Neutrino Oscillation &
Other Quantum Oscillations

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Flow of this Seminar

Part-I

* Neutrino Oscillation
* Oscillation measurements
  - Status
  - Future

Part-II

* Collection of Quantum Oscillations
  - Cabbibo Angle: θ_C ,
  - Weinberg Angle: θ_W
  - Chirality Oscillation (why μ_R^- can decay)
  - What is Parity and Isospin?

Part-I: Neutrino Oscillation

Fermion(spin=1/2) **Flavor**

Name	Charge	1^{st} generation	2^{nd} generation	3^{rd} generation
Lepton	-1	e	μ	τ
	0	ν_e	ν_μ	ν_τ
Quark	+2/3	u	c	t
	-1/3	d	s	b

← neutrino

Gauge boson(spin=1)

charge	EM	W	S
0	γ	Z^0	G
±1		$W^\pm$	

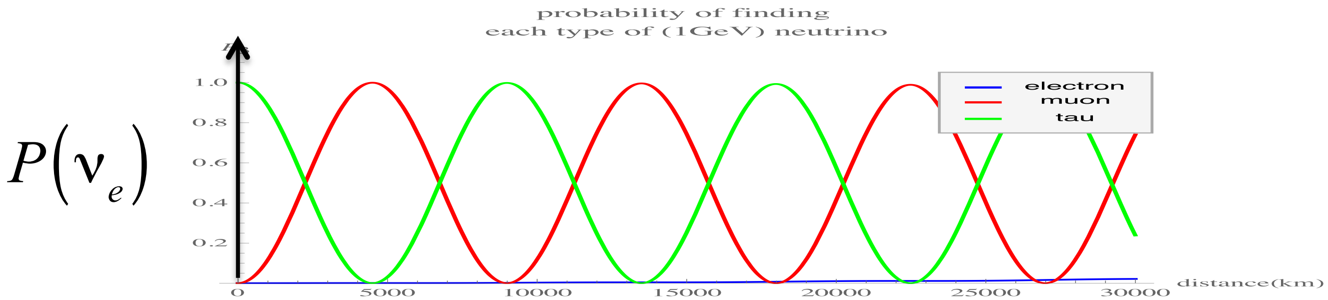
Higgs boson(spin=0)

charge	
0	H^0

What is Neutrino Oscillation?

Electron stays as electron while it travels in space.

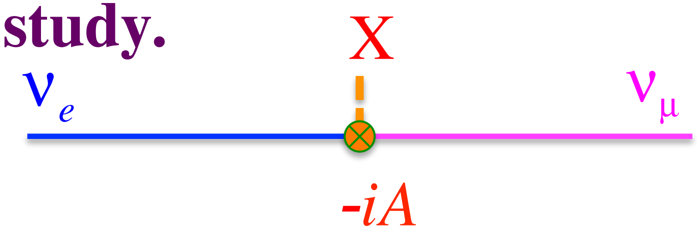
However, neutrinos change their flavors periodically.



This phenomenon is called neutrino oscillation

What causes the neutrino to oscillate?

We do not know yet.
In order for N.O. to happen, something(X) has to change the neutrino flavor. To know what is X is the important purpose of N.O. study.



"A" indicates the strength of the transition (amplitude).

In this case

State equation

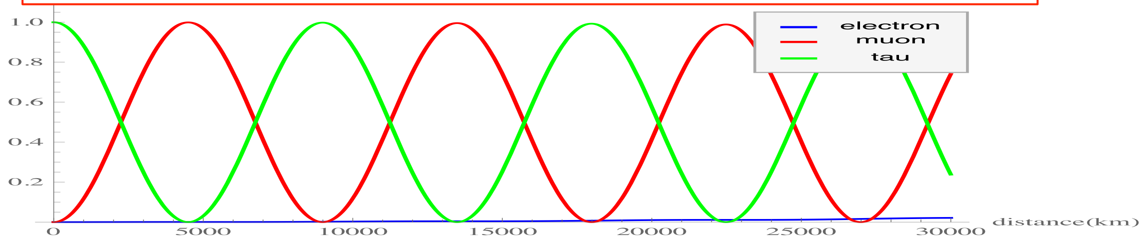
$$\frac{d}{dt}\nu_e = -iA\nu_\mu, \quad \frac{d}{dt}\nu_\mu = -iA\nu_e$$

Initial condition

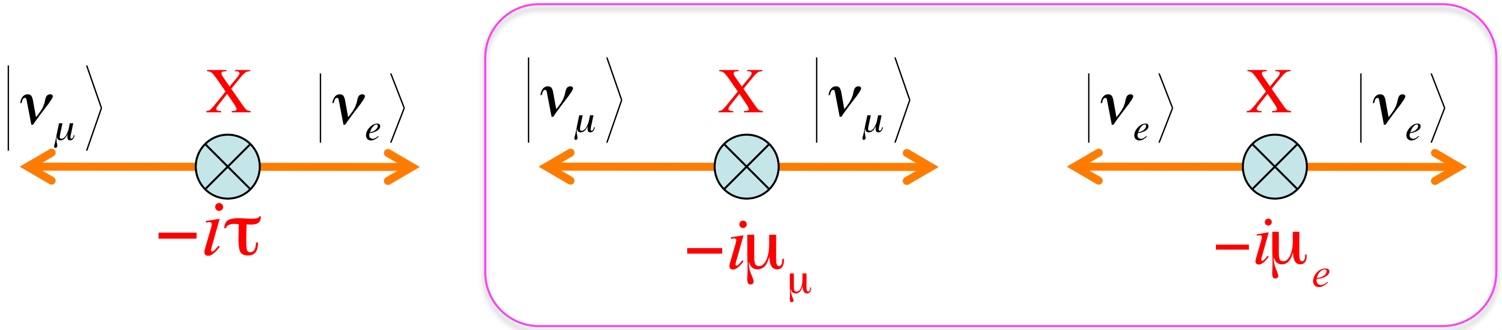
$$\nu_e[t=0]=1, \quad \nu_\mu[t=0]=0$$

Oscillation

$$|\nu_e[t]|^2 = \cos^2 At, \quad |\nu_\mu[t]|^2 = \sin^2 At$$



General transition amplitudes & mass eigenstates

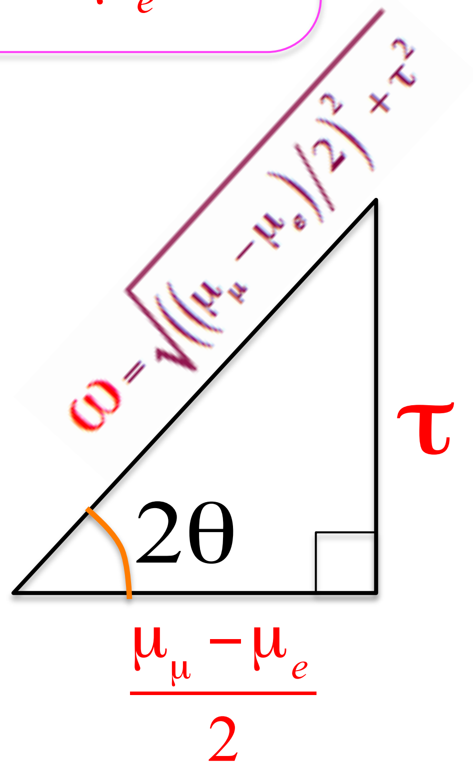


If there are self-transitions,
The mass eigenstate becomes

$$\begin{cases} \nu_1[t] = \left(\cos\theta|\nu_e\rangle - \sin\theta|\nu_\mu\rangle\right)\exp[-im_1 t] \\ \nu_2[t] = \left(\sin\theta|\nu_e\rangle + \cos\theta|\nu_\mu\rangle\right)\exp[-im_2 t] \end{cases}$$

The masses are

$$m_1 = \frac{\mu_\mu + \mu_e}{2} - \omega, \quad m_2 = \frac{\mu_\mu + \mu_e}{2} + \omega$$



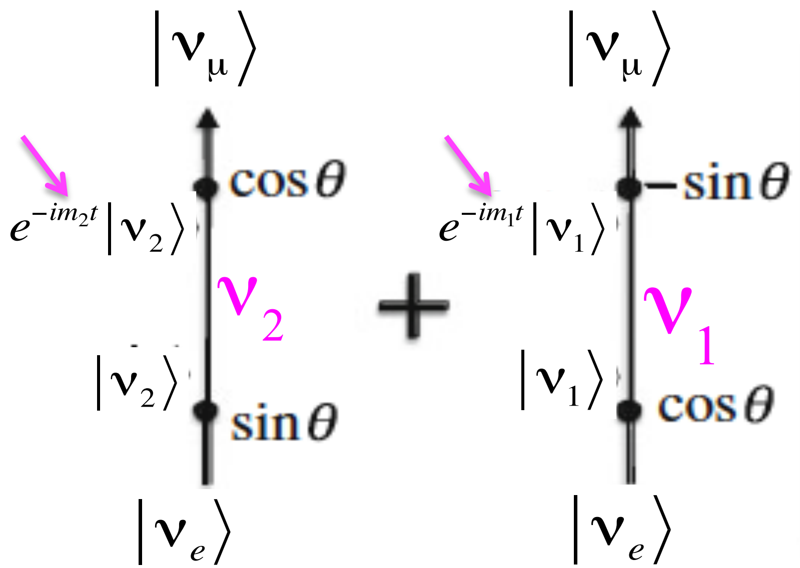
Mixing triangle

ν oscillation (non relativistic case)

A Q.M. principle: Probability for something to happen is the absolute square of the sum of amplitudes of all possible diagrams.

There are 2 amplitudes for $\nu_e \rightarrow \nu_\mu$

$P[\nu_e \rightarrow \nu_\mu] = \Big| \; \ldots \; + \; \ldots \; \Big|^2$



$$= \left| \sin\theta\cos\theta e^{-im_2 t} - \sin\theta\cos\theta e^{-im_1 t} \right|^2$$

$$= \sin^2 2\theta \sin^2 \frac{m_2 - m_1}{2} t$$

Oscillation in time

Relativistic Neutrino Oscillation

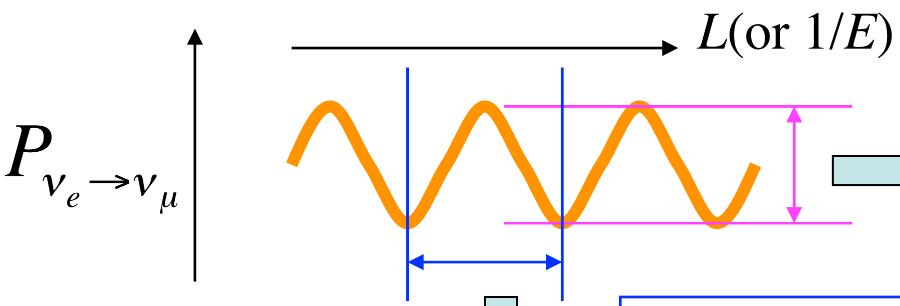
In experimental condition, neutrino is traveling relativistic

Lorentz Boost $(\gamma = E/m)$

$$mt \rightarrow m\frac{t}{\gamma} = \frac{m^2}{E}t = \frac{m^2 L}{E}$$

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \frac{\Delta m^2}{4E} L$$

What we can measure,



Amplitude ==>$\sin^2 2\theta = (\tau/\omega)^2$

Frequency => $\Delta m^2 = |m_2^2 - m_1^2| = 4\bar{\mu}\omega$

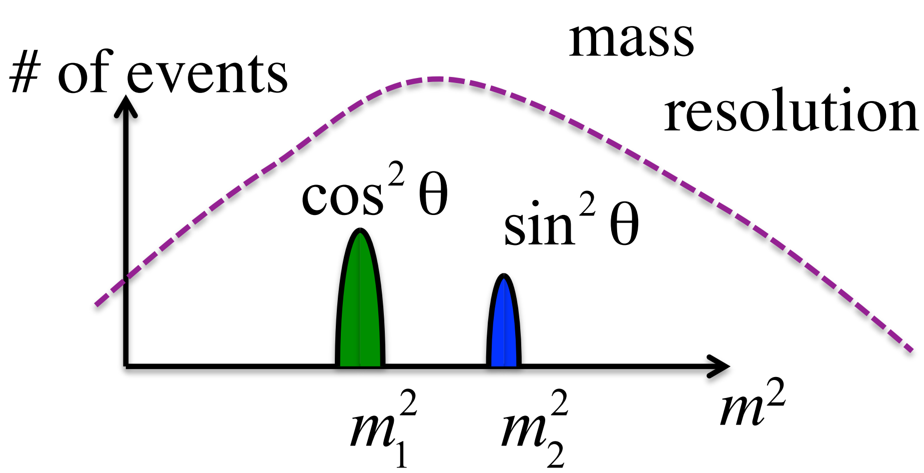
Relation to the absolute neutrino mass measurements

Absolute mass measurement:

$$^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \bar{\nu}_e \qquad \text{Measures } m^2_{\nu_e}.$$

But ν_e is not mass eigenstate. What actually is measured there?

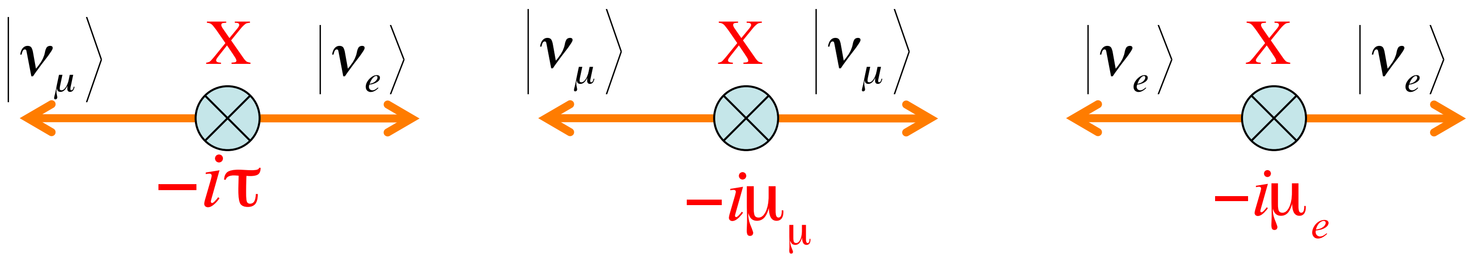
Since ν_e is a mixture of ν_1 and ν_2: $\nu_e = \nu_1 \cos\theta + \nu_2 \sin\theta$



The experiment measure the weighted average of ν_1, ν_2-mass^{2}s

$$m^2_{\nu_e} = \langle m^2 \rangle$$
$$= m_1^2 \cos^2\theta + m_2^2 \sin^2\theta = \mu_e^2 + \tau^2$$

Relation to the absolute neutrino mass measurements



*** Oscillation measurement**

$$\sin 2\theta = 2\tau \Big/ \sqrt{(\mu_\mu - \mu_e)^2 + 4\tau^2}$$

$$\Delta m^2 = 4(\mu_e + \mu_\mu)\sqrt{(\mu_\mu - \mu_e)^2 + 4\tau^2}$$

*** Absolute mass measurement**

$$m_{\nu_e}^2 = \mu_e^2 + \tau^2$$

➔Determination (μ_e, μ_μ, τ) possible.

➔ Oscillation measurements and absolute mass measurements are complementary to determine the neutrino transition amplitudes

Why we measure ν oscillations?

There are many oscillations (irrespective to it is observable or not).

* $K^0 \Leftrightarrow \overline{K^0}$ oscillation. ➔ CP violation
* $|u\bar{u}\rangle \Leftrightarrow |d\bar{d}\rangle$ oscillation in π^0, ρ, etc. ➔ Hadron mass pattern
* Cabbibo angle, quark mass ← $d' \Leftrightarrow s'$ oscillation
* Weinberg angle, W, Z^0 mass ← $B \Leftrightarrow W_3$ oscillation

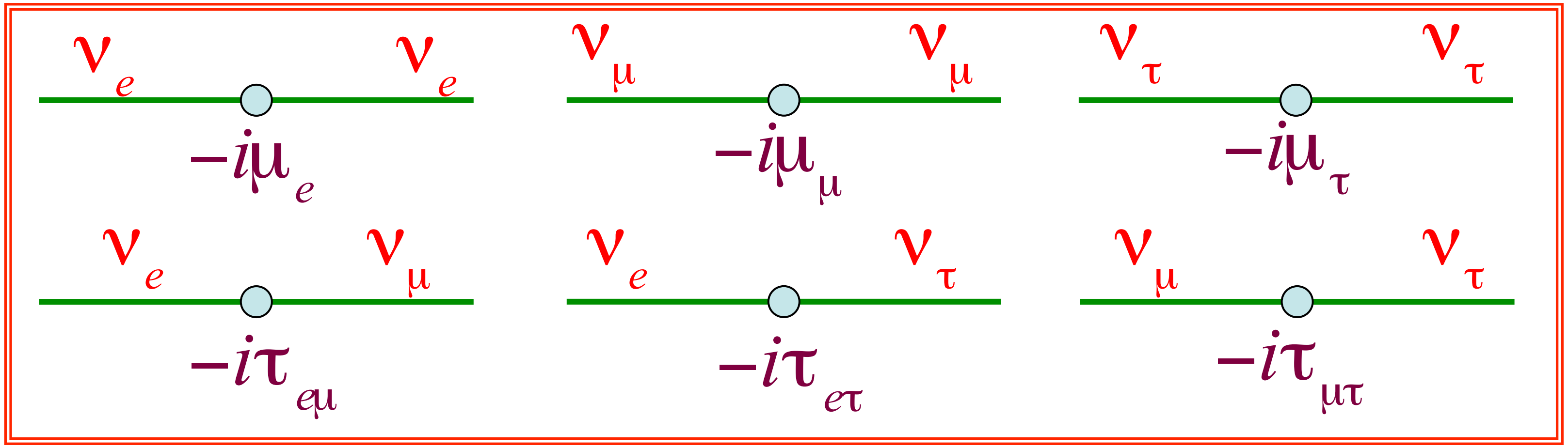
← We have learned a lot from these "Oscillations"

We can expect to learn more from ν oscillations;

$$\nu_\alpha \Leftrightarrow \nu_\beta$$

What is X??

3 Flavor Neutrino Oscillation



➔ **The mixing matrix becomes 3x3 & there are 3 masses**

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

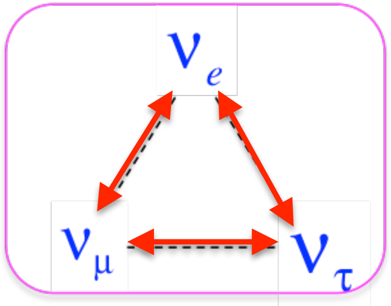
$$m_1, m_2, m_3 = \ldots$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Oscillation Parameter Measurements

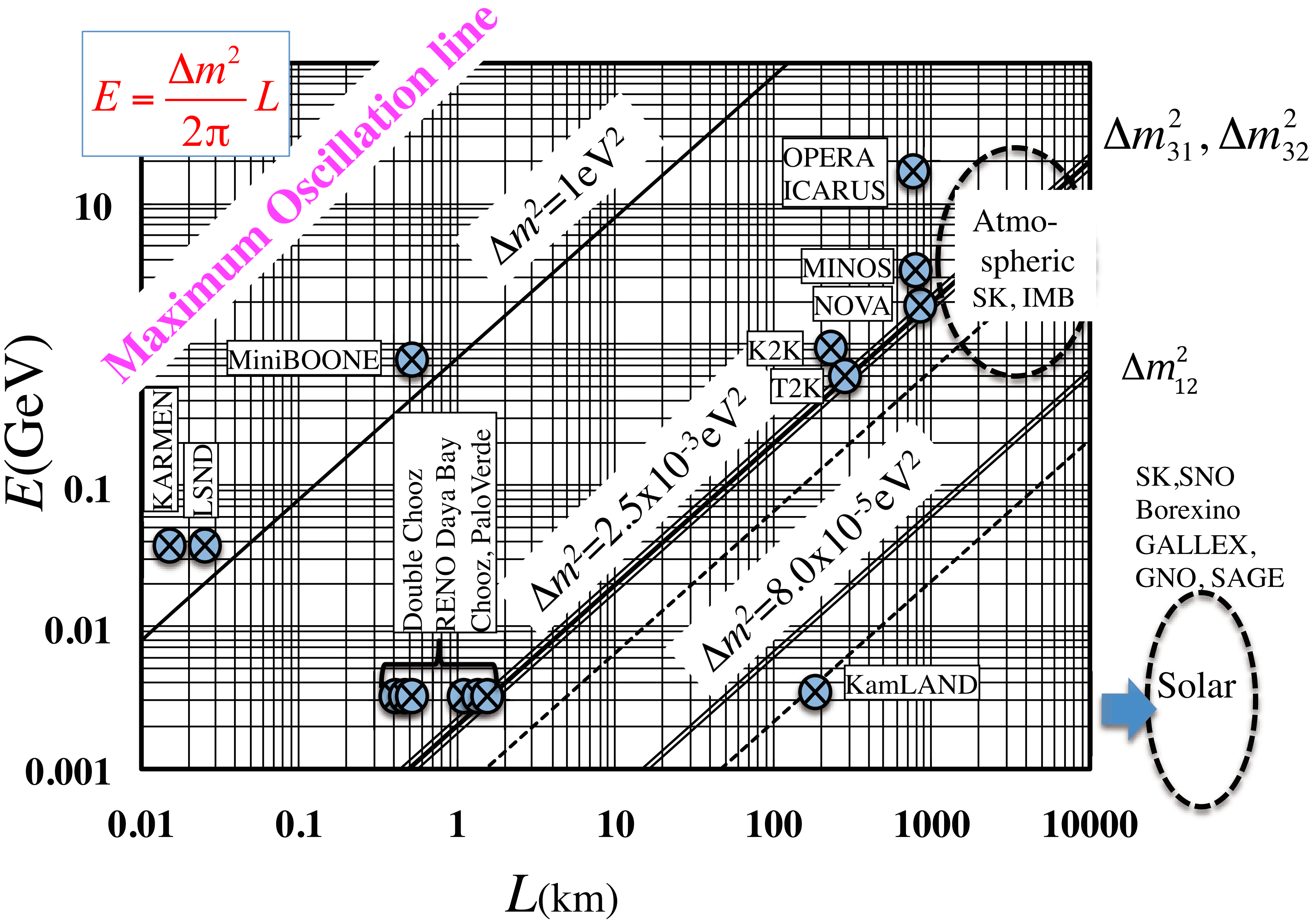
Parameters to measure

- **Mixing angles:** $\theta_{12}, \theta_{23}, \theta_{13}$
- **Square mass differences:** $\Delta m^2_{12}, \Delta m^2_{23} \left(, \Delta m^2_{13}\right)$
- **CP violation phase:** δ_{CP}



It has bee a long story .... Simplified here.

E-L relation of N.O. experiments

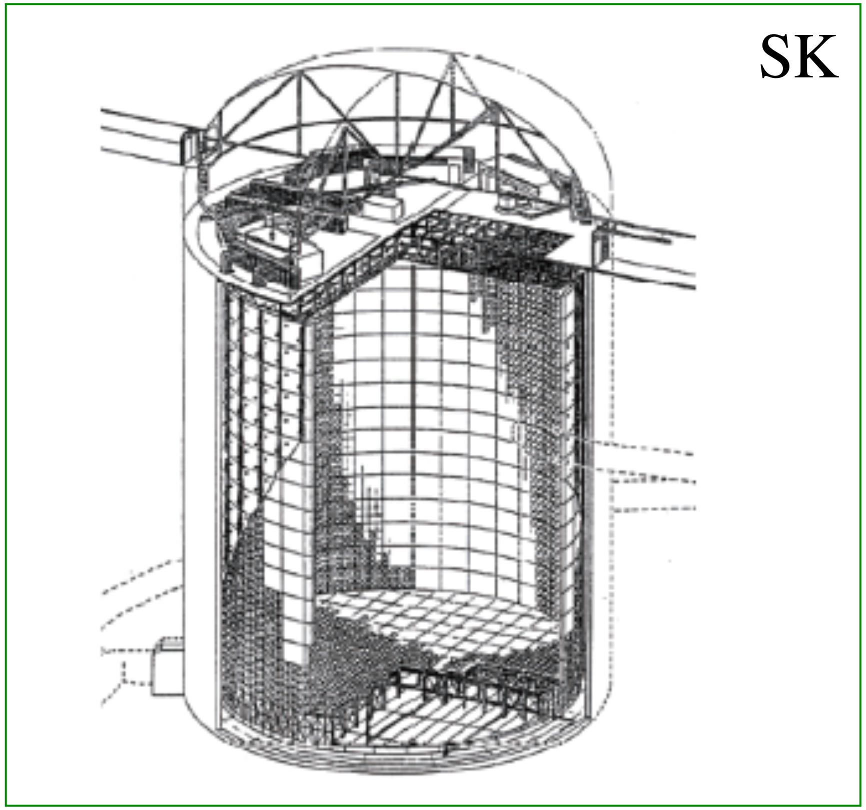


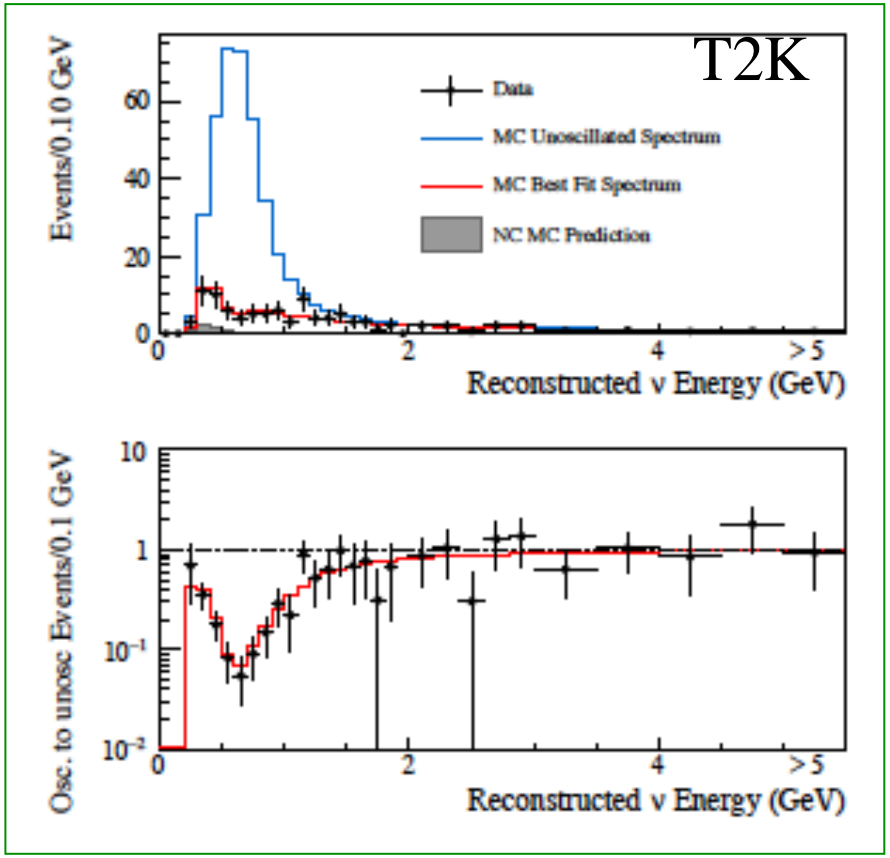
$\theta_{23}, \quad \Delta m^2_{32}$

Atmospheric (SK. etc.), T2K, MINOS, NOVA ....

$$P(\nu_\mu \to \nu_\mu) \sim 1 - \sin^2 2\theta_{23} \sin^2 \frac{\Delta m^2_{32}}{4E} L$$

arXiv:1502.01550v1

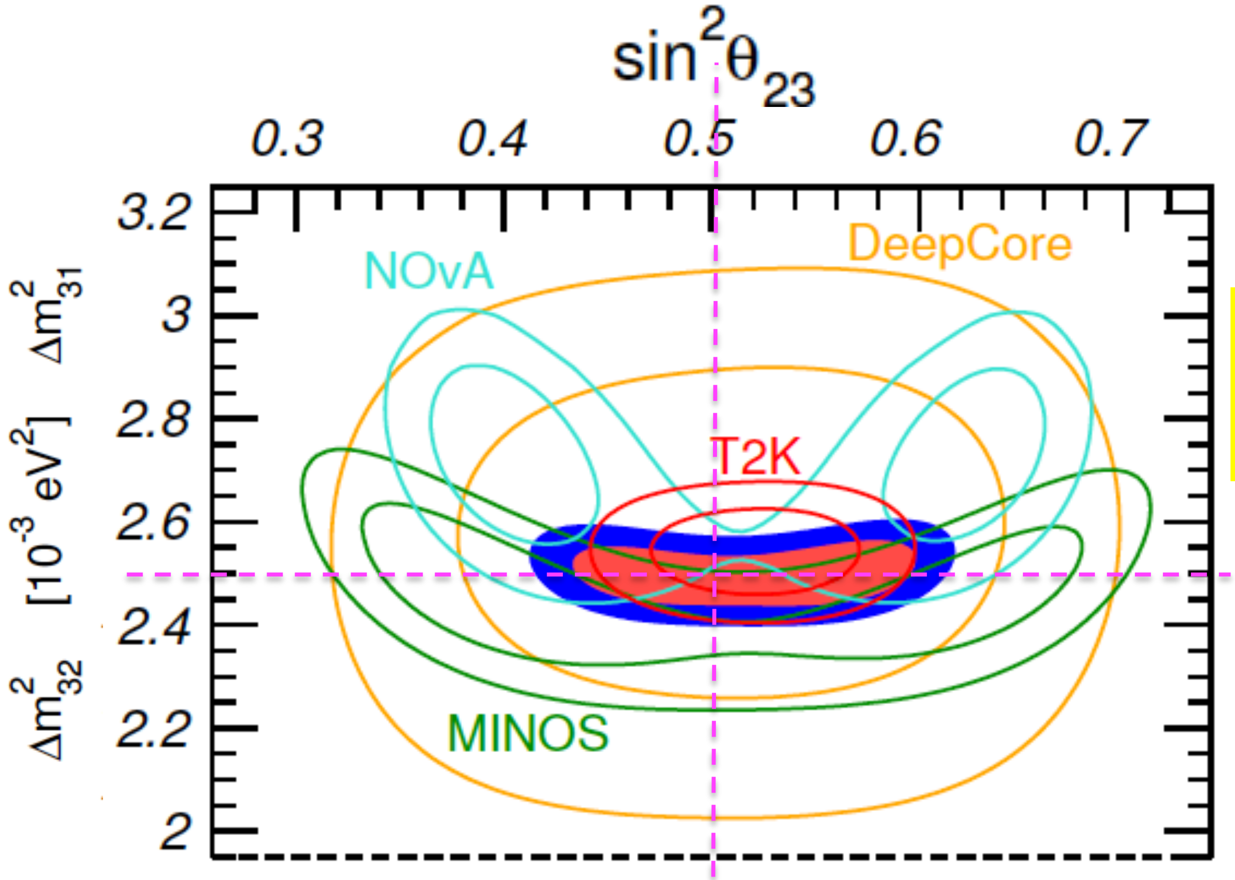




→ Discovery of N.O. : Nobel prize in 2015 (T.Kajita)

$\theta_{23}, \quad \Delta m_{32}^2$

$$P(\nu_\mu \to \nu_\mu) \sim 1 - \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{32}^2}{4E} L$$



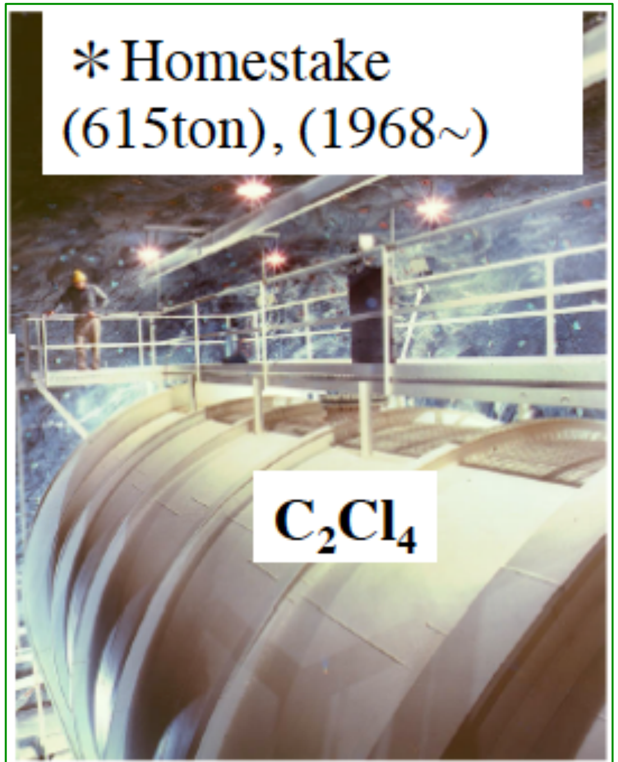
Normal Hierarchy (m_3>m_2) case

NuFIT 3.1(2017), www.nu-fit.org, JHEP01(2017)087,

$$\sin^2 2\theta_{23} \sim 1, \quad \Delta m_{32}^2 \sim 2.5 \times 10^{-3} [eV^2]$$

$\theta_{12}, \quad \Delta m^2_{12}$

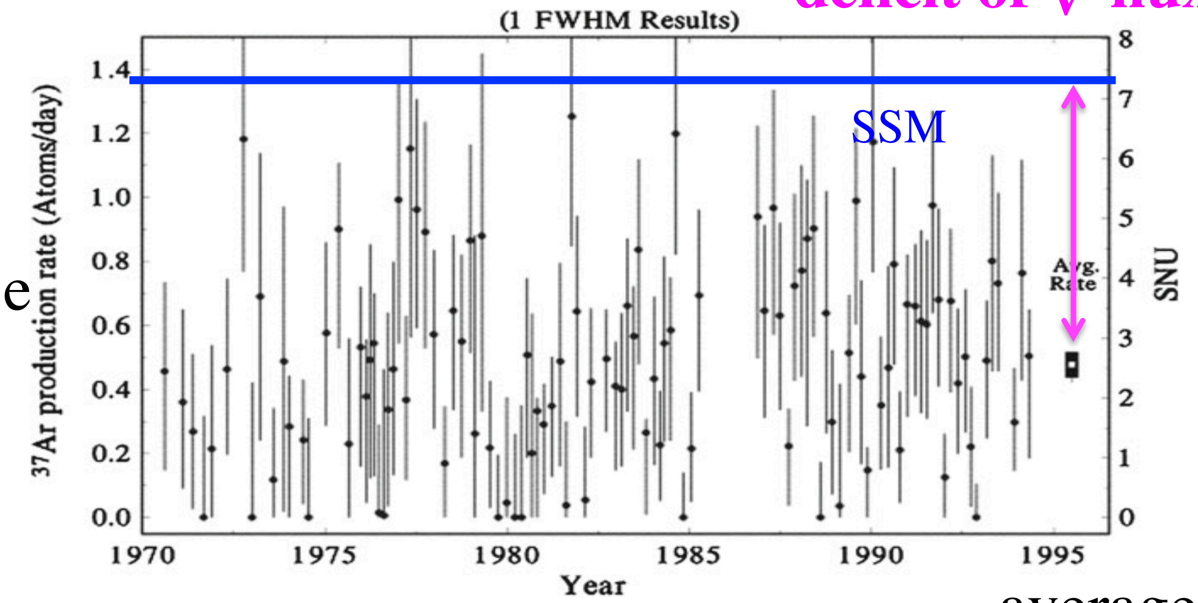
Solar Neutrino Experiments (Homestake, SuperK, SAGE, GALLEX, BNO, Borexino, SNO, etc.)



$^{37}Cl + \nu_e \rightarrow {}^{37}Ar + e^-$

$$P(\nu_e \to \nu_e; @\Delta m^2_{21}) \sim 1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m^2_{12}}{4E} L$$

deficit of ν flux

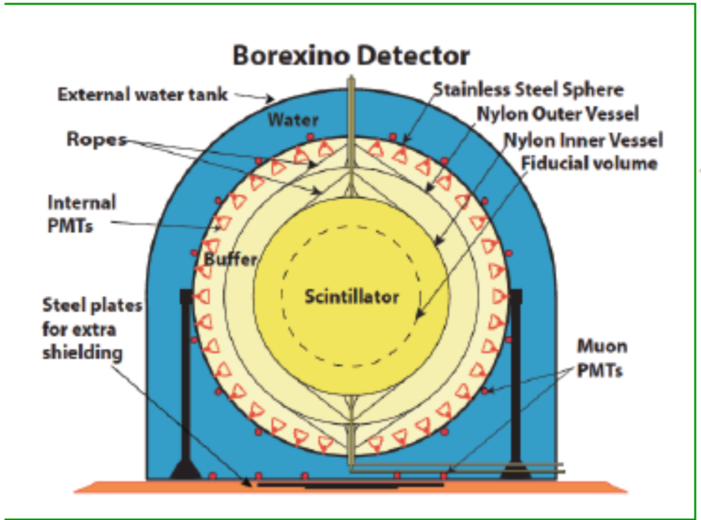


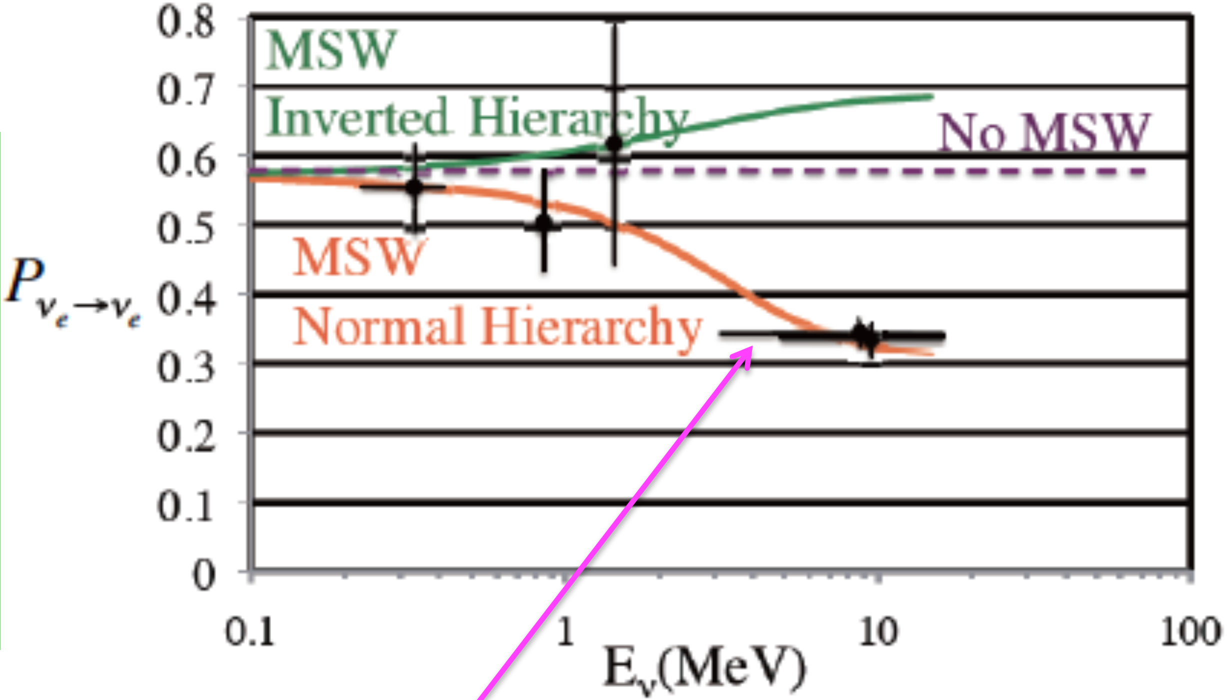
average

➔ Nobel prize in 2002 (R.Davis)

Δm^2_{21} mass hierarchy ($m_2 > m_1$ or $m_2 < m_1$)

Genuine N.O. can not resolve it but
Matter Effect depends on M.H. and it can be used.





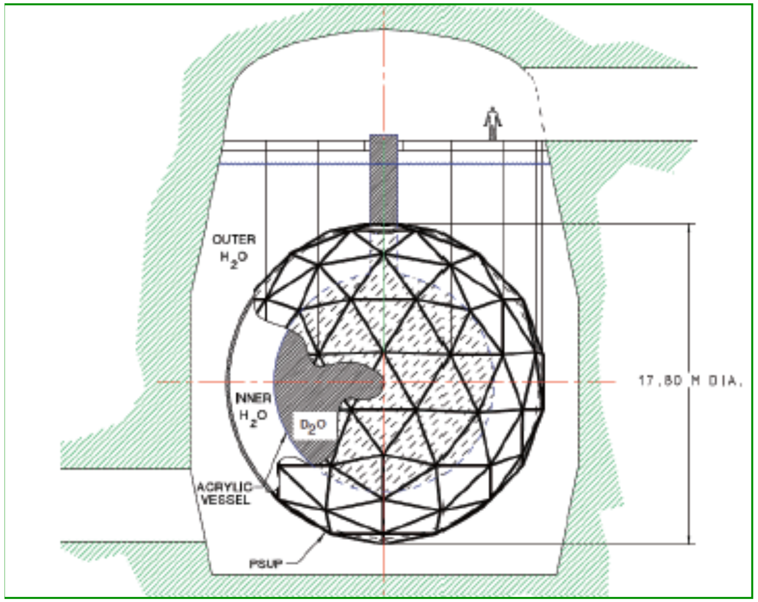
etc.

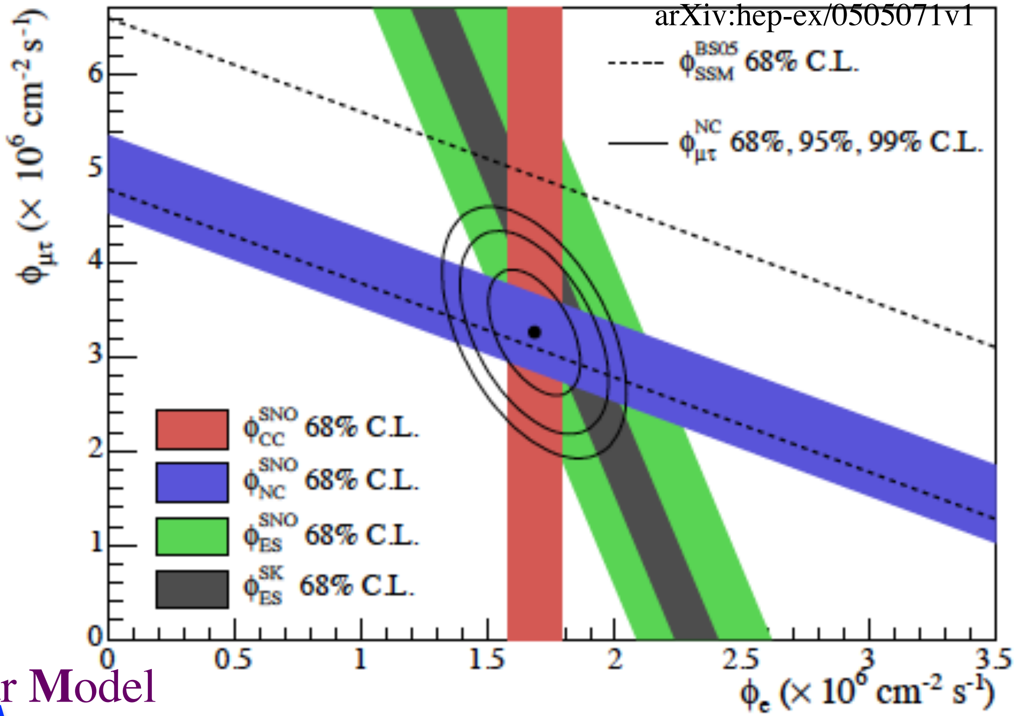
$m_2 > m_1$ determined

Flavor Transmutation: SNO experiment

$$\nu_x + D \rightarrow \nu_x + p + n$$

NC interaction: possible to count all flavors





Standard Solar Model

Although $\Phi(\nu_e) < \Phi(SSM)$, $\quad \Phi(\nu_e) + \Phi(\nu_\mu) + \Phi(\nu_\tau) = \Phi(SSM)$

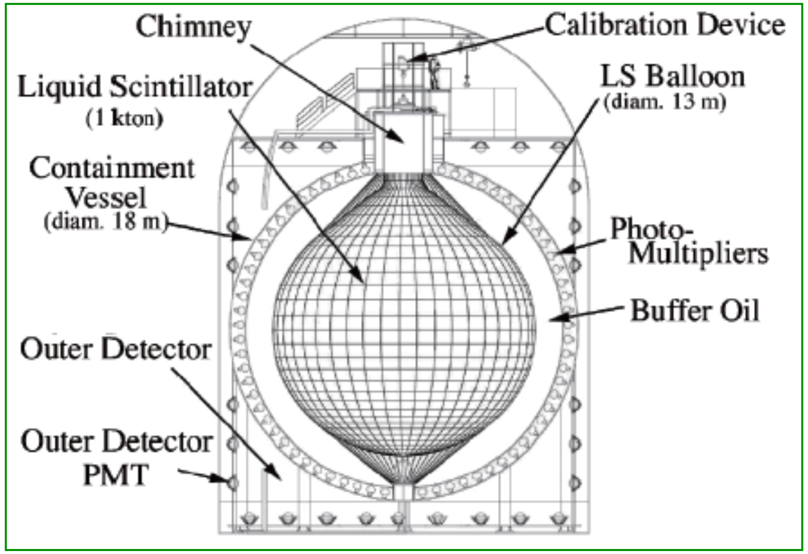
➔ Total # of ν does not change. ν_e changed to other neutrinos.

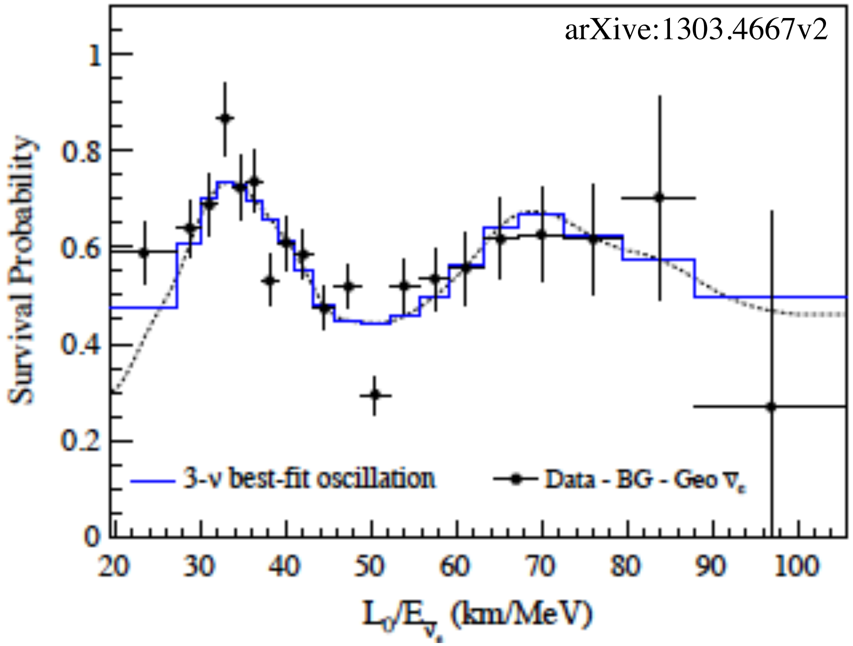
➔ Nobel prize in 2015 (A.McDonald)

$\theta_{12}, \quad \Delta m^2_{12}$

KamLAND Reactor Neutrino Oscillation

$$P\left(\bar{\nu}_e \rightarrow \bar{\nu}_e; @\Delta m^2_{21}\right) \sim 1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m^2_{12}}{4E} L$$



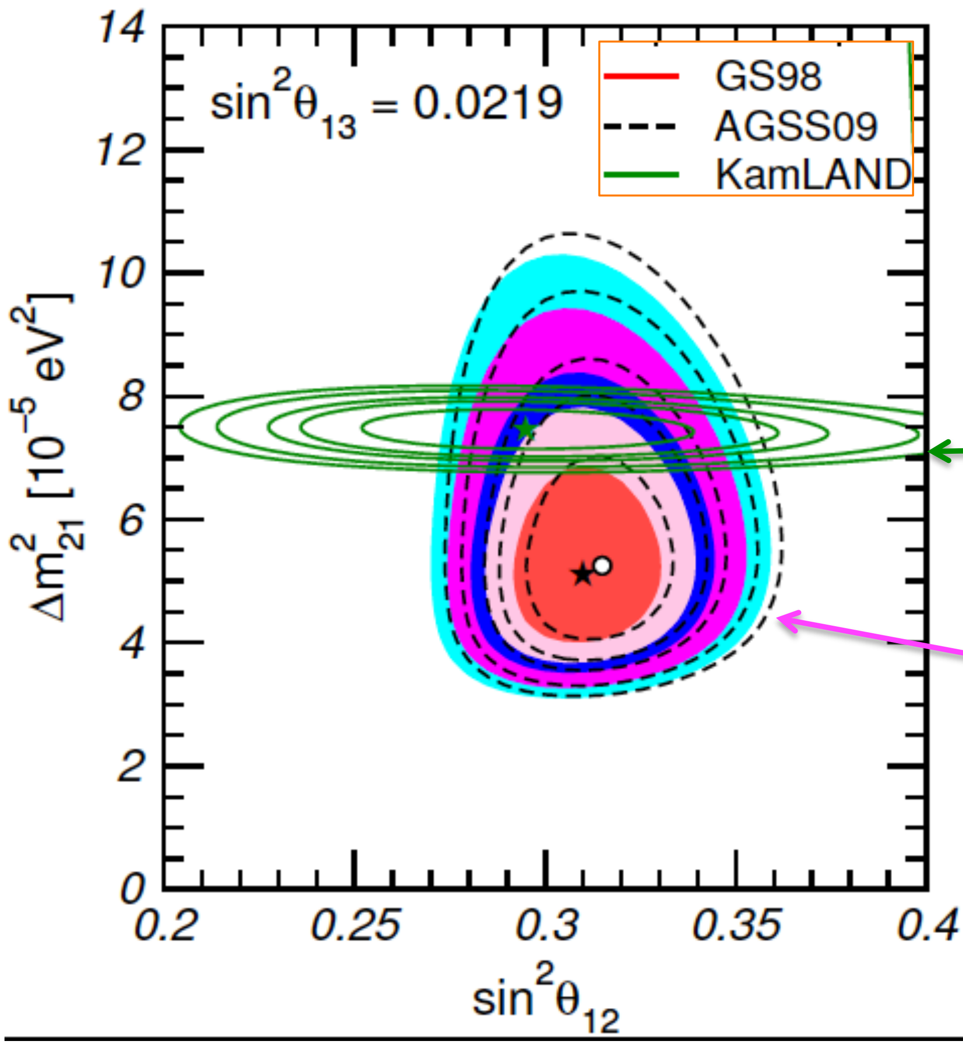


L~180km

$$\text{KamLAND}: \quad \tan^2\theta_{12} = 0.436^{+0.029}_{-0.025}, \quad \left|\Delta m^2_{21}\right| = 7.53^{+0.18}_{-0.18} \times 10^{-5} eV^2$$

θ_{12} , Δm^2_{12}

NuFIT 3.1(2017), www.nu-fit.org, JHEP01(2017)087,



KamLAND Reactor Exp.

Solar Exps.

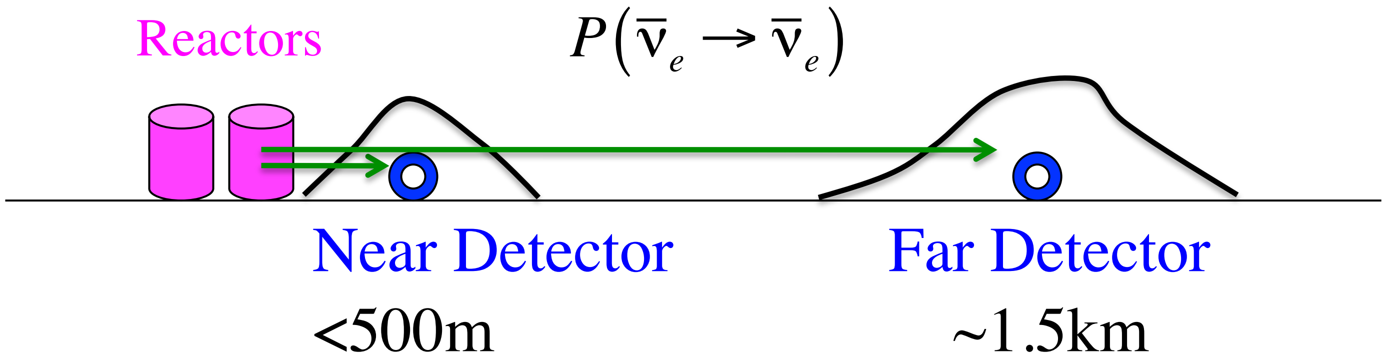
➔ There is slight tension

$\theta_{13}, \quad \Delta m^2_{31}$

Reactor-θ_{13} Experiment

➔ Reactor measurement of θ_{13}

$$P\left(\bar{\nu}_e \to \bar{\nu}_e; L \sim 1.5km\right) \sim 1 - \sin^2 2\theta_{13}$$

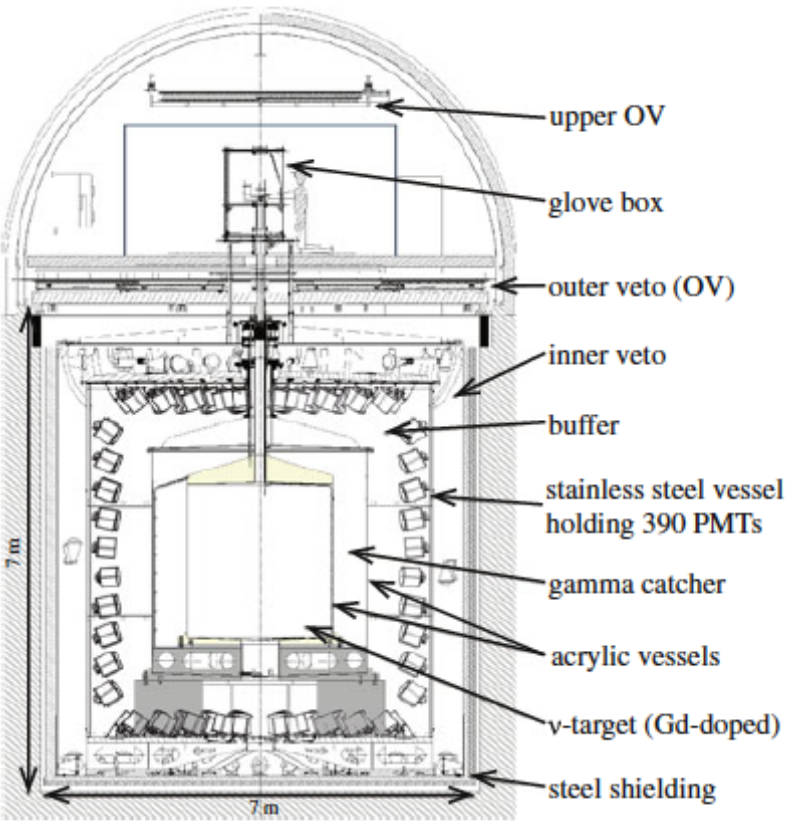


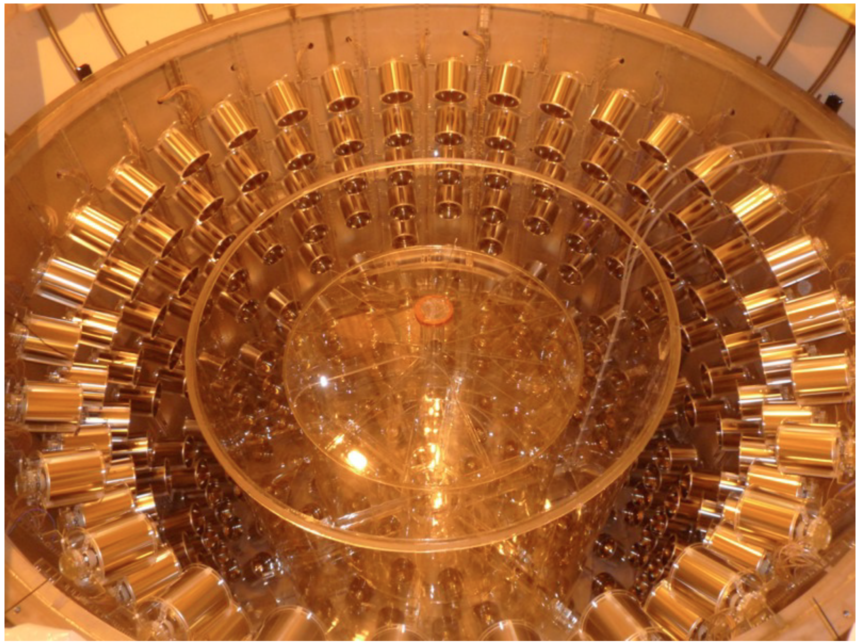
Two detector concept: Cancel uncertainty of neutrino flux and detection efficiency by comparing near & far detector

$\theta_{13}, \quad \Delta m^2_{31}:$

Our experiment: Double Chooz

@ CHOOZ-B reactors



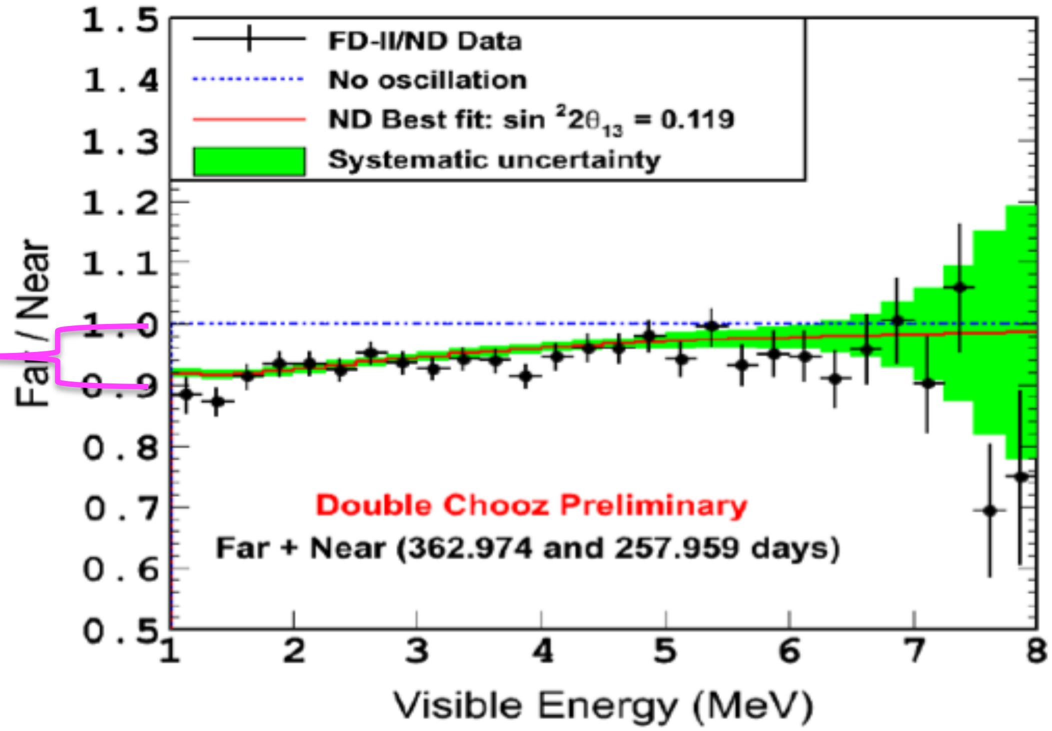


Double Chooz Oscillation fit result

Far detector/Near detector concept to cancel most of the systematics.

Far/Near ratio

Small deficit

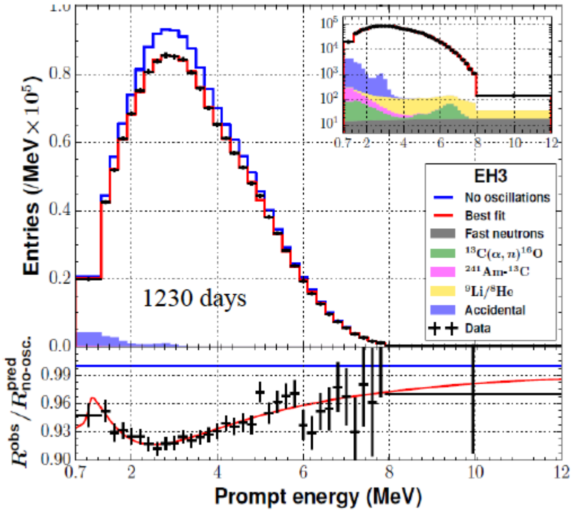


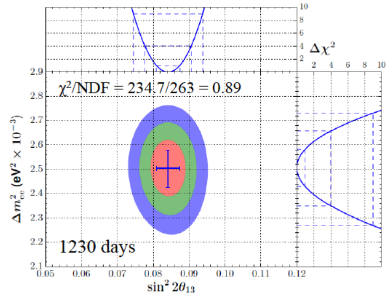
$\sin^2 2\theta_{13} = 0.119 \pm 0.016$ with χ^2/ndf = 236.2/114
(preliminary)

Daya Bay Result

Logan Lebanowski @ 2016.11 NNN16

Oscillation analysis result





$\sin^2 2\theta_{13} = 0.0841 \pm 0.0027(stat.) \pm 0.0019(syst.)$

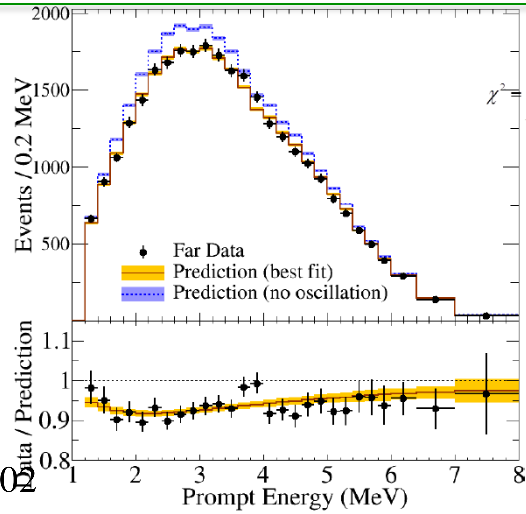
$|\Delta m^2_{ee}| = [2.50 \pm 0.06(stat.) \pm 0.06(syst.)] \times 10^{-3} eV^2$

Multiple analyses yield consistent results.

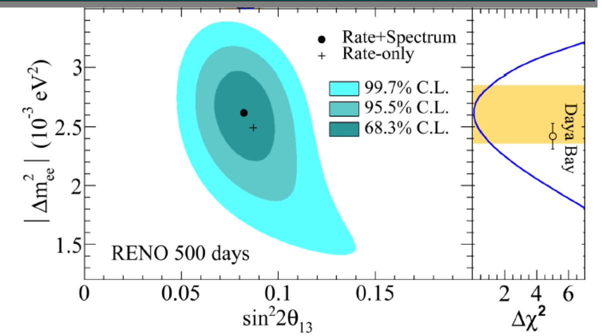
[arXiv:1610.04802]

RENO result

Hyunkwan Seo @ 2016.11 NNN16



Results from Spectral Fit



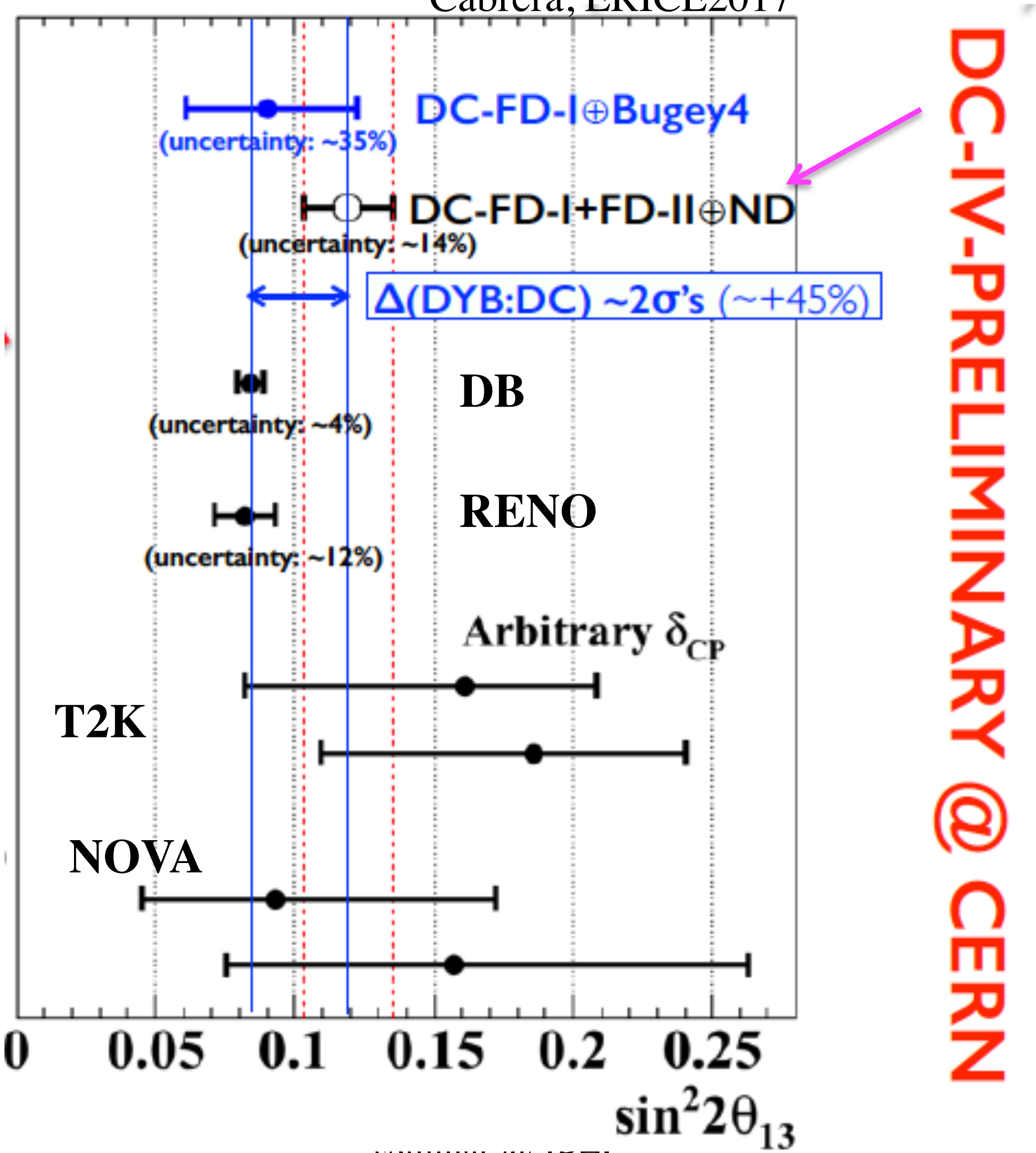
Rate+shape new results

$\sin^2 2\theta_{13} = 0.082 \pm 0.009(\text{stat.}) \pm 0.006(\text{syst.})$

$|\Delta m^2_{ee}| = 2.62^{+0.21}_{-0.23}(\text{stat.})^{+0.12}_{-0.13}(\text{syst.})\ (10^{-3} eV^2)$

There may be a tension between DC and DB, RENO

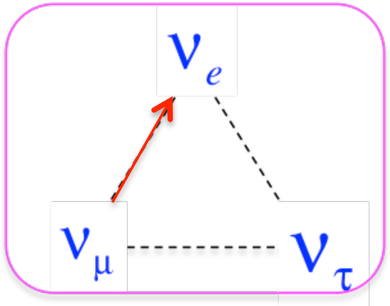
Cabrera, ERICE2017



CP violation δ

T2K and NOVA measure

$$\left(\nu_\mu \to \nu_e\right),\ \left(\bar{\nu}_\mu \to \bar{\nu}_e\right)$$



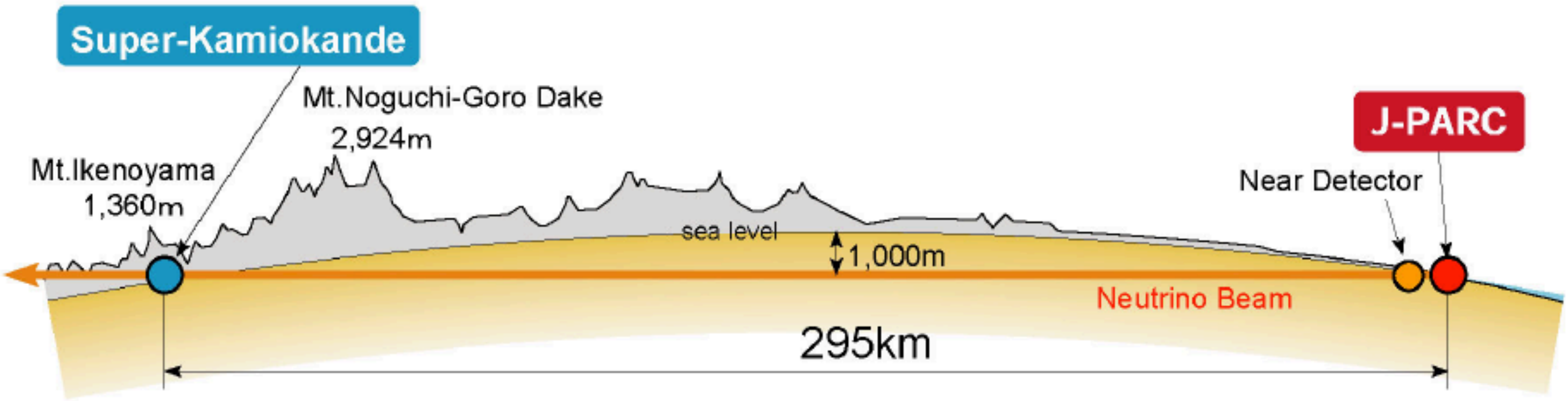
$$\nu_\mu \to \nu_e \qquad P_A \sim 0.5\sin^2 2\theta_{13} - 0.043\sin 2\theta_{13}\,\underline{\sin\delta}$$

$$\bar{\nu}_\mu \to \bar{\nu}_e \qquad \bar{P}_A \sim 0.5\sin^2 2\theta_{13} + 0.043\sin 2\theta_{13}\,\underline{\sin\delta}$$

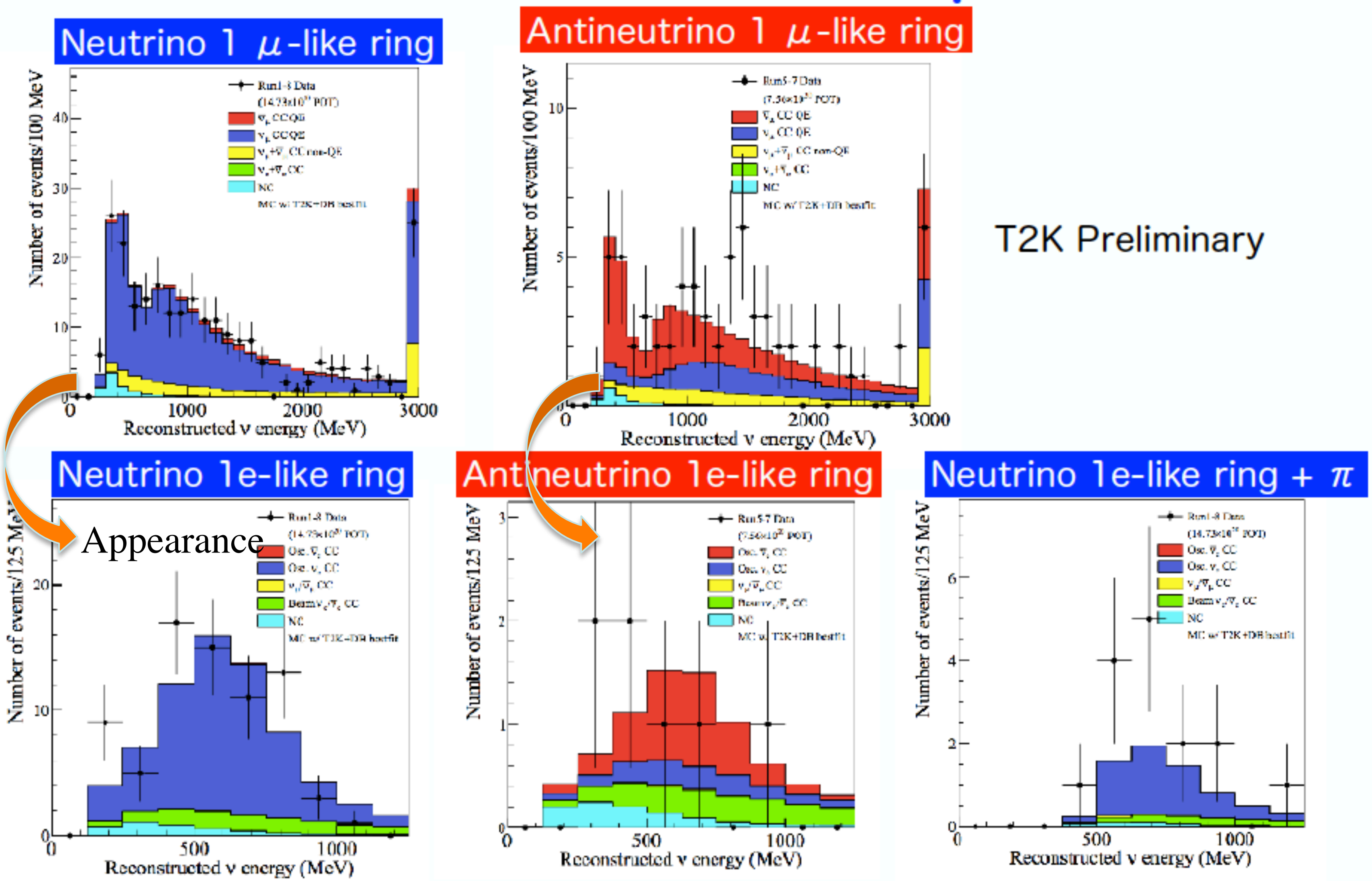
$$A_{CP} = \frac{P_A - \bar{P}_A}{P_A + \bar{P}_A} \sim 0.3\sin\delta$$

T2K Experiment

K.Iwamoto@ICHEP16



Observation at Super-K



θ_{13} and δ_{CP}

- Fit without the reactor constraint: closed contours in δ_{cp} at 90% CL
- The T2K value for $\sin^2\theta_{13}$ is consistent with the PDG 2016

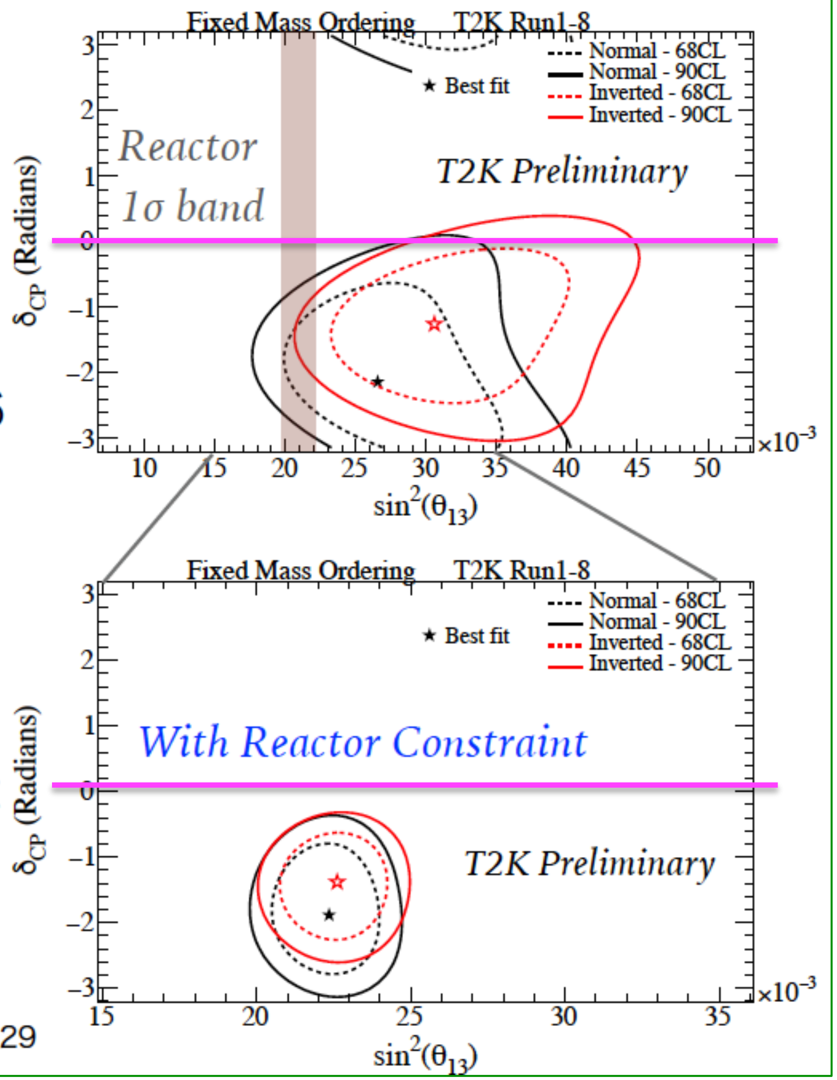
T2K Best Fit:

$$\sin^2\theta_{13} = 0.0277^{+0.0054}_{-0.0047} \text{ (NH)}$$

PDG 2016:

$$\sin^2\theta_{13} = 0.0210 \pm 0.0011$$

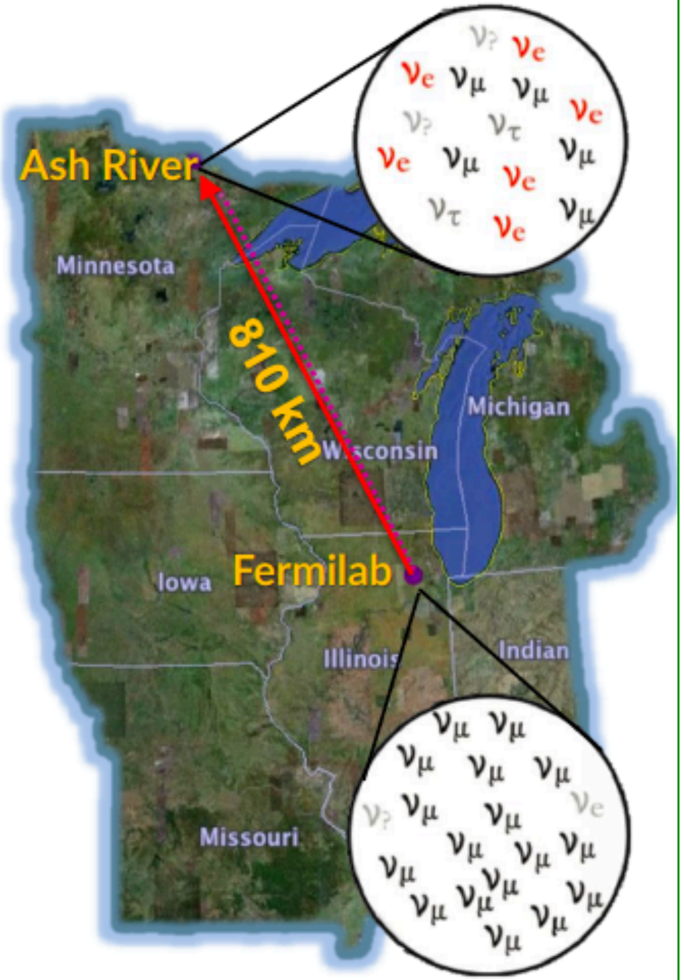
- Adding the reactor constraint improves the constraint on δ_{cp} average:



NOVA

NuMI Off-axis ν_e Appearance Experiment

- Long-baseline, two-detector ν oscillation experiment
- Looks for ν_e in ν_μ NuMI beam
- 14 mrad off-axis
- 2 liquid scintillator detectors
- FD (14 kton), ND (0.3 kton)
- Cooled APD readout (live)
- Appearance & disappearance
- Exotics, non-beam…

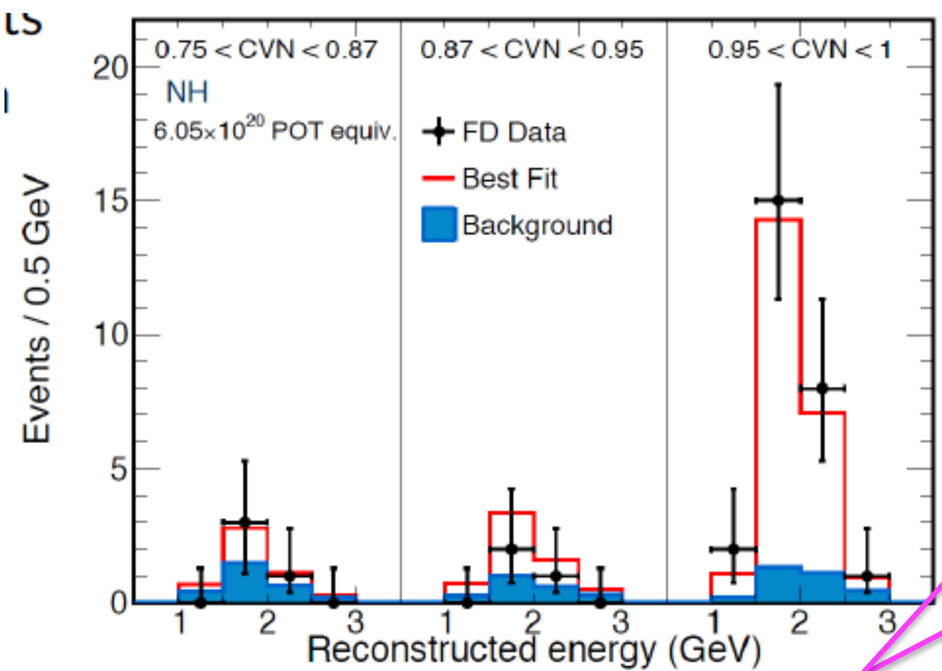


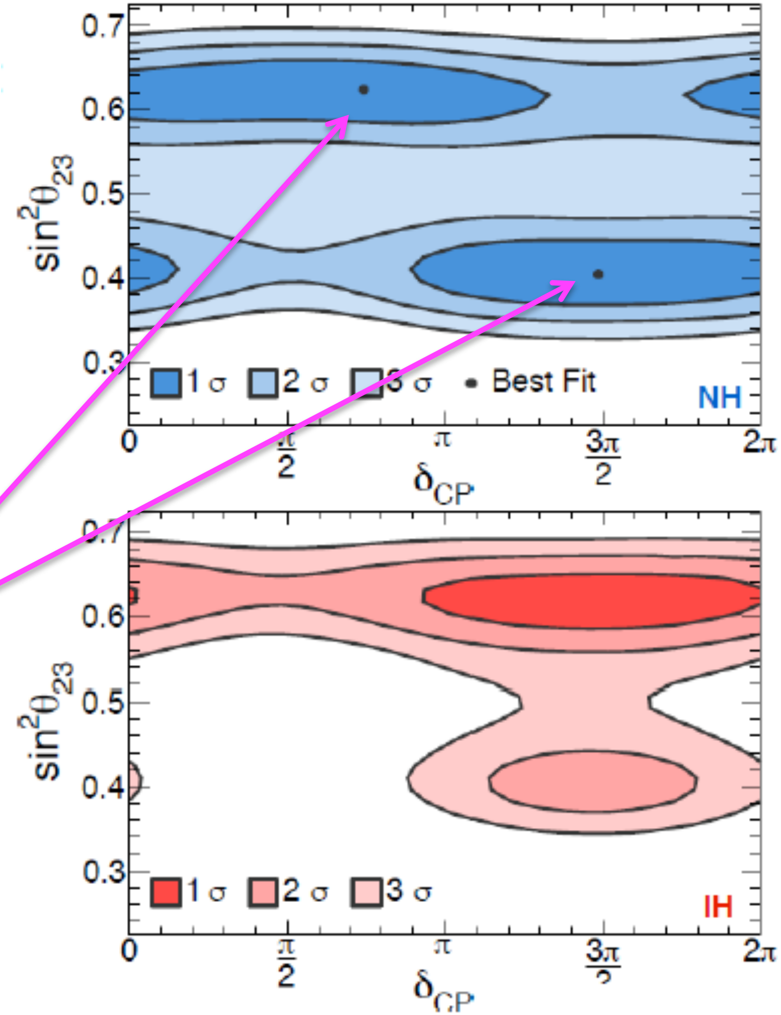
Filip Jediný - NOvA neutrino experiment 7

ν_e appearance results

NOvA

- Observe 33 events on a background of 8.2 ± 0.8



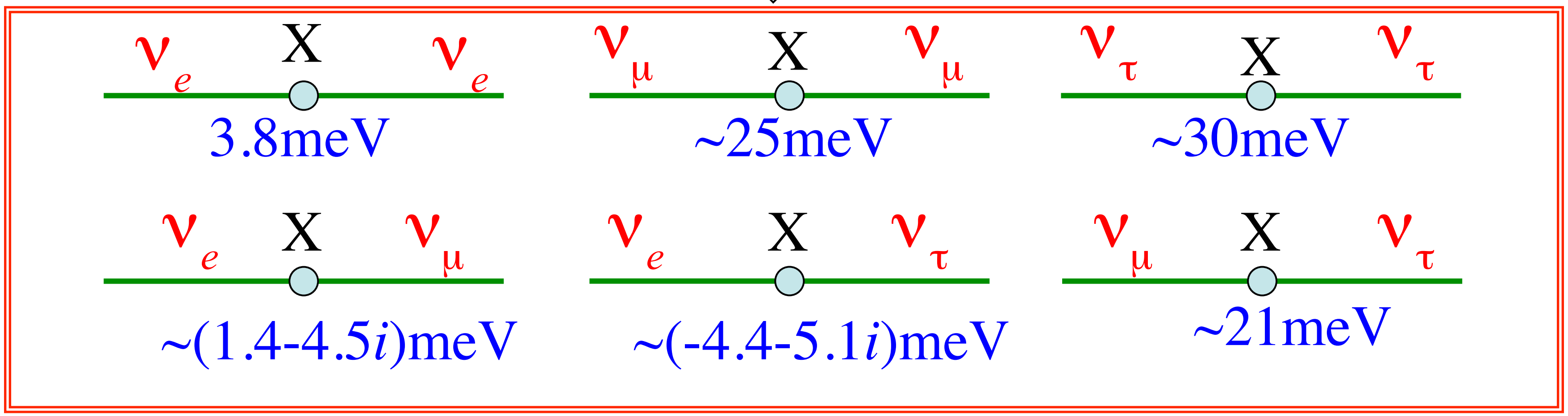


- 2 degenerate best fit points:
  - NH, $\delta_{CP} = 1.48\pi$ $\sin^2\theta_{23} = 0.404$
  - NH, $\delta_{CP} = 0.74\pi$ $\sin^2\theta_{23} = 0.623$

Our Current Knowledge of Neutrino Transition Amplitude

If we assume $m_3 > m_2 > m_1 \sim 0$, $\delta_{CP} = -\pi/2$

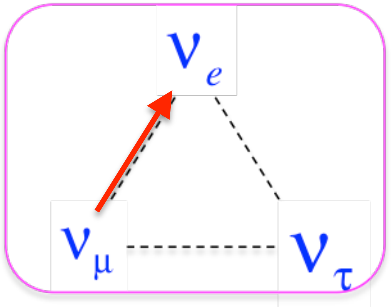
$$U_{NH} \sim \begin{pmatrix} 0.82 & 0.55 & -0.09+0.13i \\ -0.36+0.07i & 0.65+0.05i & 0.67 \\ 0.43+0.08i & -0.53+0.05i & 0.73 \end{pmatrix}$$

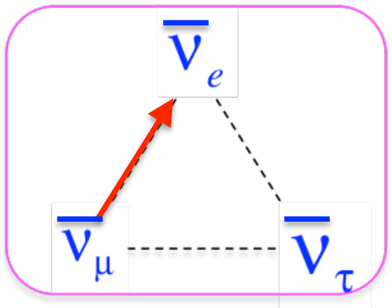


What is X and how this pattern can be explained??

Future

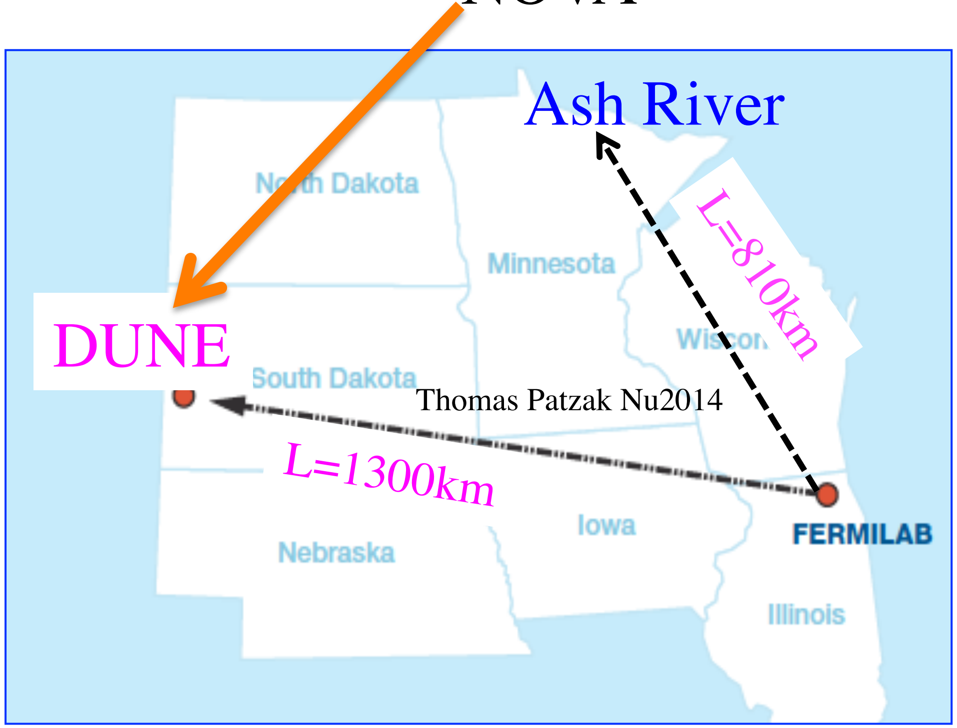
More precise CP Asymmetry
Mass Hierarchy determination
:





CP asymmetry by Future Long baseline experiments

NOVA

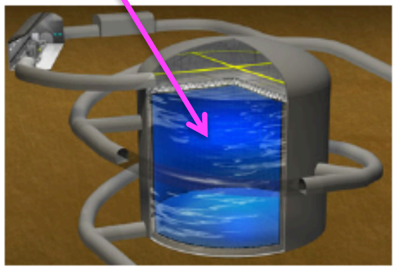


E. O'Sullivan, NuFACT 2017

Hyper Kamiokande:

516kt W.C.

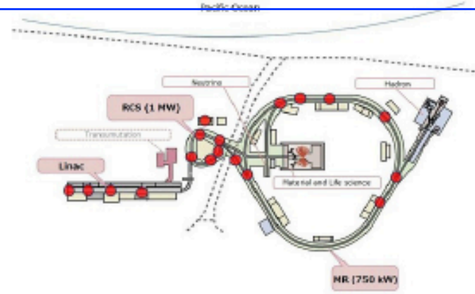
T2HK



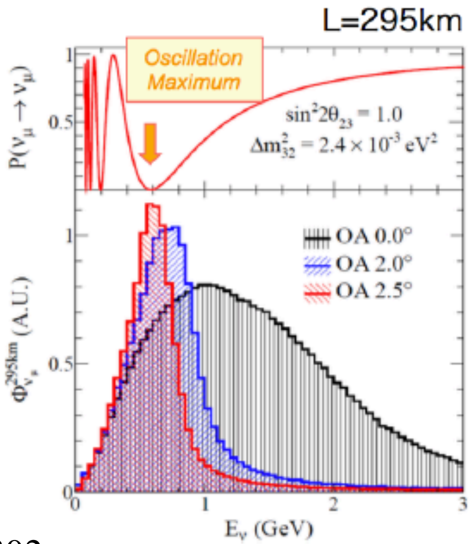
Hyper-K detector (Kamioka)

295 km

T2K→
*Larger target
*Larger acc. power



J-PARC neutrino Beam (Tokai)



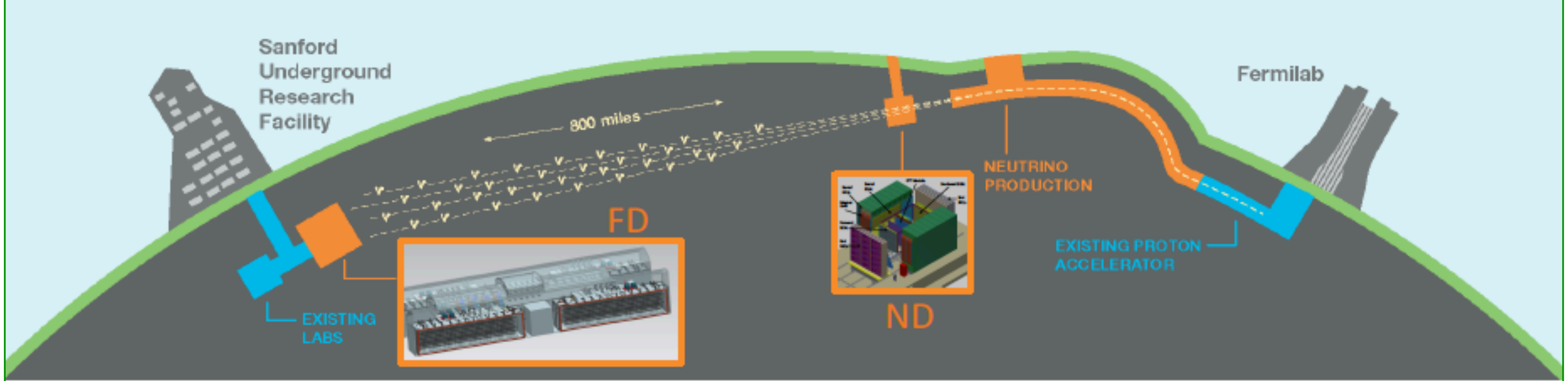
Building off the successful T2K program, Hyper-K also plans to measure neutrinos from the J-PARC neutrino beam

Data taking expected in 2026

DUNE at LBNF

Deep Underground Neutrino Experiment at the Long Baseline Neutrino Facility

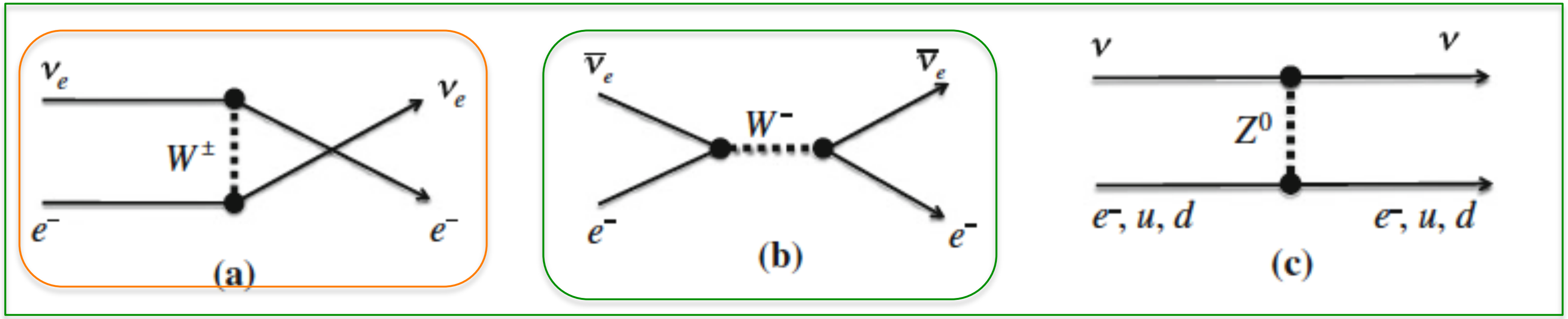
DUNE: detectors + science ⟺ **LBNF**: facilities + beam



- High intensity, wide-band, neutrino beam from Fermilab
- Highly capable neutrino near detector at Fermilab
- 40-kt fiducial mass far detector at SURF based on LAr-TPCs

2026, Neutrino Beam available

CP asymmetry with the matter effect



Difference of weak potential for ν_e and $\bar{\nu}_e$ in Earth produces a fake asymmetry

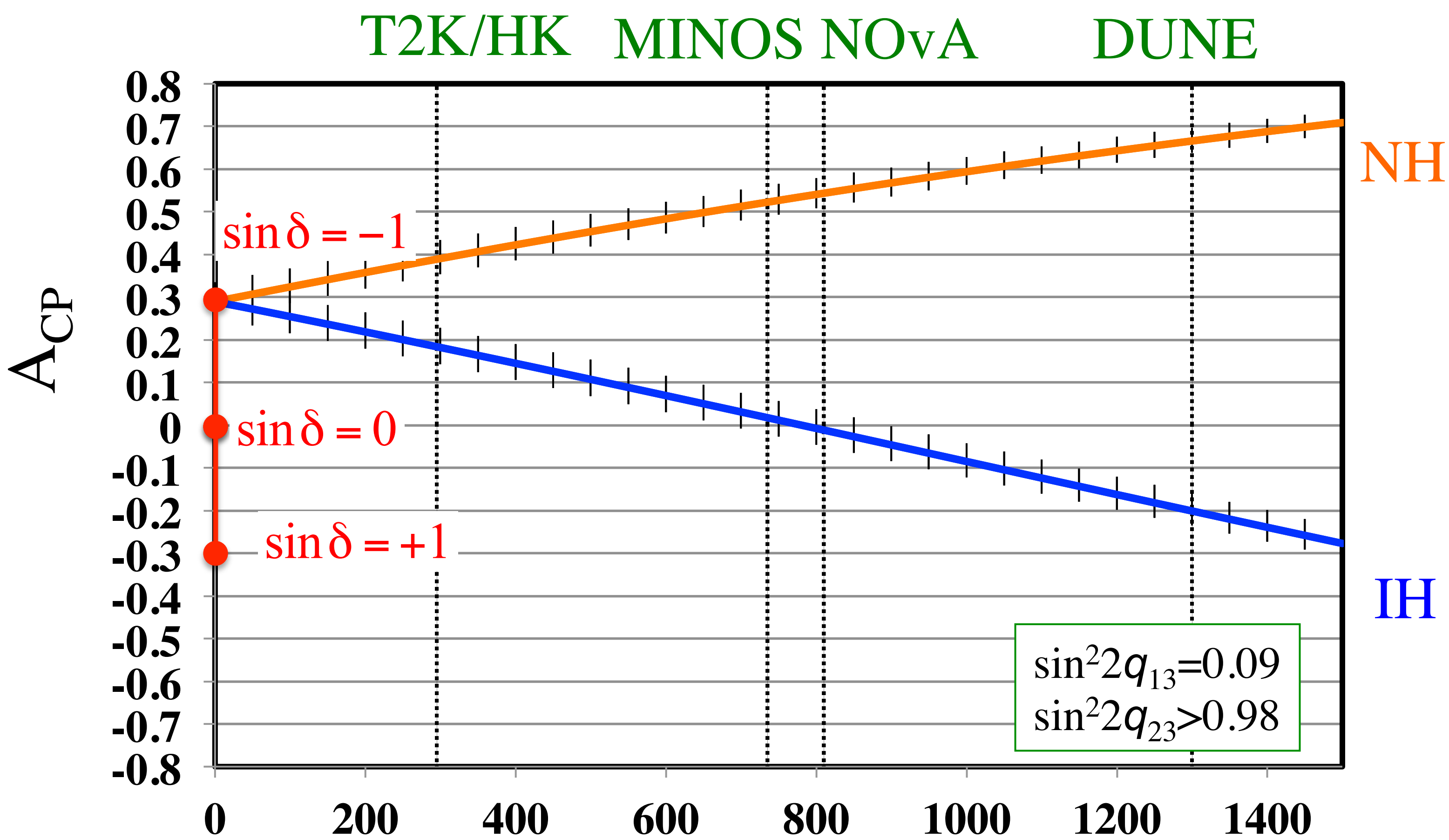
$$A_{CP}\left(@\Phi_{13}\right) \sim -0.3\sin\delta_{CP} \pm 2(L/L_0)$$

	L[km]	$A_{FK}=2(L/L_0)$
T2K/HK	295	±0.11
NOVA	810	±0.30
DUNE	1,300	±0.48

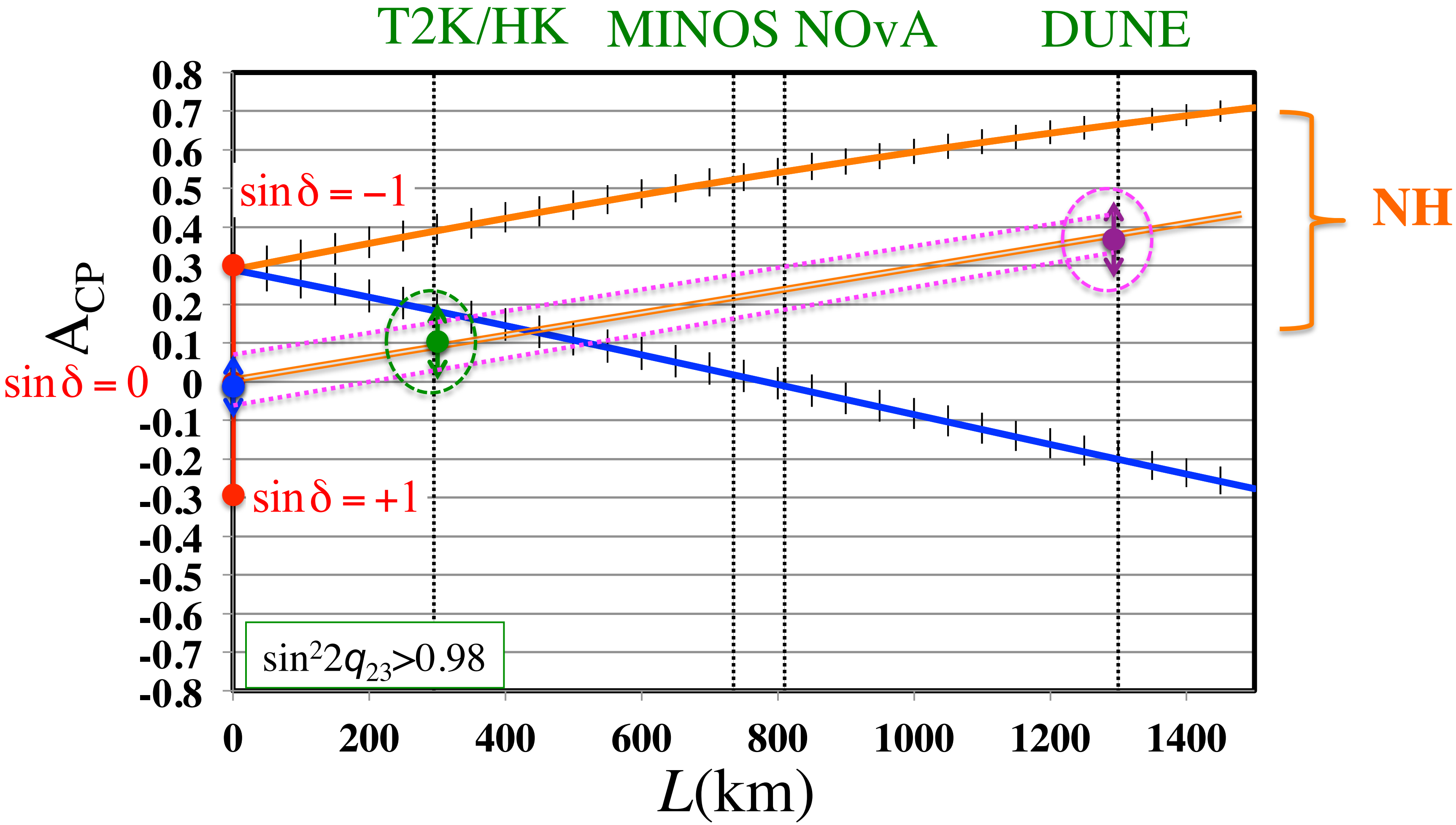
The sign depends on mass hierarchy

Baseline Dependence of Matter effect

$$A_{CP} \sim -0.29\sin\delta \pm 2\left(\frac{L}{L_0}\right)$$



HK+DUNE



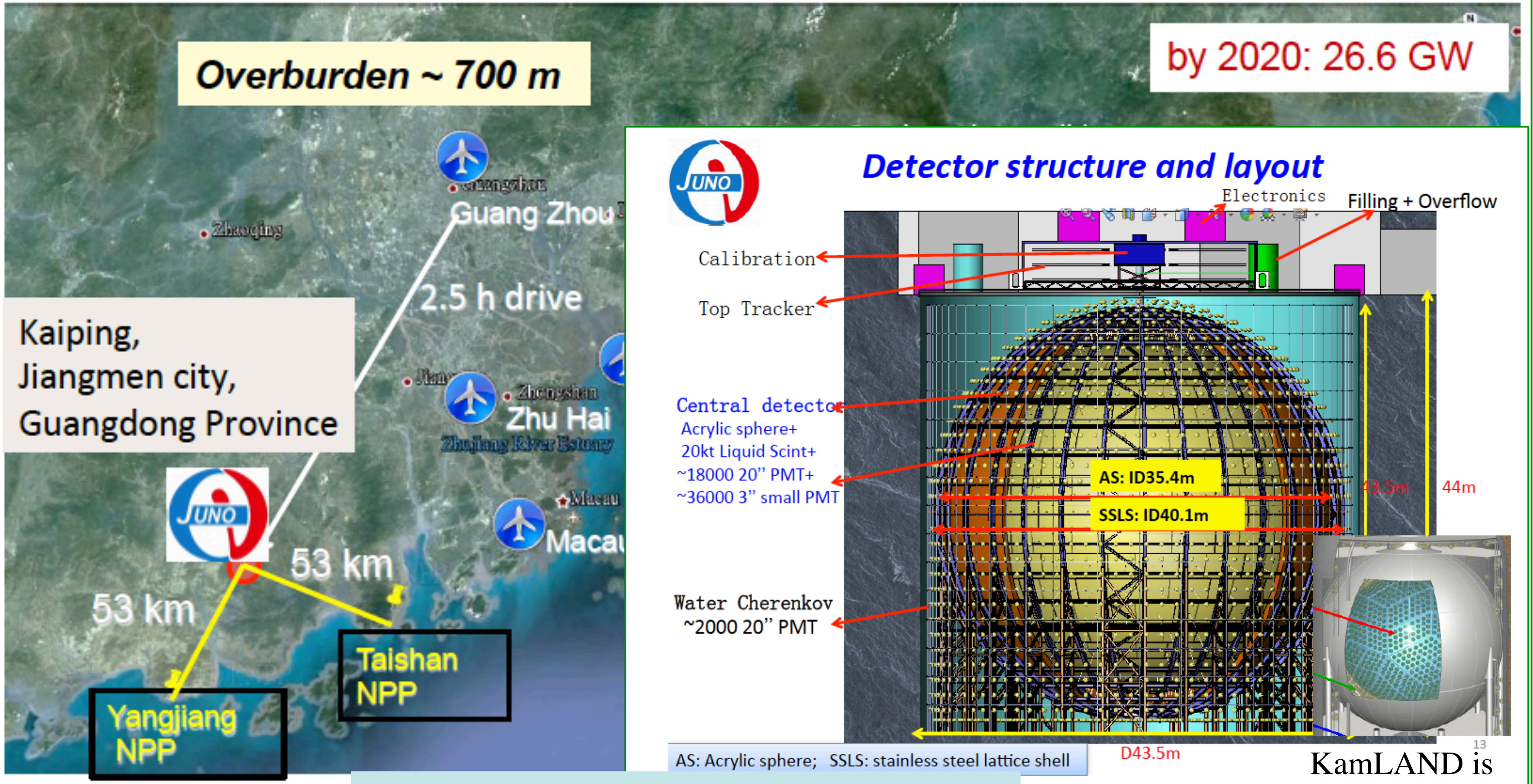
* **good to have the both experiments.**

Δm^2_{23} mass hierarchy:

Yee Bob Hsiung @ 2016.11 NNN16

Location of JUNO

NPP	Daya Bay	Huizhou	Lufeng	Yangjiang	Taishan
Status	Operational	Planned	Planned	Under construction	Under construction
Power	17.4 GW	17.4 GW	17.4 GW	17.4 GW	18.4 GW



KamLAND is this size

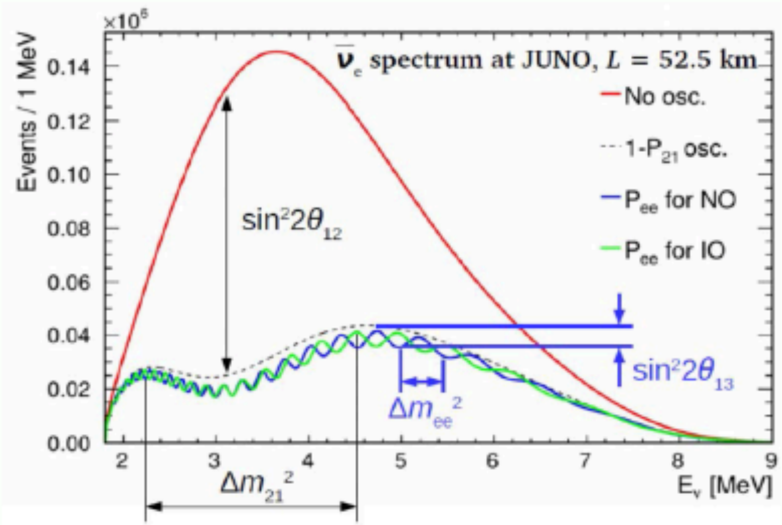
Filling and data taking 2020

Δm^2_{23} mass hierarchy:

JUNO

- The Jiangmen Underground Neutrino Observatory (JUNO) is a multipurpose experiment under construction in China:
  - Rich physics program: neutrino mass hierarchy, sub-% measurement of oscillation parameters, astrophysical neutrinos, geo-neutrinos, atmospheric neutrinos, search for exotic physics… etc.

- Main keys to accomplishing the physics goals:
  - **Optimal baseline**
  - **High statistics**
  - **Superb energy resolution (3% @ 1 MeV)**
  - **Excellent control of energy response systematics**
  - **Background reduction**



Part-I Summary:

* Thanks to the huge experimental efforts, $\theta_{12}, \theta_{23}, \theta_{13}$

  $\Delta m^2_{12}, \quad \left|\Delta \tilde{m}^2_{32}\right|, \quad \left|\Delta \tilde{m}^2_{31}\right|$ have been measured.

* Decisive measurements of δ, M.H. are planned

* There are several tensions.

  ➔ Redundant experiments to check each other are important.

  ➔ New physics might be behind them.

Part-II

Other Quantum Oscillations:

= A collection of various oscillations & mixings =

The origin of the neutrino oscillation is transitions between different flavor neutrinos, such as,

$$\nu_e \Leftrightarrow \nu_\mu$$

In fact, many kinds of transitions take place in various physics phenomena; many of them bear important physics effects. Such important physics can be understood as the same way as neutrino oscillation mechanism.

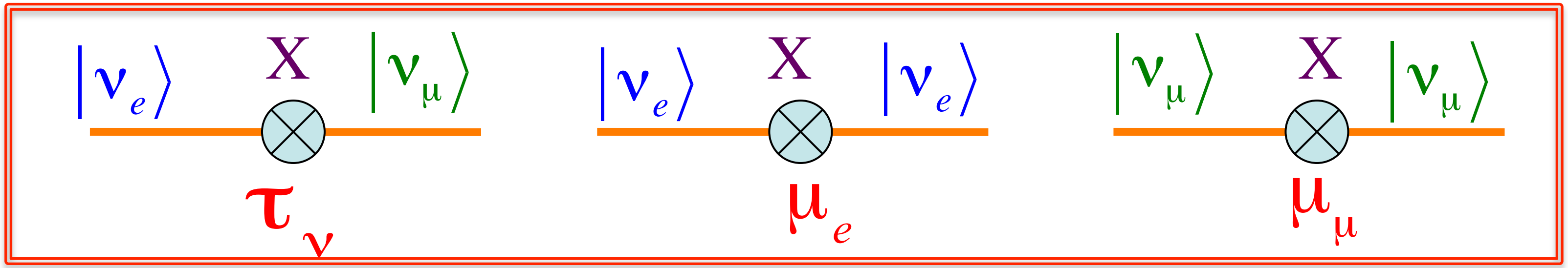
In some cases, abstract concepts, such as Parity, can be understood by a concrete idea of oscillation and mixing.

It should be useful to teach various physics using such unified and concrete point of view.

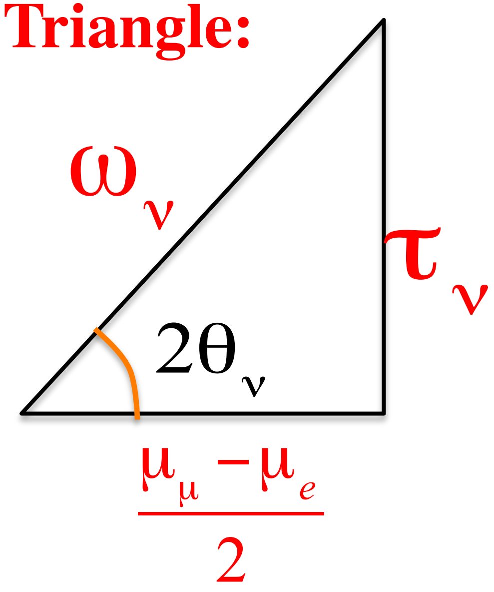
Cabbibo angle θ_C

Neutrino Oscillation case

Something (X) changes ν_μ to ν_e and gives self-transition



Mixing Triangle:



Mass eigenstate:

$$\begin{cases} \nu_1 = \left(\cos\theta_\nu |\nu_e\rangle - \sin\theta_\nu |\nu_\mu\rangle\right)\exp\left[-im_1 t\right] \\ \nu_2 = \left(\sin\theta_\nu |\nu_e\rangle + \cos\theta_\nu |\nu_\mu\rangle\right)\exp\left[-im_2 t\right] \end{cases}$$

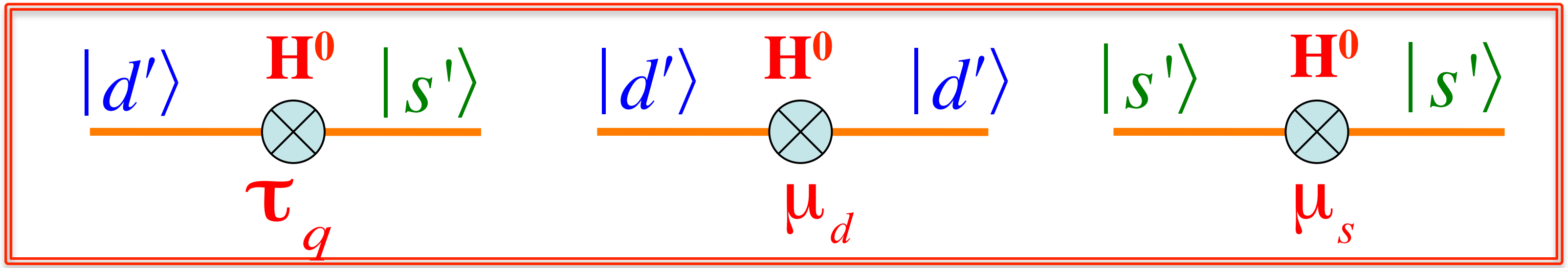
Masses:

$$m_{1,2} = \bar{\mu}_\nu \mp \omega_\nu$$

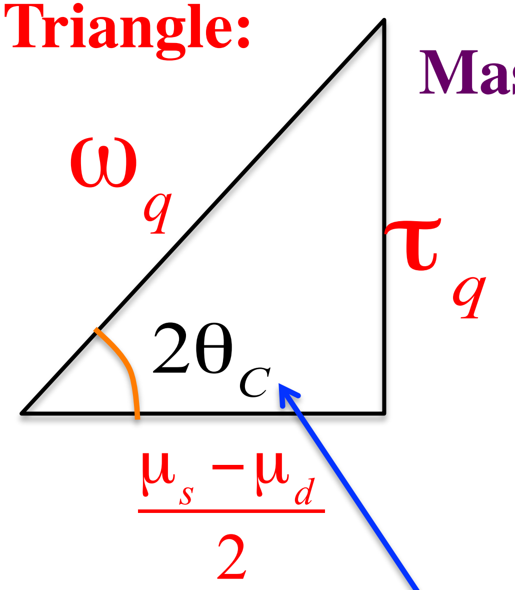
$$\bar{\mu}_\nu = \frac{\mu_e + \mu_\mu}{2}, \quad \omega_\nu = \frac{1}{2}\sqrt{\left(\mu_\mu - \mu_e\right)^2 + 4\tau_\nu^2}$$

Quark case $(\nu_e, \nu_\mu) \rightarrow (d', s')$

Higgs potential (H^0) changes d' to s' and gives self-transition



Mixing Triangle:



Mass eigenstate:

$$\begin{cases} d = \left(\cos\theta_C |d'\rangle - \sin\theta_C |s'\rangle\right)\exp\left[-im_d t\right] \\ s = \left(\sin\theta_C |d'\rangle + \cos\theta_C |s'\rangle\right)\exp\left[-im_s t\right] \end{cases}$$

Masses: $m_{d,s} = \bar{\mu}_q \mp \omega_q$

$$\bar{\mu}_q = \frac{\mu_d + \mu_s}{2}, \quad \omega_q = \frac{1}{2}\sqrt{\left(\mu_s - \mu_d\right)^2 + 4\tau_q^2}$$

Cabbibo angle is quark version of ν mixing angle

Important Difference

neutrino and quark oscillations are two extreme cases of the uncertainty principle.

$$P[t] = \sin^2 2\theta \sin^2 \frac{\Delta m^2}{4E} t$$

Oscillation length for 1GeV particle

$$\lambda_\nu \sim 10^6 m$$

$$\lambda_q \sim 10^{-14} m$$

ν: Measurement of the flavor evolution pattern possible but it is impossible to distinguish ν_1 and ν_2 by measuring masses	**q: It is possible to distinguish d and s by their masses but the oscillation is too quick to observe the evolution of d' and s'**

Weinberg Angle θ_W

The Lagrangian for the interaction of the gauge boson and Higgs fields is written as,

U(1) G SU(3) G Higgs

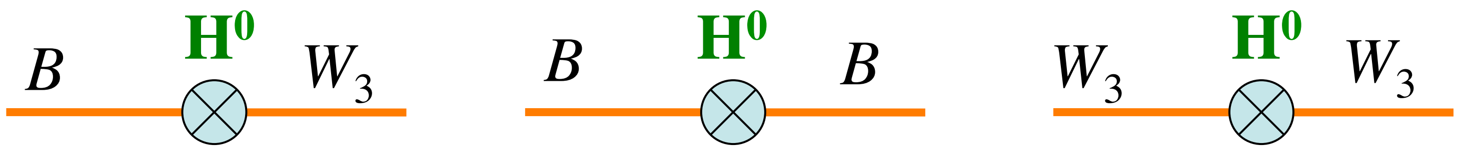
$$\mathcal{L}_{\Phi G} = \frac{1}{4}\left|(g' B^{\mu} + g(\vec{W}^{\mu}\cdot\vec{\sigma}))\Phi\right|^2 \xrightarrow{\text{SSB}}$$

Neutral component

$$\frac{(v_0+h)^2}{8}\left(g^2(|W^+|^2+|W^-|^2) + (g^2 W_3^2 + g'^2 B^2 - gg'(W_3 B + B W_3))\right)$$

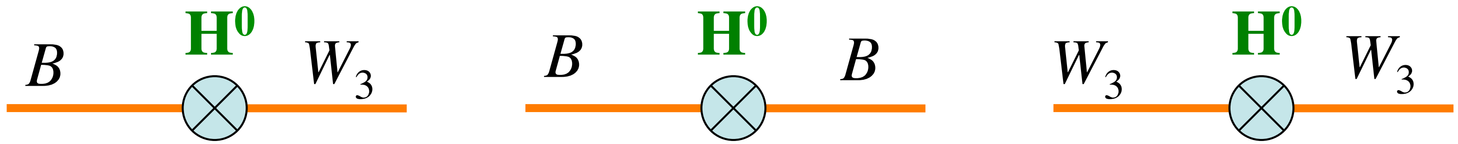
State equation,

$$i\frac{d}{dt}\begin{pmatrix} B \\ W_3 \end{pmatrix} = \frac{v_0}{2\sqrt{g^2+g'^2}}\begin{pmatrix} g'^2 & -gg' \\ -gg' & g^2 \end{pmatrix}\begin{pmatrix} B \\ W_3 \end{pmatrix}$$



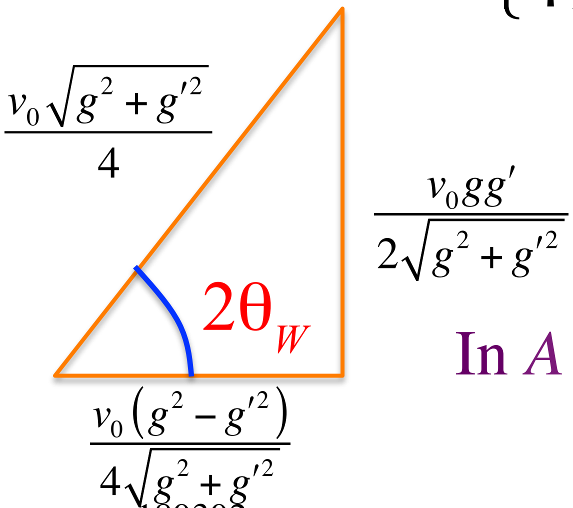
$$-gg' \times \frac{v_0}{2\sqrt{g^2+g'^2}} \qquad g'^2 \times \frac{v_0}{2\sqrt{g^2+g'^2}} \qquad g^2 \times \frac{v_0}{2\sqrt{g^2+g'^2}}$$

Oscillation View of Gauge Bosons



$$-gg' \times \frac{v_0}{2\sqrt{g^2+g'^2}} \qquad g'^2 \times \frac{v_0}{2\sqrt{g^2+g'^2}} \qquad g^2 \times \frac{v_0}{2\sqrt{g^2+g'^2}}$$

Mass eigenstate
$$\begin{cases} \psi_1 = \left(B\cos\theta_W - W_3\sin\theta_W\right)\exp\left[-i\times 0\times t\right] & \Rightarrow \quad A \\ \psi_2 = \left(B\sin\theta_W + W_3\cos\theta_W\right)\exp\left[-iM_Z t\right] & \Rightarrow \quad Z^0 \end{cases}$$



$$M_Z = \frac{v_0}{2}\sqrt{g^2+g'^2}, \quad M_A = 0$$

$$\tan 2\theta_W = \frac{2gg'}{g^2-g'^2} \text{ or } \tan\theta_W = \frac{g'}{g}$$

In A and Z^0, B and W_3 are oscillating very quickly.

$$P\left[B \Leftrightarrow W_3\right] = \sin^2 2\theta_W \sin^2\left[\frac{1}{2}M_Z t\right]$$

Chirality is Oscillating

A problem:

μ_R^- **can be produced by EM interactions.**

$\sim 2\mu s$ **after, it decays weakly.**

$$e^+ + e^- \rightarrow \gamma^* \rightarrow \mu_L^+ + \mu_R^-$$

EM

$$\hookrightarrow e^- + \nu_\mu + \overline{\nu}_e$$

Weak

But why this μ_R^- can decay weakly?

Chirality Oscillation & Muon Decay

Definition of Chirality State:

$$\begin{cases} \psi_R = \dfrac{1+\gamma^5}{2}\begin{pmatrix} u \\ v \end{pmatrix} = \dfrac{u+v}{\sqrt{2}} \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \equiv \dfrac{u+v}{\sqrt{2}} |R\rangle, \\ \psi_L = \dfrac{1-\gamma^5}{2}\begin{pmatrix} u \\ v \end{pmatrix} = \dfrac{u-v}{\sqrt{2}} \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \equiv \dfrac{u-v}{\sqrt{2}} |L\rangle \end{cases}$$

Muon satisfies the Dirac equation

$$\frac{d}{dt}\psi_\mu = -im_\mu \gamma_0 \psi_\mu$$

Wave function can be expressed in chirality basis

$$\frac{d}{dt}\psi_\mu = \dot{C}_R |\mu_R\rangle + \dot{C}_L |\mu_L\rangle$$

The right-hand side of the Dirac equation is expressed as

$$\gamma_0 \psi_\mu = \begin{pmatrix} u \\ -v \end{pmatrix} = C_L |\mu_R\rangle + C_R |\mu_L\rangle$$

Therefore, the Dirac equation can be expressed as

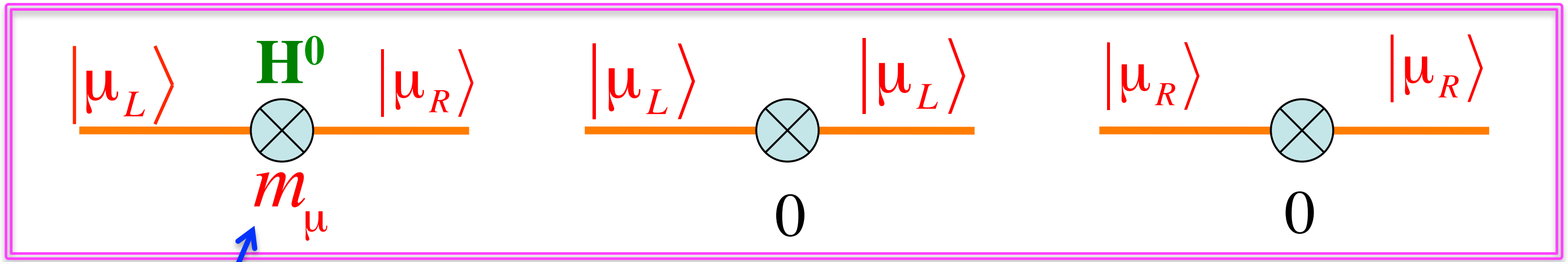
$$\Rightarrow \quad i\left(\dot{C}_L |\mu_L\rangle + \dot{C}_R |\mu_R\rangle\right) = m_\mu \left(C_R |\mu_L\rangle + C_L |\mu_R\rangle\right)$$

or,

$$i\frac{d}{dt}\begin{pmatrix} C_L \\ C_R \end{pmatrix} = \begin{pmatrix} 0 & m_\mu \\ m_\mu & 0 \end{pmatrix}\begin{pmatrix} C_L \\ C_R \end{pmatrix}$$

🡐 The Dirac equation is actually chirality swapping equation

$$i\frac{d}{dt}\begin{pmatrix} C_L \\ C_R \end{pmatrix} = \begin{pmatrix} 0 & m_\mu \\ m_\mu & 0 \end{pmatrix}\begin{pmatrix} C_L \\ C_R \end{pmatrix}$$



Dirac mass

mass eigenstates:

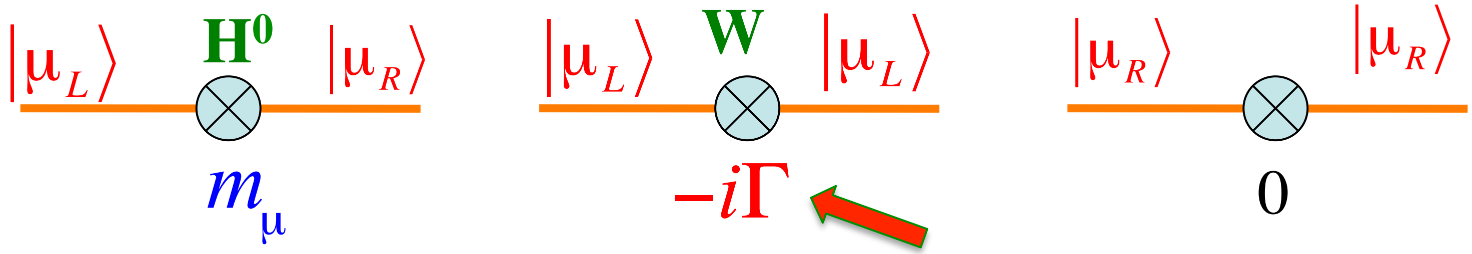
$$\begin{cases} \mu[E>0] = \frac{1}{\sqrt{2}}\left(|\mu_L\rangle + |\mu_R\rangle\right)\exp\left[-im_\mu t\right] \\ \mu[E<0] = \frac{1}{\sqrt{2}}\left(|\mu_L\rangle - |\mu_R\rangle\right)\exp\left[+im_\mu t\right] \end{cases}$$

$\mu_R \Leftrightarrow \mu_L$ **oscillation is taking place**

$$P\left[\mu_R \Leftrightarrow \mu_L\right] = \sin^2 m_\mu t$$

Muon decays weakly while it is in μ_L state

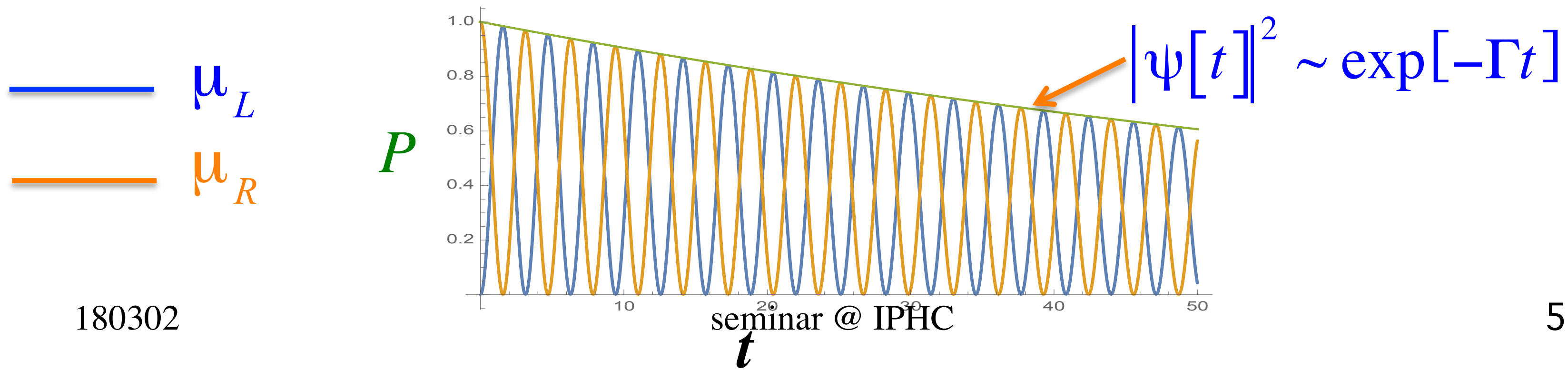
The weak decay effect can be included by putting imaginary amplitude to the μ_L self transition



State equation: $$i\begin{pmatrix}\dot{C}_L\\ \dot{C}_R\end{pmatrix}=\begin{pmatrix}-i\Gamma & m_\mu\\ m_\mu & 0\end{pmatrix}\begin{pmatrix}C_L\\ C_R\end{pmatrix}$$ $$\Gamma=\frac{1}{\tau_\mu}=3\times10^{-10}eV\ll m_\mu$$

The solution for the condition $\psi[0]=|\mu_R\rangle$ **is,**

$$\psi[t]\sim\left(\cos[m_\mu t]|\mu_R\rangle-i\sin[m_\mu t]|\mu_L\rangle\right)\exp\left[-\frac{\Gamma}{2}t\right]$$

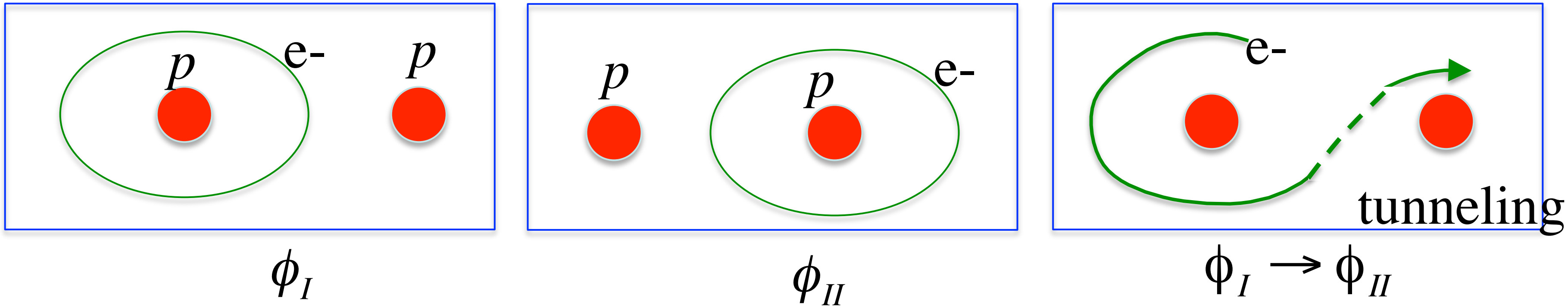


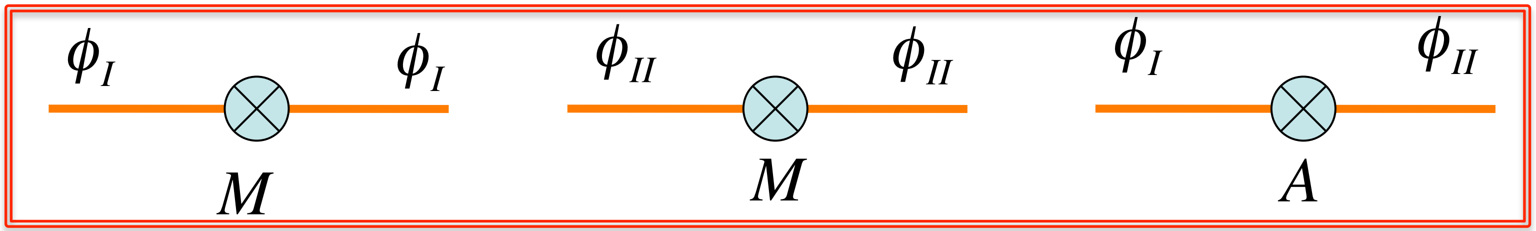
The oscillation and mixing view is useful to understand abstract properties concretely

* Parity
* C-Parity
* Isospin

What is parity? (Feynman's explanation)

An Hydrogen ion H_2^+ has two basis states: ϕ_I and ϕ_{II}

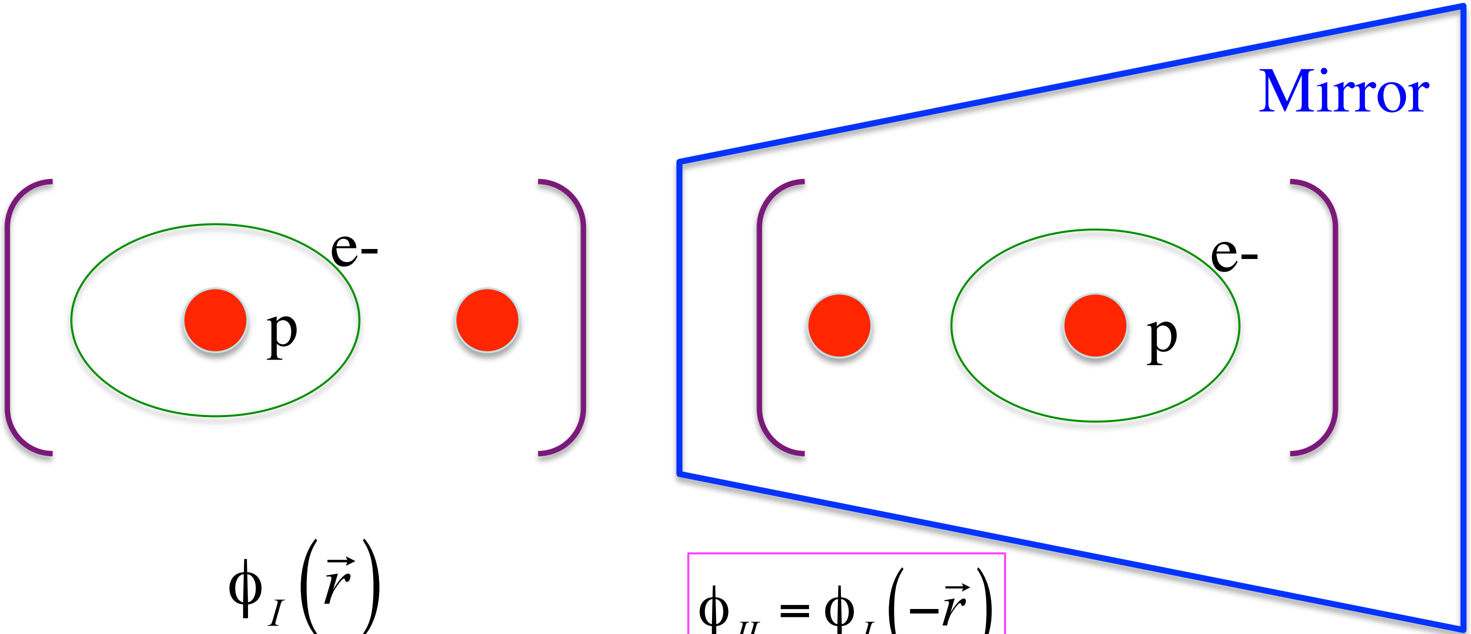




➔ **The energy eigenstates**

$$\begin{cases} \Phi_+ = \dfrac{1}{\sqrt{2}}(\phi_I + \phi_{II})e^{-i(M+A)t} \\ \Phi_- = \dfrac{1}{\sqrt{2}}(\phi_I - \phi_{II})e^{-i(M-A)t} \end{cases}$$

ϕ_I and ϕ_{II} are oscillating



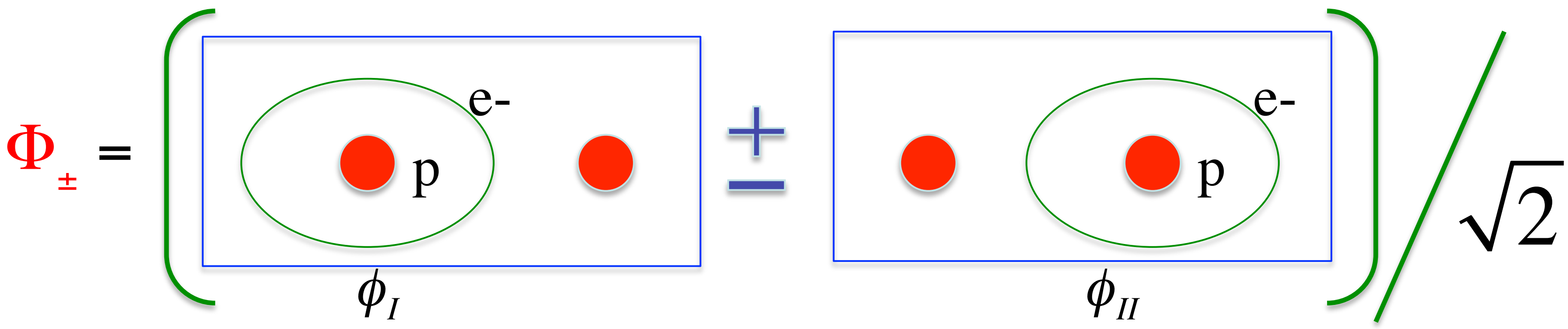
$\phi_I(\vec{r})$

$\phi_{II} = \phi_I(-\vec{r})$

Actually, ϕ_{II} is a mirror image of ϕ_I

Therefore, the energy eigenstates are

$$\begin{cases} \Phi_+(\vec{r}) = \dfrac{1}{\sqrt{2}}\left(\phi_I(\vec{r}) + \phi_I(-\vec{r})\right)\exp\left[-i(M+A)t\right] \\ \Phi_-(\vec{r}) = \dfrac{1}{\sqrt{2}}\left(\phi_I(\vec{r}) - \phi_I(-\vec{r})\right)\exp\left[-i(M-A)t\right] \end{cases}$$



If parity of the mass eigenstates is reversed

Parity = + structure

$$\begin{cases} \Phi_+(-\vec{r}) = \frac{1}{\sqrt{2}}\left(\phi_I(-\vec{r}) + \phi_I(\vec{r})\right)\exp\left[-i(M+A)t\right] = +\Phi_+(\vec{r}) \\ \Phi_-(-\vec{r}) = \frac{1}{\sqrt{2}}\left(\phi_I(-\vec{r}) - \phi_I(\vec{r})\right)\exp\left[-i(M-A)t\right] = -\Phi_-(\vec{r}) \end{cases}$$

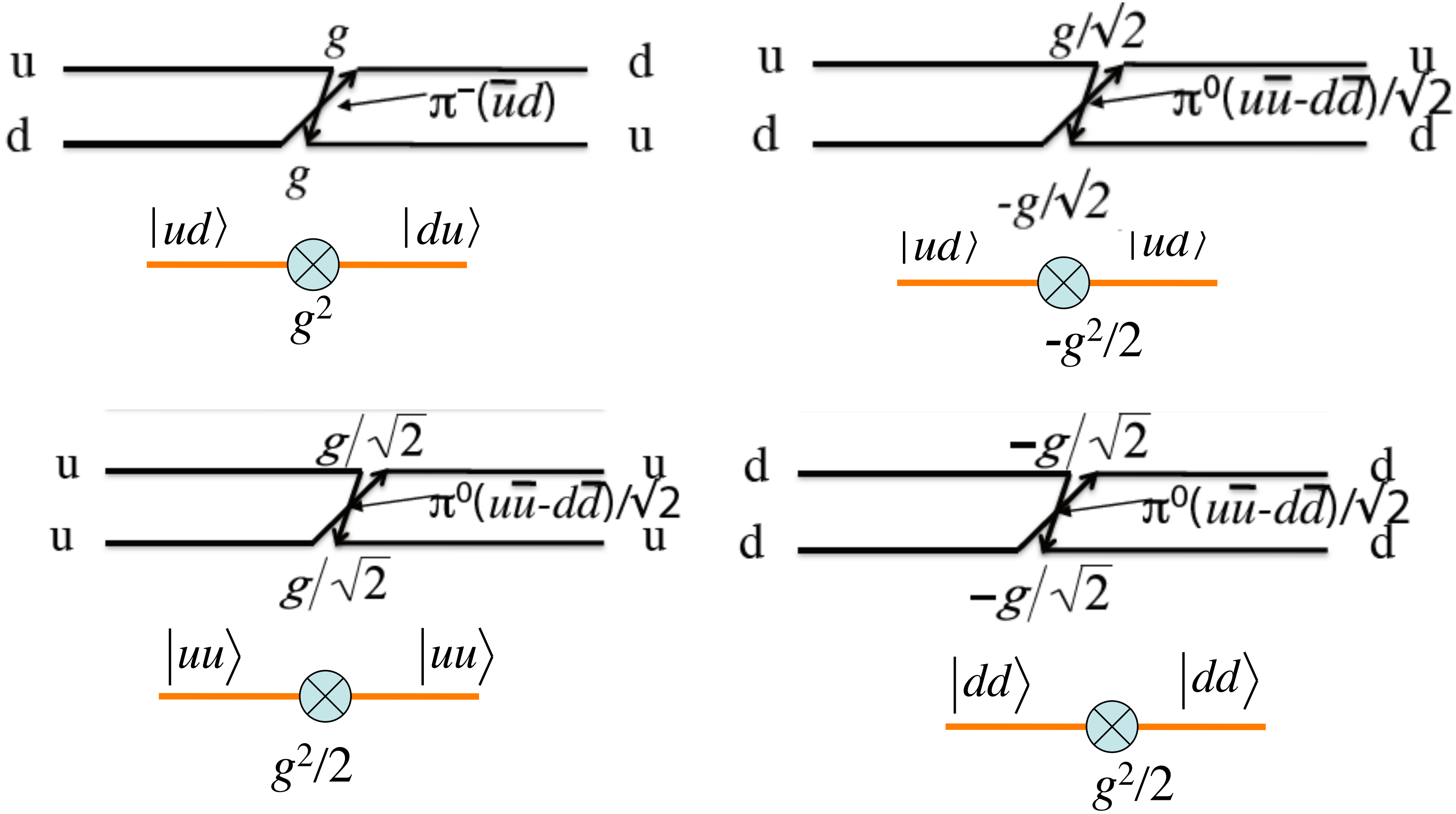
Parity =- structure

=> Energy eigenstates have fixed parities.

Isospin

Isospin

For *u,d* system, pion exchange changes the basis system



State equation

$$\frac{d}{dt}C_{uu} = -i\frac{g^2}{2}C_{uu}, \quad \frac{d}{dt}C_{dd} = -i\frac{g^2}{2}C_{dd}$$

$$\frac{d}{dt}\begin{pmatrix} C_{ud} \\ C_{du} \end{pmatrix} = -i\begin{pmatrix} -g^2/2 & g^2 \\ g^2 & -g^2/2 \end{pmatrix}\begin{pmatrix} C_{ud} \\ C_{du} \end{pmatrix}$$

← **This is the same form as spin dipole moment interaction (cf. 21cm line)**

Mass eigenstates

$$\begin{cases} \psi_U = |uu\rangle \exp\left[-i\left(M_0 + \frac{1}{2}g^2\right)t\right] \\ \psi_D = |dd\rangle \exp\left[-i\left(M_0 + \frac{1}{2}g^2\right)t\right] \\ \psi_+ = \frac{|ud\rangle + |du\rangle}{\sqrt{2}} \exp\left[-i\left(M_0 + \frac{1}{2}g^2\right)t\right] \\ \psi_- = \frac{|ud\rangle - |du\rangle}{\sqrt{2}} \exp\left[-i\left(M_0 - \frac{3}{2}g^2\right)t\right] \end{cases}$$

Analogy to the Spin combination

⇩

3 same mass state: I=1 state

$$|\Sigma^+\rangle = |uu\rangle|s\rangle$$

$$|\Sigma^-\rangle = |dd\rangle|s\rangle$$

$$|\Sigma^0\rangle = \frac{|ud\rangle + |du\rangle}{\sqrt{2}}|s\rangle$$

singlet state: I=0 → $|\Lambda\rangle = \frac{|ud\rangle - |du\rangle}{\sqrt{2}}|s\rangle$

There are a lot more interesting oscillations and mixings . . .

Name	Origin	Transition	Energy eigenstate
Neutrino Oscillation	X	$\vert\nu_e\rangle \Leftrightarrow \vert\nu_\mu\rangle \Leftrightarrow \vert\nu_\tau\rangle$	$\nu_1,\ \nu_2,\ \nu_3$
Cabbibo Angle	Higgs	$\vert d'\rangle \Leftrightarrow \vert s'\rangle$	$d,\ s$
Chirality Osc.	Higgs	$\vert L\rangle \Leftrightarrow \vert R\rangle$	$\vert R\rangle \pm \vert L\rangle$
Majorana Neutrino	X	$\vert\nu_L\rangle \Leftrightarrow \vert\overline{\nu}_R\rangle$	$\vert\nu_L\rangle \pm \vert\overline{\nu}_R\rangle$
Seesaw Mechanism	X	$\vert\nu_R\rangle \Leftrightarrow \vert\overline{\nu}_L\rangle,\ \vert\nu_L\rangle \Leftrightarrow \vert\nu_R\rangle$	$\nu = \vert\nu_L\rangle - \vert\overline{\nu}_R\rangle,\ N = \vert\nu_R\rangle + \vert\overline{\nu}_L\rangle$
Weinberg angle	Higgs	$W_3 \Leftrightarrow B$	$\gamma,\ Z^0$
Hydrogen 21 cm line	$\vec{\mu}_p \cdot \vec{\mu}_e$	$\vert p(\Uparrow)e(\Downarrow)\rangle \Leftrightarrow \vert p(\Downarrow)e(\Uparrow)\rangle$	$\vert\Uparrow\Downarrow\rangle \pm \vert\Downarrow\Uparrow\rangle$
$\pi^+ - \rho^+$ mass difference	Strong	$\vert\Uparrow\Downarrow\rangle_S \Leftrightarrow \vert\Downarrow\Uparrow\rangle_S$	$\pi^+,\ \rho^+$
CPV	Weak	$K_{CP+} \Leftrightarrow K_{CP-}$	$K_1,\ K_2$
Hydrogen Ion (H_2^+)	tunneling	$\vert(pe^-)p\rangle \Leftrightarrow \vert p(e^-p)\rangle$	$\vert(pe^-)p\rangle \pm \vert p(e^-p)\rangle$
Positronium	EM	$\vert e^+e^-\rangle \Leftrightarrow \vert e^-e^+\rangle$	o-Ps, p-Ps
Isospin	S	$\vert ud\rangle \Leftrightarrow \vert du\rangle,\ \vert u\overline{u}\rangle \Leftrightarrow \vert d\overline{d}\rangle$	$(\Lambda,\ \Sigma),\ (\rho,\omega)$
Baryon Color	Strong	$\vert RGB\rangle \Leftrightarrow \vert GRB\rangle$	$\vert RGB\rangle - \vert RBG\rangle + \vert BRG\rangle - \cdots$
ρ^0, ω, ϕ structure	Strong	$\vert u\overline{u}\rangle \Leftrightarrow \vert d\overline{d}\rangle \Leftrightarrow \vert s\overline{s}\rangle$	$\rho^0,\ \omega,\ \phi$
Spin precession in $\vec{B}$	$\vec{\mu}\vec{B}$	$\vert\Uparrow\rangle \Leftrightarrow \vert\Downarrow\rangle$	$\vert\Uparrow\rangle_\theta$
Deuteron	S	$\vert pn\rangle \Leftrightarrow \vert np\rangle$	$(\vert pn\rangle - \vert np\rangle)\ \vert\Uparrow\Uparrow\rangle$
sp^3 hybrid orbit	EM	$\psi_{2S} \Leftrightarrow \psi_{2P_i}$	$\psi_{2S} \pm \psi_{2P_x} \pm \psi_{2P_y} \pm \psi_{2P_z}$
⋮	⋮	⋮	⋮

Part-II, Summary:

* The same mechanism as neutrino oscillation is working in various other places and is playing important physics roles.

* Many important physics can be understood by analogy of neutrino oscillation mechanism (or vice versa).

* Abstract properties, such as parity, etc. can be attributed to the structure of the mixings.

* It should be educative to teach such ideas.