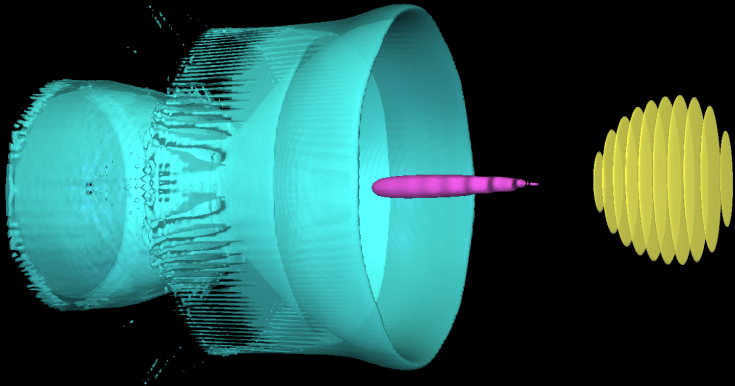


# Laser Wakefield Acceleration Numerical Simulation

Arnaud Beck, Laboratoire Leprince-Ringuet, CNRS/IN2P3  
Imen Zemzemi, Francesco Massimo, Martin Khojoyan, Arnd Specka





- 1 Laser Wakefield Acceleration
- 2 The HPC environment : Opportunities and Constraints
- 3 The PIC method and its parallelization
- 4 Accuracy issues, the example of numerical dispersion
- 5 Performances issues : the example of dynamic load balancing
- 6 Latest developments and results

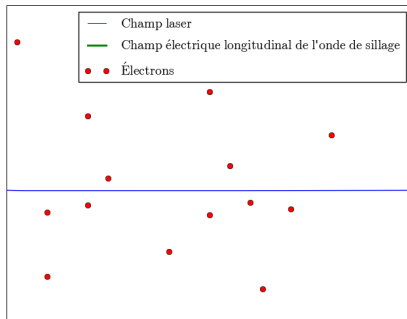




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L'ACCÉLÉRATION PAR  
SILLAGE LASER  
POUR  
**LES NULS**

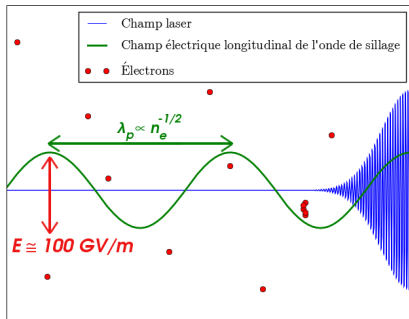


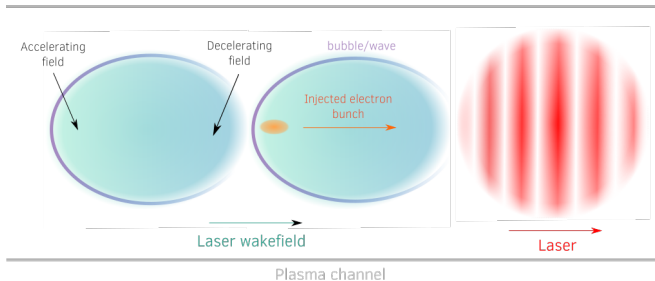


L'ACCÉLÉRATION PAR  
SILLAGE LASER  
POUR  
**LES NULS**



$$c_{TL} < \lambda_p$$



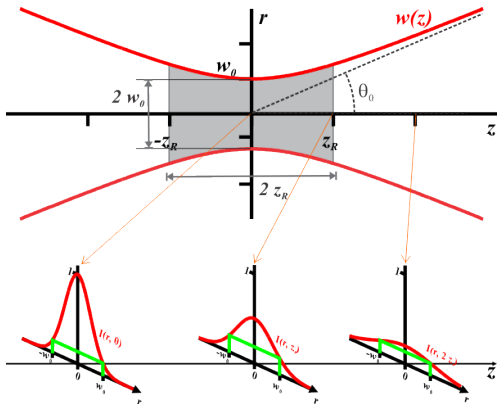




$$\mathbf{F}_p = \frac{e^2}{4\omega^2 m} \nabla E^2$$



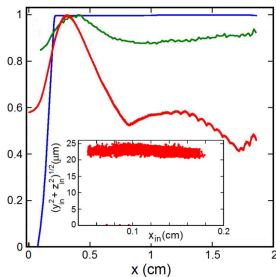
Diffraction in vacuum :



Optical indice in a plasma :

$$n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

$$\omega_p^2 = \frac{n_e e^2}{\epsilon_0 m_e}$$



*Physical processes at work in sub-30 fs, PW laser pulse-driven plasma accelerators, Beck et. al., NMIA, 2014.*

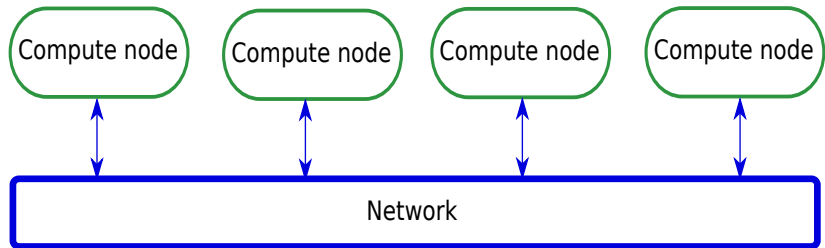


- Describe **wave-particle interactions** accurately at **small time and space scales** over a **large number of oscillations**.
- Evaluate local **relativistic** optical index.
- Solve **Maxwell equations**.
- Account for the collective behaviour of the plasma at **large scales**.
- **3D** in space and in momentum.

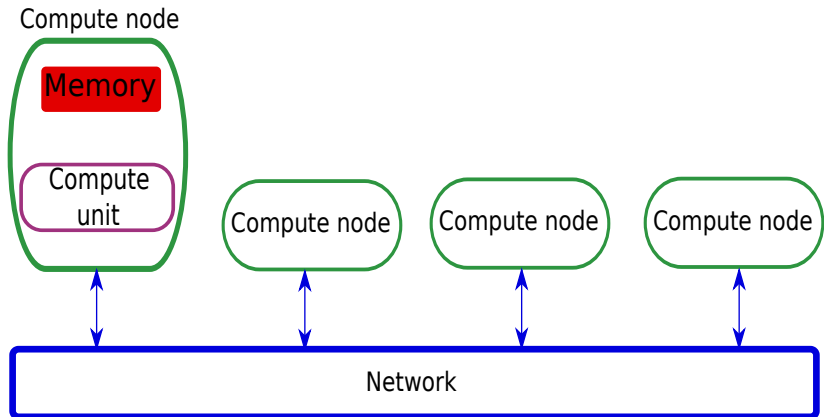




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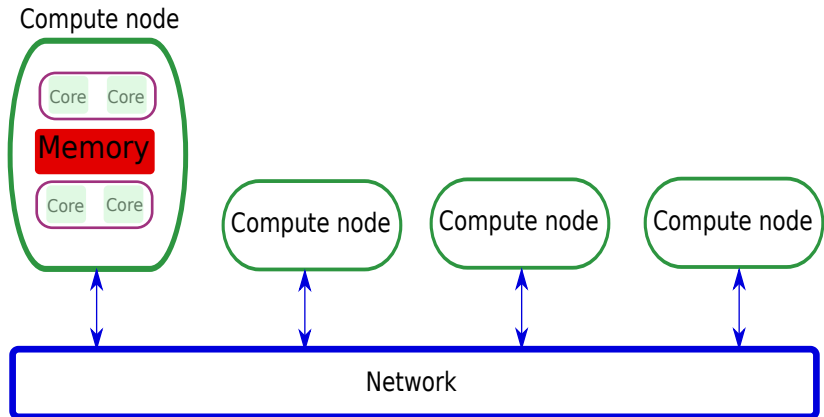


Distributed computing



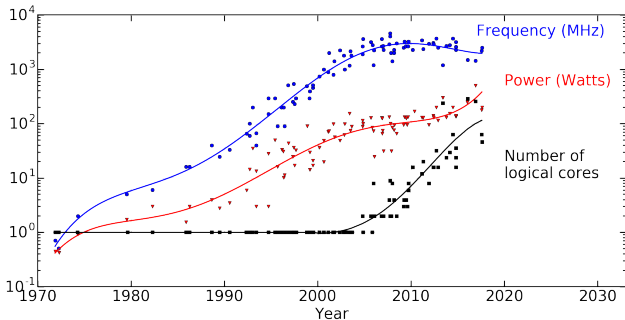
Distributed memory system

# What is a super computer ?



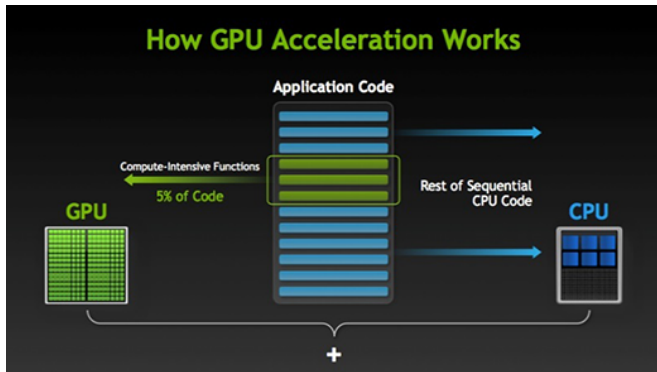
Distributed {shared memory} system

# 1) Many core



- Increased performances
- Reasonable energy budget





- Most energy efficient architecture today
- Difficult to address :
  - Libraries : Cuda, OpenCl.
  - Directives programming : OpenMP 4 ou openACC.



Compiler Case Study

## Introducing SIMD: Single Instruction, Multiple Data

### • Scalar processing

- traditional mode
- **one operation produces one result**

X

+

Y

X + Y

X

x3 x2 x1 x0

+

Y

y3 y2 y1 y0

X + Y

x3+y3 x2+y2 x1+y1 x0+y0

Intel Labs

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- Excellent potential speed up, very good power budget.
- Heavy constraints on data structure and algorithm.
- Difficult to use at its full extent in a PIC code.



## As a developer

- 1 Expose parallelism. Massive parallelization is key.
- 2 Focus on the algorithm and data structures. Not on architectures.
- 3 Reduce data movement : Computation is becoming cheaper, loads and stores not so much.
- 4 Be aware of the increasing gap between peak power and effective performances. The race to exascale is becoming a race to exaflops.

## As a user

- 1 Disclaimer. Parallelization is performed by experts.





«A collaborative, open-source, multi-purpose PIC code  
for the next generation of super-computers»

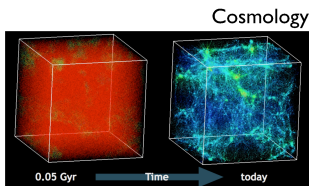


[www.maisondelasimulation.fr /smilei](http://www.maisondelasimulation.fr /smilei)

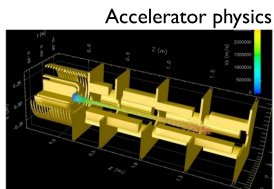


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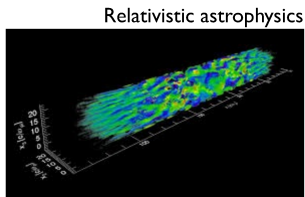
The Particle-In-Cell (PIC) method is a central tool for simulation over a wide range of physics studies



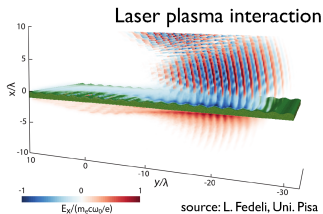
source: K. Heitmann, Argonne National Lab



source: WARP, Berkeley Lab



source: F. Fiuza, Livermore National Lab



source: L. Fedeli, Uni. Pisa

- Conceptually simple
- Efficiently implemented on (massively) parallel super-computers



$f_s(\mathbf{x}, \mathbf{v}) d\mathbf{x}d\mathbf{v}$  is the probability to find a particle of species  $s$  in the phase space point  $(\mathbf{x}, \mathbf{v})$  around  $d\mathbf{x}d\mathbf{v}$ .

Vlasov equation

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + q_s/m_s (\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0$$

No collisions

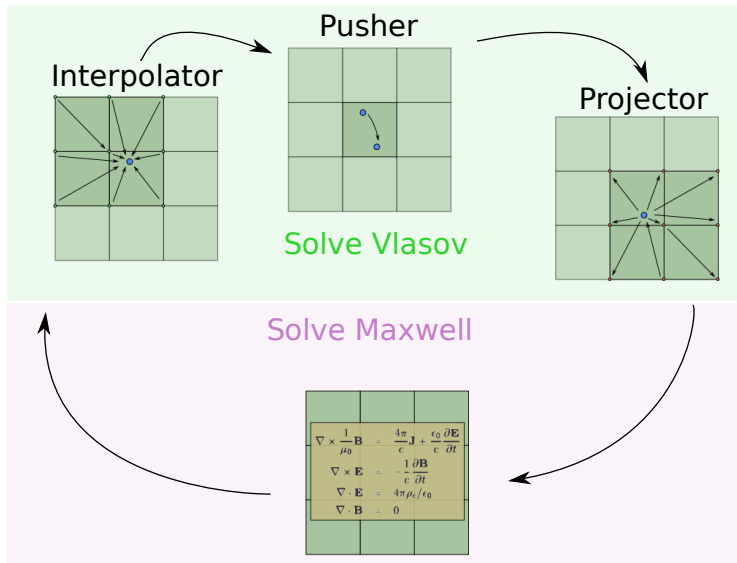
Maxwell's equations

$$\begin{cases} \nabla \cdot \mathbf{E} = 4\pi\rho \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} \mathbf{J} \end{cases}$$

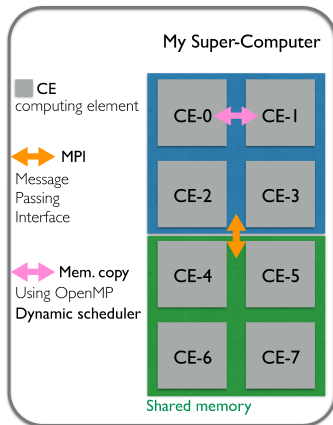
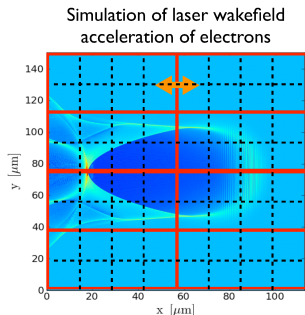
Moments equations

$$\begin{aligned} \rho &= \sum_s^{n_s} q_s \int f_s d\mathbf{v} \\ \mathbf{J} &= \sum_s^{n_s} q_s \int \mathbf{v} f_s d\mathbf{v} \end{aligned}$$

Difficulty : Equations are coupled !

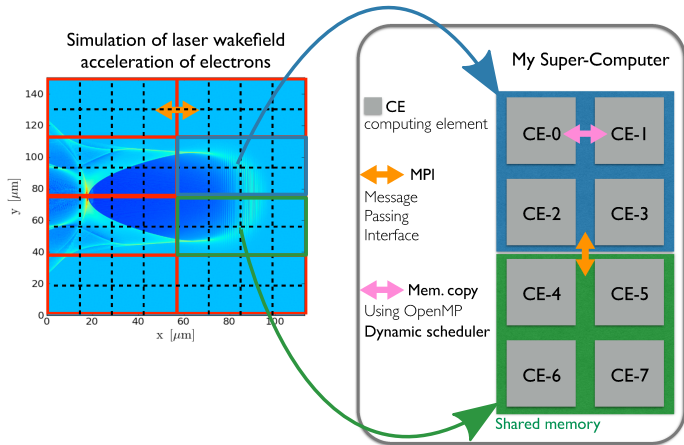


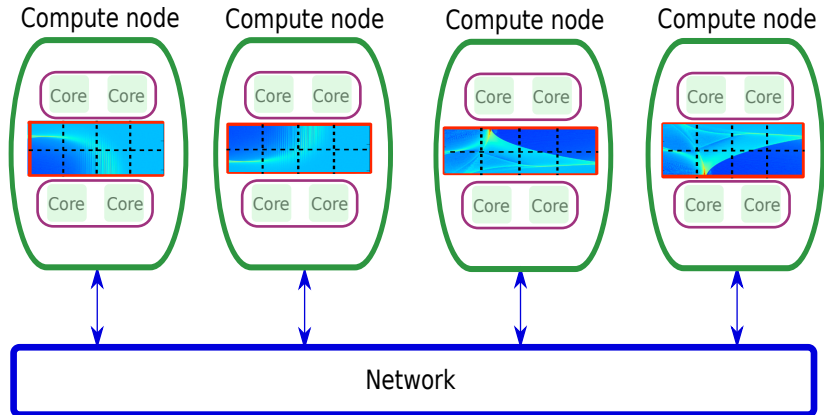
PIC code are 'easily' parallelized using **domain decomposition**





PIC code are 'easily' parallelized using **domain decomposition**

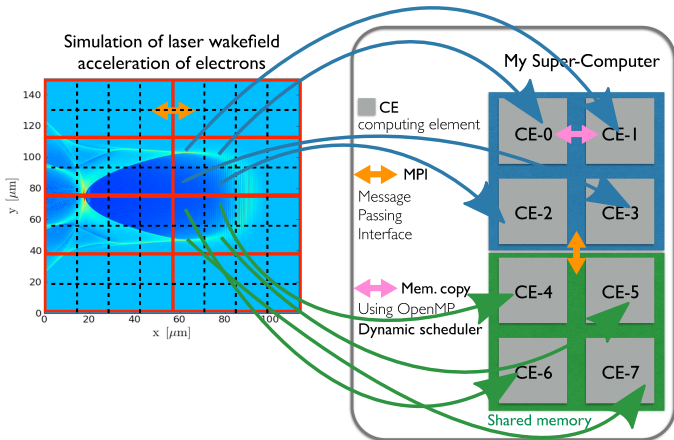






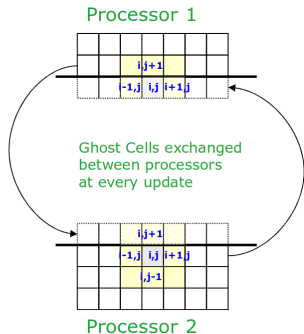
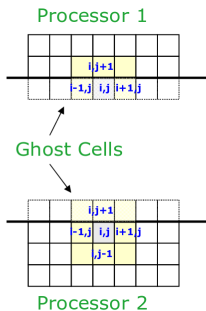
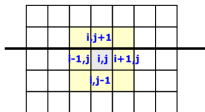


PIC code are 'easily' parallelized using **domain decomposition** + Patch





sub-domain boundaries



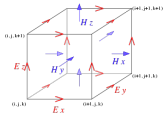
- If processors have a shared memory ==> OpenMP
- If processors have distributed memory ==> MPI
- Same logic for particles



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Discrete Maxwell equations for finite difference time domain (FDTD) scheme :

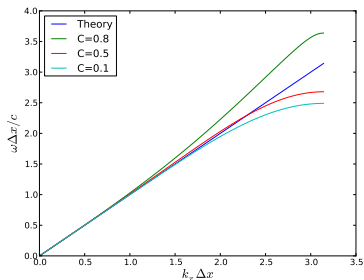


$$\begin{aligned}
 \frac{(E_x)^{n+1}_{i+\frac{1}{2},j,k} - (E_x)^n_{i+\frac{1}{2},j,k}}{\Delta t} &= -\frac{1}{\epsilon_0} (J_x)^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k} \\
 &+ c^2 \left[ \frac{(B_z)^{n+\frac{1}{2}}_{i+\frac{1}{2},j+\frac{1}{2},k} - (B_z)^{n+\frac{1}{2}}_{i+\frac{1}{2},j-\frac{1}{2},k}}{\Delta y} - \frac{(B_y)^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k+\frac{1}{2}} - (B_y)^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k-\frac{1}{2}}}{\Delta z} \right] \\
 \frac{(E_y)^{n+1}_{i,j+\frac{1}{2},k} - (E_y)^n_{i,j+\frac{1}{2},k}}{\Delta t} &= -\frac{1}{\epsilon_0} (J_y)^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k} \\
 &+ c^2 \left[ \frac{(B_x)^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}} - (B_x)^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k-\frac{1}{2}}}{\Delta z} - \frac{(B_z)^{n+\frac{1}{2}}_{i-\frac{1}{2},j+\frac{1}{2},k} - (B_z)^{n+\frac{1}{2}}_{i+\frac{1}{2},j+\frac{1}{2},k}}{\Delta x} \right] \\
 \frac{(E_z)^{n+1}_{i,j,k+\frac{1}{2}} - (E_z)^n_{i,j,k+\frac{1}{2}}}{\Delta t} &= -\frac{1}{\epsilon_0} (J_z)^{n+\frac{1}{2}}_{i,j,k+\frac{1}{2}} \\
 &+ c^2 \left[ \frac{(B_y)^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}} - (B_y)^{n+\frac{1}{2}}_{i-\frac{1}{2},j+\frac{1}{2},k+\frac{1}{2}}}{\Delta x} - \frac{(B_x)^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}} - (B_x)^{n+\frac{1}{2}}_{i,j-\frac{1}{2},k+\frac{1}{2}}}{\Delta y} \right] \\
 \frac{(B_x)^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}} - (B_x)^{n-\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}}}{\Delta t} &= -\frac{(E_z)^n_{i,j+1,k+\frac{1}{2}} - (E_z)^n_{i,j,k+\frac{1}{2}}}{\Delta y} + \frac{(E_y)^n_{i,j+\frac{1}{2},k+1} - (E_y)^n_{i,j+\frac{1}{2},k}}{\Delta z} \\
 \frac{(B_y)^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k+\frac{1}{2}} - (B_y)^{n-\frac{1}{2}}_{i+\frac{1}{2},j,k+\frac{1}{2}}}{\Delta t} &= -\frac{(E_x)^n_{i+\frac{1}{2},j,k+1} - (E_x)^n_{i+\frac{1}{2},j,k}}{\Delta z} + \frac{(E_z)^n_{i+1,j,k+\frac{1}{2}} - (E_z)^n_{i,j,k+\frac{1}{2}}}{\Delta x} \\
 \frac{(B_z)^{n+\frac{1}{2}}_{i+\frac{1}{2},j+\frac{1}{2},k} - (B_z)^{n-\frac{1}{2}}_{i+\frac{1}{2},j+\frac{1}{2},k}}{\Delta t} &= -\frac{(E_y)^n_{i+\frac{1}{2},j+\frac{1}{2},k} - (E_y)^n_{i,j+\frac{1}{2},k}}{\Delta y} + \frac{(E_x)^n_{i+\frac{1}{2},j+1,k} - (E_x)^n_{i+\frac{1}{2},j,k}}{\Delta x}
 \end{aligned}$$

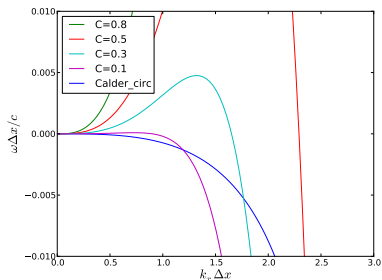
Yee lattice conveniently allows first order finite difference scheme to produce order 2 accuracy :  $\mathcal{O}(\Delta x^2)$ .



A plane wave is propagated in vacuum via the discrete equations. We obtain information on stability and accuracy.



Numerical dispersion in vacuum.



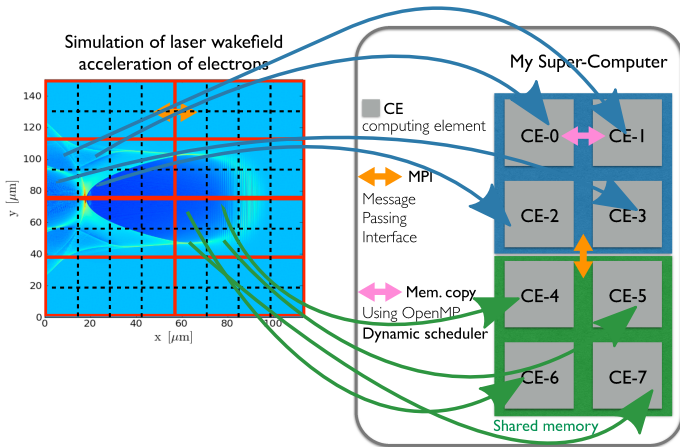
Difference between numerical and theoretical dispersion.



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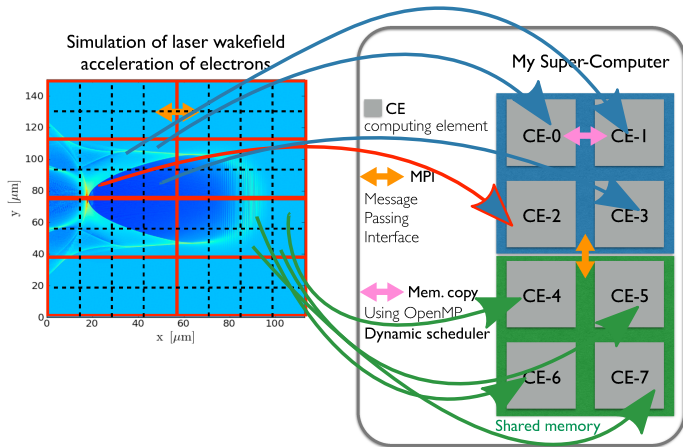


PIC code are 'easily' parallelized using **domain decomposition**





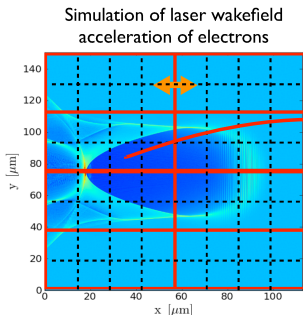
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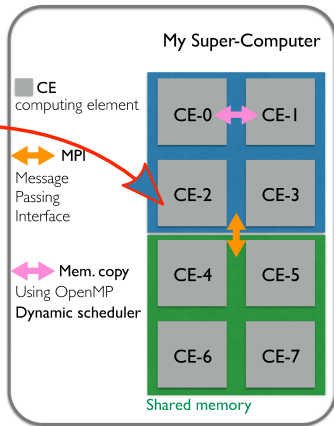


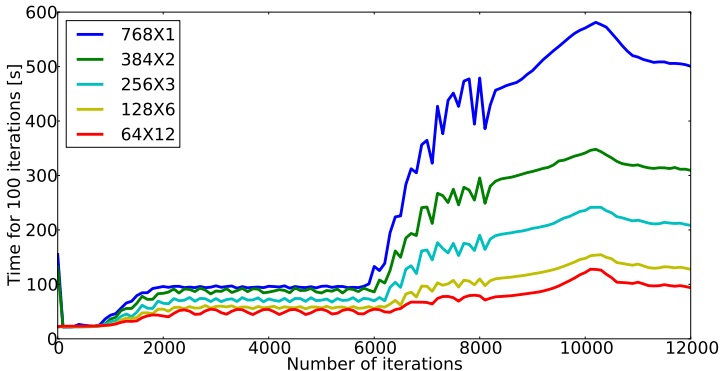
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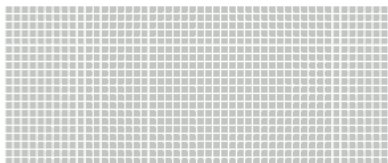
**Domain decomp. is not enough!!!**

- workload not optimally shared
- not adapted to new architectures!

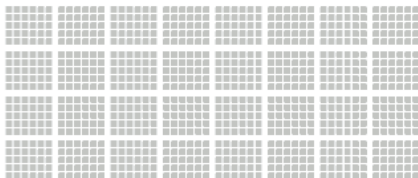


 $MPI \times OpenMP$ 

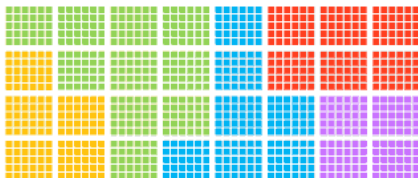
OpenMP dynamic scheduler is able to smooth the load but only at the node level.



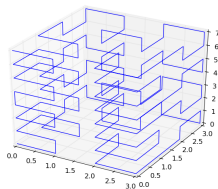
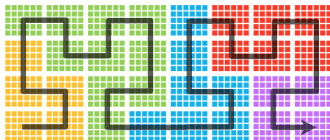
960 cells



32 patches

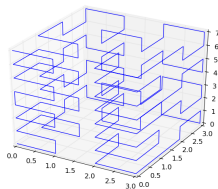
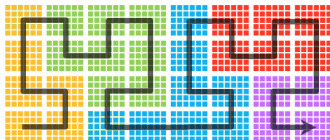


5 MPI regions



We need a policy to assign patches to MPI processes. To do so, patches are organized along a one dimensional **space-filling curve**.

- 1 Continuous curve which goes across all patches.
- 2 Each patch is visited only once.
- 3 Two consecutive patches are neighbours.
- 4 In addition we want compactness !

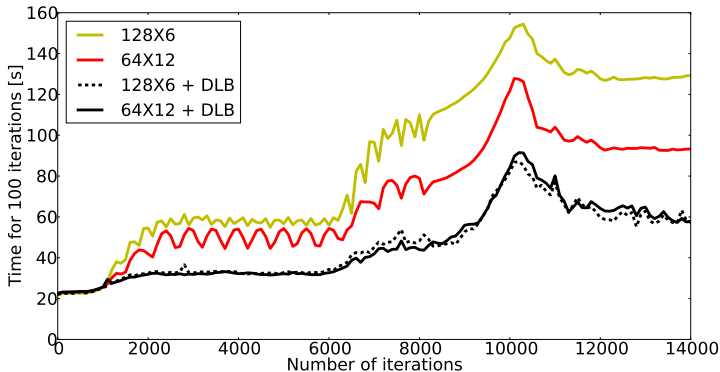


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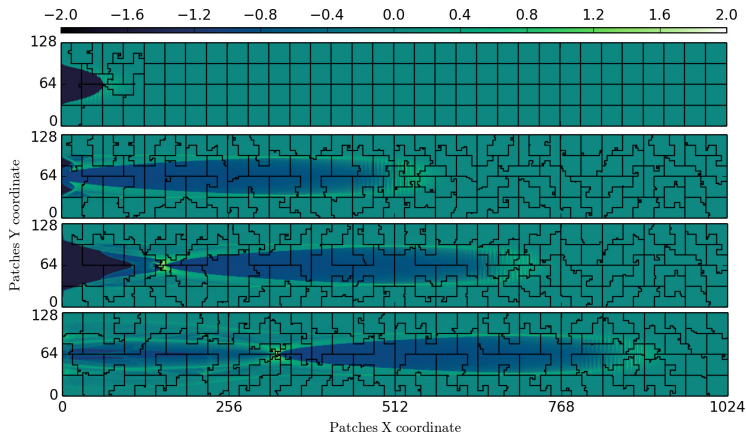
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- 3 Two consecutive patches are neighbours.
- 4 In addition we want compactness !



*MPI × OpenMP*

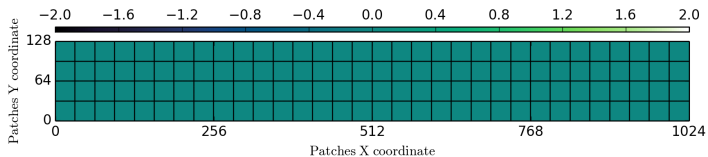


Yellow and red are copied from previous figure.



Color represents the local patch computational load imbalance

$$I_{loc} = \log_{10} (L_{loc}/L_{av})$$



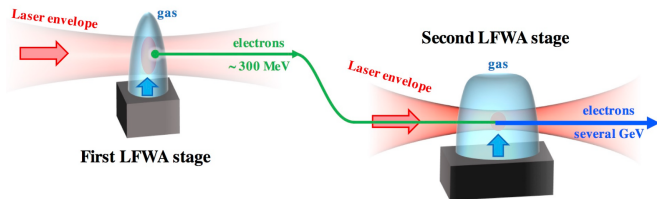
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Hypothèse:

$$\hat{A}(\mathbf{x}, t) = \text{Re} \left[ \tilde{A}(\mathbf{x}, t) e^{ik_0(x-ct)} \right]$$

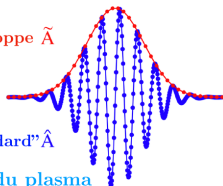
**Enveloppe Complexe**

Laser Enveloppe  $\tilde{A}$

Equation d'Enveloppe:

$$\nabla^2 \tilde{A} + 2i \left( \partial_x \tilde{A} + \partial_t \tilde{A} \right) - \partial_t^2 \tilde{A} = \chi \tilde{A} \quad \text{Laser "Standard" } \hat{A}$$

$$\chi(\mathbf{x}) = \sum_s \frac{q_s^2}{m_s} \sum_p \frac{w_p}{\tilde{\gamma}_p} S(\mathbf{x} - \bar{\mathbf{x}}_p) \quad \text{Susceptibilité du plasma}$$



Equations du Mouvement des Macroparticules:

$$\frac{d\bar{\mathbf{x}}_p}{dt} = \frac{\bar{\mathbf{u}}_p}{\tilde{\gamma}_p}$$

**Force Pondéromotrice**

$$\tilde{\gamma}_p = \sqrt{1 + \bar{\mathbf{u}}_p^2 + \frac{|\tilde{A}(\bar{\mathbf{x}}_p)|^2}{2}}$$

$$\frac{d\bar{\mathbf{u}}_p}{dt} = r_s \left( \bar{\mathbf{E}}_p + \frac{\bar{\mathbf{u}}_p}{\tilde{\gamma}_p} \times \bar{\mathbf{B}}_p \right) - r_s^2 \frac{1}{4\tilde{\gamma}_p} \nabla \left( |\tilde{A}_p|^2 \right)$$

**Force de Lorentz**

$$r_s = q_s/m_s$$

P. Mora and T. M. Antonsen Jr, Physics of Plasmas 4, 217 (1997)

B. Quesnel and P. Mora, Physics Review E 58, 3719 (1998)

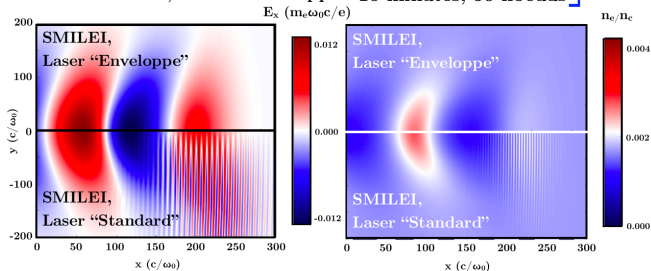
S. Sinigardi et al., ALaDyn v2017.1 zenodo (2017)



SMILEI 3D, Laser "Standard" 10.6 heures, 256 noeuds

SMILEI 3D, Laser "Enveloppe" 10 minutes, 50 noeuds

1 mm



$$L_x = 400 \text{ c}/\omega_0$$

$$L_y = L_z = 640 \text{ c}/\omega_0$$

$$\text{Particules par cellule} = 8$$

$$\frac{\text{T-processus Laser "Standard"}}{\text{T-processus Laser "Enveloppe"}} = 320 !$$



$$F(x, r, \theta) = \tilde{F}^0 + \sum_{m=1}^{+\infty} \Re \left( \tilde{F}^m(x, r) \right) \cos(m\theta) + \Im \left( \tilde{F}^m(x, r) \right) \sin(m\theta)$$

We can then rewrite Maxwell equations for each Fourier modes :

$$\frac{\partial \tilde{B}_r^m}{\partial t} = \frac{im}{r} \tilde{E}_x^m + \frac{\partial \tilde{E}_\theta^m}{\partial x}$$

Since the modes are independant, the simulation boils down to  $m$  2D simulations.



- New French super computer Irene : 79 488 cores and computational power of 6,86 Pflop/s.
- Preliminary access was granted to 20 applications and Smilei won this “Grand Challenge” : 7 million hours.
- Smilei ran and showed exceptional efficiency on 43200 cores.
- Perfect opportunity to run a partial second stage simulation to validate the enveloppe model on a long distance propagation.

Pour utilisateurs, futurs développeurs:

# Smilei) Training Workshop!

<http://www.maisondelasimulation.fr/smilei/>

**Prochaine Edition: Fevrier/Mars 2019**

