B_01: Development of the software package CKMfitter

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on behalf of CKMfitter-FJPPL collaboration

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Outline

1. Introduction
2. Averaging ϕ_3/γ for the input of CKMfit
3. Determination of Wilson Coefficients
4. Summary and Prospects

1. Introduction

- The software package CKMfitter is being developed to determine basic physics parameters such as (ρ,η) in the unitarity triangle by the simultaneous fit to various experimental measurements.

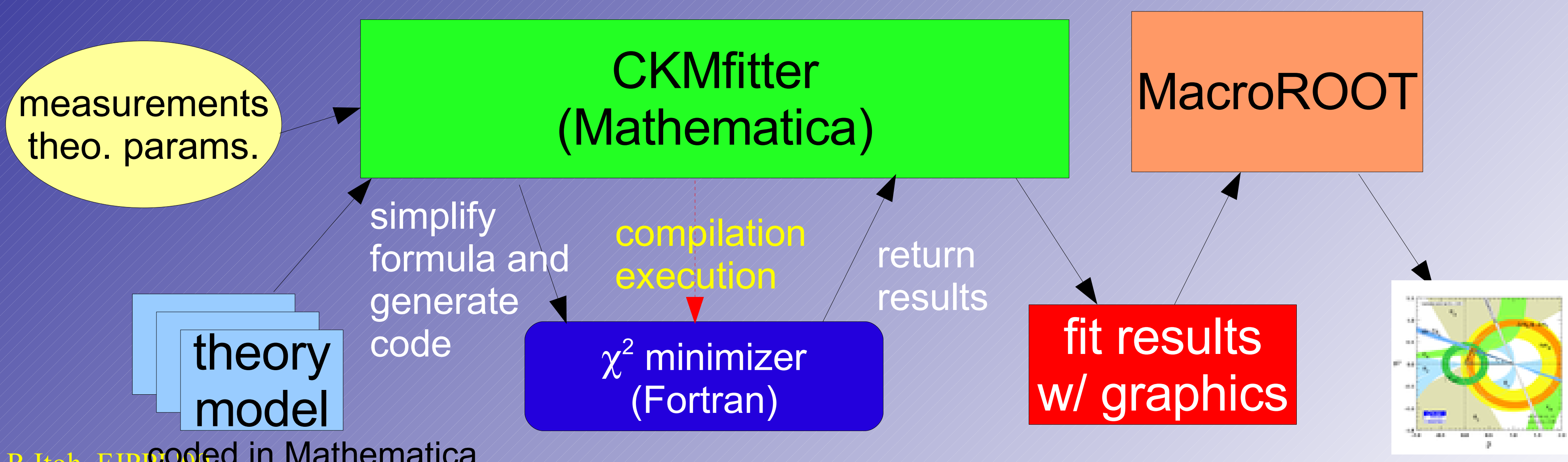
- CKMfitter was first written in Fortran ~10 years ago, but recently rewritten completely in Mathematica for the speed up. (>10 times faster).

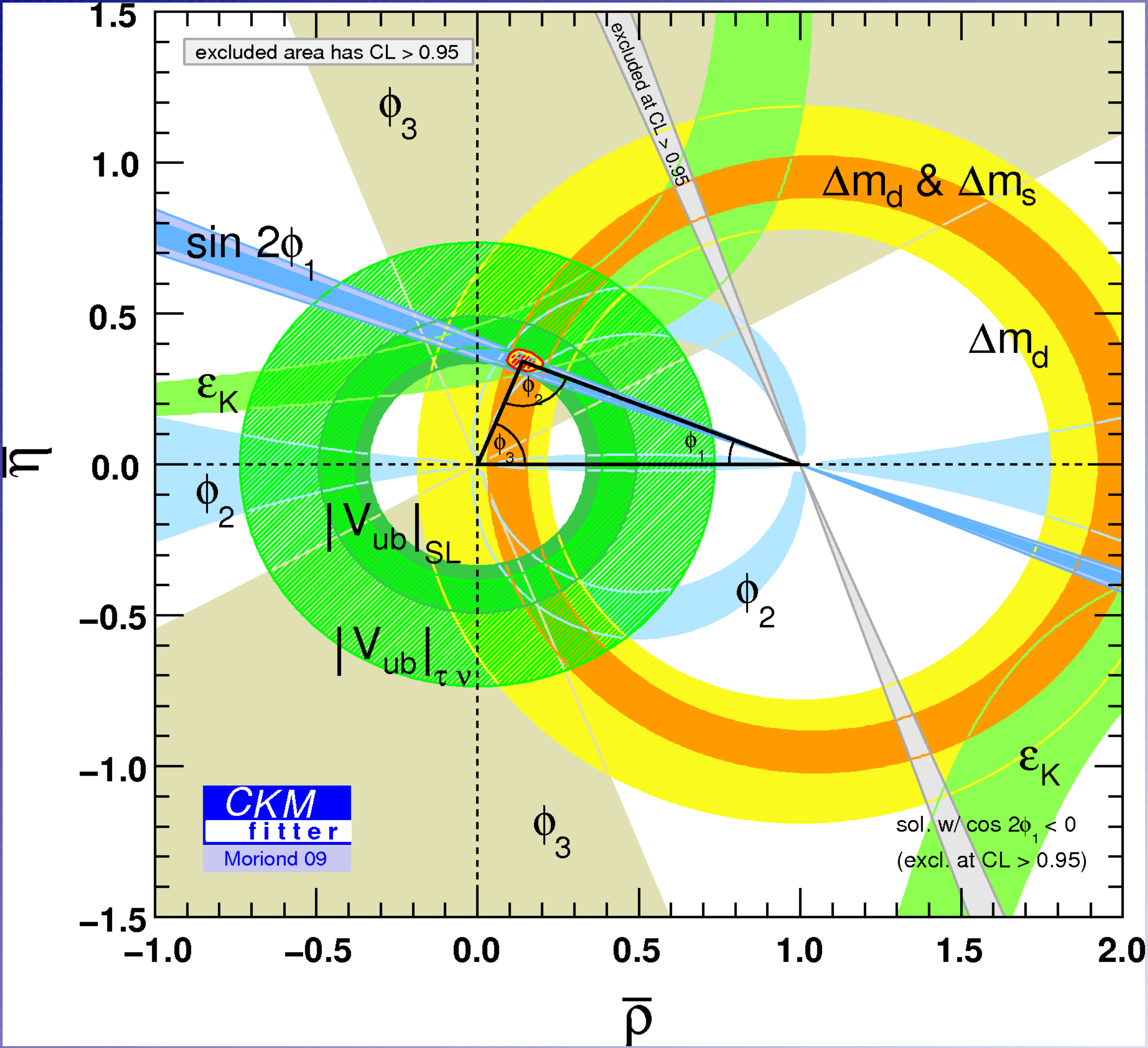
- The enhancement of CKMfitter is going on under FJPPL framework.
    * New treatment of ϕ_3/γ average in the CKM fit.
    * Determination of Wilson coefficients by the global fit.

CKMfitter : χ^2 minimizer originally developed to constrain unitarity triangle by accepting various experimental measurements ($\sin 2\phi_1$, ϕ_2, ϕ_3, V_{ub}, Δm_d, Δm_s.......)

- Statistical approach : Frequentist (Rfit)

  See Eur. Phys. J. C41, 1 (2005) for detail
- Originally coded in Fortran and specialized in calculating unitarity triangle constraints. Minimizer : MINUIT in CERNLIB
- Rewritten in Mathematica in 2006 for speed up.
  * Analytic simplification of theoretical formula helps to reduce the calculation time. (>O(10) faster than Fortran version).
  * Generalized χ^2 minimizer

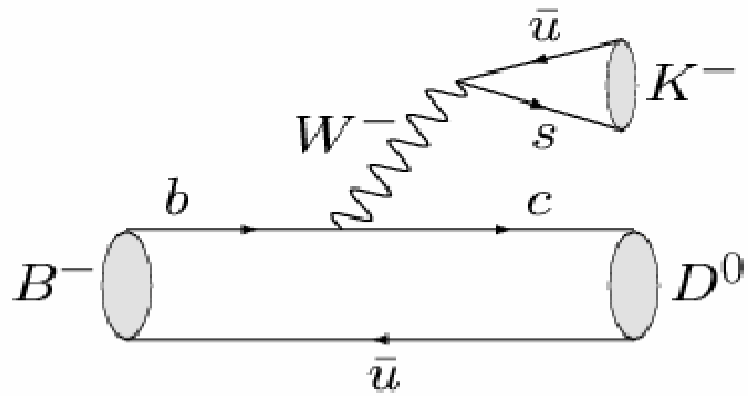




This plot was shown by Prof. Kobayashi in his Nobel prize lecture!

2. Averaging ϕ_3/γ

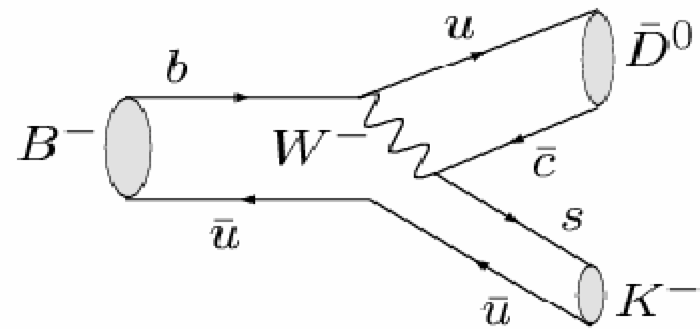
- Angle ϕ_3/γ in unitarity triangle is measured by the interference between $B \to D^0 K^-$ and $B \to \overline{D}{}^0 K^-$ at B-factories



color allowed

$B^- \to D^0 K^- \sim V_{cb} V_{us}^*$

$\sim A\lambda^3$



color suppressed

$B^- \to \overline{D}{}^0 K^- \sim V_{ub} V_{cs}^*$

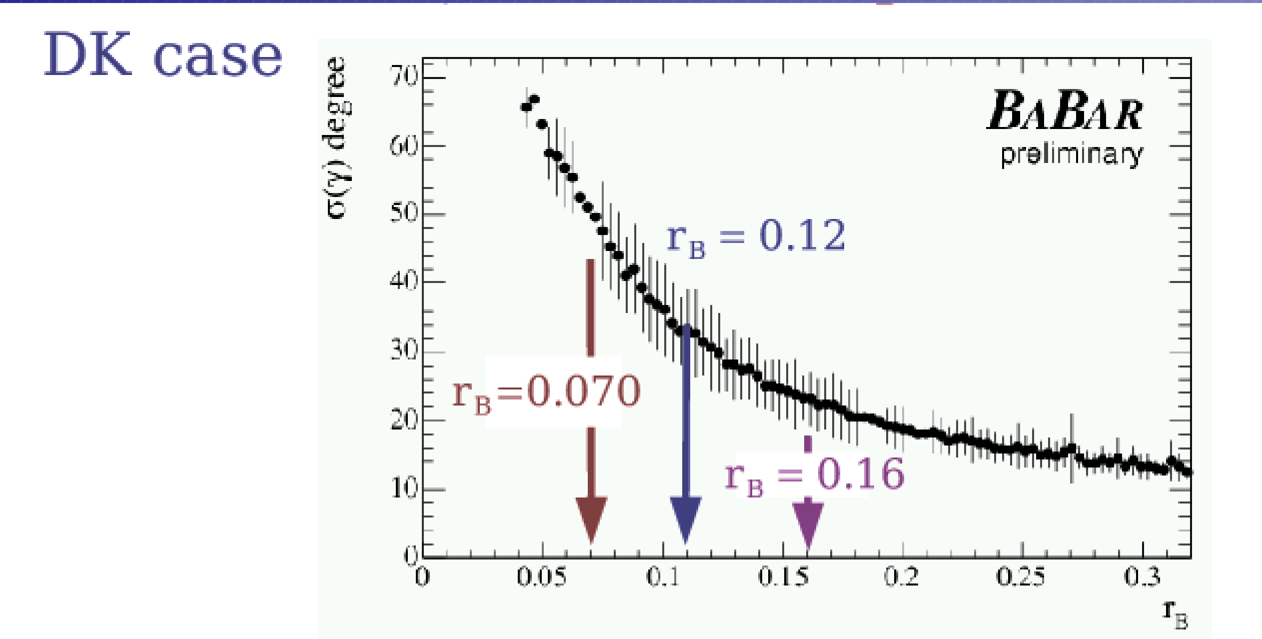
$\sim A\lambda^3(\rho + i\eta)$

$$r_B = \frac{|A_{\text{suppressed}}|}{|A_{\text{favoured}}|} \sim \frac{|V_{ub} V_{cs}^*|}{|V_{cb} V_{us}^*|} \times [\text{color supp}] = 0.1\text{-}0.2$$

- Three methods to measure ϕ_3/γ:
  1. GLW method : $D = D_{CP}$
  2. ADS method : $D = D_{WS}$
  3. GGSZ method : Dalitz analysis for $D \to K_s \pi^+ \pi^-, K_s K^+ K^-$

ϕ_3/γ and r_B

- error in ϕ_3/γ has a strong dependence on r_B



- If r_B is small, fitted r_B value is biased towards higher values which results in the biased smaller error in ϕ_3/γ.
  → undercoverage problem

- Combined treatment of ϕ_3/γ and r_B is necessary to average the measurements.
  → GLW/ADS/GGSZ implies 31 observables + 9 nuisance parameters →→ very complicated!

Methodology of averaging

a) If the measurements follow the standard χ^2 law,
the PDF is the standard cumulative distribution function(prob).
b) If not, the PDF depends on the nuisance parameters
$\mu = (r_B, \delta_B, ...)$: PDF is determined by toy MC experiments.

* Choice of nuisance parameters for b)
- μ_{hat} method: nuisance parameters are fixed at the best parameter values for the given ϕ_3/γ.
- Supremum method : choose least favored values of nuisance parameters.

Apparently case b) for ϕ_3/γ.

Try to compare both in ϕ_3/γ determination .

Coverage test

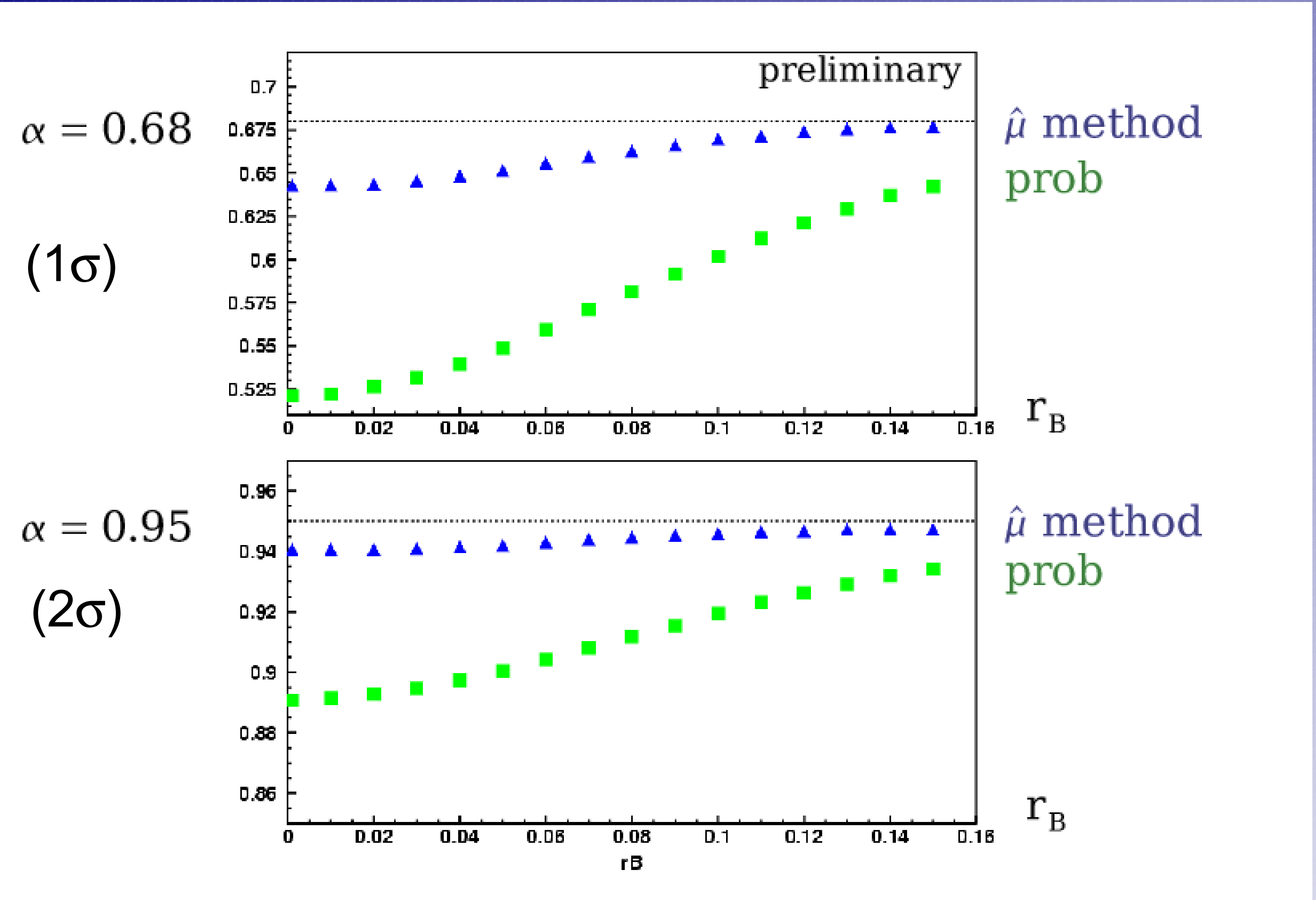
Compare the coverage of cases a) and b)

Method: toy MC

1. Fix some true value of
   - ϕ_3/γ: parameter of interest
   - $\mu = (r_B, \delta_B)$: nuisance parameters
2. Generate a large sample of toy experiments
3. For each experiment, compute CL(ϕ_3/γ) at the ϕ_3/γ = true ϕ_3/γ.
4. The fraction of experiments with CL(ϕ_3/γ)> given a gives the coverage.

- Scan nuisance parameter r_B in GGSZ analysis and compare the coverage.
  * $\phi_3/\gamma = 60^o$, $\delta_B = 140^o$, scan range for r_B : {0.001, 0.15}
  * estimate CL($\mu=\mu_{true}$) in the toyMC sample and then calculate the fraction of experiments with CL($\mu=\mu_{true}$)>α.

Scan results:



- Large undercoverage by the standard “prob” is observed for smaller r_B.
- Undercoverage is still seen in μ_{hat} method also.

Coverage test with full inputs (31 obs. + 9 nuisance par's)

Coverage results

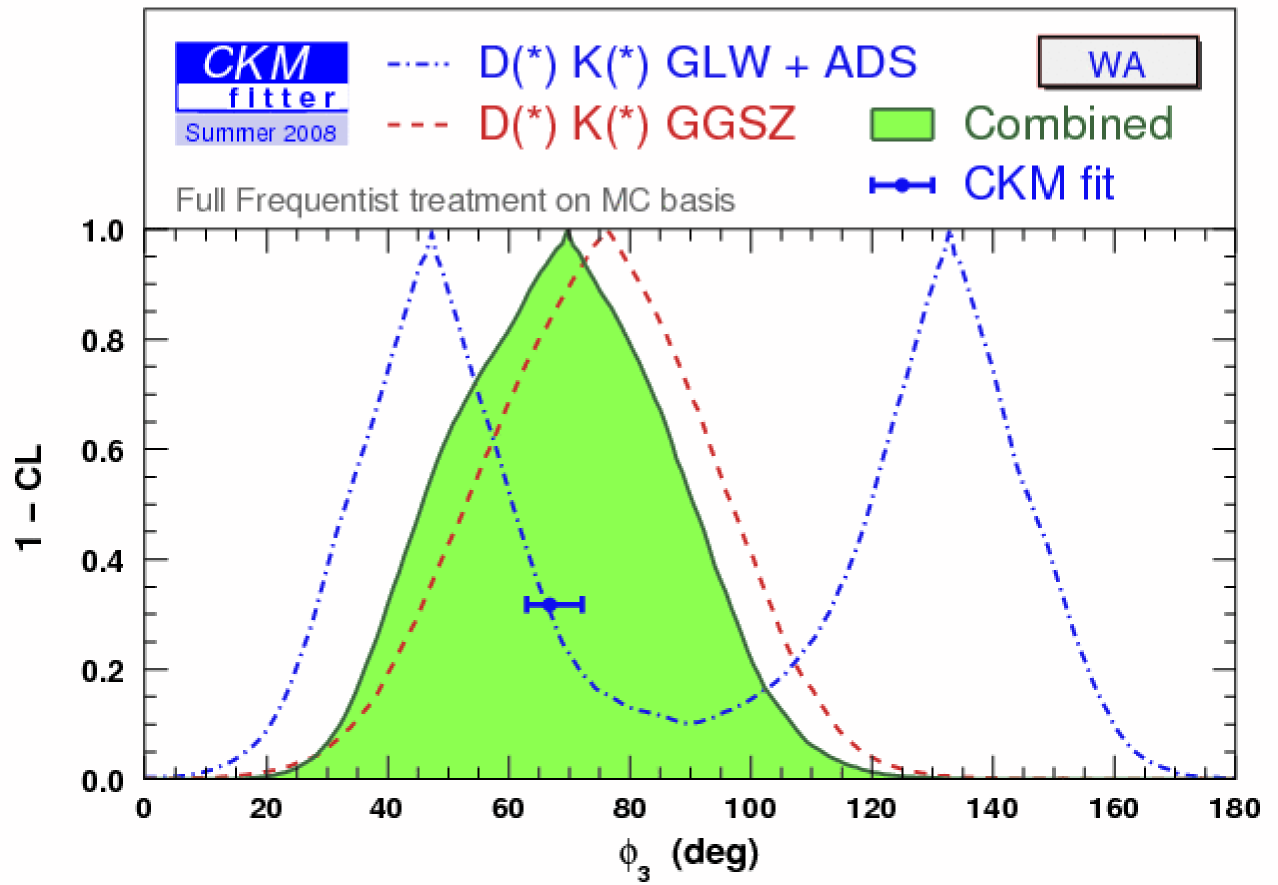
method	CL	CKM08 values Best Fit scenario	r_B's = 0.04 Small Ratio scenario
Prob	68%	$(60 \pm 1)\%$	$(41 \pm 1)\%$
	95%	$(90.5 \pm 0.5)\%$	$(77.8 \pm 0.8)\%$
	99.73%	$(99.2 \pm 0.2)\%$	$(96.2 \pm 0.4)\%$
$\hat{\mu}$	68%	$(66.7 \pm 0.9)\%$	$(55.7 \pm 0.9)\%$
	95%	$(94.4 \pm 0.4)\%$	$(91 \pm 0.5)\%$
	99.73%	$(99.80 \pm 0.09\%$	$(99.5 \pm 0.2)\%$
supremum	68%	$(87.5 \pm 0.7)\%$	$(77 \pm 1)\%$
	95%	$(98.9 \pm 0.2)\%$	$(95.9 \pm 0.6)\%$
	99.73%	$(99.92 \pm 0.05)\%$	$(99.6 \pm 0.2)\%$

We use this method although overcoverage is observed.

Current constraint on ϕ_3/γ

Combined constraint on γ

$\gamma = (70^{+27}_{-30})^\circ$ (direct) vs. $\gamma = (67.8^{+4.2}_{-3.9})^\circ$ (indirect)



3. Determination of Wilson coefficients

- In the framework of CKM fit, the NP effect is searched for in the $B_{d,s}$-$B_{d,s}$ mixing diagram ($\Delta B=2$ transition).

  i.e. comparison of ρ–η constraints by (β/ϕ_1, α/ϕ_2, $\Delta m_{d,s}$) and by (γ/ϕ_3, V_{ub})

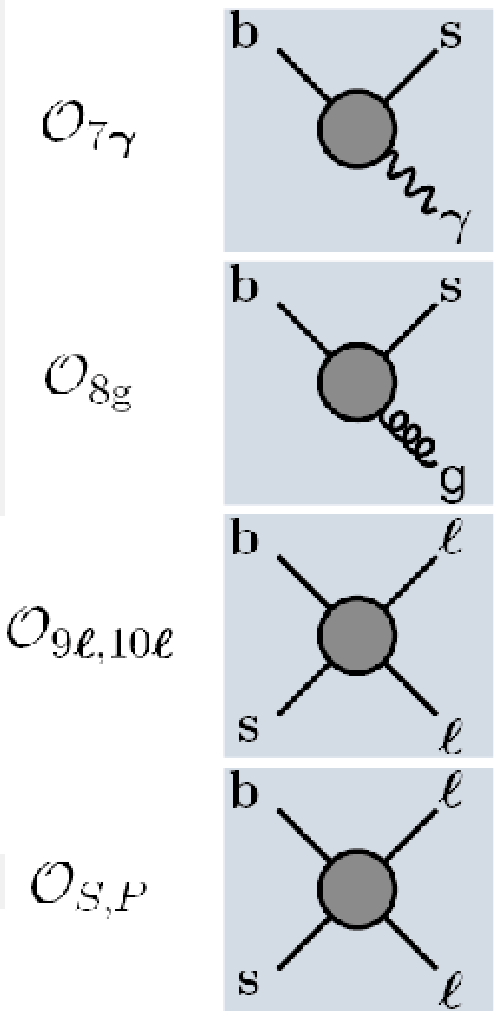
- A complementary NP search can be performed by studying the FCNC transitions like $B\to X_s\gamma$ and $B\to X_s l^+l^-$ which are governed by Wilson coefficients C_7, C_9 and C_{10} ($\Delta F=1$ FCNC).

- The determination of Wilson coefficients using various FCNC decays simultaneously in a similar manner as that for CKM fit (global fit) can be a sensitive probe to NP by comparing with the SM expectations.

S.T'Jampens

ΔF=1 FCNC: b→s transitions and OPE

G. Hiller - hep-ph/0308180



Describe *b*→*s* transitions by an effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V^*_{\mathbf{ts}} V_{\mathbf{tb}} \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)$$

- Long Distance:
  - Operators O_i
- Short Distance:
  - Wilson coef. C_i

New physics shows up as modified C_i,

(or as new operators)

Implementation of Wilson coefficient fit in CKMfitter

- Belle's measurement of A_{FB} in $B \to K^*ll$ already gives a good constraints to Wilson coefficients.
- The extension of this approach is considered by including other measurements together in the fit.

- Inputs
  * $Br(B \to K^*\gamma)$
  * $Br(B \to K^*\mu^+\mu^-)$
  * $A_{FB}(B \to K^*l^+l^-)$ as a function of q^2
- Free parameters : Wilson Coefficients C_7, C_9 and C_{10}.

Code as an add-on theory model for CKMfitter written in Mathematica.

Theory model

- Need to implement "consistent" theory models for all the measurements used in the fit.

- The models described in the paper by A.Ali, et al. (Phys. Rev. D 61, 074024 (2000)) are adoped.
  * A little bit obsolete, but it gives all calculations for $Br(B \to K^*\gamma)$, $Br(B \to K^*\mu^+\mu^-)$, and $A_{FB}(B \to K^*l^+l^-)$ in a consistent way and is a good starting point for the trial.

- Treatment of Wilson coefficients in the model:

$$C_7^{eff} = C_7$$

$$C_9^{eff}(s) = C_9 + Y(s)$$

$$C_{10}$$

1) Br(B→K*γ)

$$\Gamma(B\to K^*\gamma)=\frac{G_F^2\alpha|V_{ts}^*V_{tb}|^2}{32\pi^4}m_b^2m_B^3$$

$$\times(1-m_{K*}^2/m_B^2)^3|C_7^{\text{eff}}|^2|T_1(0)|^2,$$

2) Br(B→K*μμ)

$$\mathcal{B}(B\to K^*\mu^+\mu^-)=a_{K*}^{(nr)}|C_7^{\text{eff}}|^2+b_{K*}^{(nr)}|C_9|^2+c_{K*}^{(nr)}|C_{10}|^2$$

$$+d_{K*}^{(nr)}C_7^{\text{eff}}C_9+e_{K*}^{(nr)}C_7^{\text{eff}}+f_{K*}^{(nr)}C_9$$

$$+g_{K*}^{(nr)}. \qquad (6.9)$$

* a_K*-g_K* : Form factors calculated using LCSR

	$a_{K*}^{(nr)}$	$b_{K*}^{(nr)}$	$c_{K*}^{(nr)}$	$d_{K*}^{(nr)}$	$e_{K*}^{(nr)}$	$f_{K*}^{(nr)}$	$g_{K*}^{(nr)}$
FF(central)	21.295	0.502	0.500	3.530	1.434	0.413	0.148
FF(max)	28.183	0.630	0.633	4.577	1.859	0.520	0.183
FF(min)	16.795	0.417	0.416	2.864	1.164	0.343	0.125

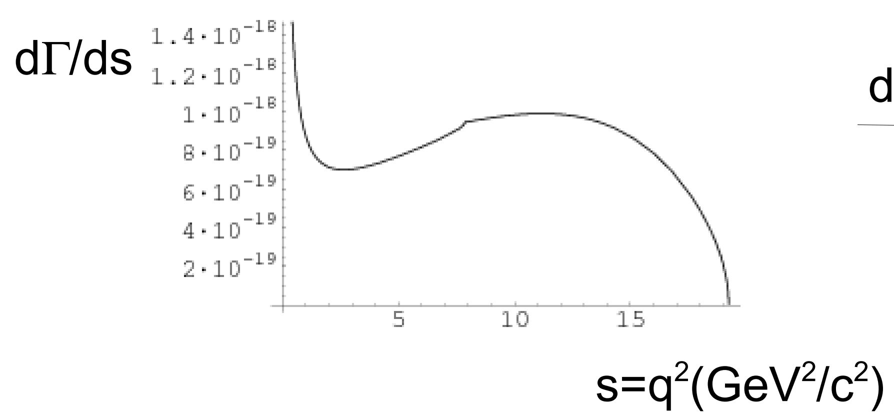
3) $A_{FB}(B \to K^* ll)$

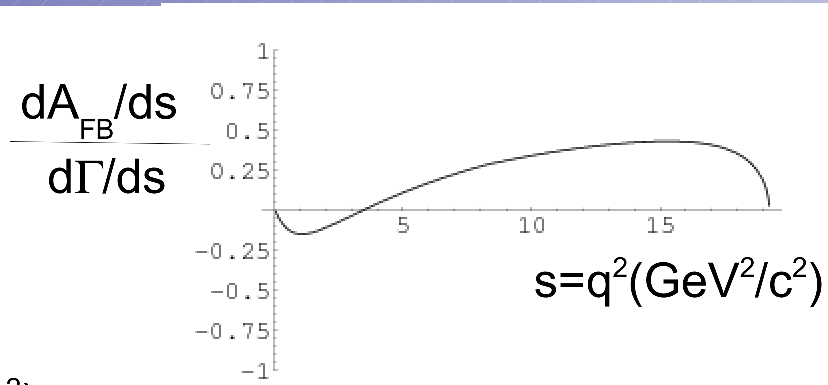
$$\frac{dA_{FB}}{d\hat{s}} = \frac{G_F^2 \alpha^2 m_B^5}{2^{10}\pi^5} |V_{ts}^* V_{tb}|^2 \hat{s}\hat{u}(\hat{s})^2 [\mathrm{Re}(BE^*) + \mathrm{Re}(AF^*)]$$

$$= \frac{G_F^2 \alpha^2 m_B^5}{2^{8}\pi^5} |V_{ts}^* V_{tb}|^2 \hat{s}\hat{u}(\hat{s})^2 C_{10} \left[\mathrm{Re}(C_9^{\mathrm{eff}}) V A_1 + \frac{\hat{m}_b}{\hat{s}} C_7^{\mathrm{eff}} (V T_2 (1 - \hat{m}_{K^*}) + A_1 T_1 (1 + \hat{m}_{K^*})) \right].$$

Normalized A_{FB} = Experimental inputs: $$\frac{d\bar{A}_{FB}}{d\hat{s}} = \frac{dA_{FB}}{d\hat{s}} \bigg/ \frac{d\Gamma}{d\hat{s}}$$

Output of coded model (C_7, C_9, C_{10} = SM values)





Confirmed to reproduce the results in the paper

4) Form factors

Based on light corn sum rule (LCSR) approach

Parametrization : $F(\hat{s}) = F(0)\exp(c_1\hat{s} + c_2\hat{s}^2 + c_3\hat{s}^3).$

parameters for K*ll (A_1, A_2, A_0, V, T_1, T_2, T_3)

central value

	f_+	f_0	f_T	A_1	A_2	A_0	V	T_1	T_2	T_3
$F(0)$	0.319	0.319	0.355	0.337	0.282	0.471	0.457	0.379	0.379	0.260
c_1	1.465	0.633	1.478	0.602	1.172	1.505	1.482	1.519	0.517	1.129
c_2	0.372	−0.095	0.373	0.258	0.567	0.710	1.015	1.030	0.426	1.128
c_3	0.782	0.591	0.700	0	0	0	0	0	0	0

maximum

	f_+	f_0	f_T	A_1	A_2	A_0	V	T_1	T_2	T_3
$F(0)$	0.371	0.371	0.423	0.385	0.320	0.698	0.548	0.437	0.437	0.295
c_1	1.412	0.579	1.413	0.557	1.083	1.945	1.462	1.498	0.495	1.044
c_2	0.261	−0.240	0.247	0.068	0.393	0.314	0.953	0.976	0.402	1.378
c_3	0.822	0.774	0.742	0	0	0	0	0	0	0

minimum

	f_+	f_0	f_T	A_1	A_2	A_0	V	T_1	T_2	T_3
$F(0)$	0.278	0.278	0.300	0.294	0.246	0.412	0.399	0.334	0.334	0.234
c_1	1.568	0.740	1.600	0.656	1.237	1.543	1.537	1.575	0.562	1.230
c_2	0.470	0.080	0.501	0.456	0.822	0.954	1.123	1.140	0.481	1.089
c_3	0.885	0.425	0.796	0	0	0	0	0	0	0

* maximum/minimum : treated in “Rfit”

Fit to experimental measurements

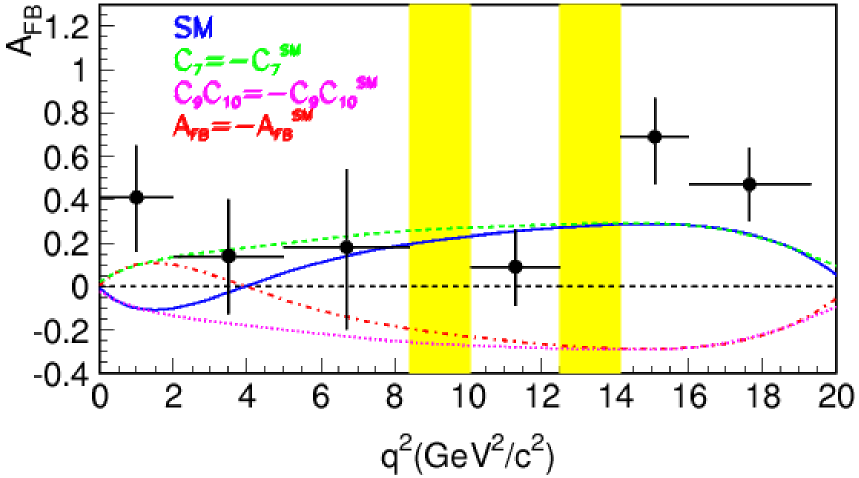
- Use HFAG08 data for Br(K*γ) and Br(K*μμ) + Belle's new A_{FB}

Br(B→K*γ) = (40.3 ± 2.6) x 10^{-6} (HFAG08)

Br(B→K*μμ) = 0.78±0.50 x 10^{-6} (HFAG08)

Belle's new A_{FB}(B→K*ll) measurements with 605/fb

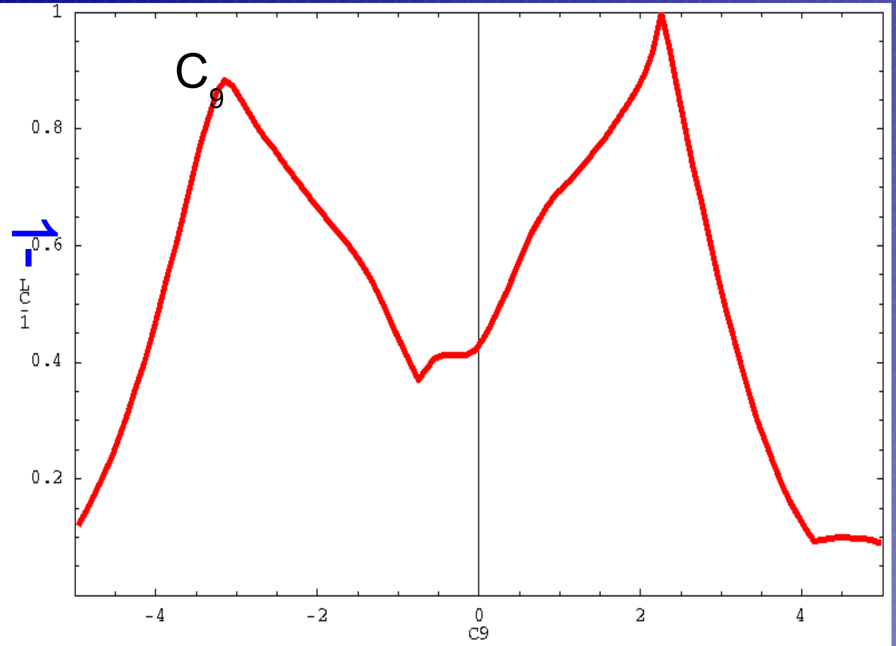
J.-T.Wei (Belle)

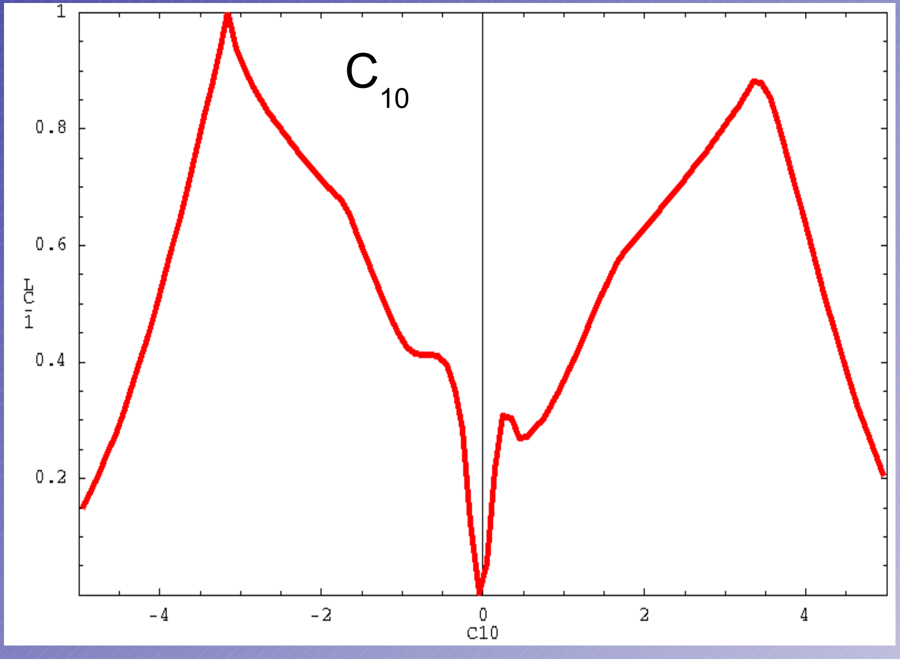


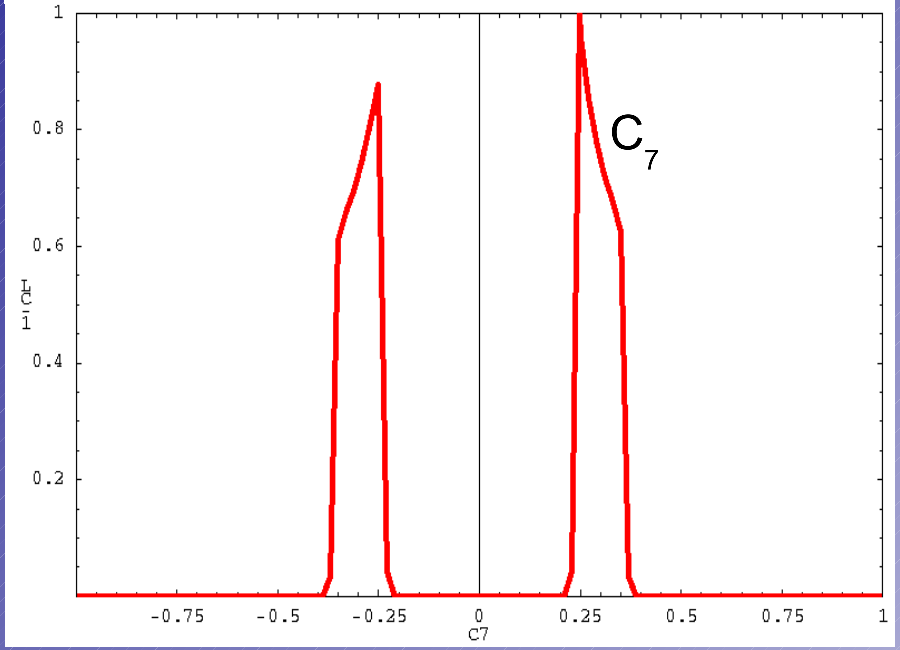
q^2 (GeV^2/c^2)	A_{FB}
0-2	$0.47^{+0.26}_{-0.32} \pm 0.03$
2-5	$0.14^{+0.20}_{-0.26} \pm 0.07$
5-8.68	$0.47^{+0.16}_{-0.25} \pm 0.14$
10.09-12.86	$0.43^{+0.18}_{-0.20} \pm 0.03$
14.18-16	$0.70^{+0.16}_{-0.22} \pm 0.10$
>16	$0.66^{+0.11}_{-0.16} \pm 0.04$

6 points
- quadratic sum of stat. and sys. errors
- errors are symmetrized

b) Fit with uncertainties in form factors







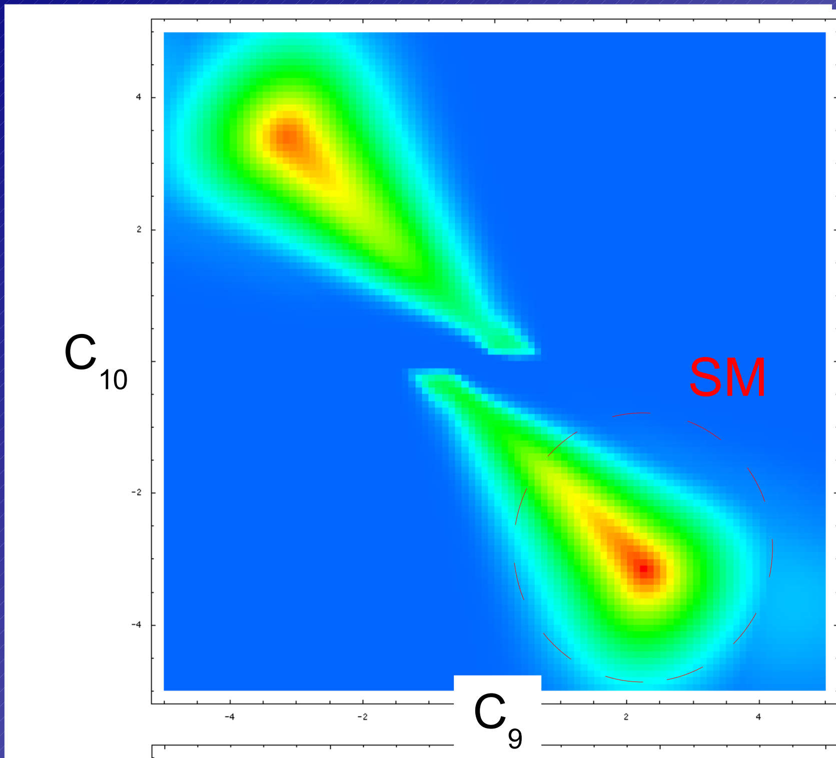
$C7 = \begin{array}{l} 0.248\ ^{+0.112}_{-0.014} \\ -0.250\ ^{+0.013}_{-0.110} \end{array} [1\sigma]$

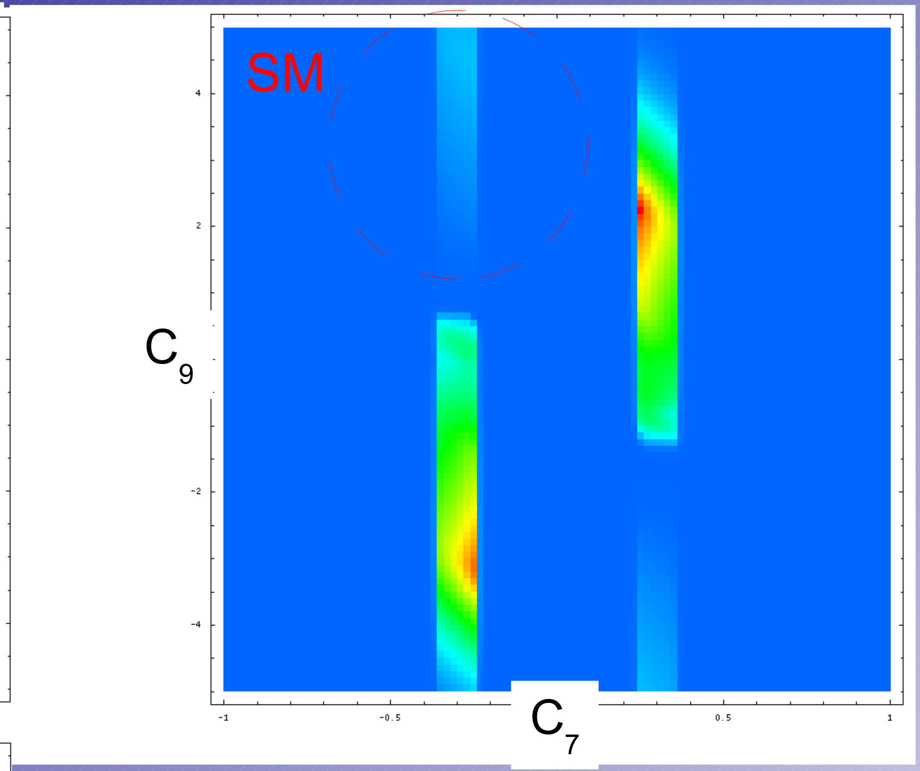
$\begin{array}{l} 0.248\ ^{+0.121}_{-0.018} \\ -0.250\ ^{+0.020}_{-0.120} \end{array} [2\sigma]$ $\begin{array}{l} 0.248\ ^{+0.125}_{-0.019} \\ -0.250\ ^{+0.023}_{-0.123} \end{array} [3\sigma]$

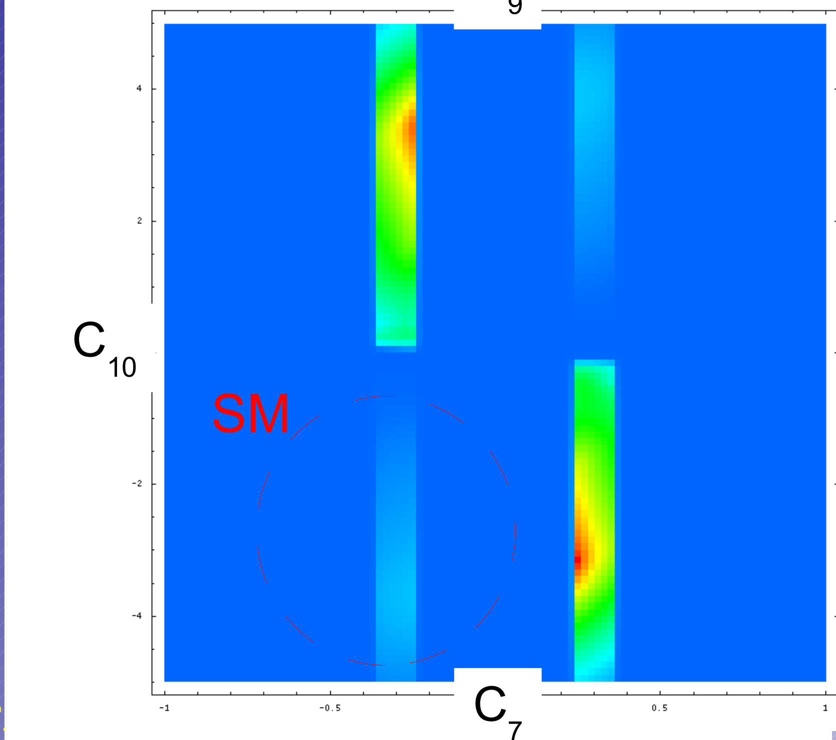
$C9 = 2.3\ ^{+1.1}_{-6.6}\ [1\sigma]$ $2.3\ ^{+\infty}_{-\infty}\ [2\sigma]$ $2.3\ ^{+\infty}_{-\infty}\ [3\sigma]$

$C10 = \begin{array}{l} -3.2\ ^{+2.9}_{-1.3} \\ 3.4\ ^{+1.3}_{-2.6} \end{array} [1\sigma]$ $\begin{array}{l} -3.2\ ^{+3.1}_{-\infty} \\ 3.4\ ^{+\infty}_{-3.4} \end{array} [2\sigma]$ $-3.2\ ^{+\infty}_{-\infty}\ [3\sigma]$

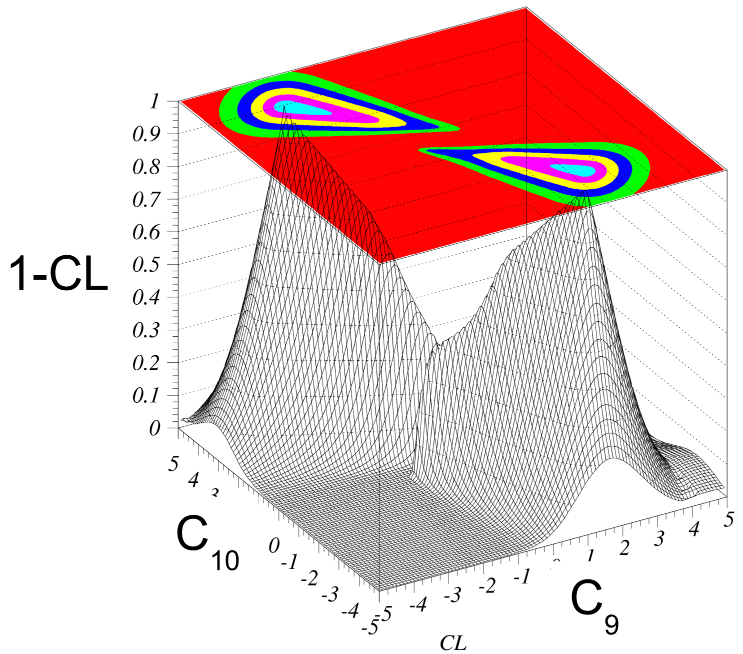
2-D scan results

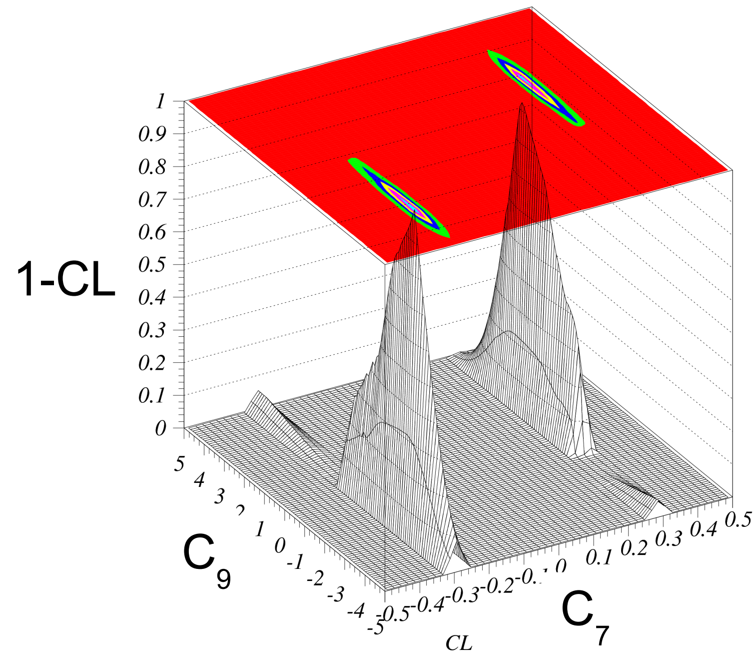


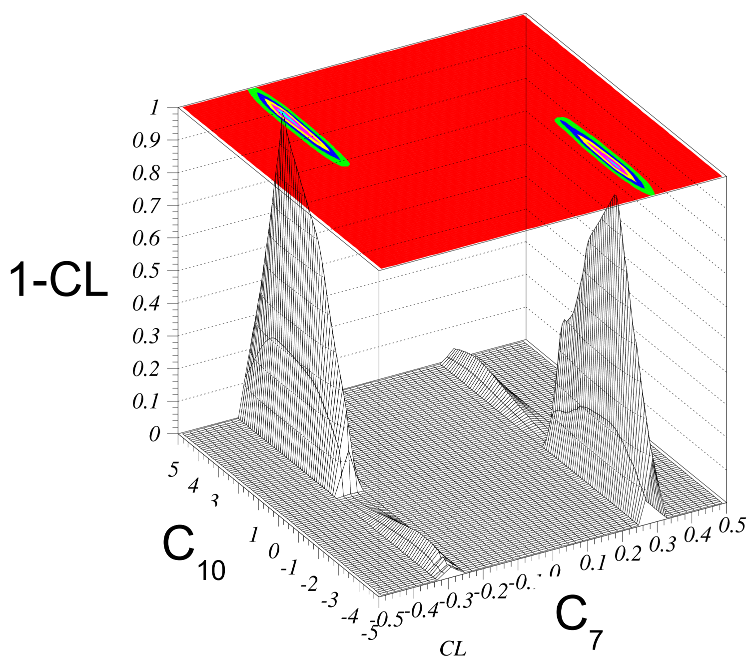




2-D scan favors
* Positive C_9
* Negative C_{10}
* Positive C_7


1-CL
C_{10}
C_9
CL


1-CL
C_9
C_7
CL


1-CL
C_{10}
C_7
CL

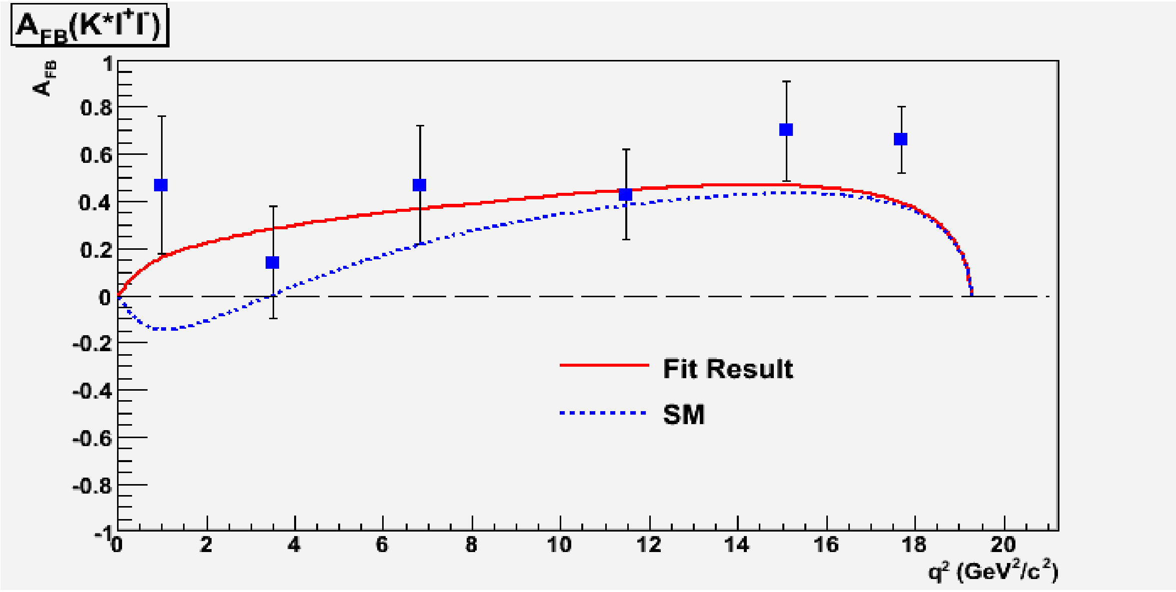
Results of fit to measurements

* No “reasonable” constraints on $C_{7,}$ C_9 nor C_{10} to state NP or SM ?????
* Preferred central values by the fit:
  C_7 = +0.25 C_9 = +2.3 C_{10} = - 3.2

Comparison of measurements and predictions with these values

	Measurements	Predictions	
$Br(B\to K^*\gamma)$	40.3 ± 2.6	29.6	x 10^{-6}
$Br(B\to K^*\mu\mu)$	0.78 ± 0.50	1.26	x 10^{-6}

$A_{FB}(B\to K^*ll)(q^2)$:



4. Summary and Prospects

* The CKMfitter development under FJPPL is actively going on.

* Two achievements have been done in JFY2008
  1) The treatment nuisance parameters in ϕ_3/γ averaging was studied: supremum method offers the best coverage and will be the default for the averaging.
  2) The first trial of Wilson coefficient determination by the global fit was made.

* Prospects for JFY2009:
  1) Extend the methodology to treat nuisance parameters to other averaging + publication.
  2) Add inclusive radiative decays to Wilson coefficient fit + update of theoretical model (NNLO).

* CKMfitter activities become more important in coming years to cope with new results by LHCb and new-generation B-factory experiments.