



***Workshop on future instruments for gravity-based  
earthquake early warning***

***January 10th 2018***



**Elasto-gravitational signals preceding direct  
seismic waves:  
Status of observations and modelling**

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# Context

## Gravitational perturbations due to mass redistribution associated with tectonic processes :

- **Static (final) gravitational perturbations:**
  - **Known solution for shear or tensile faults in half-space** [*Okubo et al.*, 1992]
  - **Observed by Earth gravimeters** [2003 Tokachi earthquake, Imanishi et al., 2004] **and space gravimetry** [static gravitational changes of the 2011 Tohoku earthquake detected by GRACE, e.g. Matsuo & Heki, 2011]
  
- **Dynamic gravitational perturbations :**
  - **Such perturbations also occur immediately after an earthquake :**
    - **The Earth masses are perturbed, both at the source location and at the places affected by the transient dilatant/compressive elastic waves**
    - **These perturbations propagate at the speed of light... even if their signature is small, the quiet period before the P-wave arrival may allow to observe them**

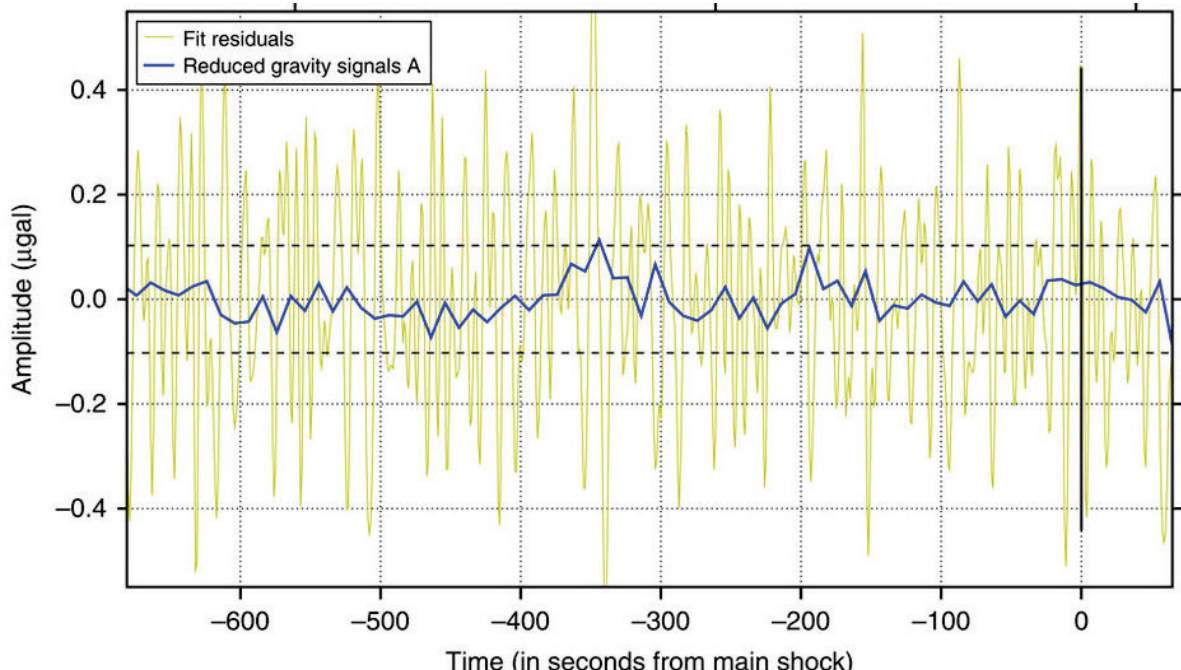
➤ Analytical solution for pre-P gravitational perturbations in full space [Harms et al., 2015]:

$$\Delta g(\mathbf{r}_0, t) = \frac{6G}{r_0^4} \underbrace{((\mathbf{e}_z \cdot \mathbf{e}_r)\mathbf{e}_x + (\mathbf{e}_x \cdot \mathbf{e}_r)\mathbf{e}_z - 5(\mathbf{e}_x \cdot \mathbf{e}_r)(\mathbf{e}_z \cdot \mathbf{e}_r)\mathbf{e}_r)}_{\text{Radiation pattern}} \underbrace{I_2[M_0](t)}_{\text{2nd integral of the moment time function}}.$$

- Large earthquakes (with rapidly increasing moment) offer the best potential
- When the earthquake is in its development phase ( $M_0 \sim t^3$ ):  $\Delta g(\mathbf{r}_0, t) \sim \frac{t^5}{r_0^4}$

➤ First detection of a perturbation [Montagner et al., 2016] for the 2011 Tohoku earthquake

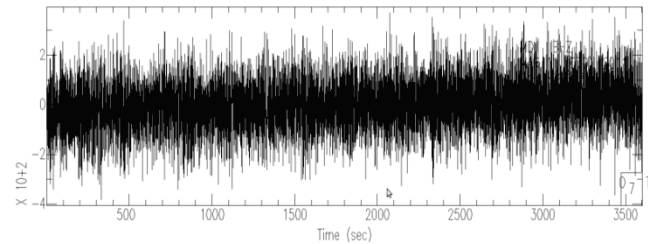
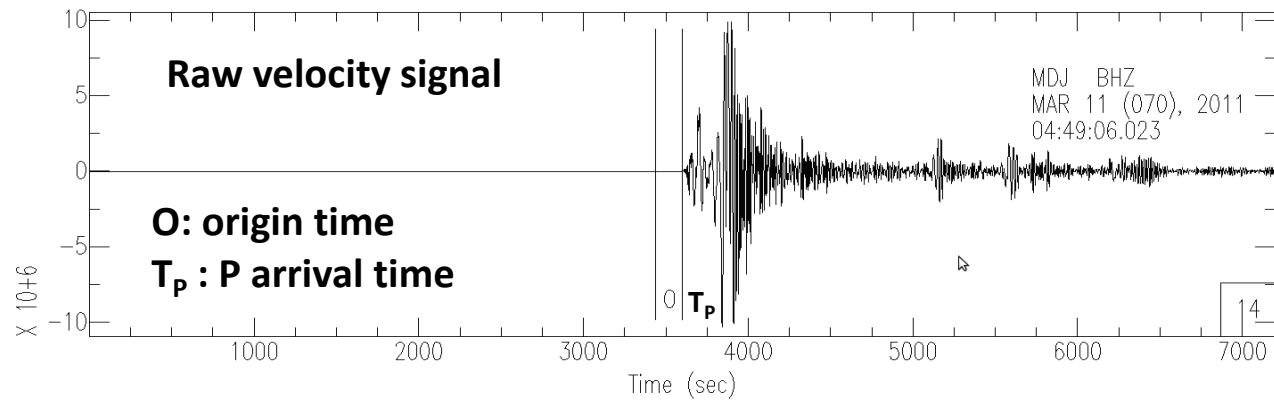
**Kamioka  
gravimeter  
in Japan,  
about  
500km  
from the  
earthquake**



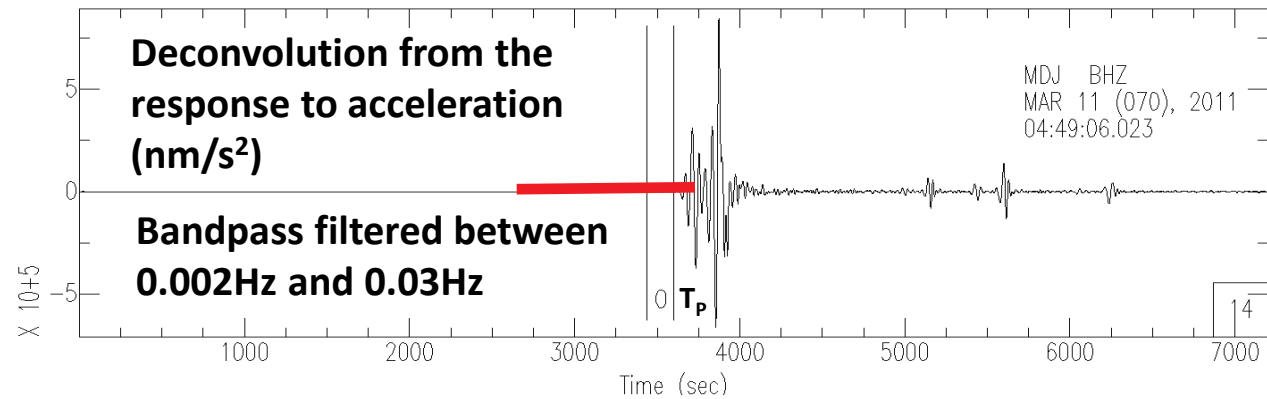
Full space theory indicates that the closest distances do not provide the best observation potential. Let's look also further away :



Observations of the signal recorded at the MDJ seismic broadband station (IC network), located ~1300km from the earthquake

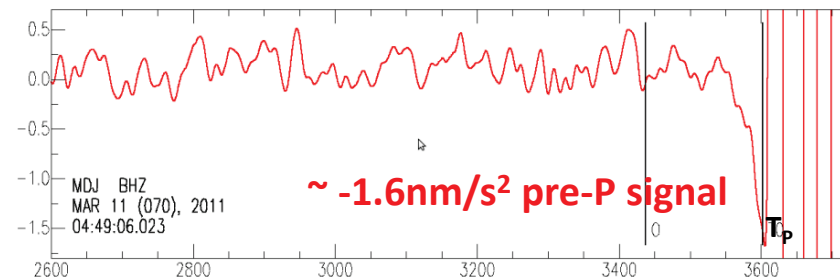


Zoom on the pre-P arrival time (scale /25000)



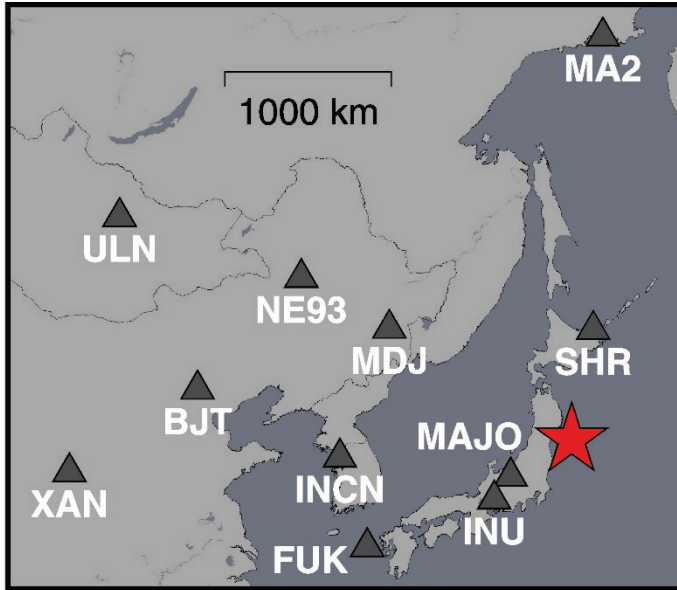
Deconvolution from the response to acceleration (nm/s<sup>2</sup>)

Bandpass filtered between 0.002Hz and 0.03Hz



Zoom on the pre-P arrival time (scale 1:600000)

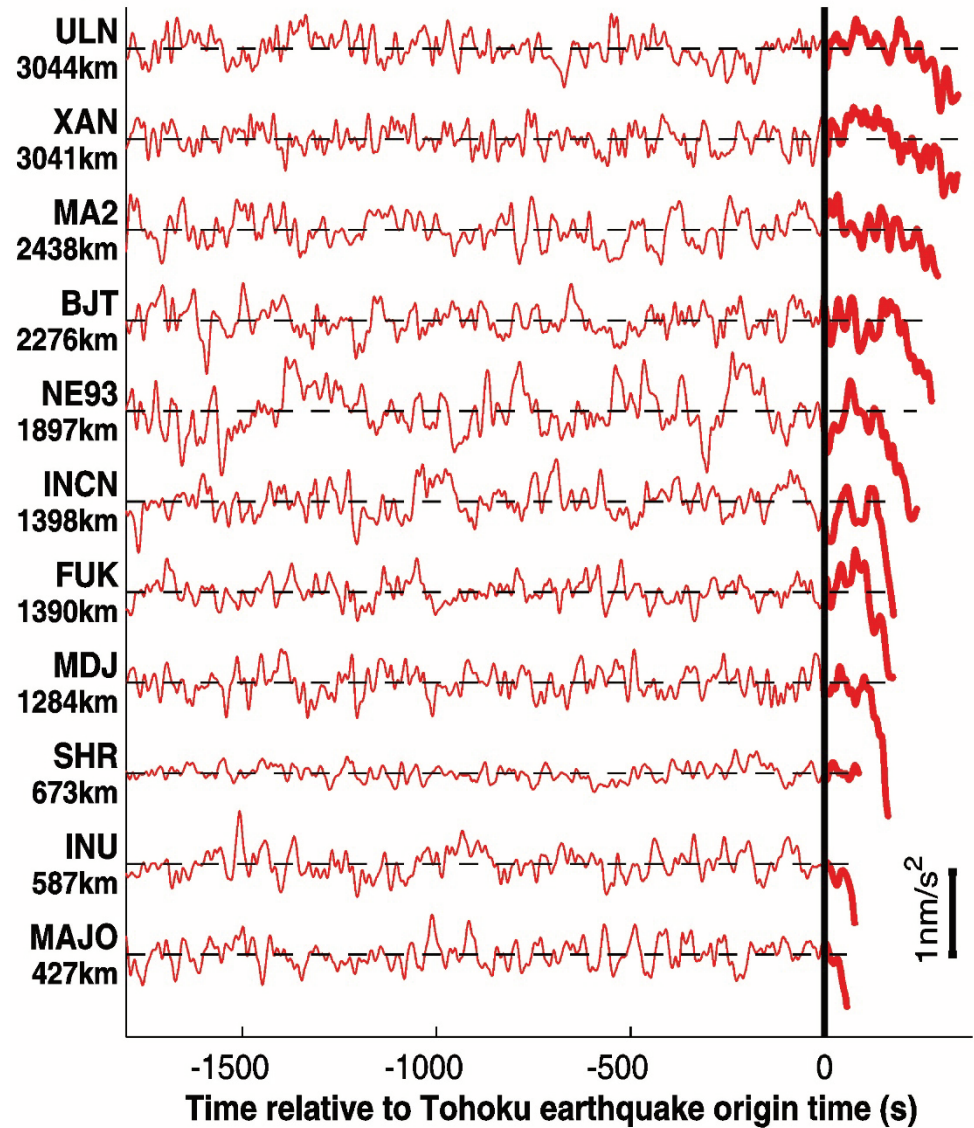
# With seismometers, such a tiny signal requires excellent stations to be recorded

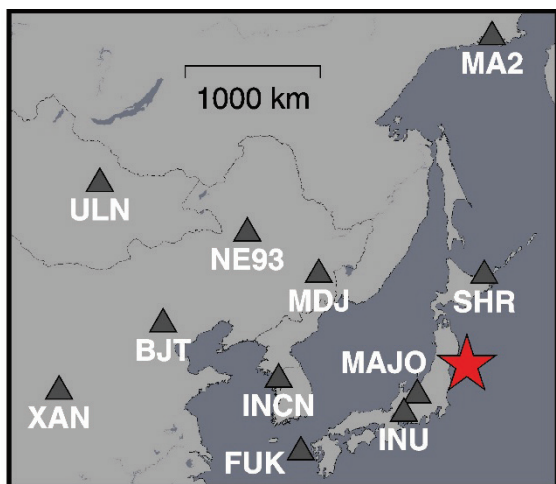


After retrieving all the available broadband stations (IRIS), map of all the stations able to detect the signal, based on a Signal-to-Noise ratio criterion

Most of the stations are FDSN stations (IRIS or GEOSCOPE) known for their high quality

Some stations from F-NET (Japan) are also included

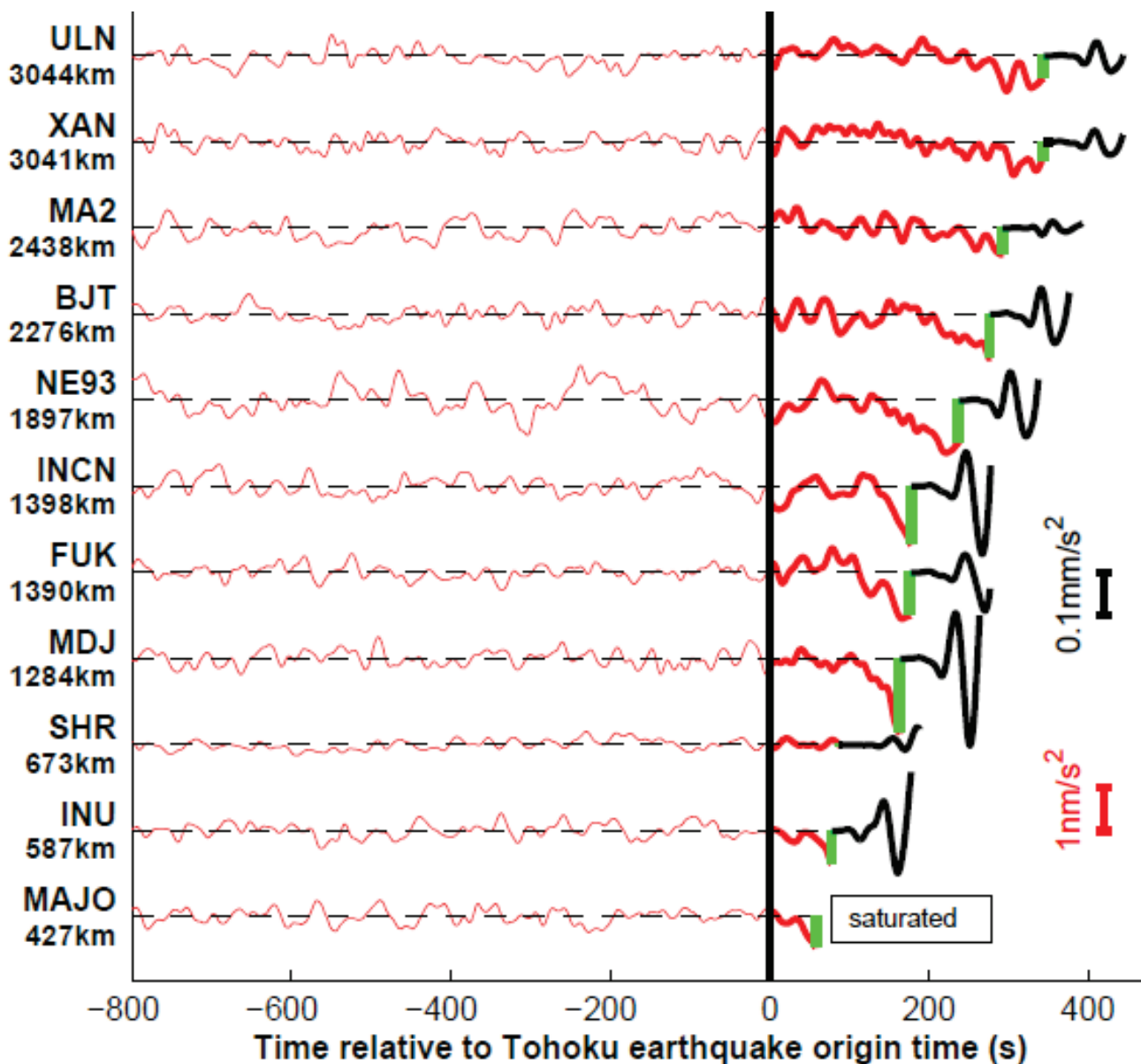




Green tick : P wave arrival

Relative amplitudes between the pre-P and the post-P signals

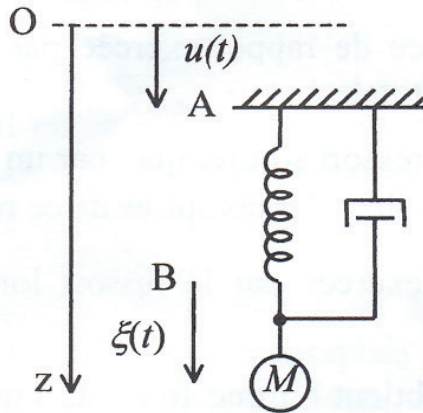
Pre-P signals are  $10^5$  to  $10^6$  smaller





But what are exactly these signals that we observe ?

What do we expect to record with a ground-attached seismometer (or gravimeter) ?



Without gravitational changes, gravity only controls the equilibrium position of the mass, and we have :

$$\frac{d^2\xi}{dt^2} + \frac{D}{M} \frac{d\xi}{dt} + \frac{k}{M} \xi = - \frac{d^2u}{dt^2} \quad (1)$$

With  $\Delta g$ , (1) is simply modified as :

$$\frac{d^2\xi}{dt^2} + \frac{D}{M} \frac{d\xi}{dt} + \frac{k}{M} \xi = \Delta g - \frac{d^2u}{dt^2} \quad (2)$$

A seismometer is therefore a **seismo-gravimeter**, which records, after correction from the instrumental response, **the difference between the ground acceleration and the gravitational perturbations**

## How to compute $\Delta g$ ?

- At a given time  $t$  ( $0 < t < T_P$ ), transient elastic displacements affect the volume  $V_s^P$  around the source

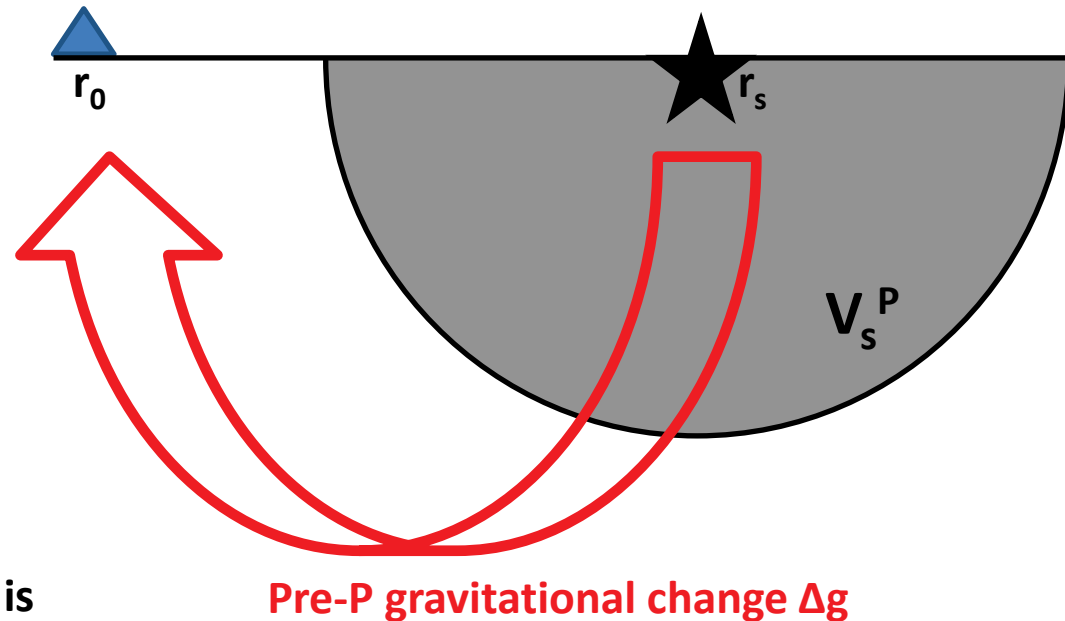
- These displacements can be calculated in every point  $r$  of  $V_s^P$  (use of AXITRA method, moment tensor version) with :

$$u_i(\mathbf{r}, t) = M_{jk} m(t) * \mathcal{G}_{ij,k}(\mathbf{r}_s, \mathbf{r}, t)$$

- The pre-P gravitational perturbation is controlled by an integral over  $V_s^P$  of the form (Dahlen & Tromp 1998):

$$\Delta g^P(\mathbf{r}, t) = G \int_{V_s^P} \frac{\rho(\mathbf{r}') [\mathbf{u}(\mathbf{r}', t) - 3(\mathbf{e}_{\mathbf{r}'\mathbf{r}} \cdot \mathbf{u}(\mathbf{r}', t)) \mathbf{e}_{\mathbf{r}'\mathbf{r}}]}{|\mathbf{r}' - \mathbf{r}|^3} d\mathbf{r}'$$

Let us consider an earthquake in  $r_s$ , starting at  $t=0$ , and generating elastic waves, with the fastest (P) one arriving at  $T_P$  at the station in  $r_0$

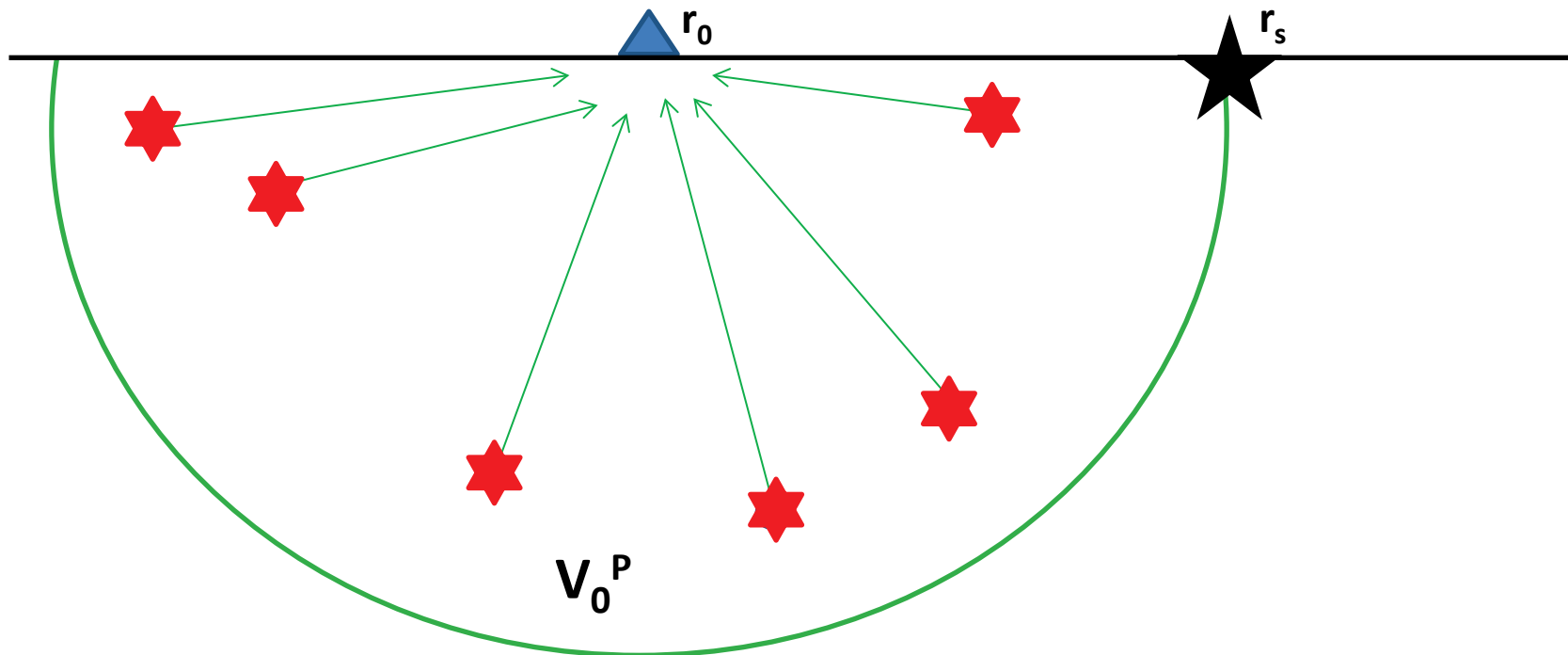


**But note that there is a gravitational perturbation not only at  $r_0$ , but everywhere in the medium**



$\Delta g$  is also a body force acting in the whole medium, which will cause the station to move **EVEN BEFORE** the arrival to the direct P wave. This can be seen as a try of the Earth to elastically re-equilibrate after the gravitational perturbations.

Concretely, all the gravitational perturbations occurring in the volume  $V_0^P$  defined by  $V_0^P = \{\mathbf{r}' \in V / T^P(\mathbf{r}_0, \mathbf{r}') < T_p\}$  can generate elastic waves arriving before the hypocentral P arrival at the station

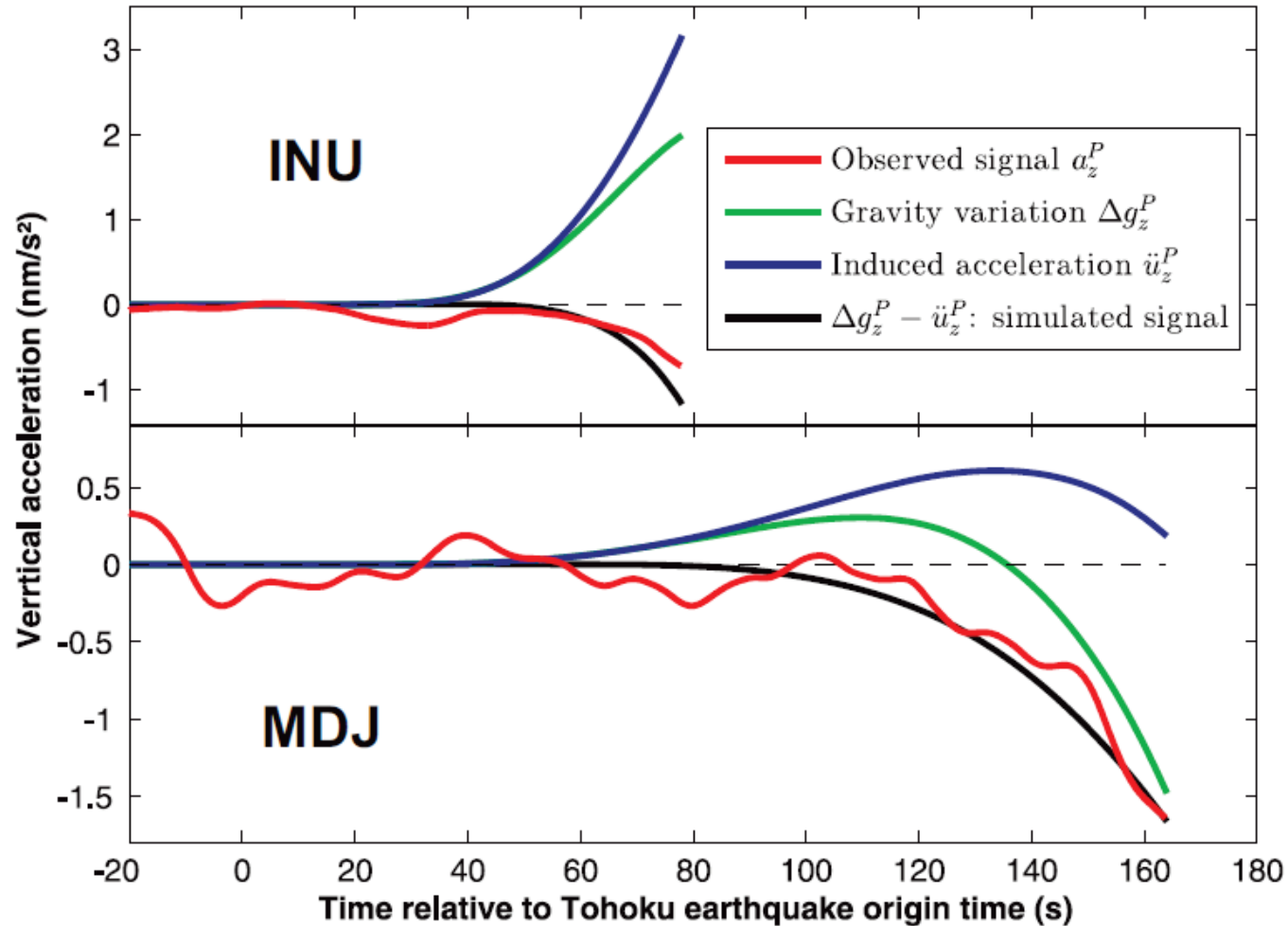


This gravitational-induced acceleration can be computed with the integral

$$\ddot{u}_z^P(\mathbf{r}, t) = \frac{d^2}{dt^2} \int_{V_0^P} \underbrace{\rho(\mathbf{r}') \Delta g_i^P(\mathbf{r}', t) * \mathcal{G}_{iz}(\mathbf{r}', \mathbf{r}, t)}_{\text{AXITRA method (single force)}} d\mathbf{r}'$$

We now have all the ingredients to compute the prompt vertical acceleration recorded by the broadband seismometers  $= \Delta g_z^P(\mathbf{r}_0, t) - \ddot{u}_z^P(\mathbf{r}_0, t)$

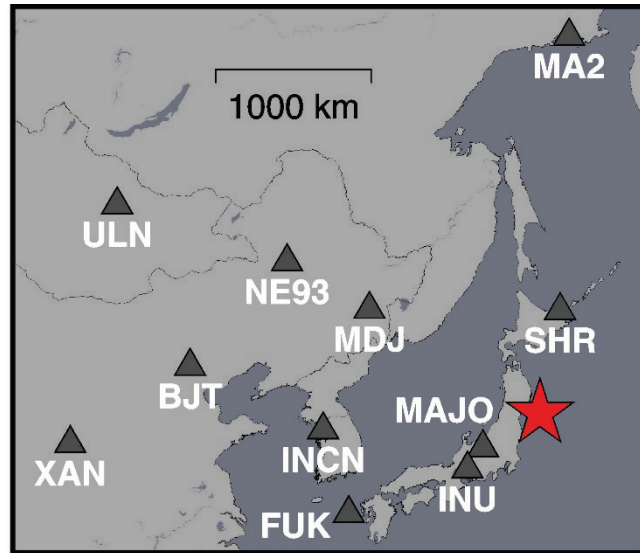
### Data & simulations at INU (GEOSCOPE, G) and MDJ (IRIS-China, IC)



*Vallée et al.,  
Science, 2017*

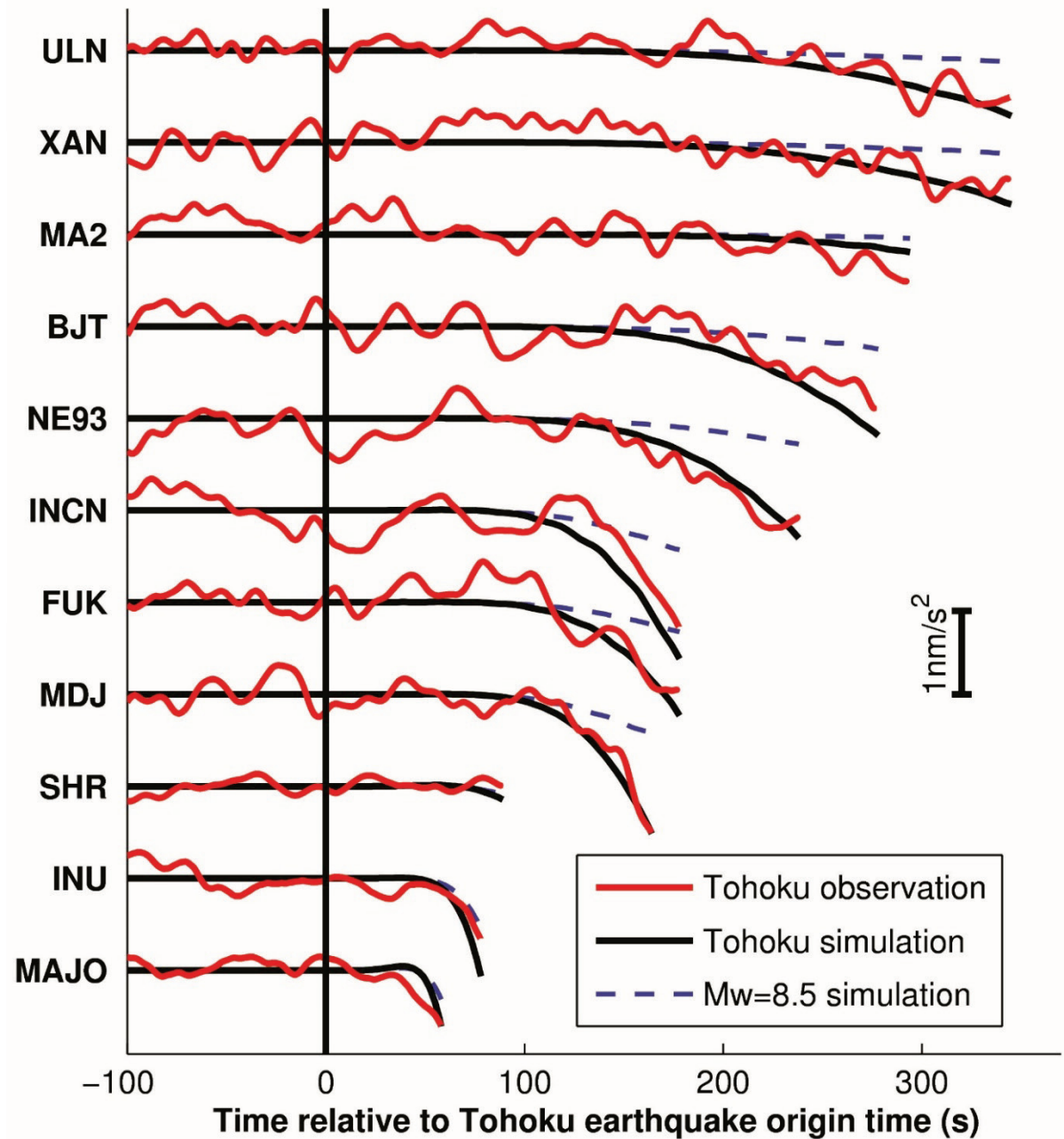
**Remark : at INU station, the recorded signal is negative while the gravitational perturbation is positive : the signal is dominated by the induced acceleration**

## Complete simulation at all stations



**Good agreement  
between observed and  
modeled signals**

**Strong sensitivity to  
earthquake magnitude  
[A smaller Tohoku-type  
earthquake ( $M_w < 8.5$ )  
would not have been  
detected]**

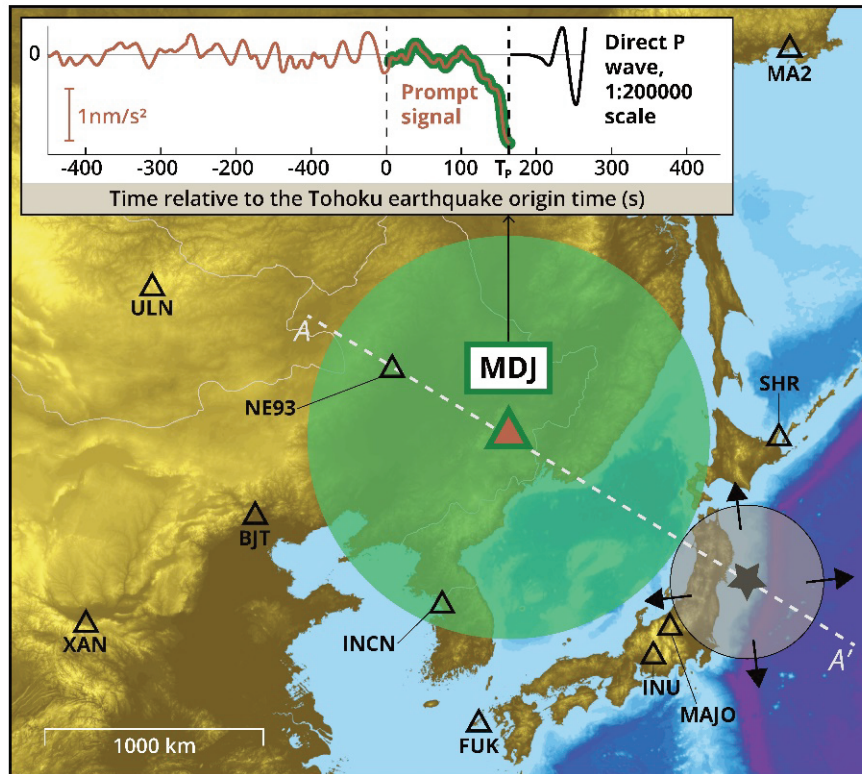


# Conclusions

- The signal preceding direct seismic waves is today **measurable with high-quality broadband seismometers in case of very large earthquakes**
  - This signal results from elasto-gravitational effects and can be modelled **with a 3-stage procedure:**
    - **Elastic displacements in the medium**
    - **Induced gravitational perturbation**
    - **Induced ground acceleration**
  - These observations, besides being original and physically modelled by a new procedure, also have **a great potential for an early determination of the earthquake magnitude, in the minutes following a very large earthquake**
- **Such a potential for early magnitude determination should even more motivate instrumental developments to increase the range of magnitude where these signals can be measured**

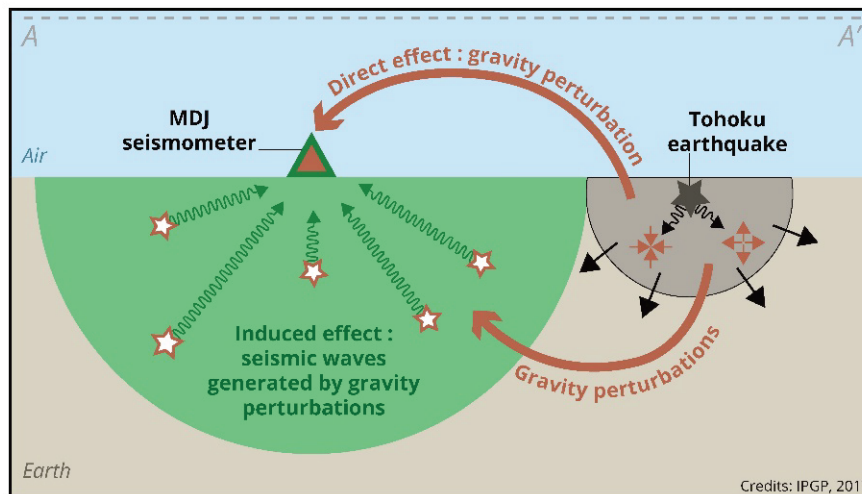


# Observation and origin of the elastogravity signal preceding direct seismic waves



The map shows the location of the seismometers (triangles) detecting the prompt signals just after the beginning of the Tohoku earthquake (Japan, 11 March 2011, magnitude 9.1), indicated by the black star. We focus here on one of the stations (MDJ), located in north-east China, 1280km away from the Tohoku earthquake. At such distances, direct seismic waves arrive about 165 s after the earthquake starts, as shown in the inset reproducing the MDJ vertical seismogram. However, a clear, even if much weaker, acceleration signal is detected by the seismometer before the direct waves arrival.

The origin of such signal can be understood by considering a time after the earthquake onset but before the arrival of the direct seismic waves. For example, about 55 s after origin time, direct waves have propagated inside the volume shown by the grey area, but are still far from arriving at MDJ station. However, inside this volume, seismic waves have caused compressions and dilations of the medium (as further indicated in the bottom cross-section), and the global contribution of all such elements whose mass has changed gives rise to a gravity perturbation, immediately detected by the seismometer (direct effect). The gravitational field is also modified everywhere in the Earth, and each of the elements affected by these perturbations is a secondary source of seismic waves (induced effect). In the green volume around the seismometer, this secondary seismic wavefield arrives before the direct waves. The seismometer therefore records a prompt elastogravity signal, due to the direct and induced effects of the gravity perturbations.





We now have all the ingredients to compute the prompt vertical acceleration recorded by the broadband seismometers  $= \Delta g_z^P(\mathbf{r}_0, t) - \ddot{u}_z^P(\mathbf{r}_0, t)$

## More practical information :

### ➤ Propagation

- Intensive use of the AXITRA (Cotton and Coutant, 1997) method
- Introduction of Earth flattening formulas (Muller, 1977) to correct for sphericity (some stations are thousands of kilometers away from the Tohoku earthquake)
- Use of the PREM model in the mantle combined with a crust thickness of 40 km

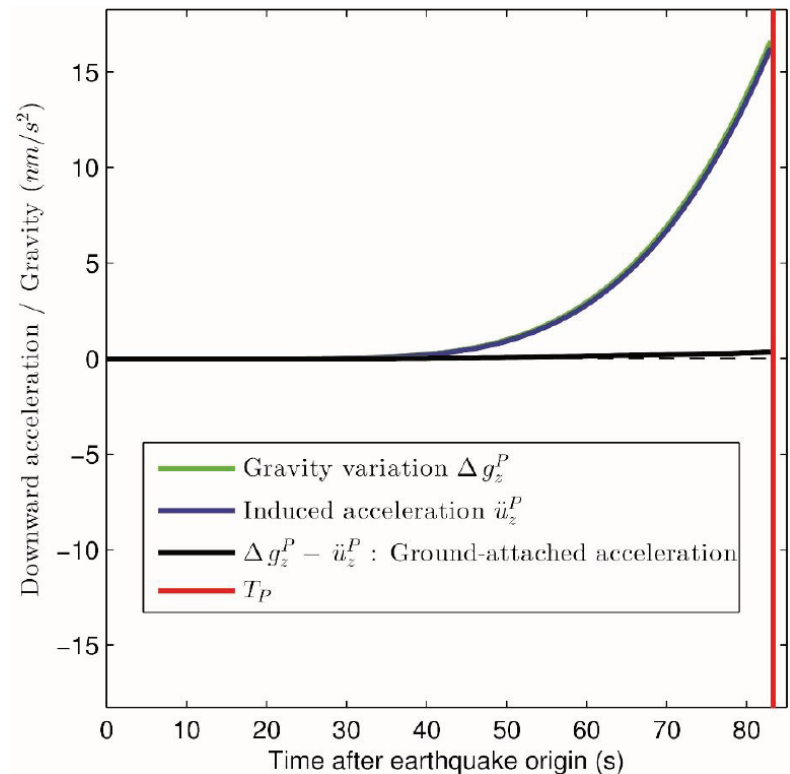
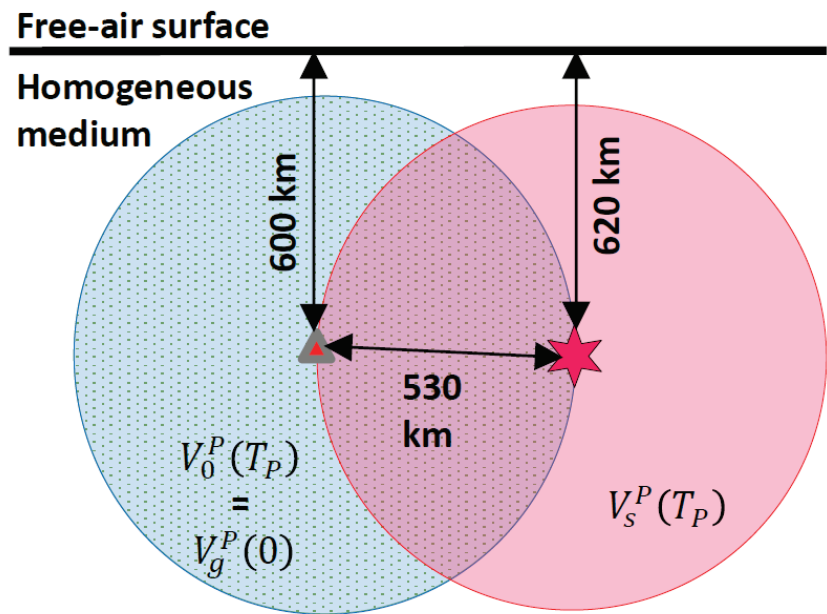
### ➤ Source

- Global CMT parameters for the source coordinates, origin time, moment tensor (strike, dip, rake = 203°, 10°, 88°)
- isosceles triangular GCMT moment rate function (140 s duration) with GCMT seismic moment ( $M_0 = 5.31 \cdot 10^{22} \text{N.m}$ ).

# The infinite homogeneous medium

A nice configuration to test the **validity and accuracy of the approach**, as we theoretically know in this case that there is an **exact cancellation between the gravity perturbation and the induced acceleration**.

Such a cancellation is very well reproduced by the numerical simulation.



## Alternative approaches ?

### ❑ Normal modes

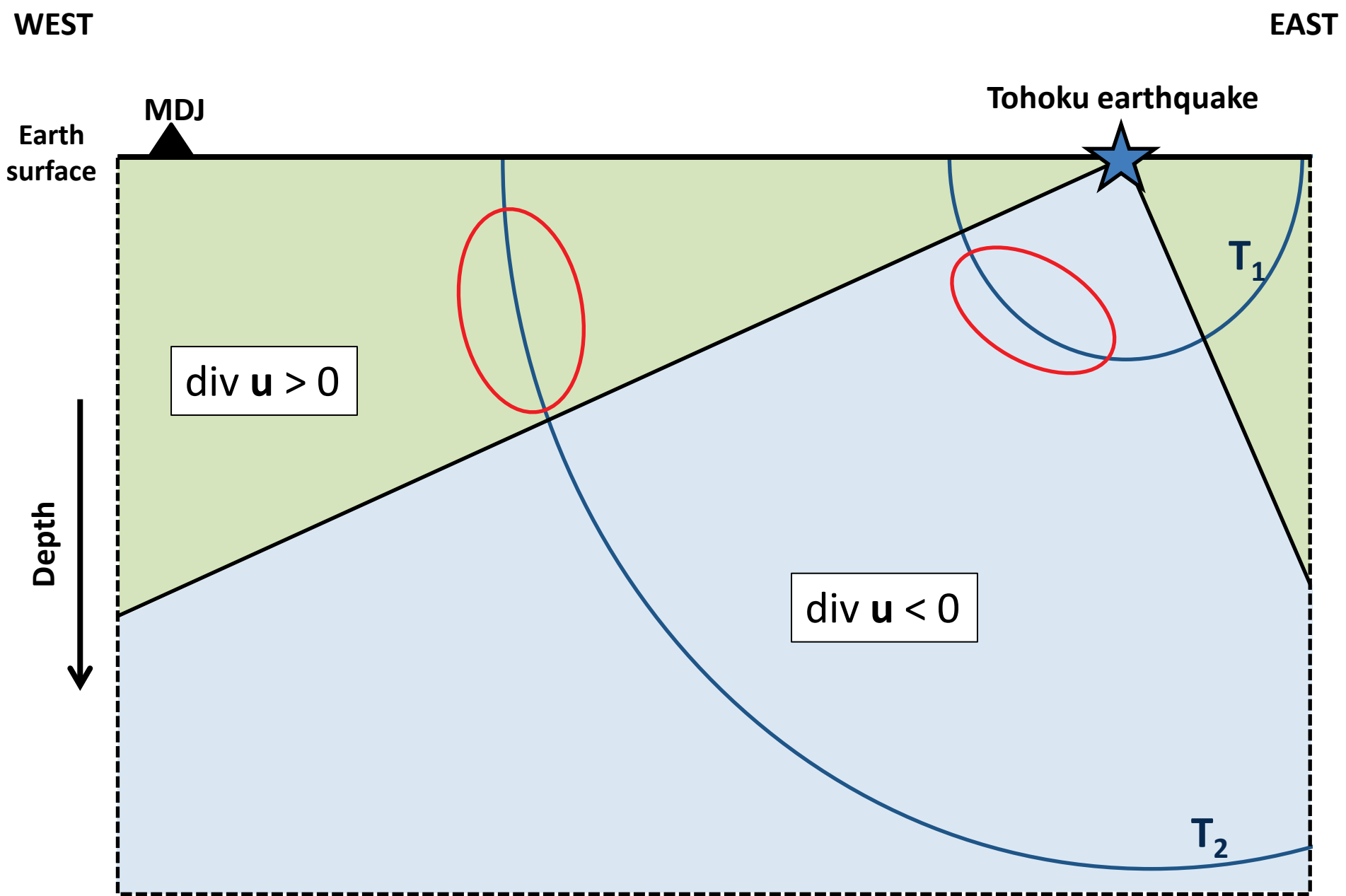
- **Normal modes** can theoretically model such signals more directly because **self-gravitation can be intrinsically included**.
- This theoretical advantage however comes with the drawback of modeling the waves at the global scale (and not around the earthquake as the 3-stage-AXITRA method does)
- Results **should not be different when frequencies are not too low (> 0.001-0.002Hz)**

### ❑ Analytical derivations

- Offer **efficient and insightful** computation of the **gravity perturbations**, both in **full-space** and **half-space** models.



These approaches have been explored in detail in the PhD of Kévin Juhel



Understanding the sign change of the vertical gravity perturbation at MDJ station