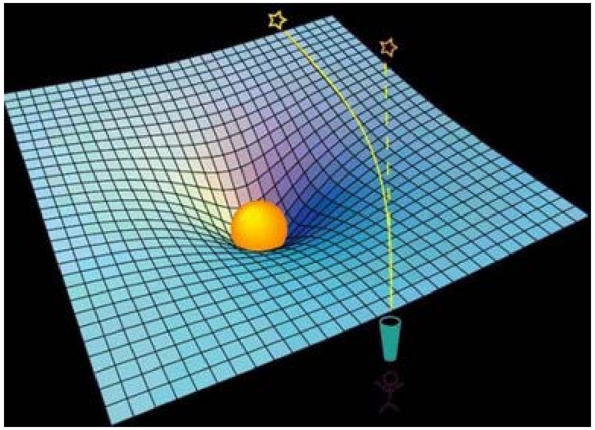
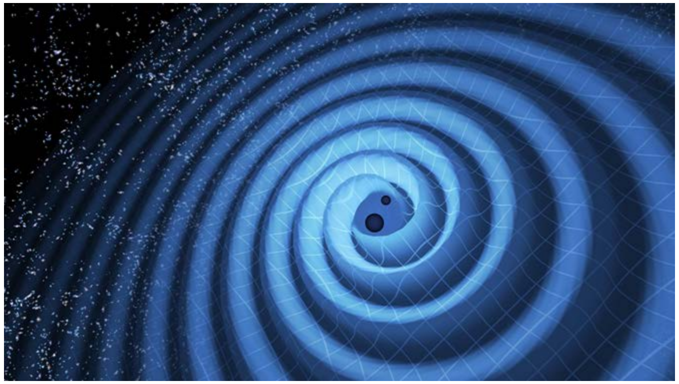
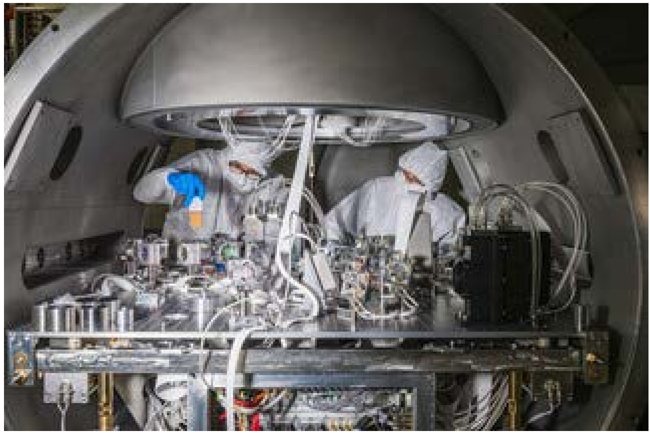




Gravitational Waves:

The challenges of the detection

Romain Gouaty
LAPP – Annecy
GraSPA summer school



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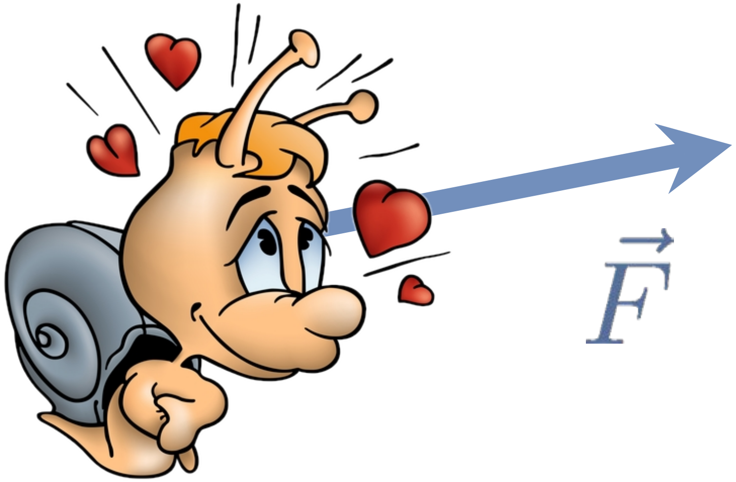
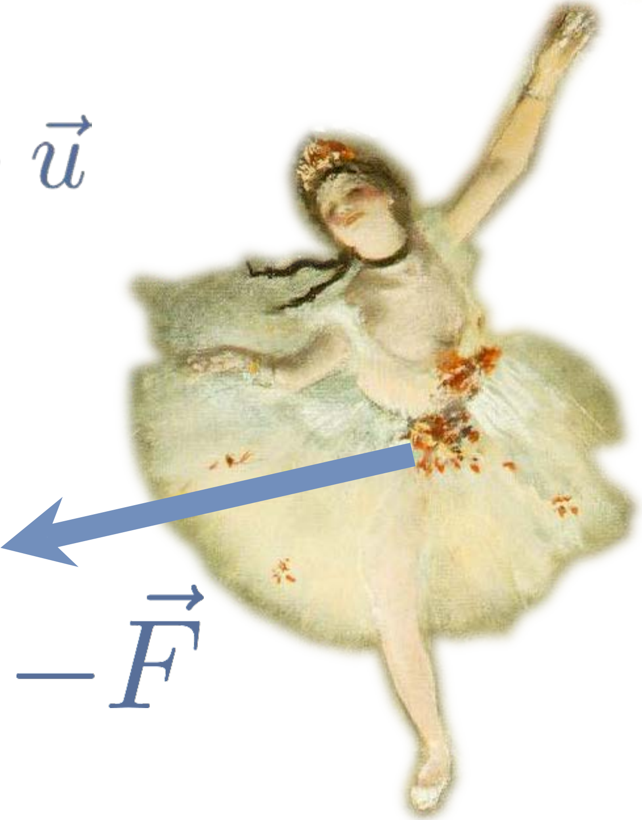
Gravitation: classical theory

- Flat space, absolute time
- Instantaneous interaction between distant masses

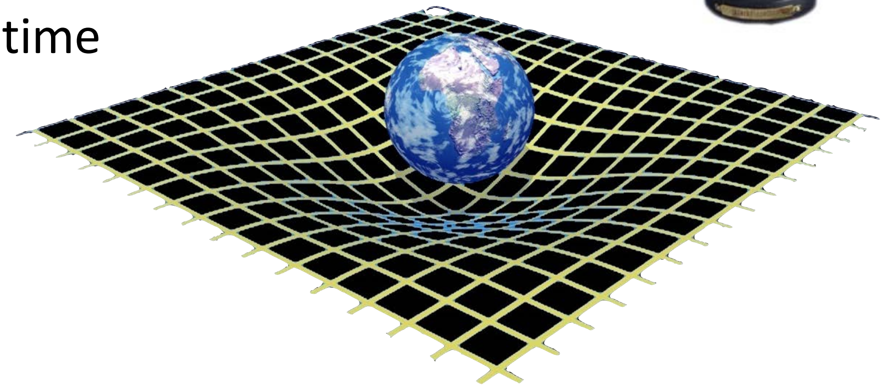
$$\vec{F} = G \cdot m_1 m_2 \cdot \frac{1}{r^2} \cdot \vec{u}$$





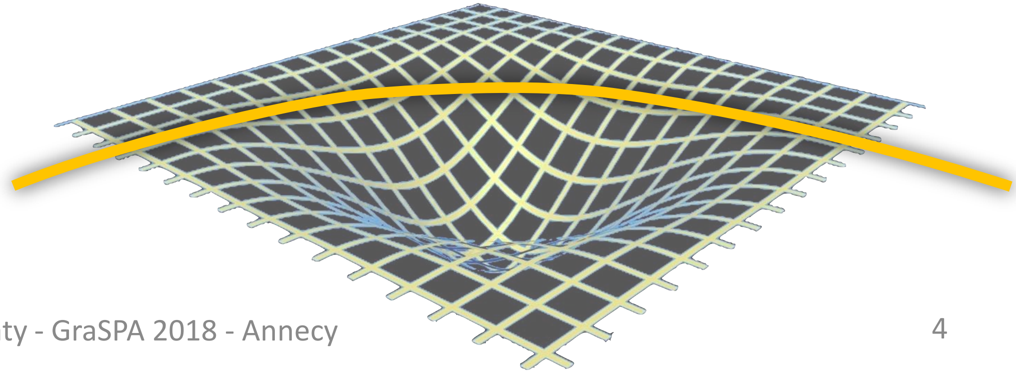
Gravitation: modern theory

- **Theory of General Relativity (GR)**
- Einstein 1915-1918 : geometric theory of gravitation
- A mass "bends" and "deforms " space-time



- The trajectory of a mass is influenced by the curvature of space-time

J. A. Wheeler : ***"Space tells matter how to move and matter tells space how to curve"***



The Einstein Field Equations

- What relation links deformation of space-time and energy-momentum ?
- Answer : the Einstein Field Equations (EFE)

with c = 1 ! would be $\frac{8\pi G}{c^4}$.

$$\underbrace{\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right)}_{\textbf{curvature term}} = 8\pi G \underbrace{\left(T_{\mu\nu}\right)}_{\textbf{energy-momentum term (includes mass)}}$$

$g_{\mu\nu}$ **metric tensor**

$R_{\mu\nu}$ **Ricci tensor (depends on $g_{\mu\nu}$ and derivatives)**

- Energy-momentum bends spacetime
- Spacetime tells mass (energy momentum) how to move
- These equations are non-linear

From Einstein Field Equations to Gravitational Waves

- Flat space-time = Minkowski metric
  - Add a perturbation $h_{\mu\nu}$ to the metric of a flat space
  - Linearize Einstein Field Equations
  - Choose a coordinate system ("Transverse Traceless" (T T) gauge)
- Obtain a wave equation

$$\boxed{(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})h_{\mu\nu} = 0}$$

(in vacuum, no $T_{\mu\nu}$)

- Solution (in vacuum) :

$$h_{\mu\nu} = A_{\mu\nu} \cdot e^{-i(\vec{k}\cdot\vec{x} - \omega\cdot t)}$$

Gravitational Waves

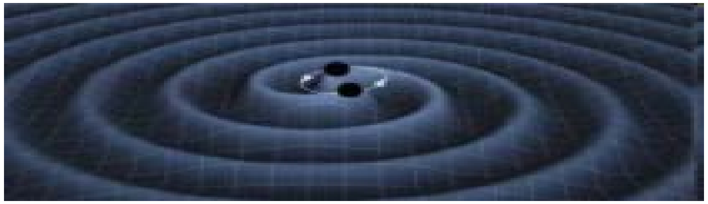
$$h_{\mu\nu} = A_{\mu\nu} \cdot e^{-i(\vec{k}\cdot\vec{x}-\omega\cdot t)}$$

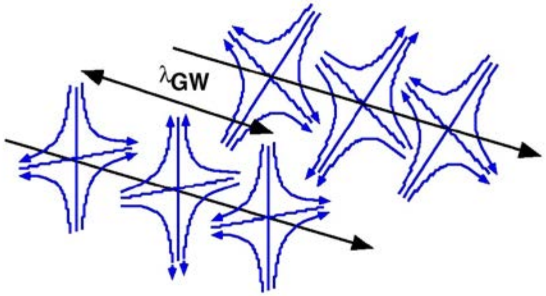
- transversal plane wave
- propagation at the light speed c
- Two polarisation states (+ and x)

Masses in motion

Space-time deformation

Gravitational wave





$$h_{\mu\nu} = h_+(t - z/c) + h_x(t - z/c)$$

Detectable effect on free fall masses

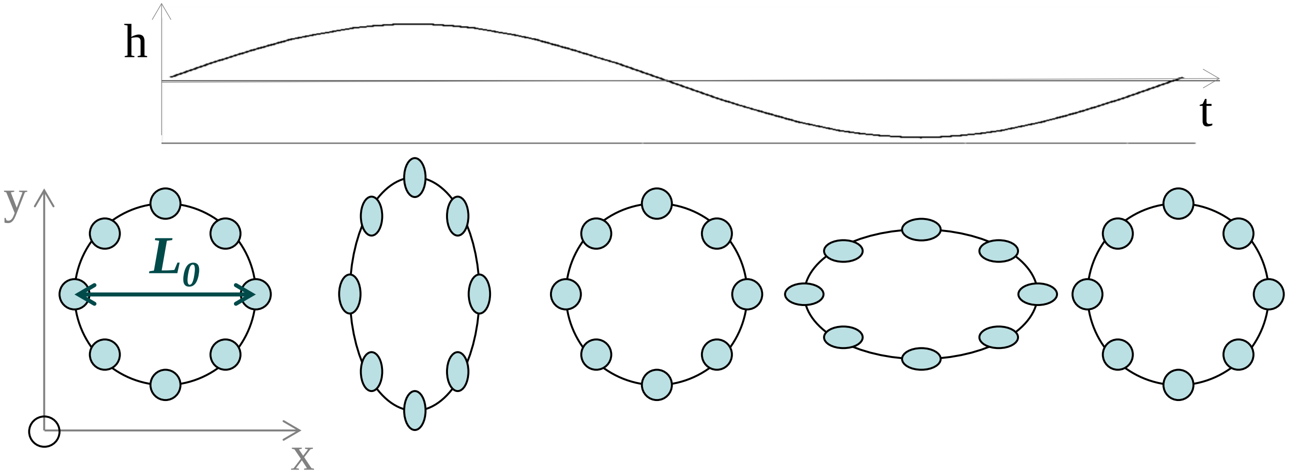


Illustration of the metric variation with free fall masses initially located along a circle, for a + polarised GW propagating along z

$$\delta L_x(t) = \frac{1}{2}\, h(t)\, L_0$$

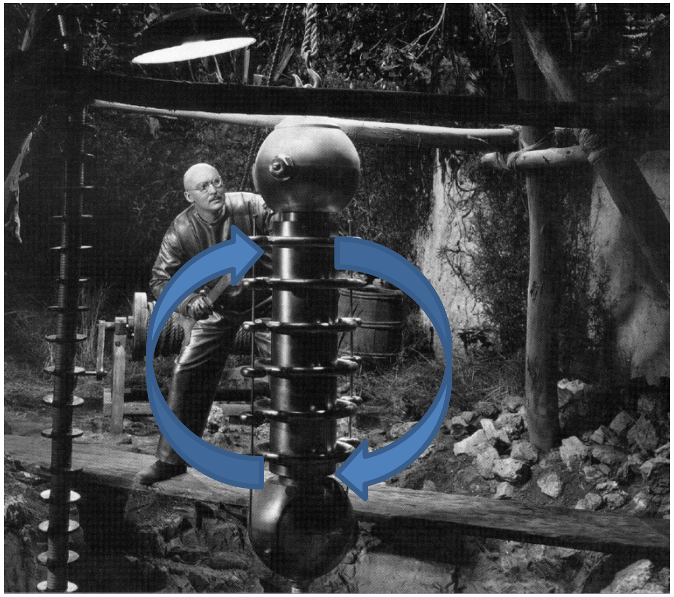
h(t): amplitude of the GW

(*h* has no dimension)

Effect on free fall masses

GW generation

- Accelerated masses, quadrupolar momentum



$$h \approx \frac{G}{c^4} \frac{E_{ns}}{r}$$

"Non spherical" kinetic energy

distance to the source

$\sim 10^{-44}\ \mathrm{m}^{-1}\ \mathrm{kg}^{-1}\ \mathrm{s}^{2}$

Examples with 2 orbiting objects: $$h \approx \frac{32\pi^2 G M R^2 f_{orb}^2}{r c^4}$$

- M = 1000 kg, R = 1 m, f = 1 kHz, r = 300 m

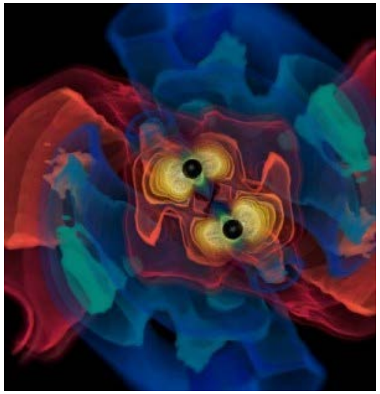
  $h \sim 10^{-35}$

- $M = 1.4\ M_\odot$, R = 20 km, f = 400 Hz, $r = 10^{23}$ m (15 Mpc = 48,9 Mlyr)

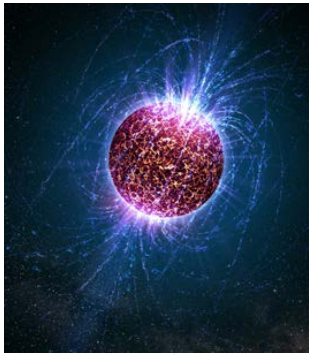
  $h \sim 10^{-21}$

- Which detectable sources?
  - No way for lab generation
  - Astrophysical sources (high masses and velocities)
    - Despite the distance penalty
    - Typical sources: compact orbiting objects

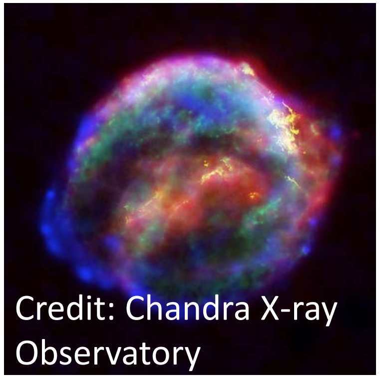
Astrophysical sources of GW


Credit: AEI, CCT, LSU

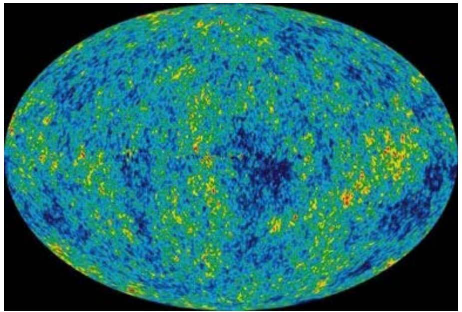
- **Binary system**
  - Need to be compact to be observed by ground based detectors
    → Neutron stars, black holes
  - Signal well modeled but rates not well known


Casey Reed, Penn State

- **Spinning neutron stars**
  - Nearly monotonic signals
  - Long duration
  - Strength not well known

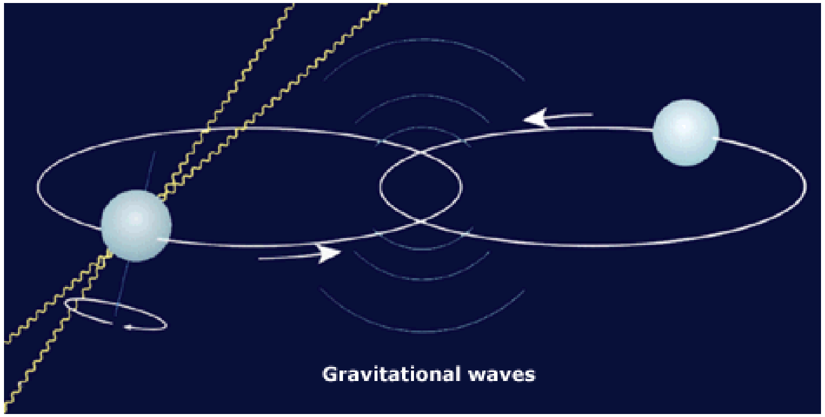

Credit: Chandra X-ray Observatory

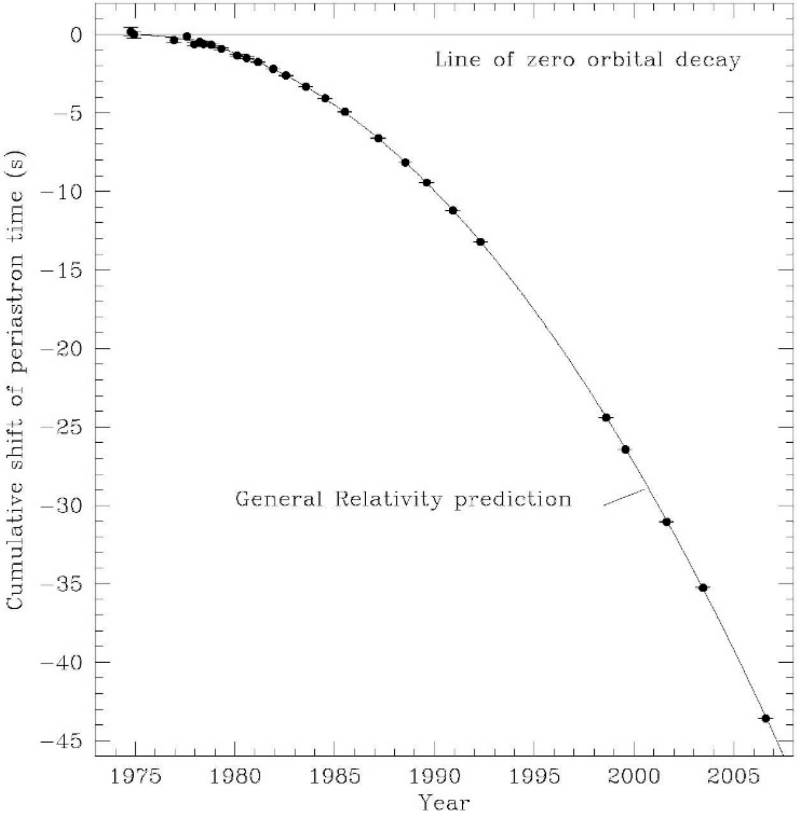
- **Asymmetric explosion**
  - Like supernovae core collapse
  - “burst” transient
  - Not well modeled


NASA/WMAP Science Team

- **Cosmic gravitational wave background**
  - Residual of the big bang/inflation
  - Stochastic background
  - Could be overlapped by superposition of transients

Indirect evidence: PSR 1913+16



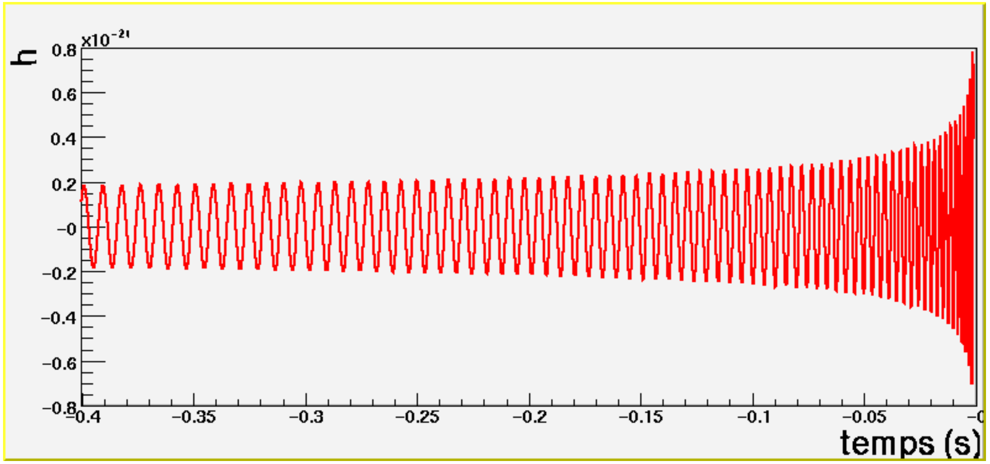


$$\dot{P}_{observe} / \dot{P}_{predit} = 1.0013 \pm 0.0021$$

- Binary system of neutron stars
- One neutron star is a radio pulsar
- Discovered in 1975 by Hulse and Taylor
- Studied by Taylor, Weisberg and co.
- **Decay of the orbital period compatible with GW emission**
- Frequency of GW emitted by PSR 1913+16: ~ 0.07 mHz
  Undetectable by ground-based detectors (bandwidth 10 Hz- 10 kHz)

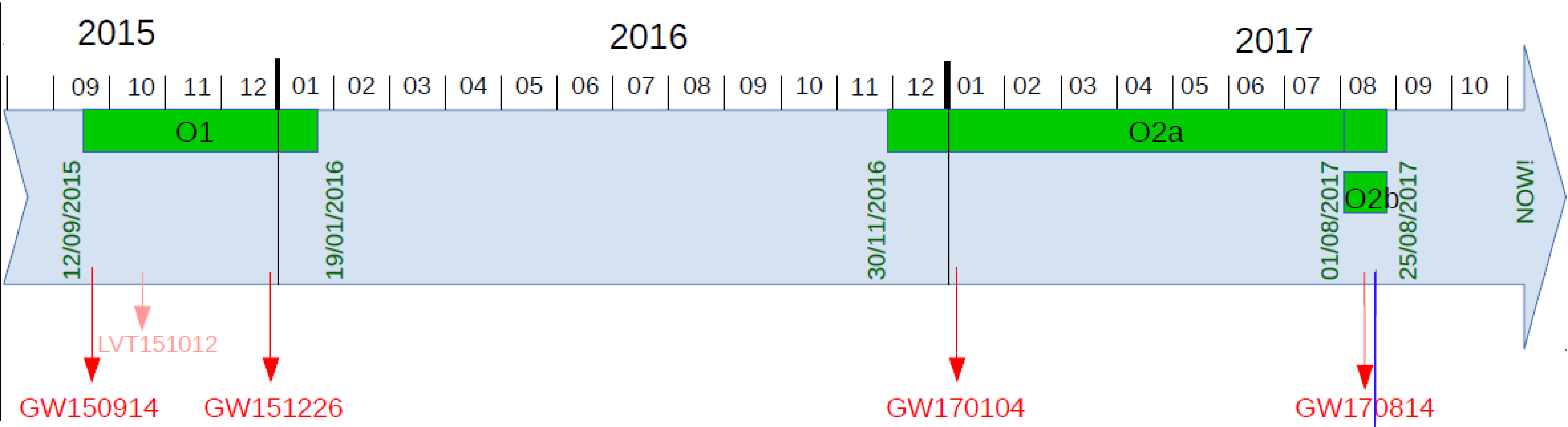
Coalescing binaries

- Binary systems of compact stars at the end of their evolution
  - Neutron stars and black holes
- Very rare phenomenum in our Galaxy
  - A few tens per million years
- Typical amplitude (for neutron stars)
  - $h \sim 10^{-22}$ à 20 Mpc
- Very distinctive waveform



First detections!

By the LIGO scientific collaboration and the Virgo collaboration



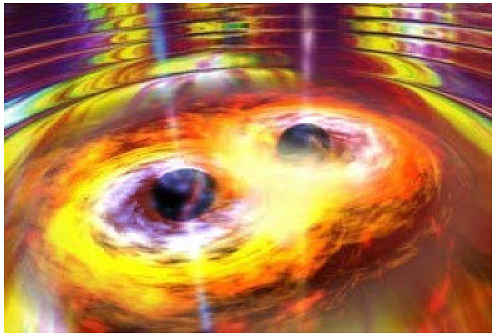
First GW detection !

First detection in triple coincidence

GW170817 + multi-messengers detections

Sources = Coalescing binaries

- **Black holes**
- **Neutron Stars**



Scientific goals (1/2)

- Confirmation of GW

Done !

- Study properties, test GR
  - Speed = c ? Really quadrupolar ?

Started !

- Measure the Hubble constant
  - Coalescing binaries should be standard candles if the redshift and distance are known

Started !

Detectors on earth,
in space

Impossible with only one detector for most of the sources

Build a worldwide observatory of gravitational waves

Scientific goals (2/2)

Started!

- Study characteristics
  - of neutron stars
  - of solar mass black holes (BH)
    - Ellipticity, vibration modes, higher order moments, population…

Detectors on earth

- Study supermassive black holes
  - cartography of space-time around a supermassive BH (Kerr).
  - study of their distribution, galactic evolution

Detectors in space

- Stochastic background of GW :
  - first moments of the universe ?
- …

Detectors on earth?

Detectors in space?

Table of Contents

GW quest: a bit of history

- Joseph Weber invents the bar detector
    - The GW changes the resonance condition of a resonant bar of a few tons
    - First claim for detection in 1969… but contested
    - Triggered large interest, at least 18 bars in 8 countries
- Evolve to cryogenic resonant bars (80-90)
- Bar not enough sensitivity:
    - h : few 10^{-21} 1/sqrt(Hz) @ 900Hz

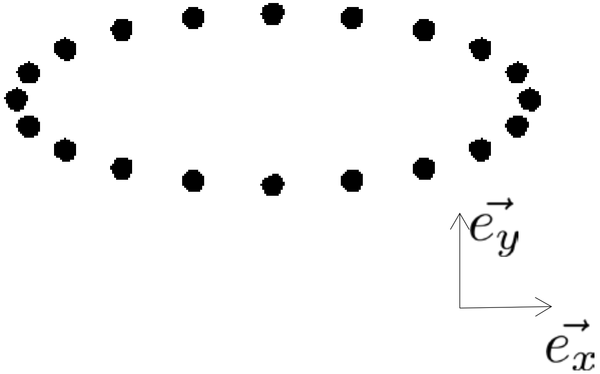
- ITF started in the 70's (Germany, Rai Weiss)
    - Broad band instrument
- Few ITF prototypes in the 80's
    - MIT, Glasgow, Garching, Caltech,...
    - ~10m long
    - Not made for detection
- Jump to km scale in early 90
    - LIGO, Virgo, GEO, TAMA

Reminder: effect of a GW on free fall masses

A gravitational wave (GW) modifies the distance between free-fall masses

$$\delta x(t) = -\delta y(t) = \frac{1}{2} h(t) L_0$$

h(t): amplitude of the GW

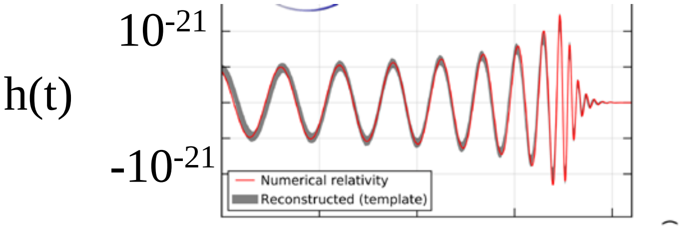


Typical amplitude of a GW crossing the Earth:
$h \sim 10^{-23}$

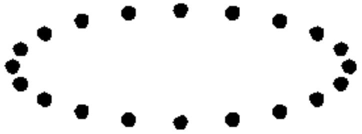
(h has no dimension/unit)

Case of a GW with polarisation + propagating along z

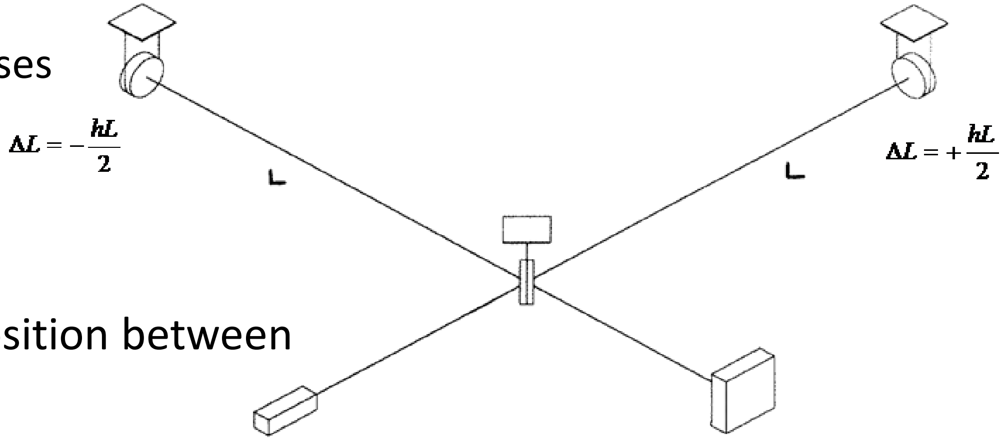
Reconstructed strain of GW150914



Terrestrial GW Interferometer: basic principle



- Measure a variation of distance between masses
  - Measure the light travel time to propagate over this distance
  - Laser interferometry is an appropriate technique
    - Comparative measurement
    - Suspended mirrors = free fall test masses



- Michelson interferometer well suited:
  - Effect of a gravitational wave is in opposition between 2 perpendicular axes
  - **Light intensity of interfering beams is related to the difference of optical path length in the 2 arms**

We need a big interferometer:

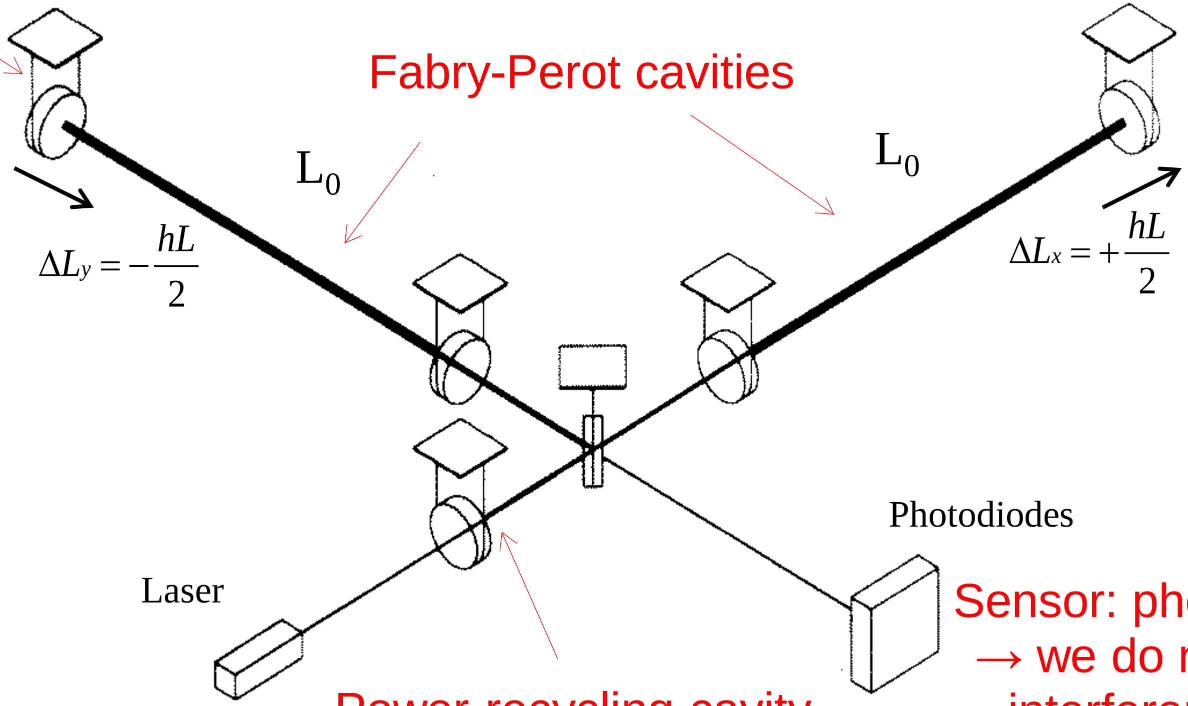
ΔL proportional to L

➔ need several km arms!

Bandwidth: 10 Hz to few kHz

Virgo/LIGO: more complicated interferometers

Suspended mirrors → Mirrors can be considered as free-falling in the ITF plane for frequencies larger than ~10 Hz



Fabry-Perot cavities

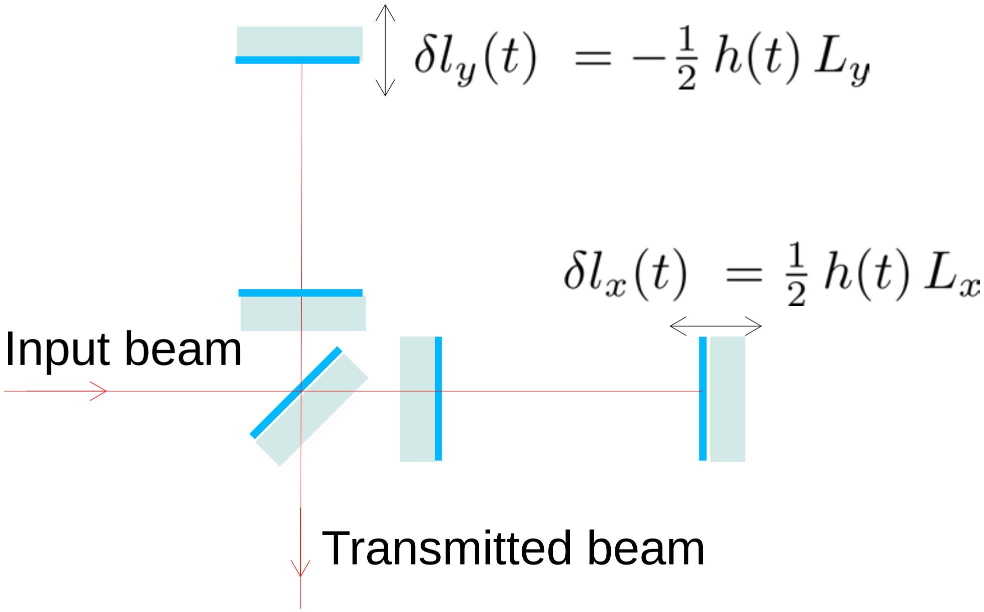
Sensor: photodiode
→ we do not image the interference pattern!

Power-recycling cavity

Infrared laser
P ~ 100 W

WARNING: STILL VERY SIMPLIFIED SCHEME!

Orders of magnitude



$$\delta l_y(t) = -\tfrac{1}{2}\, h(t)\, L_y$$

$$\delta l_x(t) = \tfrac{1}{2}\, h(t)\, L_x$$

Typical amplitude of differential arm length variations when a GW crosses the Earth:

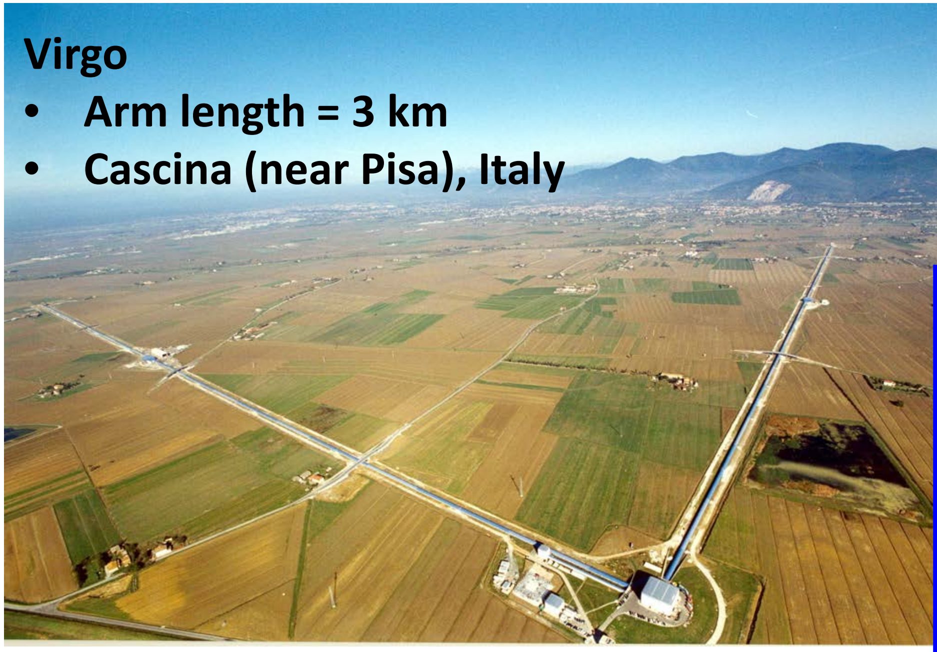
$$\delta\Delta L = \delta l_x(t) - \delta l_y(t)$$
$$= h(t)\, L_0$$

$$\mathrm{h} \sim 10^{-23} \qquad L_0 = 3\,\mathrm{km}$$

$$\rightarrow\ \delta\Delta L \sim 3 \times 10^{-20}\,\mathrm{m}$$

$$\sim \frac{\text{size of a proton}}{100000}$$

Km scale interferometers



Virgo

- Arm length = 3 km
- Cascina (near Pisa), Italy

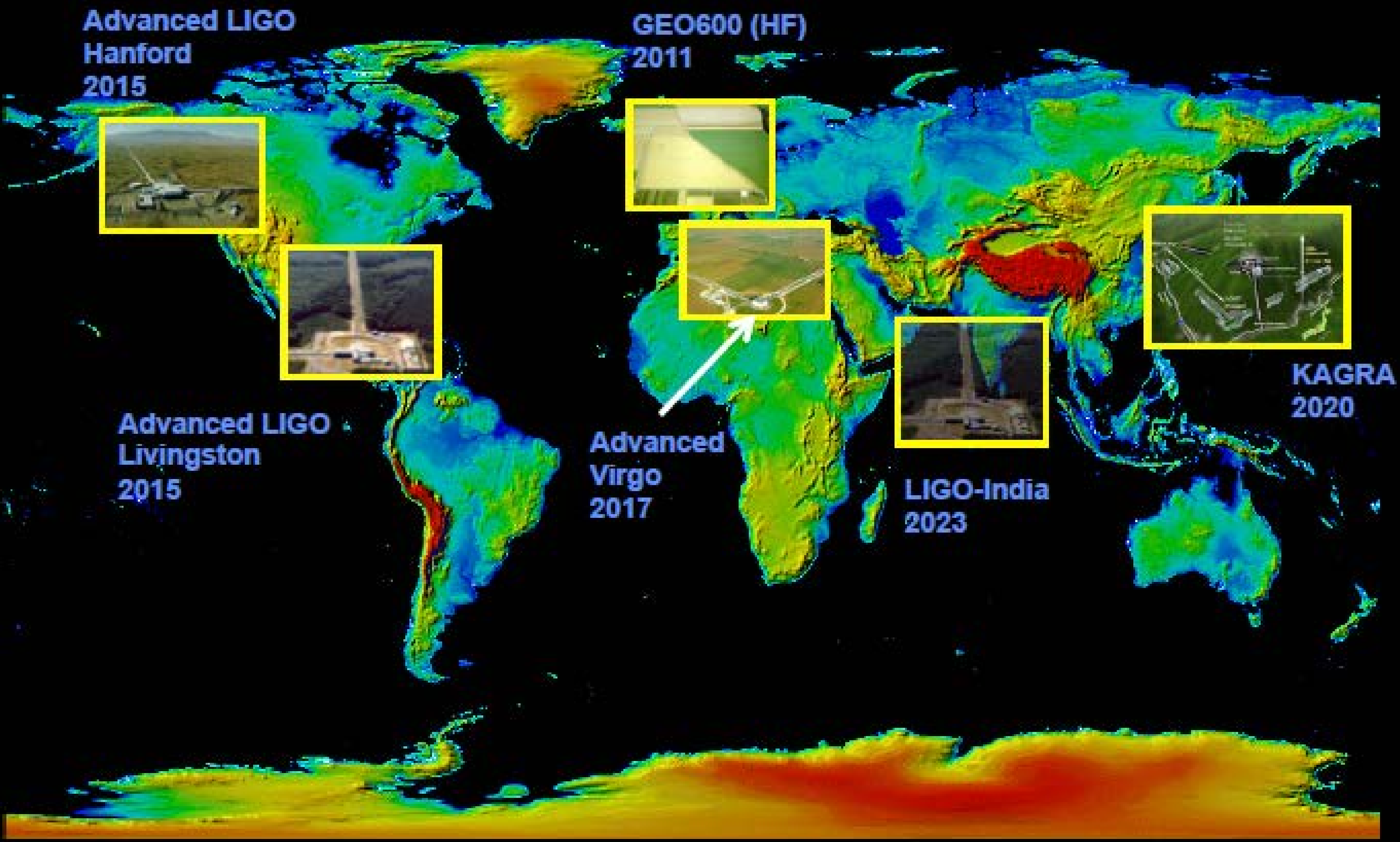
LIGO Livingston

- Arm length = 4 km
- Louisiana

LIGO Hanford

- Arm length = 4 km
- Washington State

The detector network

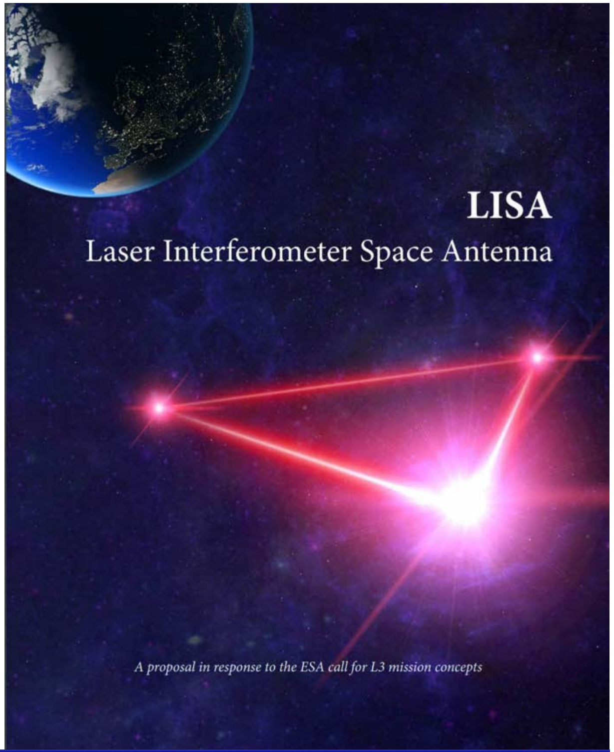


Spatial interferometer: LISA

- **Bandwidth: 0.1 mHz to 1 Hz**
- Launch of LISA in the years 2030

  → operation for 5 to 10 years

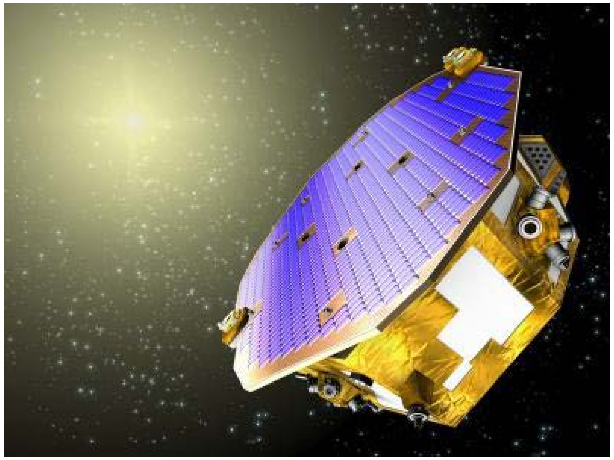
- Successful intermediate step: LISA Pathfinder
  - ➢ launched end 2015
  - ➢ test of free-fall masses
  - ➢ validation of differential motion measurements

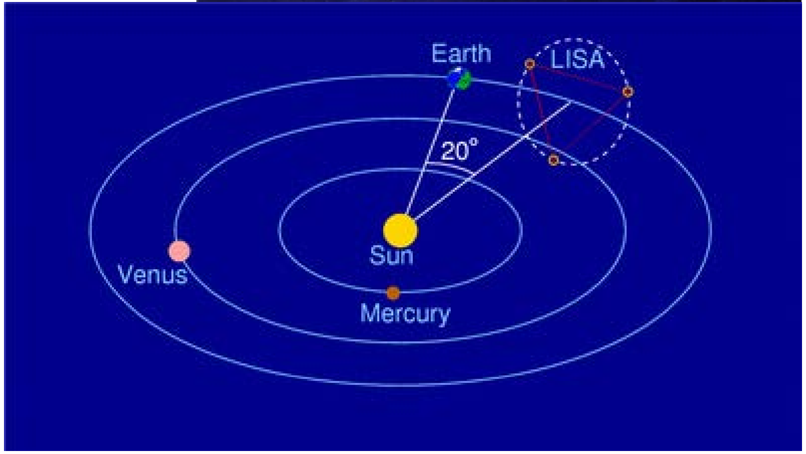


LISA

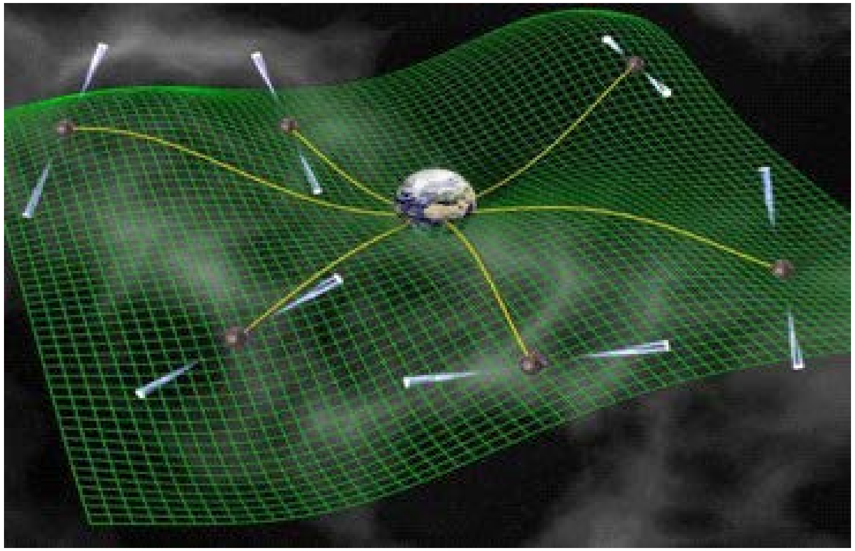
Laser Interferometer Space Antenna

A proposal in response to the ESA call for L3 mission concepts





Pulsars timing arrays



- **Bandwidth: 1 nHz to 10 nHz**
- Observation of 20 ms pulsars in radio

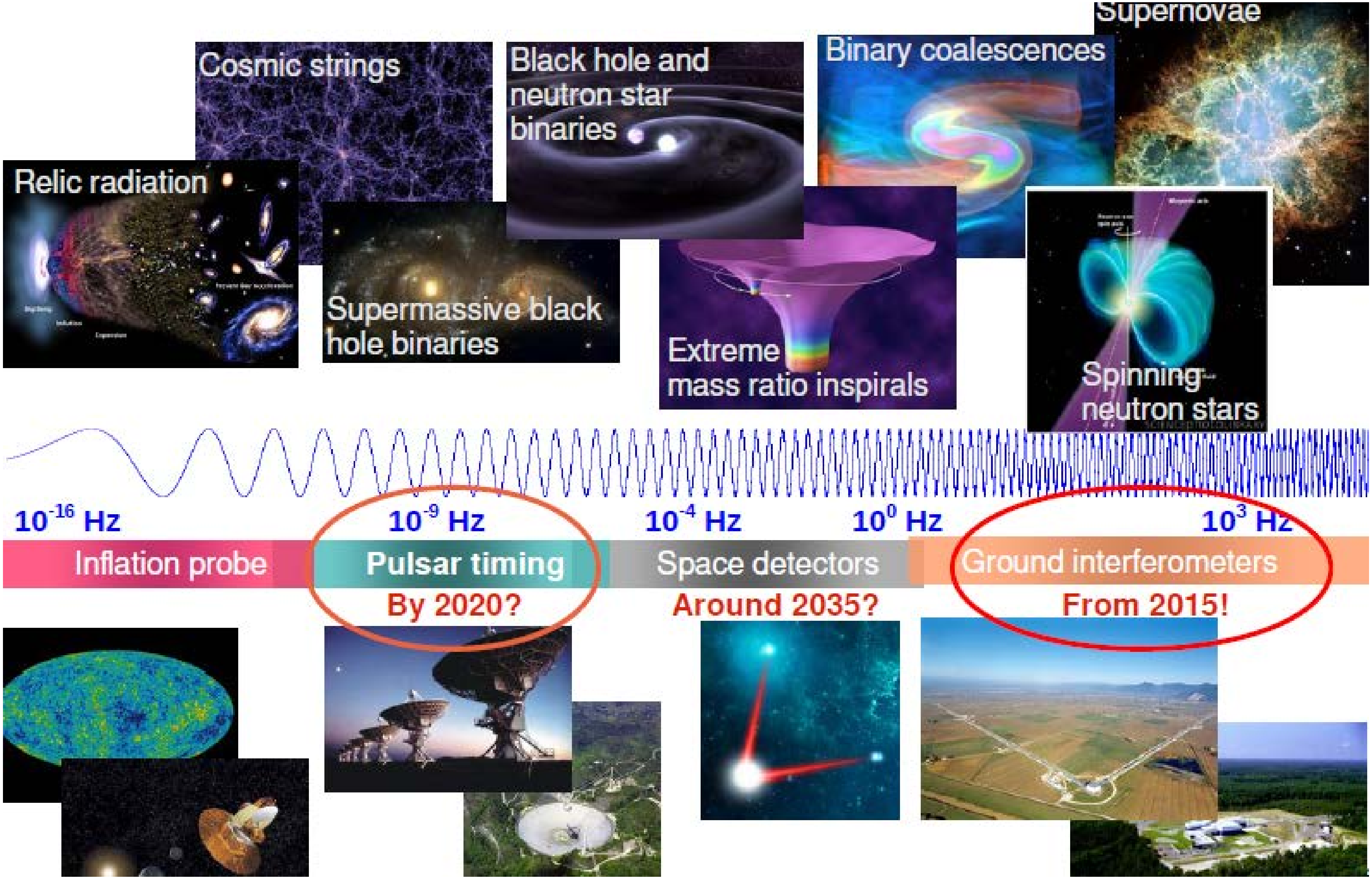
  - Residuals of modellisation < 100 ns

  - Weekly sampling over 5 years

- International network
  - Parkes PTA
  - North American NanoHertz Gravitationnal Wave Obsevatory
  - European PTA

- **First detections expected in the coming years!**

A large GW spectrum to be studied…



10^{-16} Hz | 10^{-9} Hz | 10^{-4} Hz | 10^{0} Hz | 10^{3} Hz

Inflation probe

Pulsar timing
By 2020?

Space detectors
Around 2035?

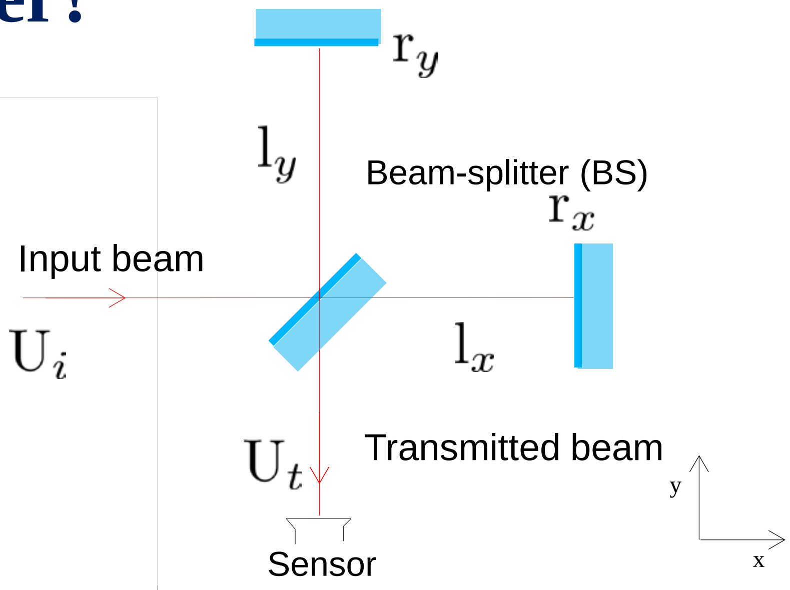
Ground interferometers
From 2015!

Table of Contents

How do we « observe » ΔL with a Michelson interferometer?

- Input wave $$\begin{aligned} U_i(x,t) &= \underline{A}_i e^{\jmath kx} \\ &= \underline{A}_i \quad \text{on BS} \end{aligned}$$
- BS located at (0,0)
- Sensor located at $(0,-y_s)$
- Amplitude reflection and transmission coefficients: r and t

→ We are interested in the beam transmitted by the interferometer: it is the sum of the two beams (fields) that have propagated along each arm



Around the mirrors:

- Radius of curvature of the beam ~ 1400 m
- Size of the beam ~ few cm

→ The beam can be approximated by plane waves

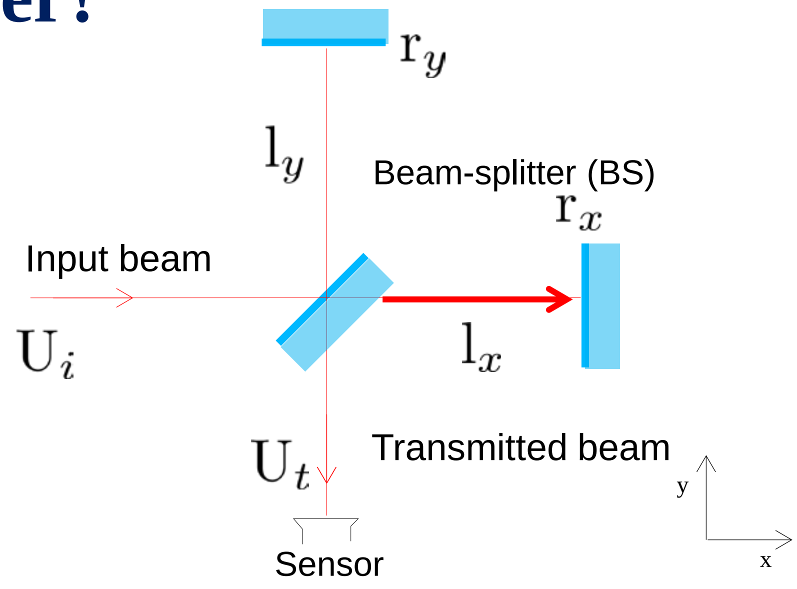
How do we « observe » ΔL with a Michelson interferometer?

- Input wave $U_i(x,t) = \underline{A}_i e^{\jmath kx}$

$$= \underline{A}_i \quad \text{on BS}$$

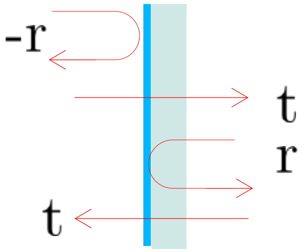
- Beam propagating along x-arm:

$$U_{tx} = \underline{A}_i \, t_{BS} \, e^{\jmath k l_x} \;\ldots\ldots$$



Sign convention for amplitude reflection and transmission coefficients

Without losses:
$t^2 + r^2 = 1$



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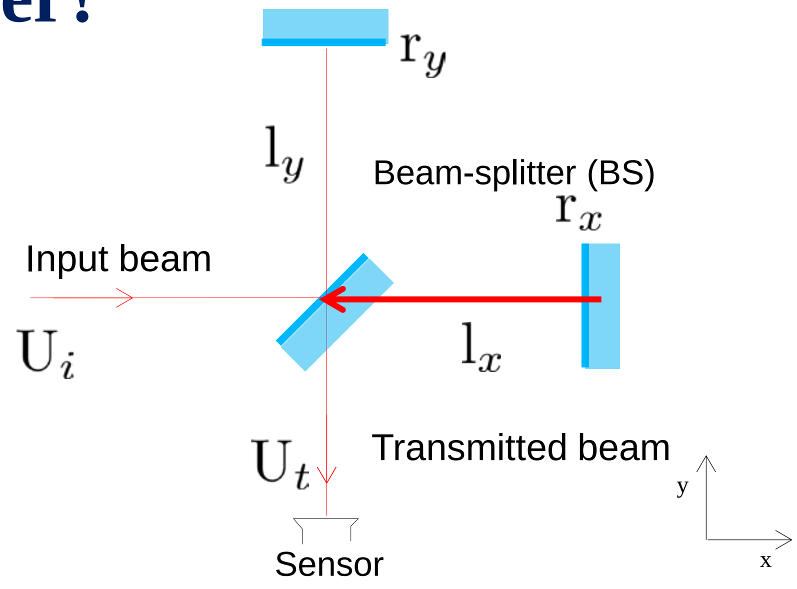

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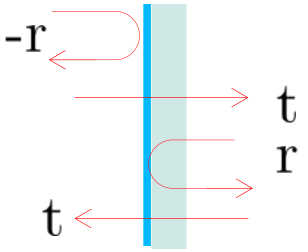
$$U_{tx} = \underline{A}_i \, t_{BS} \, e^{\jmath k l_x} \quad (-r_x) e^{\jmath k l_x} \ldots\ldots$$

Sign convention for amplitude reflection and transmission coefficients



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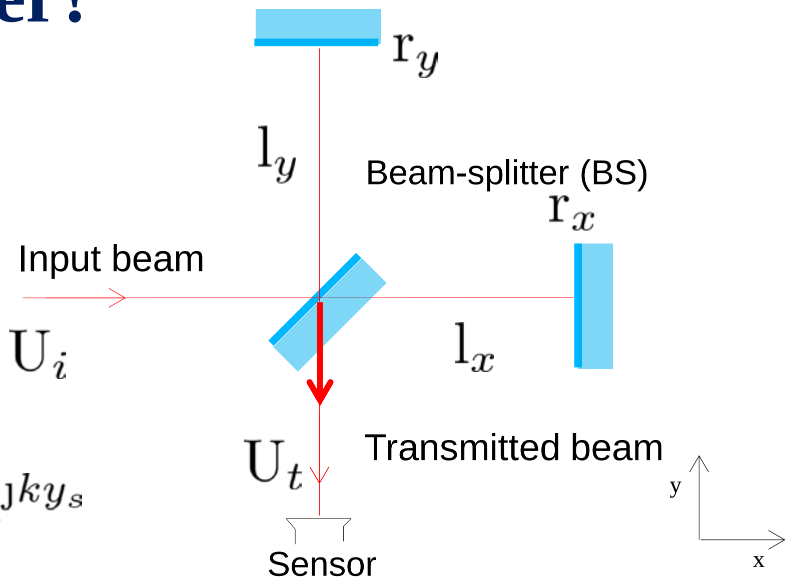

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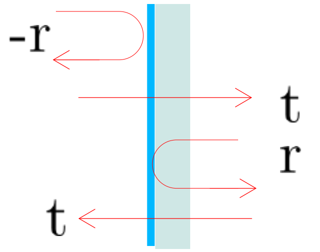
$$U_{tx} = \underline{\mathcal{A}_i}\, t_{BS}\, e^{\jmath k l_x} \quad (-r_x) e^{\jmath k l_x} \quad r_{BS}\, e^{\jmath k y_s}$$

Sign convention for amplitude reflection and transmission coefficients



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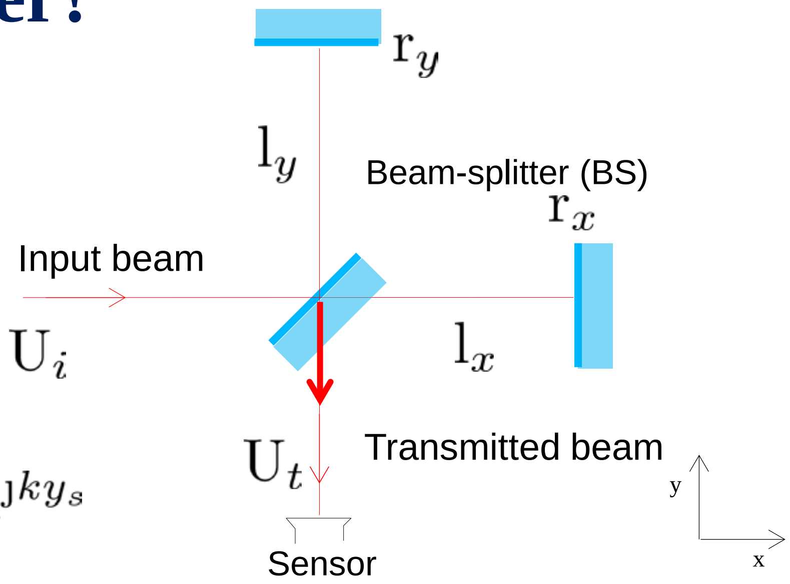
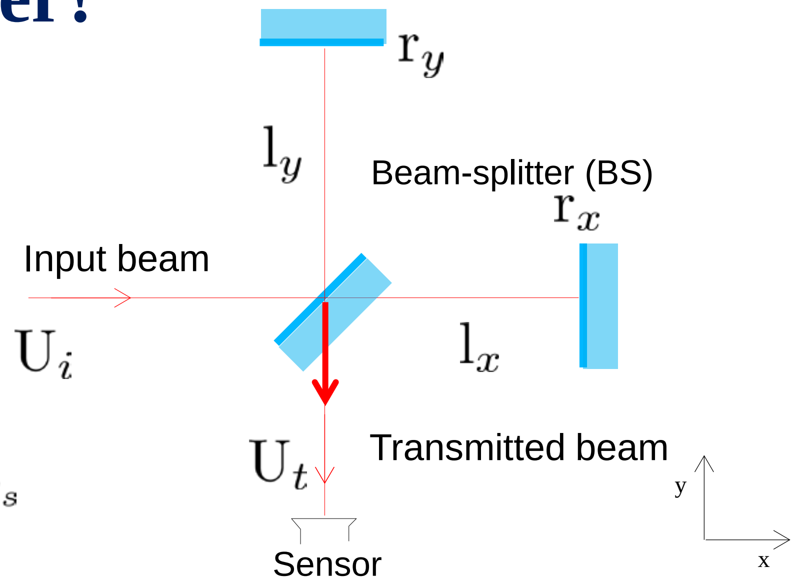


- Input wave $$U_i(x,t) = \underline{\mathcal{A}}_i e^{\jmath kx}$$ $$= \underline{\mathcal{A}}_i \quad \text{on BS}$$

- Beam propagating along x-arm:

$$\begin{aligned} U_{tx} &= \underline{\mathcal{A}}_i\, t_{BS}\, e^{\jmath k l_x} \quad (-r_x) e^{\jmath k l_x} \quad r_{BS}\, e^{\jmath k y_s} \\ &= \underline{\mathcal{A}}_i\, t_{BS}\, r_{BS}\, (-r_x)\, e^{2\jmath k l_x}\, e^{\jmath k y_s} \\ &= \frac{\underline{\underline{\mathcal{A}_i}}}{2} \times \underbrace{\left(-\, r_x\, e^{2\jmath k l_x} \right)}_{\text{Complex reflection of the x-arm}} e^{\jmath k y_s} \qquad \text{with } t_{BS} = r_{BS} = \tfrac{1}{\sqrt{2}} \end{aligned}$$

How do we « observe » ΔL with a Michelson interferometer?



- Input wave $$U_i(x,t) = \underline{\mathcal{A}_i} e^{\jmath k x}$$ $$= \underline{\mathcal{A}_i} \quad \text{on BS}$$

- Beam propagating along x-arm:

$$U_{tx} = \underline{\mathcal{A}_i}\, t_{BS}\, e^{\jmath k l_x} \quad (-r_x) e^{\jmath k l_x} \quad r_{BS}\, e^{\jmath k y_s}$$
$$= \underline{\mathcal{A}_i}\, t_{BS}\, r_{BS}\, (-r_x)\, e^{2\jmath k l_x}\, e^{\jmath k y_s}$$
$$= \frac{\underline{\mathcal{A}_i}}{2} \times \underbrace{\left(-\, r_x\, e^{2\jmath k l_x}\right)}_{\text{Complex reflection of the x-arm}} e^{\jmath k y_s}$$

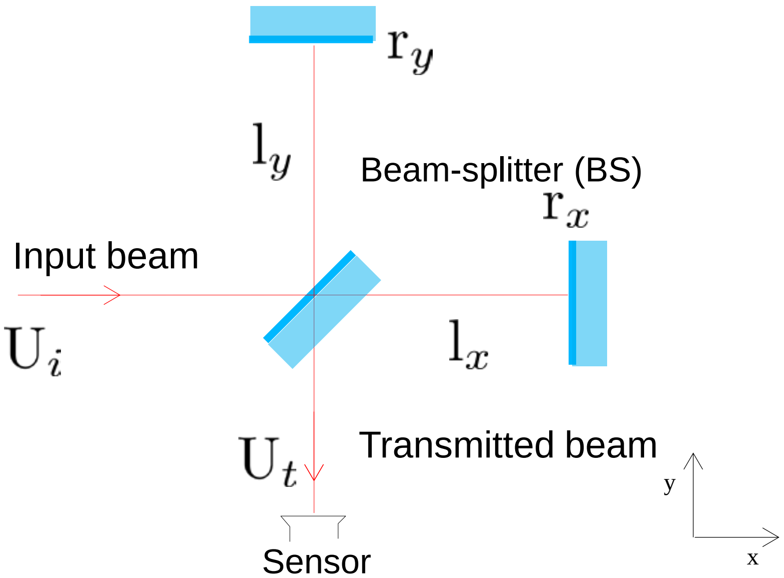
- Beam propagating along y-arm:

$$U_{ty} = -\frac{\underline{\mathcal{A}_i}}{2} \times \underbrace{\left(-\, r_y\, e^{2\jmath k l_y}\right)}_{\text{Complex reflection of the y-arm}} e^{\jmath k y_s}$$

Transmitted field:

$$U_t = U_{tx} + U_{ty}$$
$$= \frac{\underline{\mathcal{A}_i}}{2} e^{\jmath k y_s} \left(r_y\, e^{2\jmath k l_y} - r_x\, e^{2\jmath k l_x}\right)$$

Simple Michelson interferometer: transmitted power



Field transmitted by the interferometer

$$U_t = \frac{A_i}{2}\left(r_y\, e^{2\jmath k l_y} - r_x\, e^{2\jmath k l_x}\right)$$

k is the wave number, k = 2π/λ

λ is the laser wavelength (λ=1064 nm)

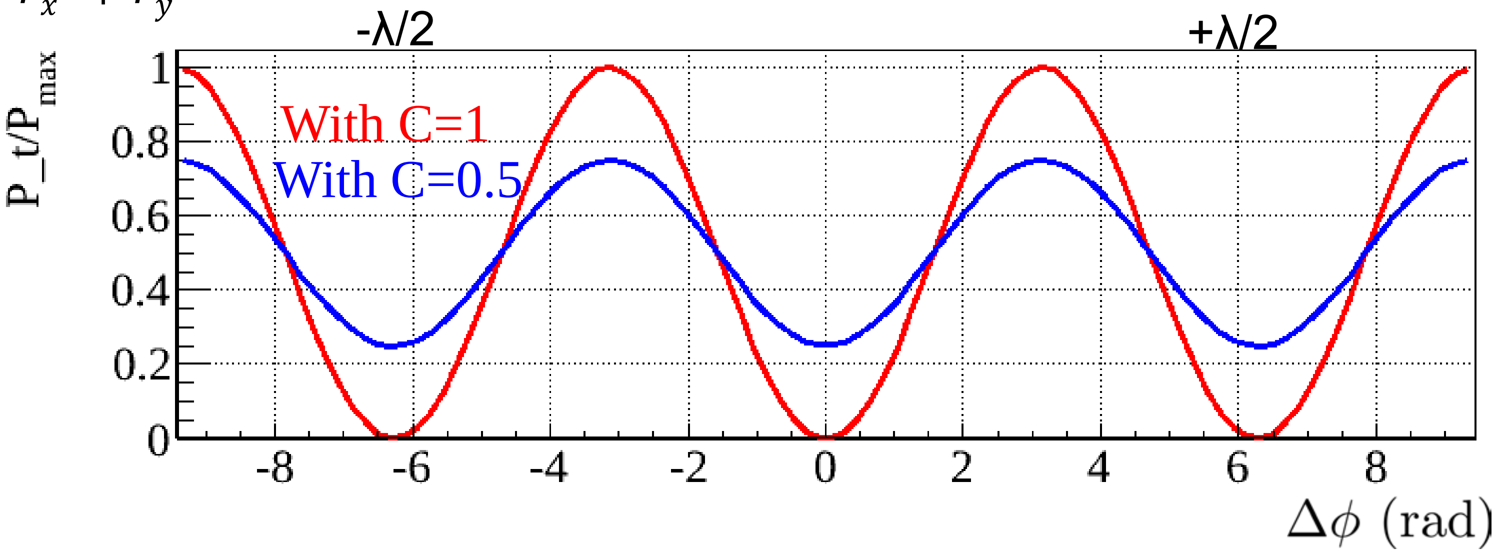
Transmitted power

$$P_t \propto |U_t|^2 = \frac{P_{max}}{2}\left(1 - C\cos(\Delta\phi)\right)$$

$$\text{where } \Delta\phi = 2k(l_y - l_x)$$

ITF contrast: $C = \dfrac{2r_x r_y}{{r_x}^2 + {r_y}^2}$

$$P_{max} = \frac{P_i}{2}(r_x^2 + r_y^2)$$



What power does Virgo measure?

- In general, the beam is not a plane wave but a spherical wave

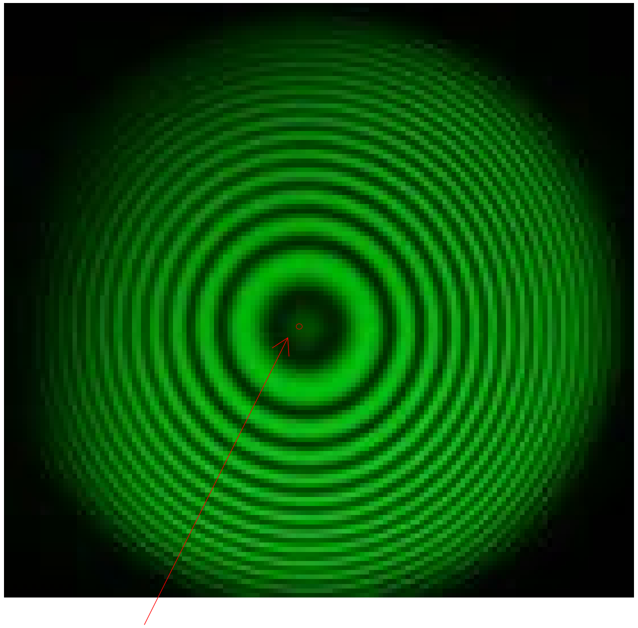
  → interference pattern

  (and the complementary pattern in reflection)

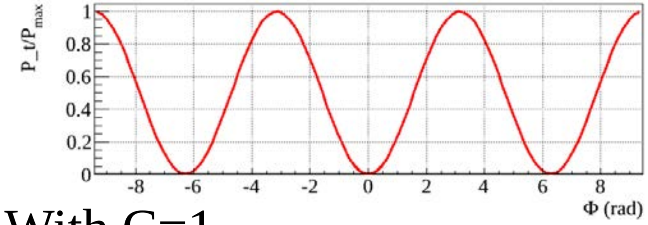
- Virgo interference pattern much larger than the beam size:

  ~1 m between two consecutive fringes

  → we do not study the fringes in nice images !



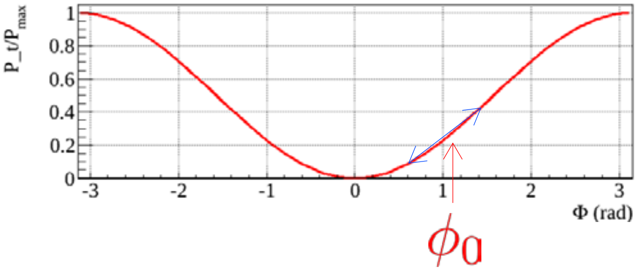
Equivalent size of Virgo beam



With C=1

Freely swinging mirrors

Setting a working point →



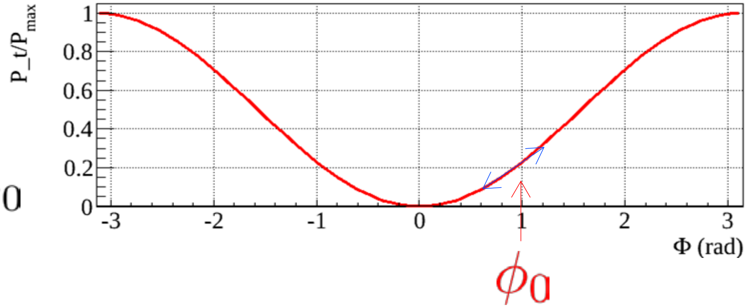
Controlled mirror positions

From the power to the gravitational wave

$$P_t = \frac{P_i}{2}\left(1 - C\cos(\phi)\right) \qquad \text{where } \phi = 2\frac{2\pi}{\lambda}(l_y - l_x)$$

- Around the working point:

$$\left.\frac{\mathrm{d}P_t}{d\phi}\right|_{\phi_0} = \frac{P_i}{2} C\sin(\phi_0) \qquad \text{where } \phi_0 = \frac{4\pi}{\lambda}\Delta L_0$$



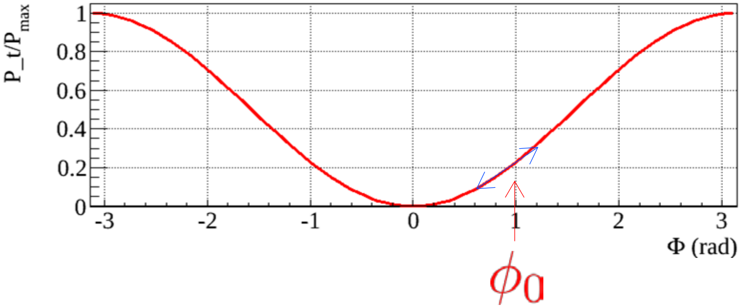
- Power variations as function of small differential length variations:

$$\delta P_t = \frac{P_i}{2} C\sin(\phi_0)\delta\phi$$

$$\delta P_t = P_i C \frac{2\pi}{\lambda}\sin\left(\frac{4\pi}{\lambda}\Delta L_0\right)\delta\Delta L$$

$\delta P_t \propto \delta\Delta L = hL_0$ around the working point !

From the power to the gravitational wave



- Around the working point:

$$\delta P_t = \underbrace{P_i\, C\, \frac{2\pi}{\lambda} \sin\left(\frac{4\pi}{\lambda}\Delta L_0\right)}\, \delta\Delta L$$

$$\delta P_t = \text{(Interferometer response)} \times \delta\Delta L$$

(W/m)

Measurable physical quantity

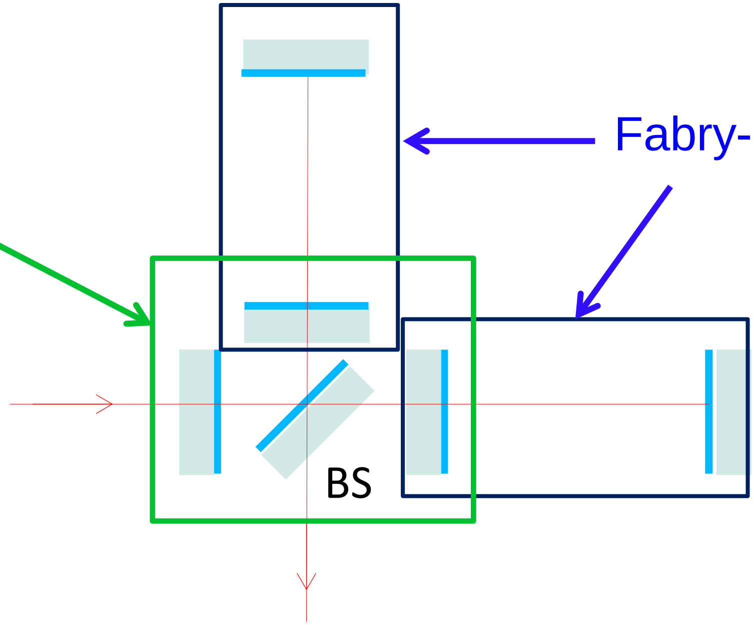
Physical effect to be detected

Improving the interferometer sensitivity

$$\delta P_t = \boxed{P_i} C \sin\left(\frac{4\pi}{\lambda}\Delta L_0\right) (\underbrace{\boxed{k}\delta\Delta L)}_{\propto\, \delta\phi}$$

Increase the input power on BS

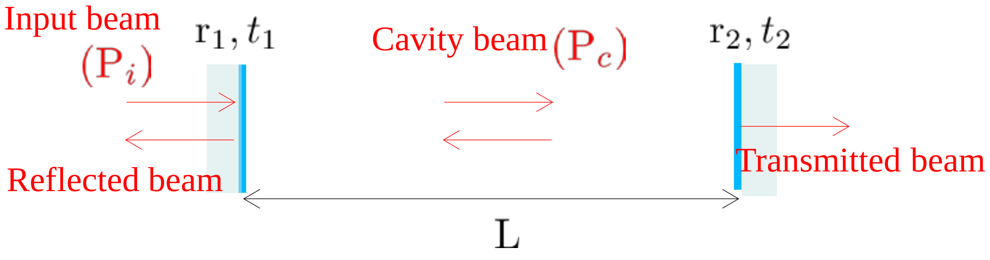
Increase the phase difference between the arms for a given differential arm length variation



Recycling **cavity**

Fabry-Perot **cavities** in the arms

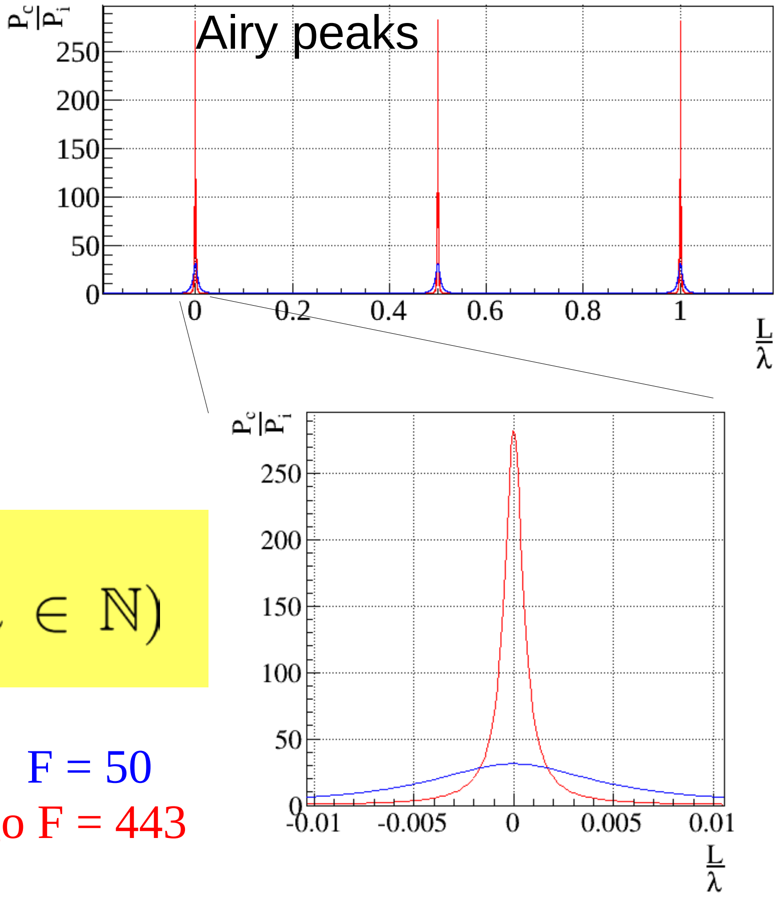
Beam resonant inside the cavities



$$P_c = P_i \frac{t_1^2}{(1-r_1r_2)^2} \frac{1}{1+\left(\frac{2\mathcal{F}}{\pi}\right)^2 \sin^2(kL)}$$

$$\text{Finesse } \mathcal{F} = \frac{\pi\sqrt{r_1r_2}}{1-r_1r_2}$$

Virgo cavity at resonance: $L = n\,\dfrac{\lambda}{2} \quad (n \in \mathbb{N})$

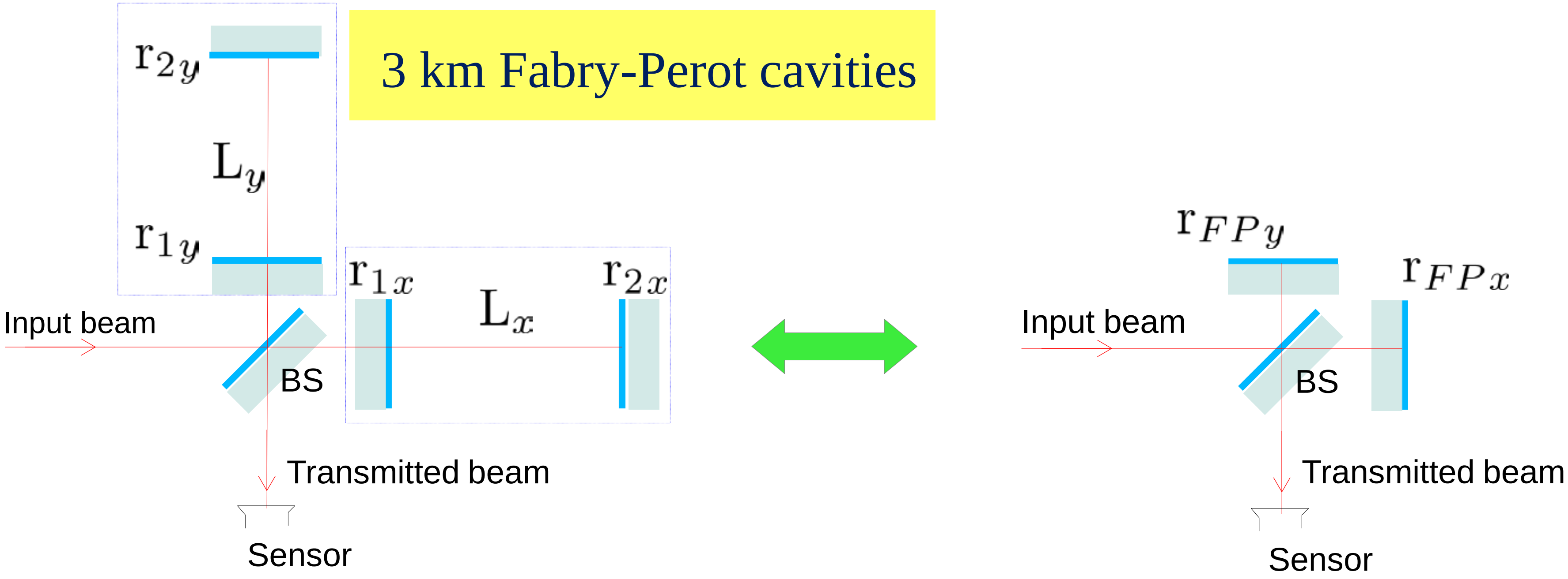


Virgo F = 50

AdVirgo F = 443

Average number of light round-trips in the cavity: $N = \dfrac{2\mathcal{F}}{\pi}$

How do we amplify the phase offset?



3 km Fabry-Perot cavities

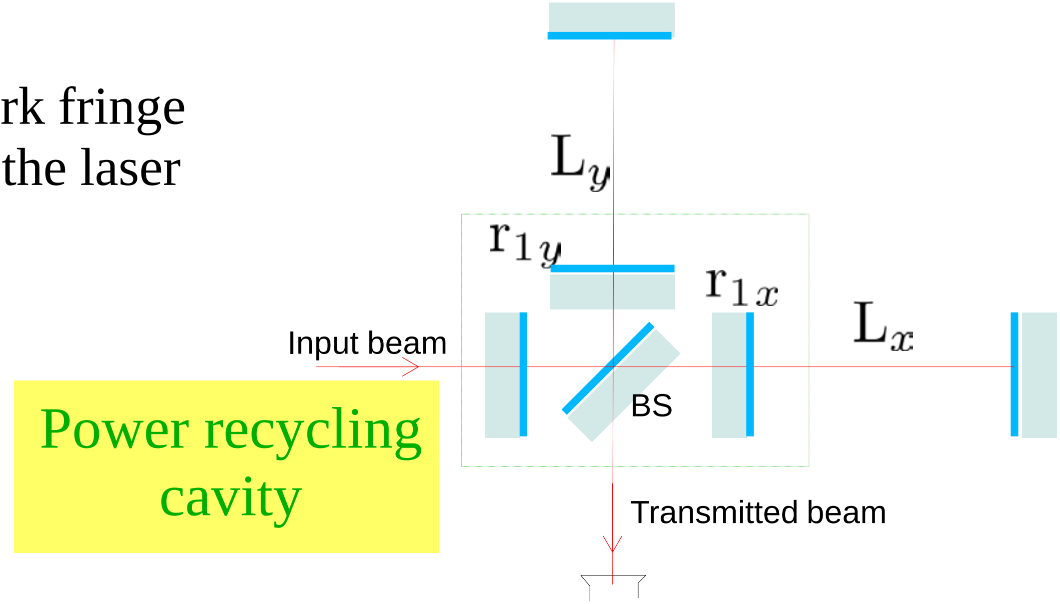
$$r_{FPx} = -1 \times e^{J \frac{2\mathcal{F}}{\pi} 2\, k\, \delta L_x}$$

~number of round-trips in the arm
~300 for AdVirgo

(instead of $r_{arm x} = -1 \times e^{J 2k(L_x + \delta L_x)}$ in the arm of a simple Michelson)

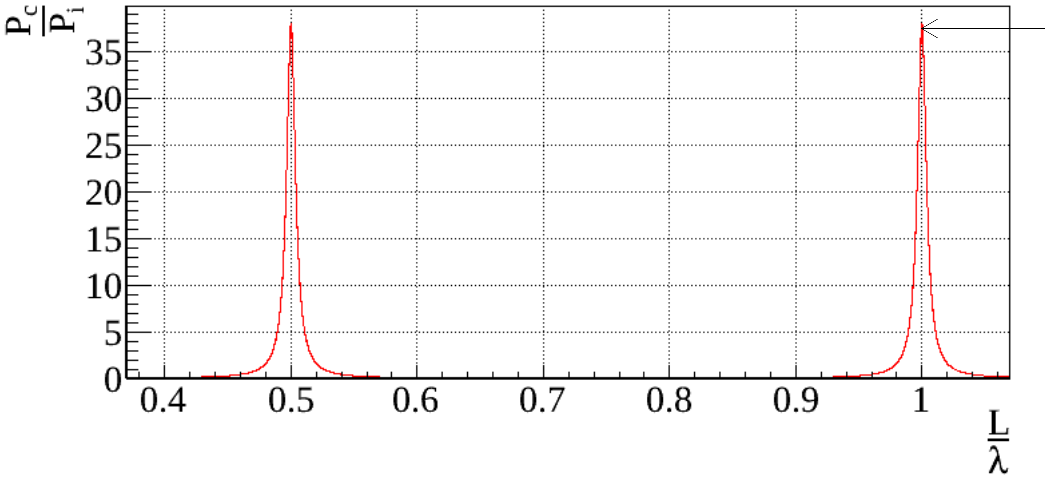
How do we increase the power on BS?

Detector working point close to a dark fringe
→ most of power go back towards the laser



Power recycling cavity

Resonant power recycling cavity



$$G_{PR} = 38 \quad (r_{PR}^2 = 0.95)$$

→ input power on BS increased by a factor 38!

Improved interferometer response

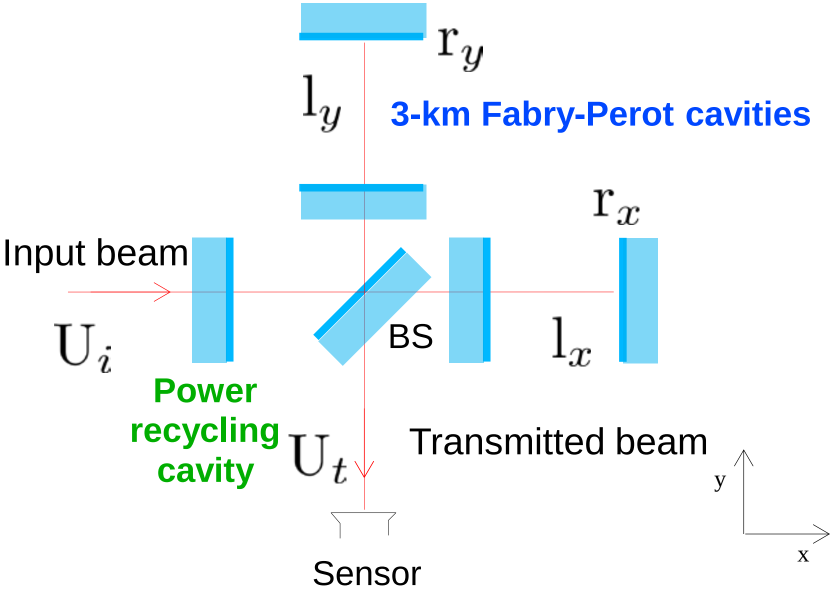
- **Response of simple Michelson:**

$$\delta P_t = \boxed{P_i\, C\, \frac{2\pi}{\lambda} \sin\left(\frac{4\pi}{\lambda}\Delta L_0\right)}\, \delta\Delta L$$

$$\delta P_t = (\text{Michelson response}) \times \delta\Delta L$$

(W/m)

- **Response of recycled Michelson with Fabry-Perot cavities:**



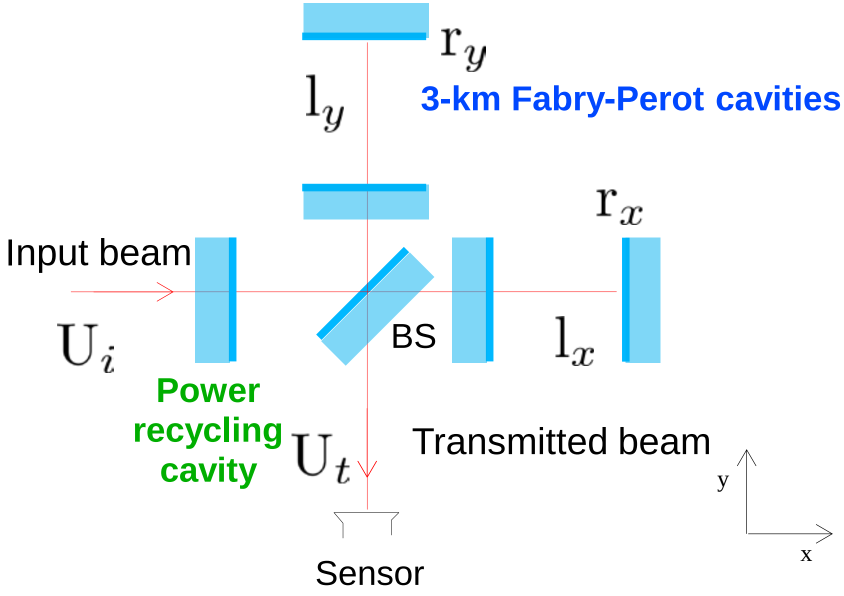
$$\delta P_t = \underbrace{G_{PR}}_{\sim 38}\, P_i\, C\, \frac{2\pi}{\lambda} \sin\left(\frac{4\pi}{\lambda}\Delta L_0\right) \underbrace{\frac{2\mathcal{F}}{\pi}}_{\sim 300}\, \delta\Delta L$$

For the same $\delta\Delta L$,
δP_t has been increased by a factor ~12000

Order of magnitude of the « sensitivity »

$$\delta P_t = G_{PR} P_i \, C \, \frac{2\pi}{\lambda} \sin\left(\frac{4\pi}{\lambda}\Delta L_0\right) \frac{2\mathcal{F}}{\pi} \, \delta\Delta L$$

Laser wavelength	$\lambda = 1064$ nm
Input power	$P_i \sim 100$ W
Interferometer contrast	$C \sim 1$
Cavity finesse	$\mathcal{F} \sim 450$
Power recycling gain	$G_{PR} \sim 38$
Working point	$\Delta L_0 \sim 10^{-11}$ m



Shot noise due to output power of $\sim 50\,\text{mW}$

$\rightarrow \delta P_{t,min} \sim 0.1\,\text{nW}$ ⟶ $\delta\Delta L_{min} \sim 5 \times 10^{-20}$ m

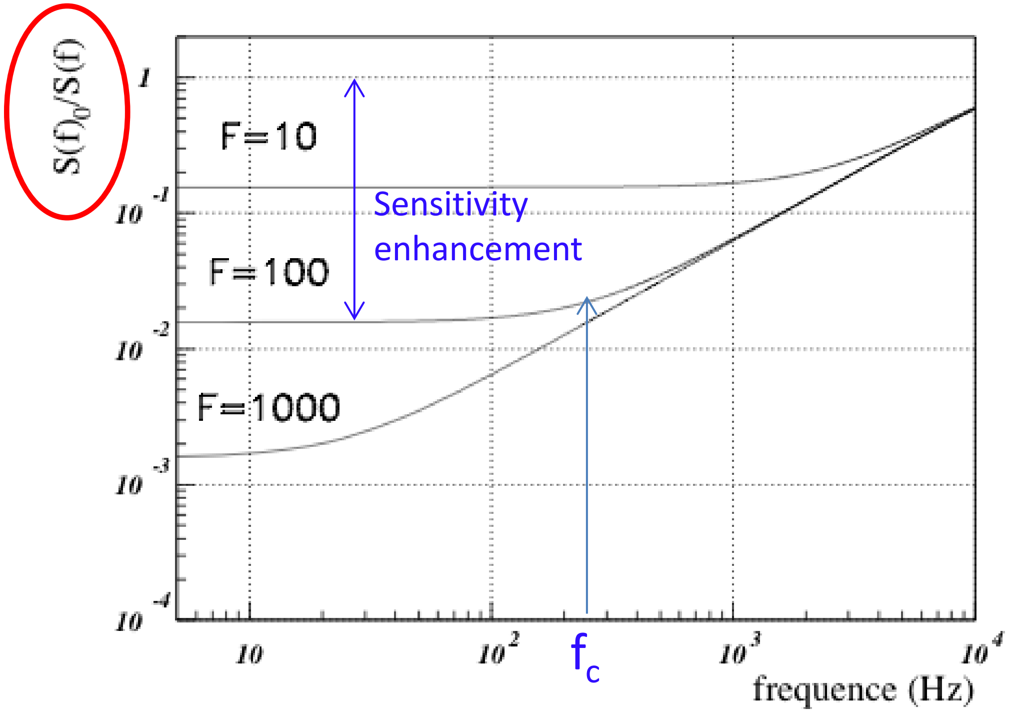
$$\rightarrow h_{min} = \frac{\delta\Delta L_{min}}{L} \sim 10^{-23}$$

In reality, the detector response depends on frequency...

Example of frequency dependency of the ITF response

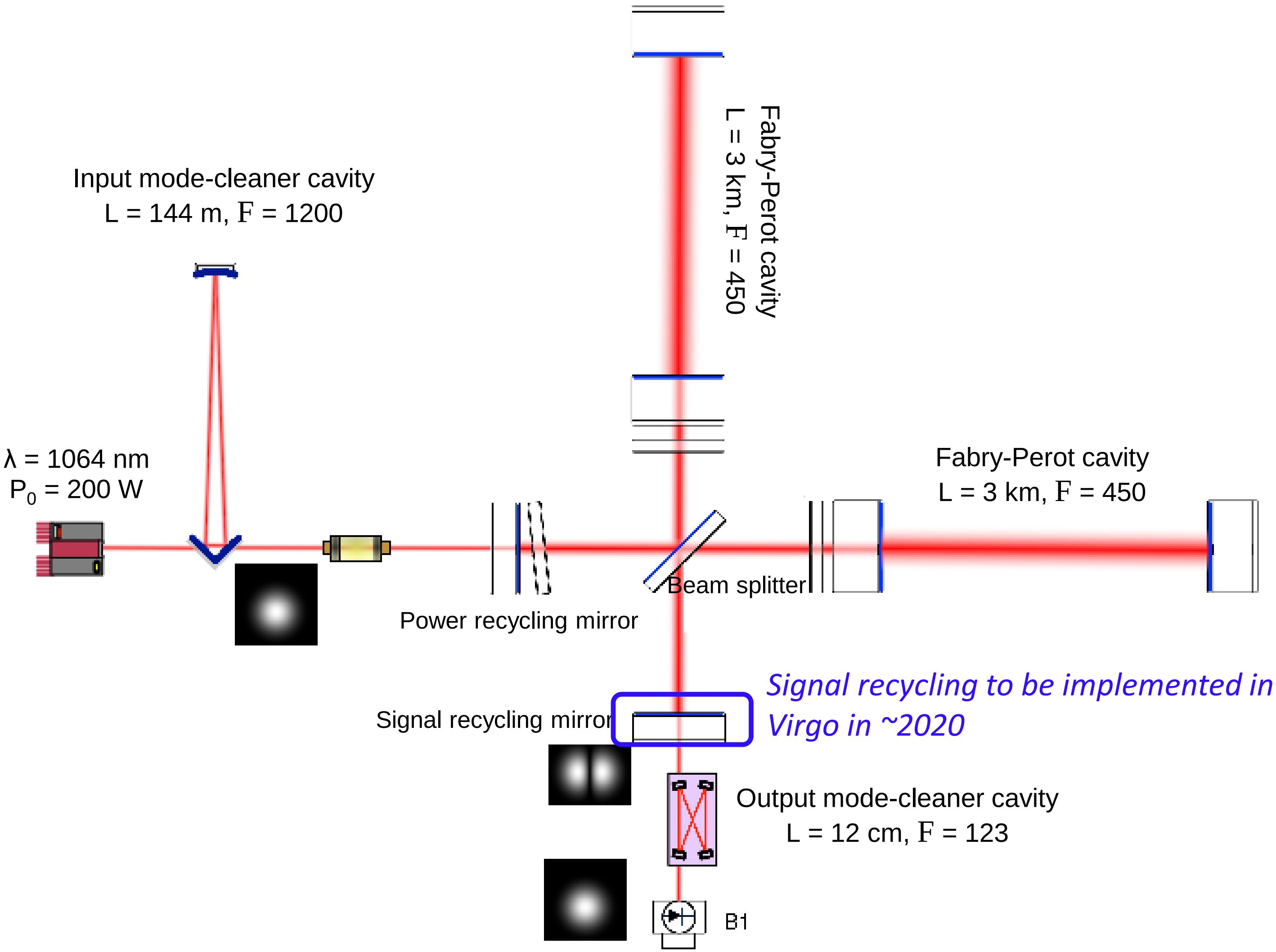
- Light travel time in the cavities must be taken into account
- Fabry-Perot cavities behave as a low pass filter
- Frequency cut-off: $f_c = \dfrac{c}{4\mathcal{F}L}$

Ratio between the sensitivity of an interferometer with Fabry-Perot cavities versus the sensitivity of an interferometer without cavities



- Finesse of Virgo Fabry Perot cavities: F = 450, L= 3 km → $f_c = 55$ Hz

Optical layout of Virgo



Input mode-cleaner cavity
L = 144 m, $\mathcal{F}$ = 1200

λ = 1064 nm
P_0 = 200 W

Fabry-Perot cavity
L = 3 km, $\mathcal{F}$ = 450

Fabry-Perot cavity
L = 3 km, $\mathcal{F}$ = 450

Beam splitter

Power recycling mirror

Signal recycling mirror

Signal recycling to be implemented in Virgo in ~2020

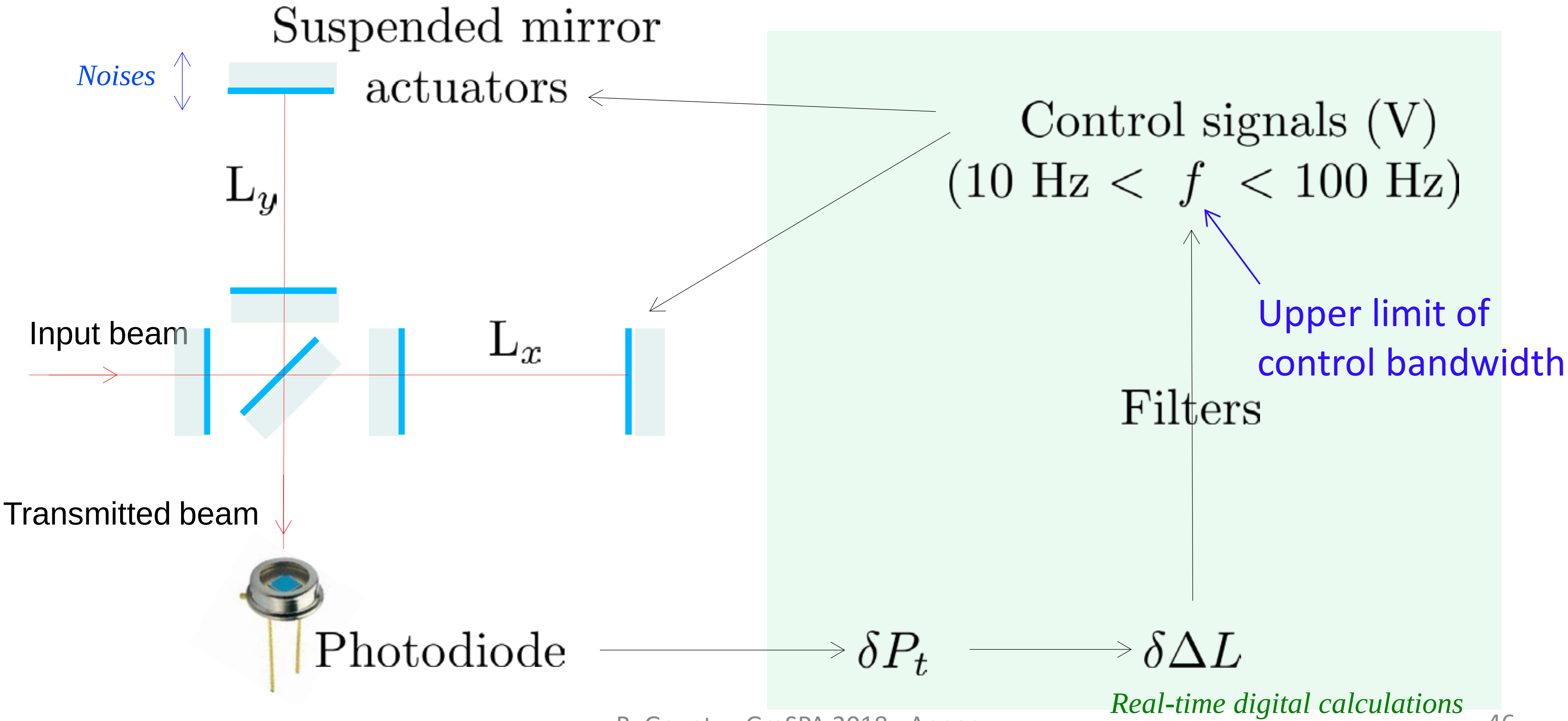
Output mode-cleaner cavity
L = 12 cm, $\mathcal{F}$ = 123

B1

How do we control the working point?

Small offset from a dark fringe: $\Delta L_0 = n\frac{\lambda}{2} + 10^{-11}\,\mathrm{m}$

- Controls to reduce the motion up to ~100 Hz
- Precision of the control $\delta\Delta L_{true}$ ~ 10^{-15} m



Suspended mirror

Noises

actuators

Control signals (V)

$(10\ \mathrm{Hz} < f < 100\ \mathrm{Hz})$

L_y

Upper limit of control bandwidth

Input beam

L_x

Filters

Transmitted beam

Photodiode

δP_t

$\delta\Delta L$

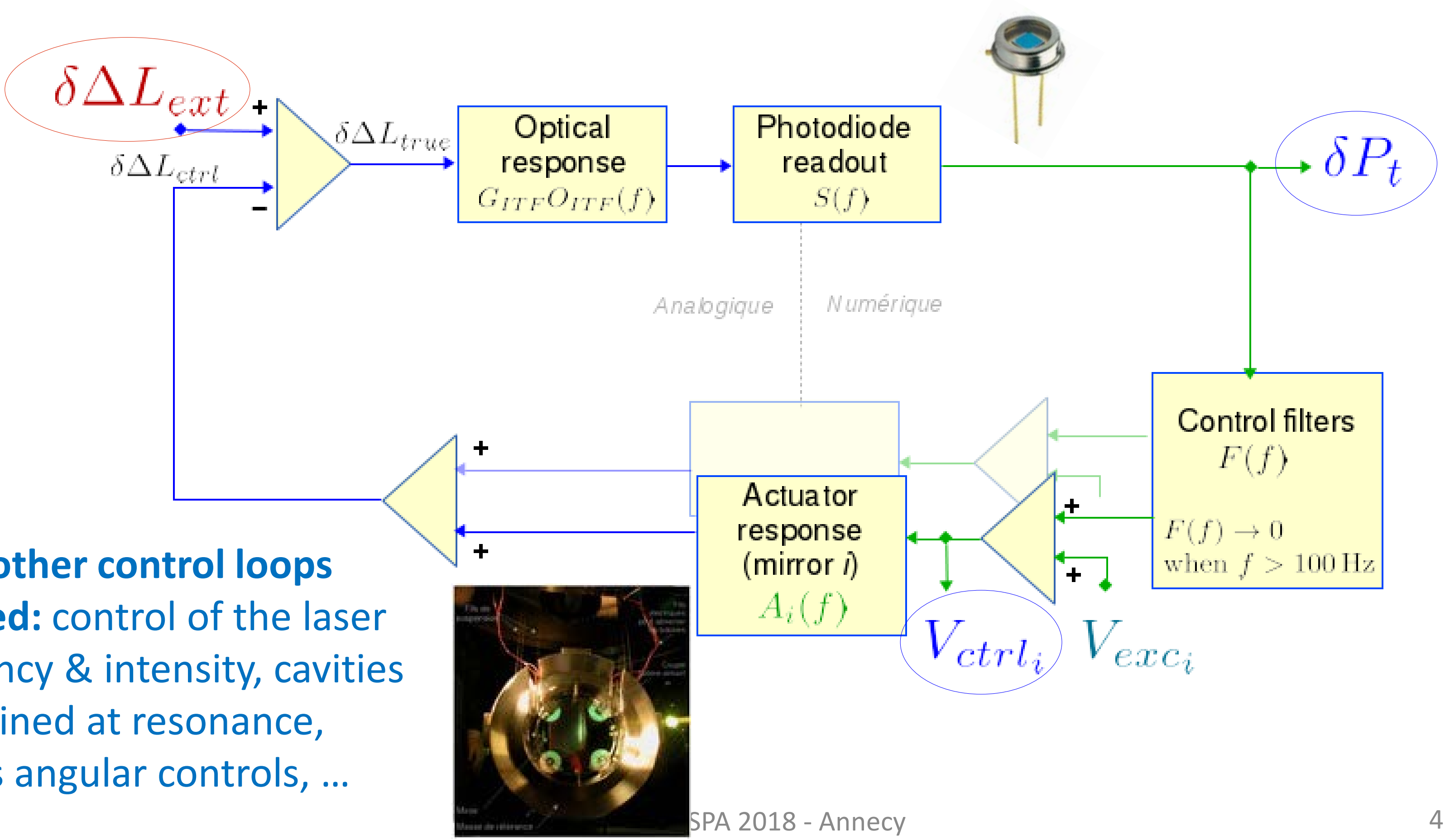
Real-time digital calculations

How do we control the working point?

Small offset from a dark fringe: $\Delta L_0 = n\frac{\lambda}{2} + 10^{-11}\,\mathrm{m}$

- Controls to reduce the motion up to ~100 Hz
- Precision of the control $\delta\Delta L_{true}$~ 10^{-15} m

$$\delta\Delta L_{ext} = \delta\Delta L_{noise} + \delta\Delta L_{GW}$$



Many other control loops required: control of the laser frequency & intensity, cavities maintained at resonance, mirrors angular controls, …

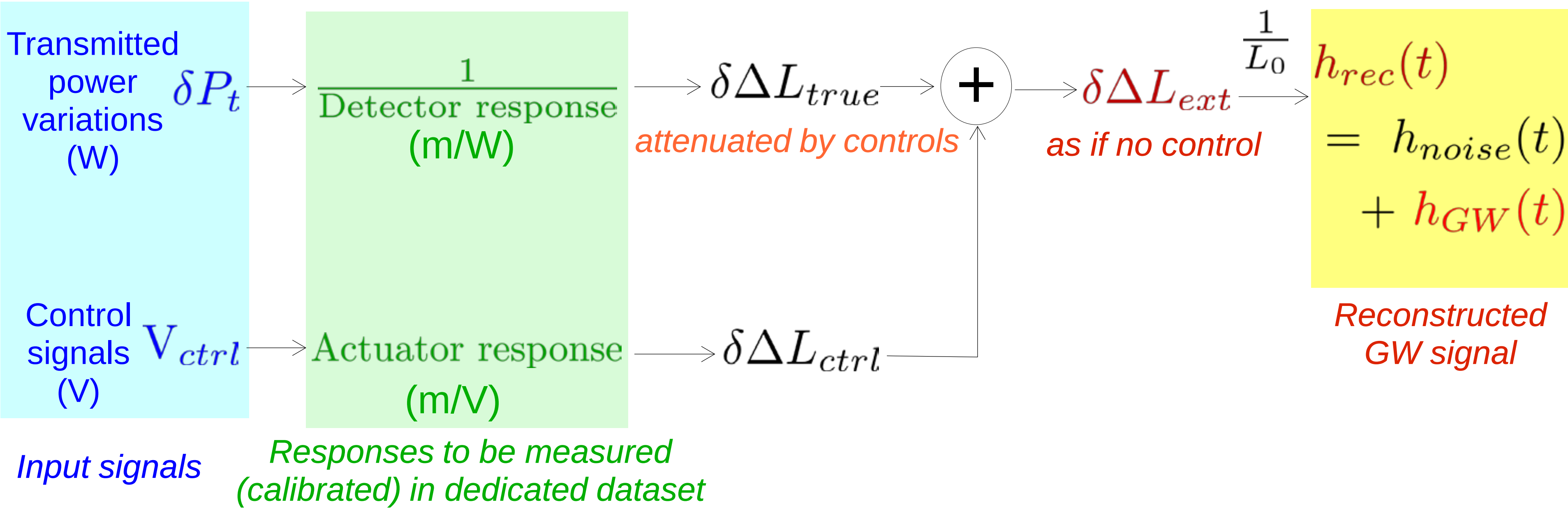
From the detector data to the GW strain h(t)

- High frequency (>100 Hz): mirrors behave as free falling masses

  → $h(t) = \frac{\delta \Delta L_{true}(t)}{L_0}$

- Lower frequency: the controls attenuate the noise... but also the GW signal!

  → the control signals contain information on *h(t)*



Transmitted power variations (W) δP_t

Control signals (V) V_{ctrl}

Input signals

$\frac{1}{\text{Detector response}}$ (m/W)

Actuator response (m/V)

Responses to be measured (calibrated) in dedicated dataset

$\delta \Delta L_{true}$

attenuated by controls

$\delta \Delta L_{ctrl}$

$+$

$\delta \Delta L_{ext}$

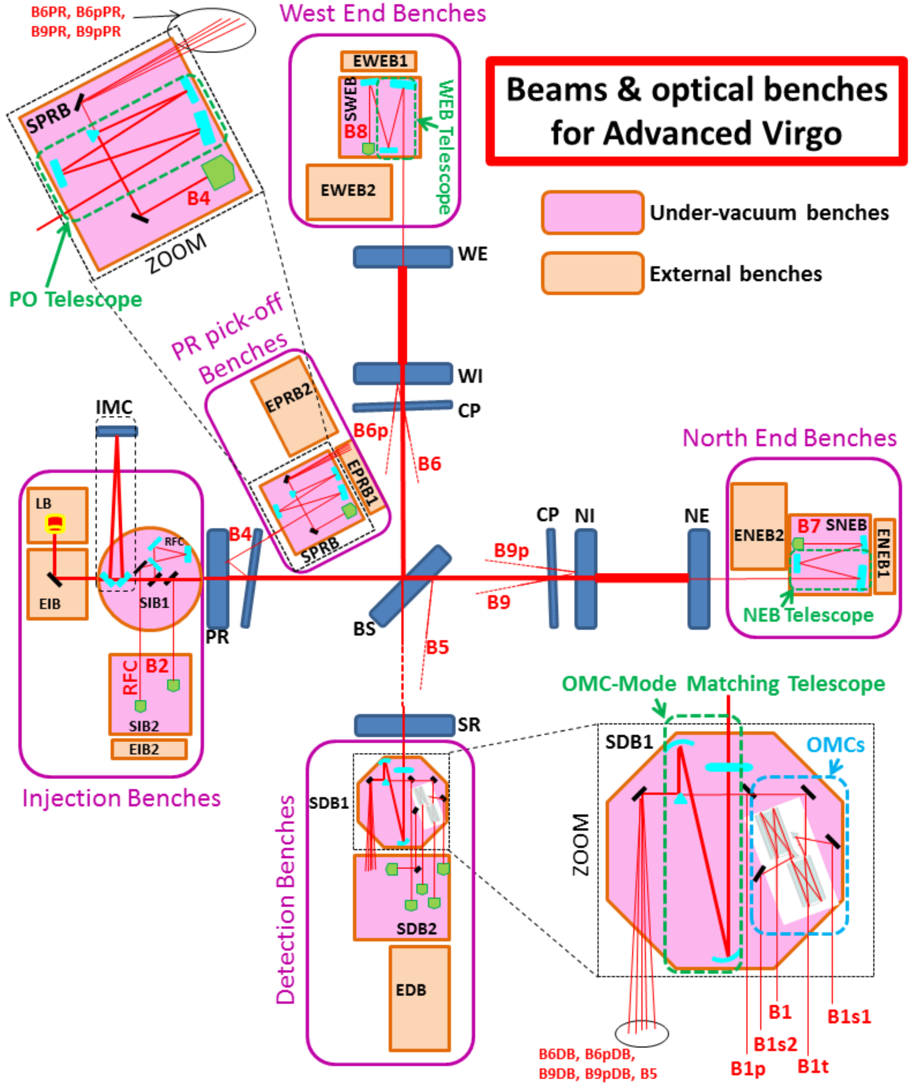
as if no control

$\frac{1}{L_0}$

$h_{rec}(t)$
$= h_{noise}(t)$
$+ h_{GW}(t)$

Reconstructed GW signal

How to extract all error signals?
Interferometer optical ports



Beams & optical benches for Advanced Virgo

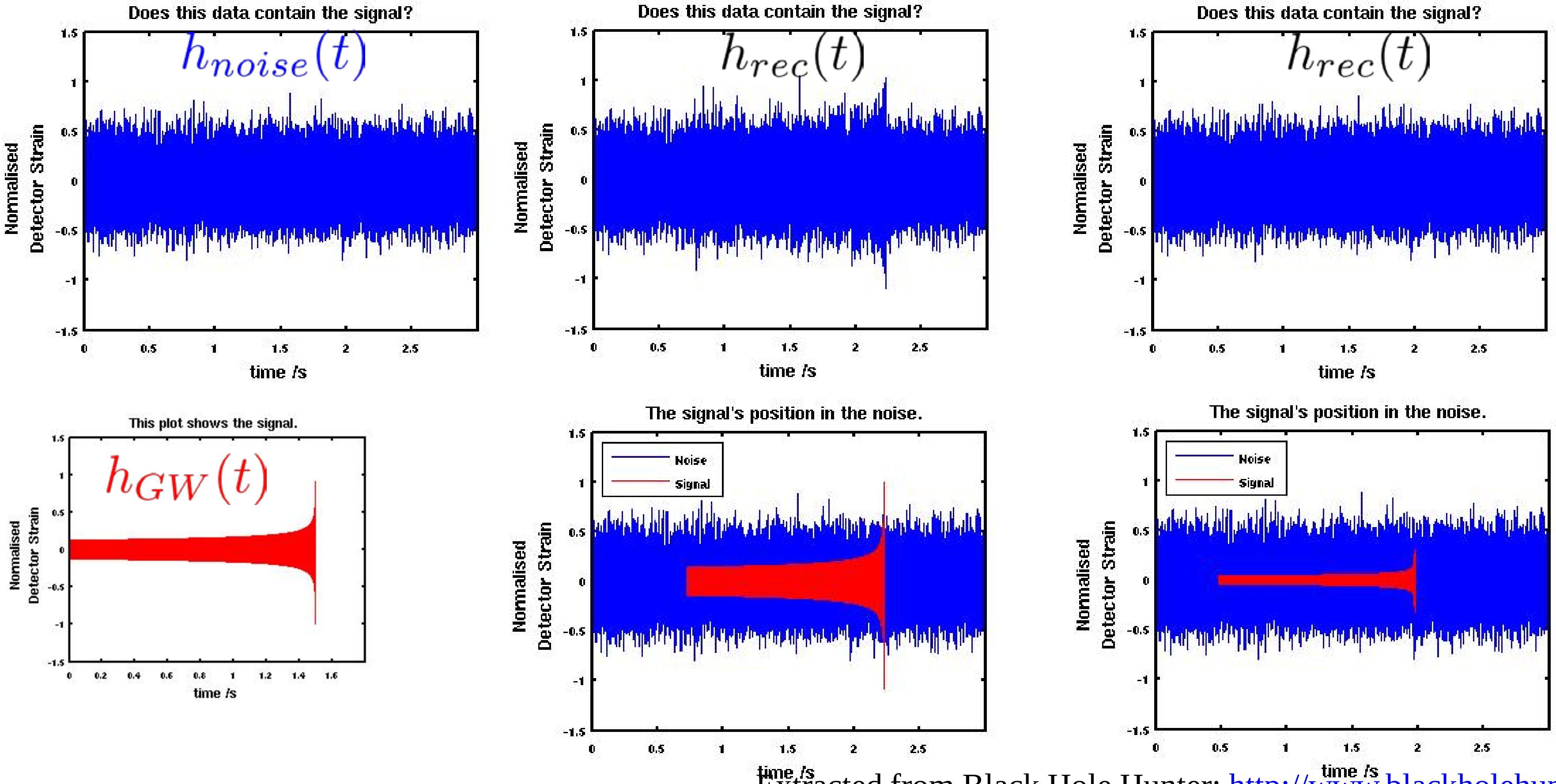
- 🟪 Under-vacuum benches
- 🟧 External benches

Noises limiting interferometer sensitivity: How to mitigate them ?

What is noise in Virgo?

- Stochastic (random) signal that contributes to the signal $h_{rec}(t)$ but does not contain information on the gravitational wave strain $h_{GW}(t)$

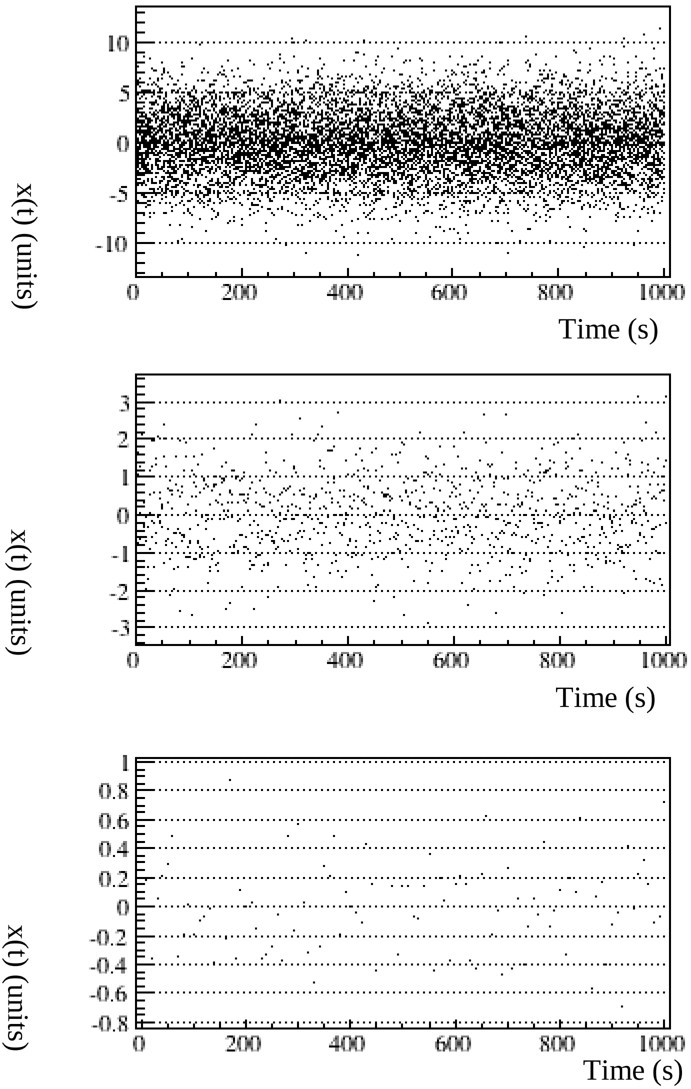
$$h_{rec}(t) = h_{noise}(t) + h_{GW}(t)$$



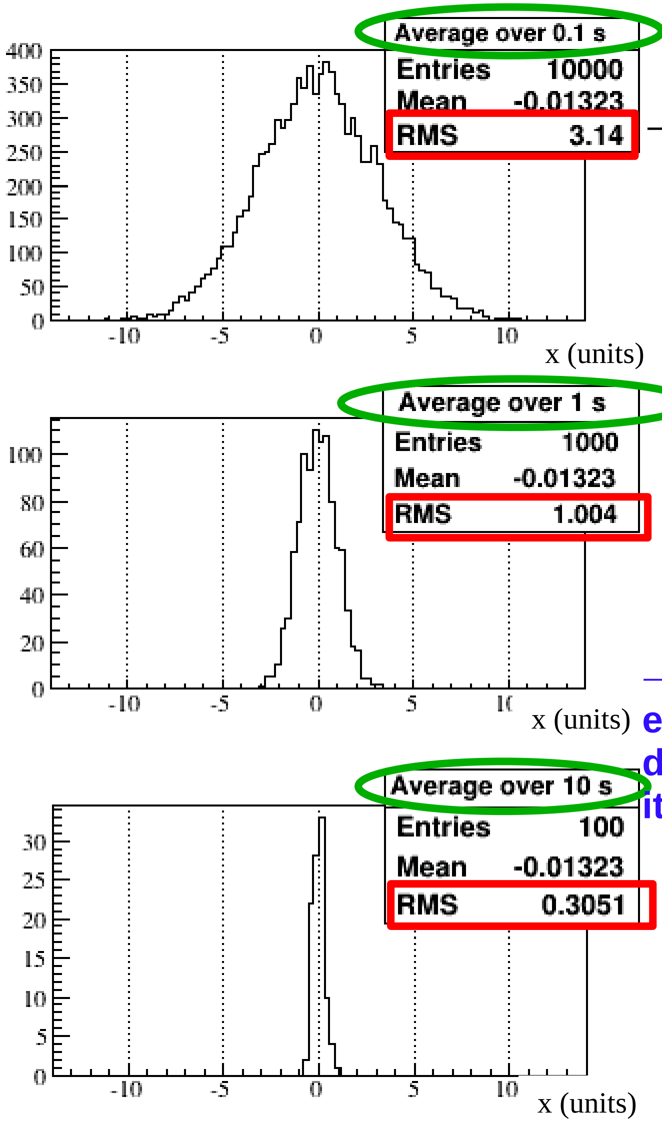
Extracted from Black Hole Hunter: http://www.blackholehunter.org/

How do we characterize noise?

Data points (noise)



Distribution of the data



→ Noise characterised by its standard deviation σ_x

$$\sigma_x = \frac{D}{\sqrt{\text{average duration}}}$$

D is in $(\text{Data units} \times \sqrt{\text{s}})$ or $\frac{\text{Data units}}{\sqrt{\text{Hz}}}$

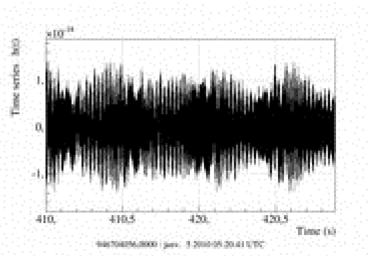
→ its absolute value is equal to the standard deviation of the noise when it is averaged over 1 s

→ D = 1 units/√Hz

From hrec(t) to Virgo sensitivity curve

1/ Reconstruction of h(t)

$$h_{rec}(t) = h_{noise}(t) + \cancel{h_{GW}(t)}$$



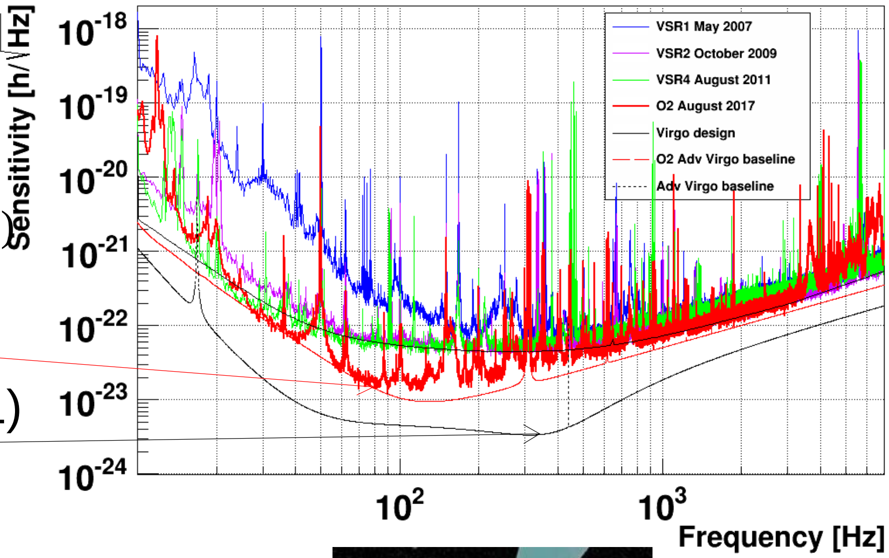
2/ Amplitude spectral density of *h(t)*
(noise standard deviation over 1 s)

$$ASD = \sqrt{PSD} = \sqrt{\frac{|DFT|^2}{T}}$$

Discrete Fourier Transform (DFT)

~5 x10^{-20} m/√Hz (Advanced Virgo O2, 2017)

~10^{-20} m/√Hz (Advanced Virgo nominal, ~2021)



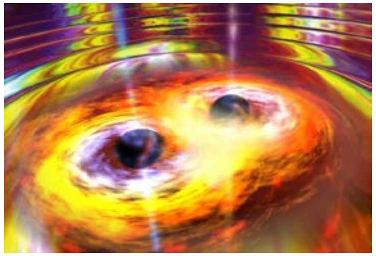


Image: Danna Berry/SkyWorks/NASA

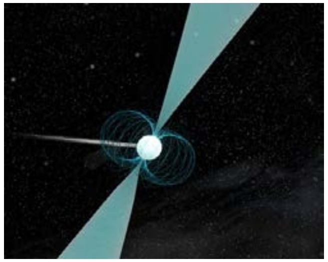


Image: B. Saxton (NRAO/AUI/NSF)

Compact Binary Coalescences

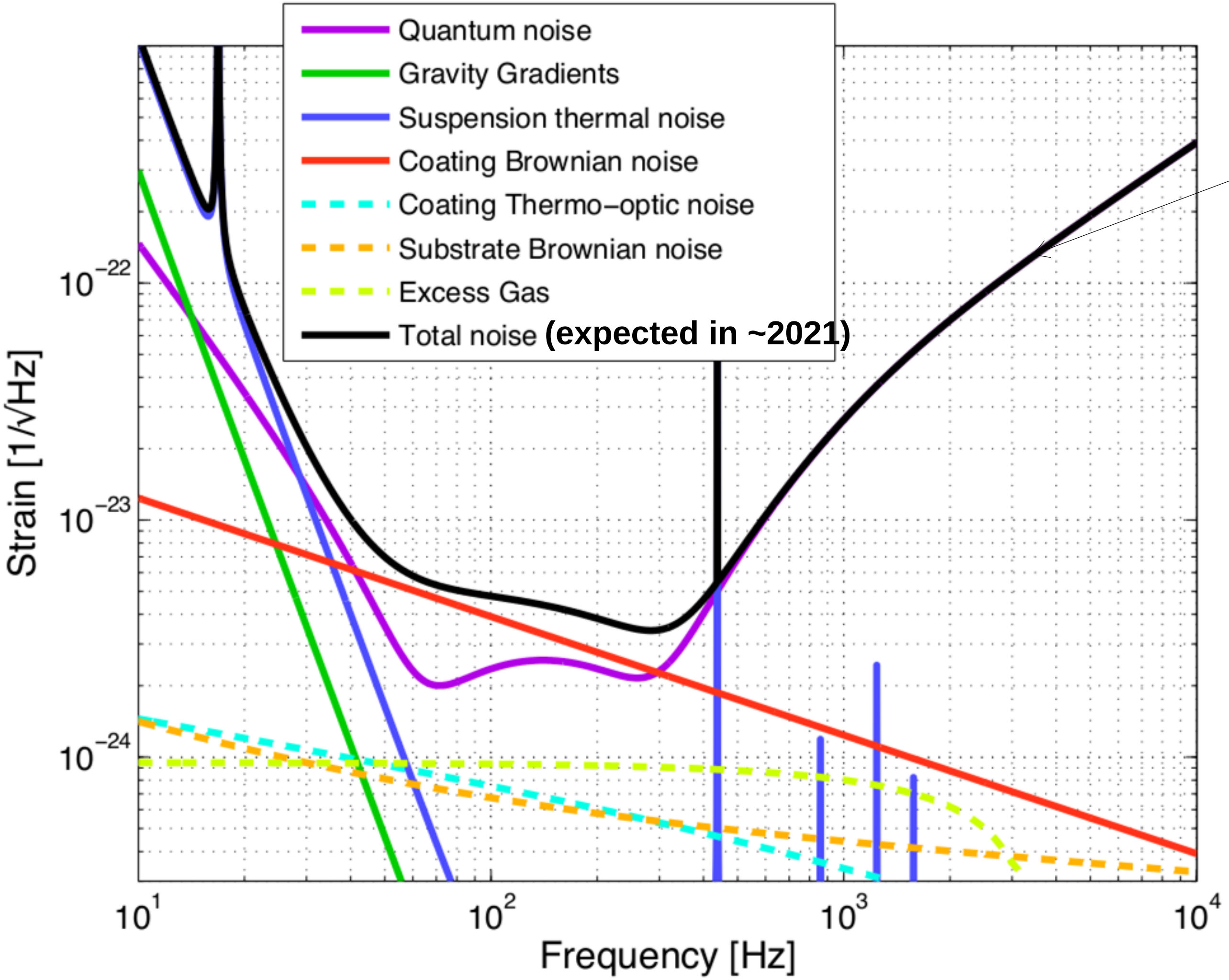
Signal lasts for a few seconds
→ can detect h ~ 10^{-23}

Rotating neutron stars

Signal averaged over days (~10^{6} s)
→ can detect h ~ 10^{-26}

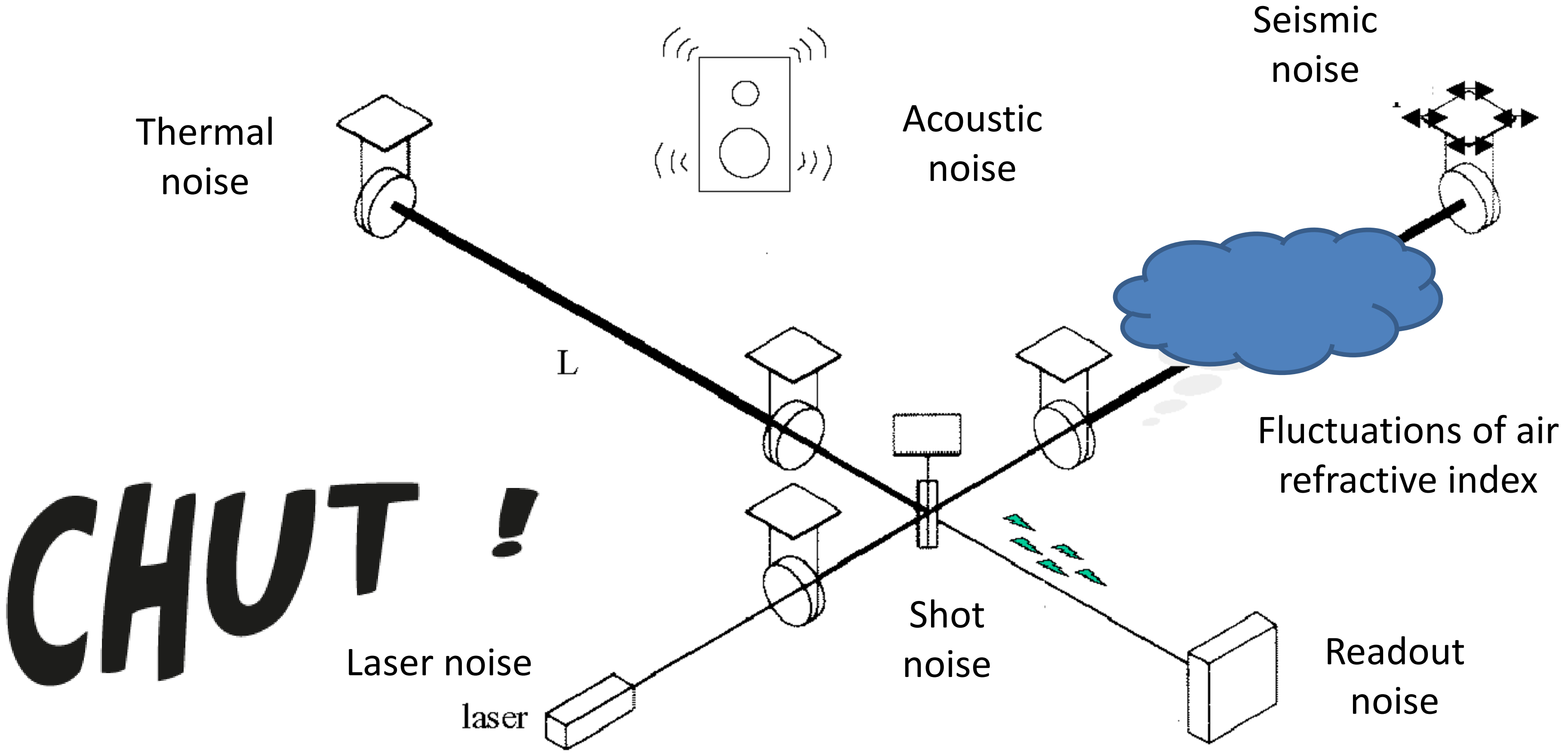
Nominal sensitivity of Advanced Virgo

Fundamental noise only
Possible technical noise not shown



Sum of all the noises

Fundamental noise sources



Shot noise

Fluctuations of arrival times of photons (**quantum noise**)

Power received by the photodiode: P_t

$\rightarrow \quad N = \frac{P_t}{\mathrm{h}\nu}$ photons/s on average.

Arrival time of single photons

Standard deviation on this number: $\sigma_N = \sqrt{N}$

$\rightarrow \quad \sigma_{P_t} = \sigma_N \times \mathrm{h}\nu = \sqrt{\frac{P}{\mathrm{h}\nu}}\mathrm{h}\nu = \sqrt{P_t \mathrm{h}\nu}$

Virgo laser: $\lambda = 1.064\,\mu\mathrm{m} \rightarrow \nu = \frac{\mathrm{c}}{\lambda} \sim 2.8 \times 10^{14}\,\mathrm{Hz}$

Working point: $P_t \sim 80\,\mathrm{mW} \quad \rightarrow \quad \sigma_{P_t} = 0.1\,\mathrm{nW}/\sqrt{\mathrm{Hz}}$

$\rightarrow$ a variation of power is interpreted as a variation of distance $\delta\Delta L$

$\delta P_t = (\text{Virgo response}) \times L_0 \times h$ $\qquad h_{equivalent} = \frac{1}{L_0}\frac{\sigma_{P_t}}{(\text{Virgo response})}$

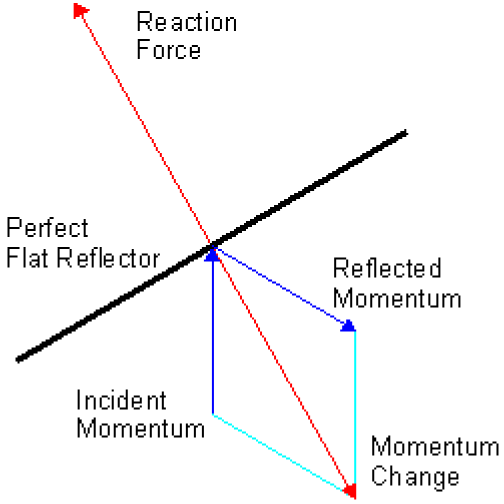
(in W/m)

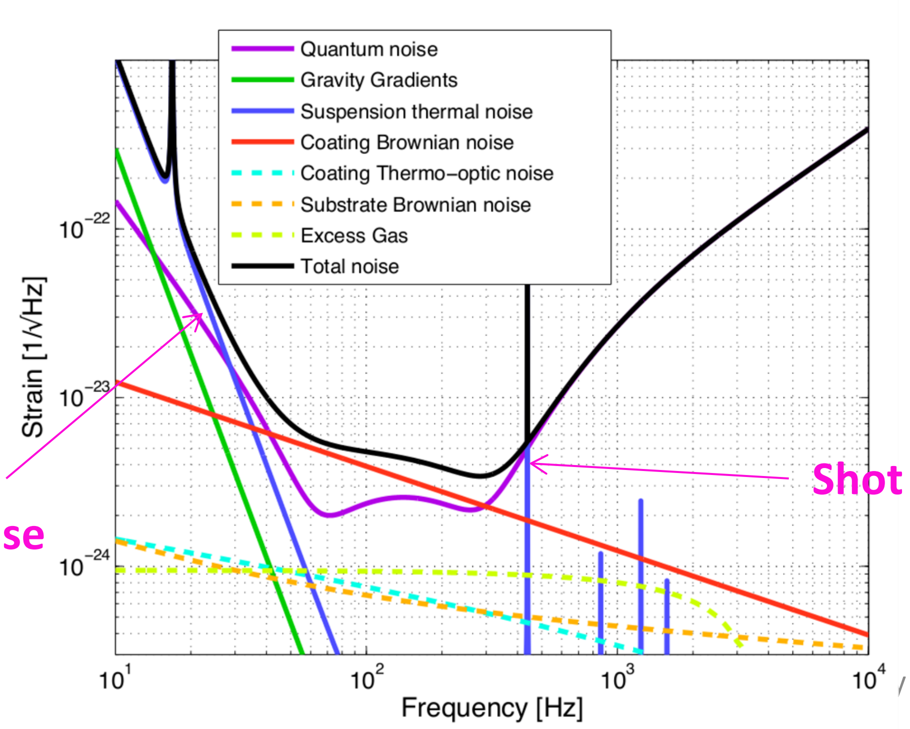
→ $\mathbf{h_{equivalent}\ \alpha\ 1/\sqrt{P_{in}}}$

Shot noise Vs radiation pressure noise

- Radiation pressure: transfer of photon’s momentum to the reflective surface (recoil force)
- Radiation pressure noise: due to fluctuations of number of photons hitting the mirror surfaces > mirror motion noise
- Radiation pressure noise impact at low frequency:

  > Mirror motion filtered by pendulum mechanical response





Radiation pressure noise

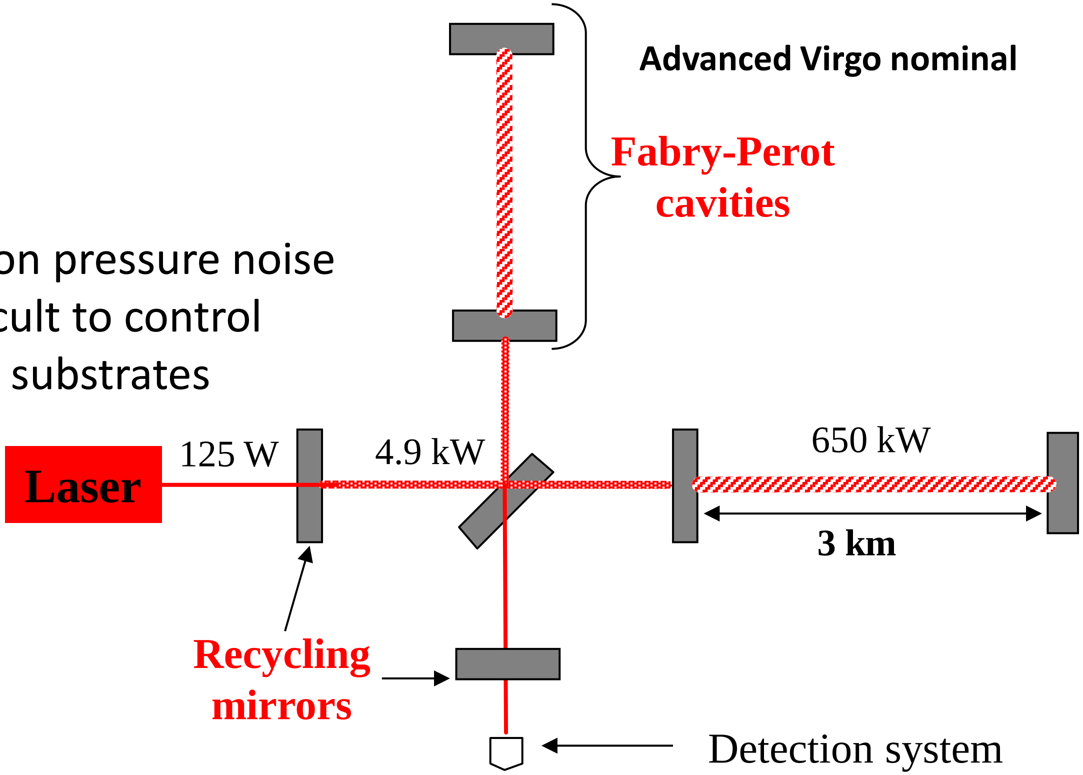
Shot noise

Minimizing impact of shot noise

- Drives **optical configuration**
- **Use high power laser**

  ⟶ limited by side-effects:

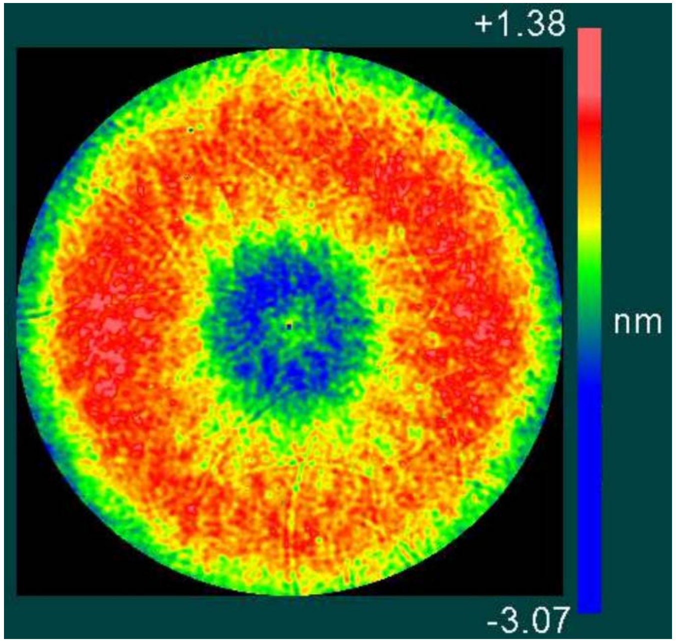
  - radiation pressure:
    - > increase of radiation pressure noise
    - > cavities more difficult to control
  - thermal absorption in mirror substrates (optical lensing)

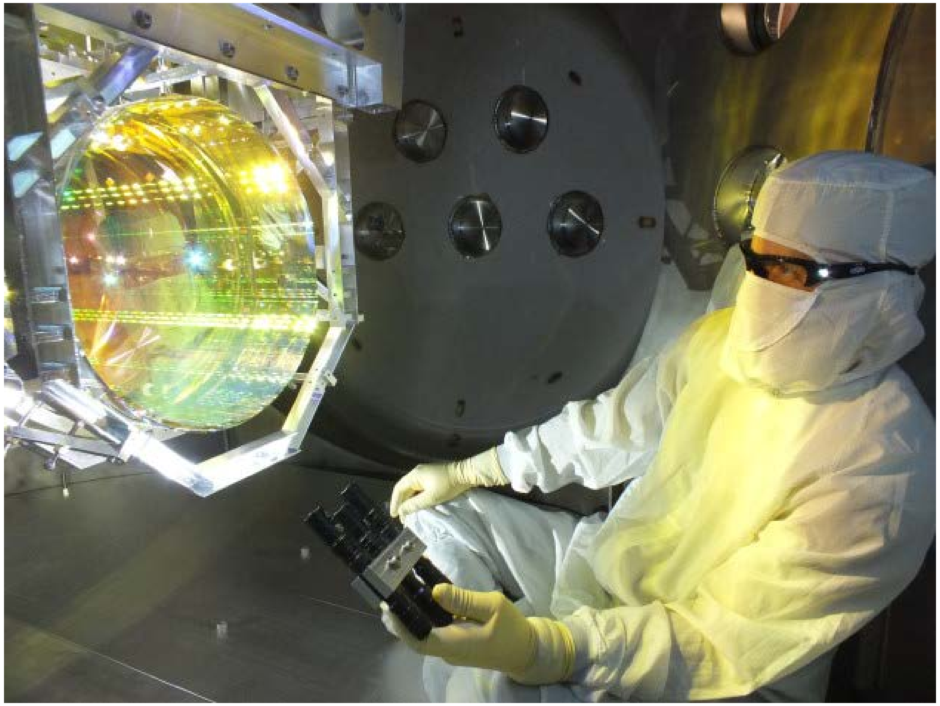


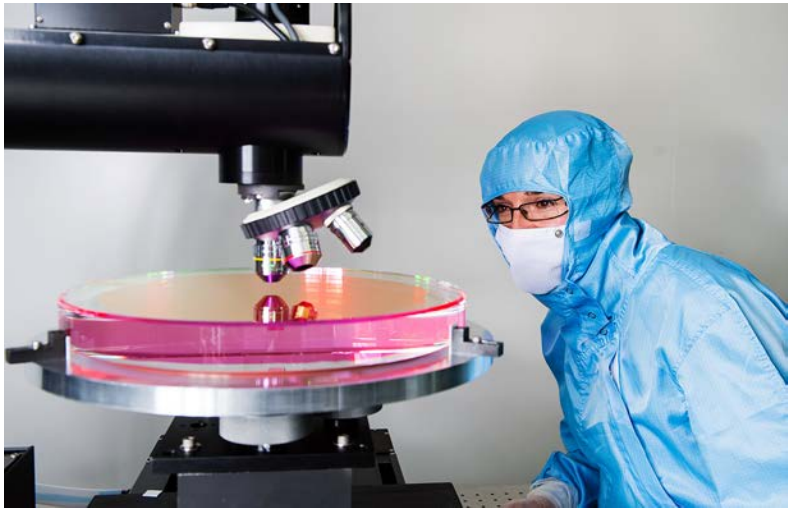
- Avoid optical losses > **high quality mirrors**
- Optimize contrast defect (C≈1) > **Output Mode Cleaner Cavity**

« Perfect » mirrors

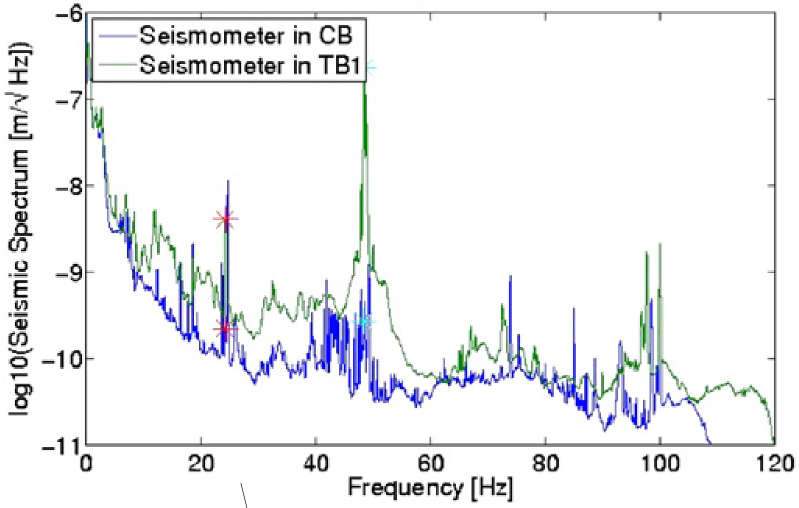
- 40 kg, 35 cm diameter, 20 cm thickness in ultra pure silica
- Uniformity of mirrors is unique in the world:
  - a few nanometers peak-to-valley
  - flatness < 0.5 nm RMS (over 150mm diameter)





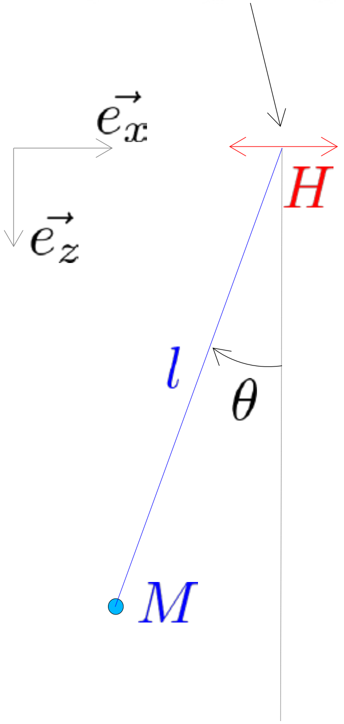


Seismic noise and suspended mirrors



Ground vibrations up to ~1 µm/√Hz at low frequency decreasing down to ~10 pm/√Hz at 100 Hz

$\gg 10^{-19}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$ needed to detect GW !!

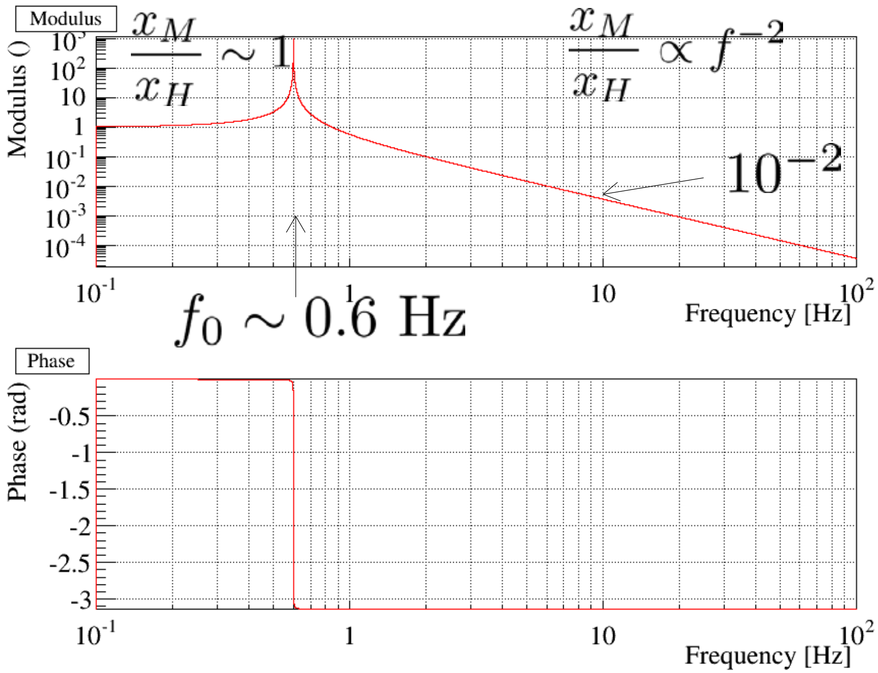


Assuming

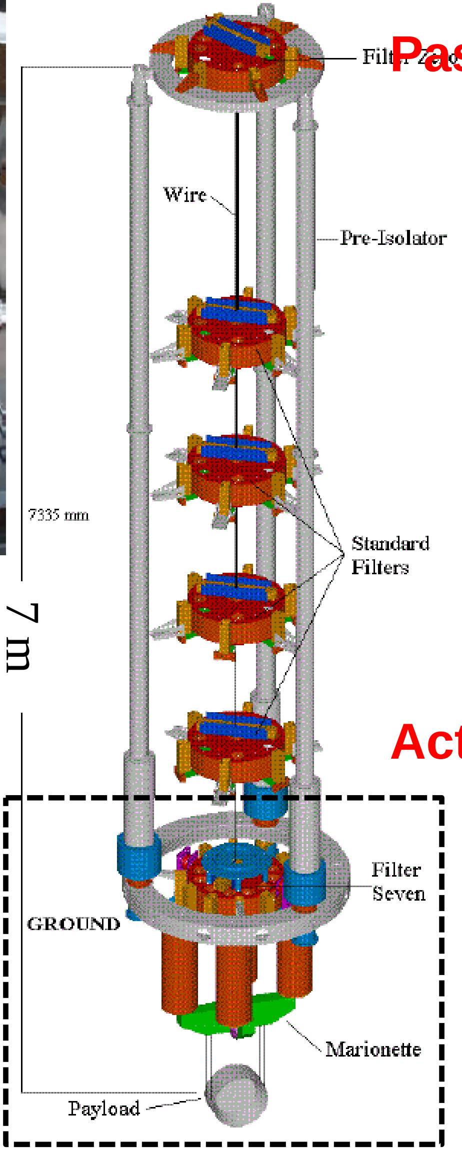
δx_H small and sinusoidal and θ small:

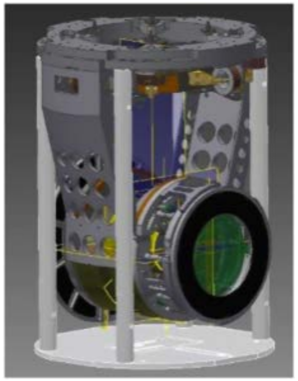
$$\underline{x_M} = \underline{\mathcal{H}} \times \underline{x_H}$$

Transfer function



Seismic noise: Virgo super-attenuators





Passive attenuation: 7 pendulum in cascade

$$\text{At 10 Hz: } \frac{x_{mirror}}{x_{ground}} \sim \left(10^{-2}\right)^7 = 10^{-14}$$

$$x_{ground} \sim 10^{-9}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$$

$$\rightarrow x_{mirror} \sim 10^{-23}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$$

This noise directly modifies the positions of the mirror surfaces, and thus $\delta\Delta L$ and $h_{rec}(t)$!

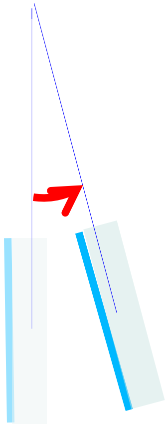
Active controls at low frequency

- Accelerometers or interferometer data
- Electromagnetic actuators
- Control loops

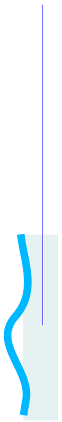
Thermal noise (pendulum and coating)

Microscopic thermal fluctuations

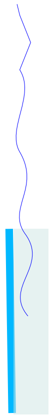
→ dissipation of energy through excitation of the macroscopic modes of the mirror



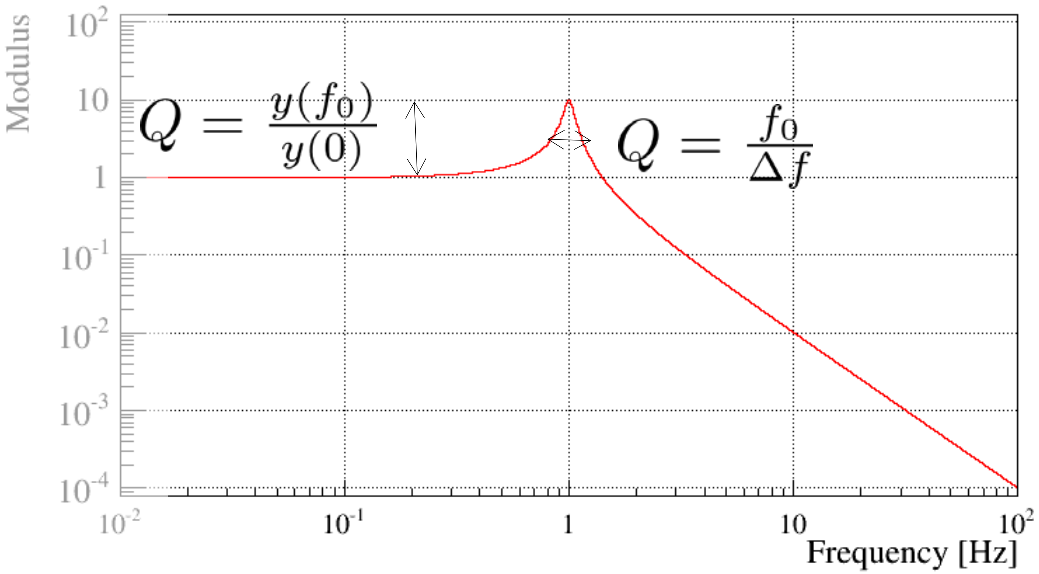
Pendulum mode
f < 40 Hz

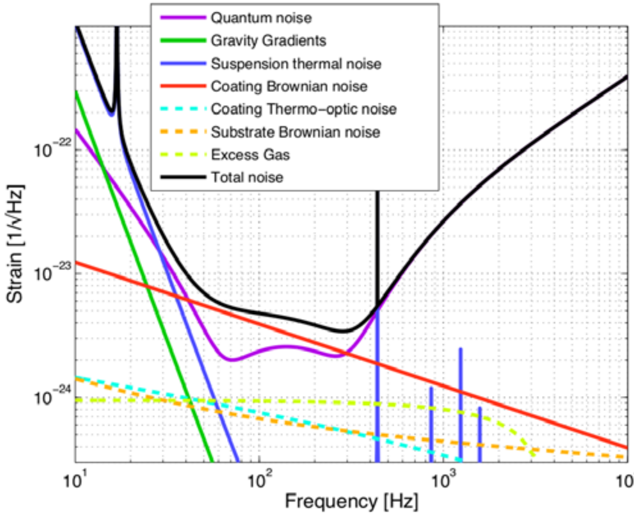


“Mirror” mode
f> few kHz



“Violin” modes
f > 40 Hz



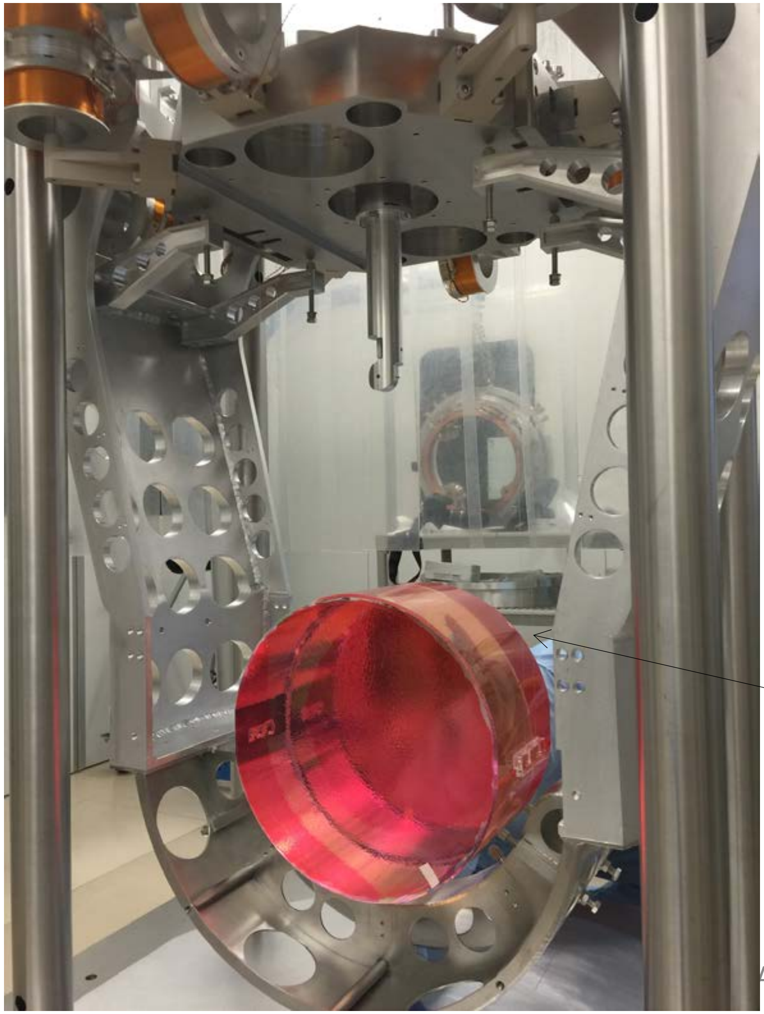


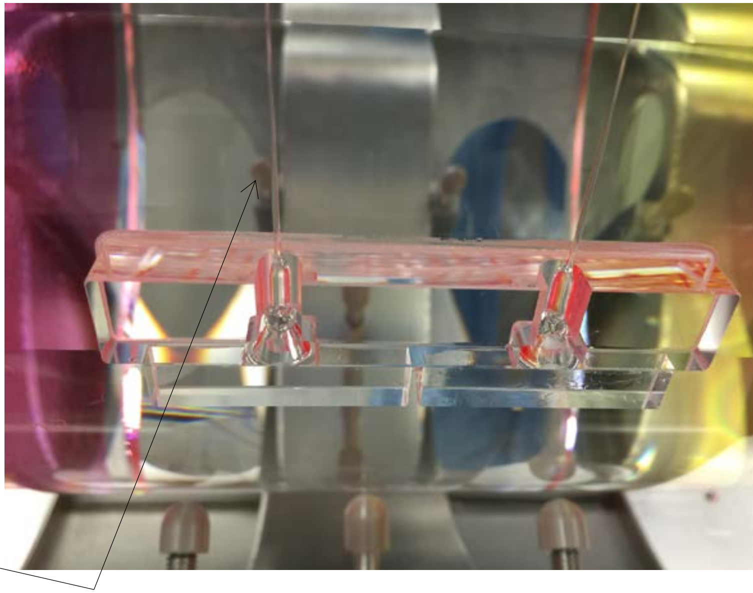
This noise directly modifies the positions of the mirror surfaces, and thus $\delta\Delta L$ and $h_{rec}(t)$!

We want high quality factors Q to concentrate all the noise in a small frequency band

Thermal noise: improving Q

- **Very high quality mirror coating** developed in a lab close to Lyon (Laboratoire des Matériaux Avancés)
- **Monolithic suspension** developed in labs in Perugia and Rome

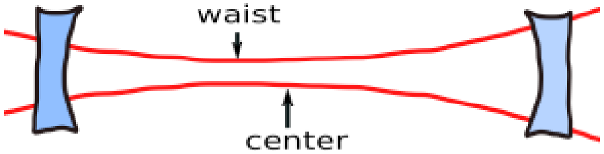




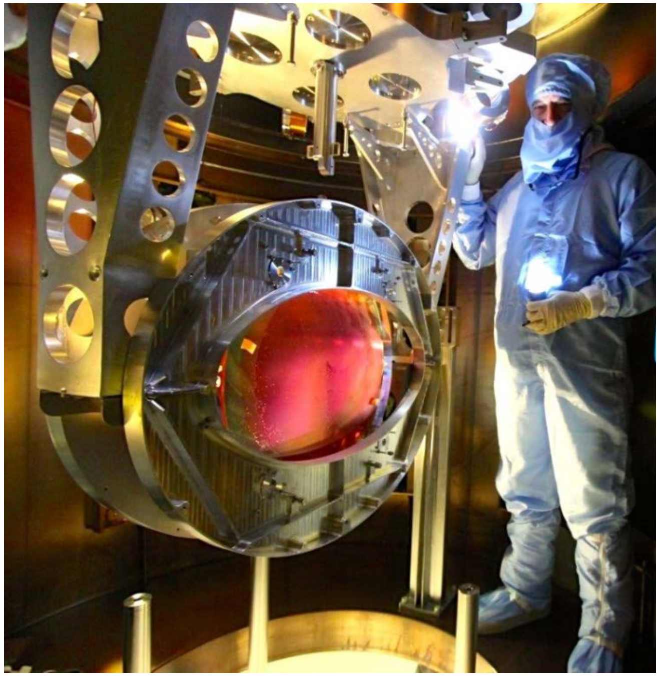
Fused-silica fibers
(diameter of 400 μm and length of 0.7 m)

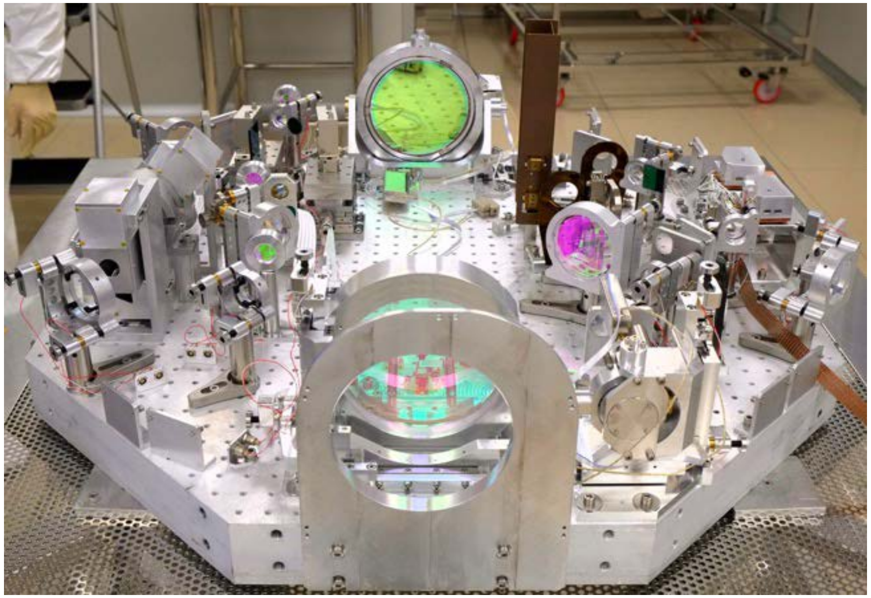
Thermal noise: coupling reduction

- Reduce the coupling between the laser beam and the thermal fluctuations
  → **use large beams**: fluctuations averaged over larger area
  → Thermal Noise ~1/D, with D = beam diameter



- Impact of large beams:
  - Require large beam splitter (diameter = 55 cm)
  - High magnification telescopes to adapt beam size to photodetectors (from w=50 mm on mirrors to w=0.3 mm on sensors) > require optical benches





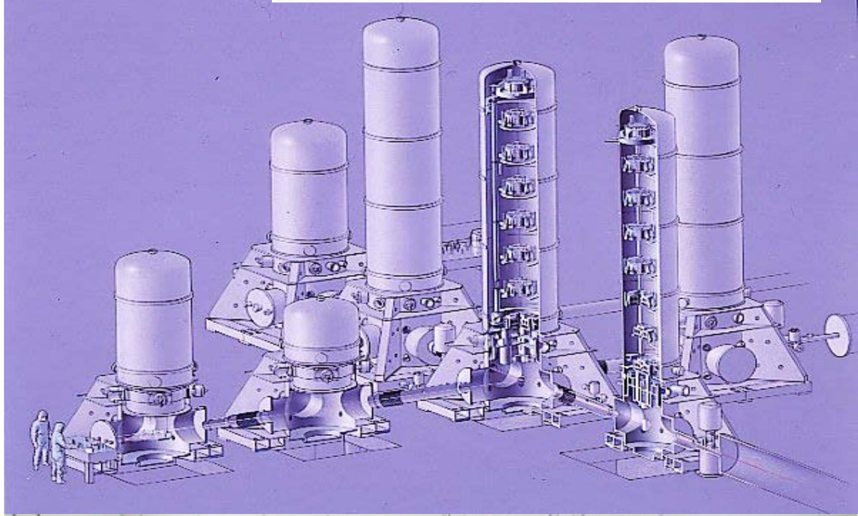
Under vacuum

Goals

- Isolation against acoustic noise
- Avoid measurement noise due to fluctuations of air refractive index
- Keep mirrors clean

Advanced Virgo vacuum in a few numbers:

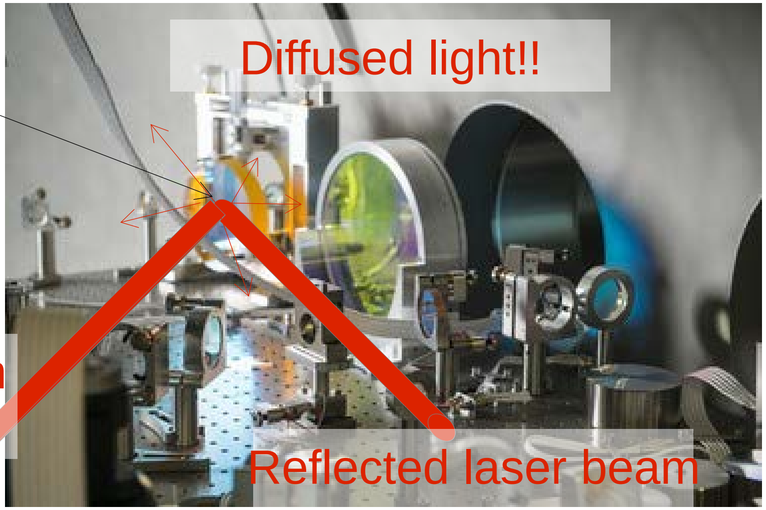
- Volume of vacuum system: 7000 m^3
- Different levels of vacuum:
  - 3 km arms designed for up to 10^{-9} mbar (Ultra High Vacuum)
  - ~10^{-6} - 10^{-7} mbar in mirror vacuum chambers (« towers »)
- Separation between arms and towers with cryotrap links



Example of technical noise: Diffused light

Optical element (mirror, lens, ...) vibrating due to seismic or acoustic noises

Incident laser beam



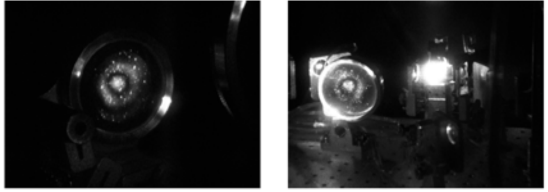
some photons of the diffused light gets recombined with the interferometer beam

↓

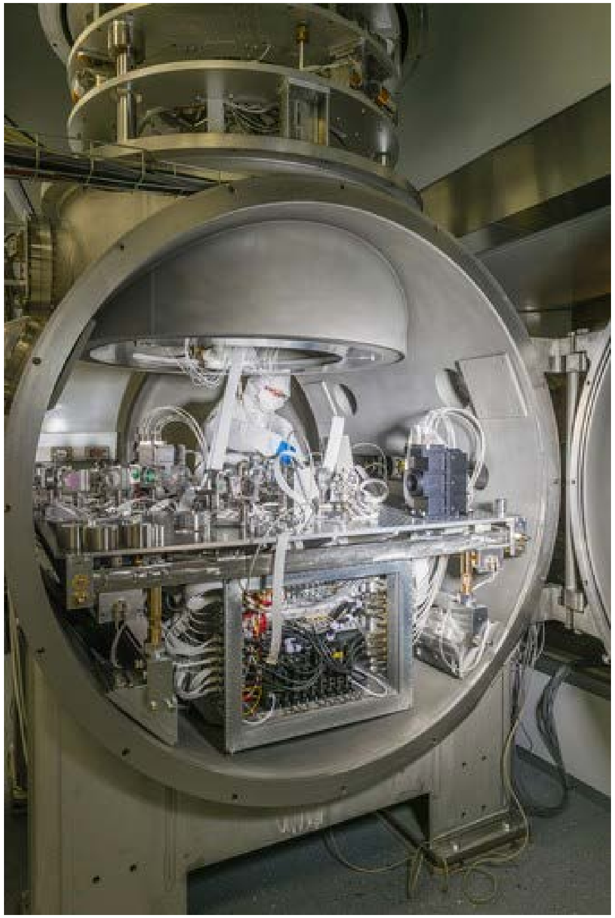
phase noise

↓

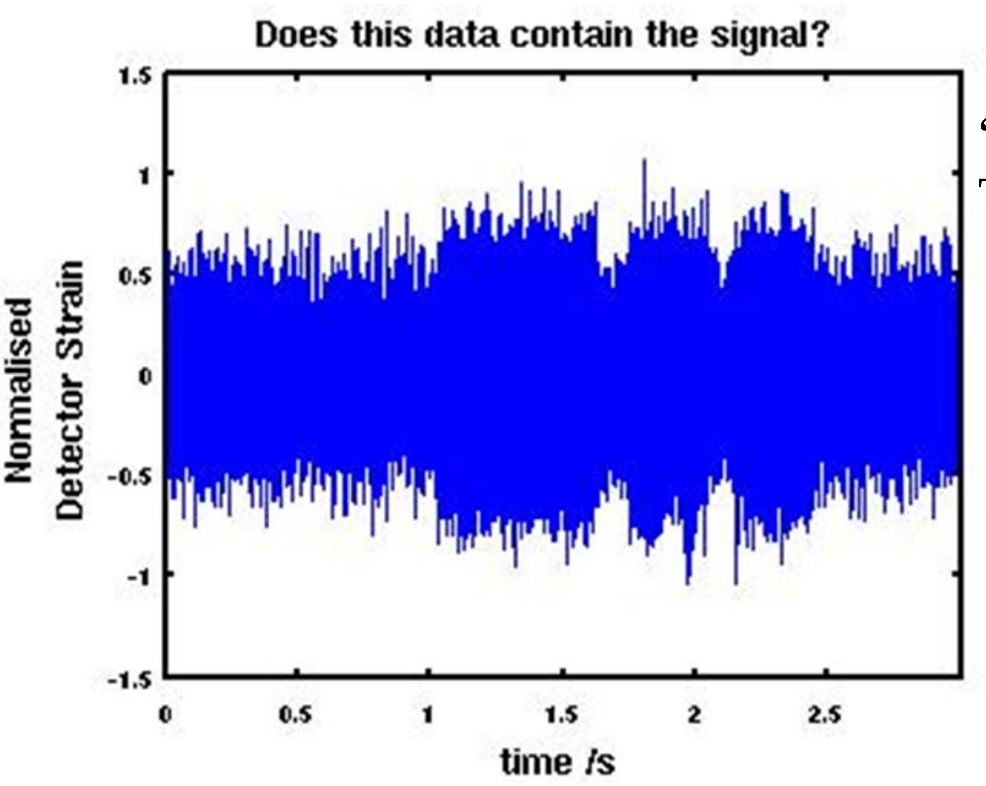
extra power fluctuations
(imprint of the optical element vibrations)



Evolution for AdVirgo: suspend the optical benches and place them under vacuum



Noises are not always stationary



"Glitches" are impulses of noise.
They might look like a transient GW signal

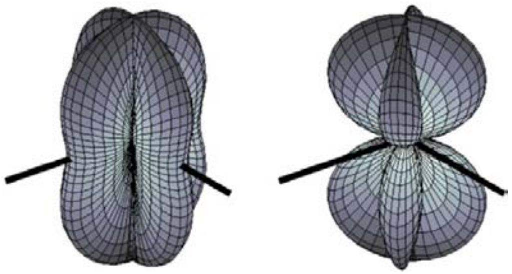
⚠

- ☐ environmental disturbances monitored with an array of sensors: seismic activities, magnetic perturbations, acoustic noises, temperature, humidity → used to veto false alarm triggers due to instrumental artifacts

- ☐ requires coincidence between 2 detectors to reduce false alarm rate

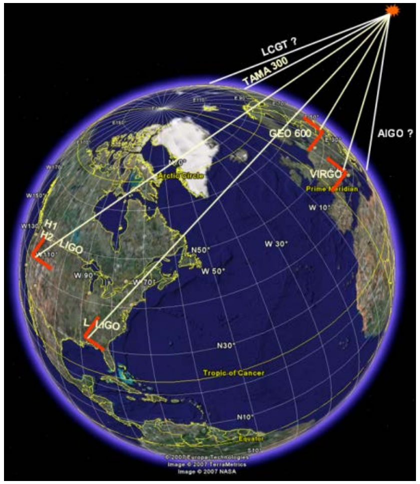
Prospectives of terrestrial interferometers

The benefits of the network



- A GW interferometer has a wide beam antenna
  - A single detector cannot localize the source
  - Need to compare the signals found in coincidence between several detectors (triangulation):

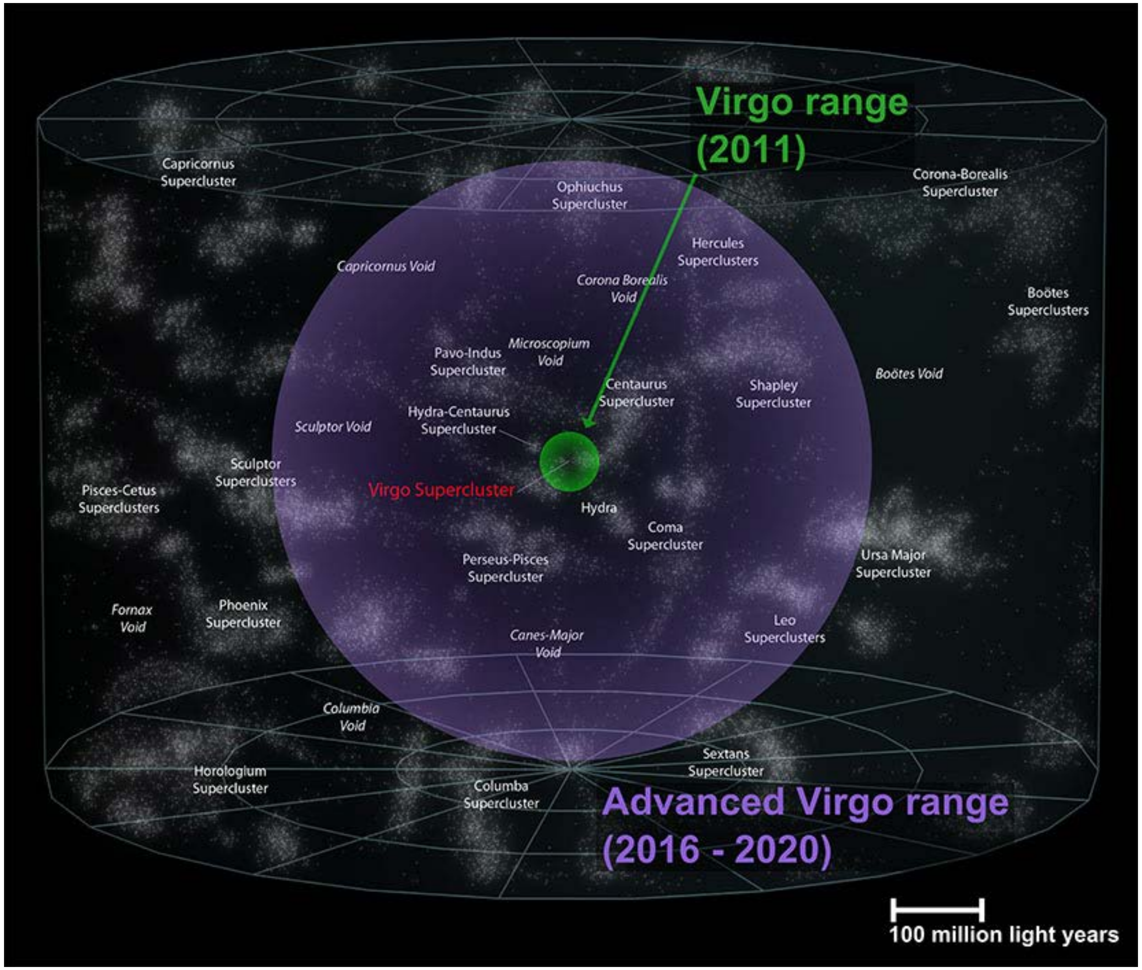
    → **allow to point towards the source position in the sky**



- Looking for rare and transient signals: can be hidden in detector noise

  → requires observation in coincidence between at least 2 detectors
- Since 2007, Virgo and LIGO share their data and analyze them jointly

Horizon of Advanced detectors



Distance at which a neutron star binary coalescence (1.4 M$_\odot$ - 1.4 M$_\odot$) can be seen with signal-to-noise ratio of 8

Improving the sensitivity (or horizon) by a factor 10

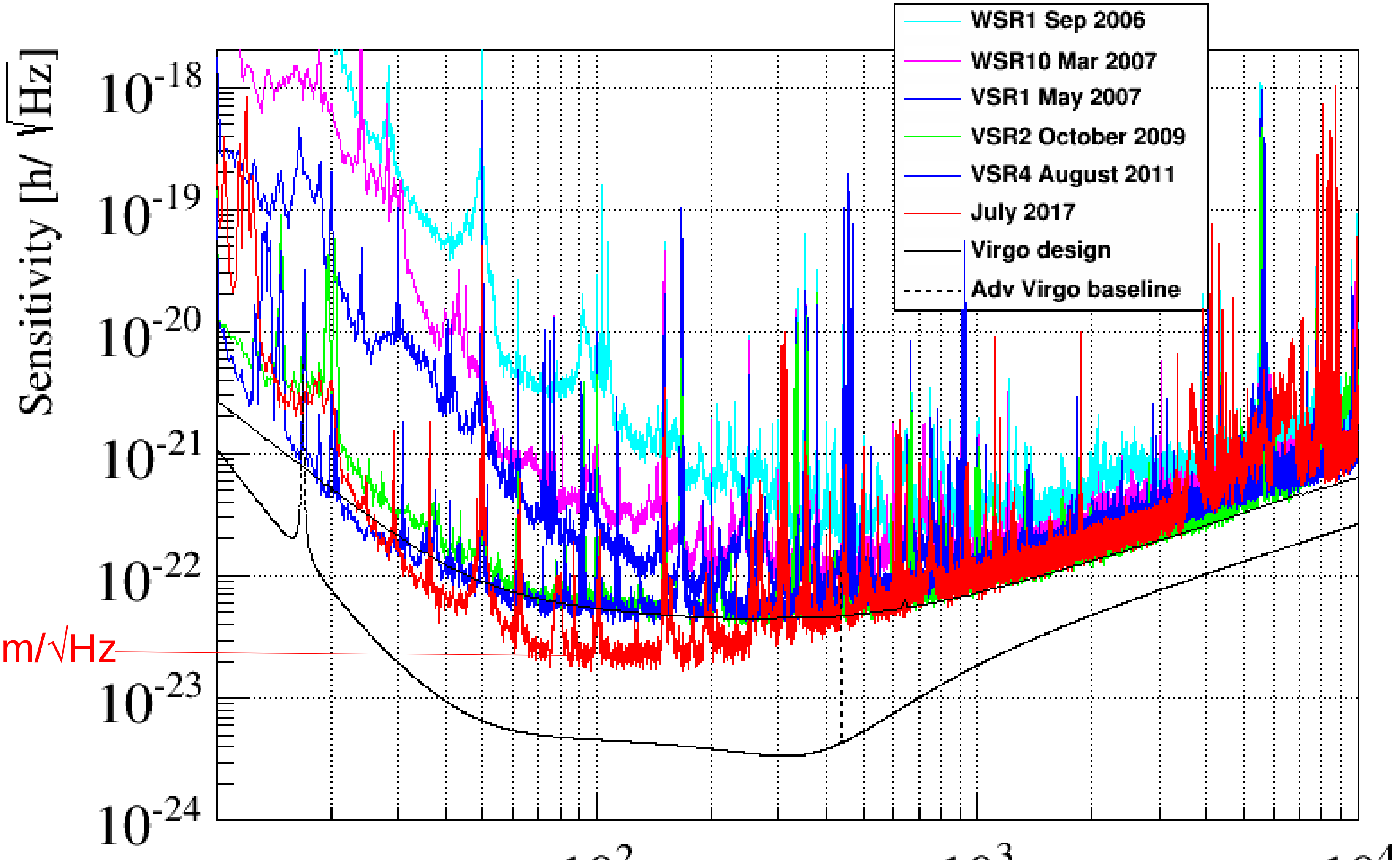
↓

Increase the volume (or event rate) by 10^3 = 1000

	low	realistic	high
Initial Virgo/LIGO	0.0002	**0.02**	0.2
Advanced Virgo/LIGO	0.4	**40**	400

Expected neutron star binary coalescence detection rate (event/year)

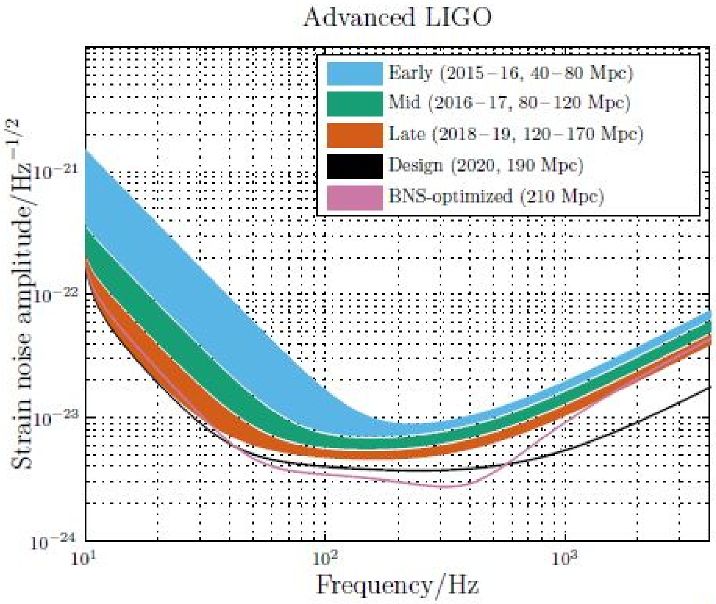
Advanced Virgo starting observations

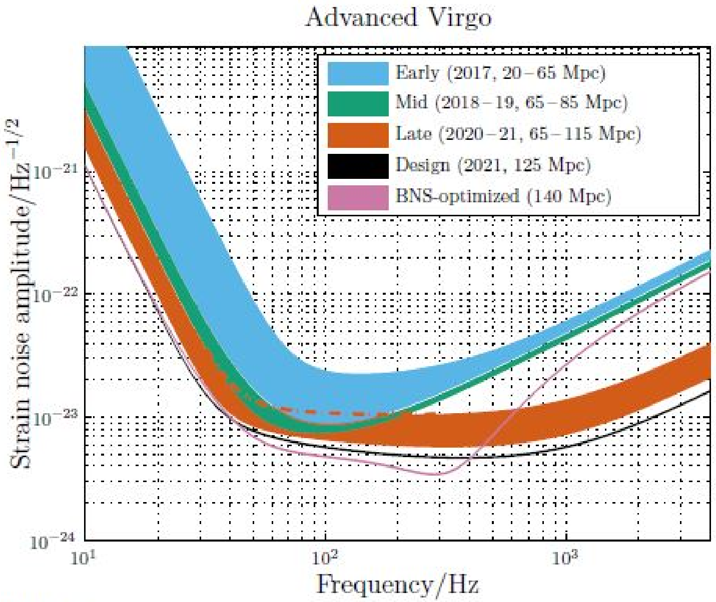


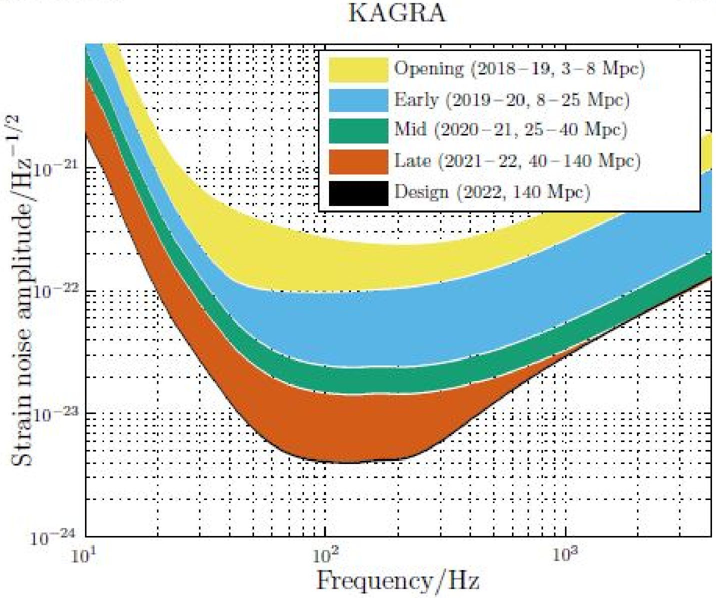
~3x10^{-20} m/√Hz

Going towards the Advanced Virgo nominal sensitivity….

A wider network of more sensitive detectors

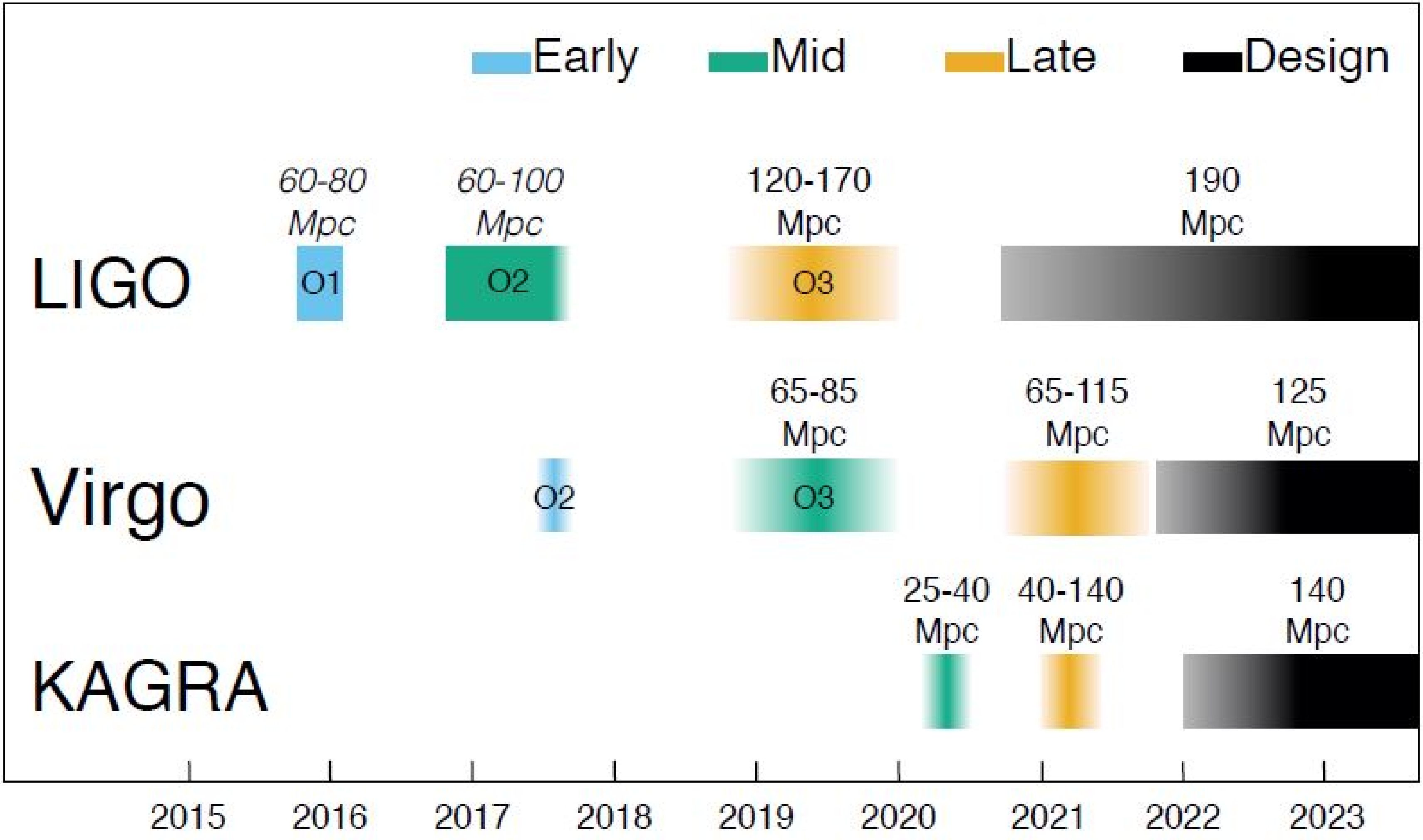






+ 3rd LIGO detector to be installed in India, operations > 2024

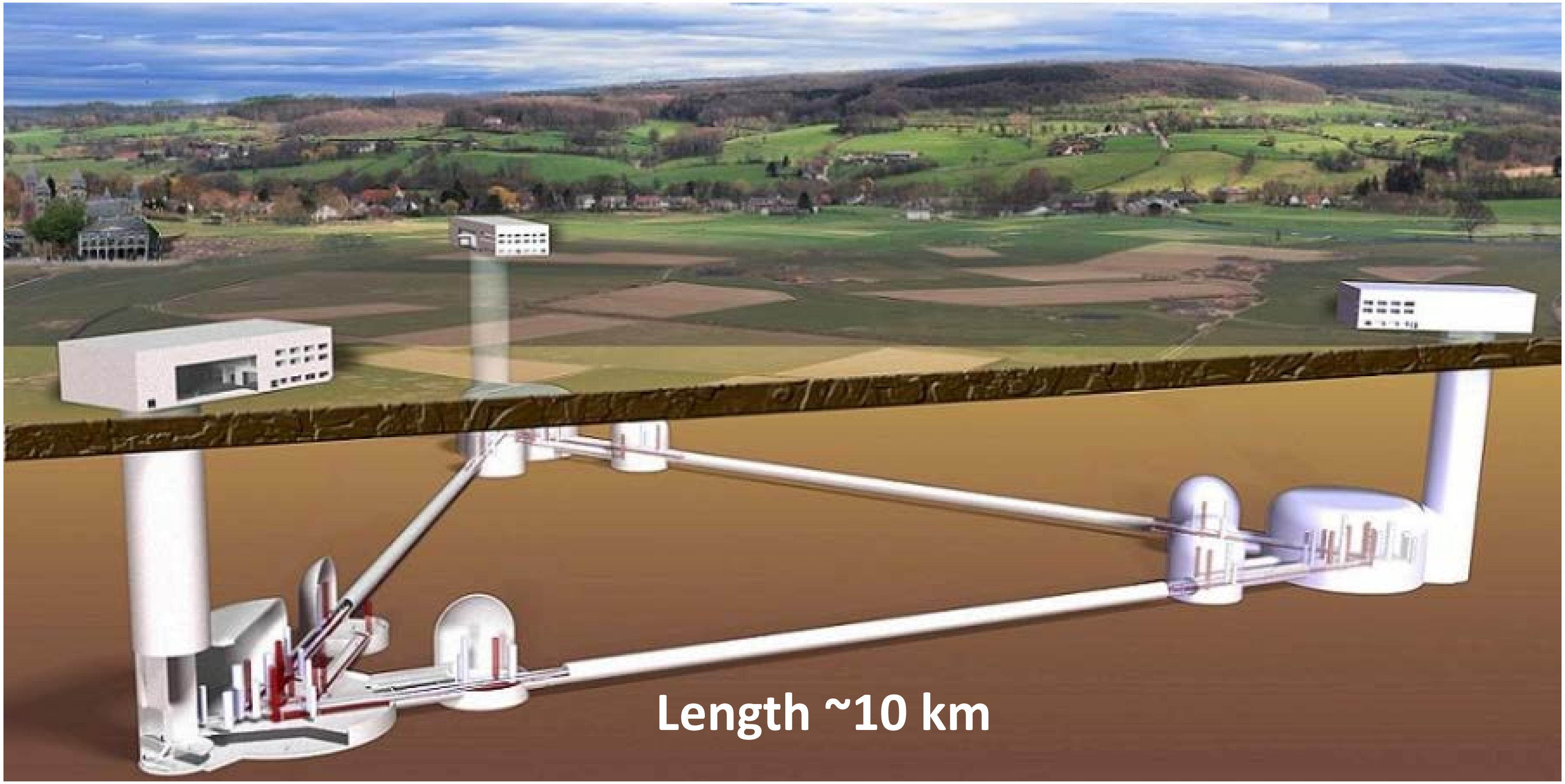
Future observing runs



+ Plans starting for 2.5G and 3G detectors

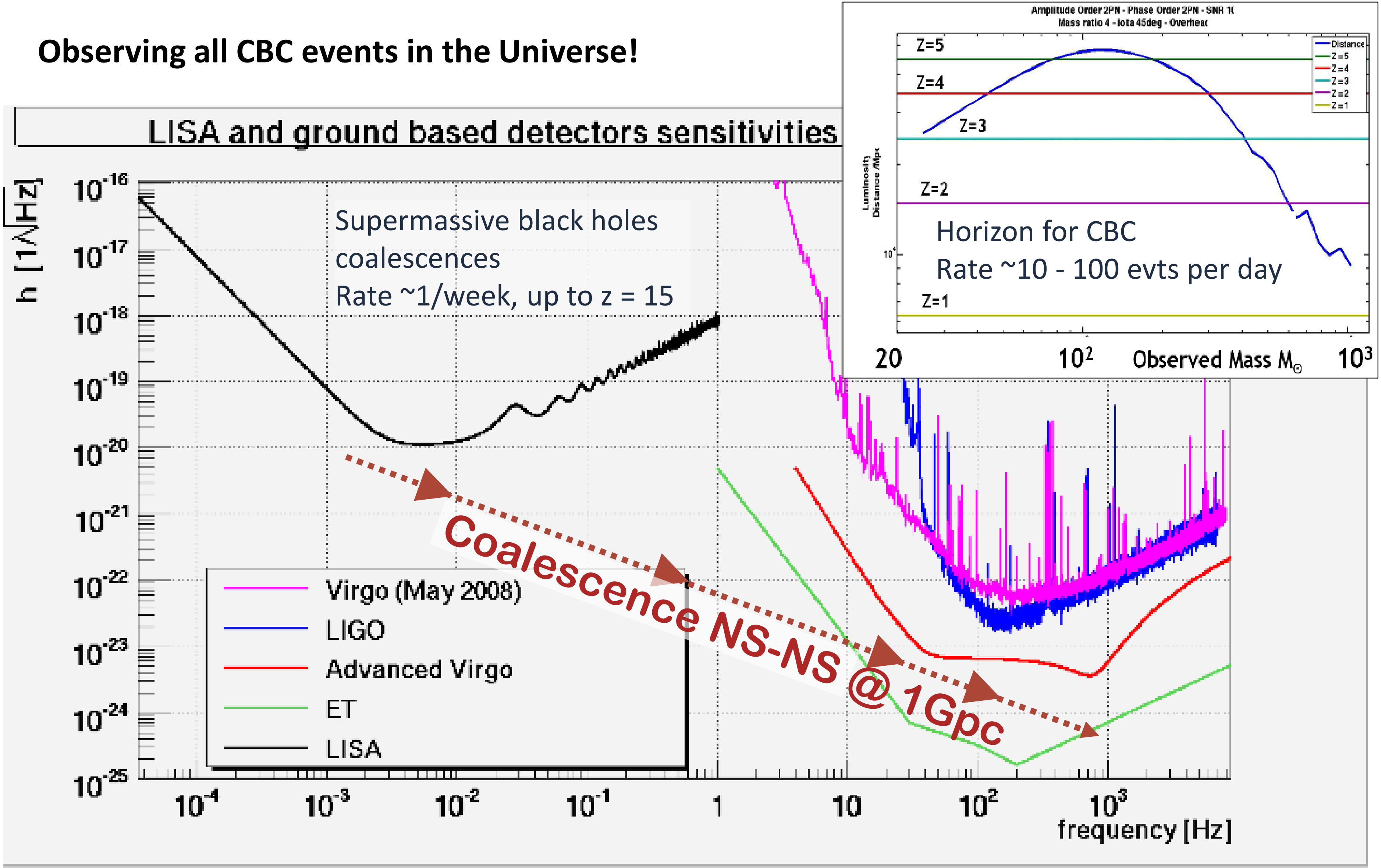
Einstein Telescope

- Third generation interferometer
- Located underground, ~10 km arms
- Technical design to be written in ~2024 -2025, detector operational after 2030?



ET and LISA performances

Observing all CBC events in the Universe!



Supermassive black holes coalescences
Rate ~1/week, up to z = 15

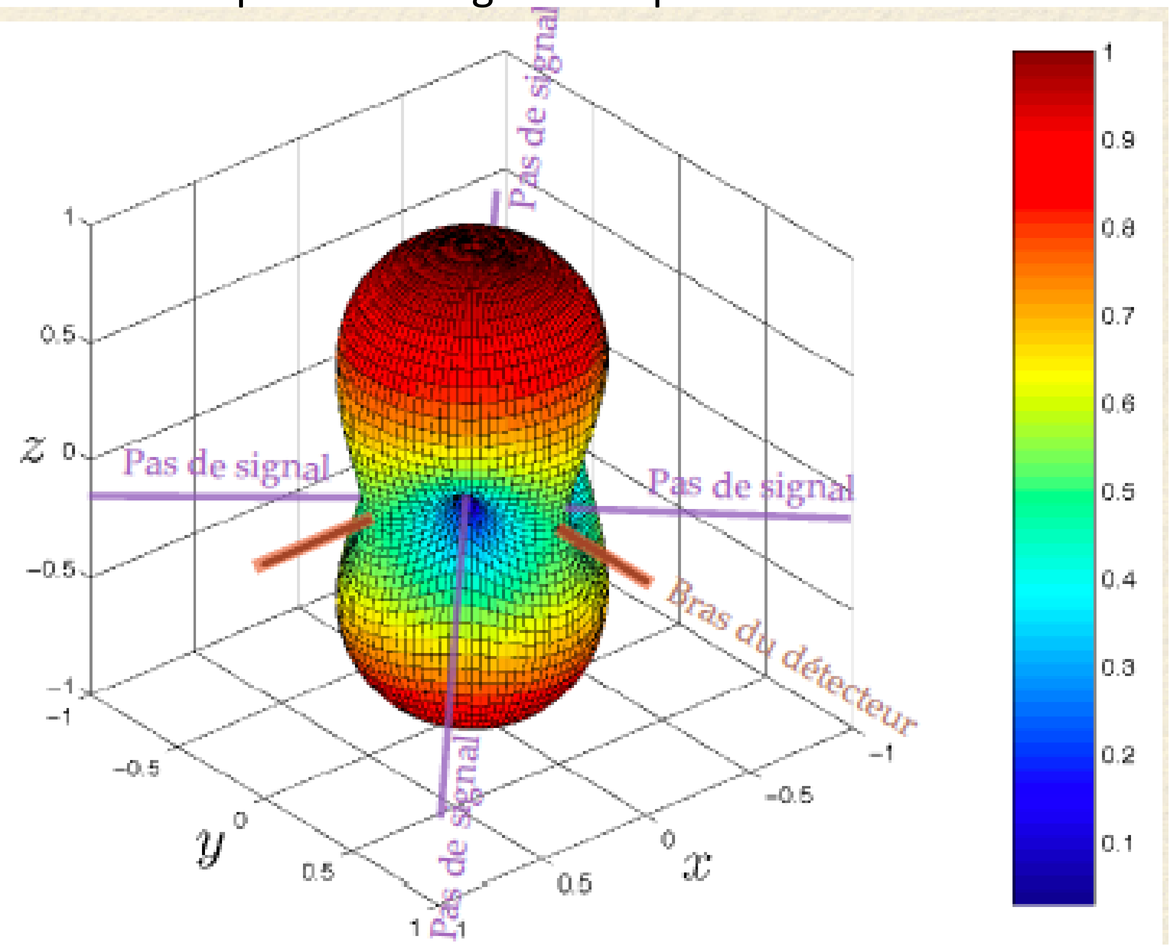
Coalescence NS-NS @ 1Gpc

Horizon for CBC
Rate ~10 - 100 evts per day

SPARES

Angular response of the interferometer

Detector response averaged over polarization of incident GW



- Interferometers have a broad angular response: behave more like an antenna than a traditional telescope
- A few blind spots

How to extract all error signals?
Phase modulation

- Use of DC signal (power measured by photodiodes) not sufficient to control all degrees of freedom
- Technique to get more error signals: phase modulation of the laser light:
  - Use of a EOM (electro-optical modulator):

    usually a Pockels cell: a crystal with a tunable optical length via a driven voltage
  - The EOM is driven with a sinusoidal signal which is converted in a variation of phase of the transmitted laser beam

ω = kc (laser pulsation)

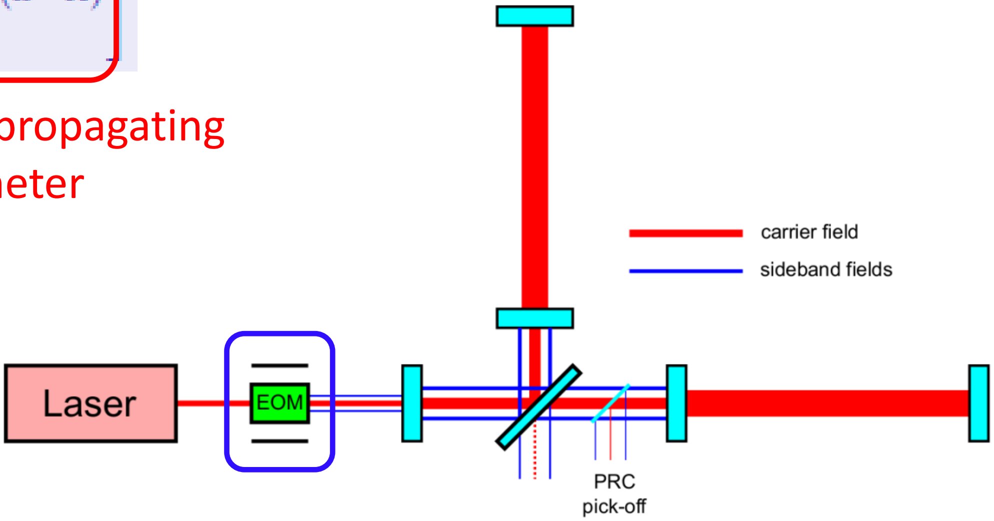
Ω : modulation frequency

β : modulation depth

$$E_{\text{inc}} = E_0 \, \mathrm{e}^{-i(\omega t + \beta \sin \Omega t)}$$

$$E_{\text{inc}} \simeq E_0 \left[\mathrm{e}^{-i\omega t} + \boxed{\frac{\beta}{2} \, \mathrm{e}^{-i(\omega+\Omega)} - \frac{\beta}{2} \, \mathrm{e}^{-i(\omega-\Omega)}} \right]$$

Side band fields propagating in the interferometer



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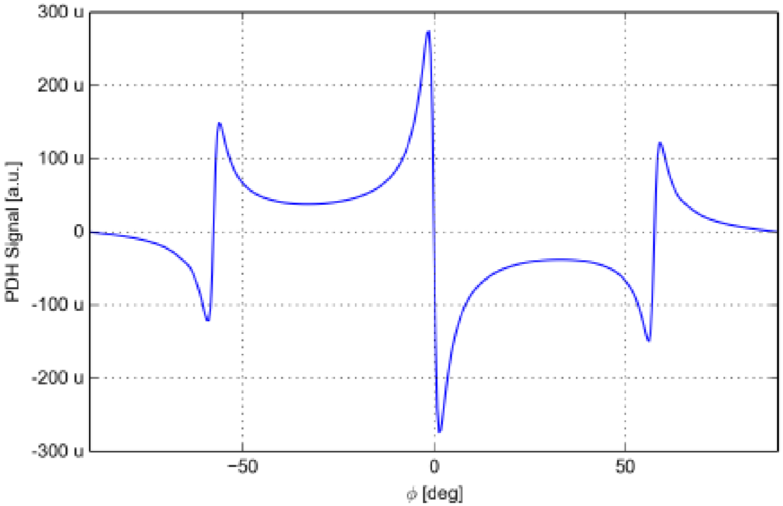
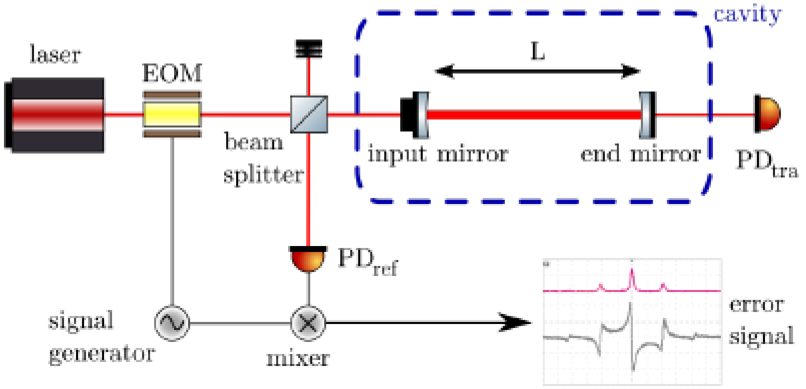
- Technique to get more error signals: phase modulation of the laser light:
  - Use of a EOM (electro-optical modulator):

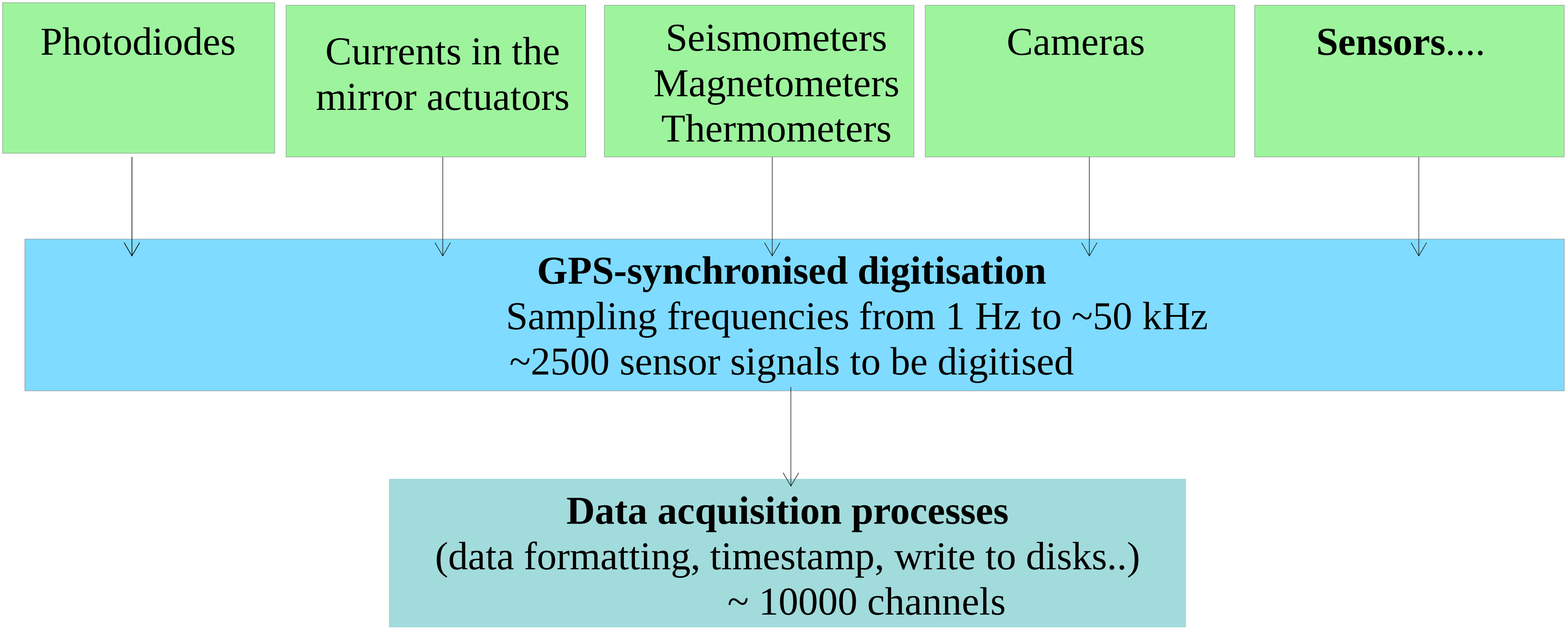
    usually a Pockels cell: a crystal with a tunable optical length via a driven voltage
  - The EOM is driven with a sinusoidal signal which is converted in a variation of phase of the transmitted laser beam
- Photodiodes signals demodulated at the modulation frequency (Pound-Drever-Hall technique)

  > give a linear error signal near resonance to control cavities lengths





Virgo data acquisition summary



Photodiodes

Currents in the mirror actuators

Seismometers
Magnetometers
Thermometers

Cameras

Sensors....

GPS-synchronised digitisation
Sampling frequencies from 1 Hz to ~50 kHz
~2500 sensor signals to be digitised

Data acquisition processes
(data formatting, timestamp, write to disks..)
~ 10000 channels

Continuous flow of ~3 TBytes/day (40 to 60 MBytes/s)
Disk space on Virgo site: ~400 TB for 4 months of data

Longer storage: data sent via Ethernet to computing centers (Lyon, Bologna)

Noise characterized in frequency domain

$$s(n) \xrightarrow{\text{Discrete Fourier Transform (DFT)}} S(k) = A(k)\mathrm{e}^{\mathrm{j}\Phi(k)}$$

Sampled signal

Fourier spectrum

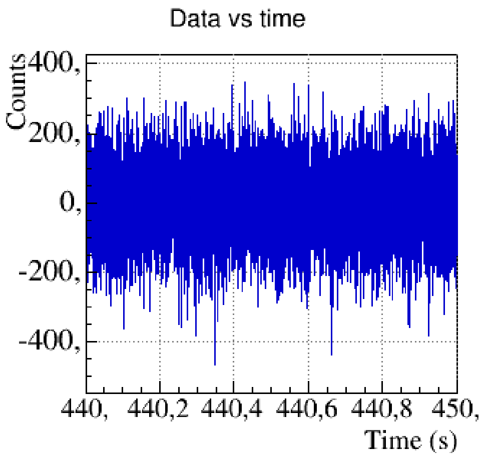
→ Noise characterised by the fluctuations of its Fourier spectrum

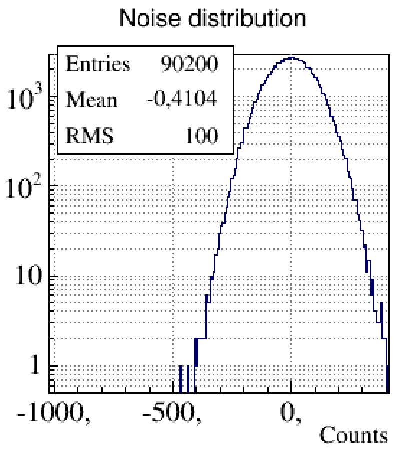
$\longrightarrow D(k)$ in units/√Hz

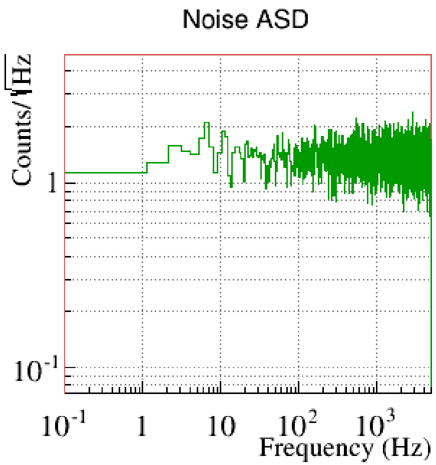
Assumption: noise is random and ergodic

→ noise characterised by its amplitude spectral density (ASD) $ASD = \sqrt{PSD} = \sqrt{\frac{|DFT|^2}{T}}$

Random gaussian noise
1 count/√Hz
Sampled at 10 kHz

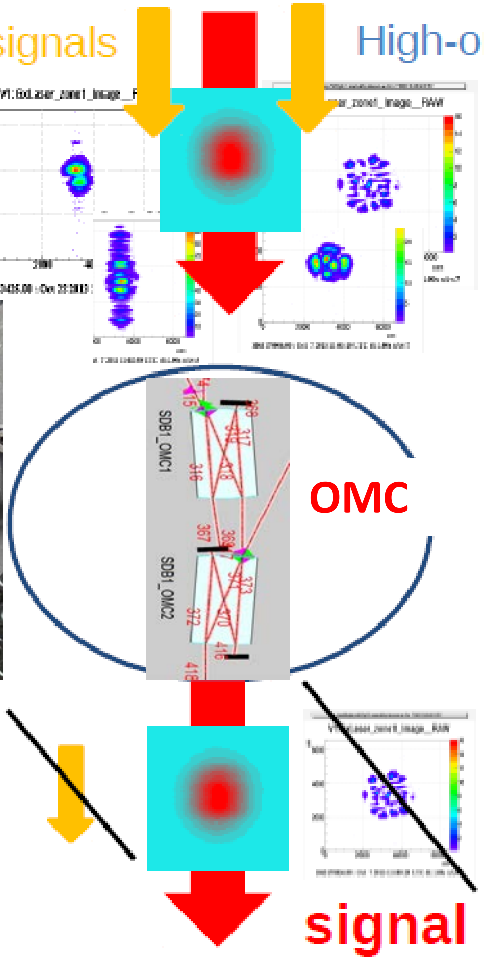

Data vs time


Noise distribution


Noise ASD

Output Mode Cleaner

- 2 bow-tie Fabry Perot cavities:
  - Get rid of high order modes and controls signals.



Control signals

High-order modes

OMC

signal