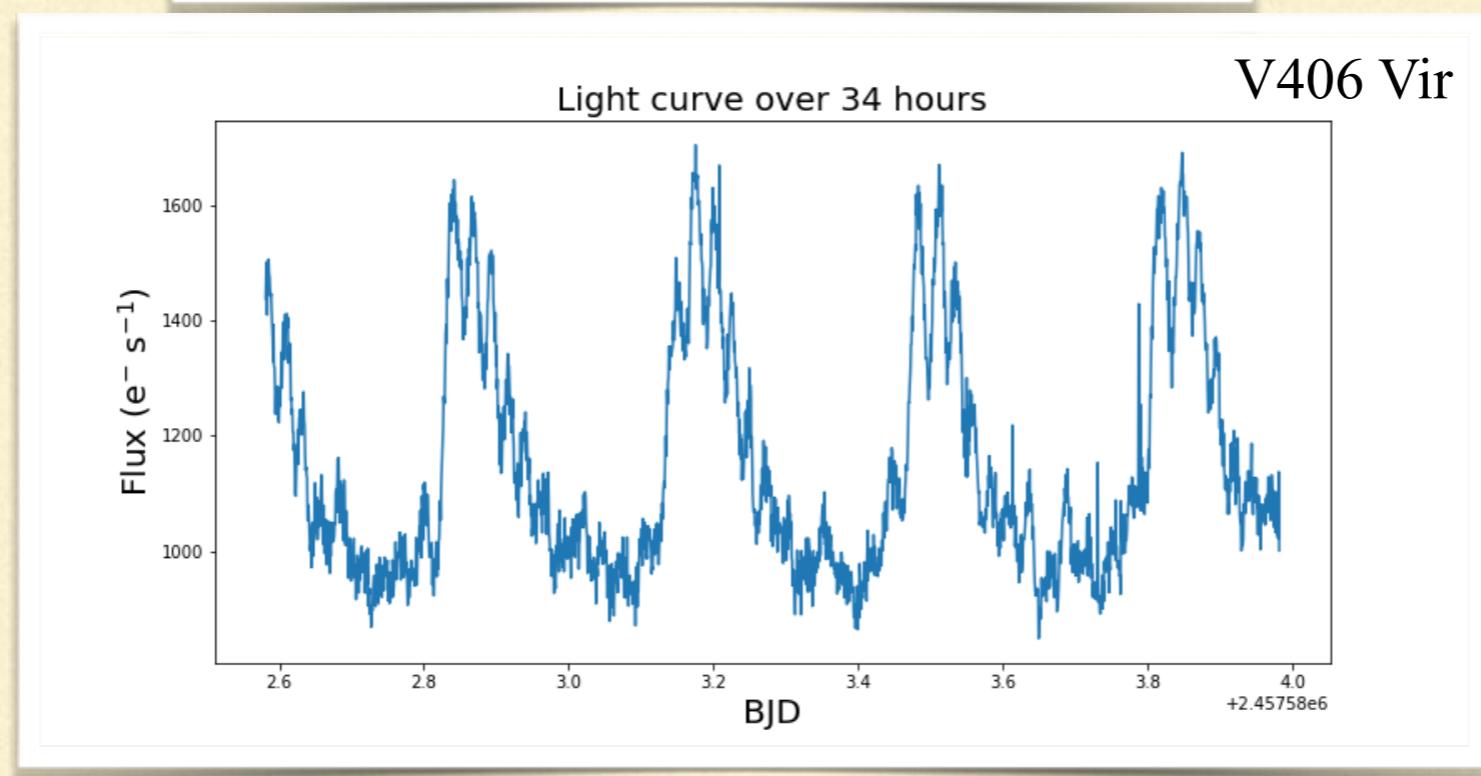


Coleman et al. 2018

# Outbursts in convective dwarf-nova discs



Coleman et al. 2017

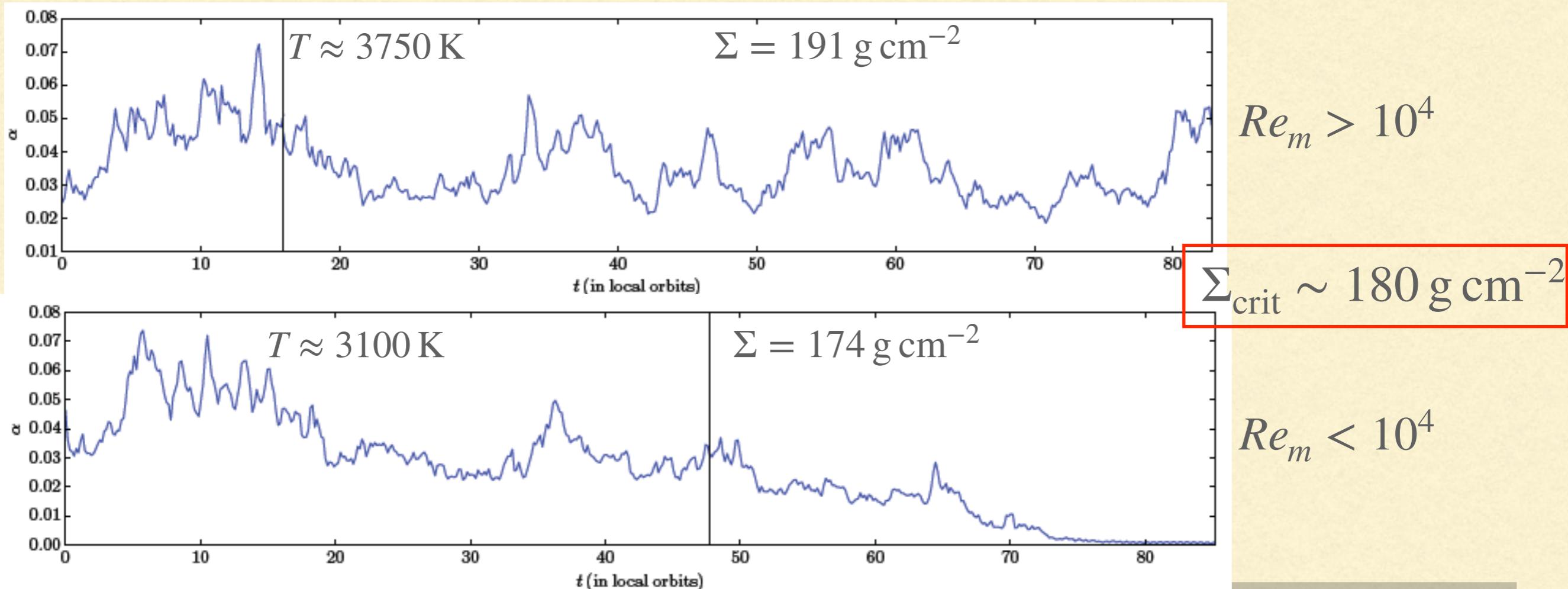


## $\alpha$ - problem II. : MRI decays at low ionisation fraction

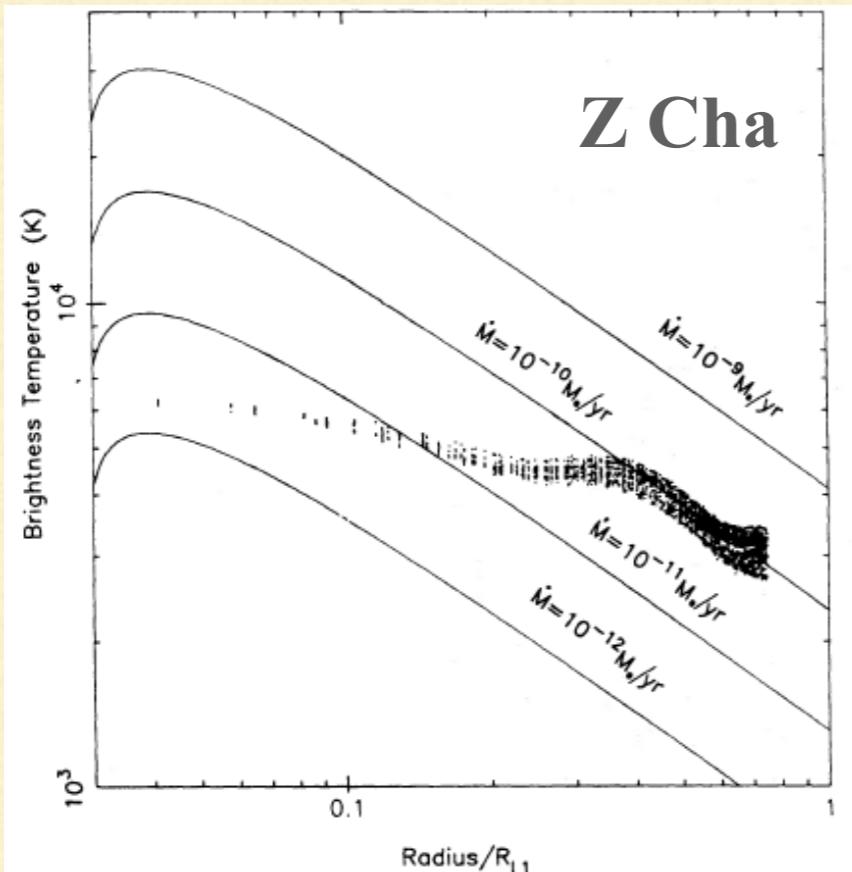
Non-ideal MHD effects:

- Ohmic resistivity
- Hall effect
- Ambipolar diffusion
- Dust

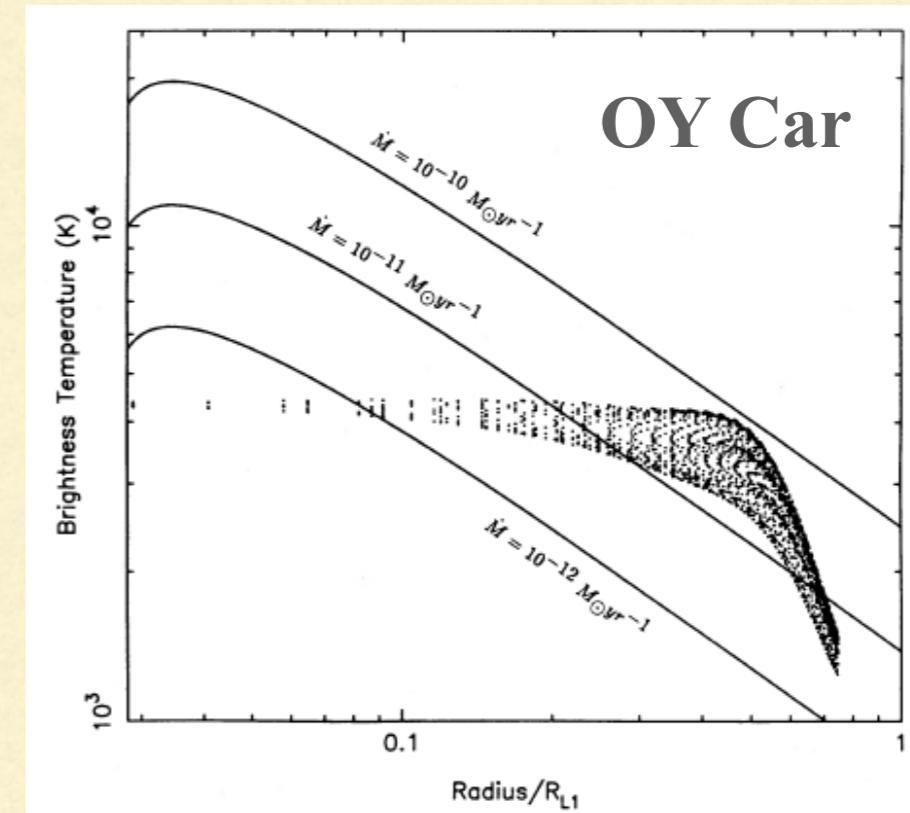
*In dwarf-nova & XRT discs ohmic resistivity is dominant.*



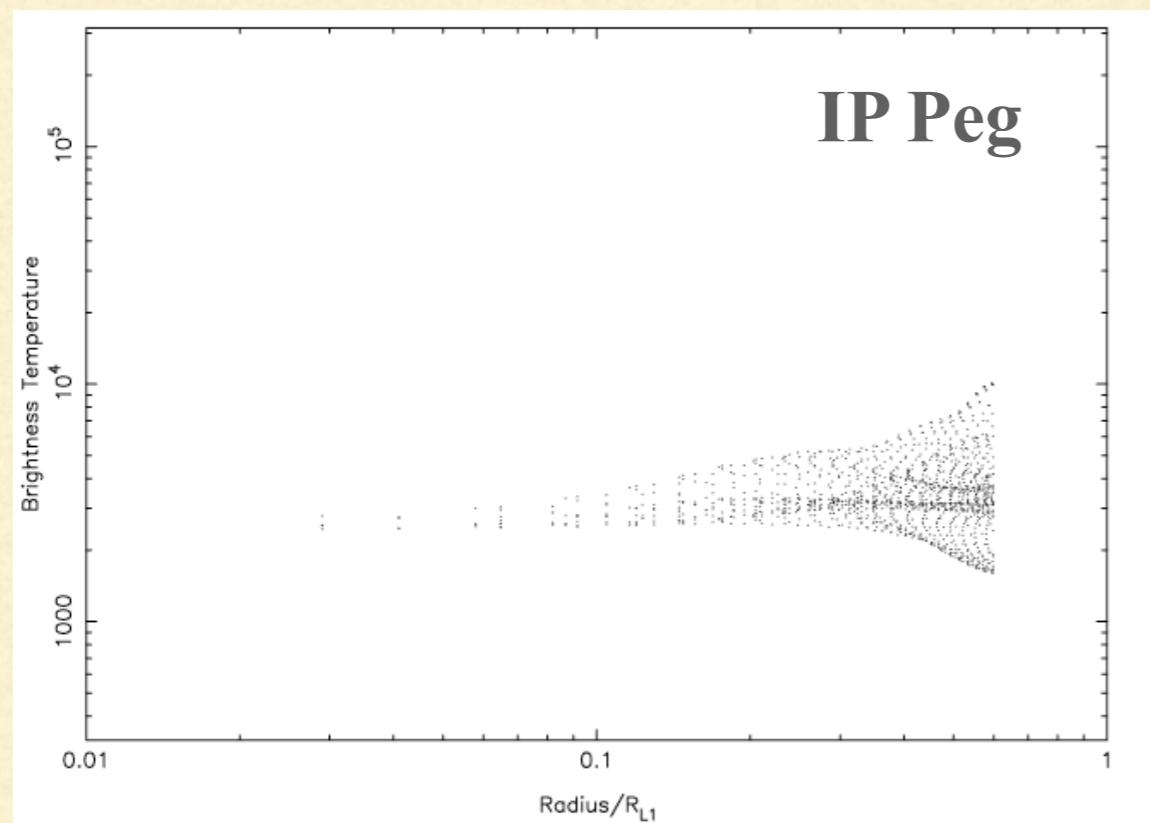
# Observations of quiescent dwarf novae



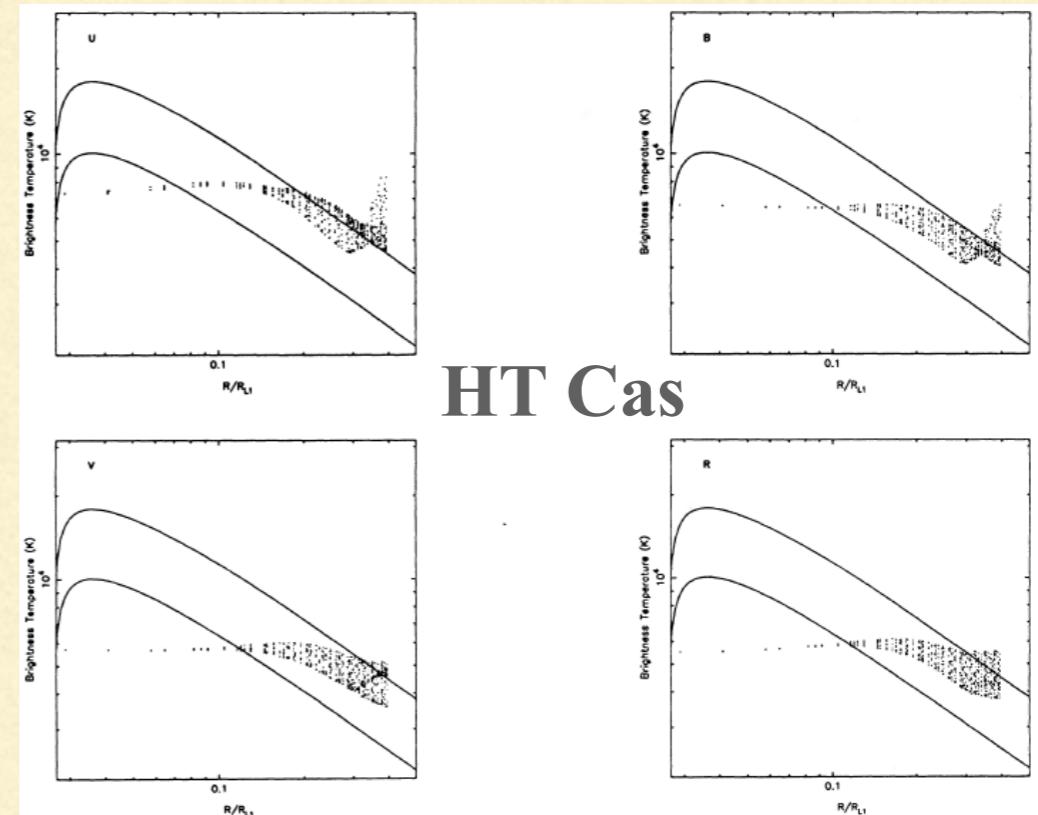
Z Cha



OY Car



IP Peg

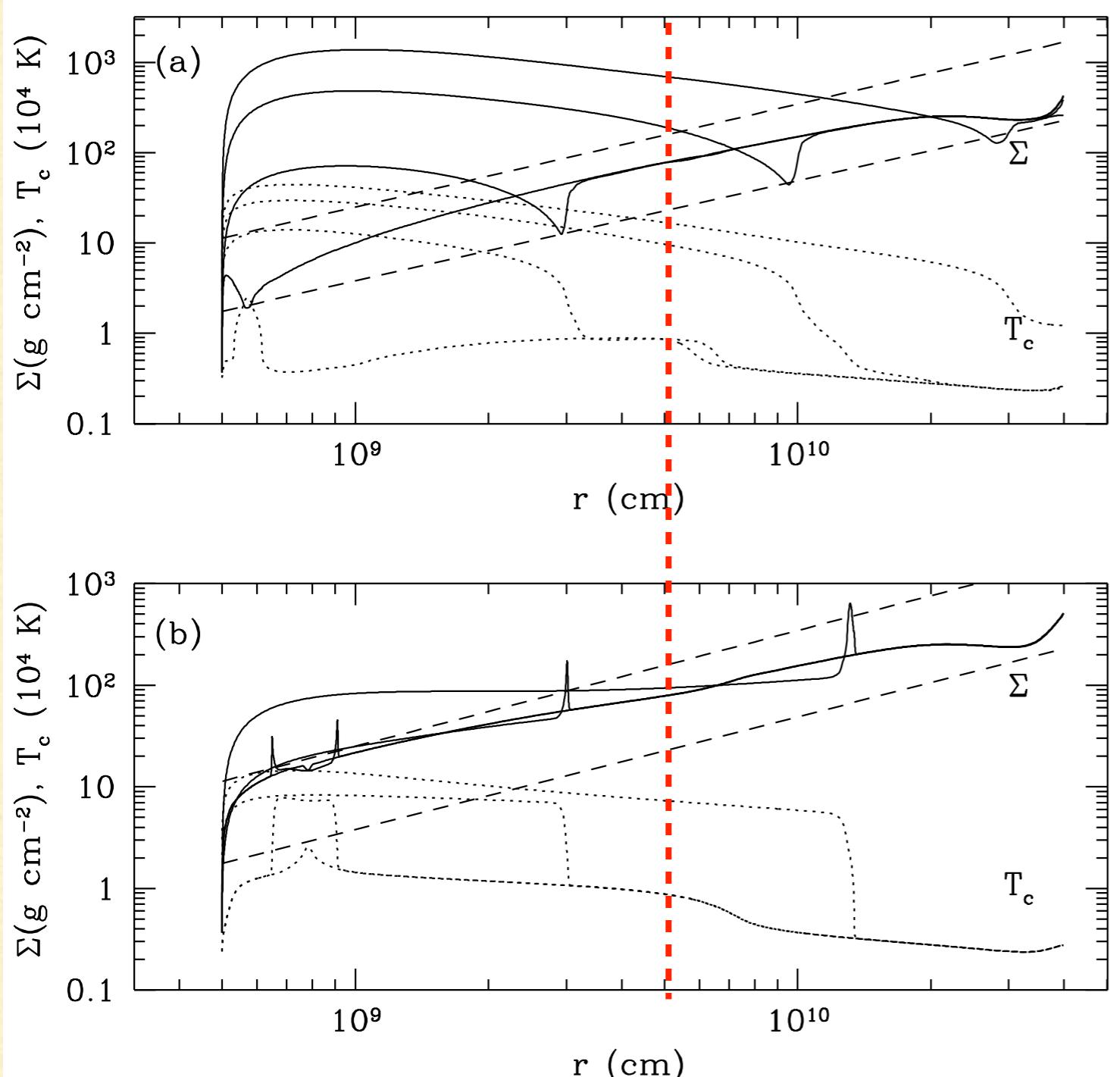


HT Cas

$T \leq 5000 \text{ K}$

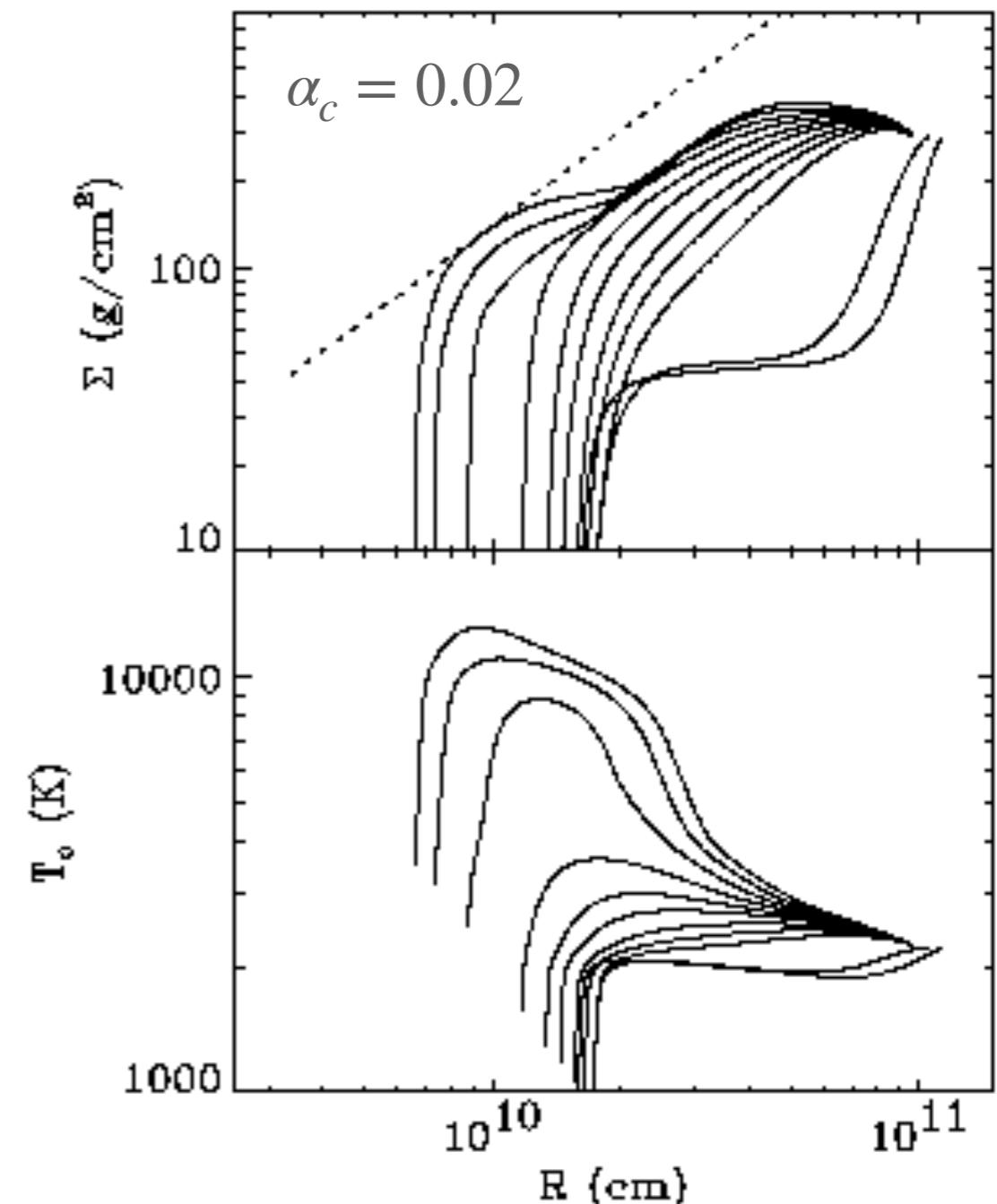
# The effect of inner-disc truncation

## Dwarf-nova



$$T_{\text{eff}}^- = 5210 R_{10}^{-0.1} \text{K}$$

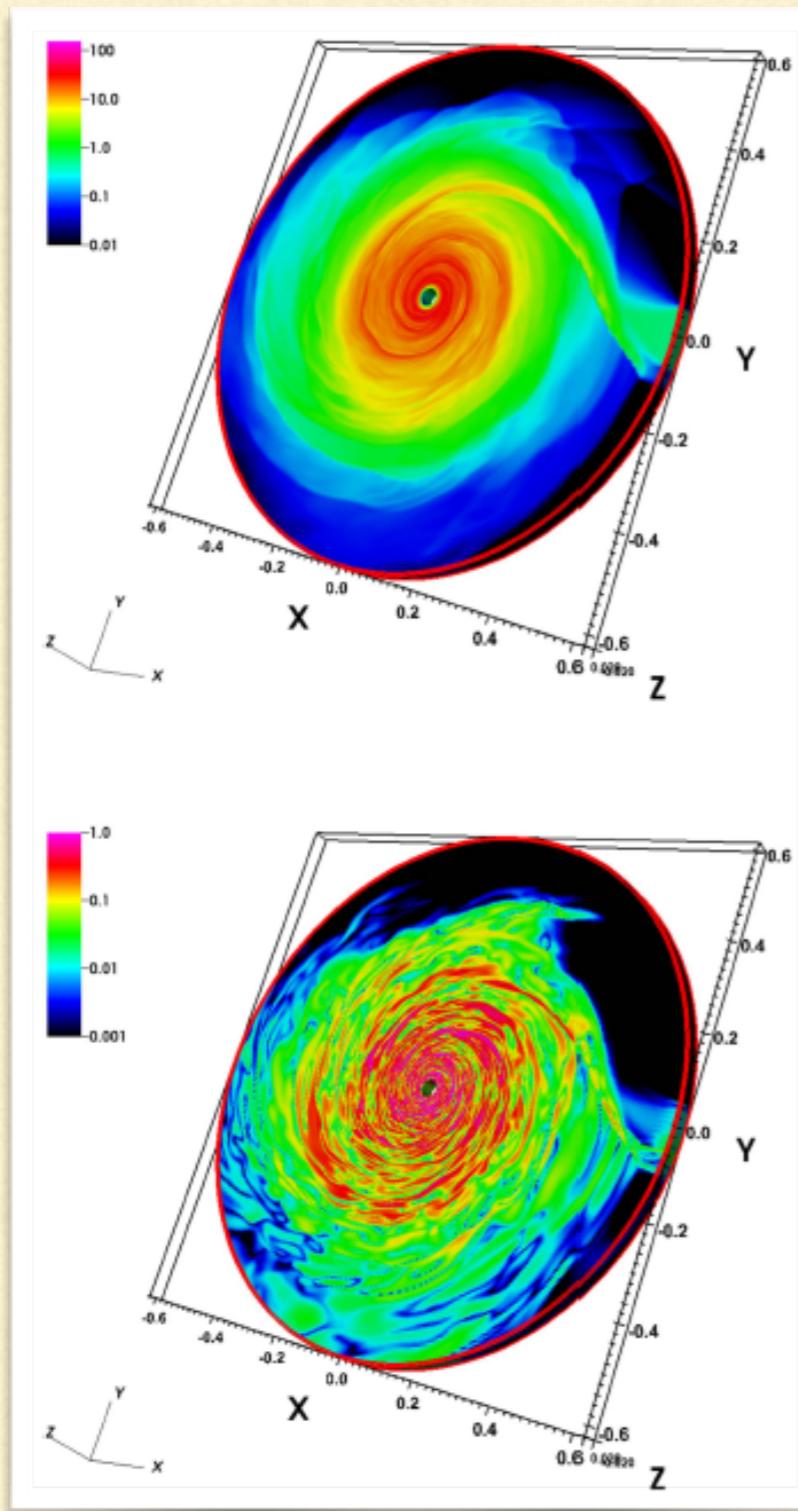
## XRT



$$T_e^- = 11374 \alpha_{0.01}^{0.14} R_{10}^{-0.1} \text{K}$$

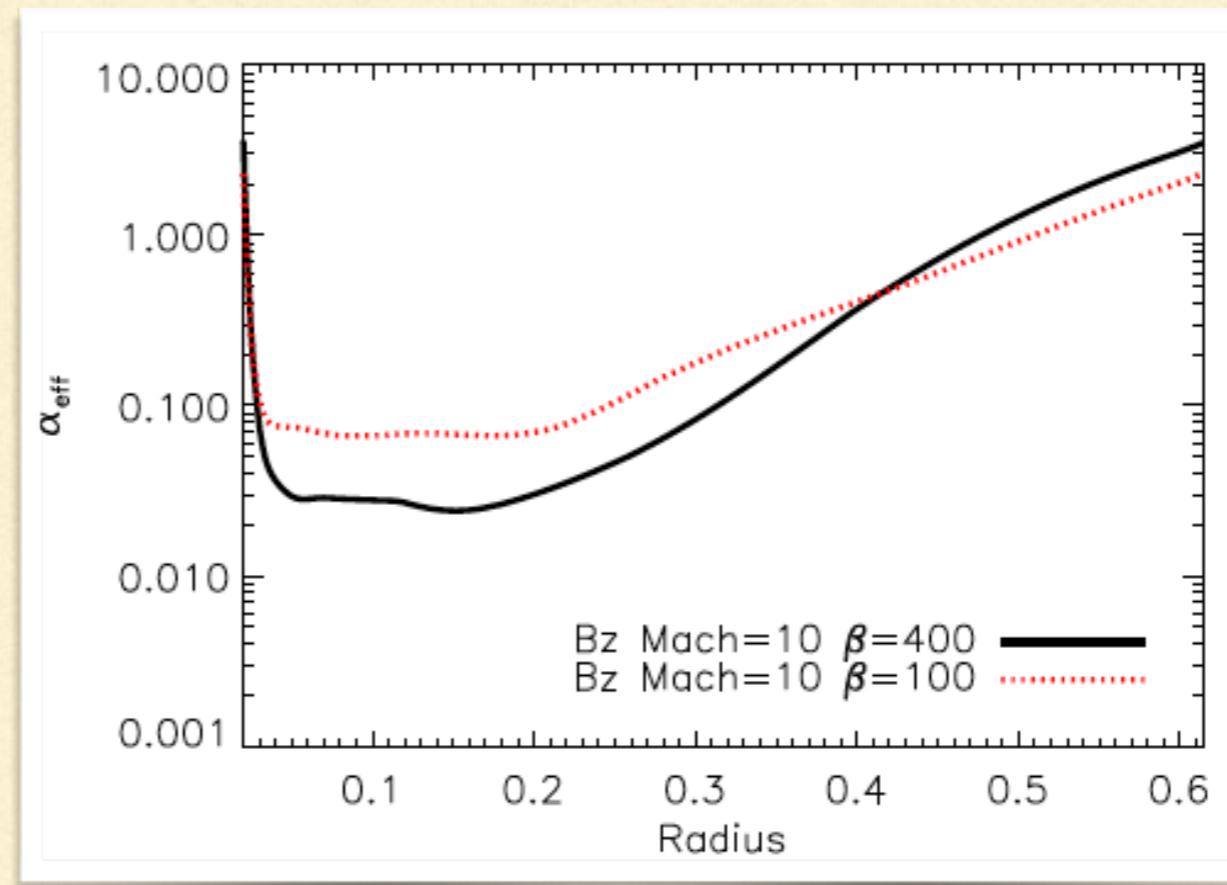
Non-turbulent part truncated ?

# Spiral waves (shocks)



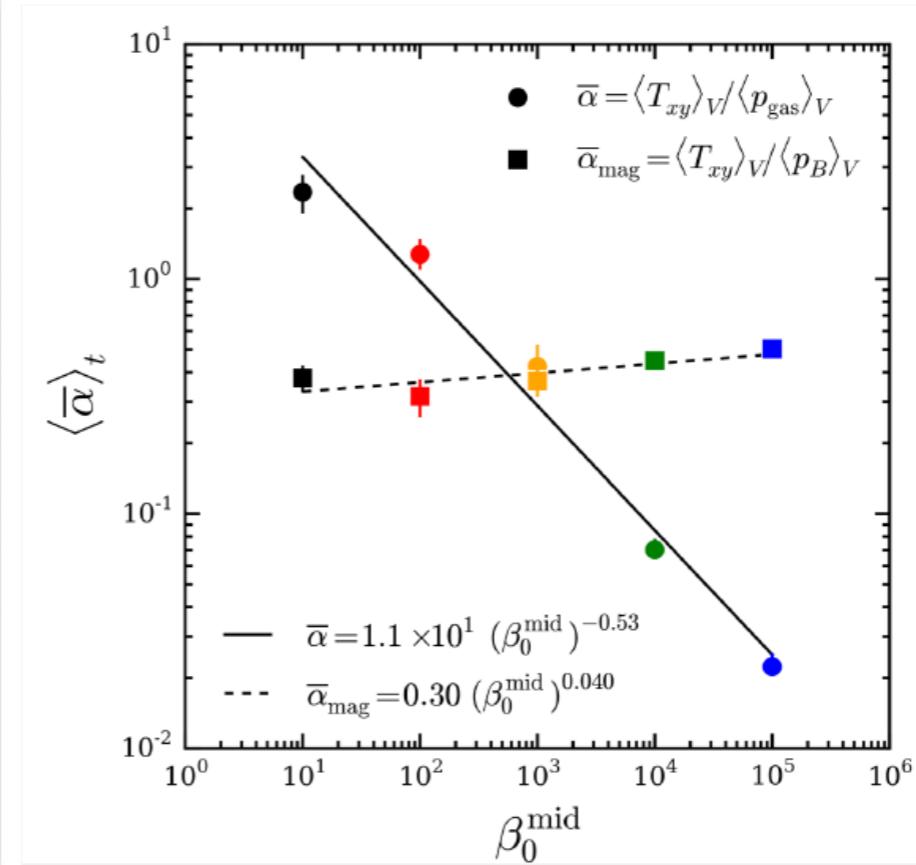
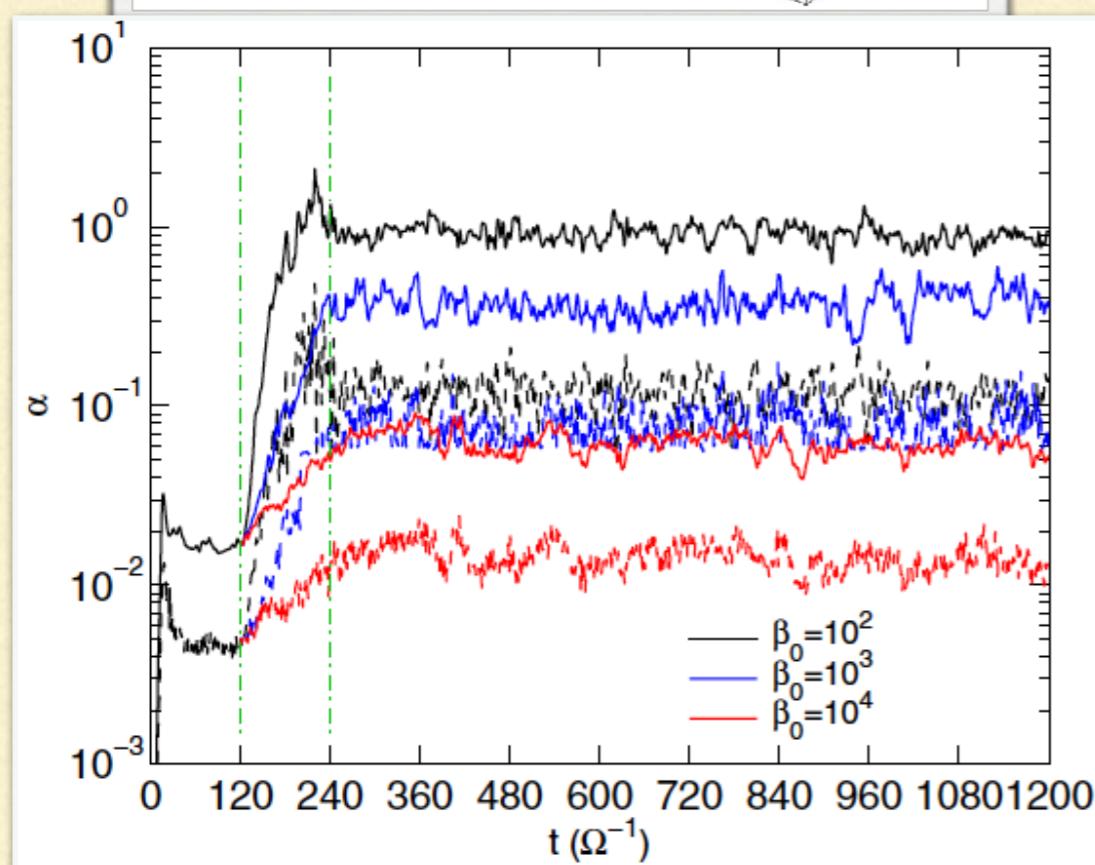
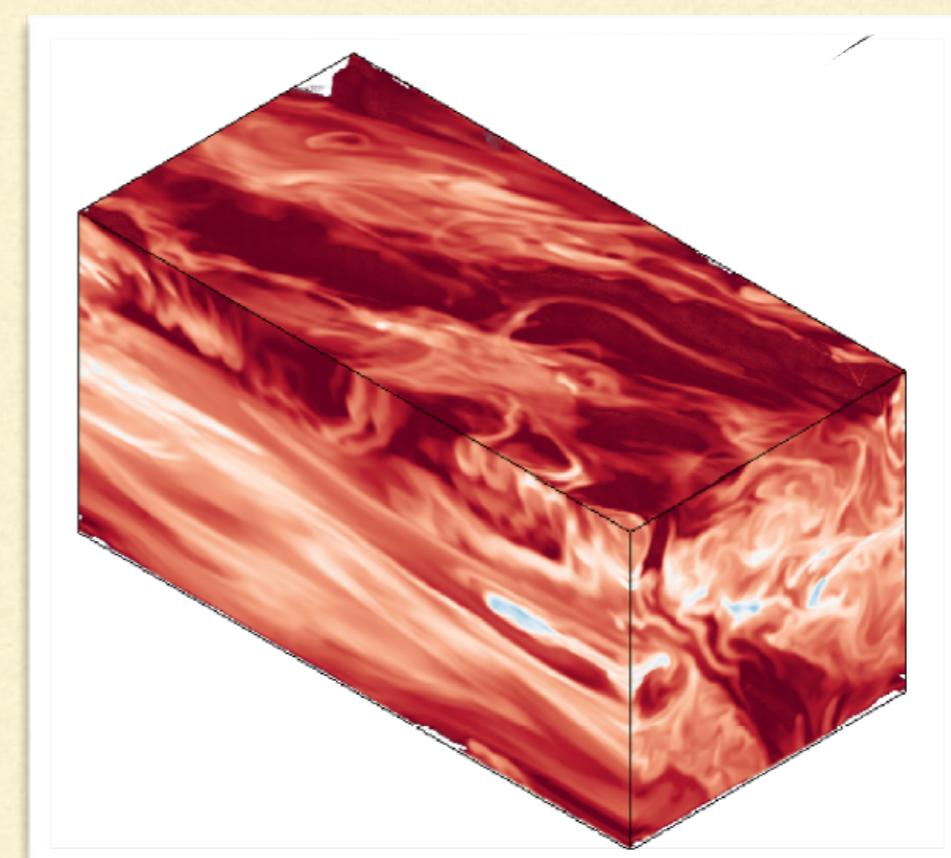
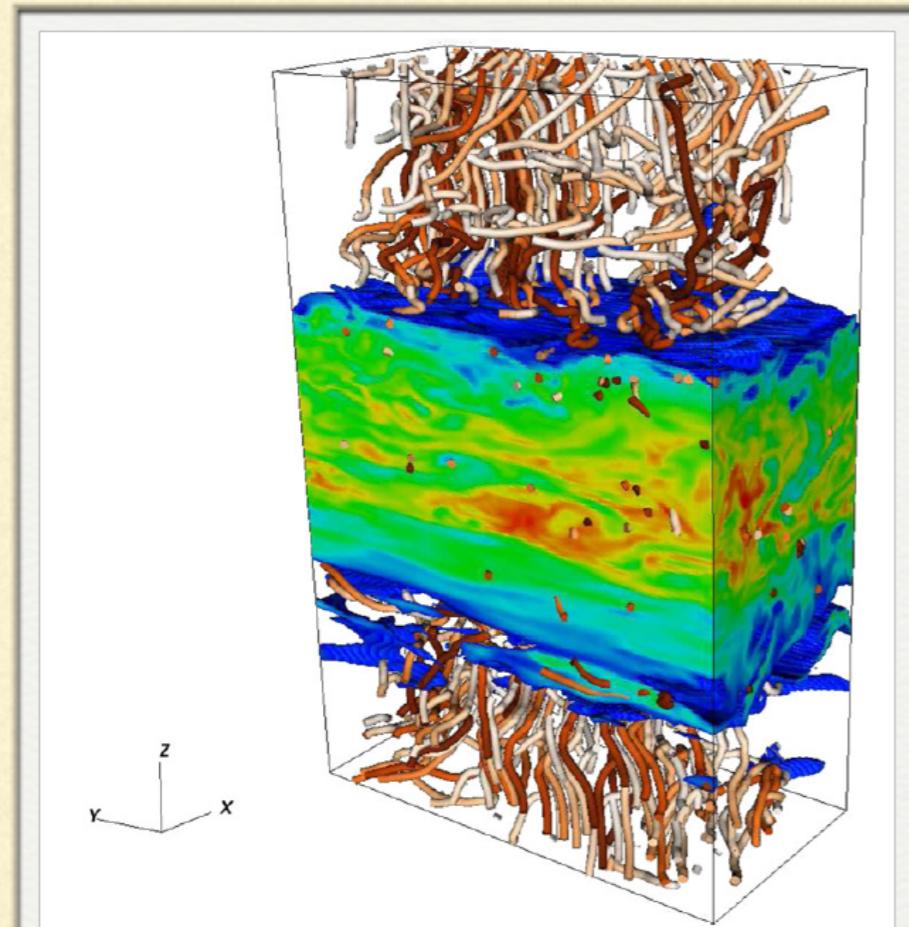
$$\mathcal{M} := \frac{v_K}{c_s} \approx \frac{R}{H} \quad \alpha_{\text{eff}} = \frac{\dot{M}}{3\pi\Sigma c_s H}$$

$$\mathcal{M} = 98 \left( \frac{R}{10^{10} \text{ cm}} \right)^{-1/2} \left( \frac{T}{10^4 \text{ K}} \right)^{-1/2}$$



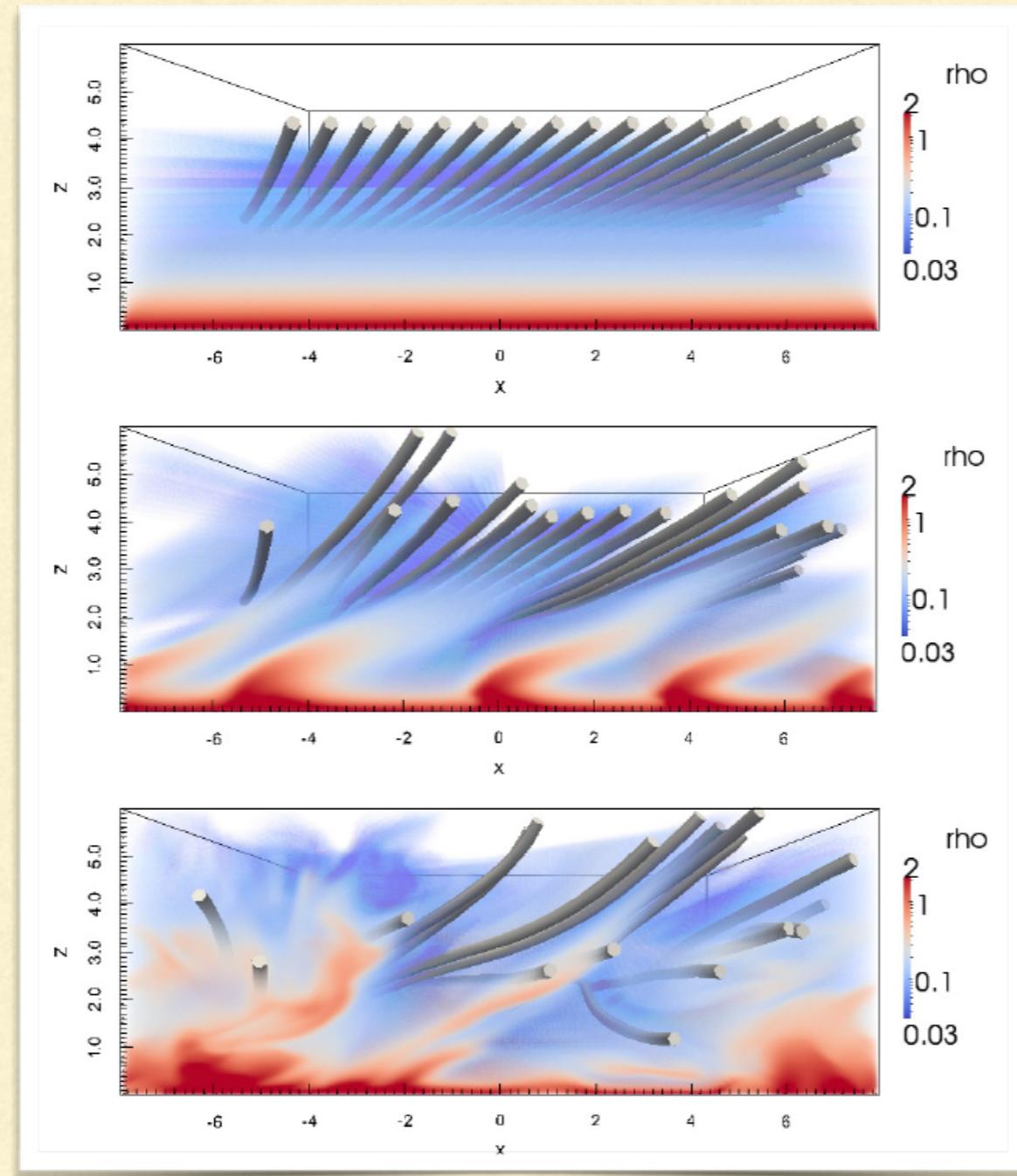
But when  $\mathcal{M} \sim 10^2$  spiral shock dissipate rapidly and do not lead to any accretion in the inner parts of the disc.

# MRI in magnetised discs



# Magnetically (MRI) driven outflows

Large scale MRI modes for  $\beta=10$



Lesur, Ferreira, Ogilvie 2013

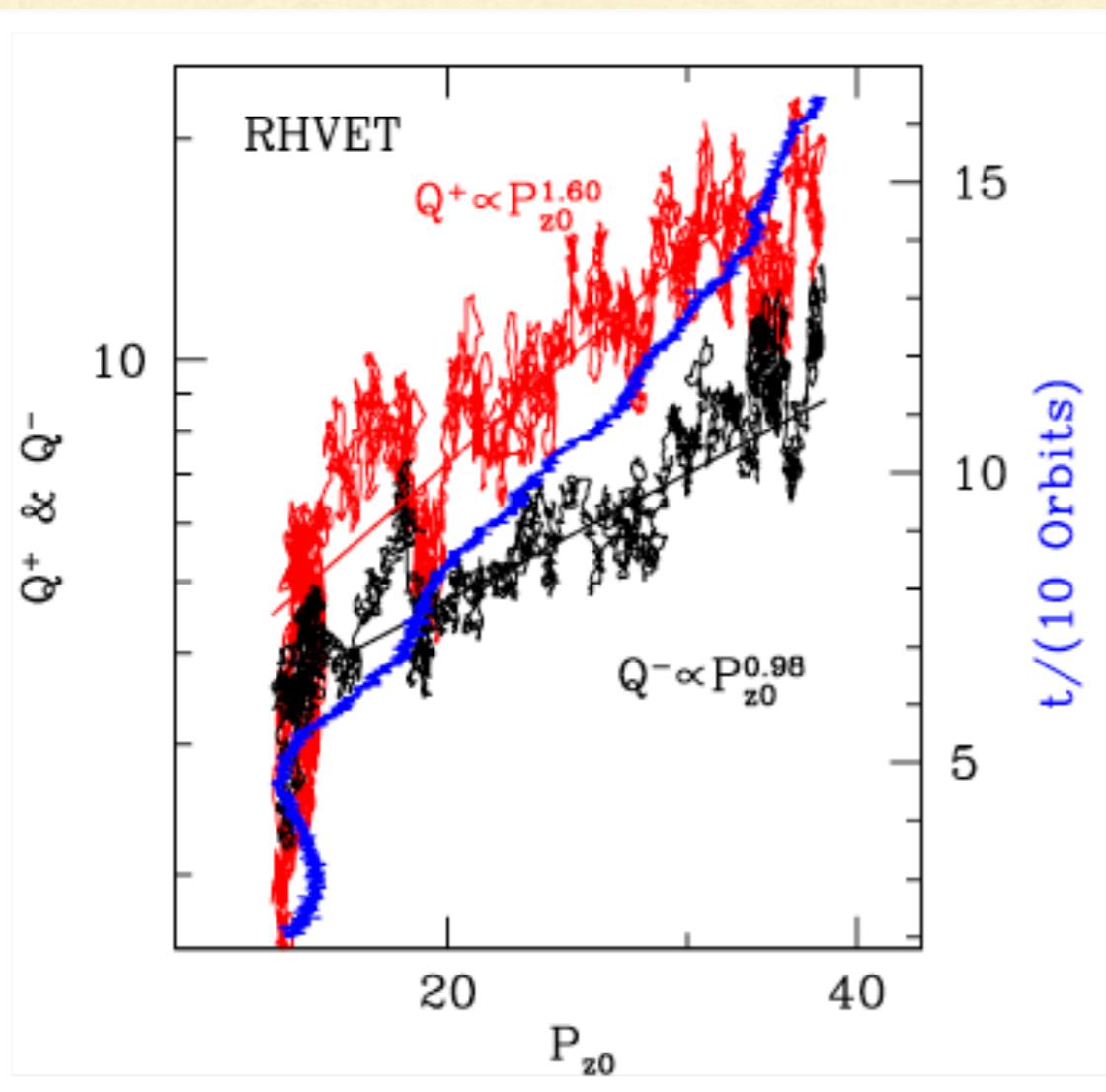
# Radiation pressure instability

$$P_{\text{rad}} \gg P_{\text{gas}}$$
$$\kappa_{\text{es}} \gg \kappa_{\text{abs}}$$

Instability:

$$\left. \frac{\partial \ln Q^+}{\partial \ln P_t} \right|_{\Sigma} > \left. \frac{\partial \ln Q^-}{\partial \ln P_t} \right|_{\Sigma}$$

Piran 1979



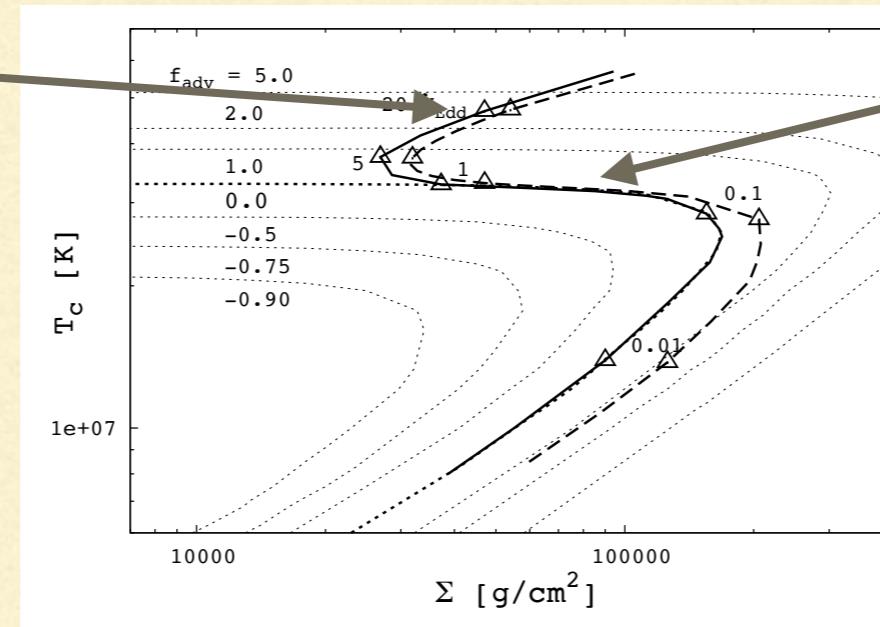
$\alpha$ -disc:  $Q^+ \sim P_t^2$   
 $Q^- \sim P_t$

*(The claim that the radiation pressure instability is suppressed in MRI discs resulted from using on a too-narrow shearing box.)*

Jiang et al. 2013

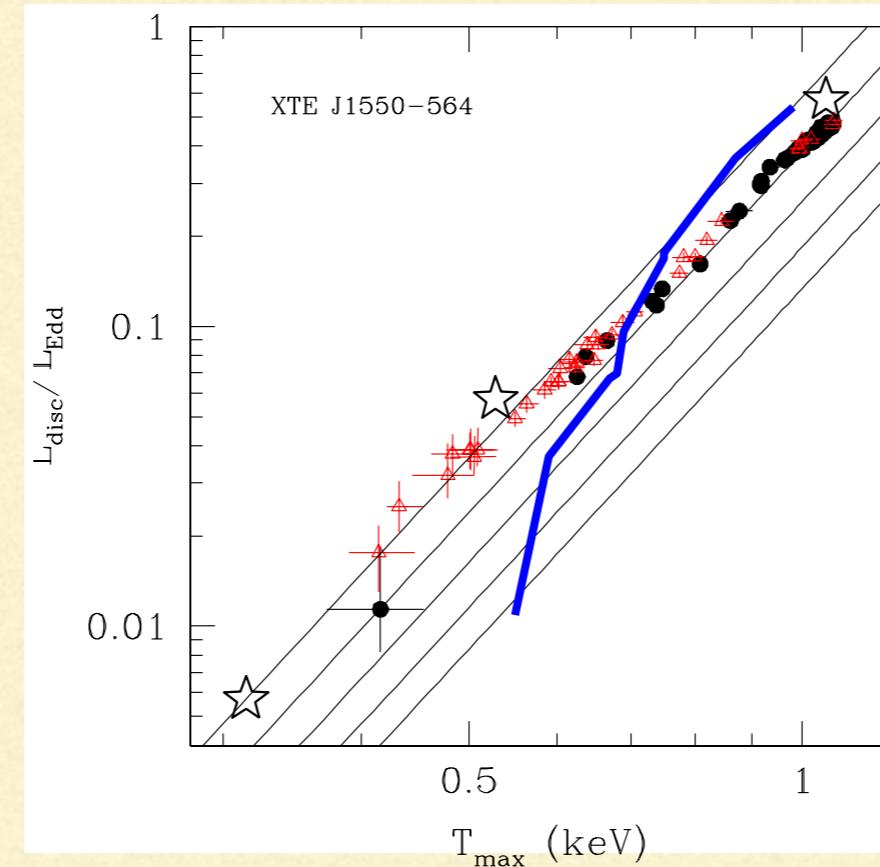
# But the radiation pressure instability at $\sim 0.01 L_{\text{Edd}}$ has never been observed !

*Slim discs*



$P_{\text{rad}} - \text{unstable}$

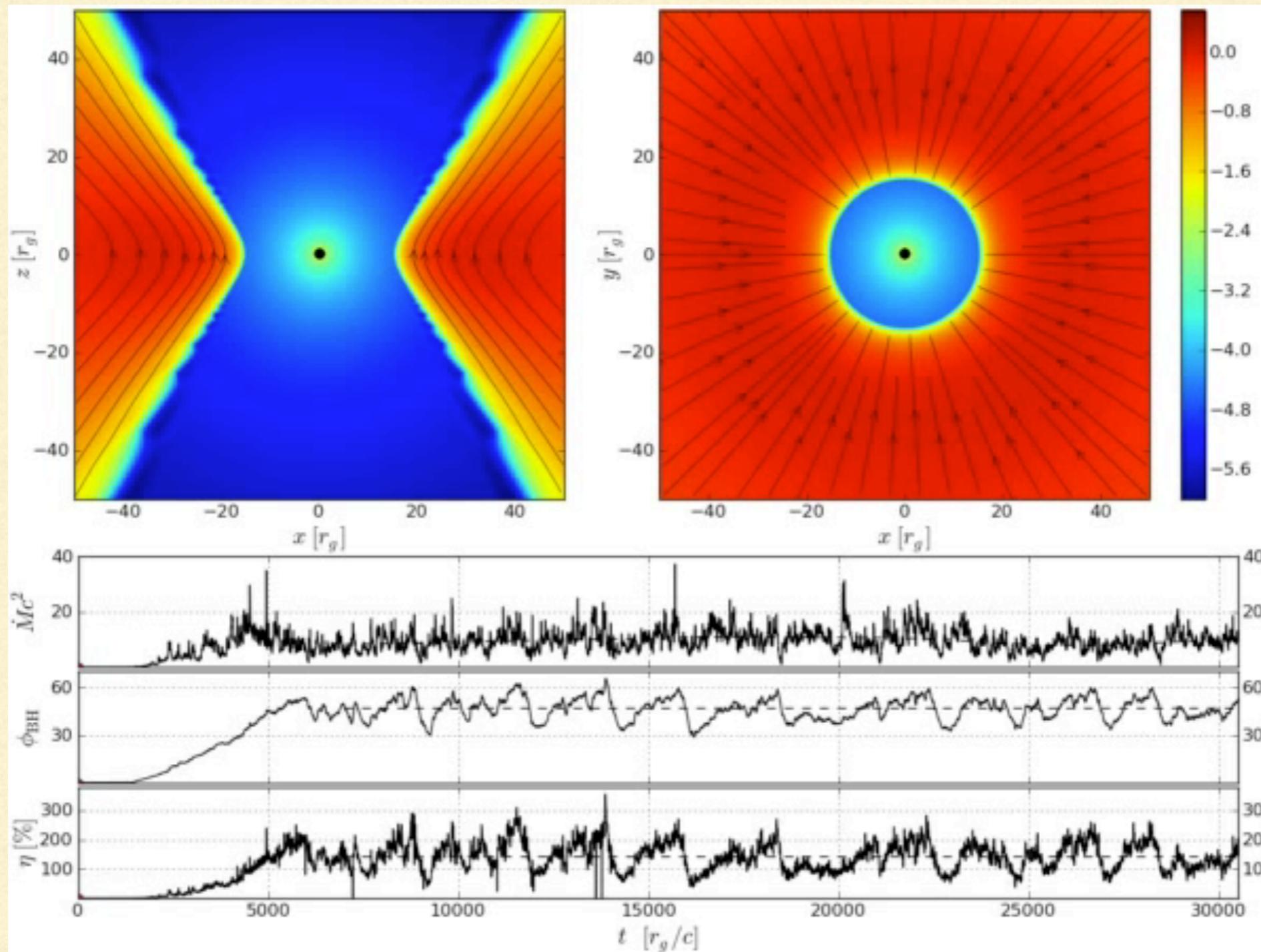
Sądowski et al. 2010



Gierliński & Done 2004

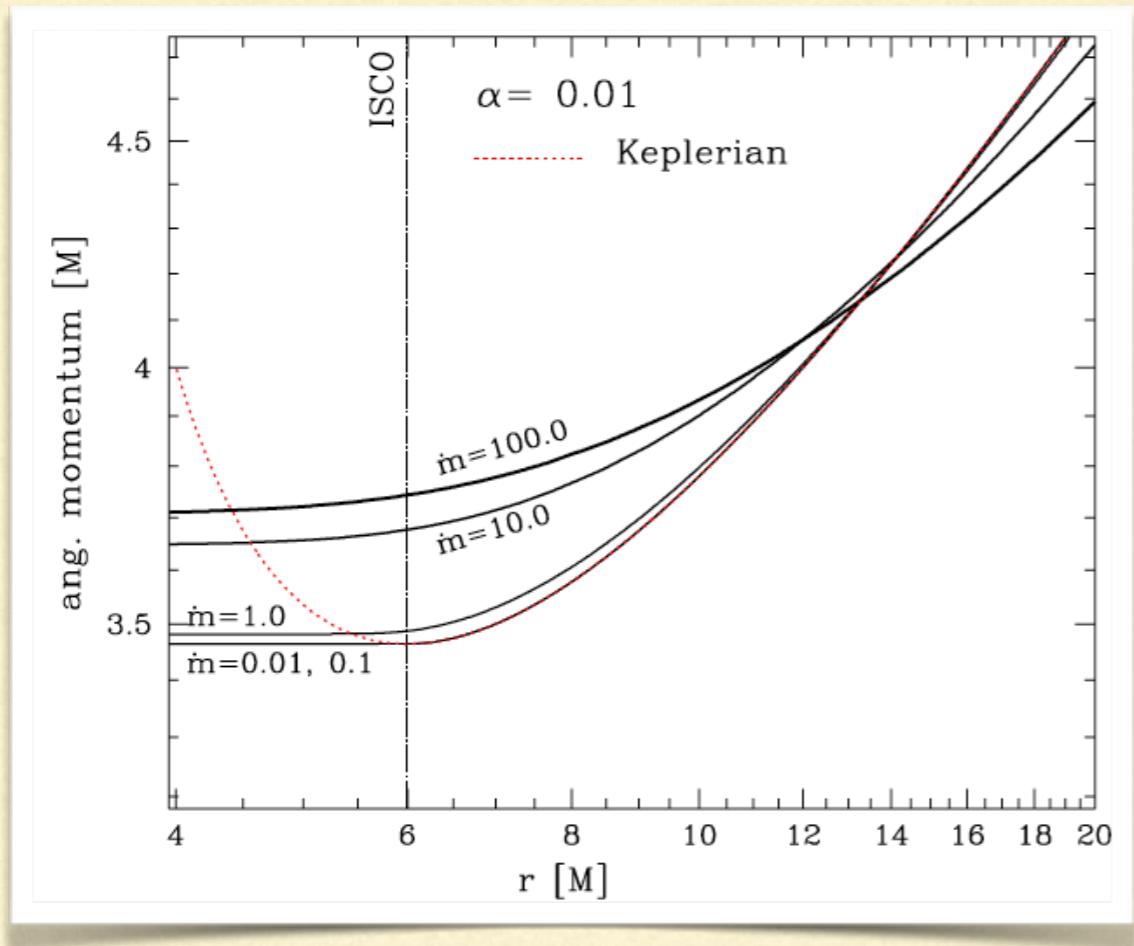
# Magnetically Arrested Discs simulations

Jets from Penrose-Blandford-Znajek mechanism



Accretion through Rayleigh-Taylor "gates".

# Effect of background flow on turbulence efficiency



Shear

$$\sigma = \frac{1}{\sqrt{2}} \left| \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right| = \frac{q\Omega_0}{\sqrt{2}}$$

Vorticity

$$\omega = \frac{1}{\sqrt{2}} \left| \frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i} \right| = \frac{(2-q)q\Omega_0}{\sqrt{2}}$$

$$\Omega \propto R^{-q}$$

$$\mathbf{U} = \boldsymbol{\Omega} \times \mathbf{r} + \mathbf{u}_{\text{shear}}$$

Shear tensor

$$\sigma_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{1}{3} \delta_{ij} \frac{\partial U_k}{\partial x_k}$$

Vorticity tensor

$$\omega_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i} \right)$$

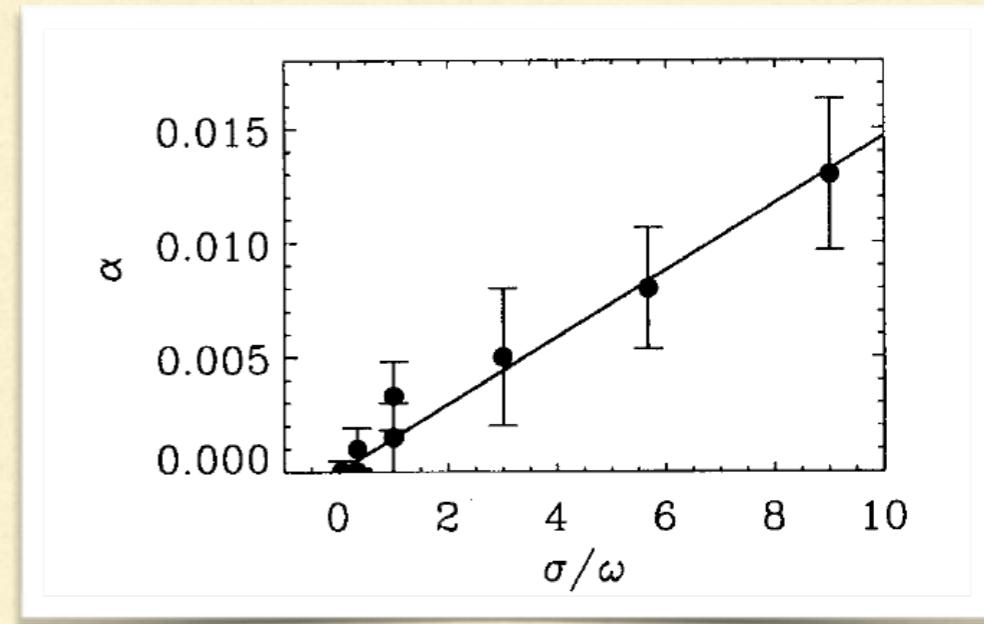
$$\langle \rho u_x u_y - B_x B_y \rangle = \nu_t(\rho) r \frac{\partial \Omega}{\partial r}$$

$$\nu_t = \alpha c_s H$$

Numerical simulations:

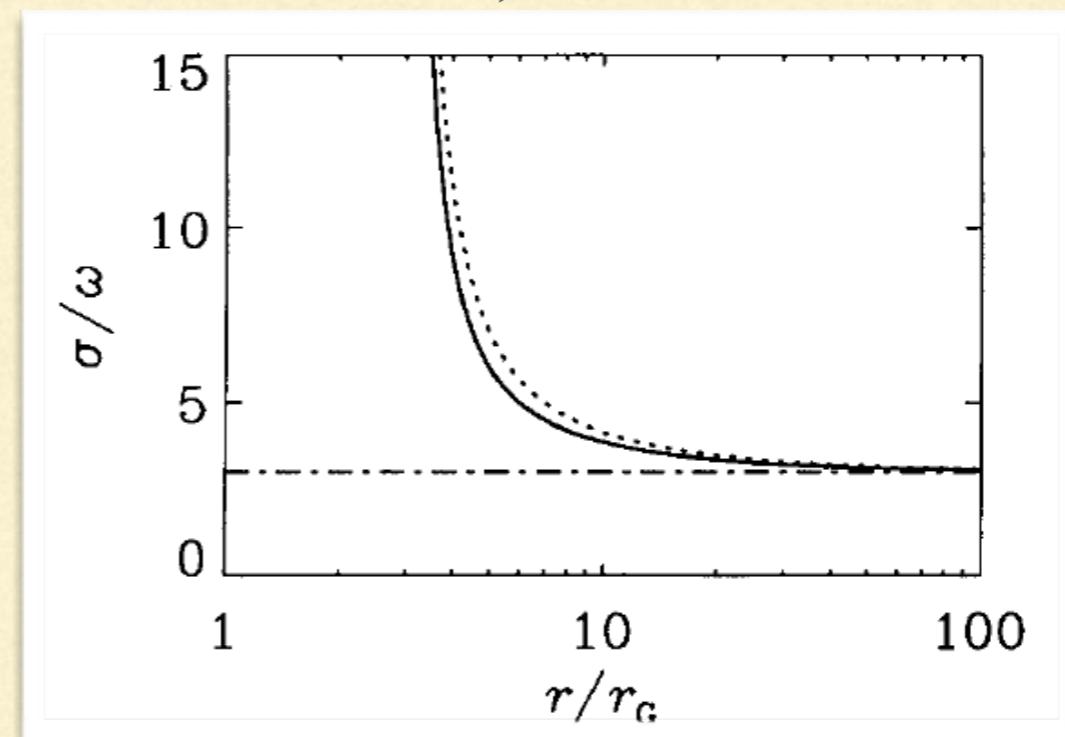
$$\alpha = \alpha_0 \frac{\sigma}{\omega}$$

$$\alpha_0 \approx 0.0015$$



⌚ Relativistic effects (Schwarzschild BH):

$$\frac{\sigma}{\omega} = 3 \frac{r - r_s}{r_3 - r_s}$$



⌚ Boundary layers ...

---

# *Quels mécanismes transportent la matière dans les disques d'accrétion ?*

- ➊ Turbulence due à l'instabilité magnéto-rotationnelle.
- ➋ Dans certains cas (systèmes) vents magnétisés.
- ➌ Instabilités magnétiques près des trous noirs ( $B$  très forts.)