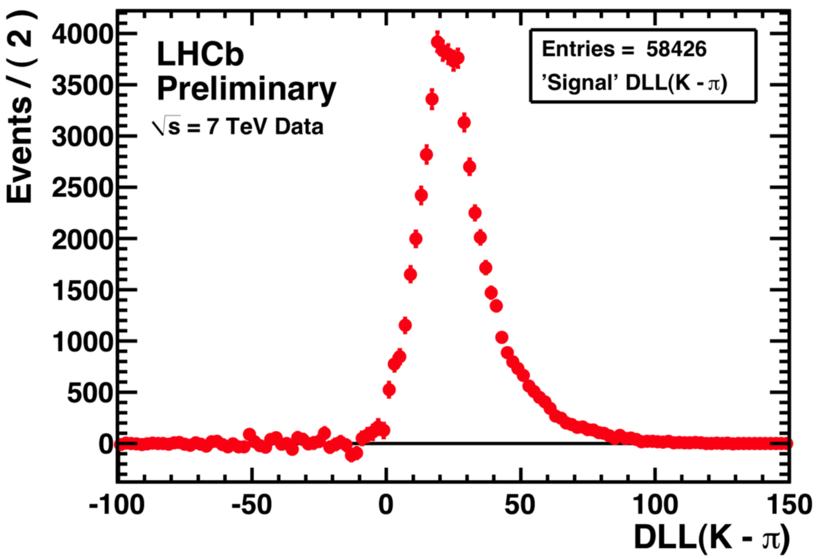
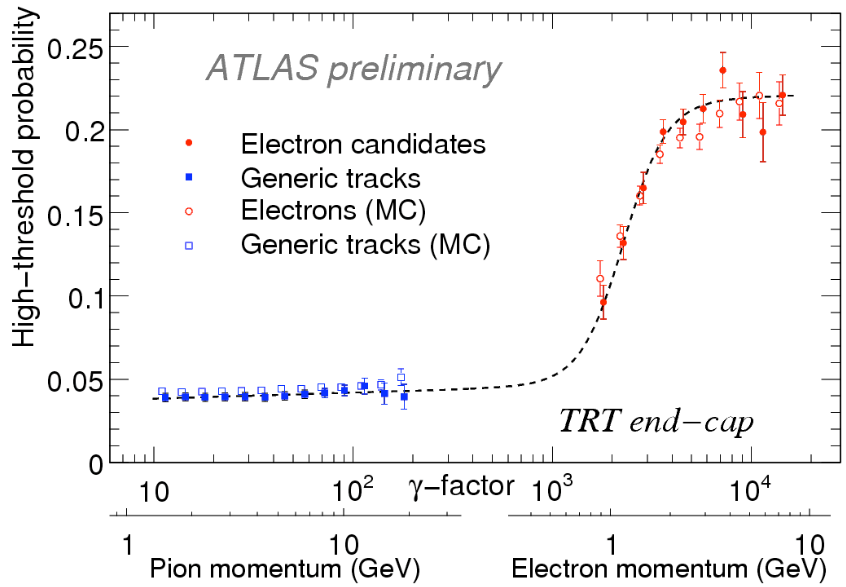
Particle identification with a track fit χ^2

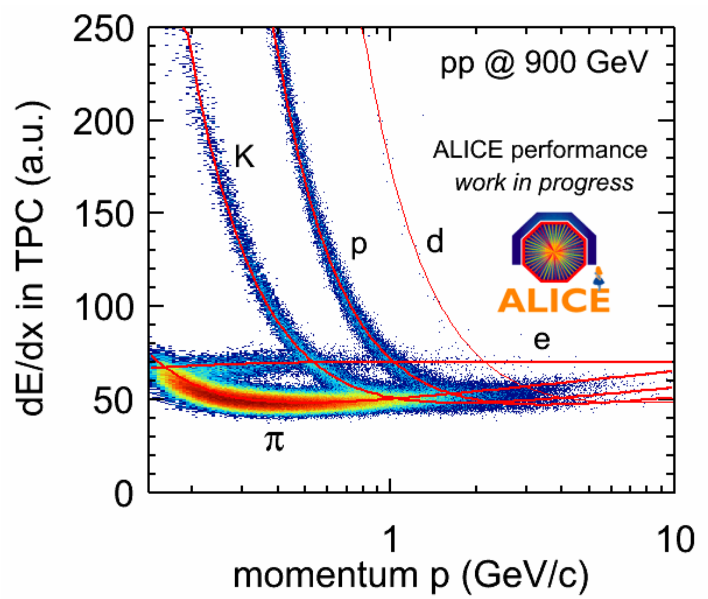
Ferenc Siklér
KFKI Research Institute for Particle and Nuclear Physics, Budapest, Hungary

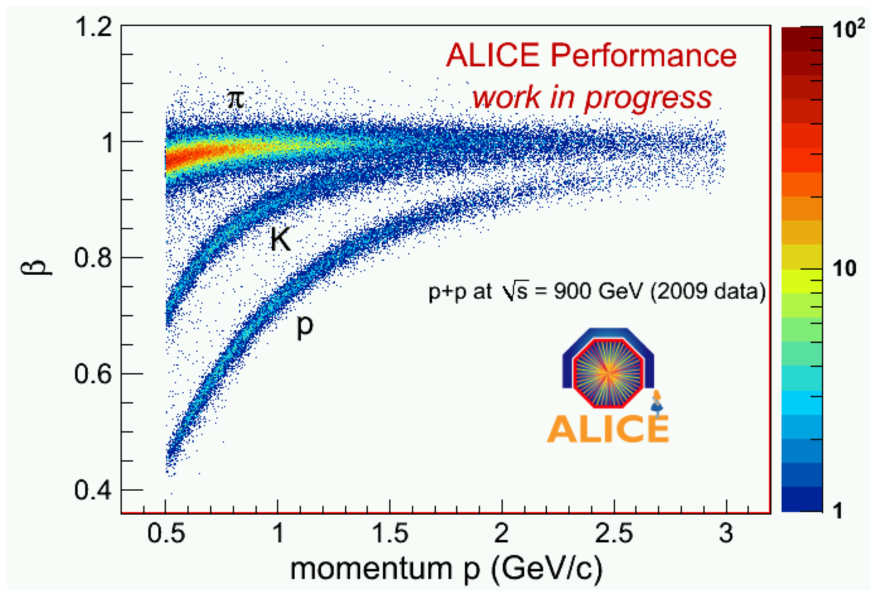
RICH2010, Cassis, France
6 May 2010

Motivation – some PID techniques







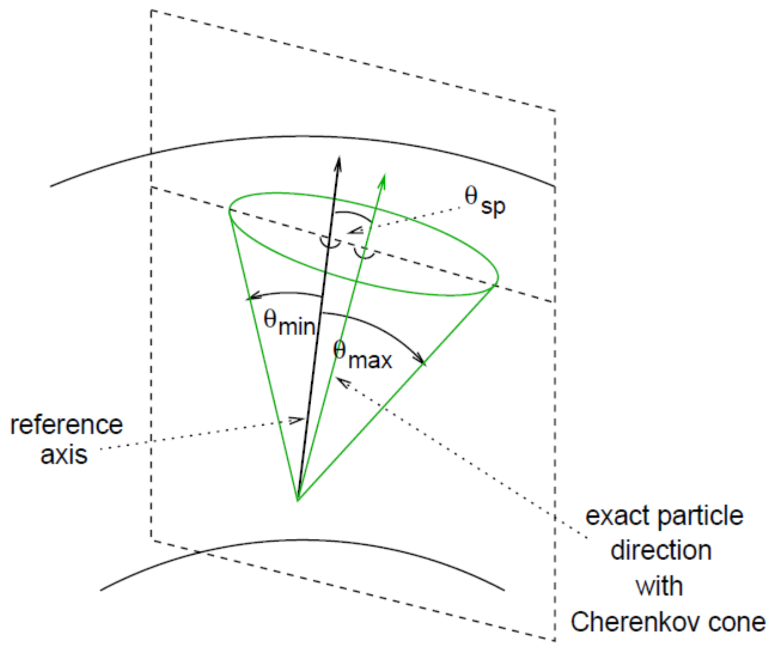


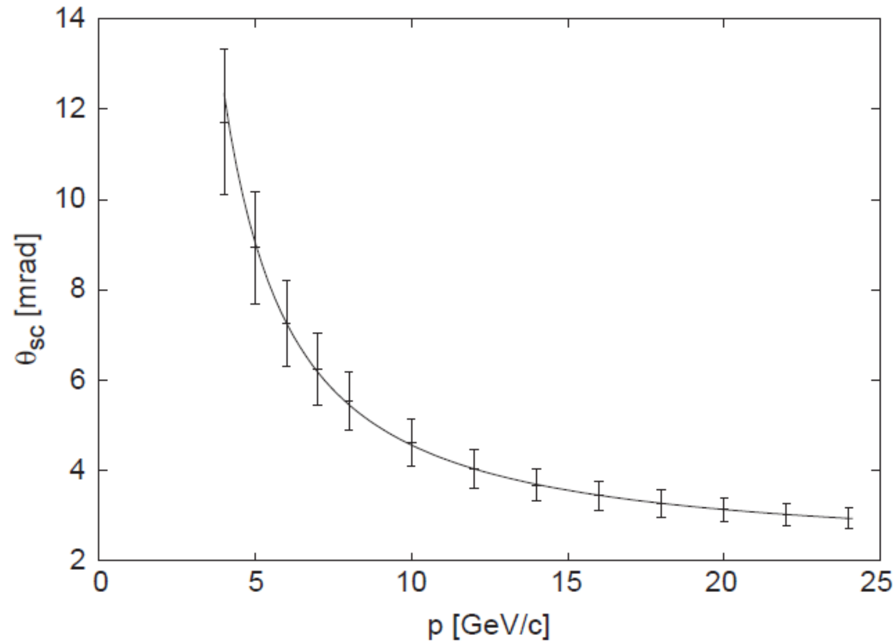
Are there more basic methods?

Plots from DIS'2010 and RICH2010

Motivation – RICH + multiple scattering

- AQUA-RICH experiment
  - Cherenkov photons used to determine the direction of the particle
  - With few mrad resolution, access to deflections due to multiple scattering
  - Use the distribution of scattering angles to determine momentum p
  - Particle type or $\beta \approx 1$ must be assumed





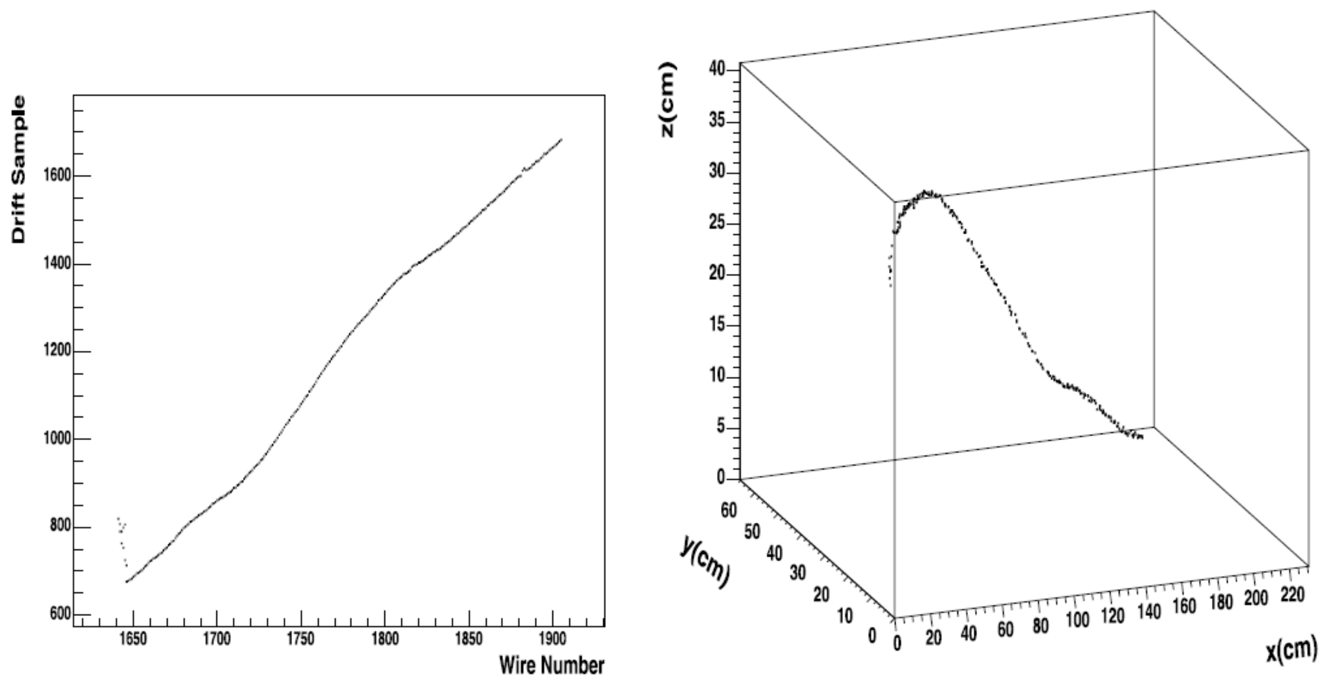
A. GrossHeim and K. Zuber, Nucl. Instr. and Meth. A 533 (2004) 532

Momentum is underestimated since particle also loses momentum

But: what to do if you have a tracker detector only?

Motivation – multiple scattering

- ICARUS T600 TPC
  - 600 ton liquid Ar time projection chamber
  - Atmospheric muons, so assume that particles are muons
  - Reconstruct particle momentum by looking at multiple scatters



A. Ankowski, Eur. Phys J C 48 (2006) 667

But: how to identify particles with tracker only?

Tracker + magnetic field

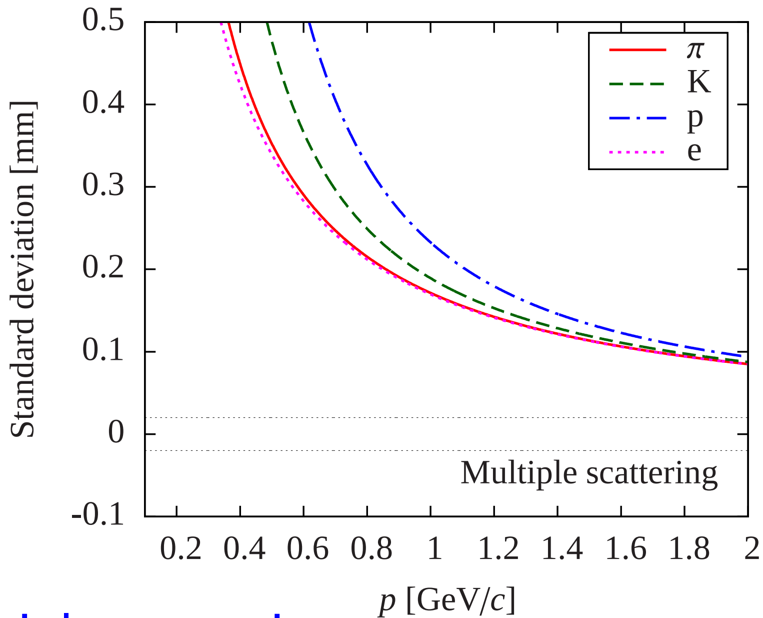
- Where can that be useful?
  - particle identification, or at least unfolding of yields
  - dE/dx is not always available (e.g. ATLAS pixel detector)
  - supplementary measurement

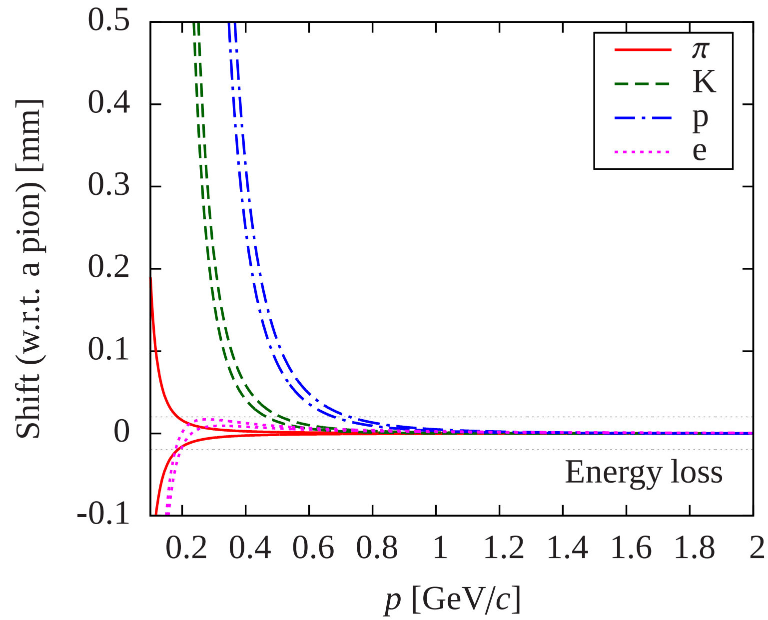
- How to do that?
  - multiple Coulomb scattering
  - measure the scattering angles during layer traversal
  - But: position measurement (2D) has uncertainties, also covariance
  - But: there is energy loss, comparable to the effect above

- What can you expect?
  - Reasonable π–K and π–p separation at low momentum, for $p < 1$ GeV/c

Need for a coherent framework

Physical effects





- Multiple scattering
  - Planar scattering angle (Gaussian approximation)

$$\theta_0 = \frac{13.6\ \mathrm{MeV}}{\beta c p} z \sqrt{x/X_0}\left[1 + 0.038 \ln(x/X_0)\right], \qquad \sigma_{ms} \approx l\theta_0 \propto \frac{1}{\beta p}$$

- Energy loss
  - Most probable energy loss and full width approximated as

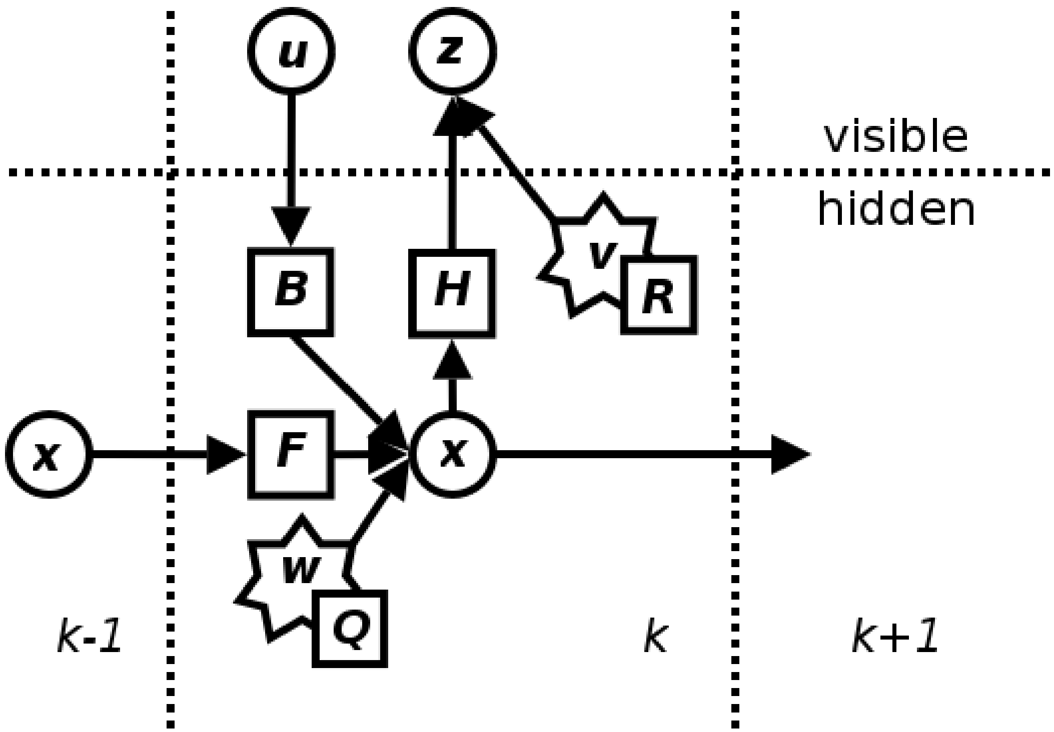
$$\Delta_p = \xi \left[\ln \frac{2mc^2\beta^2\gamma^2\xi}{I^2} + 0.2000 - \beta^2 - \delta\right]$$

$\Gamma_\Delta = 4.018\xi$, where $\xi = \frac{K}{2} z^2 \frac{Z}{A} \rho \frac{x}{\beta^2}$; $\qquad \delta_{el} \approx -\frac{0.3Bl^2}{2}\frac{\Delta}{\beta p^2}$

Shown: $B = 3.8$ T magnetic field, $x/X_0 = 2\%$ Si, then 5 cm flight to the next layer

Compare with a local position resolution of 25 μm (for Si, dotted)

Tracking with Kalman – overview



- Kalman filter
  - widely used method of track and vertex fitting
  - handling of known physics effects (transport F; process noise w, Q)
  - handling of measurement uncertainties (measurement noise v, R)
  - measurement (H, z)
  - equivalent to the global linear least square method, optimal
  - prediction $+$ filtering steps, followed by smoothing

R. Frühwirth, Nucl. Instr. and Meth. A 262 (1987) 444

Note: outliers (δs, noise, fake hits) are suppressed during pattern recognition

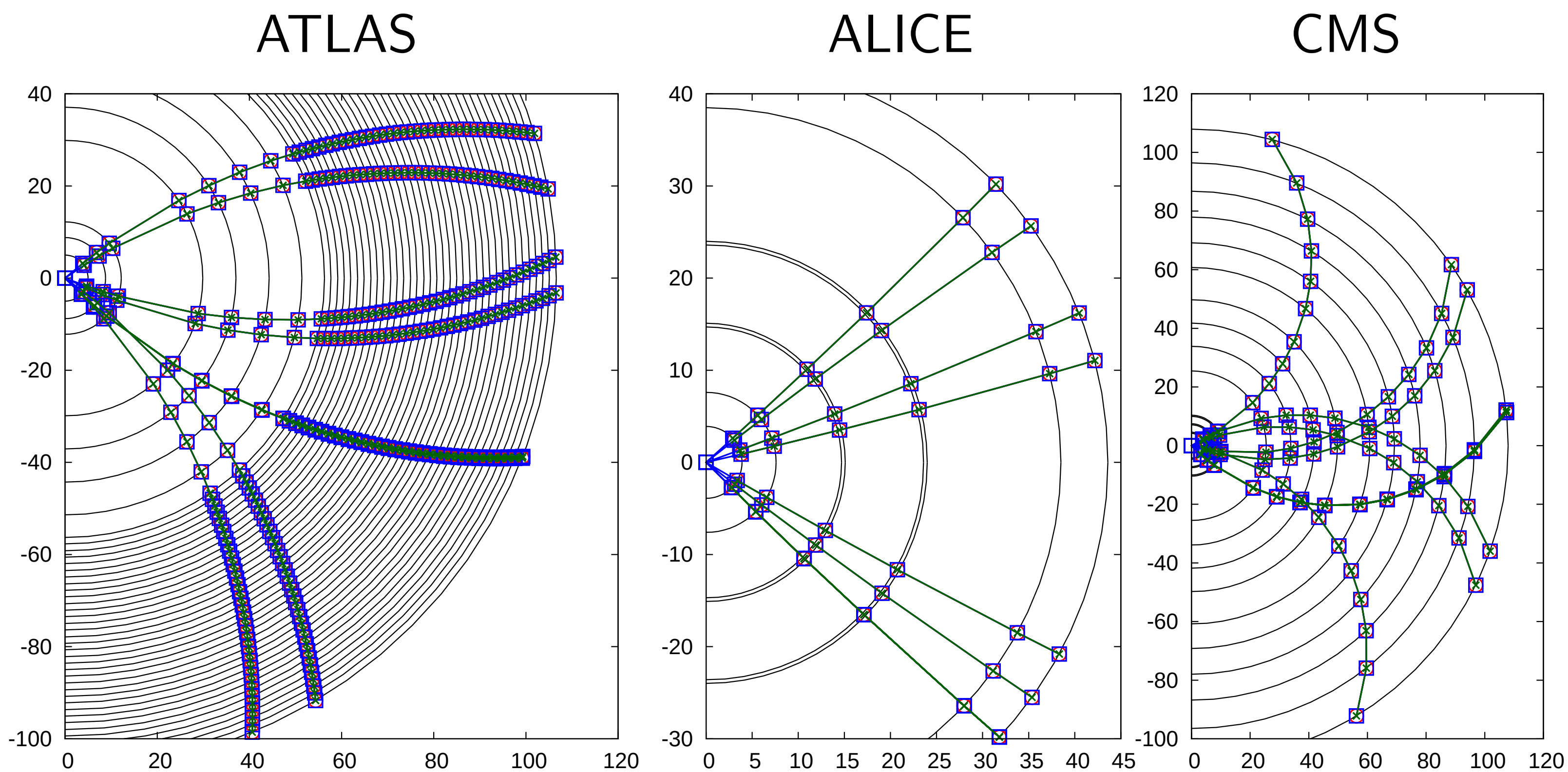
Tracking with Kalman – model

- Kalman filter
  - the state vector $x = (\kappa, \theta, \psi, r\phi, z)$ 5 dimensional, where

$$\kappa = q/p \qquad \text{(signed inverse momentum)}$$
$$\theta = \theta(\mathbf{p}) \qquad \text{(local polar angle)}$$
$$\psi = \phi(\mathbf{p}) \qquad \text{(local azimuthal angle)}$$
$$r\phi = r\phi(\mathbf{r}) \qquad \text{(global azimuthal position)}$$
$$z = r_L \qquad \text{(global longitudinal position)}$$

  - measurement vector $$m = (r\phi, z)$$
  - measurement operator
$$H = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$
  - the covariance of the process noise
$$Q = (F_\kappa \otimes F_\kappa^T)\sigma_\kappa^2 + (F_\theta \otimes F_\theta^T)\sigma_\theta^2 + (F_\psi \otimes F_\psi^T)\sigma_\psi^2$$
  - the covariance of the measurement noise
$$V = \begin{pmatrix} \sigma_{r\phi}^2 & 0 \\ 0 & \sigma_z^2 \end{pmatrix}$$

Tracking with Kalman



$p_T = 0.8$ GeV/c, 10 particles

Different curvatures due to the differing magnetic fields

2 T 0.4 T 3.8 T

Tracking – χ^2

- Kalman filter
  - merit function of trajectory

$$\chi^2(m_0) = \sum_k r_k^T R_k^{-1} r_k$$

    where r_k is the residual of kth hit (measured – predicted),
    R_k is the local covariance
  - during propagation a mass m_0 was assumed (usually m_π)
  - if R is closely diagonal, rewrite

$$\chi^2(m_0) \approx \sum_i \left(\frac{x_i - \mu_i(m_0)}{\sigma_i(m_0)}\right)^2 = \sum_i \left(\frac{\sigma_i(m)}{\sigma_i(m_0)}\right)^2 \left(\frac{x_i - \mu_i(m_0)}{\sigma_i(m)}\right)^2 = \sum_i a_i z_i$$

    where i runs on (if needed, split) 1D hits
  - linear combination of non-central χ^2 distributed random variables
  - the distribution of the z_is follows $f_X(z_i; 1, \lambda_i)$

$$\underbrace{a_i = \left(\frac{\sigma_i(m)}{\sigma_i(m_0)}\right)^2}_{\text{Weights}}, \qquad \underbrace{\lambda_i = \left(\frac{\mu_i(m) - \mu_i(m_0)}{\sigma_i(m)}\right)^2}_{\text{Shifts}}$$

Tracking – χ

- $\chi^2 \to \chi$
  - The use of $\chi \equiv \sqrt{\chi^2}$ is more practical
  - Can be approximated by a scaled non-central χ distribution: $1/\alpha f(\chi/\alpha; r, \lambda)$

$$\alpha^2 = \frac{\sum_i a_i^2}{\sum_i a_i}, \quad r = \frac{(\sum_i a_i)^2}{\sum_i a_i^2} - n_p, \quad \lambda^2 = \sum_i \lambda_i$$

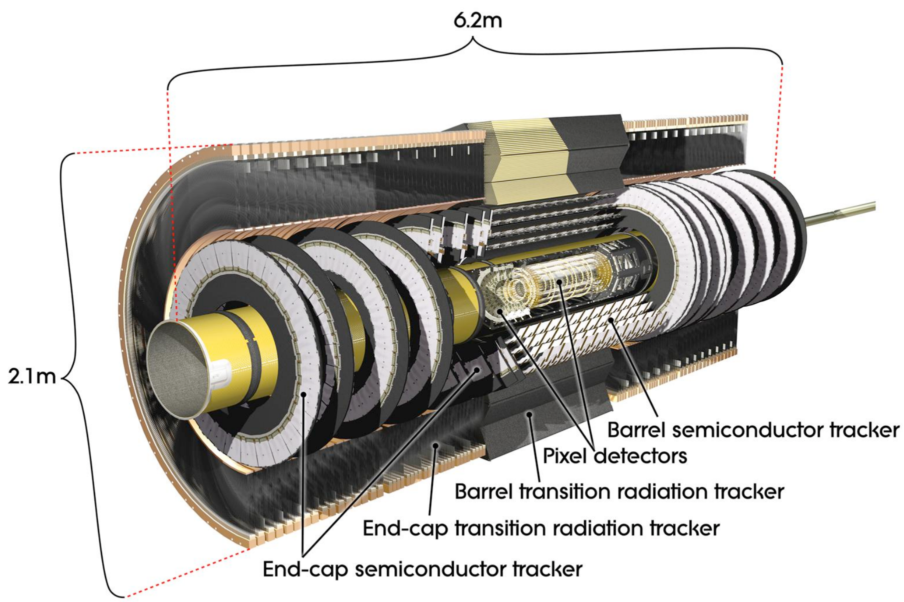
  - the scale factor is $\alpha \approx \beta(m_0)/\beta(m)$
  - r is the ndof, n_p is the number of parameters, λ is usually small

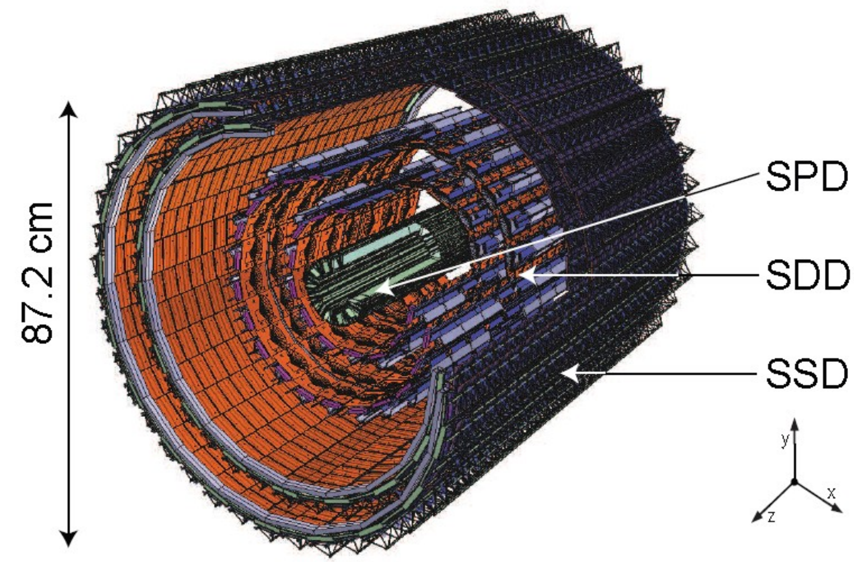
  $\Rightarrow$ that can be well approximated by a Gaussian

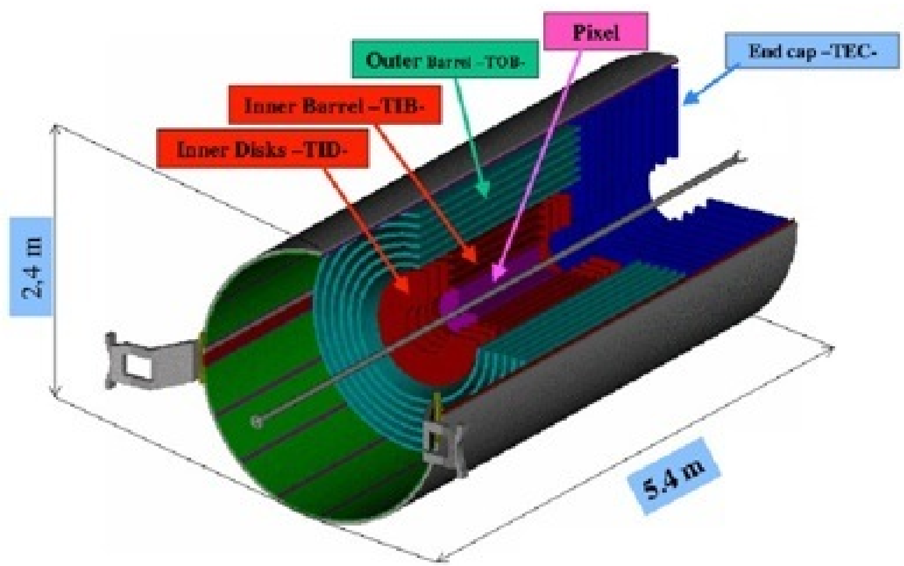
$$\mu_\chi = \alpha\sqrt{r - \frac{1}{2} + \lambda^2}, \qquad \sigma_\chi = \alpha\sqrt{\frac{1}{2}}$$

- What to do?
  - during track fitting determine χ with assuming m_0
  - since μ_χ and σ_χ depend on the ratio $\sigma_i(m)/\sigma_i(m_0)$, the distribution of χ will be mass dependent

Detectors – some trackers at LHC







- ATLAS

  3 Si pixels, 4 Si strips,
  and many ($\leq$ 36) straws

- ALICE

  2 Si pixels, 2 Si drifts, 2 Si strips,
  and a large gas TPC

- CMS

  3 Si pixels, 10 Si strips

Detectors – properties

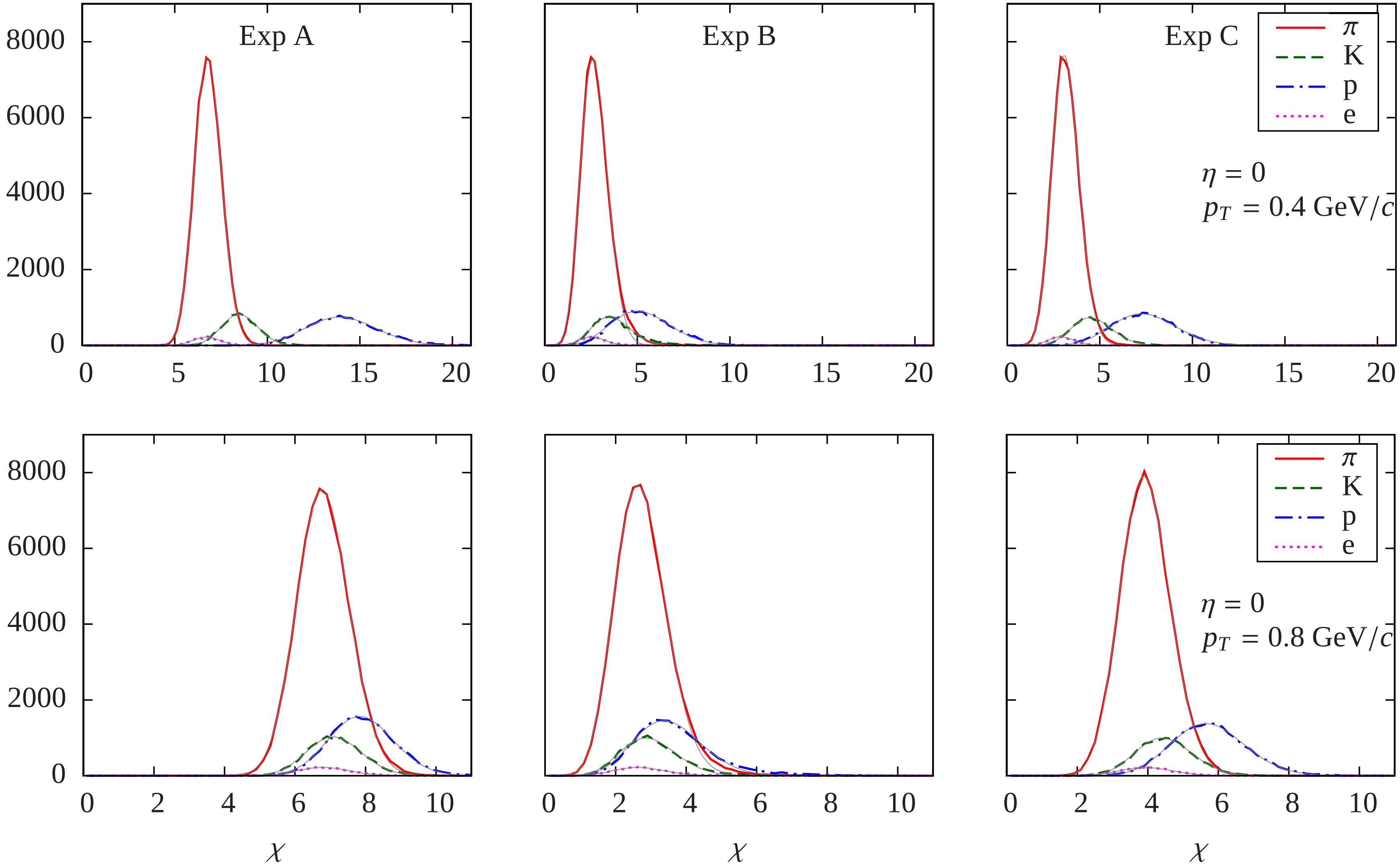
	B [T]	Subdetector	Radius of layers [cm]	$\sigma_{r\phi}$ [μm]	σ_z [μm]	x/X_0 [%]	$\zeta_{r\phi}$	ζ_z	Split meas.
Exp A	2	pixels (barrel)	5.0, 8.8, 12.2	10	115	4	0.1	1	50
		strips (SCT)^s	29.9, 37.1, 44.3, 51.4	17	580	4	0.1	3	
		straw (TRT)	56.3 – 106.6 ($\leq$ 36 hits)	130	–	0.5	10	–	
Exp B	0.4	pixels (SPD)	3.9, 7.6	12	100	1	0.2	2	12
		drifts (SDD)	14.9, 23.8	35	23	1	0.3	0.2	
		strips (SSD)^s	38.5, 43.6	15	730	1	0.1	7	
		[gas (TPC)	84.5 – 246.6 ($\leq$ 159 hits)	900	900	10^{-3}	$10^3 - 10^4$]		
Exp C	3.8	pixels (PXB)	4.4, 7.3, 10.2	15	15	3	0.2	0.2	20
		strips (TIB)^s	25.5, 33.9	$23/\sqrt{2}$	230	4	0.1	0.8	
		strips (TIB)	41.8, 49.8	35	–	2	0.2	–	
		strips (TOB)^s	60.8, 69.2	$53/\sqrt{2}$	530	4	0.1	2	
		strips (TOB)	78.0, 86.8, 96.5, 108.0	53, 35	–	2	0.2	–	

Exp A = ATLAS, Exp B = ALICE, Exp C = CMS

- Sensitivity
  - If the deviations are dominated by multiple scattering and local position measurement: $\zeta = \sigma_{pos}/\sigma_{ms}(m_0)$
  - Shown for pions at $p = 1$ GeV/c
  - Only silicons and straws contribute to result

Fast simulation results follow

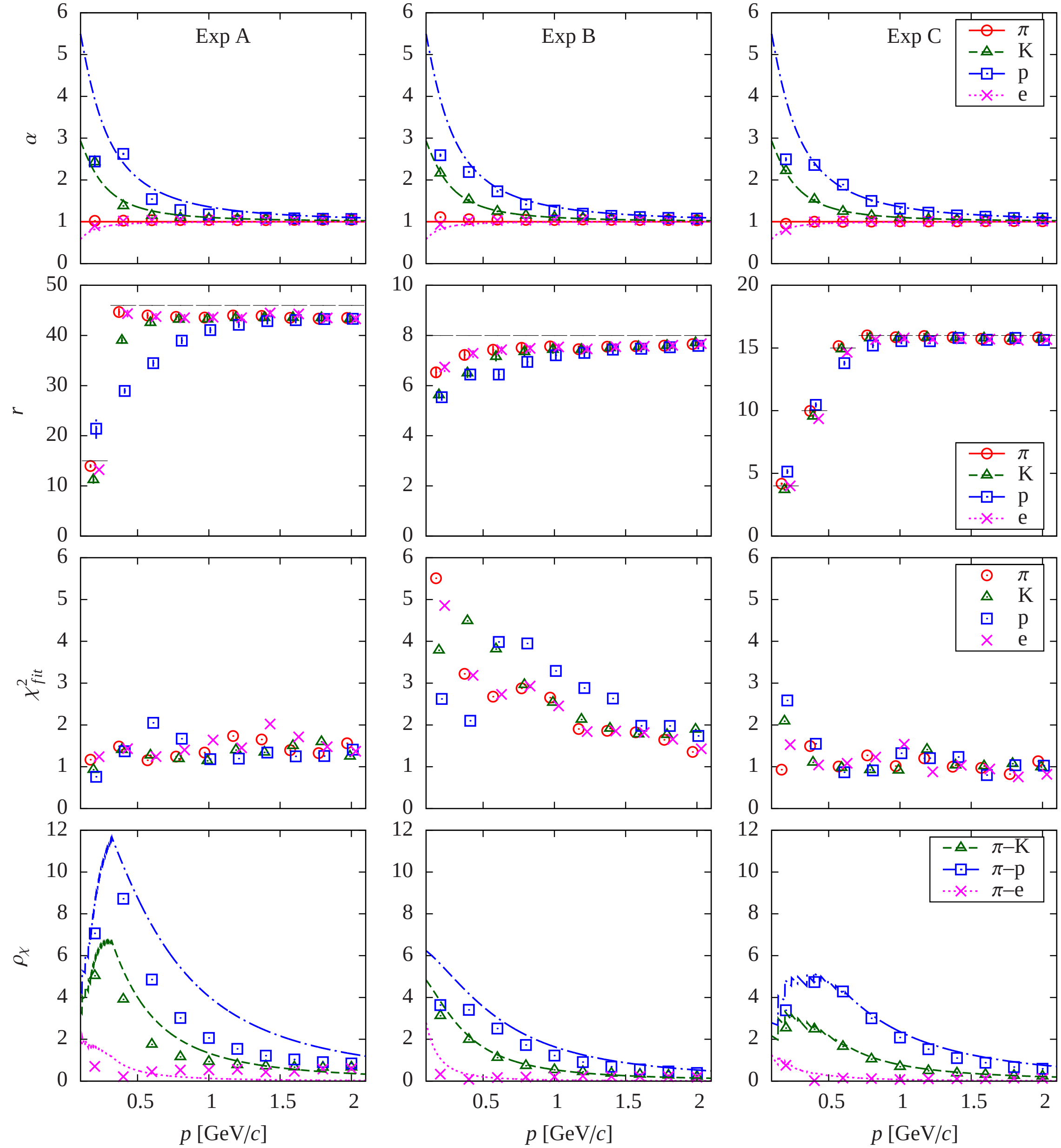
Results – χ distributions



Assumed particle composition: $\pi : \mathrm{K} : \mathrm{p} : \mathrm{e} = 70 : 10 : 18 : 2$

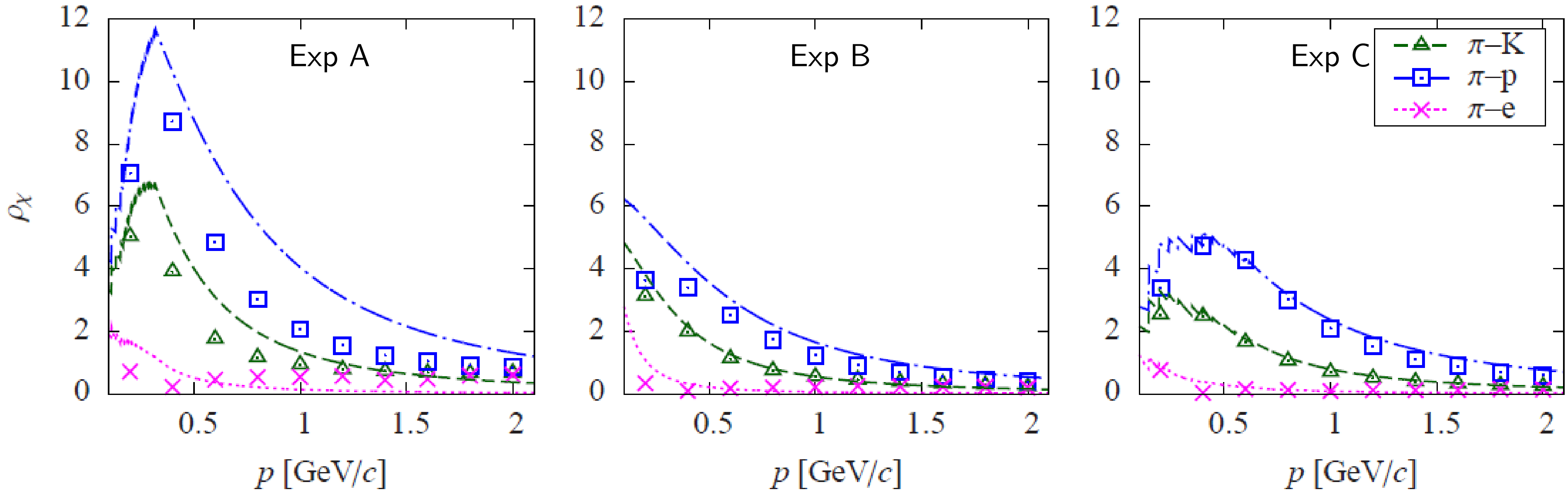
Expected distributions at low and high p_T

Results – performance



α – scale factor, r – ndof, χ^2_{fit} – goodness of χ fit, ρ_χ – separation

Results – separations



- Separation power
  - between two particles (m_1 and m_2)

$$\rho_\chi = \frac{2[\mu_\chi(m_1) - \mu_\chi(m_2)]}{\sqrt{\sigma_\chi^2(m_1) + \sigma_\chi^2(m_2)}}$$

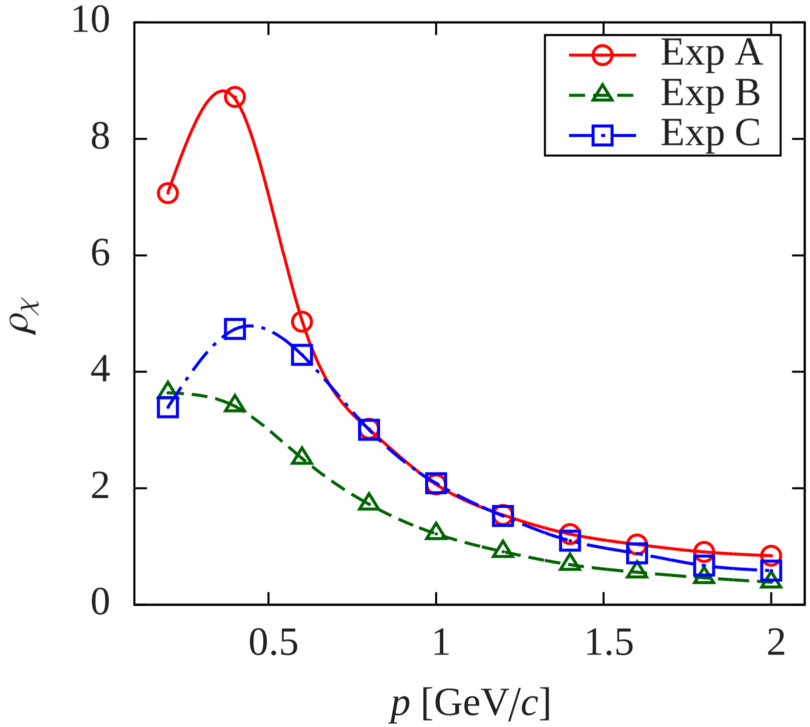
  - with approximations

$$\rho_\chi \approx 2\sqrt{2r-1}\,\frac{1-\beta(m)/\beta(m_0)}{\sqrt{1+[\beta(m)/\beta(m_0)]^2}}$$

- Comments
  - For not very low p and good local position resolution:
    no dependence on magnetic field, radii, material thickness
  - The mean and σ of the Gaussians are determined by p and mass via β
  - Even at very low p the variances still stay the same

Results – separation

π–p separation



- Comments
  - r is sometimes smaller than expected: low sensitivity measurements many straws with $\zeta_{r\phi} = 10$ (Exp A); strip layers with $\zeta_z = 7$ (Exp B)
  - Outliers and their removal procedure introduces shifts in the fitted value of r and in the resolution ρ_χ
  - The steps are due to the changing number of crossed layers with varying p

Summary

- Particle identification with track fit χ^2
  - Tracker + magnetic field
  - Perform global linear χ^2 fit with a mass hypothesis
  - One widely used option is Kalman-filter
  - Knowledge of detector material and local position resolution
  - Sensitivity to detector alignment
  - Use for particle identification or check material budget

Performance is determined by the number of good sensitivity measurements

For details see NIM A paper http://dx.doi.org/10.1016/j/nima.2010.03.098 [arXiv:0911.2624]