



Higher-order QCD corrections to Higgs pair production at the LHC

Terascale IRN Meeting, Strasbourg

01/06/2018, Julien Baglio



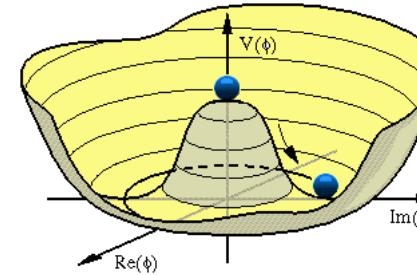
Outline

1. Overview of the higher-order QCD corrections to HH production
2. NLO QCD corrections with exact top-quark mass effects
3. Outlook

The SM ultimate test: probing the scalar potential

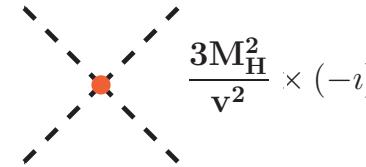
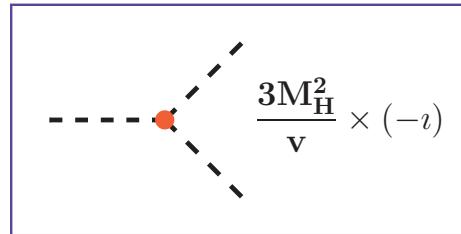
From the scalar potential before EWSB (ϕ as the Higgs field):

$$V(\phi) = -m^2|\phi|^2 + \lambda|\phi|^4$$



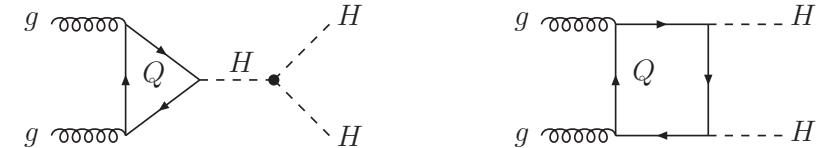
To $V(\phi)$ after EWSB, with $M_H^2 = 2m^2$, $v^2 = m^2/\lambda$:

$$\phi = \begin{pmatrix} 0 \\ \frac{v + H(x)}{\sqrt{2}} \end{pmatrix} \Rightarrow V(H) = \frac{1}{2}M_H^2H^2 + \frac{1}{2}\frac{M_H^2}{v}H^3 + \frac{1}{8}\frac{M_H^2}{v^2}H^4 + \text{constant}$$



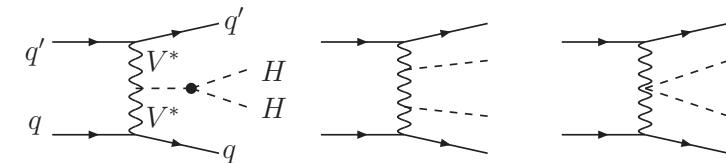
Overview of HH production channels

■ Gluon fusion: focus of the talk

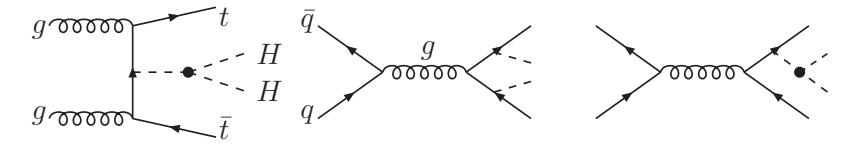


■ Vector boson fusion

NLO QCD [1,2], NNLO QCD [3]

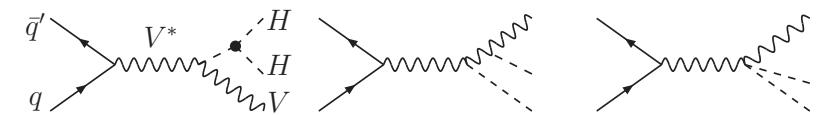


■ $t\bar{t}HH$ production NLO QCD [2]



■ Double Higgs-strahlung

NLO QCD [1,2], NNLO QCD [1,4]



[1] J.B., Djouadi, Gröber, Mühlleitner, Quevillon, Spira, JHEP 1304 (2013) 151

[2] Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, Zaro, PLB 732 (2014) 142

[3] Ling, Zhang, Ma, Guo, W.H. Li, X.Z. Li, PRD 89 (2014) 073001

[4] H.T. Li, Wang, PLB 765 (2017) 265; H.T. Li, C.S. Li, Wang, PRD 97 (2018) 074026

Overview of HH production channels

- **Gluon fusion: focus of the talk**

- **Vector boson fusion**

NLO QCD [1,2], NNLO QCD [3]

- **$t\bar{t}HH$ production** NLO QCD [2]

- **Double Higgs-strahlung**

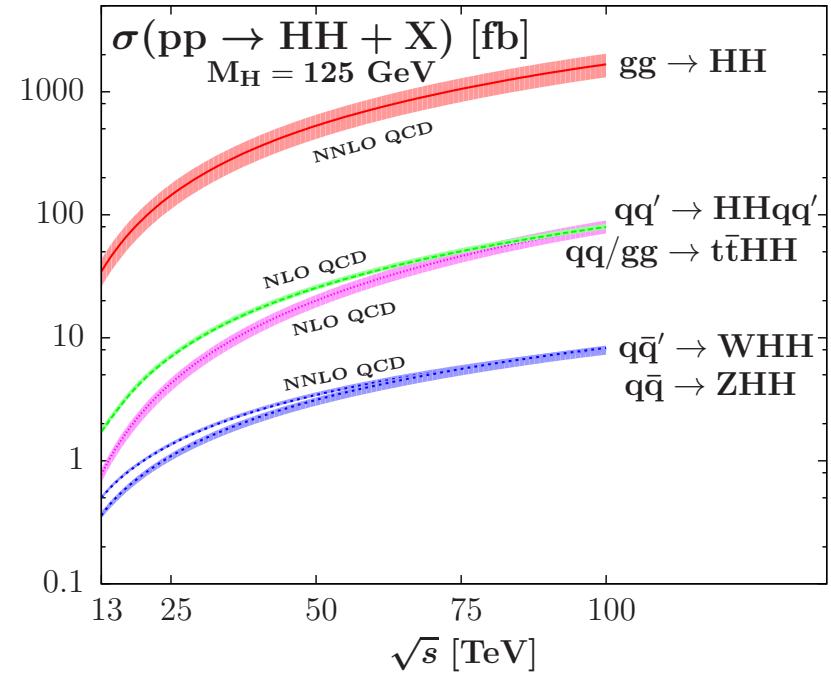
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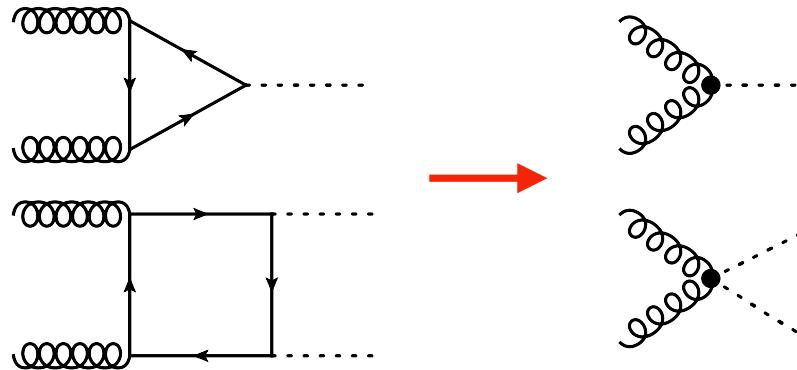


[from J.B., Djouadi, Quevillon, Rept.Prog.Phys. 79 (2016) 116201]

Heavy quark limit (HQL) calculation

HQL $\equiv m_t \rightarrow +\infty$

- Effective tree-level ggH and $ggHH$ couplings
- Reduce the number of loop by one at each perturbative order



- HQL valid for $\hat{s} \ll 4m_t^2$, but HH production threshold $4M_H^2 \leq \hat{s}$
 \Rightarrow narrow energy range for which HQL is valid!
- **Born-improved NLO QCD HQL:** improve HQL result with

$$d\sigma_{\text{NLO}} \simeq d\sigma_{\text{NLO}}^{\text{HQL}} \times \frac{d\sigma_{\text{LO}}^{\text{full}}}{d\sigma_{\text{LO}}^{\text{HQL}}} \quad [\text{Dawson, Dittmaier, Spira, PRD 58 (1998) 115012}]$$



Gluon fusion: Where we stand in 2018

- **LO QCD (1-loop):** Dominated by top-quark loops [Eboli, Marques, Novaes, Natale, PLB 197 (1987) 269; Glover, van der Bij, NPB 309 (1988) 282; Dicus, Kao, Willenbrock, PLB 203 (1988) 457]
- **NLO QCD HQL (1-loop): +93% correction** [Dawson, Dittmaier, Spira, PRD 58 (1998) 115012]
- **Toward full NLO QCD (2-loop):**
 - NLOFT_{approx}, m_t -effects in real radiation: **-10%** [Maltoni, Vryonidou, Zaro, JHEP 1411 (2014) 079]
 - $\mathcal{O}(1/m_t^{12})$ terms in virtual amplitudes: **$\pm 10\%$** [see e.g. Grigo, Hoff, Steinhauser, NPB 900 (2015) 412]
 - **Full NLO calculation: See next slide!**
- **NNLO QCD HQL (2-loop): +20%** [De Florian, Mazzitelli, PLB 724 (2013) 306; PRL 111 (2013) 201801]
- **NNLO QCD improvements:**
 - 3-loop matching coefficient [Grigo, Melnikov, Steinhauser, NPB 888 (2014) 17; Spira, JHEP 1610 (2016) 026], NNLL+NNLO matching $\Rightarrow +1\%$ at $\mu = M_{HH}/2$ [De Florian, Mazzitelli, JHEP 1509 (2015) 053]
 - Differential NNLO HQL [De Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev, JHEP 1609 (2016) 151]
 - **Best NNLO QCD prediction: See later in this talk!**



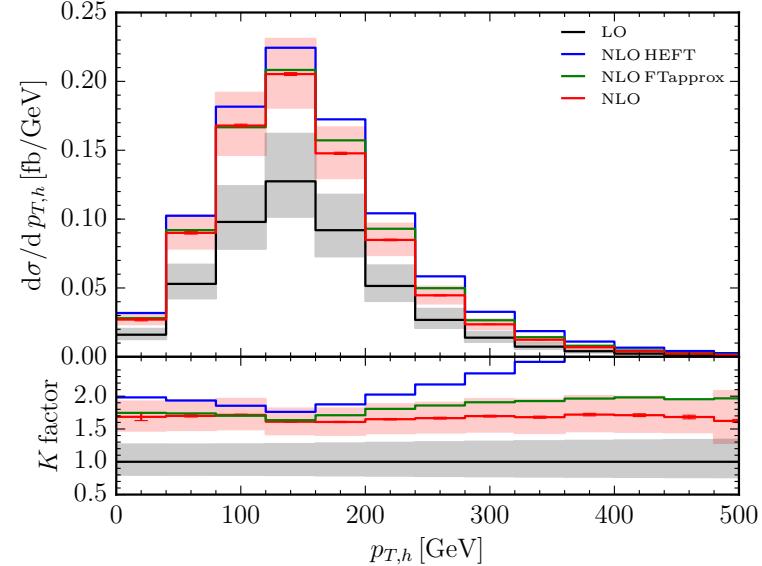
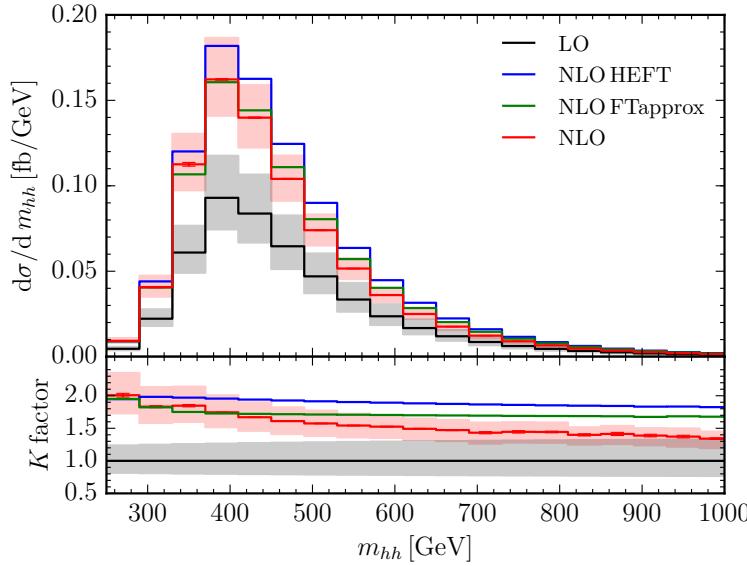
Full NLO QCD: Status in the literature (I)

- **Full NLO QCD corrections including m_t effects in the virtuals:** -4% in the total cross section, down to $\sim -25\%$ in the tails of differential distributions! [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke, PRL 117 (2016) 012001; Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Zirke, JHEP 1610 (2016) 107]
- **New independent result coming soon** [J.B., Campanario, Glaus, Mühlleitner, Spira, Streicher, 2018, to appear] ⇒ **Focus of this talk!**
- **Interface with parton shower:** Available in POWHEG and MG5_aMC@NLO [Heinrich, Jones, Kerner, Luisoni, Vryonidou, JHEP 1708 (2017) 088] and in SHERPA [Jones, Kuttmalai, JHEP 1802 (2018) 176]
- **Public tools:**
 - **NLO fixed order grid:** <https://github.com/mppmu/hhgrid/>
 - **MG5_aMC@NLO:** Contact person is eleni.vryonidou@cern.ch
 - **POWHEG:** Process ggHH available in the POWHEG-BOX-V2
 - **SHERPA:** Contact person is silvan@slac.stanford.edu

NLO QCD: Status in the literature (II)

Distributions at 14 TeV

[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Zirke, JHEP 1610 (2016) 107]



- Reduction to master integrals, sector decomposition, contour deformation
- **Large mass effects in the tail up to $\sim -30\%$ w.r.t. HQL**
- Born-improved HQL outside full NLO scale variation for $m_{hh} > 410$ GeV, $p_{T,h} > 160$ GeV
- $\text{NLOFT}_{\text{approx}}$ outside full NLO scale variation for $m_{hh} > 610$ GeV

Going to NNLO QCD

New calculation in 2018 with (partial) mass effects

[Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli, JHEP 1805 (2018) 059]

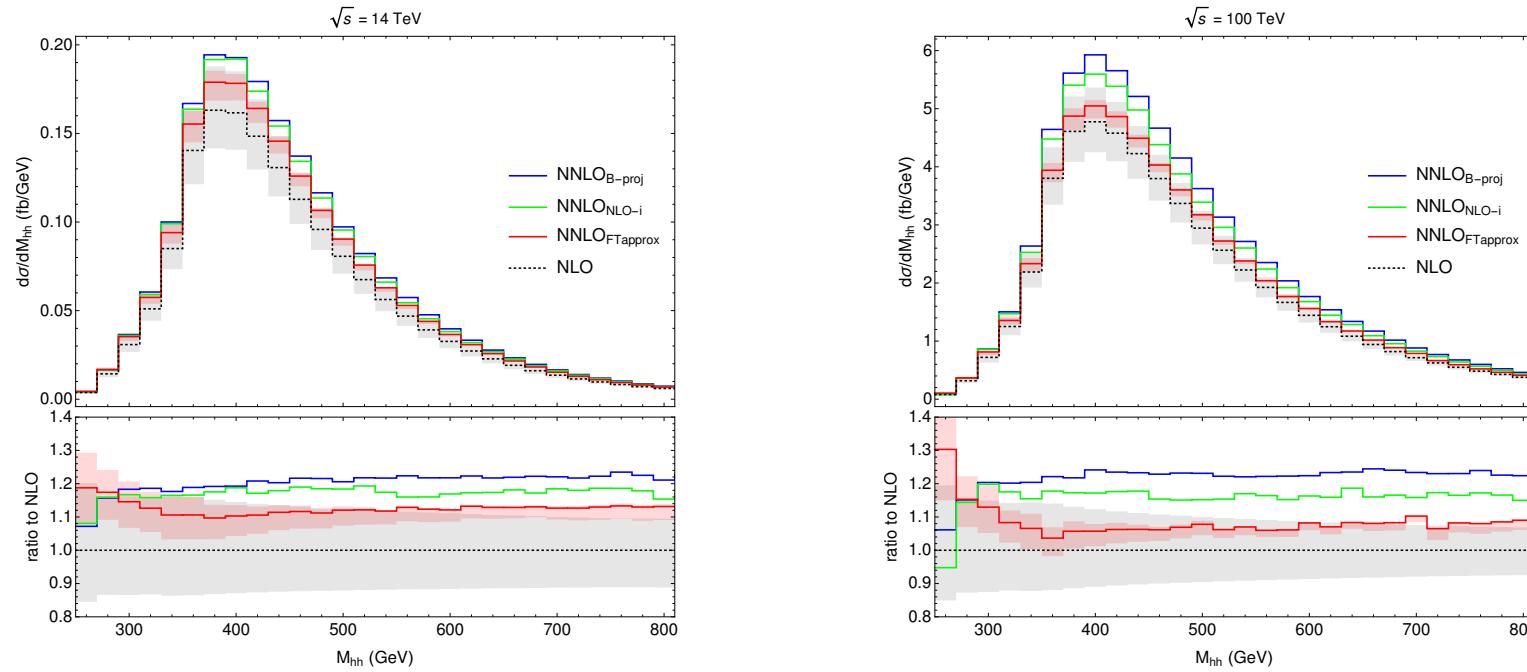
■ Building the calculation:

- Full m_t for the 1-loop real corrections (NLO, NNLO) and 1-loop HQL amplitudes with OpenLoop [see Buccioni, Pozzorini, Zoller, arXiv:1801.03772]
- Full m_t for the 2-loop NLO virtual corrections with an interpolated grid based on [Heinrich, Jones, Kerner, Luisoni, Vryonidou, JHEP 1708 (2017) 088]
- Analytical NNLO 2-loop virtual corrections in HQL [De Florian, Mazzitelli, PLB 724 (2013) 306]
- NNLO q_T -subtraction [Catani, Grazzini, PRL 98 (2007) 222002] implemented in MATRIX [Grazzini, Kallweit, Wiesemann, arXiv:1711.06631]

■ Three different approximations compared:

- NNLO_{NLO-i}: Bin-reweighting $d\sigma_{\text{NNLO}} = d\sigma_{\text{NLO}}^{\text{full}} \times (d\sigma_{\text{NNLO}}^{\text{HQL}} / d\sigma_{\text{NLO}}^{\text{HQL}})$
- NNLO_{B-proj}: Event-reweighting $d\sigma_{\text{NNLO}} = d\sigma_{\text{NNLO}}^{\text{HQL}} \times |\mathcal{M}_{\text{LO}}^{\text{full}}|^2 / |\mathcal{M}_{\text{LO}}^{\text{HQL}}|^2$
- **The best prediction, NNLO_{FTapprox}**: For each n -loop squared amplitude, reweighting $d\sigma_{ij,\text{NNLO}} = d\sigma_{ij,\text{NNLO}}^{\text{HQL}} \times |\mathcal{M}_{ij,\text{Born}}^{\text{full}}|^2 / |\mathcal{M}_{ij,\text{Born}}^{\text{HQL}}|^2$

HQL NNLO QCD distributions



- $\text{NNLO}_{\text{NLO-i}}$ and $\text{NNLO}_{\text{B-proj}}$ similar, overestimate NLO corrections by $\sim 10\%$ above 300 GeV
 $\sigma_{\text{NNLO}}^{\text{FTapprox}} = 36.69^{+2.1\%}_{-4.9\%} \text{ fb @ } 14 \text{ TeV}, \sigma_{\text{NNLO}}^{\text{FTapprox}} = 1.224^{+0.9\%}_{-3.2\%} \text{ pb @ } 100 \text{ TeV}$
- Scale uncertainties significantly reduced at NNLO
- Overlap of $\text{NNLO}_{\text{FTapprox}}$ prediction with NLO scale uncertainty band

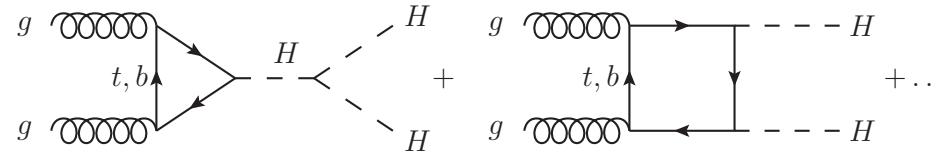


Full NLO QCD corrections to $gg \rightarrow HH$

J.B., Campanario, Glaus, Mühlleitner, Spira, Streicher, 2018, to appear]

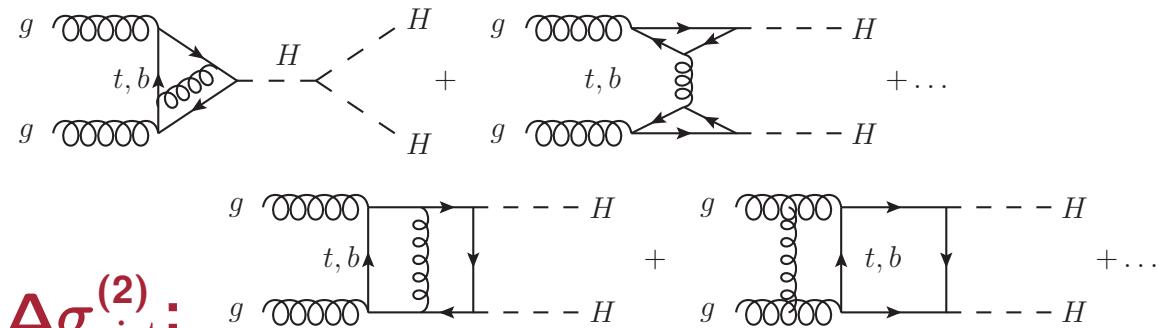
Overview of the calculation

$$\sigma_{\text{NLO}}(pp \rightarrow HH + X) = \sigma_{\text{LO}} + \Delta\sigma_{\text{virt}}^{(1)} + \Delta\sigma_{\text{virt}}^{(2)} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}}$$

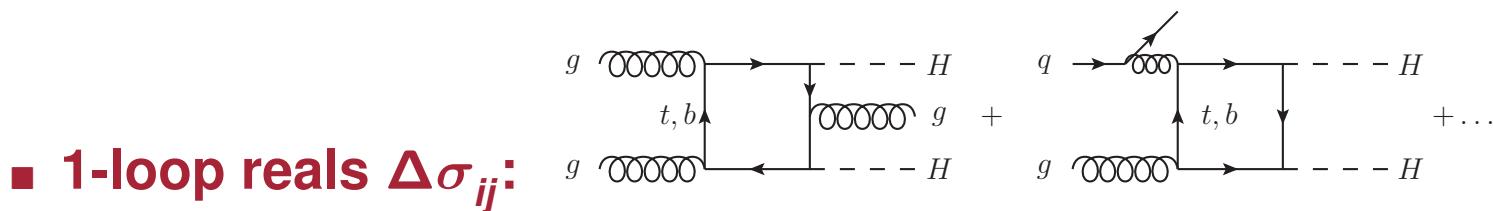


■ 1-loop LO σ_{LO} :

■ 2-loop triangle + 1-particle reducible $\Delta\sigma_{\text{virt}}^{(1)}$:



■ 2-loop box $\Delta\sigma_{\text{virt}}^{(2)}$:



Technical setup for the virtual corrections

- Triangle from single Higgs, 1-particle reducible analytically calculated
[see also Degrassi, Giardino, Gröber, EPJC 76 (2016) 411]
- Classification of the **47 2-loop tensor box diagrams** into 6 topologies
(+ corresponding fermion-flow reversed diagrams)
- With dimensional regularization $D = 4 - 2\epsilon$:
calculate the **matrix-element form factors F_1 and F_2** using
FORM/Reduce/Mathematica for $g(k_1)g(k_2) \rightarrow H(k_3)H(k_4)$,

$$\mathcal{M} = \varepsilon_\mu^*(k_1)\varepsilon_\nu^*(k_2) (F_1 \mathbf{T}_1^{\mu\nu} + F_2 \mathbf{T}_2^{\mu\nu}),$$

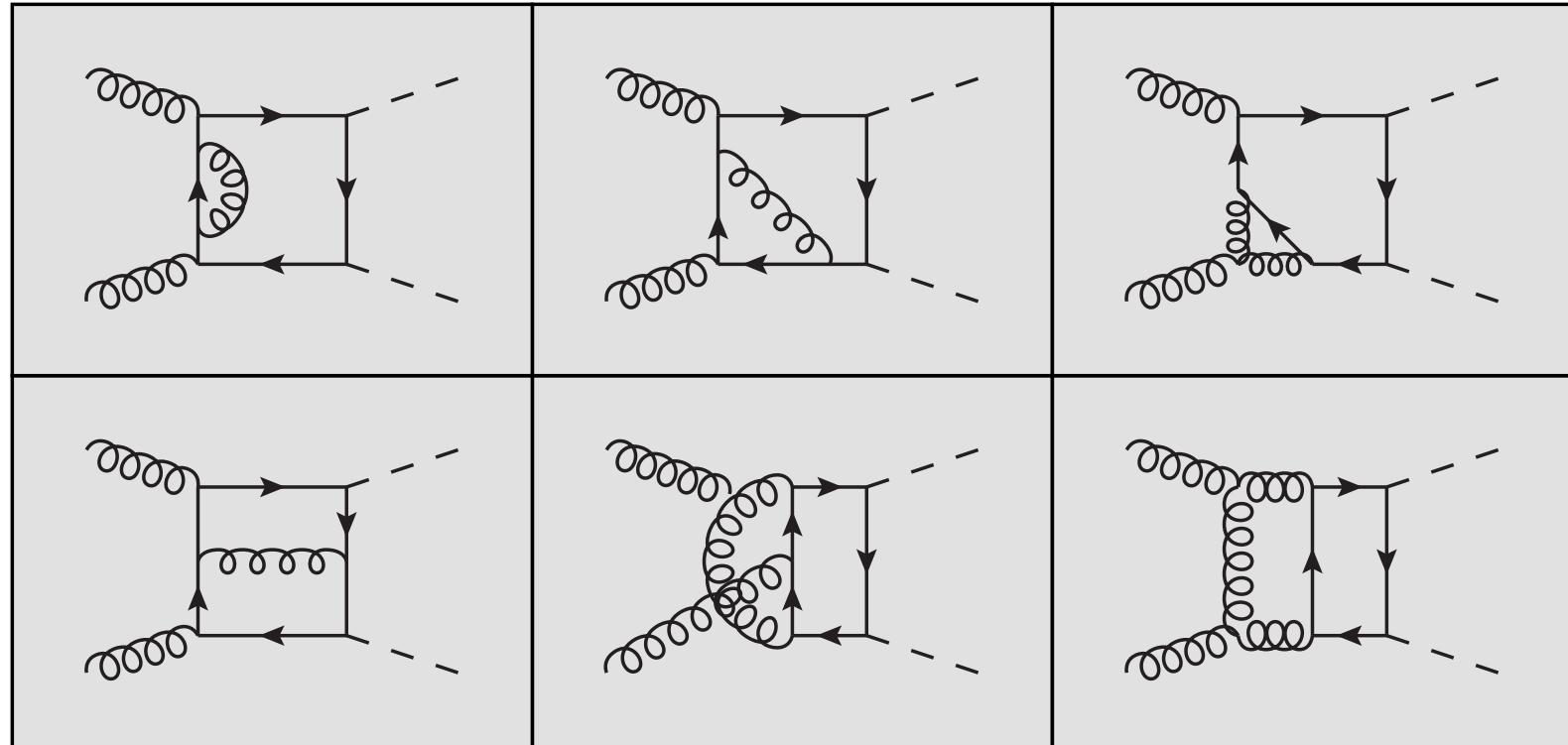
$$\mathbf{T}_1^{\mu\nu} = g^{\mu\nu} - \frac{k_2^\mu k_1^\nu}{k_1 \cdot k_2}, \quad p_T^2 = 2 \frac{(k_2 \cdot k_3)(k_1 \cdot k_3)}{k_1 \cdot k_2} - k_3^2,$$

$$\mathbf{T}_2^{\mu\nu} = g^{\mu\nu} + \frac{k_2^\mu k_1^\nu}{(k_1 \cdot k_2)p_T^2} k_3^2 - \frac{2}{p_T^2} \left[\frac{k_2 \cdot k_3}{k_1 \cdot k_2} k_3^\mu k_1^\nu + \frac{k_1 \cdot k_3}{k_1 \cdot k_2} k_2^\mu k_3^\nu - k_3^\mu k_3^\nu \right]$$

- **Perform Feynman parametrization**
→ 6-dimensional integrals to be (numerically) evaluated



2-loop virtual box corrections



2-loop virtual box corrections

■ Extraction of ultraviolet (UV) divergences:

Endpoint subtraction of the Feynman integrals

$$\int_0^1 dx \frac{f(x)}{(1-x)^{1-\epsilon}} = \frac{f(1)}{\epsilon} + \int_0^1 dx \frac{f(x) - f(1)}{1-x} + \mathcal{O}(\epsilon)$$

■ Infrared (IR) divergences in the middle of the range

⇒ Subtraction of the integrand and analytical integration

- Generic denominator $N = ar^2 + br + c$, $N_0 = br + c$
- Feynman parameters (x, s, t) and $\rho = \frac{\hat{s}}{m_t^2}$
- $a = \mathcal{O}(\rho)$, $b = 1 + \mathcal{O}(\rho)$, $c = -\rho x(1-x)(1-s)t$

$$\int_0^1 dx dr \frac{rH(x, r)}{N^{3+2\epsilon}} = \int_0^1 dx dr \left[\left(\frac{rH(x, r)}{N^{3+2\epsilon}} - \frac{rH(x, 0)}{N_0^{3+2\epsilon}} \right) + \frac{rH(x, 0)}{N_0^{3+2\epsilon}} \right]$$

Handling threshold instabilities

- **Threshold at $\hat{s} = M_{HH}^2 = 4m_t^2$:**

\Rightarrow Analytical continuation in the complex plane with

$$m_t^2 \rightarrow m_t^2 (1 - i\tilde{\epsilon}), \quad \tilde{\epsilon} \ll 1$$

- **Enhance stability above threshold with integration by parts** With $N = a + bx$,

$$\int_0^1 dx \frac{2b f(x)}{N^3} = \frac{f(0)}{a^2} - \frac{f(1)}{(a+b)^2} + \int_0^1 dx \frac{f'(x)}{N^2}$$

$$\int_0^1 dx \frac{4b f(x) \log N}{N^3} = \left[-\frac{f(x)(1 + 2 \log N)}{N^2} \right]_0^1 + \int_0^1 dx \frac{f'(x)(1 + 2 \log N)}{N^2}$$



UV renormalization and IR subtraction

■ α_s and m_t input parameters to renormalize

- $\overline{\text{MS}}$ renormalization for α_s with 5 active flavors, δ_{α_s} ,
- Top-quark contribution to the external gluon self-energies,
 δ_g
- On-shell renormalization for m_t , δ_{m_t}

■ IR subtraction, δ_{IR} :

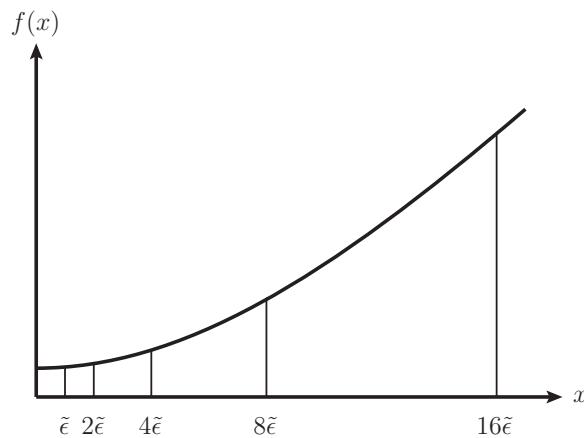
Subtraction of Born-improved HQL virtual corrections to box diagrams \Rightarrow IR-safe virtual mass-effects



Richardson extrapolation

- Goal: From $m_t^2 (1 - i\tilde{\epsilon})$, obtain the limit $\tilde{\epsilon} \rightarrow 0$
- Solution: Richardson extrapolation of the result!

Assuming $f(\tilde{\epsilon}) - f(0)$ polynomial for small $\tilde{\epsilon}$, method to accelerate the convergence of $f(\tilde{\epsilon})$ to $f(0)$



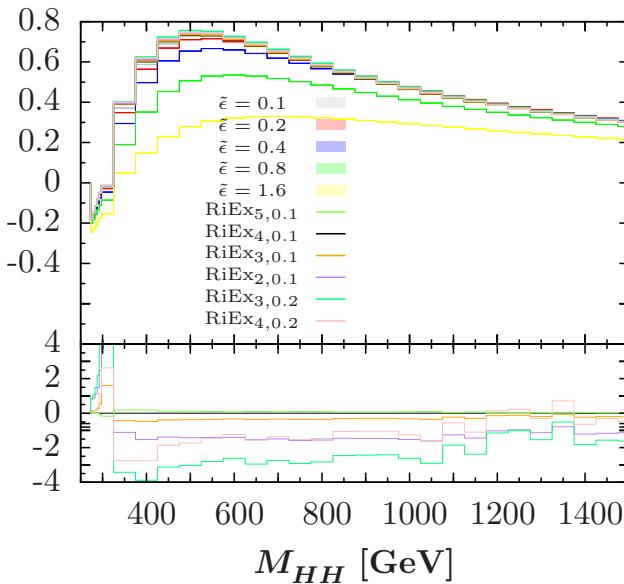
$$\begin{aligned}
 \text{RiEx}_{2,\tilde{\epsilon}} &= 2f(\tilde{\epsilon}) - f(2\tilde{\epsilon}) = f(0) + \mathcal{O}(\tilde{\epsilon}^2) \\
 \text{RiEx}_{3,\tilde{\epsilon}} &= \frac{1}{3} [8f(\tilde{\epsilon}) - 6f(2\tilde{\epsilon}) + f(4\tilde{\epsilon})] = f(0) + \mathcal{O}(\tilde{\epsilon}^3) \\
 \text{RiEx}_{4,\tilde{\epsilon}} &= \frac{1}{21} [64f(\tilde{\epsilon}) - 56f(2\tilde{\epsilon}) + 14f(4\tilde{\epsilon}) - f(8\tilde{\epsilon})] \\
 &= f(0) + \mathcal{O}(\tilde{\epsilon}^4) \\
 \text{RiEx}_{5,\tilde{\epsilon}} &= \frac{1}{315} [1024f(\tilde{\epsilon}) - 960f(2\tilde{\epsilon}) + 280f(4\tilde{\epsilon}) \\
 &\quad - 30f(8\tilde{\epsilon}) + f(16\tilde{\epsilon})] \\
 &= f(0) + \mathcal{O}(\tilde{\epsilon}^5)
 \end{aligned}$$

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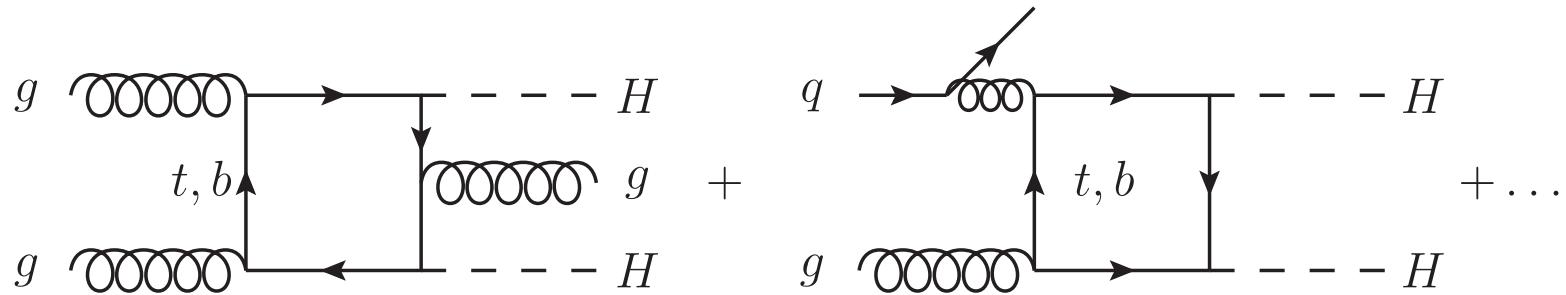
Box 47, sum of all form factors



$$\begin{aligned}
 RiEx_{2,\tilde{\epsilon}} &= 2f(\tilde{\epsilon}) - f(2\tilde{\epsilon}) = f(0) + \mathcal{O}(\tilde{\epsilon}^2) \\
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 &\quad - 30f(8\tilde{\epsilon}) + f(16\tilde{\epsilon})] \\
 &= f(0) + \mathcal{O}(\tilde{\epsilon}^5)
 \end{aligned}$$

Calculation of the real corrections

Partonic sub-processes $gg \rightarrow HHg$, $gq/\bar{q} \rightarrow HHq/\bar{q}$, $q\bar{q} \rightarrow HHg$



- Full matrix elements generated with FeynArts/FormCalc [see Hahn, PoS ACAT2010 (2010) 078], evaluated with 1-loop library COLLIER [Denner, Dittmaier, Hofer, CPC 212 (2017) 220]
- Then subtracted with Born-improved HQL matrix-element squared calculated analytically

Putting everything together

- Numerical integration performed with VEGAS on a cluster, \hat{t} -integration
- **Final hadronic result:**

$$\Delta\hat{\sigma}_{\text{virt}} = \int d\Phi_{2 \rightarrow 2} \left[(\delta_{\alpha_s} + \delta_g + \delta_{m_t} + \delta_{\text{IR}} + \mathcal{M}_{\text{virt}}^{\square})(\mathcal{M}_{\text{LO}})^* \right] + \Delta\hat{\sigma}_{\text{virt}}^{\triangle} + \Delta\hat{\sigma}_{\text{virt}}^{1\text{PR}}$$

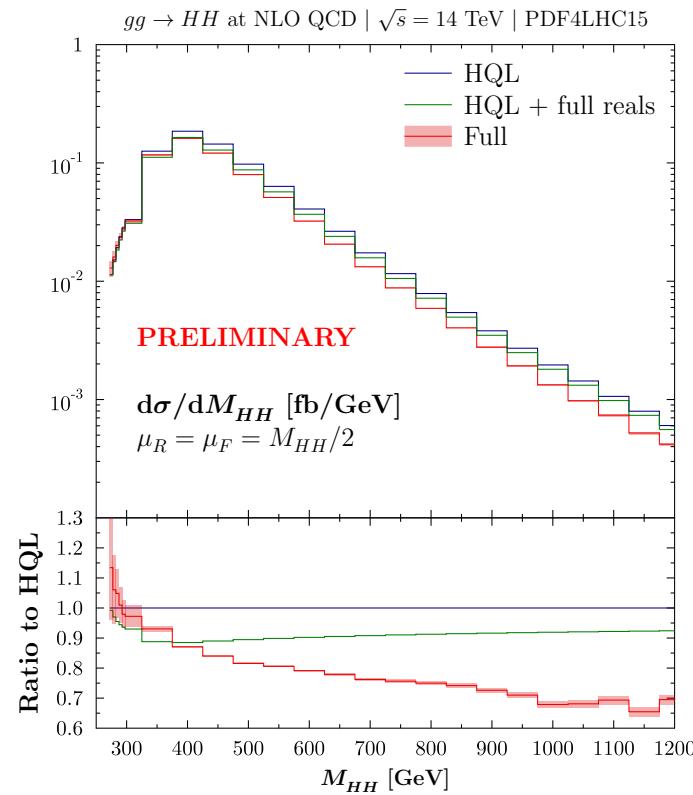
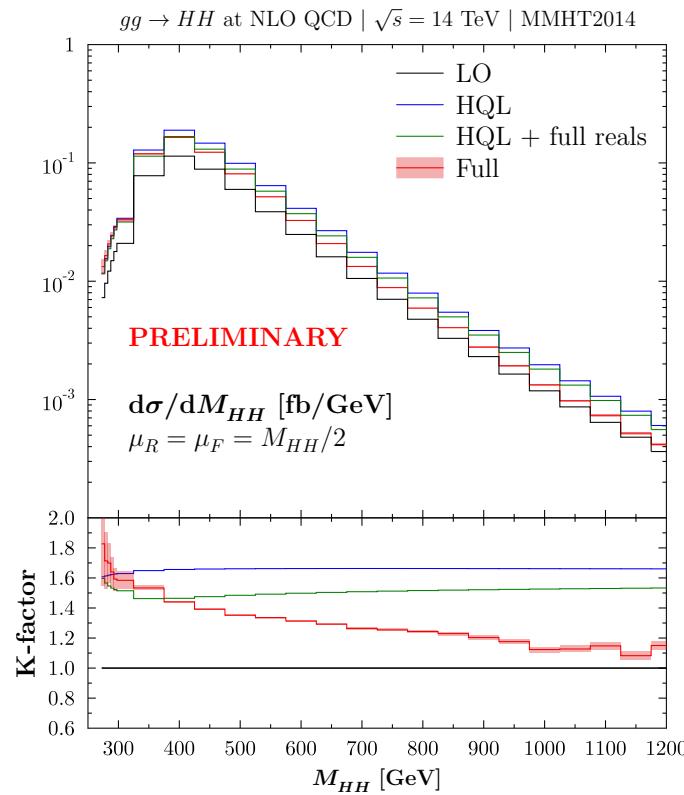
$$M_{HH}^2 \frac{d\sigma_{\text{NLO}}}{dM_{HH}^2} = M_{HH}^2 \frac{d\sigma_{\text{HPAIR}}}{dM_{HH}^2} + M_{HH}^2 \frac{d\Delta\sigma_{\text{virt}}}{dM_{HH}^2} + M_{HH}^2 \frac{d\Delta\sigma_{\text{reals}}}{dM_{HH}^2}$$

HQL hadronic result calculated with HPAIR [Spira, 1996]

- **Input parameters: can be freely chosen!** PDG values for M_W and M_Z , $M_H = 125 \text{ GeV}$, $m_t = 172.5 \text{ GeV}$, $G_F = 1.1663787 \times 10^{-5} \text{ GeV}^{-2}$, $\sqrt{s} = 14 \text{ TeV}$



Preliminary results



- Mass effects in the real corrections $\sim -10\%$ as in [Maltoni, Vryonidou, Zaro, JHEP 1411 (2014) 079]
- Mass effects in the virtual corrections $\sim -25\%$ at $M_{HH} = 1$ TeV
- First independent cross-check of the results in [Borowka *et al.*, PRL 117 (2016) 012001; JHEP 1610 (2016) 107]



Conclusion and outlook

HH production, status in 2018: Precision QCD rules the game!

- All production channels known at least at NLO QCD including differential distributions
- Focus on gluon fusion: NNLO differential predictions with approximate NNLO top-quark mass effects available!
- Full NLO QCD corrections for gluon fusion: Calculated for the first time in 2016, now interfaced with parton-shower programs
 - Public code available to be used, POWHEG, MG5_aMC@NLO SHERPA
- New in 2018: First independent cross-check for the full 2-loop NLO QCD corrections in gluon fusion!
 - Complete different method compared to the 2016 calculation
 - Code flexible: m_t , M_H not fixed a priori, can be changed at will
- Outlook: Extension to BSM physics and in particular EFT and 2HDM models



Backup slides

Details for the renormalization

■ UV renormalization: δ_{α_s} , δ_g , δ_{m_t}

→ $\overline{\text{MS}}$ renormalization for α_s with 5 active flavors $N_F = 5$

$$\frac{\delta \alpha_s}{\alpha_s} = \frac{\alpha_s}{\pi} \Gamma(1 + \epsilon) (4\pi)^\epsilon \left[-\frac{33 - 2(N_F + 1)}{12\epsilon} + \frac{1}{6} \log \left(\frac{\mu_R^2}{m_t^2} \right) \right], \quad \delta_{\alpha_s} = \frac{\delta \alpha_s}{\alpha_s} \mathcal{M}_{\text{LO}}$$

→ Top-quark contribution to the external gluons self-energies

$$\delta_g = \frac{\alpha_s}{\pi} \Gamma(1 + \epsilon) \left(\frac{4\pi\mu_R^2}{m_t^2} \right)^\epsilon \left(-\frac{1}{6\epsilon} \right) \mathcal{M}_{\text{LO}}$$

→ On-shell renormalization for m_t

$$\frac{\delta m_t}{m_t} = \frac{\alpha_s}{\pi} \Gamma(1 + \epsilon) \left(\frac{4\pi\mu_R^2}{m_t^2} \right)^\epsilon \left(\frac{1}{\epsilon} + \frac{4}{3} \right), \quad \delta_{m_t} = -2 \frac{\delta m_t}{m_t} m_t^2 \frac{\partial \mathcal{M}_{\text{LO}}}{\partial m_t^2}$$

■ IR subtraction:

$$\delta_{\text{IR}} = \frac{\alpha_s}{\pi} \frac{\Gamma(1 - \epsilon)}{\Gamma(1 - 2\epsilon)} \left(\frac{4\pi\mu_R^2}{-M_{HH}^2} \right)^\epsilon \left[\frac{3}{2\epsilon^2} + \frac{33 - 2N_F}{12\epsilon} \left(\frac{\mu_R^2}{-M_{HH}^2} \right)^{-\epsilon} - \frac{11}{4} + \frac{\pi^2}{4} \right] \mathcal{M}_{\text{LO}}$$