DM@NLO

Recent progress in precision dark matter calculation

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GDR/IRN Terascale — 30th may - 1st june 2018 — Strasbourg

Dark matter relic abundance — freeze-out picture

Time evolution of number density of the relic particle described by Boltzmann equation
— key ingredient from particle physics: **(co-)annihilation cross-section**

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -3Hn - \langle\sigma_{\mathrm{ann}}v\rangle\left(n^2 - n_{\mathrm{eq}}^2\right)$$

→

$$\Omega_\chi h^2 = \frac{m_\chi n_\chi}{\rho_{\mathrm{crit}}} \sim \frac{1}{\langle\sigma_{\mathrm{ann}}v\rangle}$$

↕ **(dis)favoured parameter regions…?**

$$\Omega_{\mathrm{CDM}}h^2 = 0.1199 \pm 0.0022$$

Relic abundance very precisely known

Planck collaboration 2015

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$$\langle \sigma_{\mathrm{ann}} v \rangle = \sum_{i,j} \sigma_{ij} v_{ij} \frac{n_i^{\mathrm{eq}}}{n_\chi^{\mathrm{eq}}} \frac{n_j^{\mathrm{eq}}}{n_\chi^{\mathrm{eq}}}$$

(dis)favoured parameter regions…?

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$$\frac{n_i^{\mathrm{eq}}}{n_\chi^{\mathrm{eq}}} \sim \exp\left\{ -\frac{m_i - m_\chi}{T} \right\}$$

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$$\Omega_{\rm CDM} h^2 = 0.1199 \pm 0.0022$$

Relic abundance very precisely known
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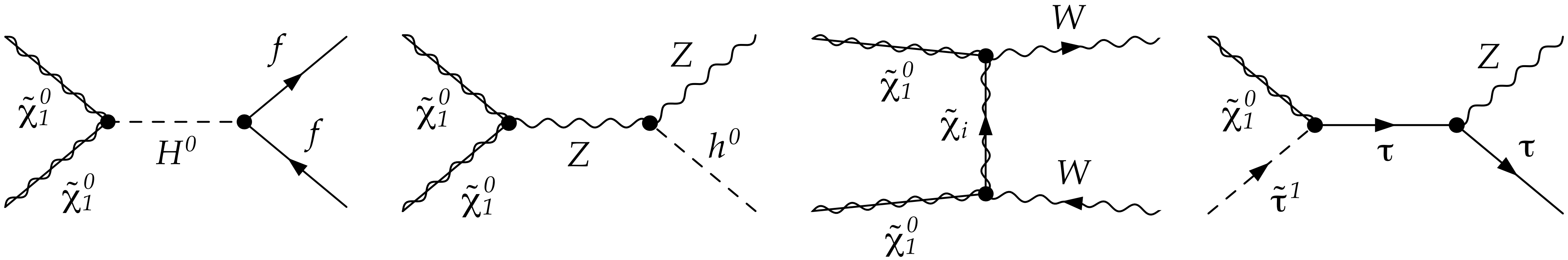
Computational tools allow an efficient calculation of the (neutralino) relic density:
DarkSUSY Bergström, Edsjö, Gondolo *et al.* 2004-2018, **micrOMEGAs** Bélanger, Boudjema, Pukhov *et al.* 2003-2018, **SuperIsoRelic** Arbey, Mahmoudi 2008, **MadDM** Backovic, Maltoni, Mantani, Mattelart *et al.* 2015-2018, …

Motivation for higher order corrections

All processes implemented in public codes — **but only at the (effective) tree-level**



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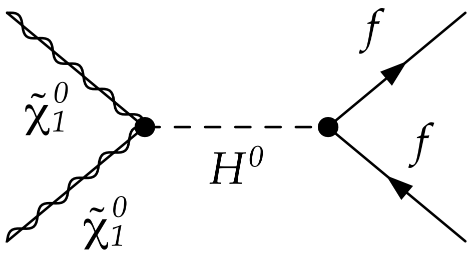
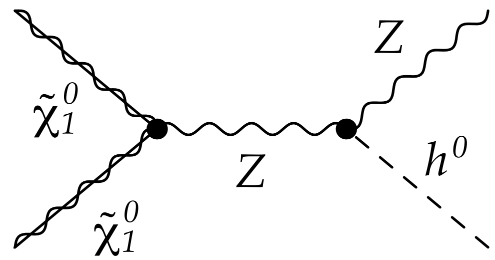
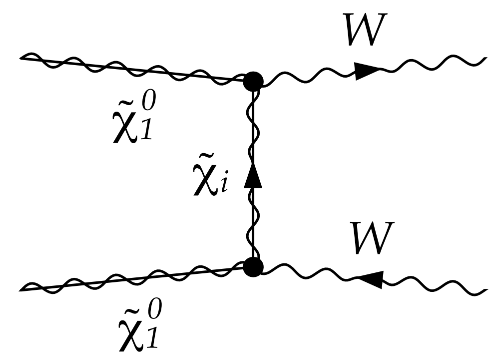
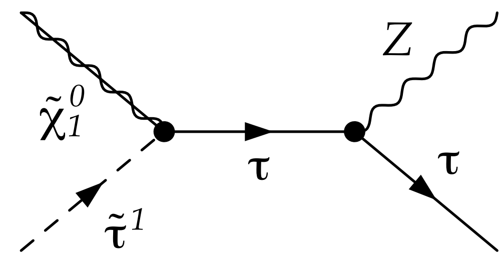




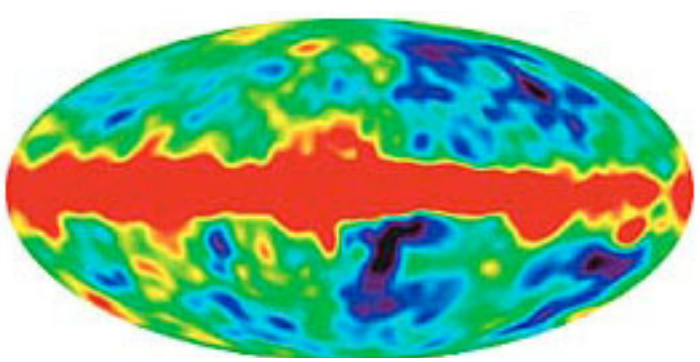


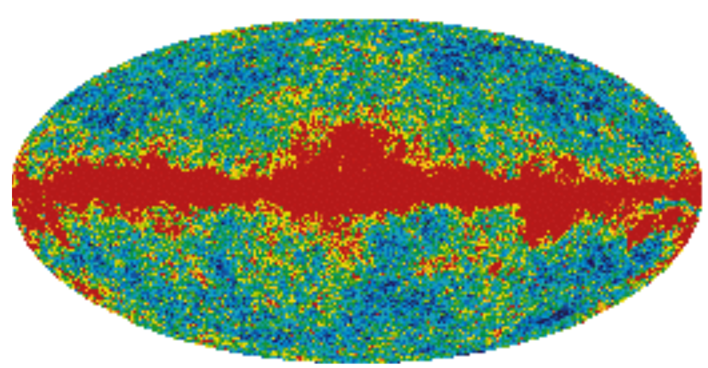


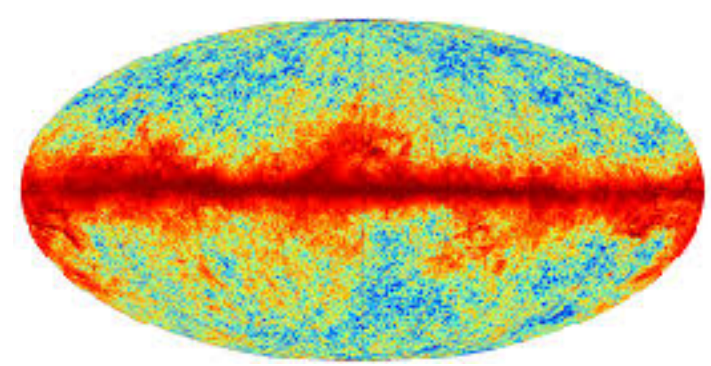
Higher-order loop corrections can give important contributions to cross-sections

In particular, sizeable impact from QCD corrections due to strong coupling constant

More precise theoretical predictions needed to keep up with experimental improvements


COBE 1989


WMAP 2002


Planck 2013

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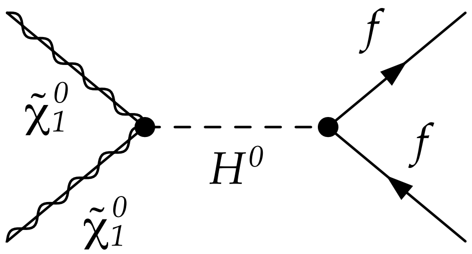

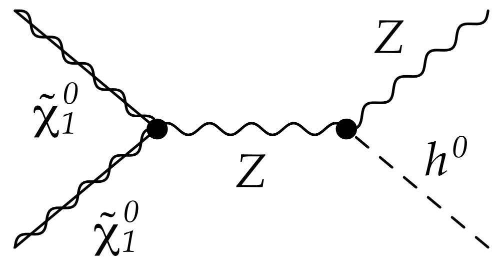

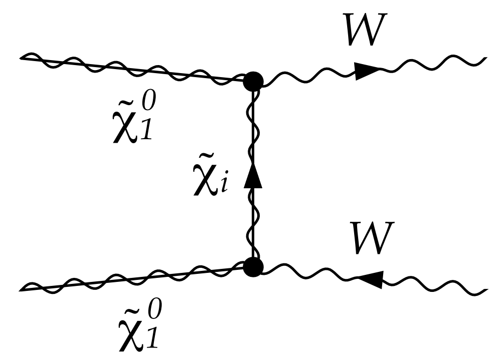

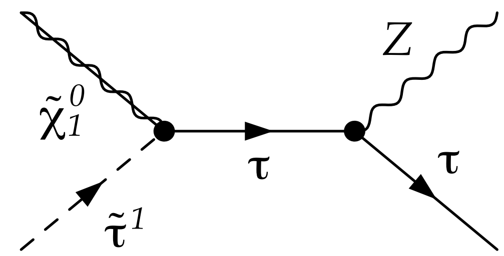

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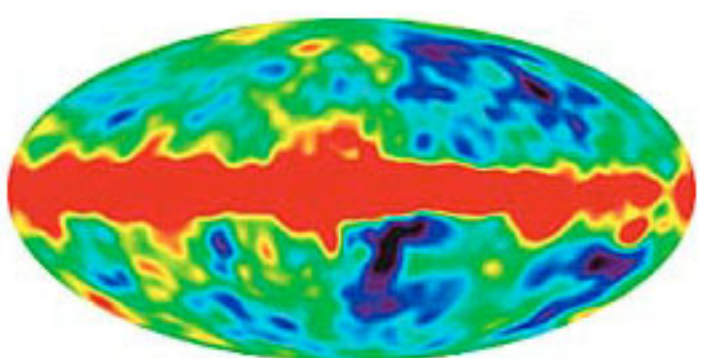

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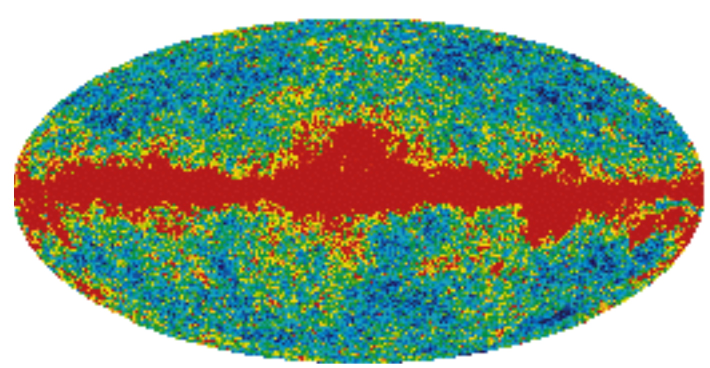

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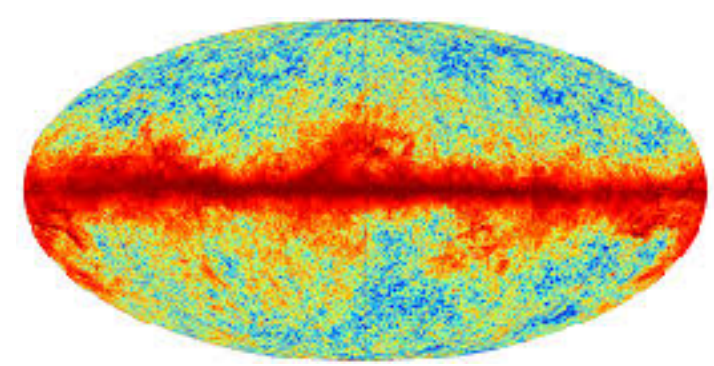
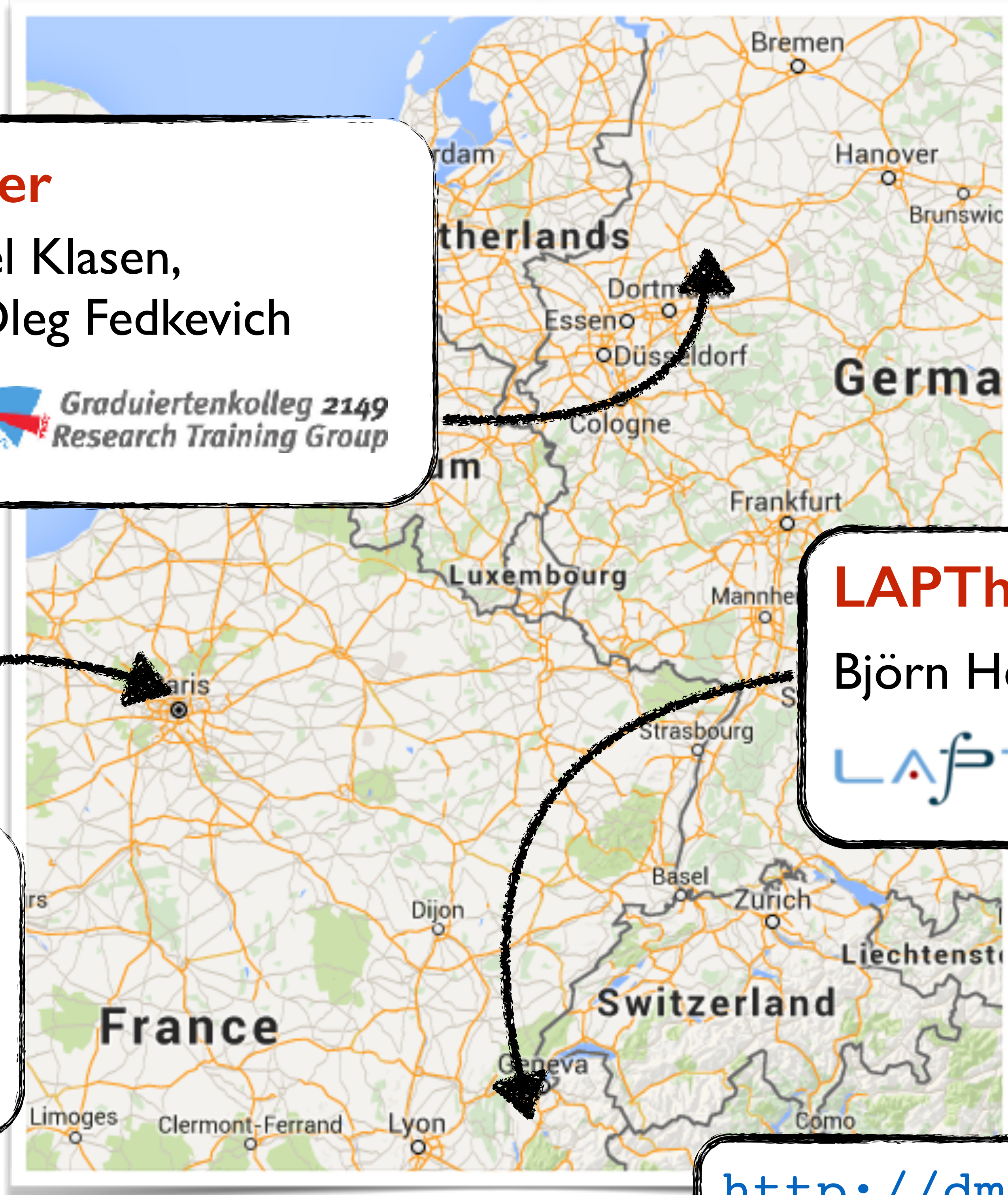


Planck 2013

DM@NLO project — **Provide calculation of σ_{ann} including QCD corrections**

— Extension to public codes (e.g. micrOMEGAs, DarkSUSY)…

DM@NLO — Collaboration



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Saskia Schmiemann, Oleg Fedkevich

Westfälische Wilhelms-Universität Münster

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Julia Harz

ILP — Sorbonne Université, Créateurs de futurs depuis 1257 — CNRS

http://dmnlo.hepforge.org

DM@NLO — Status

Provide a **next-to-leading order calculation** (in QCD) for the following (co-)annihilation cross sections (and thus for the dark matter relic density)

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$\tilde{\chi}\tilde{\chi}' \to q\bar{q}'$	**implemented micrOMEGAs + DarkSUSY**
$\tilde{\chi}\tilde{q} \to q'H/q'V$	**implemented micrOMEGAs**
$\tilde{q}\tilde{q}^* \to HH/HV/VV$	**implemented micrOMEGAs**
$\tilde{\chi}\tilde{\chi}' \to gg/\gamma\gamma$	**done but not fully checked — relevant…?**
$\tilde{q}\tilde{q}^* \to q\bar{q}'$	**implemented micrOMEGAs** ***(to be published soon!)***
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$\tilde{q}\tilde{q} \to qq$	**work in progress…**
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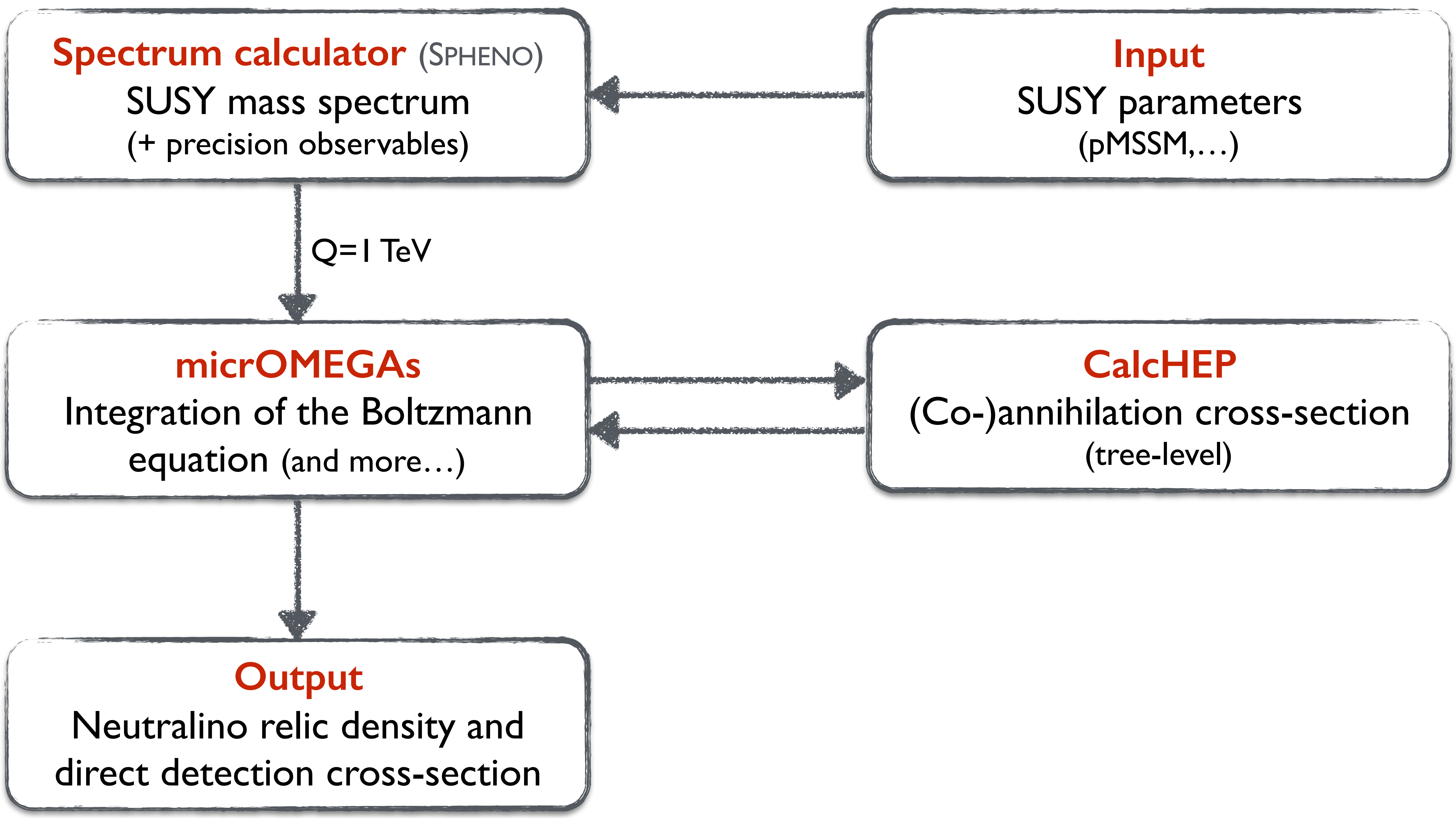
Definition and implementation of a dedicated **renormalization scheme**

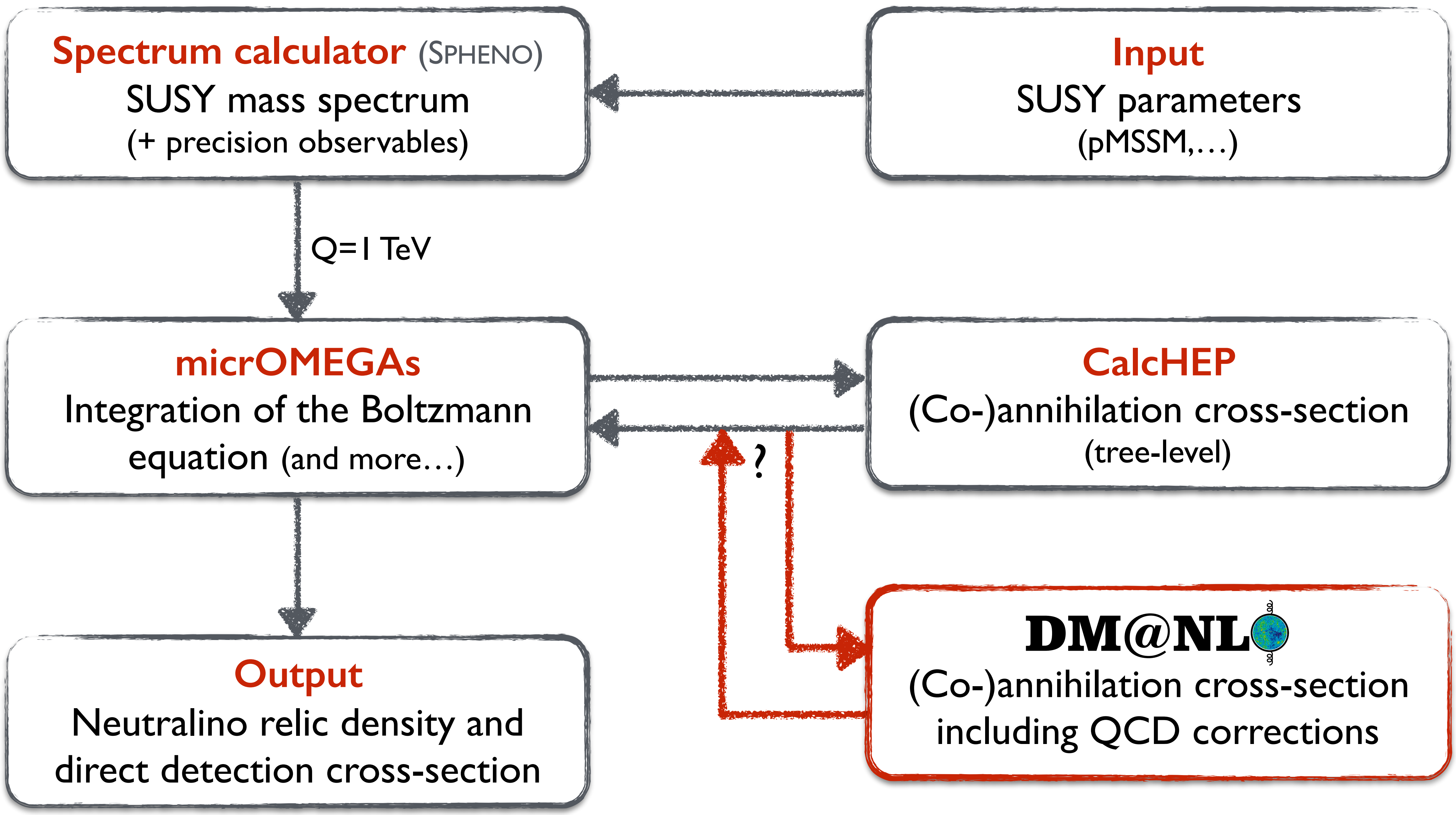
Infrared treatment — phase space slicing and dipole subtraction à la Catani-Seymour

Resummation of **Coulomb corrections** for stop-stop annihilation

DM@NLO — Setup


Spectrum calculator (SPHENO)
SUSY mass spectrum
(+ precision observables)
Input
SUSY parameters
(pMSSM,...)
Q=1 TeV
micrOMEGAs
Integration of the Boltzmann equation (and more...)
CalcHEP
(Co-)annihilation cross-section
(tree-level)
Output
Neutralino relic density and direct detection cross-section

DM@NLO — Setup



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DM@NLO
(Co-)annihilation cross-section including QCD corrections
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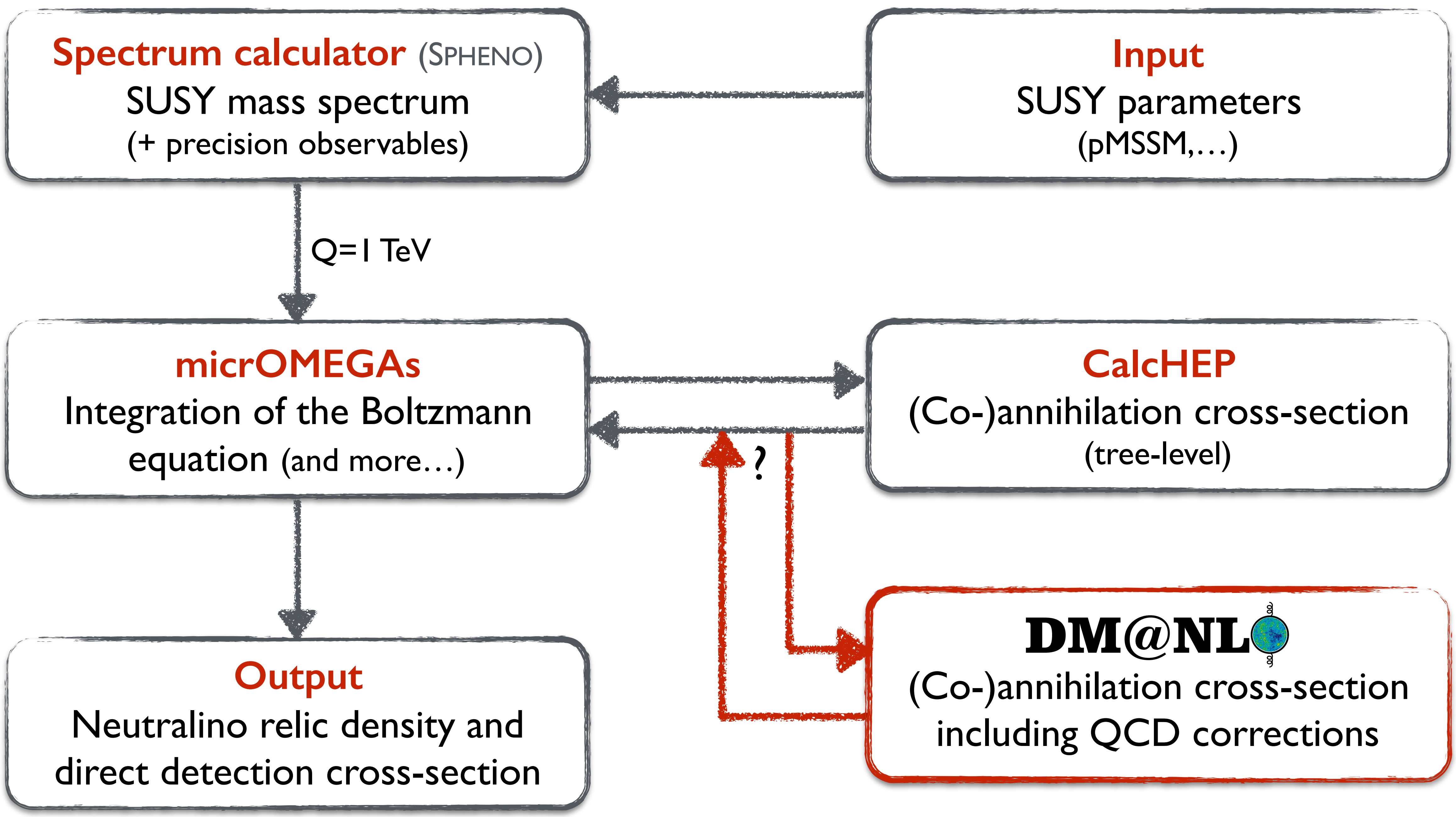
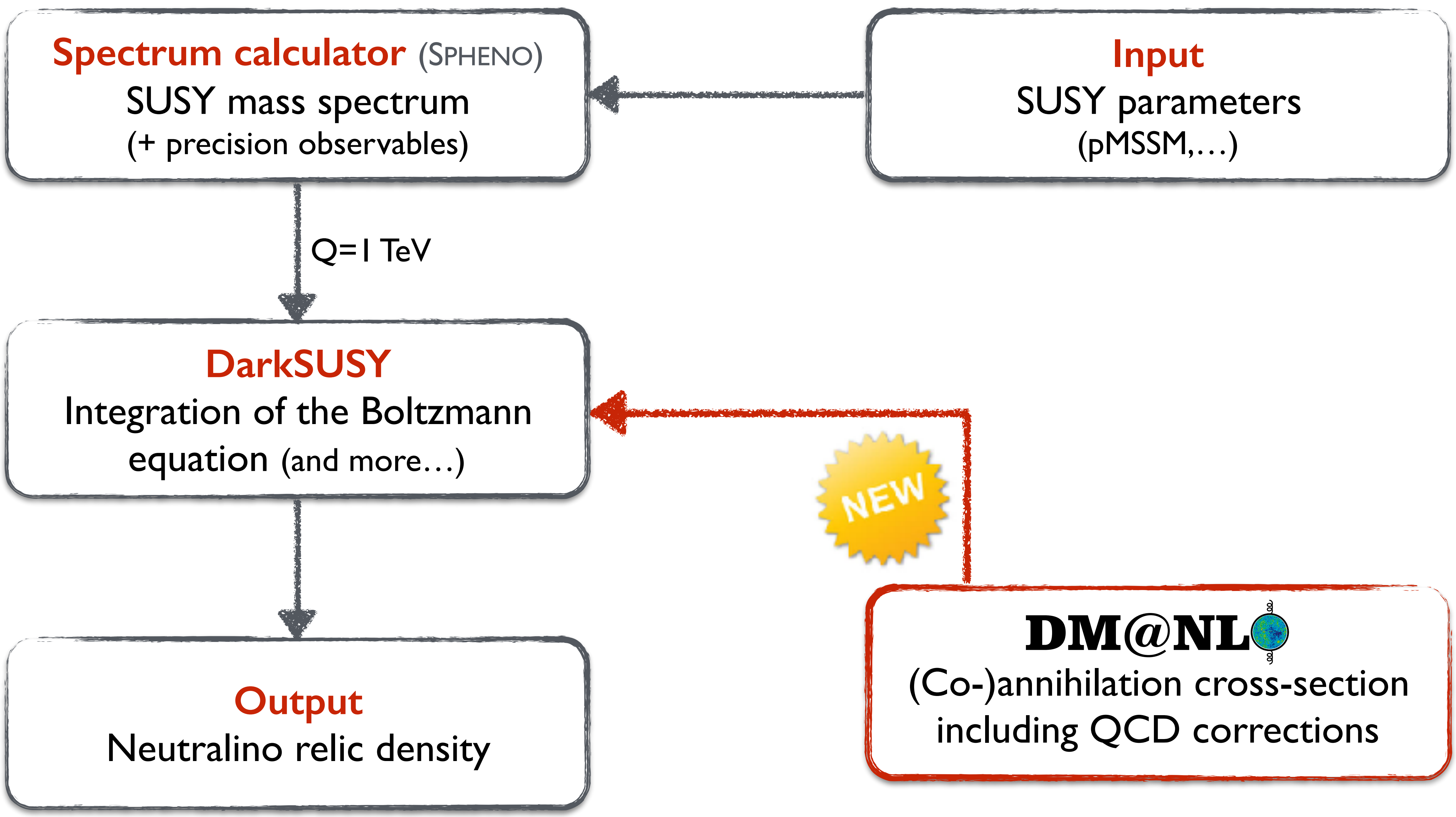


Rather general interface applicable to all (co-)annihilation channels.

Thanks to A. Pukhov!

DM@NLO — Setup



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DarkSUSY
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NEW
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DM@NLO
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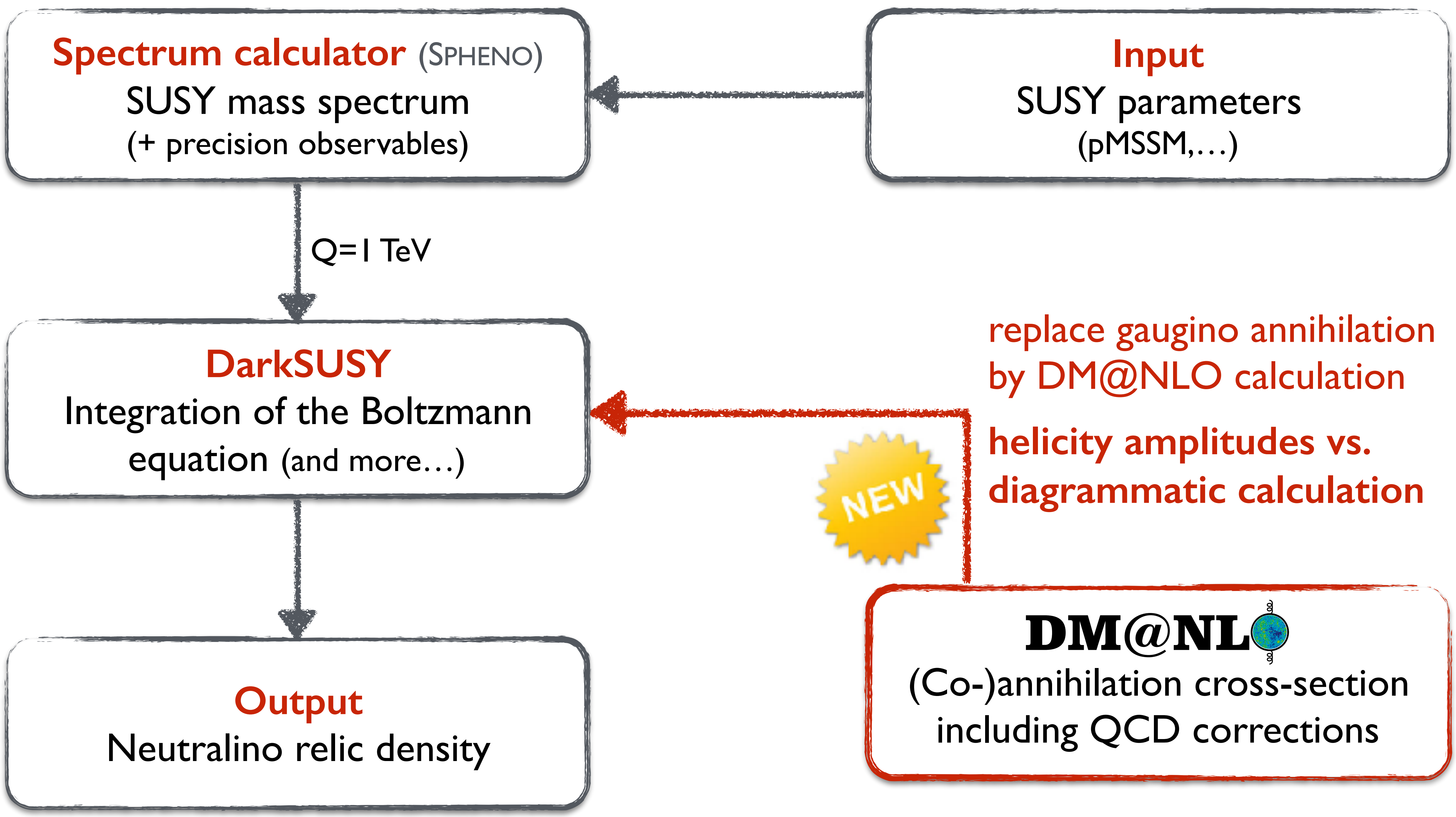
DM@NLO — Setup



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replace gaugino annihilation by DM@NLO calculation
helicity amplitudes vs. diagrammatic calculation
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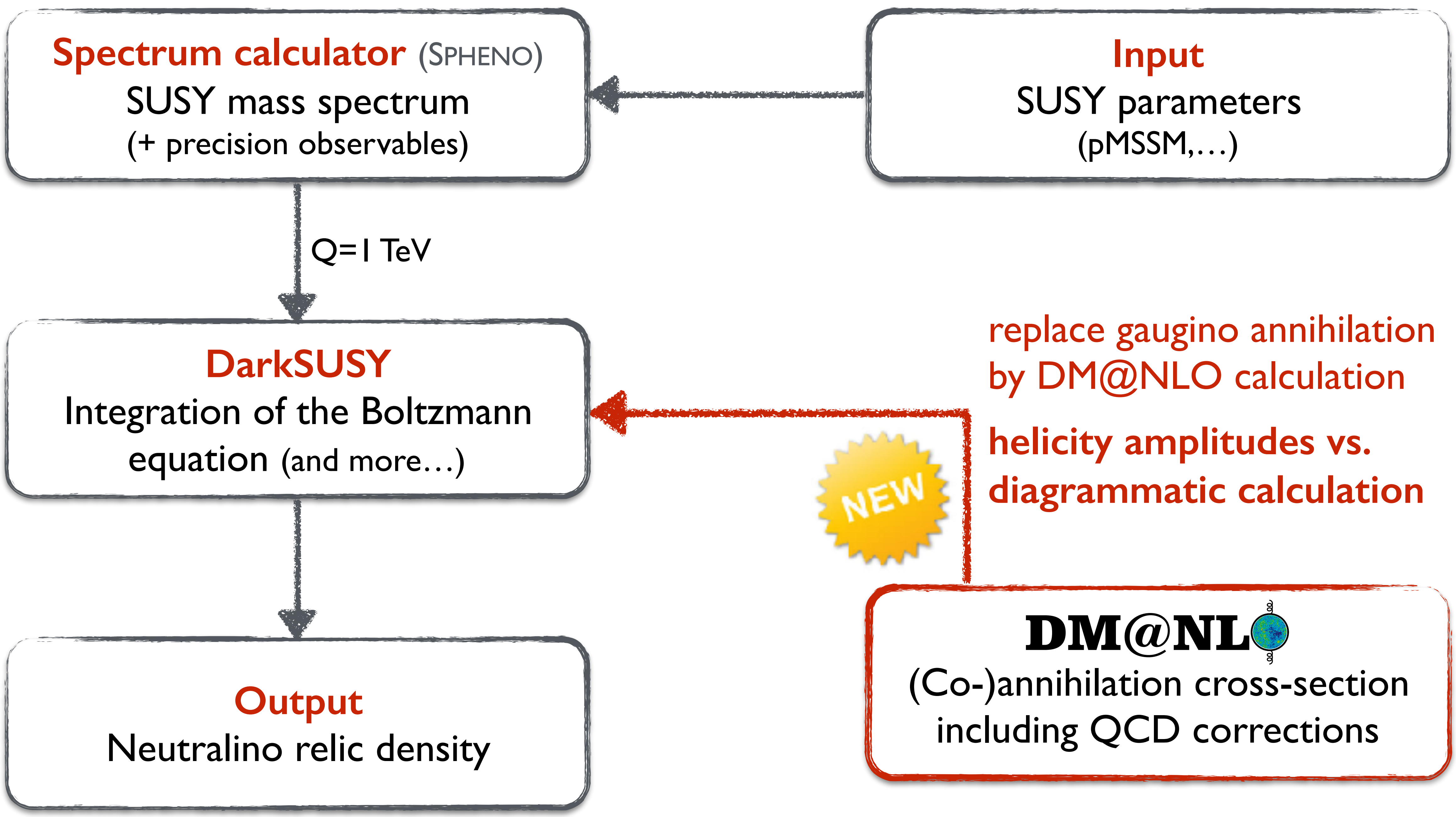


Ultimate goal: use of DM@NLO within GAMBIT studies…

C. Niblaeus, J. Harz, B. Herrmann, J. Edsjö

Outline

Motivation

Corrections to the neutralino (co)annihilation cross-section and impact on relic density

Application to direct dark matter detection

Scale dependence and theoretical uncertainty

Conclusion and Outlook

M. Klasen, K. Kovařík, P. Steppeler — Phys.Rev. D94: 095002 (2016) — arXiv:1607.06396 [hep-ph]

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, P. Steppeler — Phys. Rev. D 93: 114023 (2016) — arXiv:1602.08103 [hep-ph]

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, M. Meinecke — Phys. Rev. D 91: 034012 (2015) — arXiv:1410.8063 [hep-ph]

J. Harz, B. Herrmann, M. Klasen, K. Kovařík — Phys. Rev. D 91: 034028 (2015) — arXiv:1409.2898 [hep-ph]

B. Herrmann, M. Klasen, K. Kovařík, M. Meinecke, P. Steppeler — Phys. Rev. D 89: 114012 (2014) — arXiv:1404.2931 [hep-ph]

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, Q. Le Boulc'h — Phys. Rev. D 87: 054031 (2013) — arXiv:1212.5241 [hep-ph]

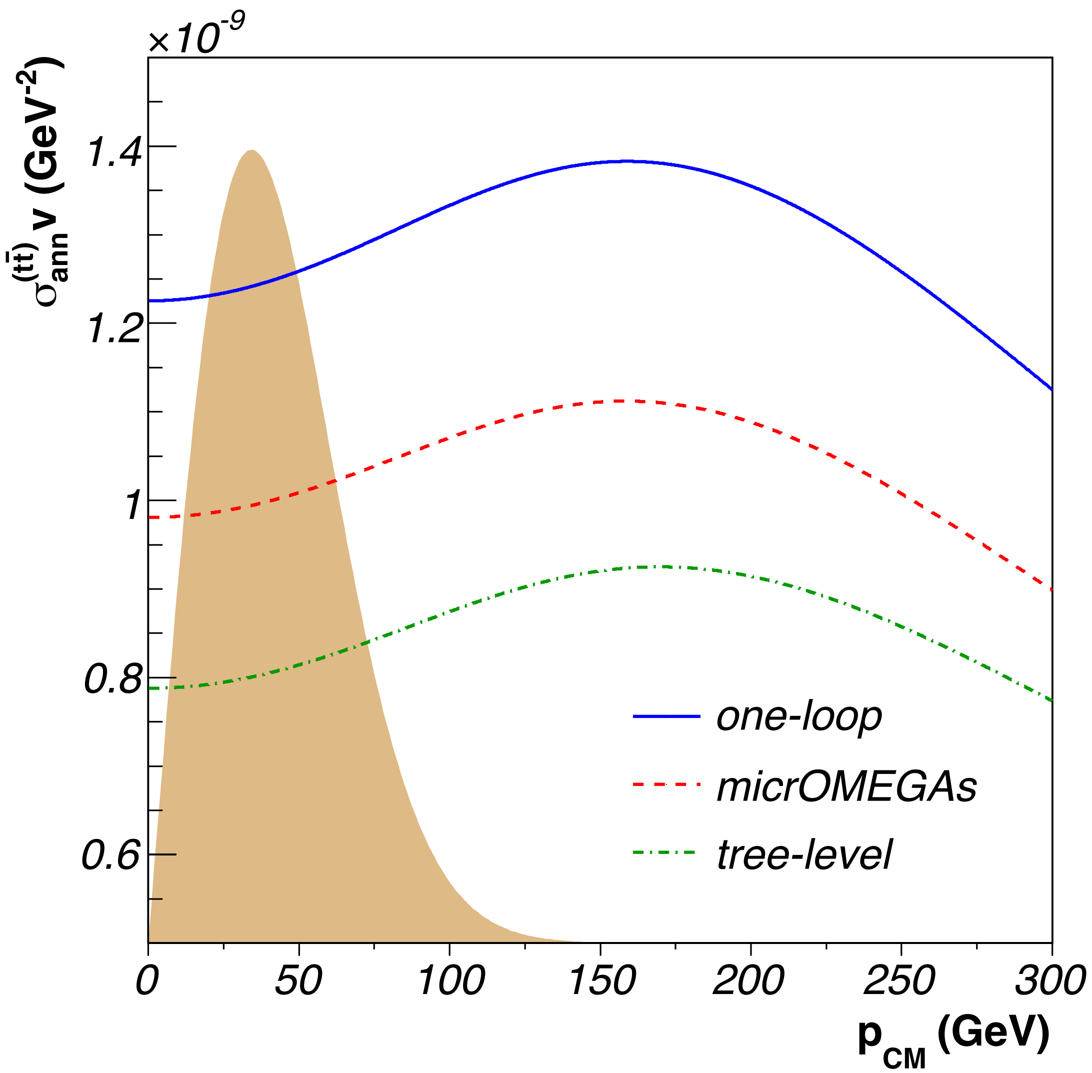
B. Herrmann, M. Klasen, K. Kovařík — Phys. Rev. D 79: 061701 (2009) — arXiv:0901.0481 [hep-ph]

B. Herrmann, M. Klasen, K. Kovařík — Phys. Rev. D 80: 085025 (2009) — arXiv:0907.0030 [hep-ph]

B. Herrmann, M. Klasen — Phys. Rev. D 76: 117704 (2007) — arXiv:0709.0043 [hep-ph]

Corrections to neutralino (co-)annihilation and impact on the relic density

Neutralino pair annihilation into top quarks



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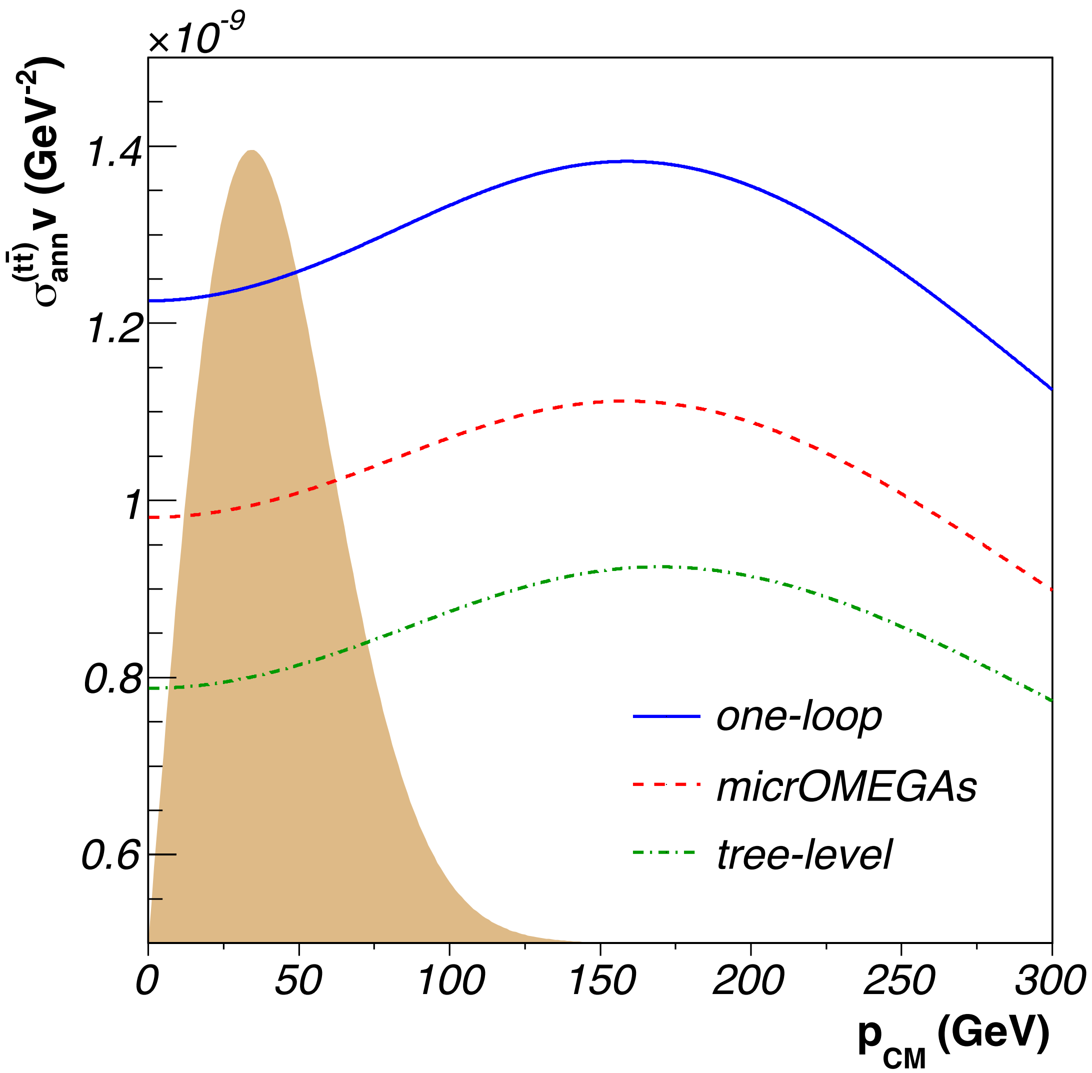

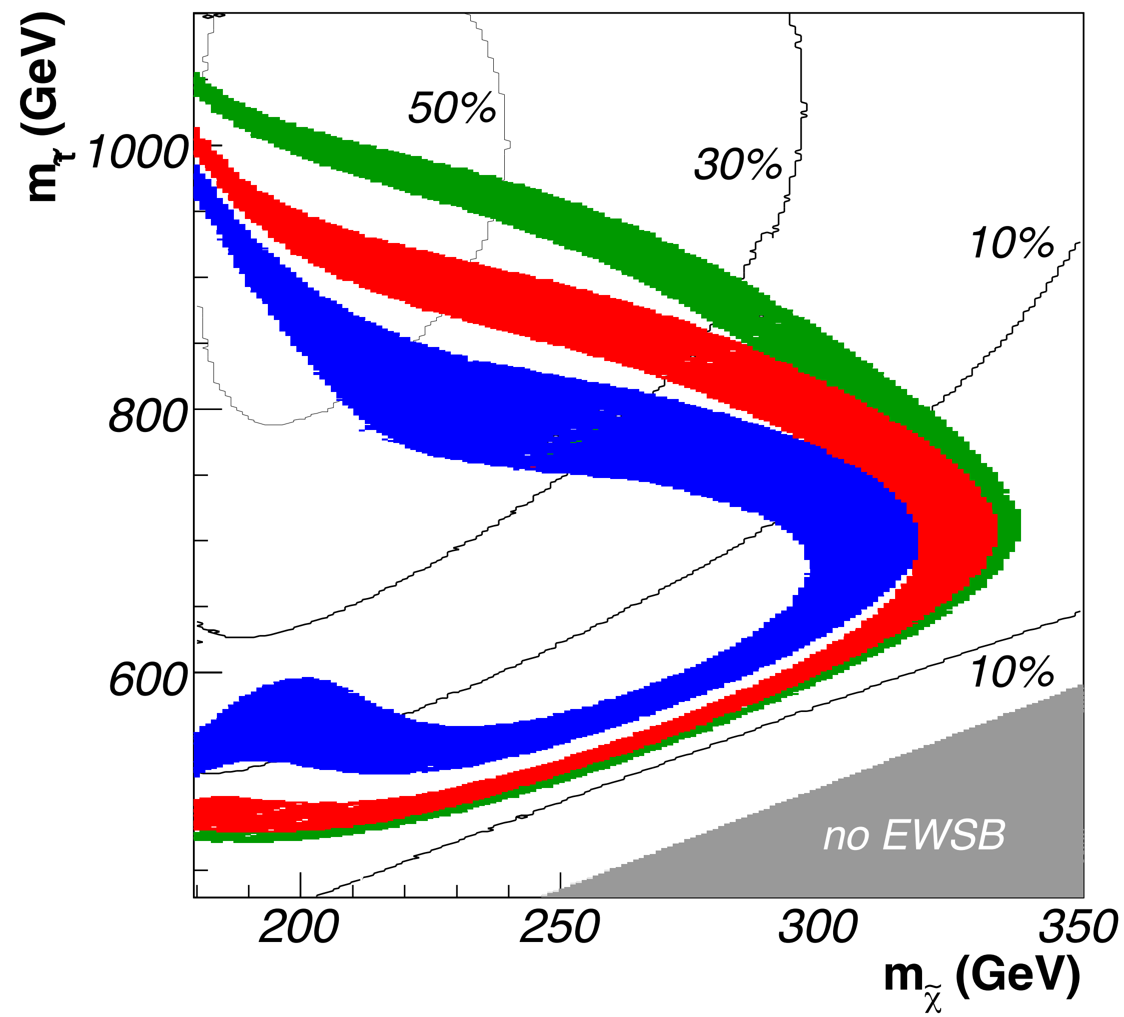

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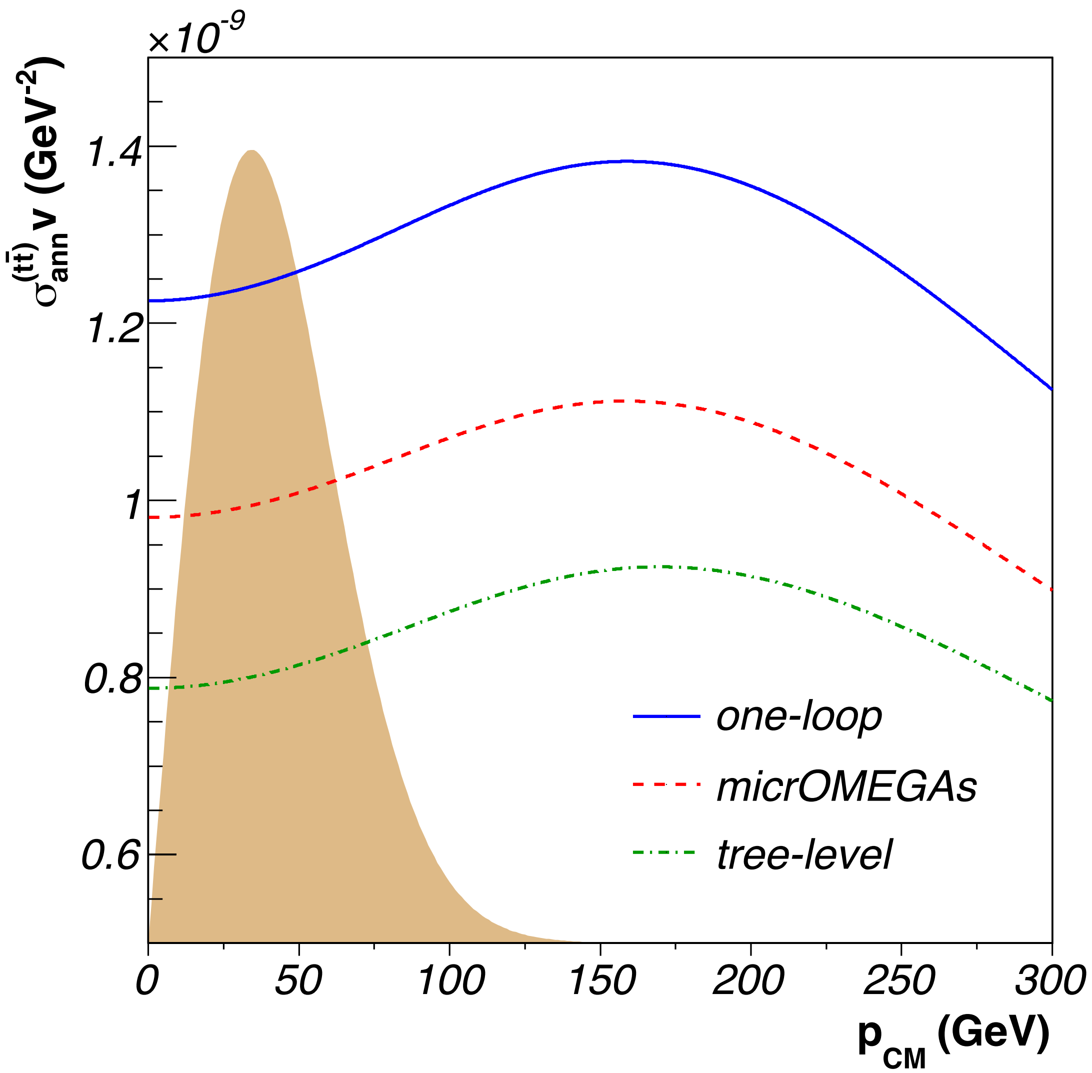
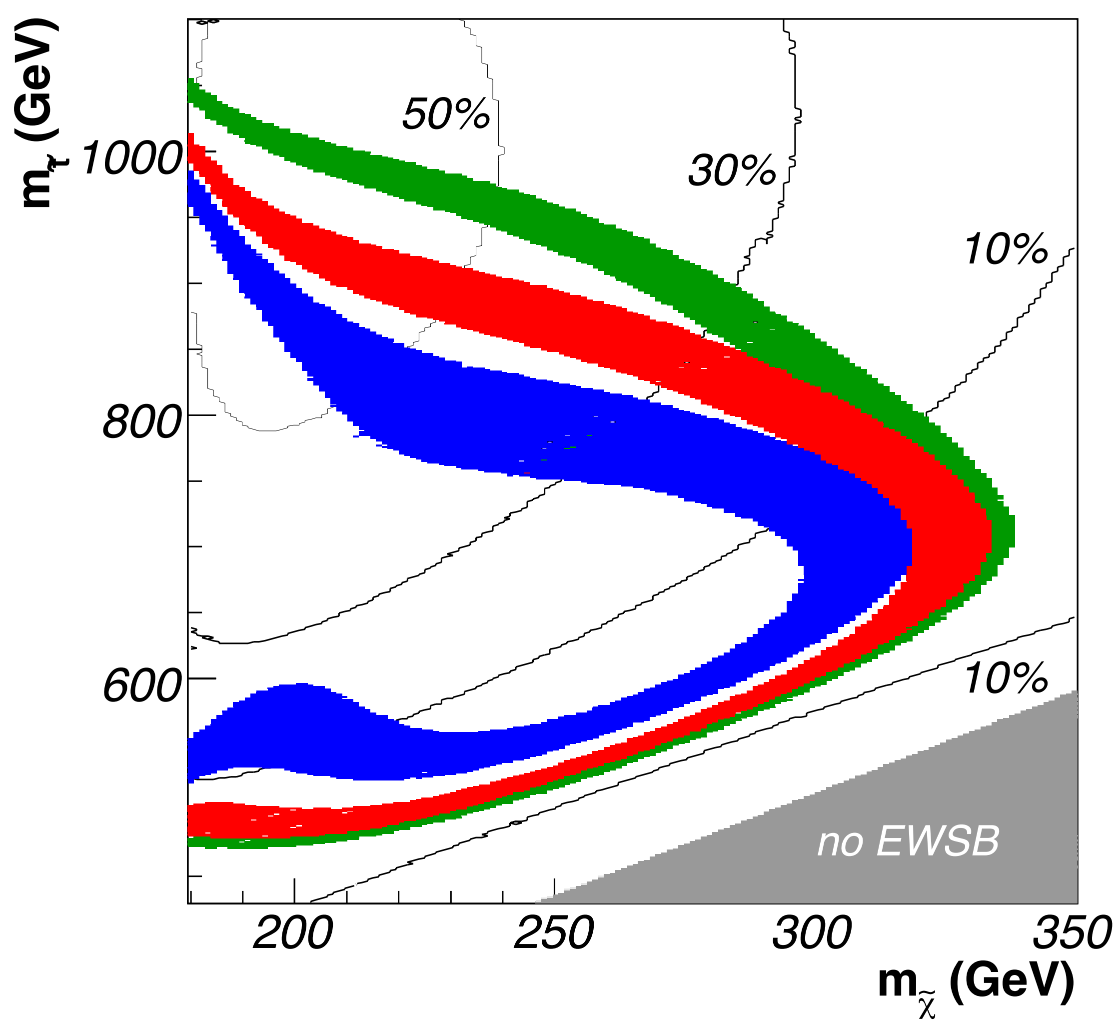


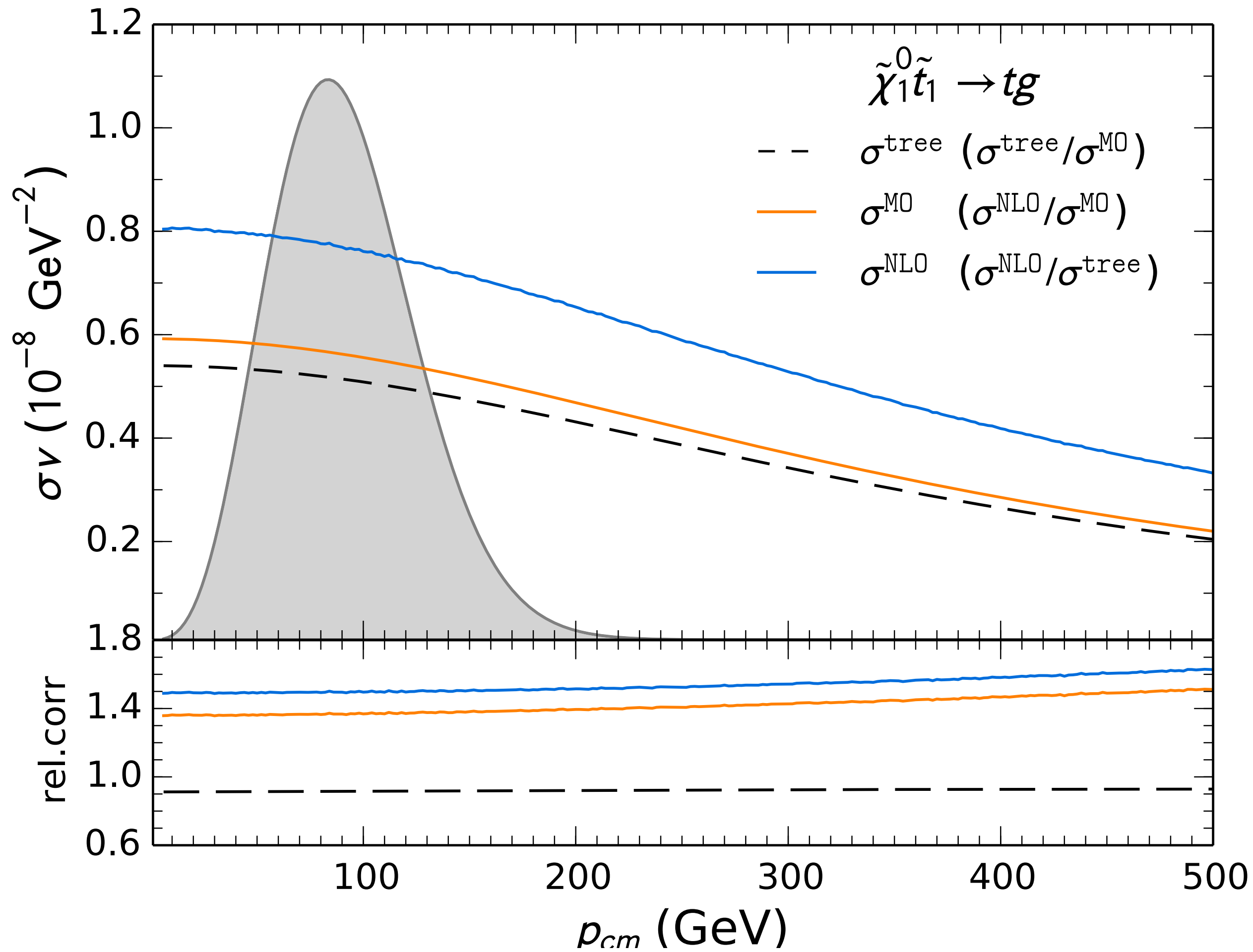


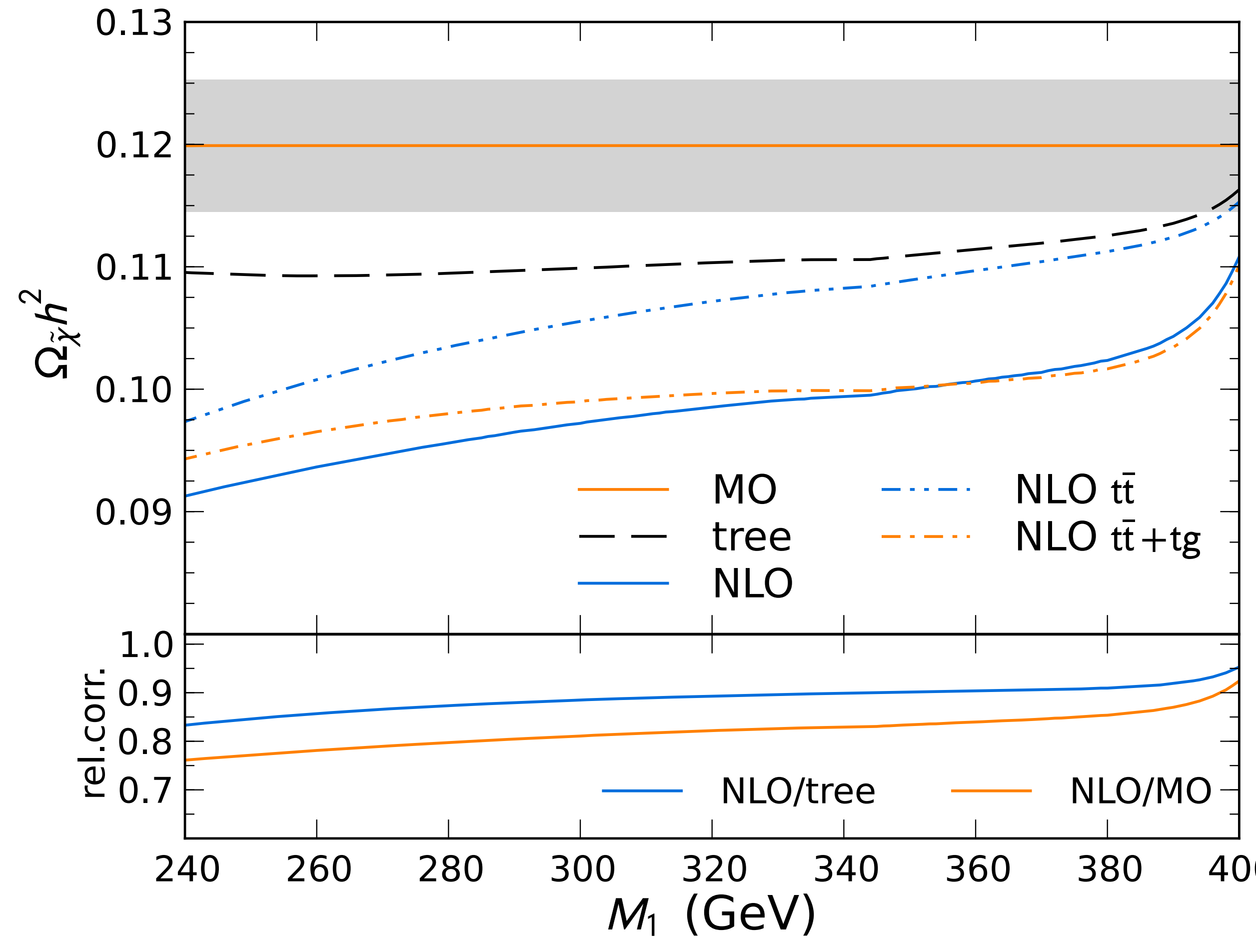
Annihilation cross-section enhanced by up to 50% by radiative corrections
Corrections can lead to **important shifts for preferred regions** (e.g. ~200 GeV for m_{stop})

Effective Yukawa couplings (as e.g. in micrOMEGAs) very good approximation around Higgs-resonances, **but other sub-channels can be dominant** (here: Z^0/squark-exchange)

B. Herrmann, M. Klasen, K. Kovařík — Phys. Rev. D 80: 085025 (2009) — arXiv:0907.0030 [hep-ph]
B. Herrmann, M. Klasen, K. Kovařík, M. Meinecke, P. Steppeler — Phys. Rev. D 89: 114012 (2014) — arXiv:1404.2931 [hep-ph]

Neutralino-stop co-annihilation



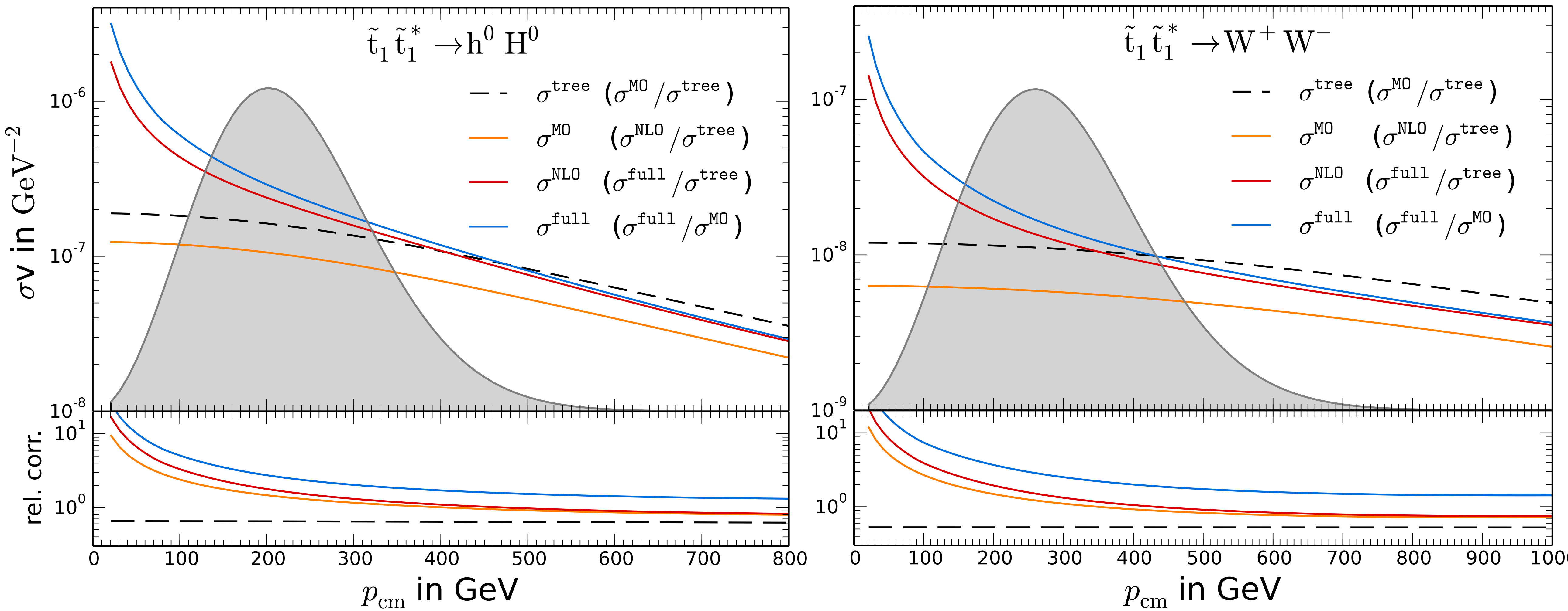


Relative corrections of up to 40-50% observed for the co-annihilation cross-section, leading to a **numerically important shift** for the predicted **neutralino relic density** (up to about 25% — more than Planck uncertainty!)

Co-annihilation into **SM-like Higgs** and gluon most important (other final states generally subdominant)

Harz, Herrmann, Klasen, Kovařík, Le Boulc'h — Phys. Rev. D 87: 054031 (2013) — arXiv:1212.5241 [hep-ph]
Harz, Herrmann, Klasen, Kovařík — Phys. Rev. D 91: 034028 (2015) — arXiv:1409.2898 [hep-ph]

Stop pair annihilation — electroweak final states



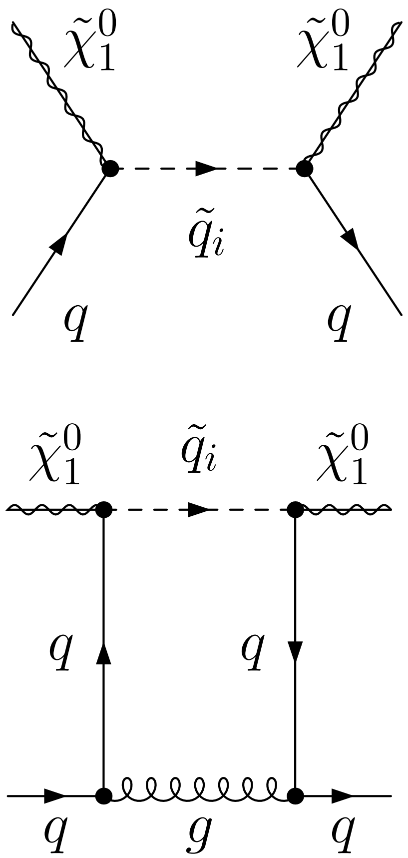
Coulomb corrections **dominant for small values of p_{cm}** (Coulomb singularity), while fixed-order corrections dominant for high-momentum region

Resulting relic density receives corrections of up to 40% (more important than Planck uncertainty!)

Harz, Herrmann, Klasen, Kovařík, Meinecke — Phys. Rev. D 91: 034012 (2015) — arXiv:1410.8063 [hep-ph]
Harz, Herrmann, Klasen, Kovařík, Schmiemann — *to be published…*

Application to direct detection

Corrections to direct dark matter detection



Full theory
(LO + NLO)
valid at Q~1 TeV

Klasen, Kovařík, Steppeler — Phys.Rev. D94: 095002 (2016) — arXiv:1607.06396 [hep-ph]

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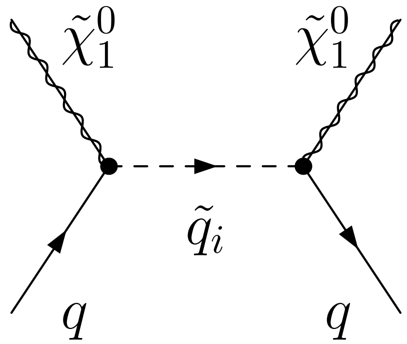

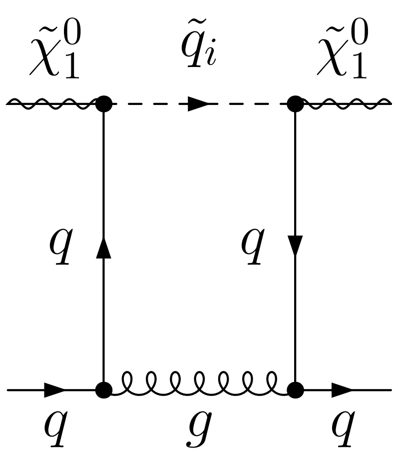

Full theory
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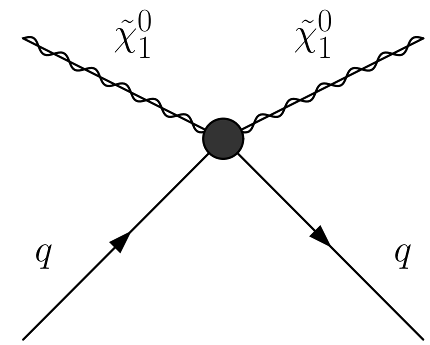
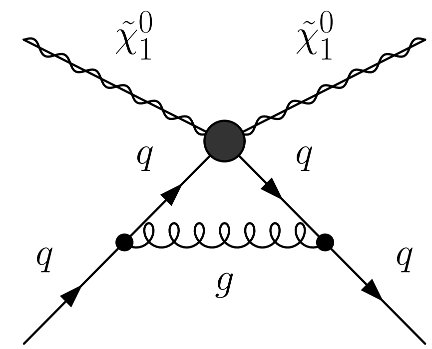
Matching
at Q~1 TeV

Effective theory
(LO + NLO)
valid at Q~5 GeV





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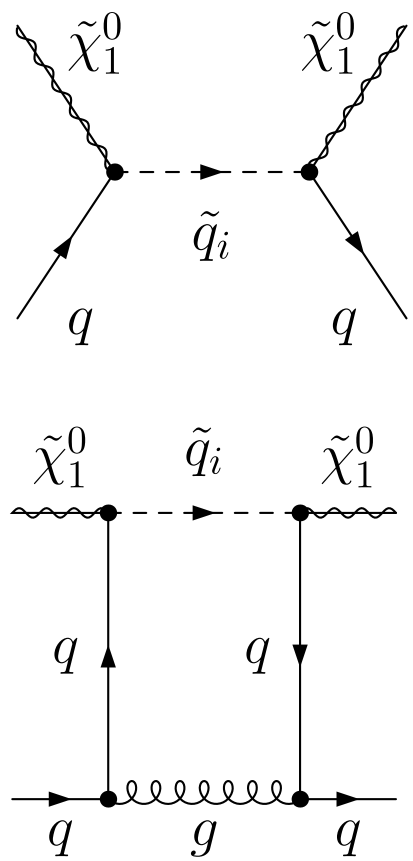

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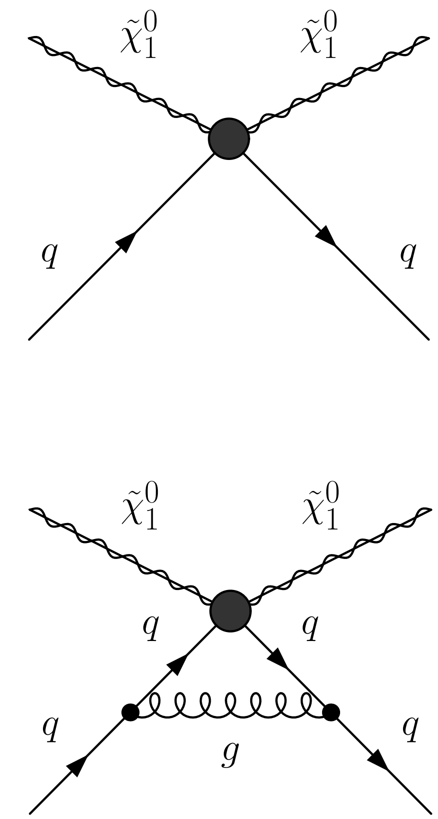
⟷ **Matching** at Q~1 TeV

Effective theory
(LO + NLO)
valid at Q~5 GeV



Renormalization (same scheme as before) in order to treat ultraviolet divergencies

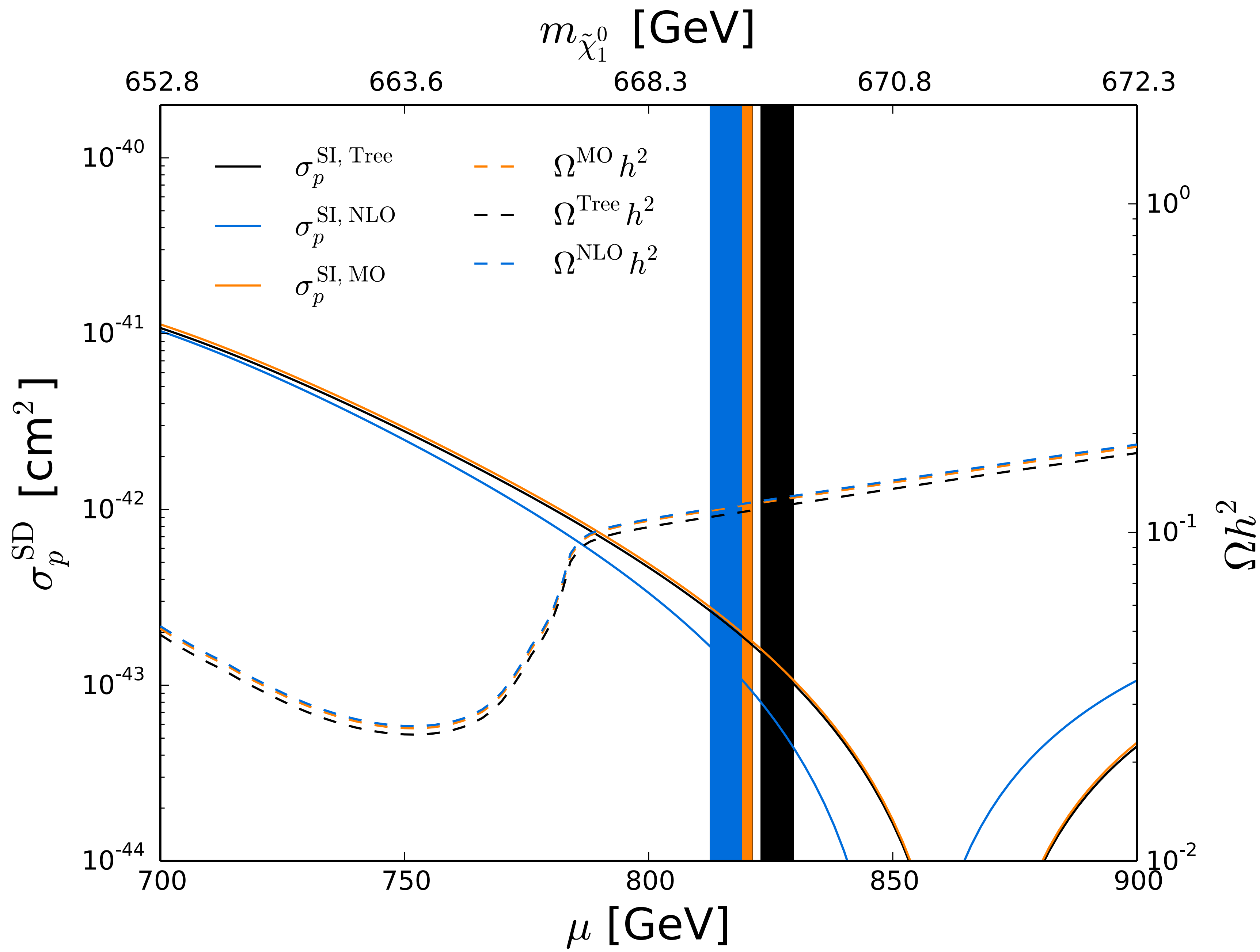
Infrared divergencies cancel between the different contributions

Dedicated **integral reduction procedure applicable to zero-velocity limit**

Renormalization group running of effective theory from Q~1 TeV to Q~5 GeV

Klasen, Kovařík, Steppeler — Phys.Rev. D94: 095002 (2016) — arXiv:1607.06396 [hep-ph]

Corrections to direct dark matter detection



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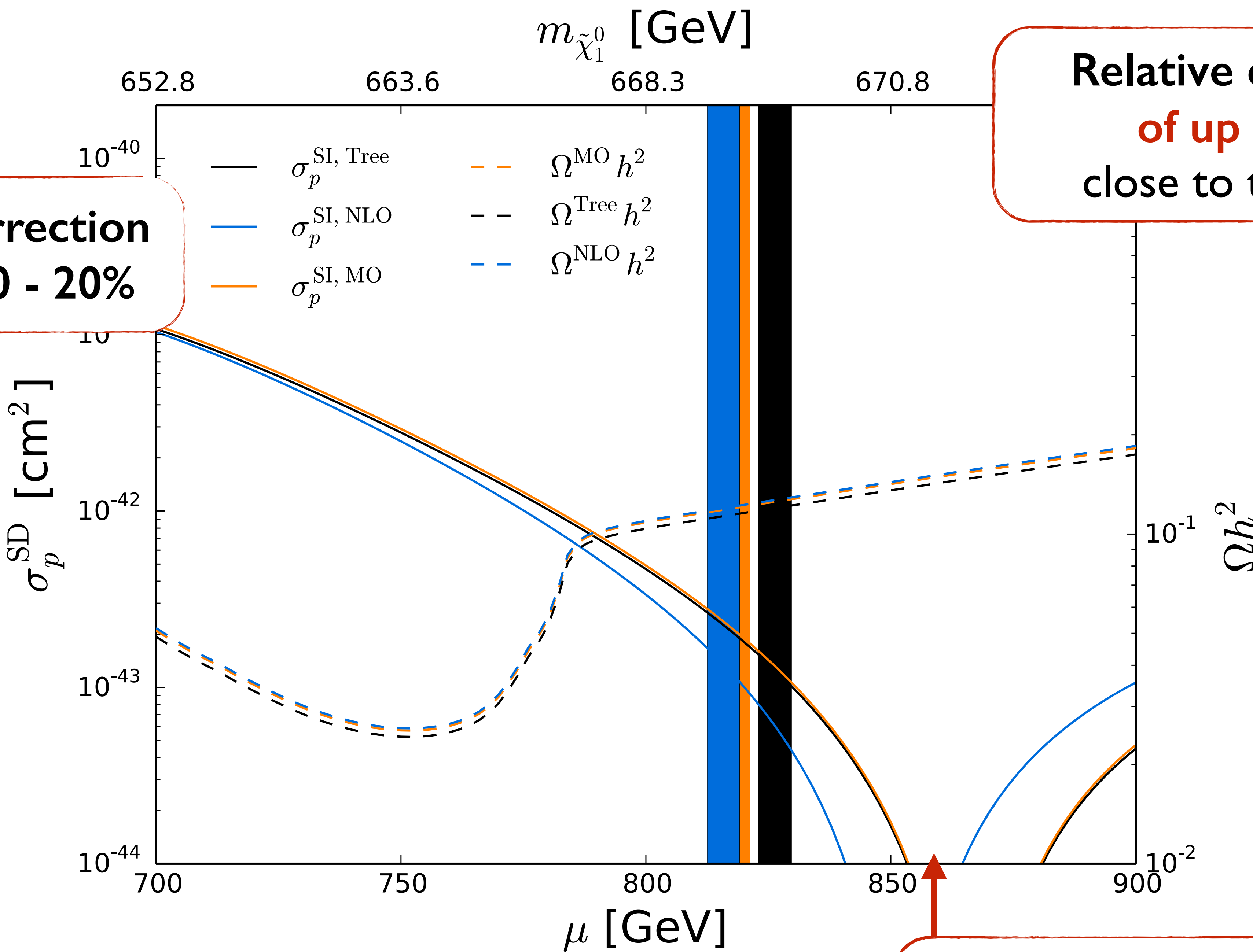

Relative correction of about 10 - 20%

Relative correction
of up to 50%
close to the "drop"

Destructive interference "higgsino-bino switch"

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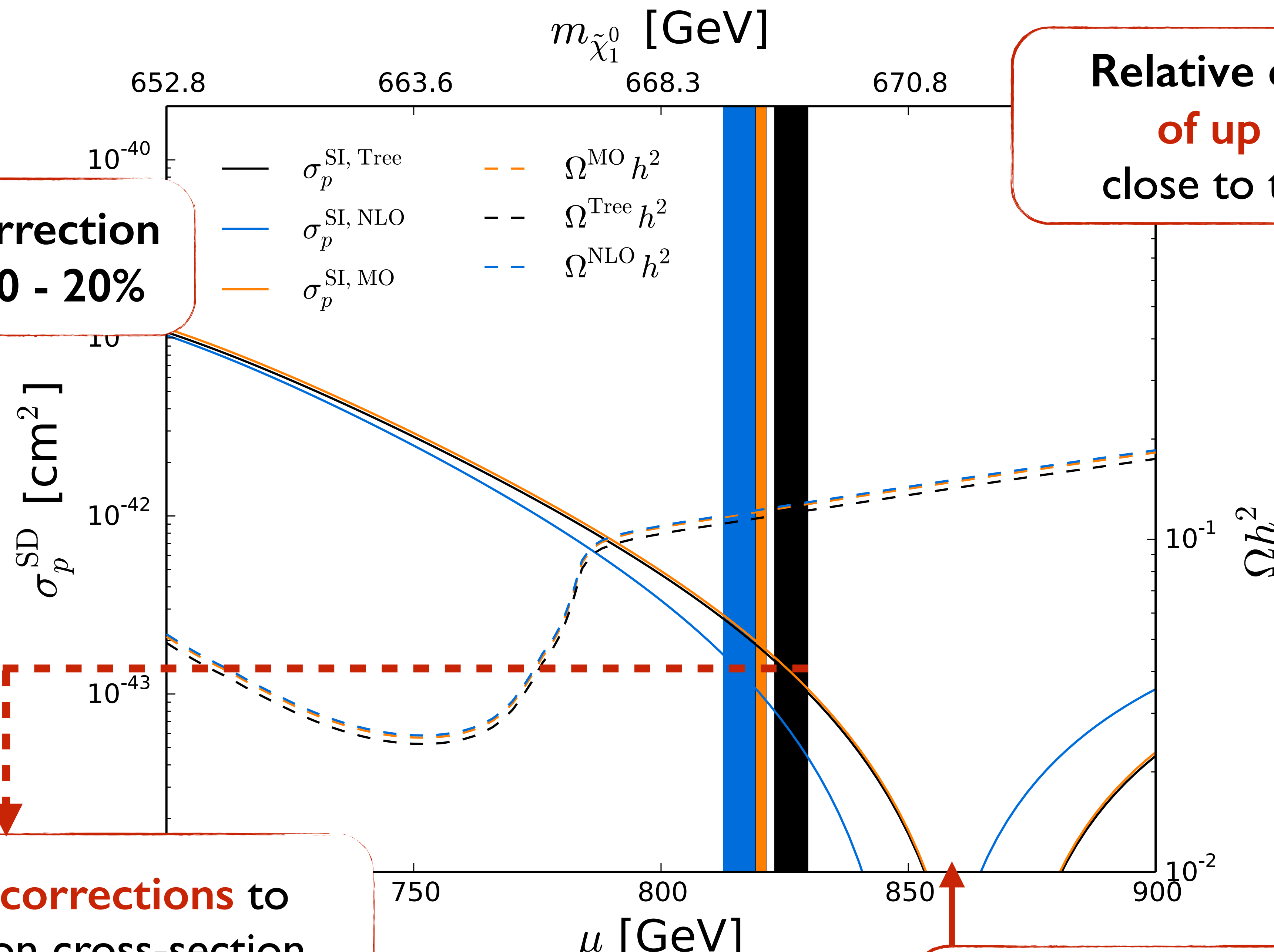

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Relative correction **of up to 50%** close to the "drop"

Effect of corrections to DM-nucleon cross-section and to relic density **are of the same order**

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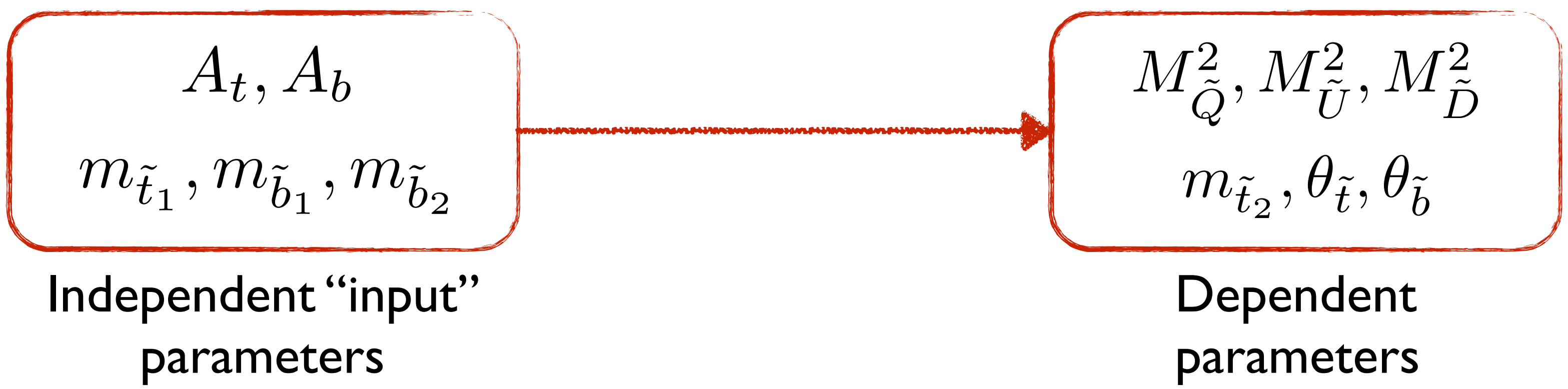
Scale dependence and theoretical uncertainty

Interlude — a few technical details

Loop diagrams include UV-divergent integrals → **Renormalization!**

Hybrid on-shell/$\overline{\text{DR}}$ renormalization scheme for the squark sector (3rd generation), which is applicable to all (co)annihilation processes



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$$A_t, A_b \qquad m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{b}_2}$$

Independent "input" parameters

→

$$M^2_{\tilde{Q}}, M^2_{\tilde{U}}, M^2_{\tilde{D}} \qquad m_{\tilde{t}_2}, \theta_{\tilde{t}}, \theta_{\tilde{b}}$$

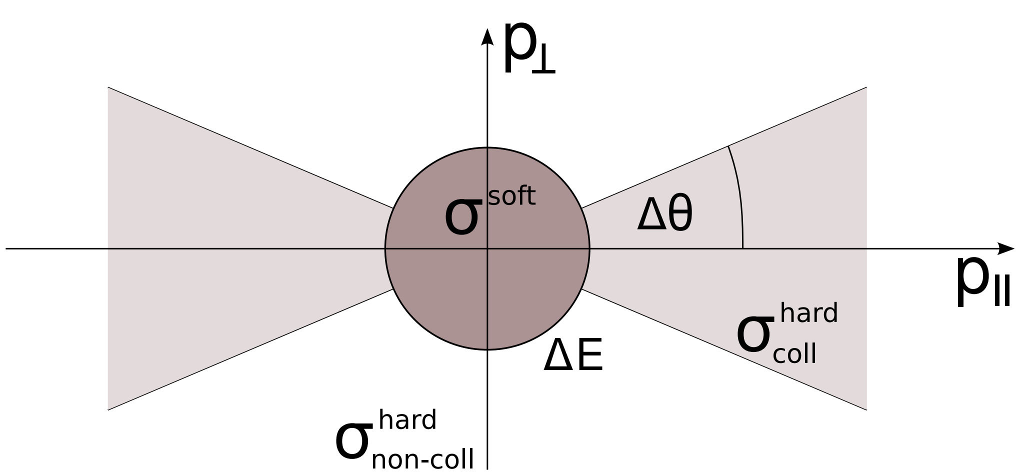
Dependent parameters

Loop diagrams contain **IR-divergencies** (soft and/or collinear), which vanish when taking into account the real emission of a gluon (2→3 processes)

Dipole Subtraction Method and **Phase Space Slicing**

Catani, Seymour (2001)

$$\sigma_{\text{NLO}} = \int_3 \left[\mathrm{d}\sigma^{\text{R}} \Big|_{\epsilon=0} - \mathrm{d}\sigma^{\text{A}} \Big|_{\epsilon=0} \right] + \int_2 \left[\mathrm{d}\sigma^{\text{V}} + \int_1 \mathrm{d}\sigma^{\text{A}} \right]_{\epsilon=0}$$

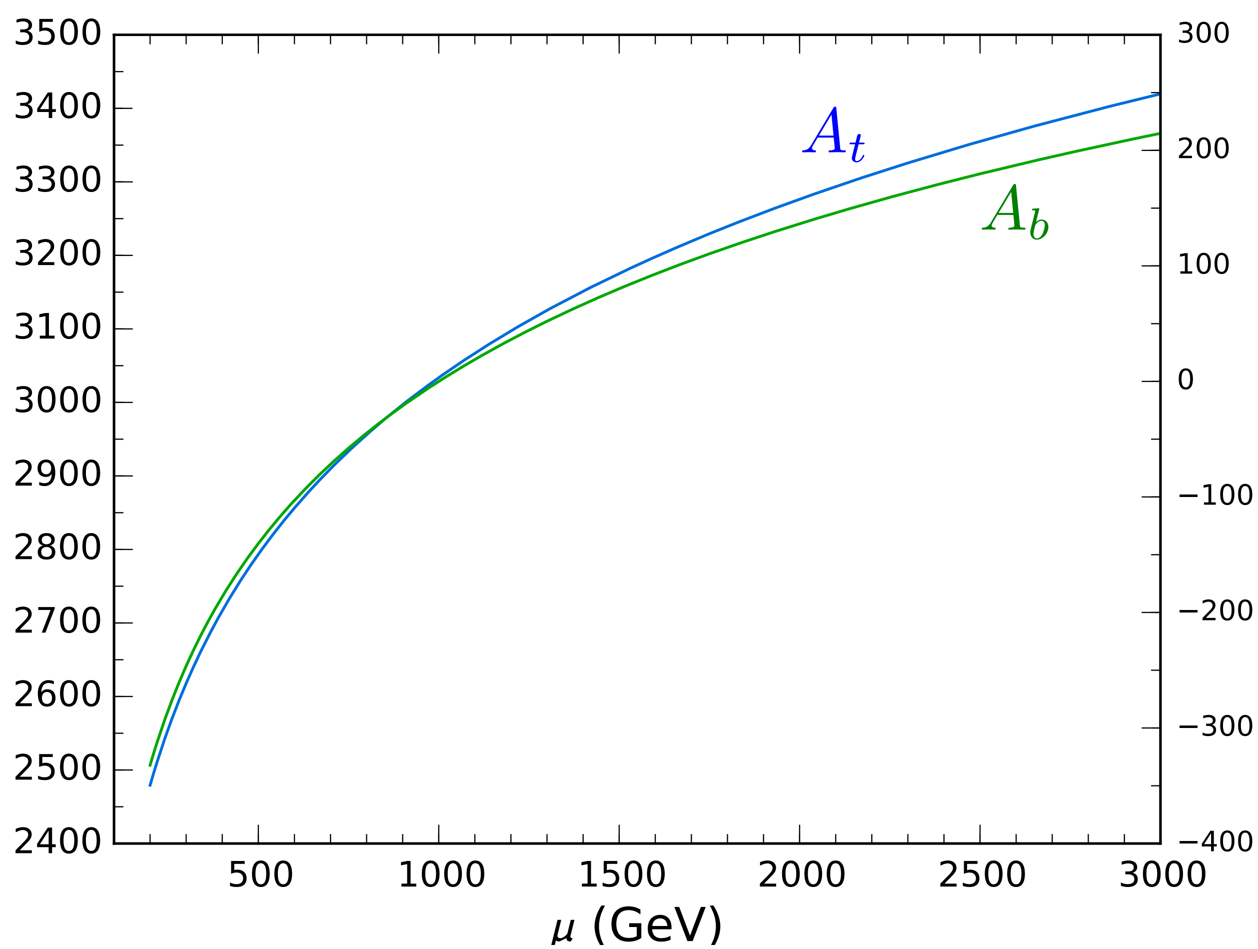


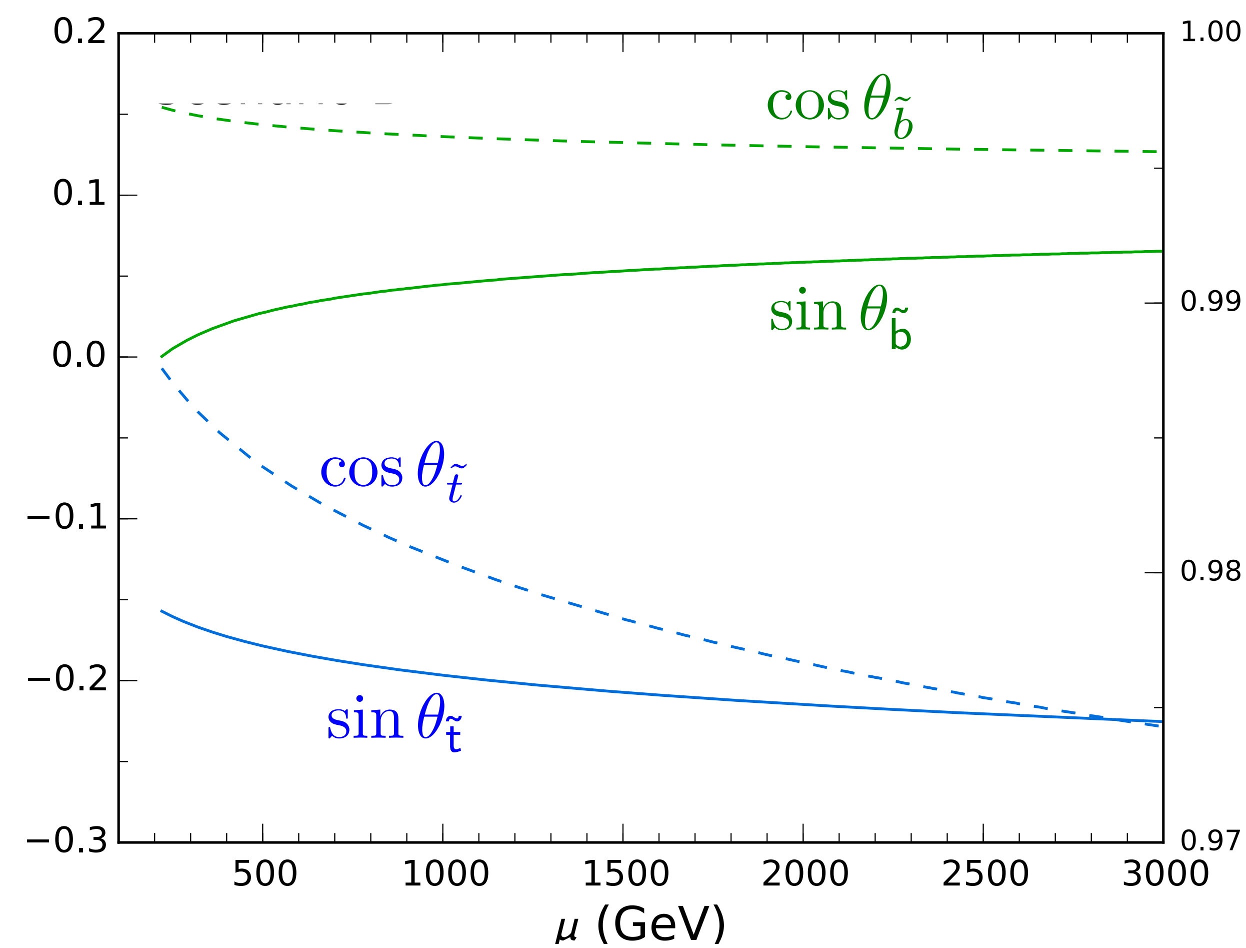
Scale dependence and theoretical uncertainty

Evaluation of theoretical uncertainty by **varying** (unphysical) **renormalization scale**
— hybrid on-shell / DRbar renormalization scheme designed for neutralino (co-)annihilation

$\mu_R = 500 \ldots 2000$ GeV → $A_t, \quad A_b, \quad \theta_{\tilde{t}}, \quad \theta_{\tilde{t}}, \quad \alpha_s, \quad m_b$

scale-dependent parameters





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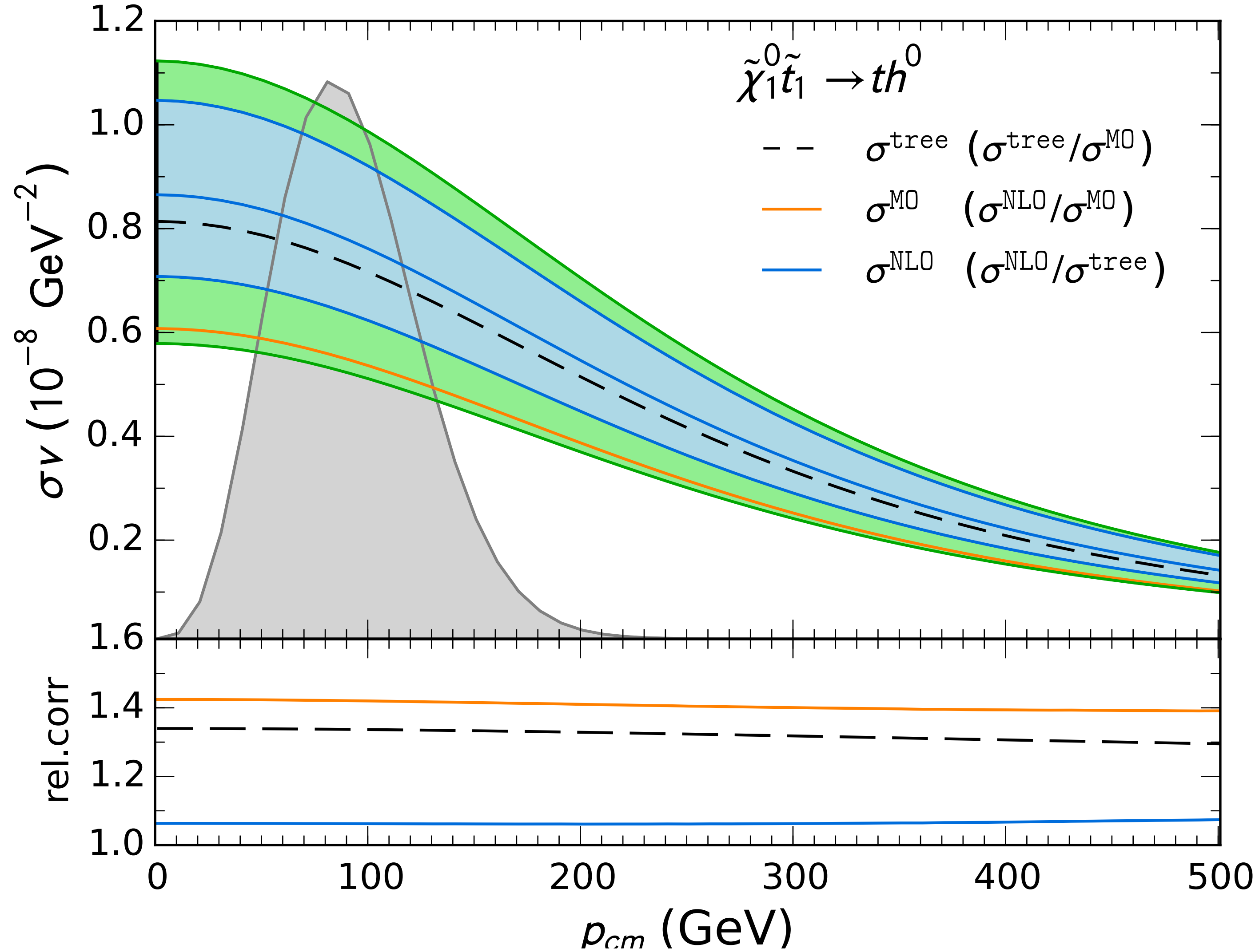

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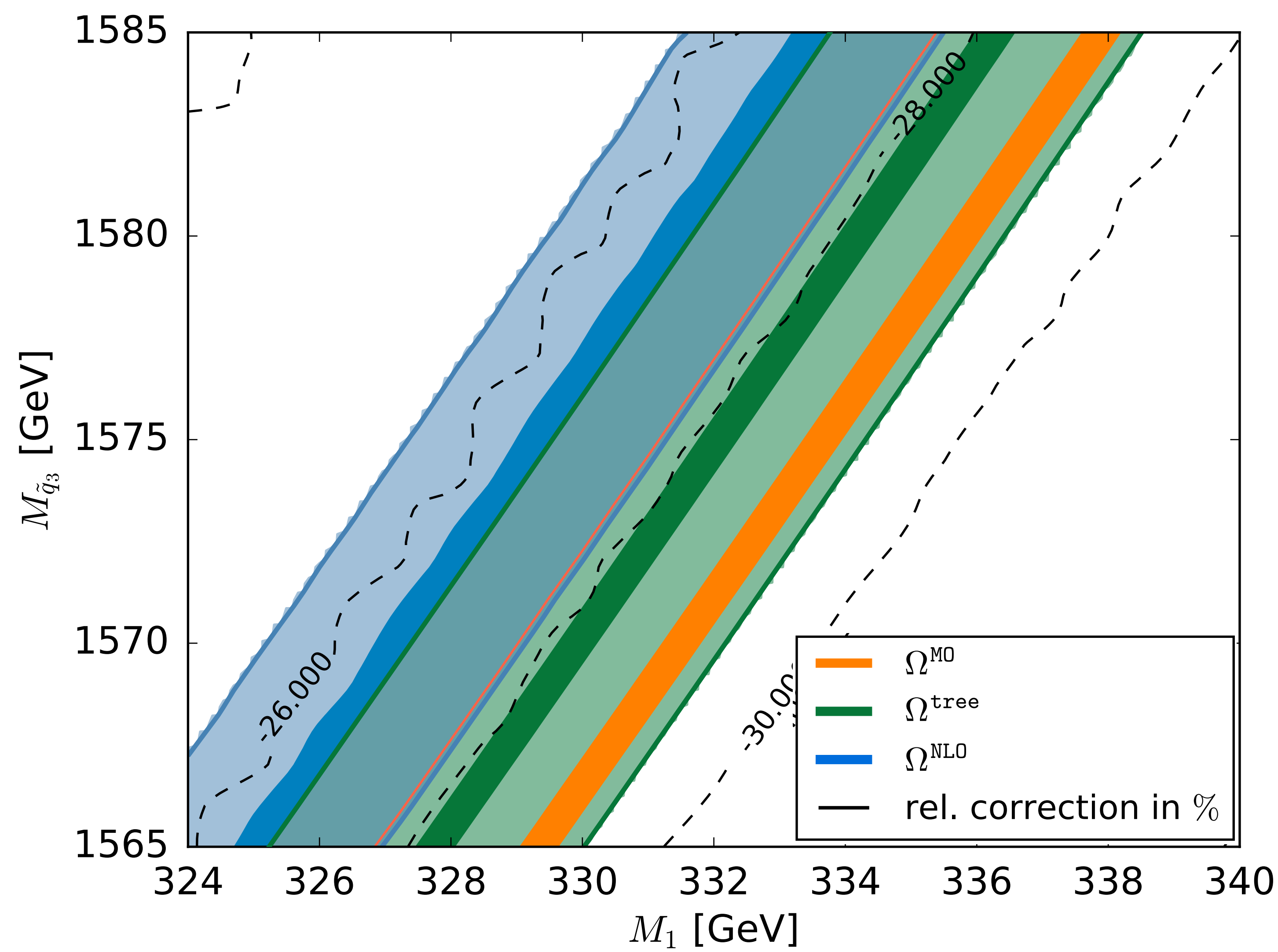

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Conclusion

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