Large mass hierarchies in strongly coupled field theories from gauge-gravity duality

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Glueballs on the Baryonic Branch of Klebanov-Strassler: dimensional deconstruction and a light scalar particle
(DE, Maurizio Piai, arXiv:1703.10158 [hep-th])

Calculable mass hierarchies and a light dilaton from gravity duals
(DE, Maurizio Piai, arXiv:1703.09205 [hep-th])

Motivation

Electro-weak symmetry breaking:

- We have observed 125 GeV Higgs boson
- Higgs mechanism in the Standard Model:

$$|D_\mu\varphi|^2 - \lambda\left(|\varphi|^2 - \frac{v^2}{2}\right)^2, \quad D_\mu = \partial_\mu - ig\frac{\tau^a}{2}W_\mu^a - ig'\frac{1}{2}B_\mu$$

  EWSB due to non-zero VEV: $\langle\varphi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$

- Quadratic divergence of mass term for Higgs implies fine-tuning: $m_H^2 \sim \Lambda^2$
- What about EWSB induced by strongly coupled dynamics?

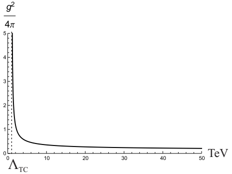
Motivation

New strongly coupled physics wishlist:

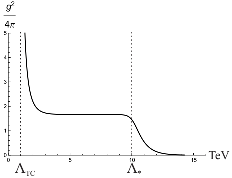
- Dynamically generate $\Lambda_{\rm EW} \ll \Lambda_{\rm Pl}$ (big hierarchy problem)
- Explain the lightness of the Higgs (is it a PNGB?)
- Satisfy experimental constraints such as flavor changing neutral currents (FCNC) and electro-weak precision parameters

Motivation

We know that QCD-like dynamics cannot work (top mass, FCNC, electroweak precision parameters):



What about more complex RG flows leading to multi-scale dynamics?



Large anomalous dimensions (γ) due to strongly coupled dynamics in *walking* region – observables can depend on $\left(\frac{\Lambda_*}{\Lambda_{TC}}\right)^{\gamma}$

Motivation

Light pseudo-dilaton:

- Spontaneously broken approximate scale invariance – light scalar, the dilaton (pseudo-Goldstone of dilatations), in the spectrum?
- Such a light scalar would couple to the Standard Model fields in a similar way as the Higgs, and therefore it would be hard to distinguish the two at low energies

Theoretical interest: strongly coupled theories with multi-scale dynamics allow for a rich set of phenomena since observables can depend non-trivially on ratios of energy scales

Motivation

Questions:

- Can we construct theories with multi-scale/walking dynamics?
- Under which circumstances is there a light scalar in the spectrum?

Approach:

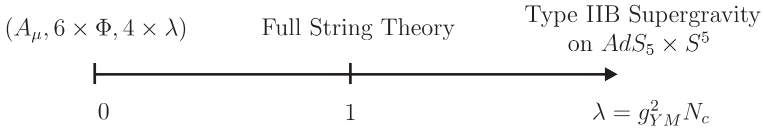
- Use gauge-gravity duality to compute at strong coupling
- Study models built from the top-down, embedded in supergravity/string theory

Gauge-gravity duality

How to compute at strong coupling?

- Canonical example: AdS/CFT is a duality between $\mathcal{N} = 4$ SYM with gauge group $SU(N_c)$ and Type IIB String Theory on $\mathrm{AdS}_5 \times S^5$:

  (Maldacena, 1997; Gubser, Klebanov, Polyakov, 1998; Witten, 1998)



- Large N_c corresponds to the classical limit on the string theory side ($\lambda / N_c = 4\pi g_s$)
- Large 't Hooft coupling λ corresponds to the low energy limit of string theory ($\lambda = R^4 / l_s^4$)
- We will take both these limits, allowing us to study strongly coupled field theory using classical supergravity

Gauge-gravity duality

- Many generalizations (confinement, chiral symmetry breaking)
- The extra bulk dimension (the radial coordinate r) is related to energy scale in the field theory, and thus the bulk is in a sense a geometrical representation of the RG flow of the dual theory
- There is a *dictionary* for translating between field theory and bulk quantities. Fields in the bulk map to operators in the field theory: $\phi \leftrightarrow \mathrm{Tr}F^2$, $g_{\mu\nu} \leftrightarrow T_{\mu\nu}$, ...
- Correlators can be computed by using

$$\begin{aligned} \langle e^{\int \mathrm{d}^4x\, \phi_0(x^\mu)\mathcal{O}(x^\mu)} \rangle_{QFT} &= \mathcal{Z}_{bulk}[\phi(x^\mu, r)|_{r=\infty} = \phi_0(x^\mu)] \\ S^{(bulk)}_{on-shell}[\phi_0] &= W_{QFT}[\phi_0] \qquad (N_c \gg 1) \end{aligned}$$

  differentiating with respect to the boundary value of the bulk field

Gauge-gravity duality

Bottom-up models:

- Phenomenologically motivated 5d models, where the action and field content is chosen by hand to get the desired dynamics
- Flexible for model building, qualitatively captures strongly coupled dynamics
- Given a 5d model, there does not necessarily exist a corresponding dual 4d field theory
- Complementary to top-down approach

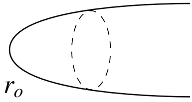
Gauge-gravity duality

Top-down models:

- Gravity side of the duality is described by a higher-dimensional theory: 10d type-IIA/IIB supergravity, 11d supergravity/M-theory
- In addition to the field theory coordinates and the radial coordinate r, there is also an internal manifold M whose shape can depend on r (isometries of M related to global symmetries of the dual field theory)

Advantages of top-down approach:

- In many cases, the exact form of the field theory dual is known
- Confinement can be modelled dynamically - the geometry can pinch off (at some $r = r_o$) leading to an IR scale (area law)



Holographic computation of spectrum

How to compute mass spectrum of composite states holographically?

- Often, there exists a consistent truncation to a sigma model consisting of a number of scalars coupled to gravity in five dimensions:

$$\mathcal{S} = \int \mathrm{d}^4x\mathrm{d}r\sqrt{-g}\left[\frac{R}{4} - \frac{1}{2}G_{ab}(\Phi)\partial_M\Phi^a\partial^M\Phi^b - V(\Phi)\right]$$

- Expand EOMs to linear order in fluctuations of the metric and the scalars around the background
- Impose appropriate BCs on fluctuations in the IR and UV
- The values of four-momenta $k^2(= -m^2)$ for which solutions exist give us the spectrum

Holographic computation of spectrum

- ADM-formalism: write the metric as (lapse function n and shift vector n^μ)

$$\mathrm{d}s^2 = (n^\mu n_\mu + n^2)\mathrm{d}r^2 + 2n_\mu \mathrm{d}x^\mu \mathrm{d}r + \tilde{g}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$$

- Expand to linear order in fluctuations $\{\varphi^a, \nu, \nu^\mu, \mathfrak{e}^\mu{}_\nu, h, H, \epsilon^\mu\}$ around the background:

$$\begin{aligned}
\Phi^a &= \bar{\Phi}^a + \varphi^a, \\
n &= 1 + \nu, \\
n^\mu &= \nu^\mu, \\
\tilde{g}_{\mu\nu} &= e^{2A}(\eta_{\mu\nu} + h_{\mu\nu}),
\end{aligned}$$

with

$$h^\mu{}_\nu = \mathfrak{e}^\mu{}_\nu + \partial^\mu \epsilon_\nu + \partial_\nu \epsilon^\mu + \frac{\partial^\mu \partial_\nu}{\Box} H + \frac{1}{3}\delta^\mu{}_\nu h$$

Holographic computation of spectrum

Linearized equation of motion for the scalar fluctuations:

- Written in terms of the gauge-invariant variable $\mathfrak{a}^a = \varphi - \frac{\bar{\Phi}'^a}{6A'}h$, the EOMs decouple: (Berg, Haack, Mück, 2005; DE, 2009)

$$\left[D_r^2 + 4A'D_r + e^{-2A}m^2\right]\mathfrak{a}^a -$$

$$\left[V^a{}_{|c} - \mathcal{R}^a{}_{bcd}\bar{\Phi}'^b\bar{\Phi}'^d + \frac{4(\bar{\Phi}'^a V_c + V^a\bar{\Phi}'_c)}{3A'} + \frac{16V\bar{\Phi}'^a\bar{\Phi}'_c}{9A'^2}\right]\mathfrak{a}^c = 0$$

What boundary conditions should we impose? (DE, Piai, 2010)

- We put $\varphi^a = 0$ in the IR and UV, corresponding to

$$-\frac{2\Phi'^a\Phi'_b}{3A'}D_r\mathfrak{a}^b\Big|_{r_{\mathrm{IR,UV}}} = \left[e^{-2A}m^2 + \frac{A'}{2}\partial_r\left(\frac{A''}{A'^2}\right)\right]\mathfrak{a}^a\Big|_{r_{\mathrm{IR,UV}}}$$

- This imposes regularity in the IR and picks the subleading mode in the UV (picks poles of two-point function)

Baryonic branch of Klebanov-Strassler

DE & Maurizio Piai

Baryonic branch of Klebanov-Strassler

Klebanov-Strassler field theory:

(Klebanov, Strassler, 2000)

- 4d theory with $SU(N+M) \times SU(N)$ gauge group, $\mathcal{N} = 1$ SUSY, bifundamental matter A_i and B_i ($i = 1, 2$) in representations $(N+M, \overline{N})$ and $(\overline{N+M}, N)$, superpotential $W \sim \mathrm{Tr}(A_i B_j A_k B_l)\epsilon^{ik}\epsilon^{jl}$
- Gravity dual is known

Rich dynamics:

- UV duality cascade (of Seiberg dualities): $SU(N+M) \times SU(N) \to SU(N) \times SU(N-M) \to \cdots$
- The theory is confining
- Non-trivial moduli space – there is a baryonic branch parametrized by the VEV of a dim-2 operator
- For large dim-2 VEV, the theory effectively becomes six-dimensional over a range of energies (deconstruction)

Baryonic branch of Klebanov-Strassler

Baryonic branch (perturbative analysis): (Maldacena, Martelli, 2009)

For gauge group $SU(qN) \times SU((q+1)N)$, the F- and D-term equations are solved by $B_i = 0$ and

$$A_1 = c \begin{pmatrix} \sqrt{q} & 0 & \cdots & 0 & 0 & 0 \\ 0 & \sqrt{q-1} & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \sqrt{2} & 0 & 0 \\ 0 & 0 & \cdots & 0 & 1 & 0 \end{pmatrix},$$

$$A_2 = c \begin{pmatrix} 0 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & \sqrt{q-1} & 0 \\ 0 & 0 & \cdots & 0 & 0 & \sqrt{q} \end{pmatrix}$$

dim-2 operator $\mathcal{U} = \frac{1}{q(q+1)N} \mathrm{Tr} \left[A_i^\dagger A_i - B_i B_i^\dagger \right] = |c|^2$
Higgsing: $SU(qN) \times SU((q+1)N) \to SU(N)$
Deconstruction of the sphere: spectrum of gauge bosons yields $m^2 = g^2 |c|^2 \lambda_{\ell,\pm}$ with

$$\lambda_{\ell,\pm} = q + \frac{1}{2} \pm \sqrt{\left(q + \frac{1}{2}\right)^2 - \ell(\ell+1)},$$

$$\lambda_{\ell,-} = \frac{\ell(\ell+1)}{2q+1} \qquad (\ell \ll q)$$

Non-perturbative analysis of moduli space (Dymarsky, Klebanov, Seiberg, 2006)

Baryonic branch of Klebanov-Strassler

Summary of type-IIB supergravity dual descriptions:

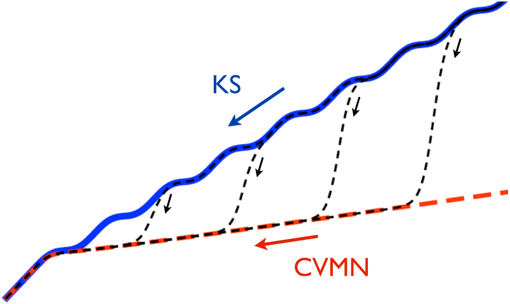
- The special case of gauge group $SU(M) \times SU(M)$ is dual to the Klebanov-Witten solution with geometry $\mathrm{AdS}_5 \times T^{1,1}$ – conformal
  (Klebanov, Witten, 1998)
- At the origin of the moduli space, the dual is given by the Klebanov-Strassler solution – confinement scale Λ_{conf}
  (Klebanov, Strassler, 2000)
- The baryonic branch is described by a one-parameter (dim-2 VEV) family of solutions – two energy scales Λ_{conf} and Λ_{VEV}
  (Butti, Grana, Minasian, Petrini, Zaffaroni, 2004)
- The limiting case of infinite dim-2 VEV is dual to the Maldacena-Nunez solution – confining, 6d dual theory
  (Maldacena, Nunez, 2000; Chamseddine, Volkov, 1997)

Idea: Go far out on the baryonic branch such that $\Lambda_{\mathrm{VEV}} \gg \Lambda_{\mathrm{conf}}$ (spontaneous vs explicit breaking of conformal invariance)

Questions: Light pseudo-dilaton in the spectrum? Deconstruction?

Baryonic branch of Klebanov-Strassler

Pictorial representation of RG flows:



Baryonic branch of Klebanov-Strassler

Papadopoulos-Tseytlin ansatz in type-IIB supergravity: (Papadopoulos, Tseytlin, 2001)

$$
\begin{aligned}
ds_{10}^2 &= e^{2p-x} ds_5^2 + \underbrace{(e^{x+\tilde{g}} + a^2 e^{x-\tilde{g}})(e_1^2 + e_2^2)}_{S^2} + \underbrace{e^{x-\tilde{g}} \left(e_3^2 + e_4^2 + 2a(e_1 e_3 + e_2 e_4)\right) + e^{-6p-x} e_5^2}_{S^3}, \\
ds_5^2 &= dr^2 + e^{2A} dx_{1,3}^2, \\
F_3 &= N\left[-e_5 \wedge (e_4 \wedge e_3 + e_2 \wedge e_1 + b(e_4 \wedge e_1 - e_3 \wedge e_2)) + dr \wedge (\partial_r b(e_4 \wedge e_2 + e_3 \wedge e_1))\right], \\
H_3 &= -h_2 e_5 \wedge (e_4 \wedge e_2 + e_3 \wedge e_1) + dr \wedge \Big[\partial_r h_1 (e_4 \wedge e_3 + e_2 \wedge e_1) - \\
& \quad \partial_r h_2 (e_4 \wedge e_1 - e_3 \wedge e_2) + \partial_r \chi(-e_4 \wedge e_3 + e_2 \wedge e_1)\Big], \\
F_5 &= \tilde{F}_5 + \star \tilde{F}_5, \quad \tilde{F}_5 = -\mathcal{K} e_1 \wedge e_2 \wedge e_3 \wedge e_4 \wedge e_5
\end{aligned}
$$

Vielbeins:

$$
\begin{aligned}
e_1 &= -\sin\theta\, d\phi, \quad e_2 = d\theta, \quad e_3 = \cos\psi \sin\tilde{\theta}\, d\tilde{\phi} - \sin\psi\, d\tilde{\theta}, \\
e_4 &= \sin\psi \sin\tilde{\theta}\, d\tilde{\phi} + \cos\psi\, d\tilde{\theta}, \quad e_5 = d\psi + \cos\tilde{\theta}\, d\tilde{\phi} + \cos\theta\, d\phi
\end{aligned}
$$

Constraints:

$$
\begin{aligned}
\mathcal{K} &= M + 2N(h_1 + b h_2), \\
\partial_r \chi &= \frac{(e^{2\tilde{g}} + 2a^2 + e^{-2\tilde{g}} a^4 - e^{-2\tilde{g}}) \partial_r h_1 + 2a(1 - e^{-2\tilde{g}} + a^2 e^{-2\tilde{g}}) \partial_r h_2}{e^{2\tilde{g}} + (1-a^2)^2 e^{-2\tilde{g}} + 2a^2}
\end{aligned}
$$

Baryonic branch of Klebanov-Strassler

To find solutions:

- Write down first-order BPS equations for the background fields $\{\tilde{g}, p, x, \phi, a, b, h_1, h_2, A\}$
- These can be repackaged into a single second order differential equation: (Hoyos, Nunez, Papadimitriou, 2008)

$$P'' + P'\left[\frac{P'+Q'}{P-Q} + \frac{P'-Q'}{P+Q} - 4\coth(2\rho)\right] = 0,$$

$$Q(\rho) = N_c(2\rho\coth(2\rho) - 1)$$

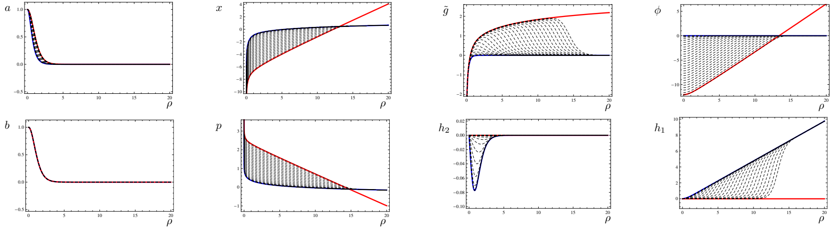
- Require KS asymptotics in the UV
- Require regularity in the IR (deformed conifold – S^2 shrinks):

$$P = (2 + e^{-\alpha})\rho + \cdots$$

- The baryonic branch is parametrized by α

Baryonic branch of Klebanov-Strassler

Background functions:



The solutions interpolate between the two limiting cases:
Klebanov-Strassler ($\alpha \to -\infty$) and Maldacena-Nunez ($\alpha \to +\infty$)

Baryonic branch of Klebanov-Strassler

To compute the spectrum, we use the 5d sigma model defined by $(\Phi^a = (\tilde{g}, p, x, \phi, a, b, h_1, h_2))$

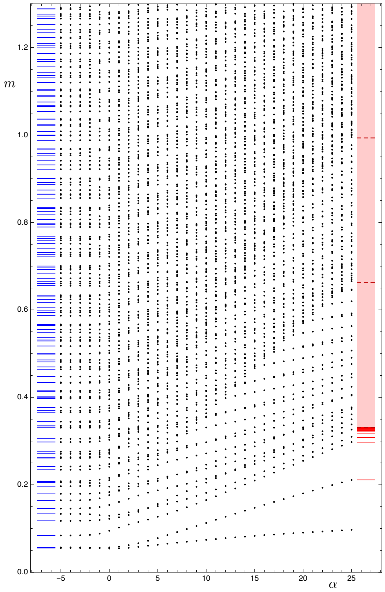
$$
\begin{aligned}
G_{ab}\partial_M\Phi^a\partial_N\Phi^b \quad = \quad & \frac{1}{2}\partial_M\tilde{g}\partial_N\tilde{g} + \partial_M x\partial_N x + 6\partial_M p\partial_N p \\
& + \frac{1}{4}\partial_M\phi\partial_N\phi + \frac{1}{2}e^{-2\tilde{g}}\partial_M a\partial_N a + \frac{1}{2}N^2 e^{\phi-2x}\partial_M b\partial_N b \\
& + \frac{e^{-\phi-2x}}{e^{2\tilde{g}} + 2a^2 + e^{-2\tilde{g}}(1-a^2)^2}\left[\frac{1}{2}(e^{2\tilde{g}} + 2a^2 + e^{-2\tilde{g}}(1+a^2)^2)\partial_M h_2\partial_N h_2\right. \\
& \left. +(1+2e^{-2\tilde{g}}a^2)\partial_M h_1\partial_N h_1 + 2a(e^{-2\tilde{g}}(a^2+1)+1)\partial_M h_1\partial_N h_2\right],
\end{aligned}
$$

$$
\begin{aligned}
V(\Phi^a) \quad = \quad & -\frac{1}{2}e^{2p-2x}(e^{\tilde{g}} + (1+a^2)e^{-g}) + \frac{1}{8}e^{-4p-4x}(e^{2\tilde{g}} + (a^2-1)^2 e^{-2\tilde{g}} + 2a^2) \\
& + \frac{1}{4}a^2 e^{-2\tilde{g}+8p} + \frac{1}{8}N^2 e^{\phi-2x+8p}\left[e^{2\tilde{g}} + e^{-2\tilde{g}}(a^2-2ab+1)^2 + 2(a-b)^2\right] \\
& + \frac{1}{4}e^{-\phi-2x+8p}h_2^2 + \frac{1}{8}e^{8p-4x}(M + 2N(h_1 + bh_2))^2
\end{aligned}
$$

Baryonic branch of Klebanov-Strassler

Spectrum of spin-0 states:
Blue is the spectrum of KS
Red is the spectrum of MN

- In the limit $\alpha \to -\infty$:
  spectrum of KS is reproduced
- In the limit $\alpha \to +\infty$:
  a) approaches spectrum of MN
  b) intermediate regimes with densely packed states approaching continua (deconstruction)
- For large dim-2 VEV, there is a light state!



Conclusions & Open questions

- We found evidence of deconstruction in the non-perturbative regime
- We found a parametrically light state
- The spectrum should organize itself into supersymmetric multiplets: partners of the light state?
- The full treatment requires also turning on fluctuations of pseudo-scalars and vectors in the 5d consistent truncation
- We relied on the existence of a non-trivial moduli space – how much of a role does SUSY play? Non-SUSY generalization?
- Can we embed electro-weak symmetry breaking? Probe D-branes?