Central charges and their constraints

Rules for drawing the maps of QFTs

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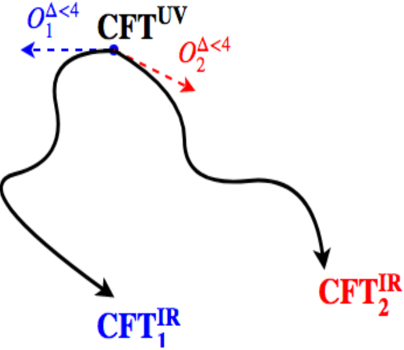
Overview

1. RG flows and anomalies
   - The trace anomaly
   - The a-theorem
   - Gradient flow equation
2. Perturbative Gauge-Yukawa theories
   - Very large N_c
   - Moderately large N_c
3. Conformal window

RG flows generalities



- In the UV, IR theory is conformal described by operator dimensions, central charges etc.
  → conformal data
- Deforming by *relevant* operators triggers RG flow
- Different deformations lead to different CFTs in the IR
  → phase diagram (cf. Francesco's talk)

Most popular examples

- UV theory free with IR interacting fixed point → *asymptotic freedom*
- IR theory free with UV interacting fixed point → *asymptotic safety*

QFT cartography

- Our goal is to draw a map of QFTs- I.e understand the flows between various CFTs.
- Can we say something about which flows are allowed based solely on their asymptotics?
- Want to define a universal quantity ($\exists$ in any generic theory) that describes the theory along the flow.
- Rules:
  - Unitarity-gives bounds.
  - Renormalizability-guarantees dependence on *finite* number of renormalized parameters.
  - Perturbativity (optional)- calculable at weak coupling.

Anomalies

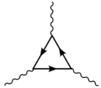
- Classical symmetries are broken by quantum effects
- Corresponding Noether currents have anomalous divergence:

$$\partial^\mu j_\mu = c\,\partial\mathbf{O}$$

- The anomaly c is usually related to type/number of d.o.f

Example: Chiral anomaly when gauging a global symmetry current j^5

$$\langle \partial^\mu j^5_\mu \rangle \propto c\,\mathbf{F}_{\mu\nu}\tilde{\mathbf{F}}^{\mu\nu} \sim$$



- c_{UV} is calculated from massless triangle graphs - depends on the fermion content of the UV theory
- t'Hooft anomaly matching equates c_{UV} with c_{IR}
- Gives a non-perturbative constraint (higher loop effects cancel due to Adler-Bardeen theorem).

Trace anomaly

- Gauge the conformal symmetry by coupling the theory to a background metric $g_{\mu\nu}$, which transforms

$$g_{\mu\nu} \to e^{-2\alpha(x)} g_{\mu\nu}$$

- Conformal anomaly is given in terms of the trace of E-M tensor ($\partial \cdot j_D = T^{\mu}_{\mu}$)

$$\langle T^{\mu}_{\mu} \rangle = a\, E_4 + c\, W^2 + b\, R^2 + \tilde{b} \Box R$$

- $a, b, c, \tilde{b}$ are the central charges 4D QFT
- Unlike the chiral anomaly case central charges run even in perturbation theory.

Bounds from unitarity

- The Weyl squared coefficient satisfies

$$c^{\text{UV,IR}} \sim \text{Im}\langle TT\rangle^{\text{UV,IR}} > 0$$

- Positive energy condition $\langle E\rangle_{S^2} \geq 0$ implies the collider bounds **[Hofman, Maldacena '08]**

$$\frac{1}{3} \leq \frac{a^{\text{UV,IR}}}{c^{\text{UV,IR}}} \leq \frac{31}{18}$$

- The quantity $\tilde{b}$ is ambiguous in CFT but nevertheless $\Delta\tilde{b}$ is well defined through a sum-rule **[Anselmi '99]**, **[VP, Zwicky '17]**

$$\Delta\tilde{b} = \frac{1}{3\cdot 2^7}\int d^4x x^4\langle\Theta\Theta\rangle$$

The a-theorem

(weak) a-theorem

$$\Delta a = a_{UV} - a_{IR} \geq 0$$

Strong a-theorem

$$\dot{\tilde{a}} > 0 \; ; \; \tilde{a}_{UV,IR} = a_{UV,IR}$$

- Effective number of d.o.f decreases along the RG flow.
- First stated/proven in 2D by Zamolodchikov **[Zamolodchikov '86]**

$$\langle T^{\mu}_{\mu} \rangle_{2d} = cR$$

- Zamolodchikov found $\dot{\tilde{c}} \sim \mathrm{Im}\langle \Theta\Theta \rangle > 0$
- Cardy proved Euclidean version $\Delta c \sim \int x^2 \langle \Theta(x)\Theta(0) \rangle$

The (weak) a-theorem in $4d$

- The proof **[Komargodski, Schwimmer '11]**, **[Komargodski '12]** introduces external compensator field τ to restore scale invariance via

$$\mu \to \mu e^{\tau}$$

- Integrating out the dynamical fields leaves us with an IR effective action (cf. Claudio's talk)

$$\Delta a \, \Gamma_{WZ} + \text{non-local} \overset{e.o.m}{=} \Delta a \, (\partial\tau)^4 + \ldots$$

- Since τ corresponds to the source for $\Theta = T^{\mu}_{\mu}$

$$\Delta a \overset{on-shell}{\sim} \text{Im}\langle \Theta\Theta\Theta\Theta \rangle \geq 0$$

- Uses Minkowski methods → Euclidean proof a'la $2d$ not clear → would be useful to lattice simulations

The gradient flow

- We will consider a set of marginally relevant operators $\{O_i\}$ couplings $\{g_i\}$ generating the trace of EMT

$$\Theta \equiv T^{\mu}_{\mu} = \beta^I O_I$$

- Strongest version of the $a-$theorem asserts that

$$\partial_I \tilde{a} = \equiv G_{IJ}\beta^J \; ; \; G_{IJ} = G_{JI}$$

- Implies (Weyl) consistency conditions

$$\partial_I \beta_J = \partial_J \beta_I$$

- Implies the beta functions follow from the gradient of $\tilde{a}$.
- At leading order in perturbation it was proven in **[Osborn '89]**

$$G_{IJ} = \chi_{IJ} + O(\beta_I)$$

- Weyl consistency implies relation between perturbative beta functions and correct counting 2-1-0, 3-2-1 etc. **[Sannino at al '13]**

Gauge-Yukawa theories at LO

- We will at the most generic standard model-like 2-1-0 setup

$$\beta_g^a = -\frac{g_a^3}{(4\pi)^2}\left[b_0^a + \frac{(b_1)^{ab}}{(4\pi)^2}g_b^2 + \frac{(b_y)^a_{IJ}}{(4\pi)^2}y^I y^J\right],$$
$$\beta_y^I = \frac{1}{(4\pi)^2}\left[(c_1)^I_{JKL}y^J y^K y^L + (c_2)^{bI}_J g_b^2 y^J\right],$$

- The leading contribution to the metric χ_{IJ} is given

$$\chi = \begin{pmatrix} \frac{\chi_{g_a g_a}}{g_a^2}\left(1 + \frac{A_a}{(4\pi)^2}g_a^2\right) & 0 \\ 0 & \chi_{y_I y_I} \end{pmatrix}$$

- Want to use the above to compute the conformal data- critical exponents, OPE coefficients, a,c to LO accuracy at the given fixed point.

a^*, c^* at LO

- Solving the 2-1-0 Weyl consistency condition and taking the fixed point limit gives **[Dondi, VP, Sannino '17]**

$$a^* = \tilde{a}^* = a^{\text{free}} - \frac{1}{4}\frac{1}{(4\pi)^2}\sum_a b_0^a \chi_{g_a g_a} g_a^{*2}\left(1 + \frac{A^a g_a^{*2}}{(4\pi)^2}\right) + \mathcal{O}(g_a^{*6}, y_l^{*6});$$

$$a^{\text{free}} = \frac{1}{360(4\pi)^2}\left(n_\phi + \frac{11}{2}n_\psi + 62 n_v\right)$$

- The a–function at fixed depends solely on the gauge coupling at this order.
- The quantity c only known to $O(g_a^2, y_l^2)$

$$c^* = c^{\text{free}} - O(g_a^2, y_l^2);$$

$$c^{\text{free}} = \frac{1}{(4\pi)^2}\frac{1}{20}(2n_v + n_\psi + \frac{1}{6}n_\phi)$$

Asymptotically free(safe) theories at very large N_c

- At very large N_c perturbative fixed point arises provided

$$\frac{|b_0^a|}{N_c} \equiv \epsilon \ll 1$$

- Expansion in powers of ϵ is under control
- Free examples: Banks-Zaks, CAF
- Safe examples: Litim-Sannino, Pelaggi-Sannino-Strumia-Vigiani
- The $a-$theorem is trivially satisfied since

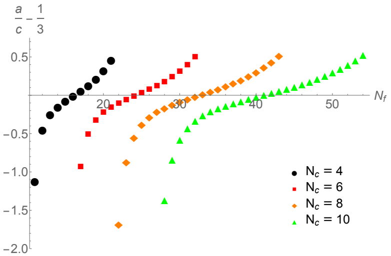
$$\Delta a = \pm \frac{1}{4} \frac{1}{(4\pi)^2} b_0^a \chi_{gg} g_a^{*2} + O(\epsilon^3) \, .$$

  where the plus (minus) applies to asymptotically free coupling with $b_0^a > 0$ (safe coupling with $b_0^a < 0$)
- c is given by its free-field value for small ϵ

CAF with charged scalars

Fields	$[SU(N_c)]$	$SU_L(N_f)$	$SU_R(N_f)$	$U(N_s)$
ψ	$\square$	$\square$	1	1
$\tilde{\psi}$	$\overline{\square}$	1	$\square$	1
ϕ	$\square$	1	1	$\square$

The collider bound is most constraining in this case:



CAS in Georgi-Glashow models **[Molgaard, Sannino '17]**

Fields	$[SU(N_c)]$	$SU(N_c \mp 4 + p)$	$SU(p)$
ψ	$\square$	1	$\square$
$\tilde{\psi}$	$\overline{\square}$	$\overline{\square}$	1
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1	1
M	1	$\square$	$\overline{\square}$
H	$\overline{\square}$	1	1

The model is anomaly free with the interactions between chiral fermions and scalars are described via the following Lagrangian terms

$$\begin{aligned} \mathcal{L}_H &= y_H f_a \,\bar{\psi}_a A\, H + h.c. \\ \mathcal{L}_M &= y_M [\delta_{ab} - f_a f_b]\, \bar{\psi}_a M_{bc} \psi_c + y_1 f_a f_b\, \bar{\psi}_a M_{bc} \psi_c + h.c. \end{aligned} \tag{1}$$

Theories with moderately large N_c

Constraints on the Georgi-Glashow CAS model

	$N_c = 5, p = 26$	$N_c = 6, p = 30$	$N_c = 8, p = 39$
α_g^*	1.41	0.0325	0.0481
α_H^*	6.12	0.151	0.241
α_M^*	0.652	0.0155	0.0233
α_1^*	0.312	0.00652	0.00801
θ_{UV}	-0.0428	-0.00585	-0.00602
$a \times (4\pi)^2$	-1311	14.7	21.6
$c \times (4\pi)^2$	710	47.5	126
a/c	-1.84	0.296	0.171
Δa	-1321	-0.537	-4.27

Comments

- $a-$theorem and the collider bounds seem to give strongest constraints in these cases
- These arguments giver limits similar to perturbative unitarity rather than ruling out the models completely
- The approach has been followed up **[Barducci, Fabbrichesi, Nieto, Percacci, Skrinjar '18]**

a−theorem in the conformal window

- Δa in the chirally broken phase of QCD can be computed purely from the free-field value (IR consists of free pions) → doesn't give any interesting constraints
- The chiral phase is in the conformal window → beta function expected to have strongly coupled IR fixed point
- Relevant to composite Higgs models and their lattice realizations
- $\Delta a > 0$ expression could provide limits on the number of flavours
- **Idea:** Find a formula for Δa suitable for lattice simulations
- **The result:** For gauge theories in conformal window we have **[VP, Zwicky '18]**

$$\Delta a = \frac{1}{3 \cdot 2^8} \int d^4 x x^4 \langle \Theta(x) \Theta(0) \rangle = \frac{1}{16} \int \beta^2 \chi^R_{gg} d \ln \mu$$

where the QCD trace anomaly gives $\Theta = \frac{\beta}{2}[\frac{1}{g_0^2} G^2]$

Proof of the relation

- For a gauge theory without scalars $O_g = \frac{1}{g_0^2} G^2$
- Start from the general relation

$$\Delta a = \frac{1}{16} \int \beta^2 (\chi^R_{gg} - \frac{\beta}{2} \chi^R_{ggg}) \tag{2}$$

where R stands for subtraction scheme of 2,3 point functions

- Under a generic change of subtraction constant by a finite constant $\omega_{gg(g)}(g^2)$ we have

$$\chi^{R'}_{gg(g)} = \chi^R_{gg(g)} + 2\mathcal{L}_\beta \omega_{gg(g)}$$

- Can we find scheme change that eliminates χ^R_{ggg}?

Proof of the relation

- For AF, it is possible to find an all order solution in the vicinity of Gaussian UV fixed point

$$\omega_{ggg}(a_s) = \frac{1}{2\beta^3} \int_0^{a_s} \beta^2(u) \chi^R_{ggg}(u) \frac{du}{u}$$

- This solution is finite provided χ^R_{ggg} vanishes at the Gaussian FP
- Remarkably this is the case as can be shown by direct computation of $\langle G^2 G^2 G^2 \rangle$
- Similarly a solution exists near non-trivial IR fixed point a_s^*

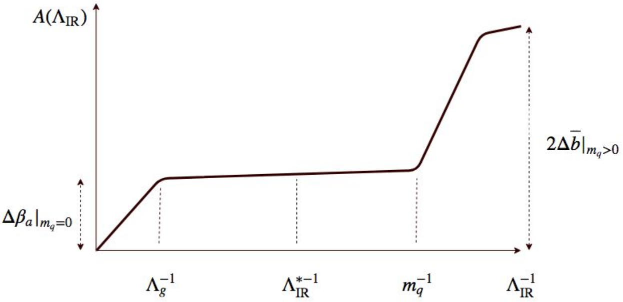
$$\omega_{ggg}(a_s) = \frac{1}{2\beta^3} \int_{a_s^*}^{a_s} \beta^2(u) \chi^R_{ggg}(u) \frac{du}{u}$$

- This is again finite due to power-like behaviour near a_s^*
- Patch the two solutions to find ω_{ggg} everywhere
- For AS the proof can be repeated with UV-IR roles inverted

Lattice application

- On the lattice one has finite quark mass m_q
- Define a 'lattice $a-$function' with IR cutoff

$$A(\Lambda_{\rm IR}, m_q, L) \equiv \frac{1}{3 \cdot 2^8} \int_0^{\Lambda_{IR}^{-1}} d^4x\, x^4 \langle \Theta(x)\Theta(0)\rangle_c$$



New roads, new horizons

- Before any model is tested in lab/collider it has to satisfy various theoretical consistency constraints
- We asked whether central charges are useful to say something about currently used models
- The topic is worth pushing in various directions- lattice, large N_f etc.