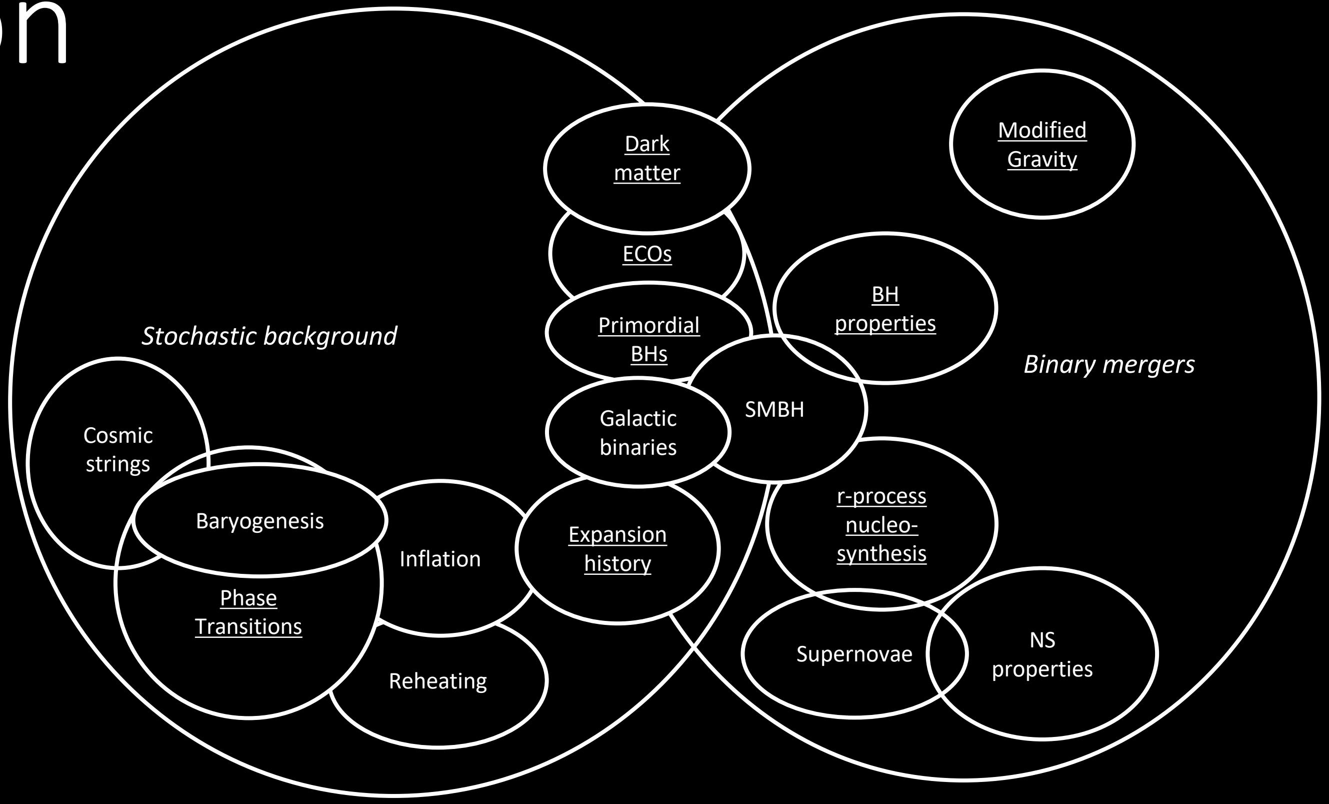
Model discrimination
in the
SGWB
from
Phase Transitions



Djuna Lize Croon

Dartmouth

Helsinki, June 2018

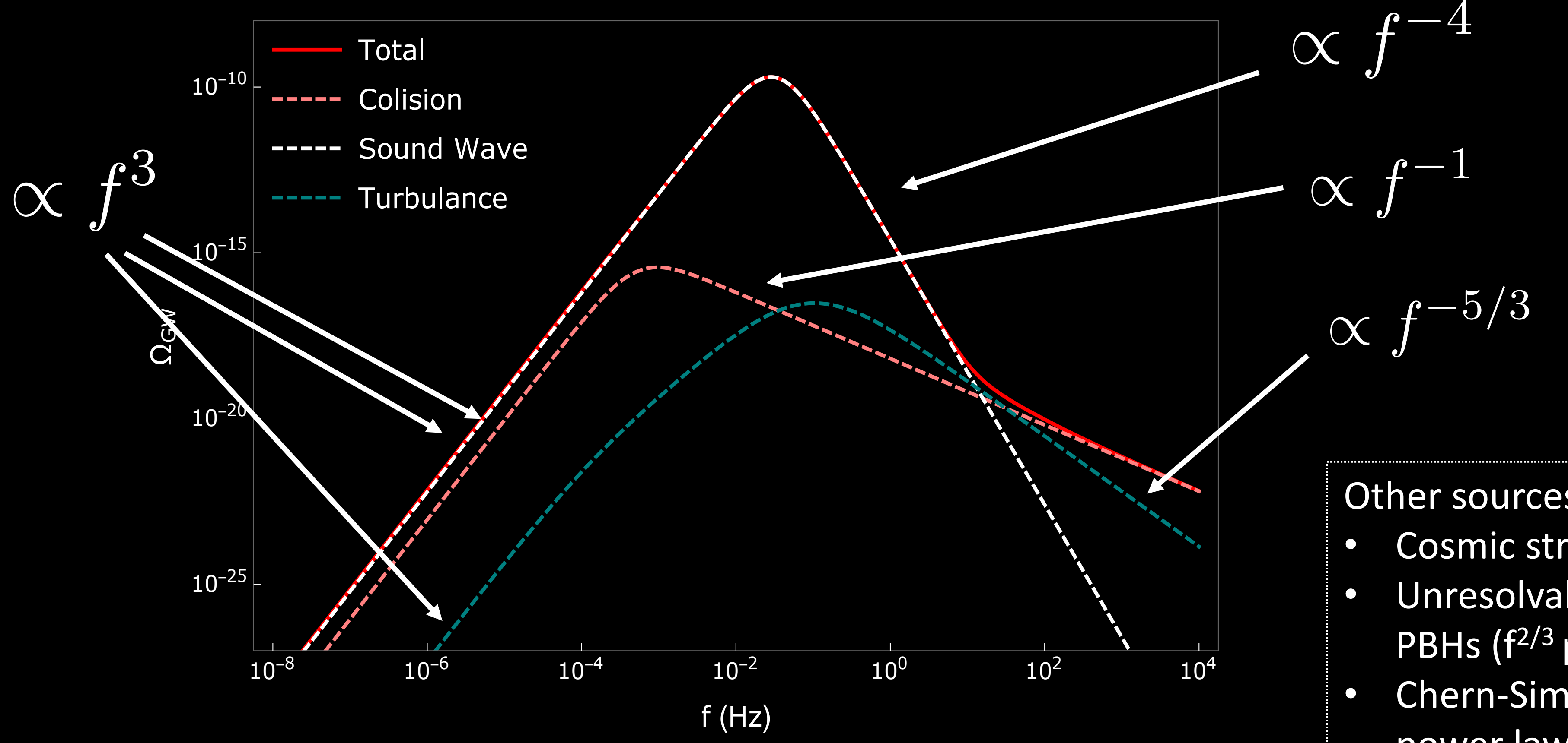
Based on

DC, V. Sanz and G. White [arXiv:1806.02332]

DC, G. White [arXiv:1803.05438, JHEP]

Model discrimination in the SGWB?

Spectrum from a typical (non-runaway) PT



(Model “A” in Mark’s talk)

Other sources @LISA:

- Cosmic strings (~flat GW profile)
- Unresolvable binary mergers, SMBH, PBHs ($f^{2/3}$ power law profile)
- Chern-Simons inflaton coupling (f^{n_T} power law profile)

GW spectra from thermal parameters

Weir, arXiv:1705.01783

$$h^2\Omega_{\rm sw} = 8.5 \times 10^{-6} \left(\frac{100}{g_*}\right)^{-1/3} \Gamma^2 \bar{U}_f^4(\alpha) \left(\frac{\beta}{H}\right)^{-1} v_w S_{\rm sw}(f)$$

$$S_{\rm sw} = \left(\frac{f}{f_{\rm sw}}\right)^3 \left(\frac{7}{4+3\left(\frac{f}{f_{\rm sw}}\right)^2}\right)^{7/2}$$

$$f_{\rm sw} = 8.9 \times 10^{-7}\mathrm{Hz}\frac{1}{v_w}\left(\frac{\beta}{H}\right)\left(\frac{T_N}{\mathrm{Gev}}\right)\left(\frac{g_*}{100}\right)^{1/6}$$

- Thermal parameters

$$p(t_N)t_N^4 = 1 \qquad p(T) = T^4\, e^{-S_E/T}$$

$$\frac{\beta}{H} = T\frac{\partial}{\partial T}\left(\frac{S_E}{T}\right)$$

Normalized rate

$$\alpha = \frac{\Delta\rho}{\rho^*}$$

Latent heat

$$\frac{S_E}{T_N} \sim 177 - 4\ln\frac{T_N}{\mathrm{GeV}} - 2\ln g_*$$

Nucleation temperature

GW spectra from thermal parameters

Weir, arXiv:1705.01783

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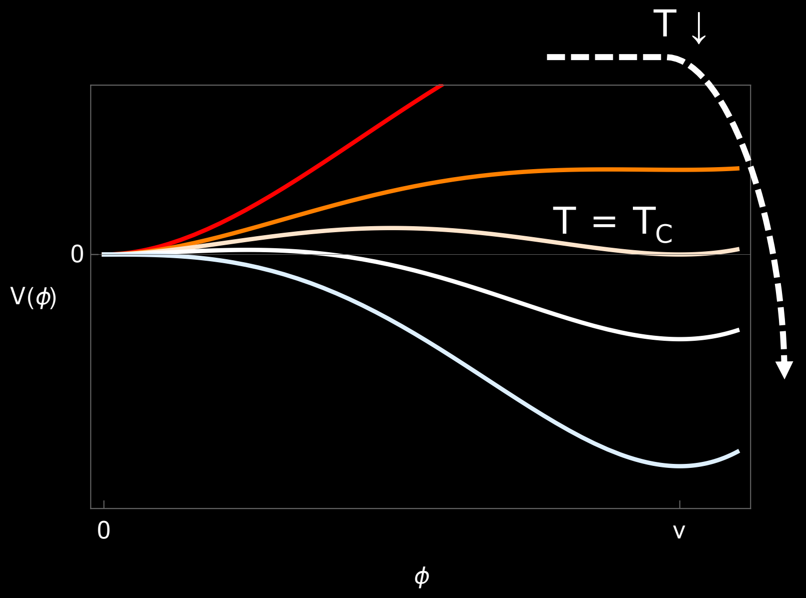
- Two parameters:
  - **Ω** and **f**
  - Peak amplitude and frequency
- How much model discrimination is possible?

Effective models for a first order PT

- Minimal model for a PT: double well potential → three terms in the effective potential, with relative signs



- We consider two limiting cases:
  1. $V(h_D, T) = \frac{1}{2}m(T)^2 h_D^2 - c_3(T) h_D^3 + \frac{1}{4}\lambda(T) h_D^4$
  2. $V(h_D, T) = \frac{1}{2}m(T)^2 h_D^2 - \frac{1}{4}\lambda(T) h_D^4 + c_6(T) h_D^6$

- Models: SSB in a dark gauge sector

SSB: SU(N) → SU(N-1) through h_D getting a VEV
N_f coupled fermions

PT from renormalizable operators (1)

$$V(H,T) = \Lambda^4 \left[-\frac{1}{2}\left(\frac{h_D}{v}\right)^2 + \frac{1}{4}\left(\frac{h_D}{v}\right)^4 \right] + V_{\rm CW}(h_D)$$

Zero temperature potential

$$+ \frac{T^4}{2\pi^2}\left[\sum_{i\,\in\,\rm bosons} n_i J_B\left(\frac{m_i^2}{T^2}\right) + \sum_{j\,\in\,\rm fermions} n_j J_F\left(\frac{m_j^2}{T^2}\right) \right]$$

Finite temperature correction

(see for example Quiros, [hep-ph/9901312])

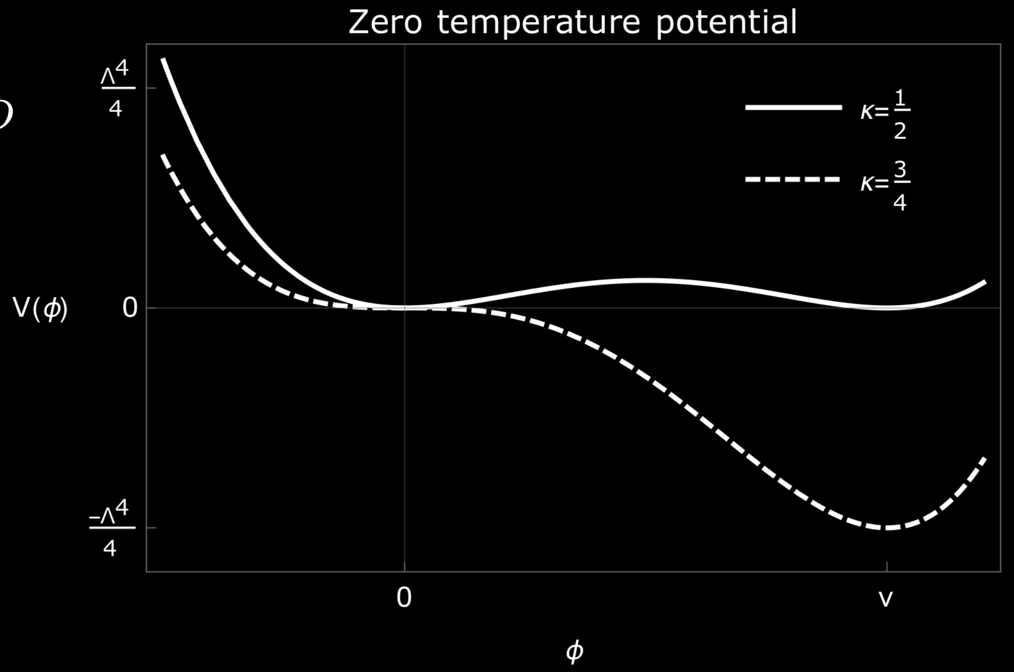
High temperature expansion of the thermal functions

$$V(H,T) = \Lambda^4 \left[\left(-\frac{1}{2} + \left(\frac{1}{8} + \frac{N_G}{24}\right)\frac{T^2}{v^2} + \frac{3}{24} N_{GB} \frac{g^2}{4}\frac{T^2 v^2}{\Lambda^4} + y^2 N_f \frac{T^2}{24}\frac{v^2}{\Lambda^4} \right)\left(\frac{h_D}{v}\right)^2 - \left(N_{GB}\left(\frac{g^2}{4}\right)^{3/2}\frac{1}{4\pi}\frac{v^3 T}{\Lambda^4} \right)\left(\frac{h_D}{v}\right)^3 + \frac{1}{4}\left(\frac{h_D}{v}\right)^4 \right]$$

v/Λ, g, N_{GB}, N_f x y^2

PT from non-renormalizable operators (2)

$$V(h_D,T)=\frac{1}{2}m(T)^2h_D^2-\frac{1}{4}\lambda(T)h_D^4+c_6(T)h_D^6$$



$$m^2(0)=(2-3\kappa)\,\frac{\Lambda^4}{v^2}$$
$$\lambda(0)=4\,\frac{\Lambda^4}{v^4}$$
$$c_6(0)=\kappa\,\frac{\Lambda^4}{v^6}$$

Reparameterization

$$V(H,T)=\Lambda^4\left[\left(2-3\kappa-\left[\frac{1}{2}+N_G\frac{1}{6}\right]\frac{T^2}{v^2}+\frac{3}{24}N_{GB}\frac{g^2}{4}\frac{T^2v^2}{\Lambda^4}+y^2N_f\frac{T^2}{24}\frac{v^2}{\Lambda^4}\right)\left(\frac{h_D}{v}\right)^2-\left(1-\frac{(30+6N_G)\kappa T^2}{v^2}\right)\left(\frac{h_D}{v}\right)^4+\kappa\left(\frac{h_D}{v}\right)^6\right]$$

v/Λ, κ, g, N_{GB}, N_f x y^2

Aside: EWPT

- Up to dimension-6 operators, models of the EWPT can be captured by the previous potential

$$V_6(h,T) = \left(a_T T^2 - \frac{\mu^2}{2}\right) h^2 + \left(b_T T^2 - \frac{\lambda}{4}\right) h^4 + \frac{1}{8\Lambda_6^2} h^6$$

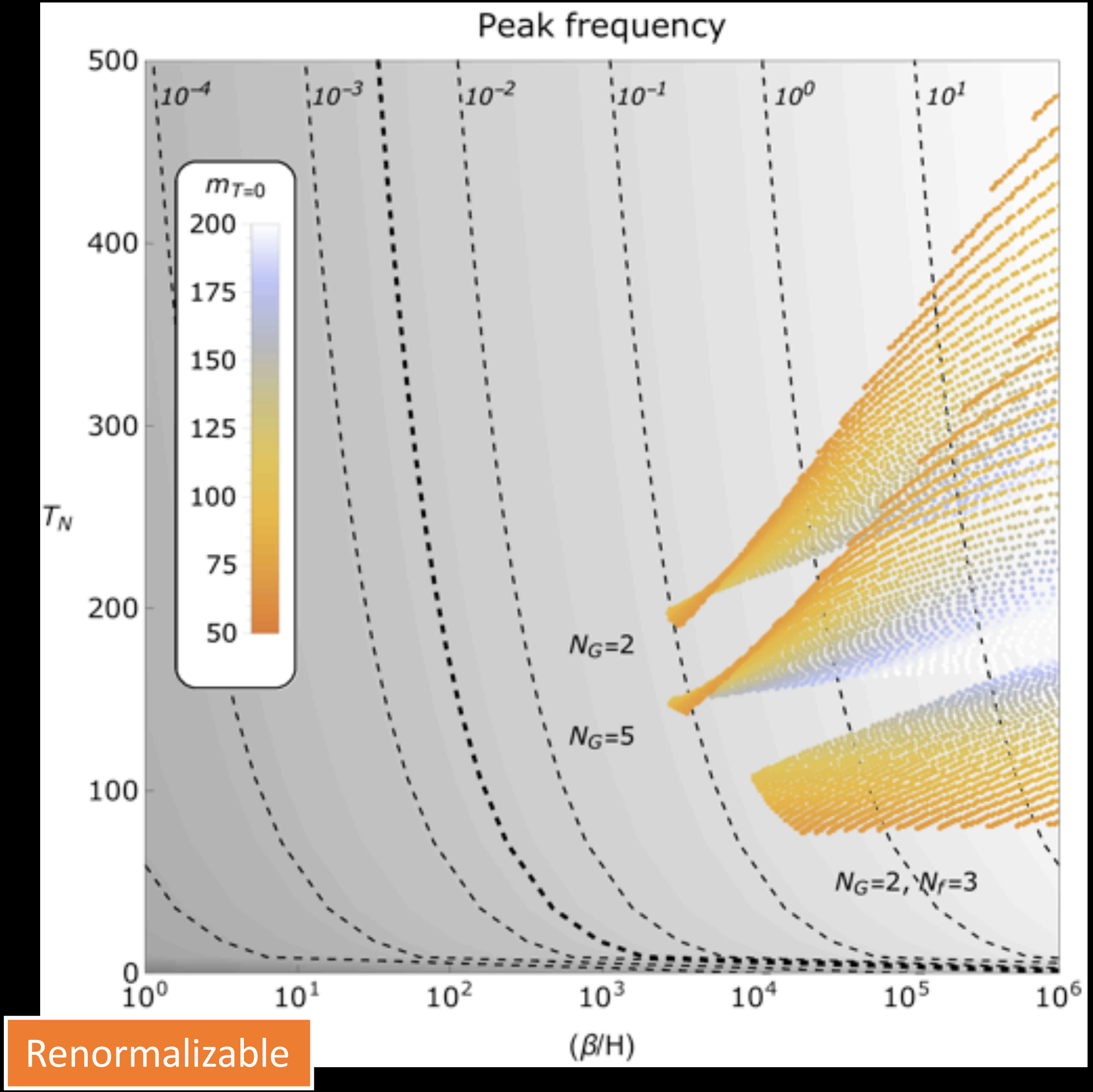
$$a_T = \frac{y_t^2}{8} + 3\frac{g^2}{32} + \frac{g\prime^2}{32} - \frac{\lambda}{4} + \frac{v_0^2}{\Lambda_6^2}\frac{3}{4}$$

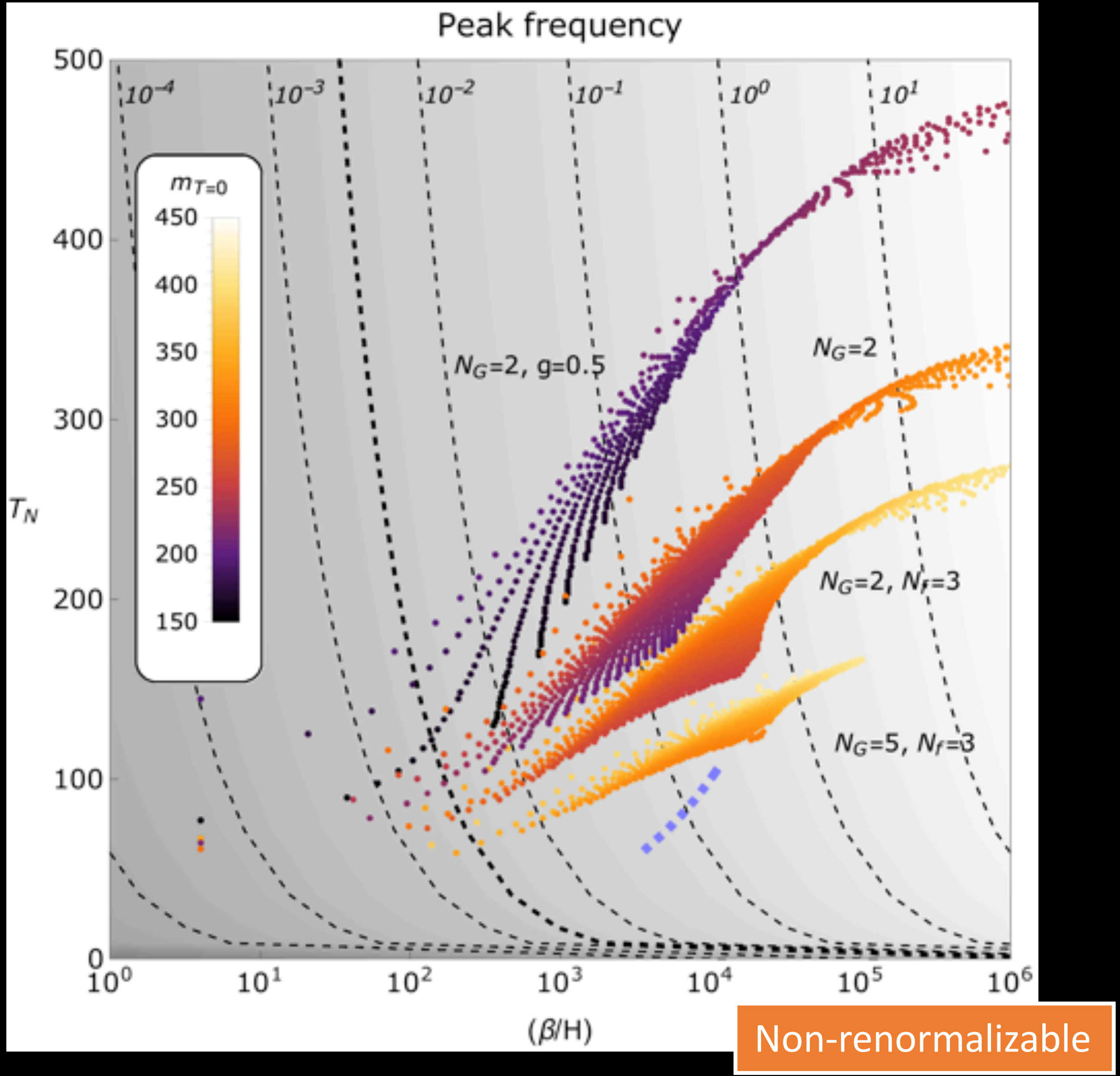
$$b_T = \frac{1}{4}\frac{1}{\Lambda_6^2}$$

Scale of new physics; a singlet, an extra doublet, …

Peak frequency

$$f_{\rm sw} = 8.9 \times 10^{-7}{\rm Hz}\frac{1}{v_w}\left(\frac{\beta}{H}\right)\left(\frac{T_N}{\rm Gev}\right)\left(\frac{g_*}{100}\right)^{1/6}$$





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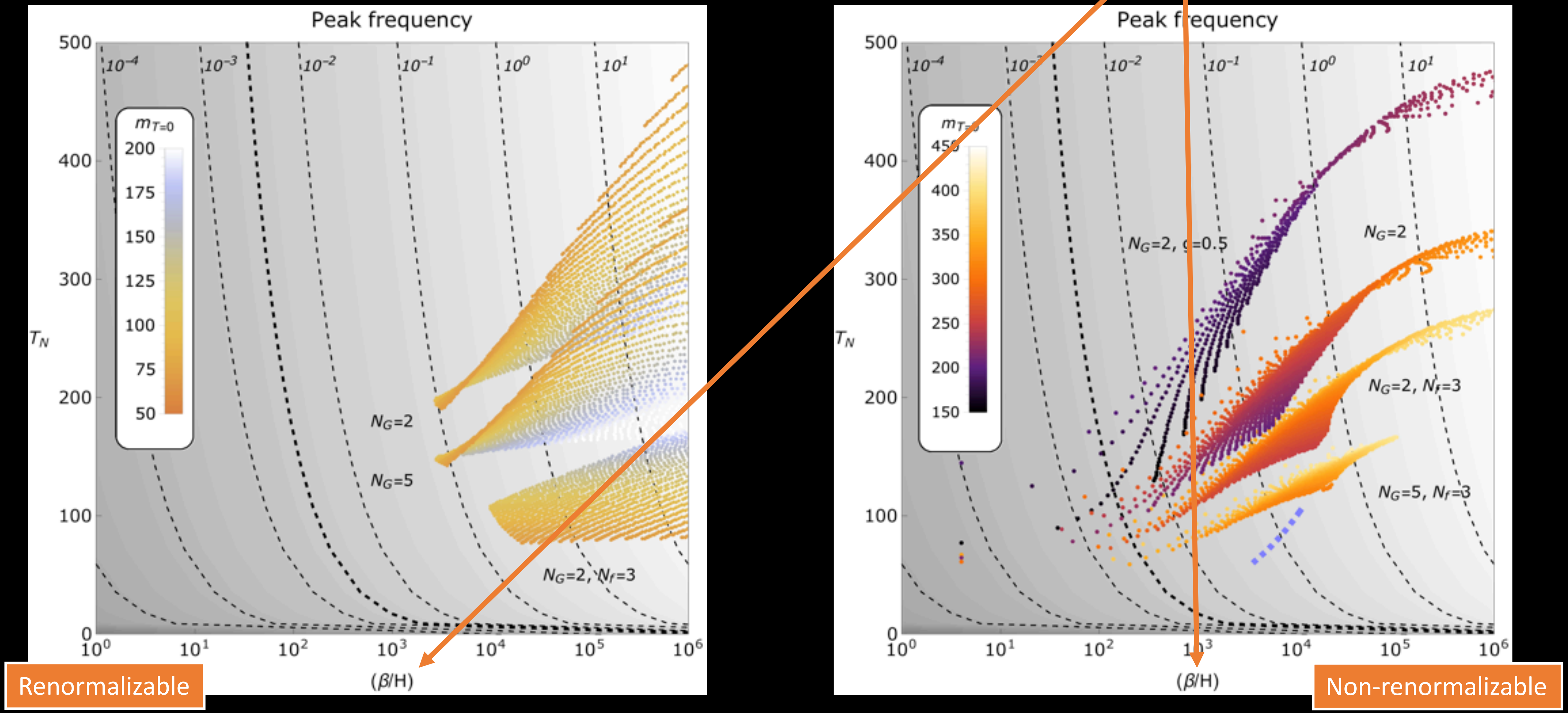

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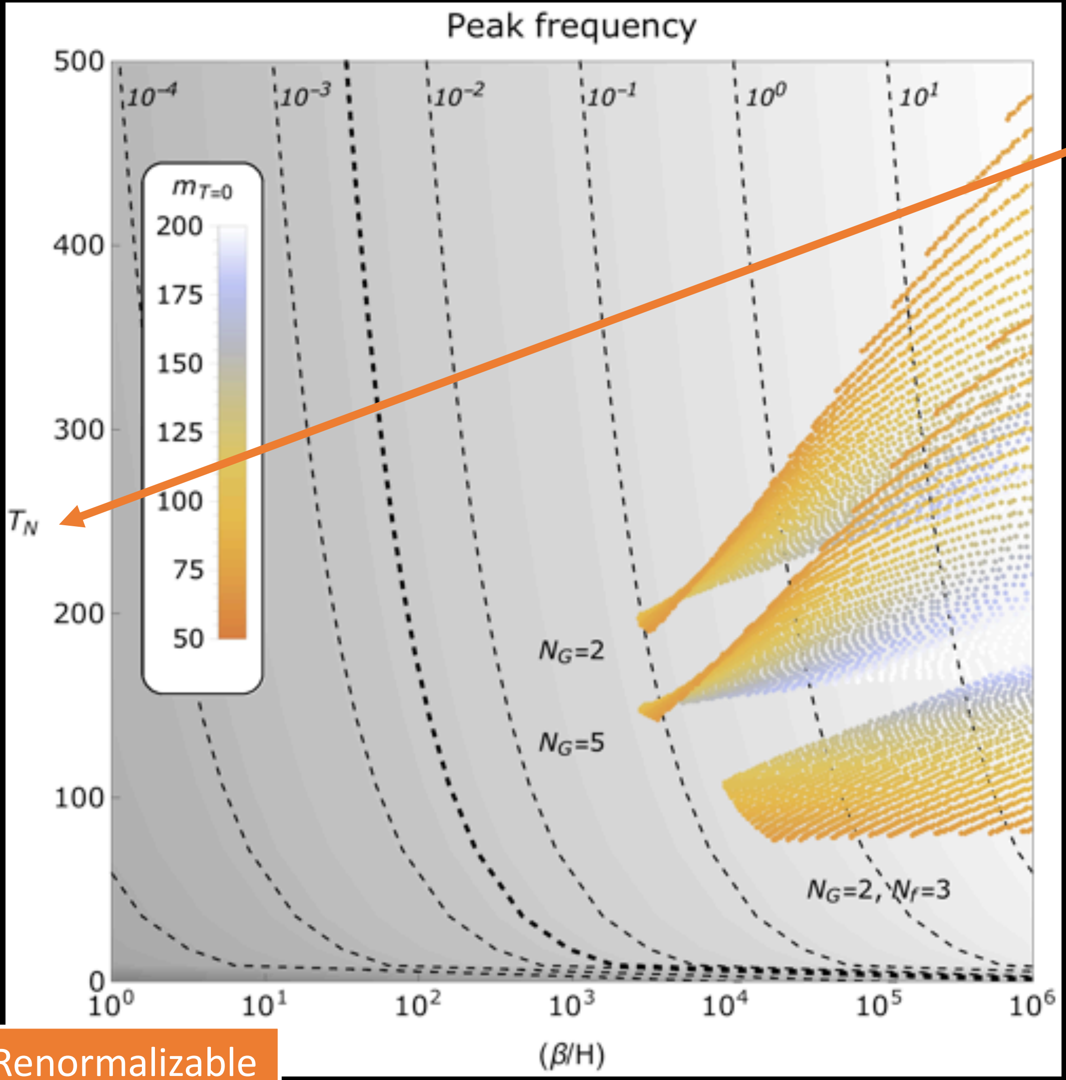
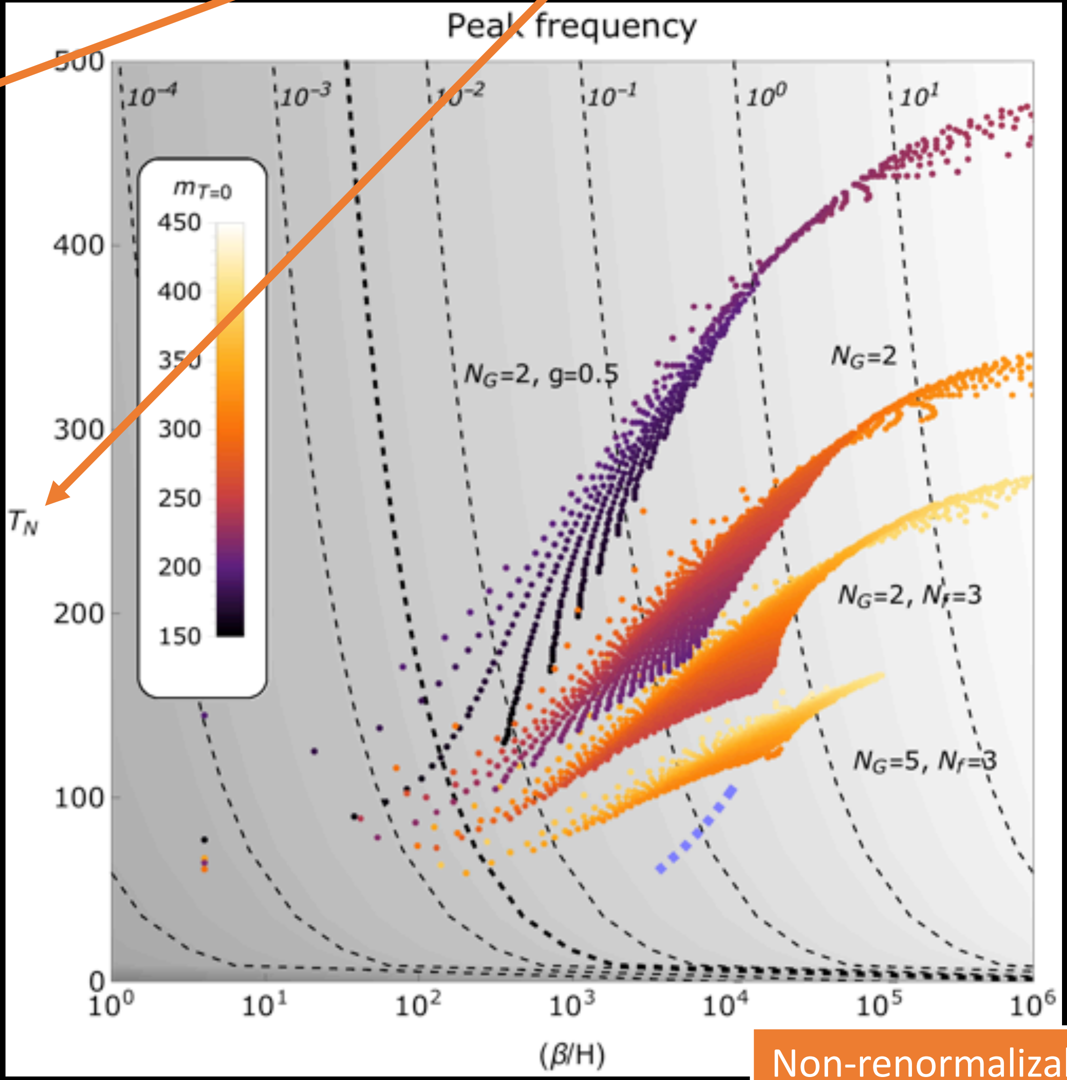


Renormalizable



Non-renormalizable

Peak frequency

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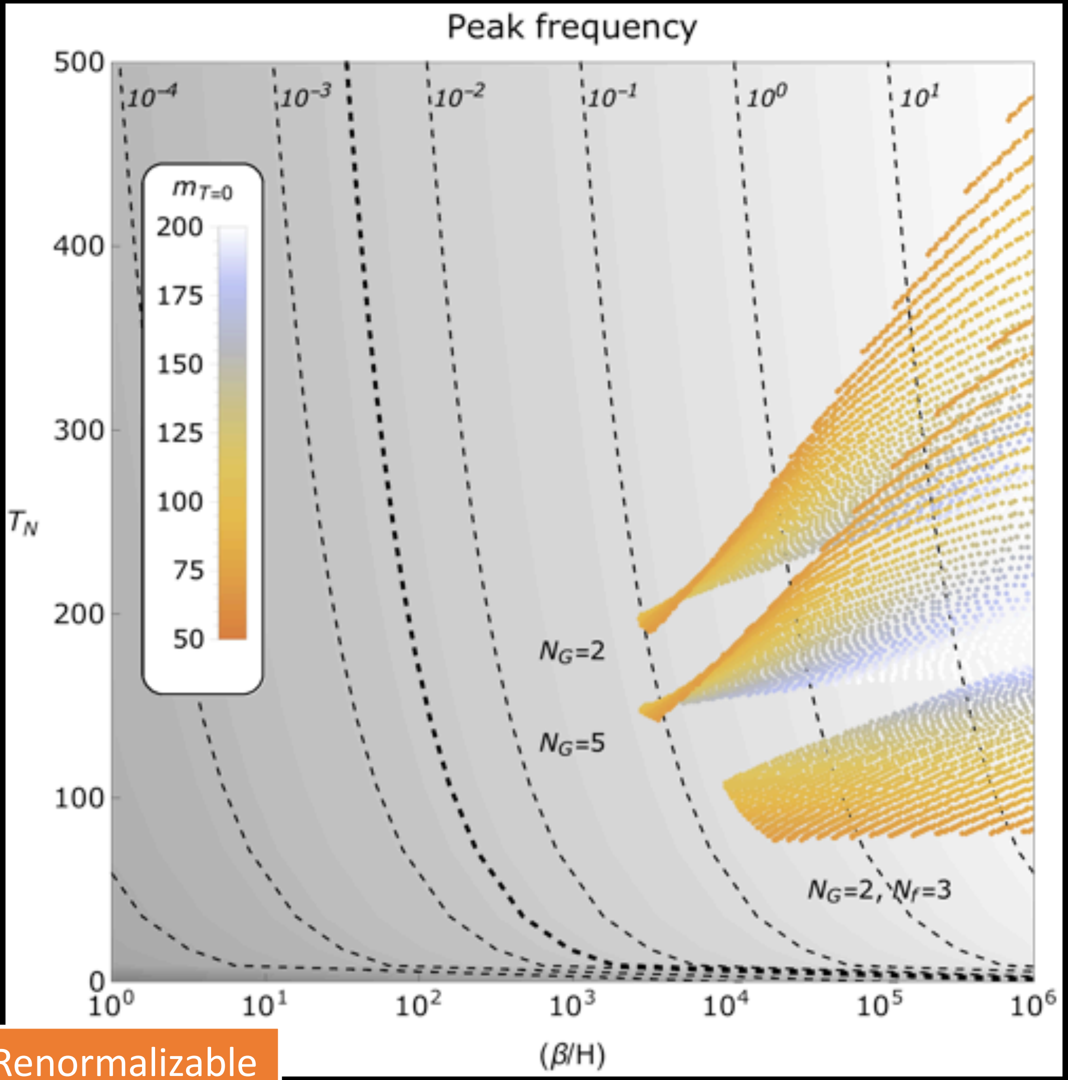
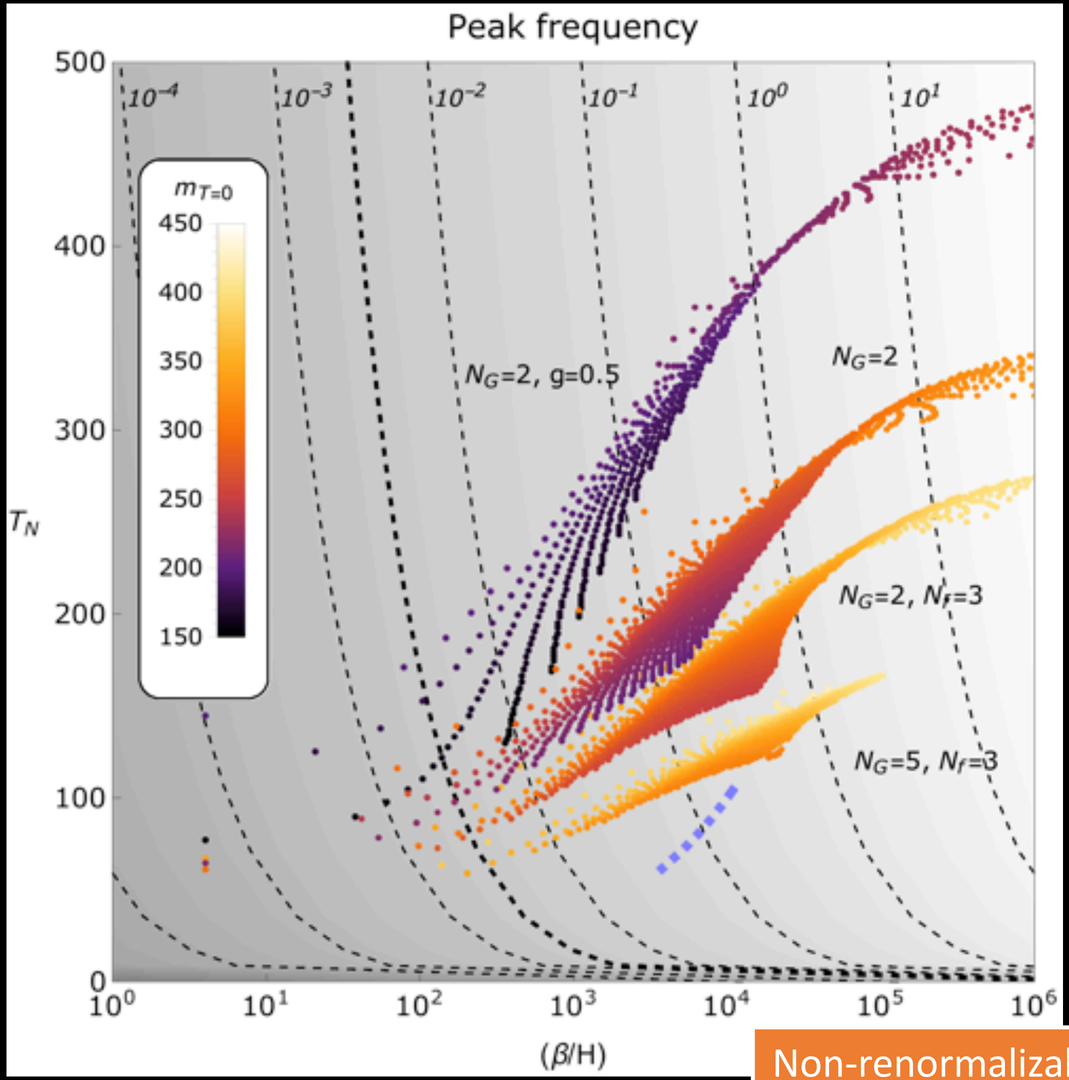


Renormalizable



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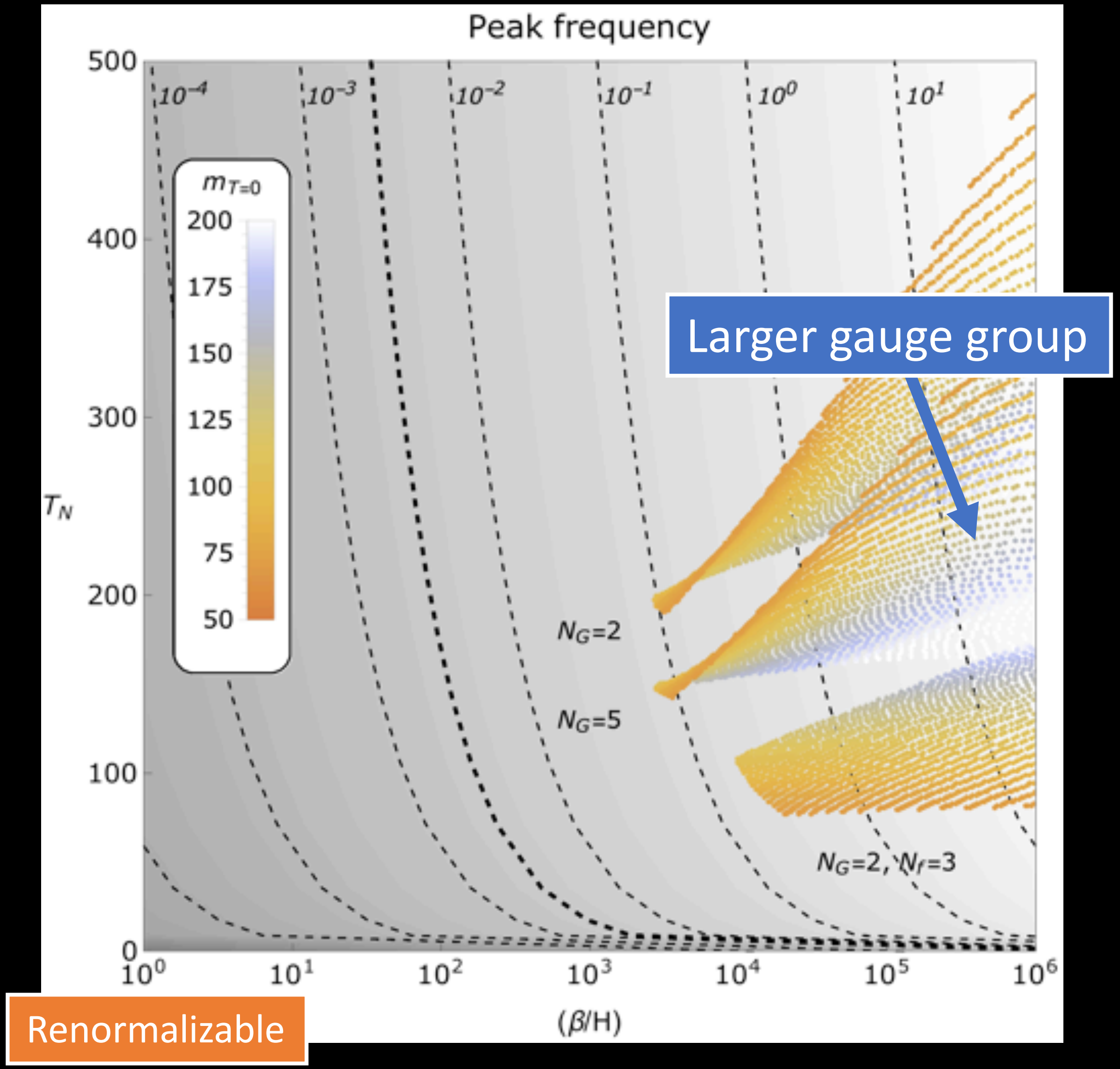

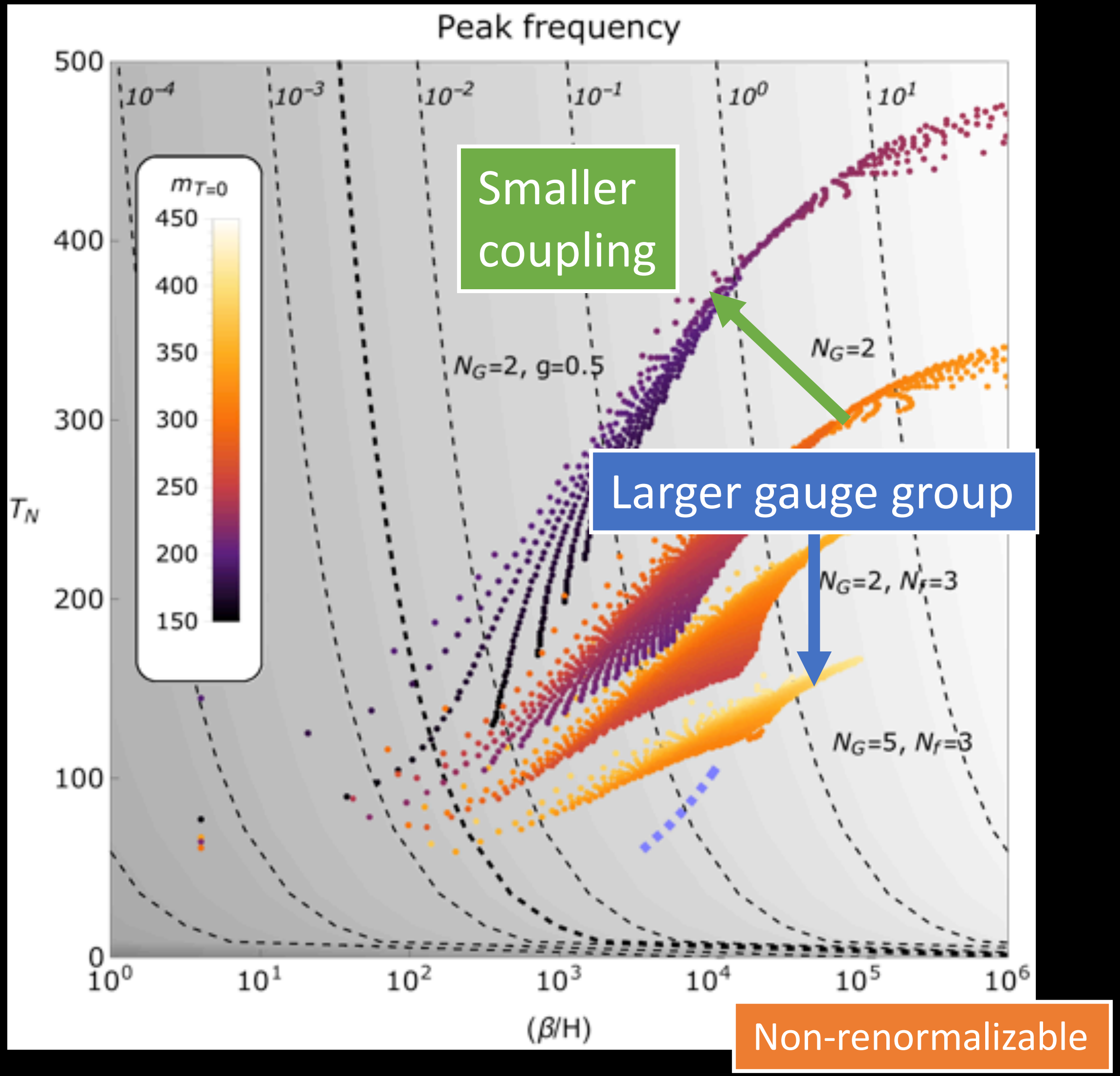

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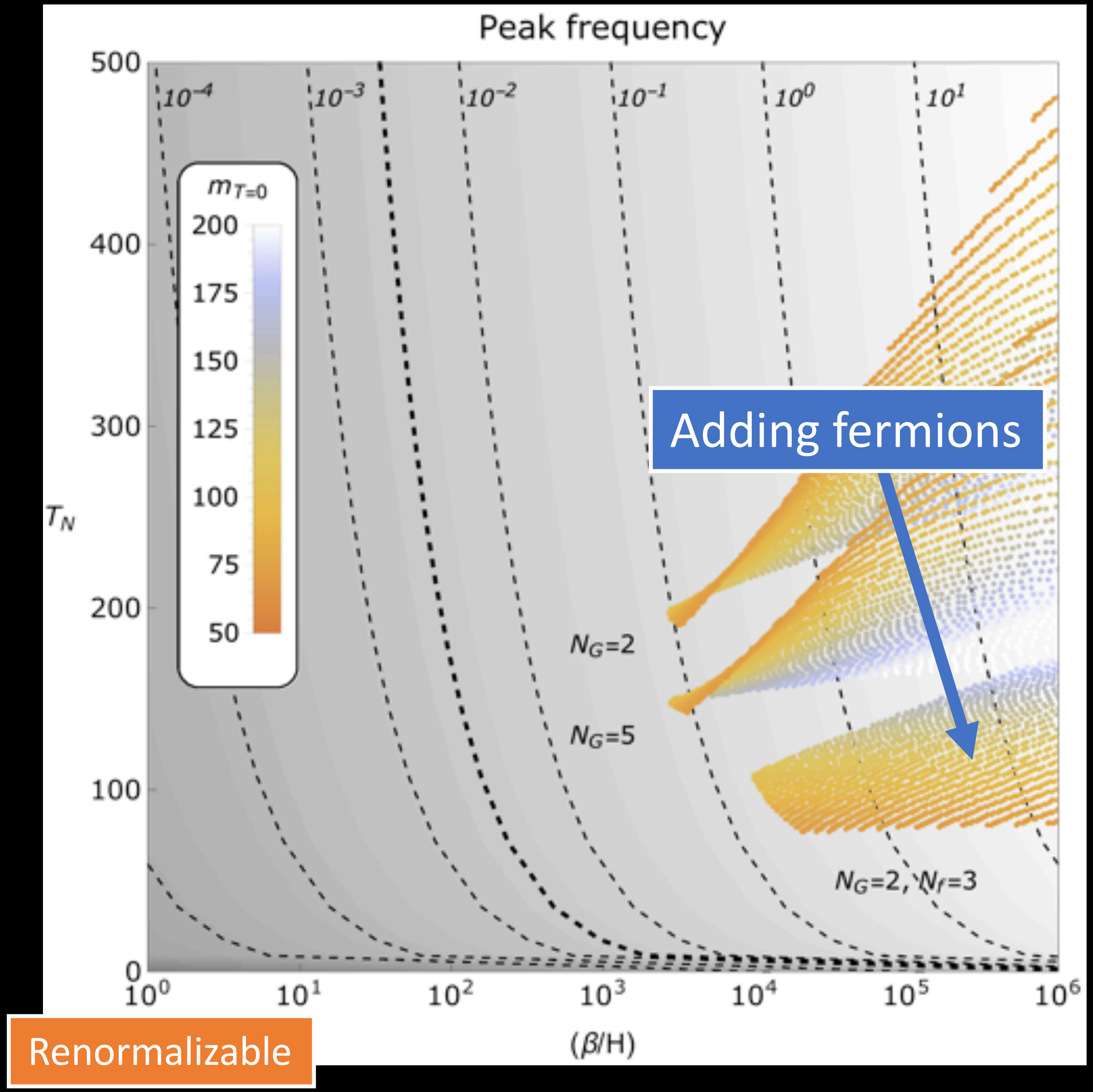
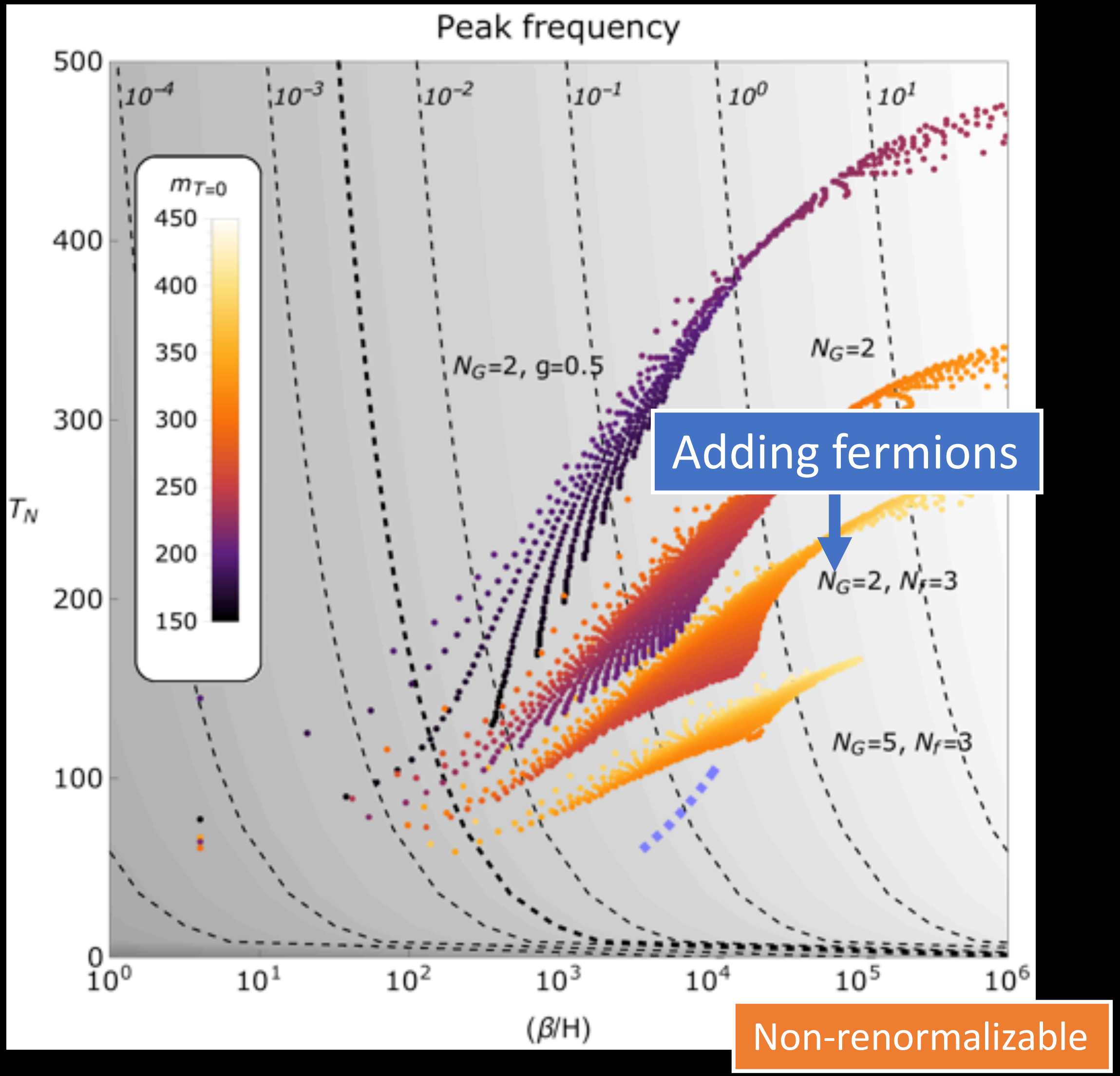
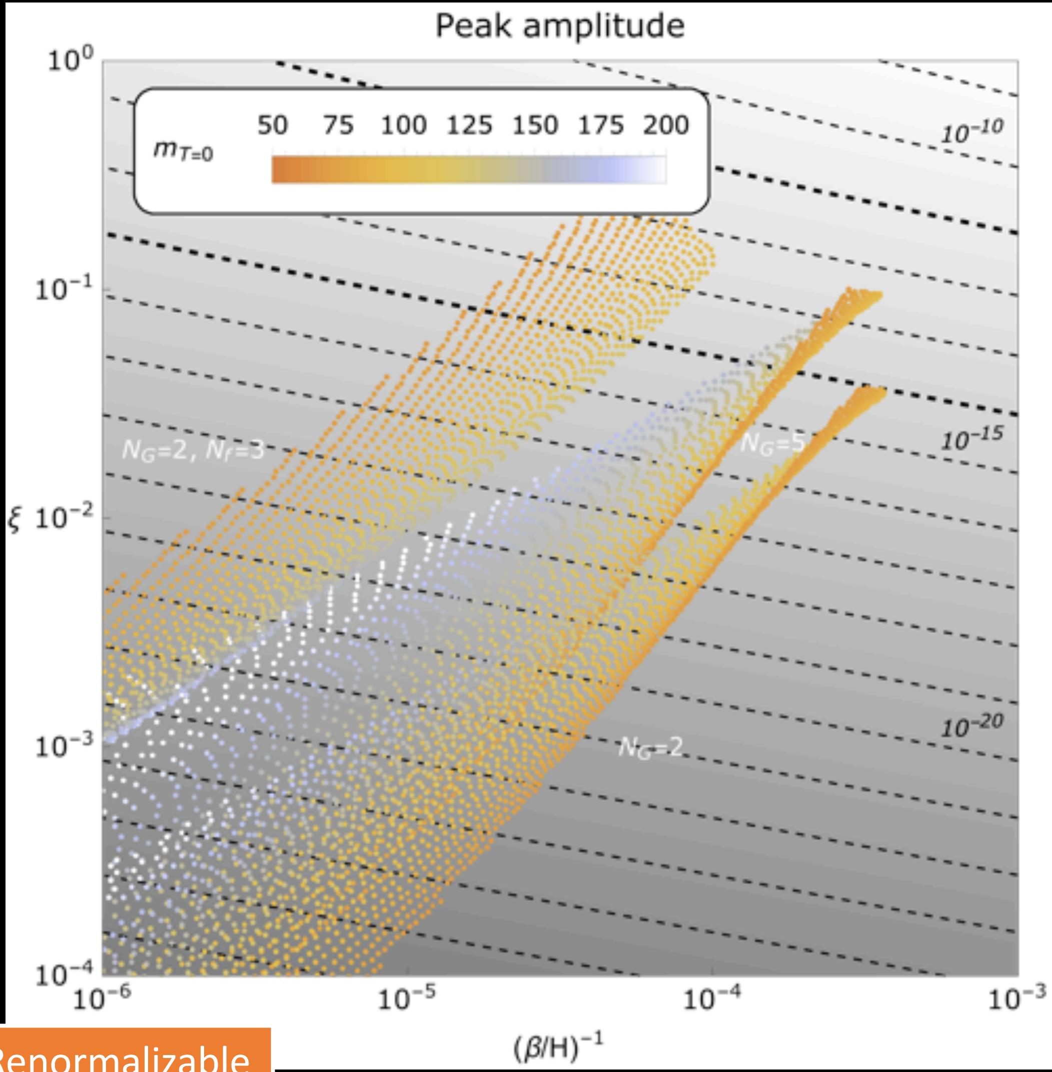




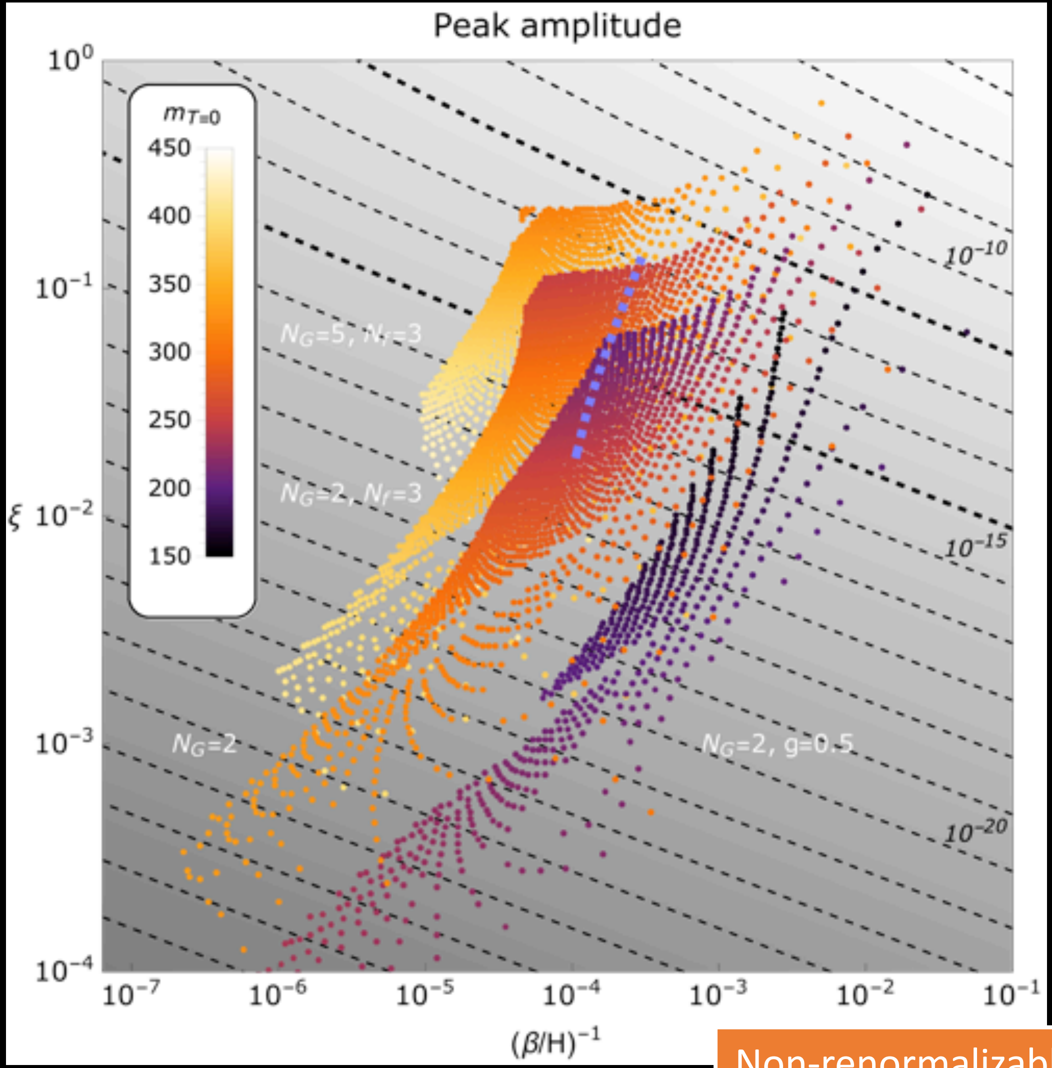
Romano, Thrane; arXiv:1310.5300

Amplitude

$$h^2\Omega_{\rm sw} = 8.5\times10^{-6}\left(\frac{100}{g_*}\right)^{-1/3}\Gamma^2\bar{U}_f^4(\alpha)\left(\frac{\beta}{H}\right)^{-1} v_w S_{\rm sw}(f)$$



Renormalizable



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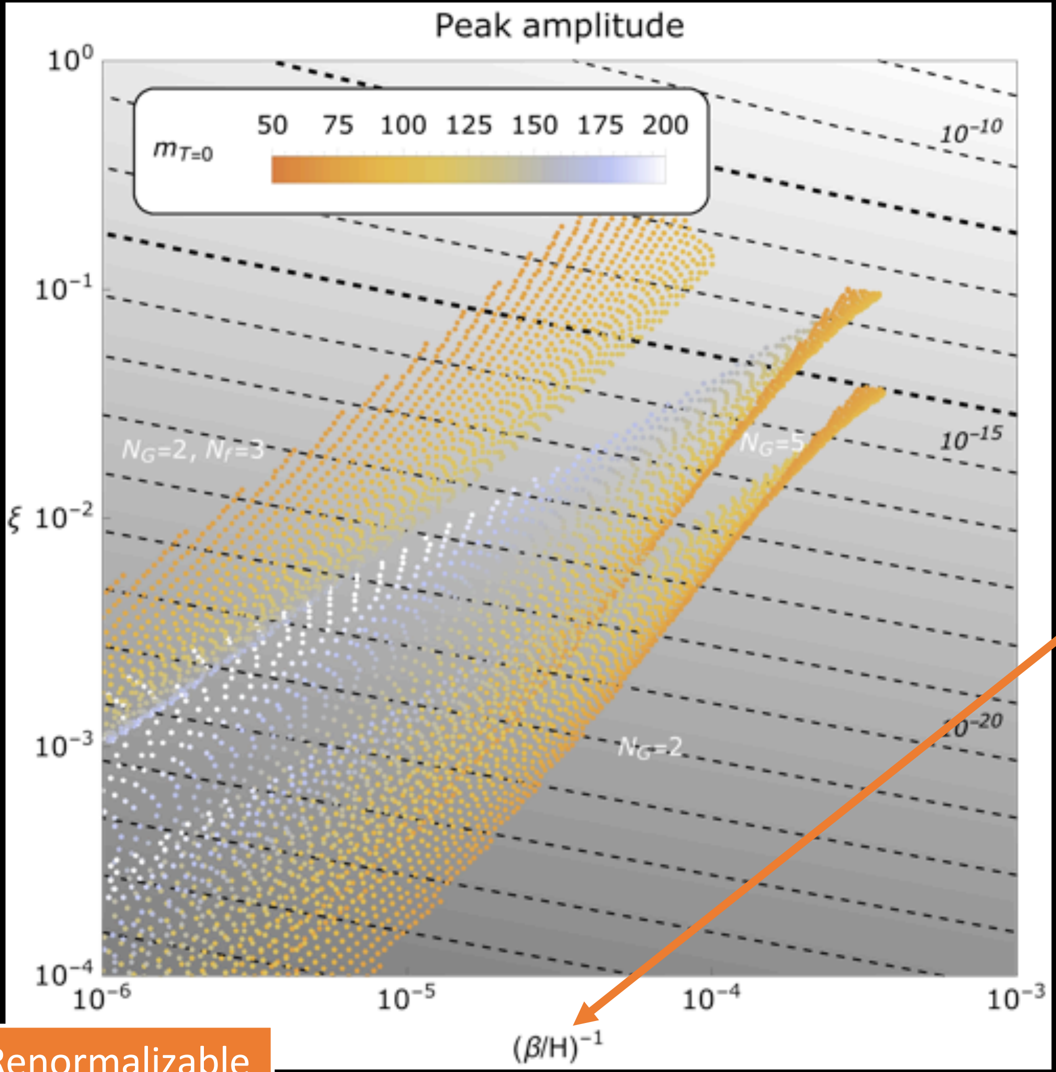
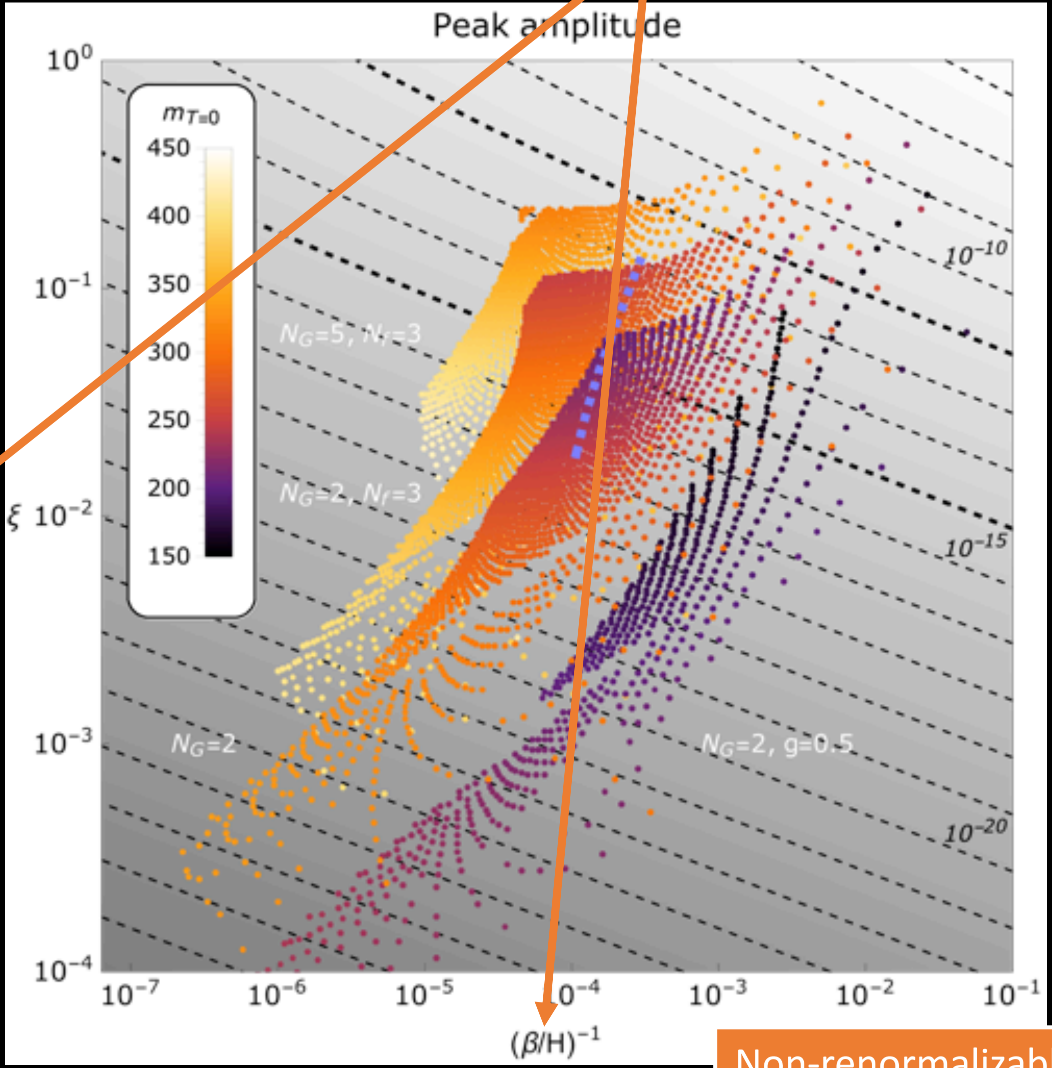


Renormalizable



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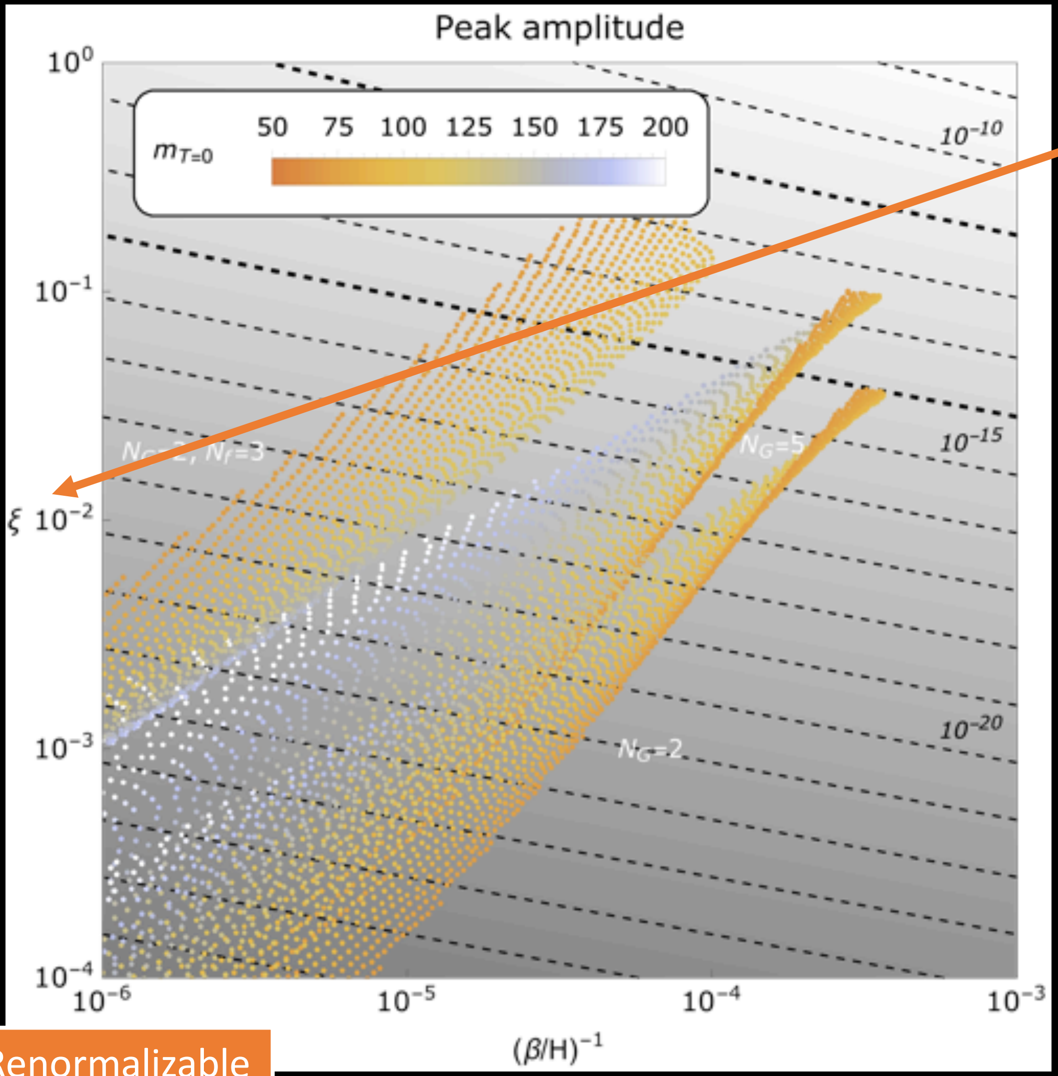

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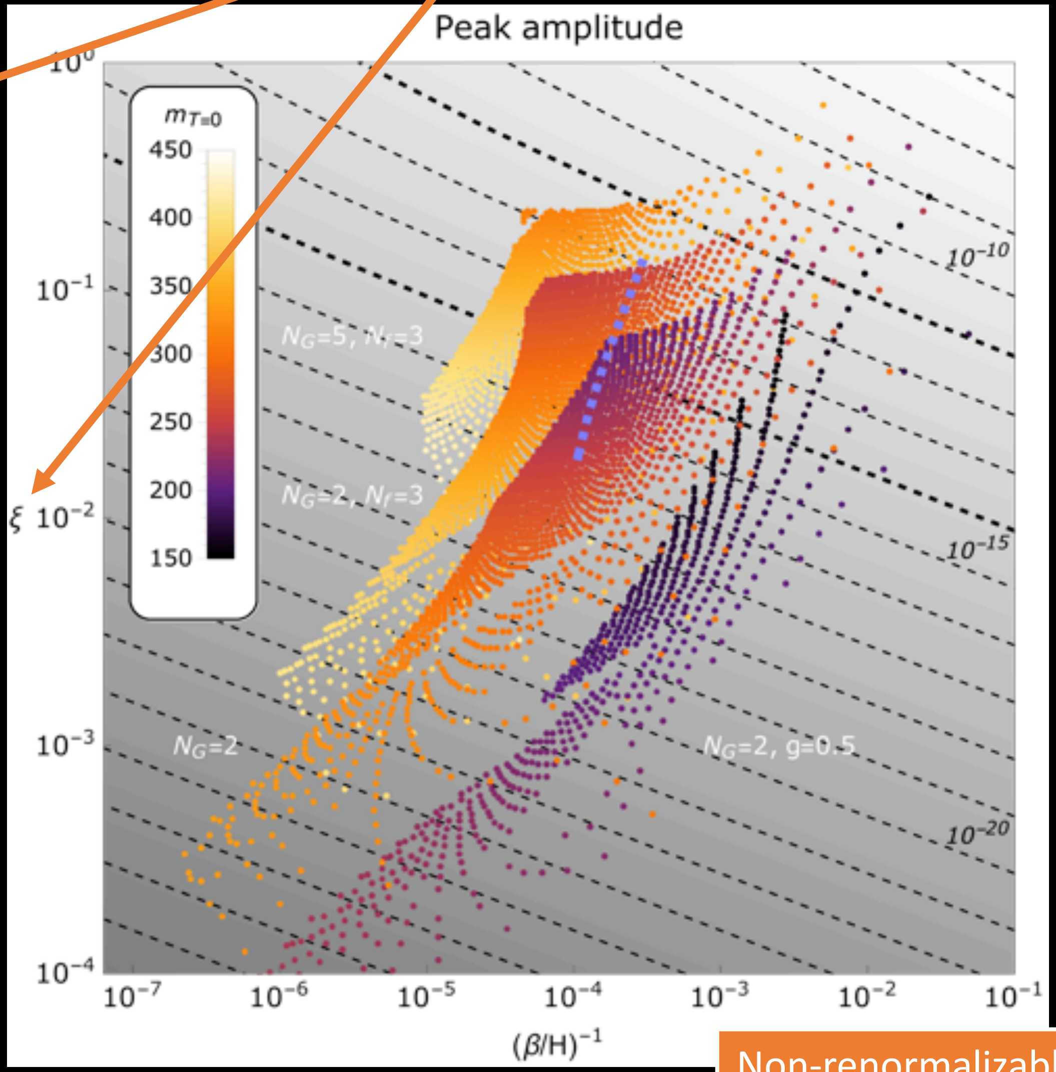

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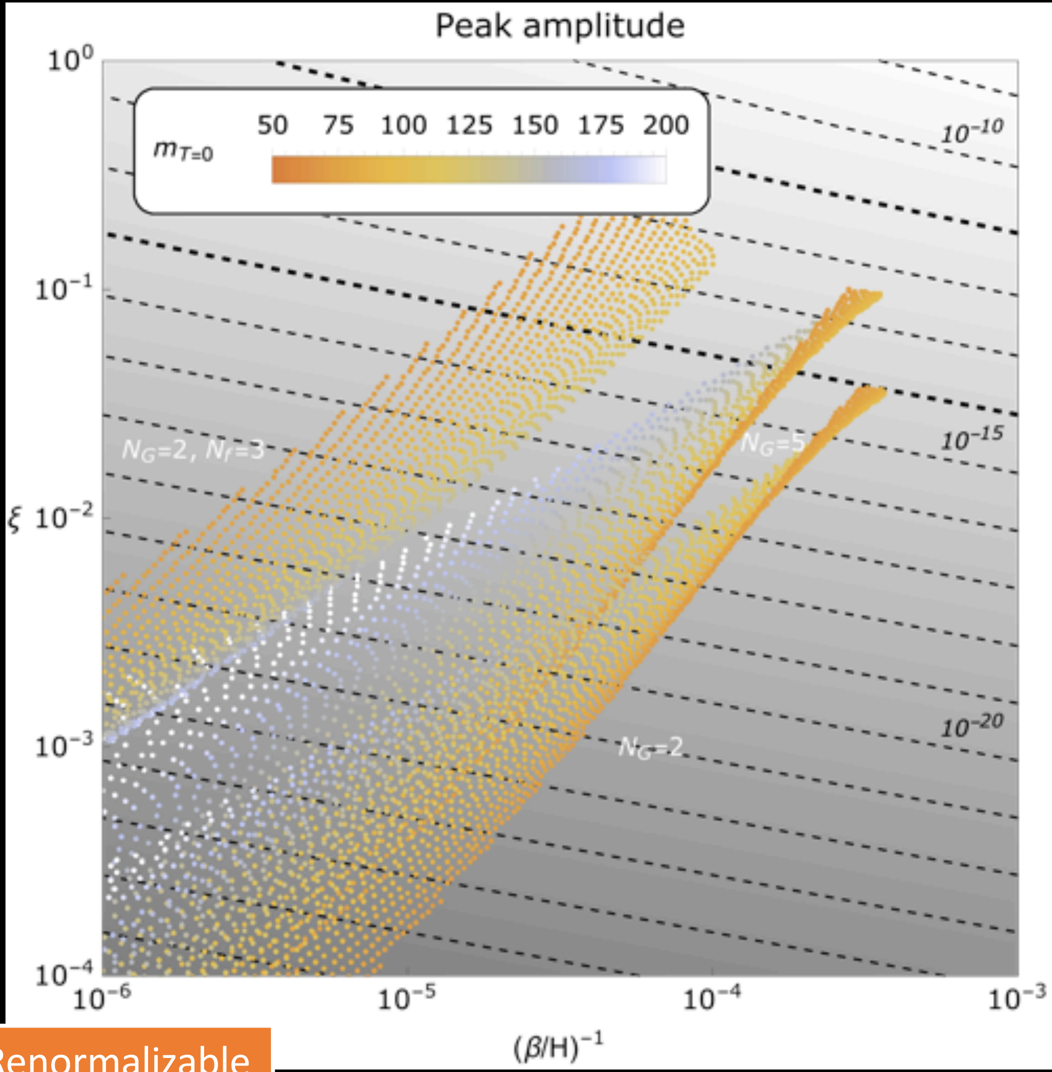
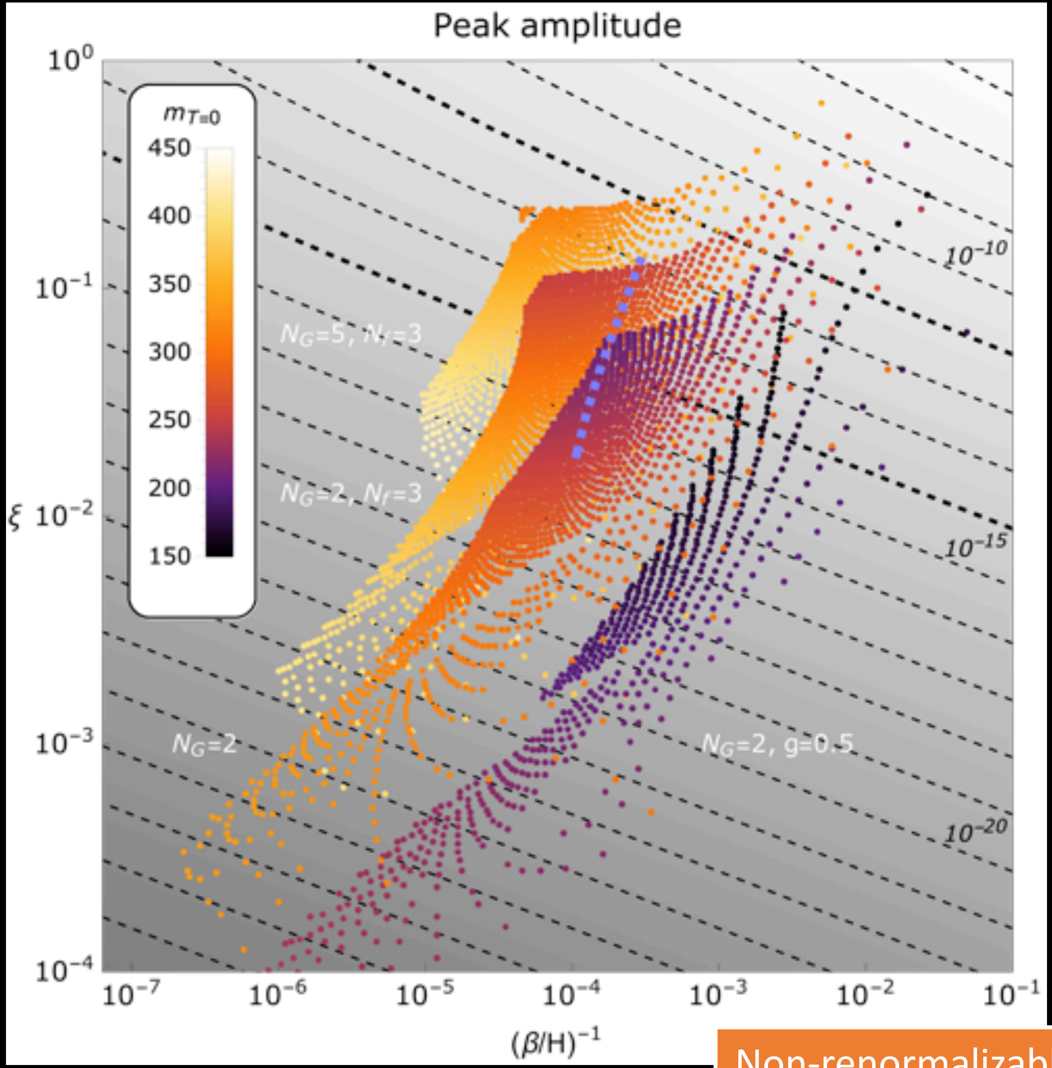


Renormalizable



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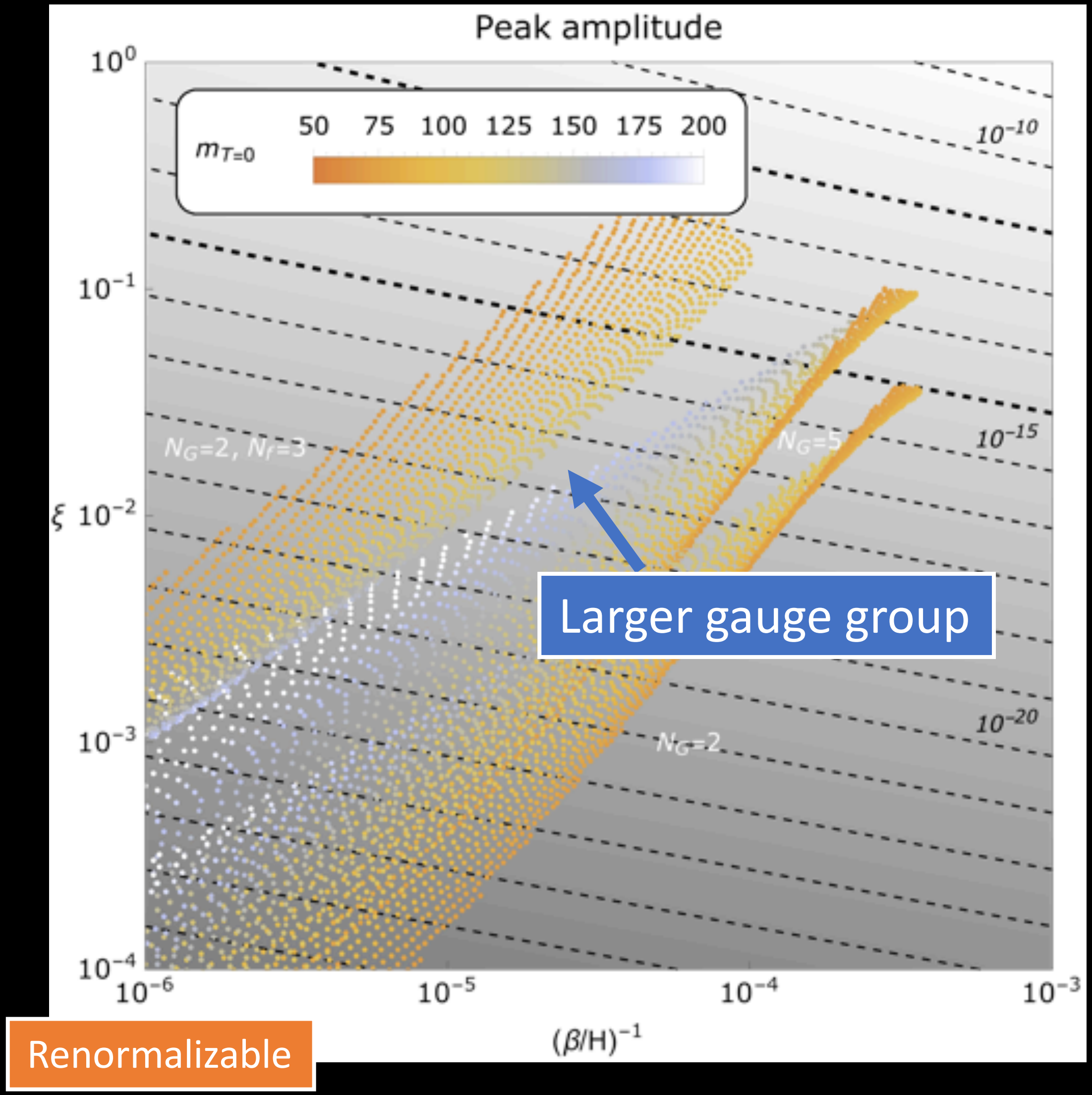

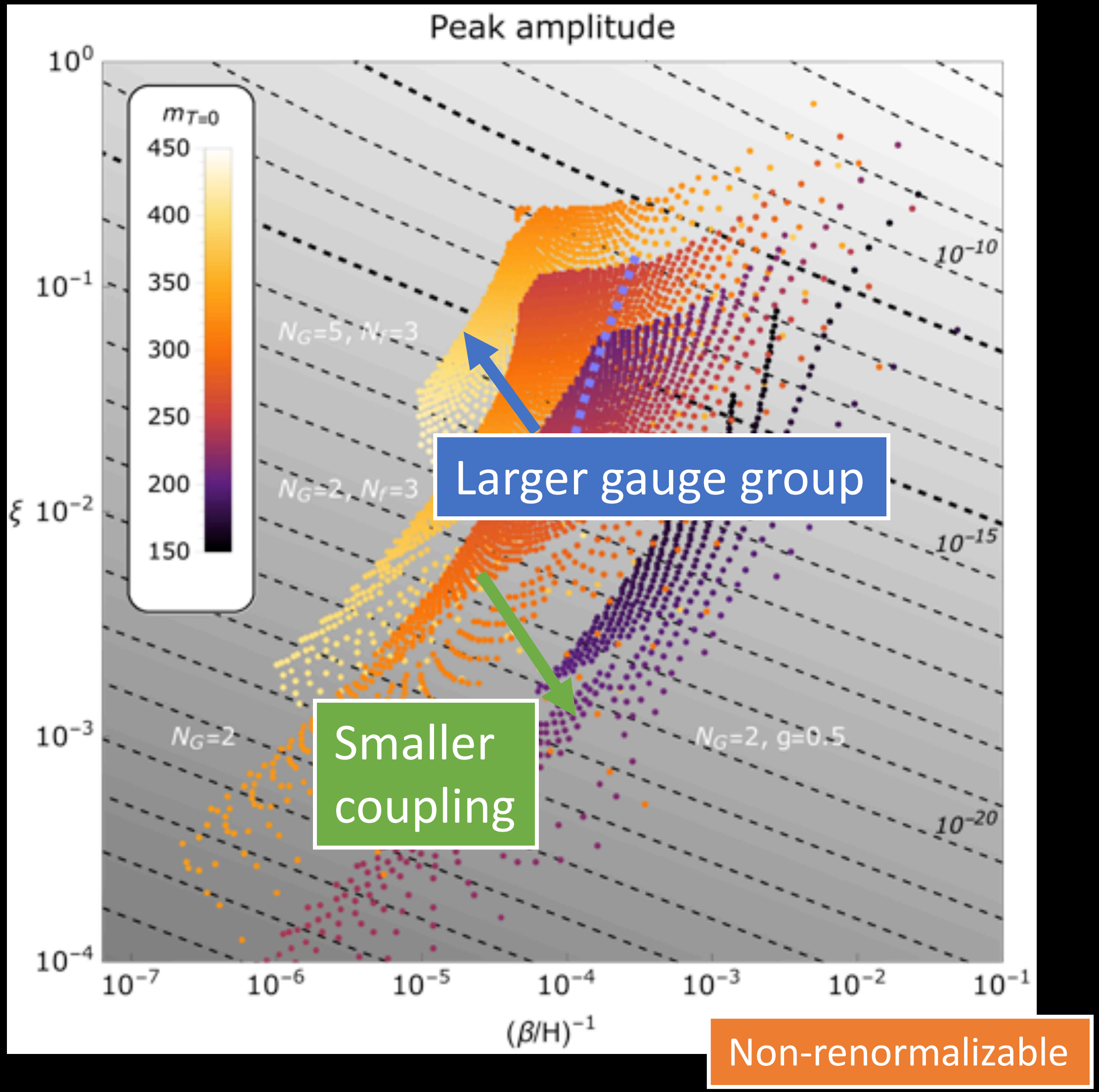

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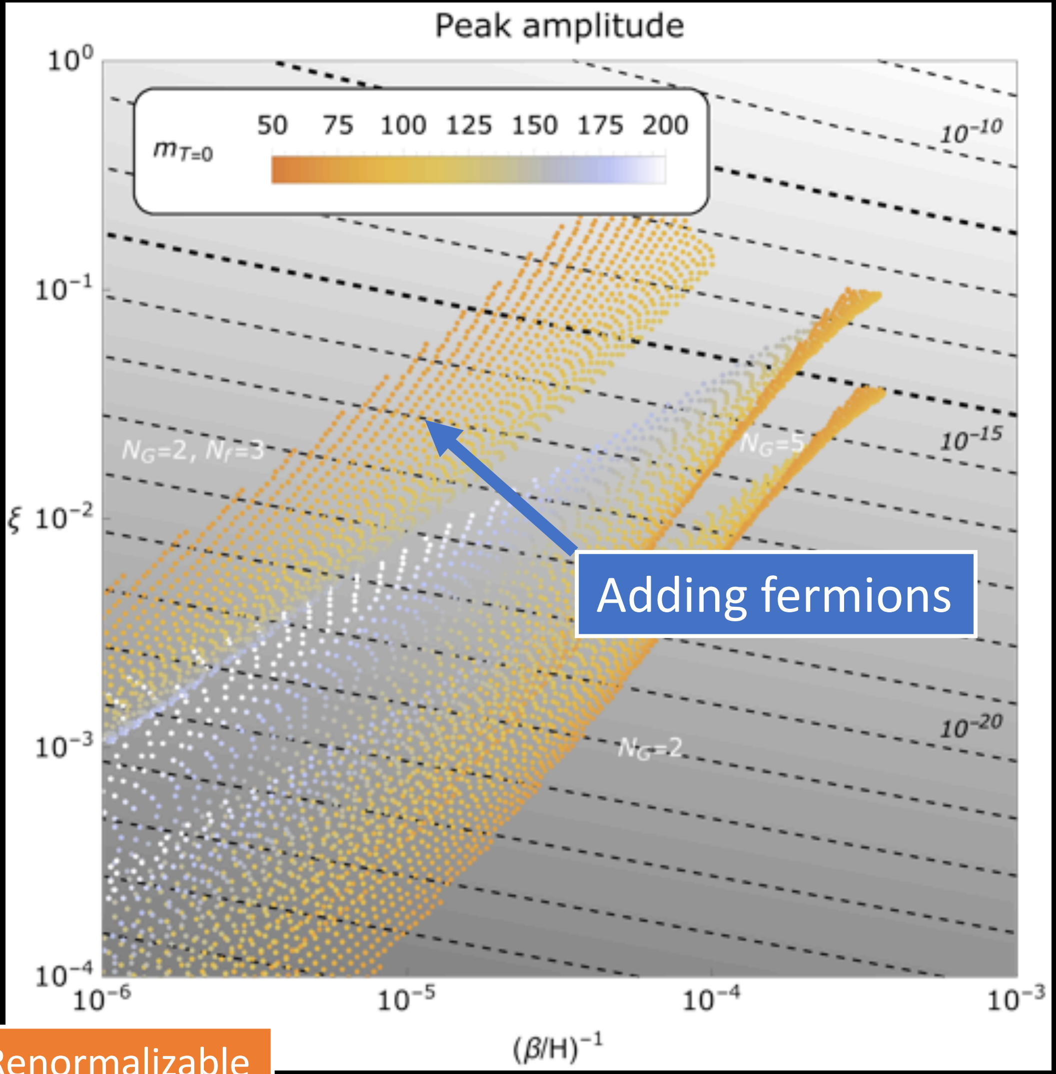
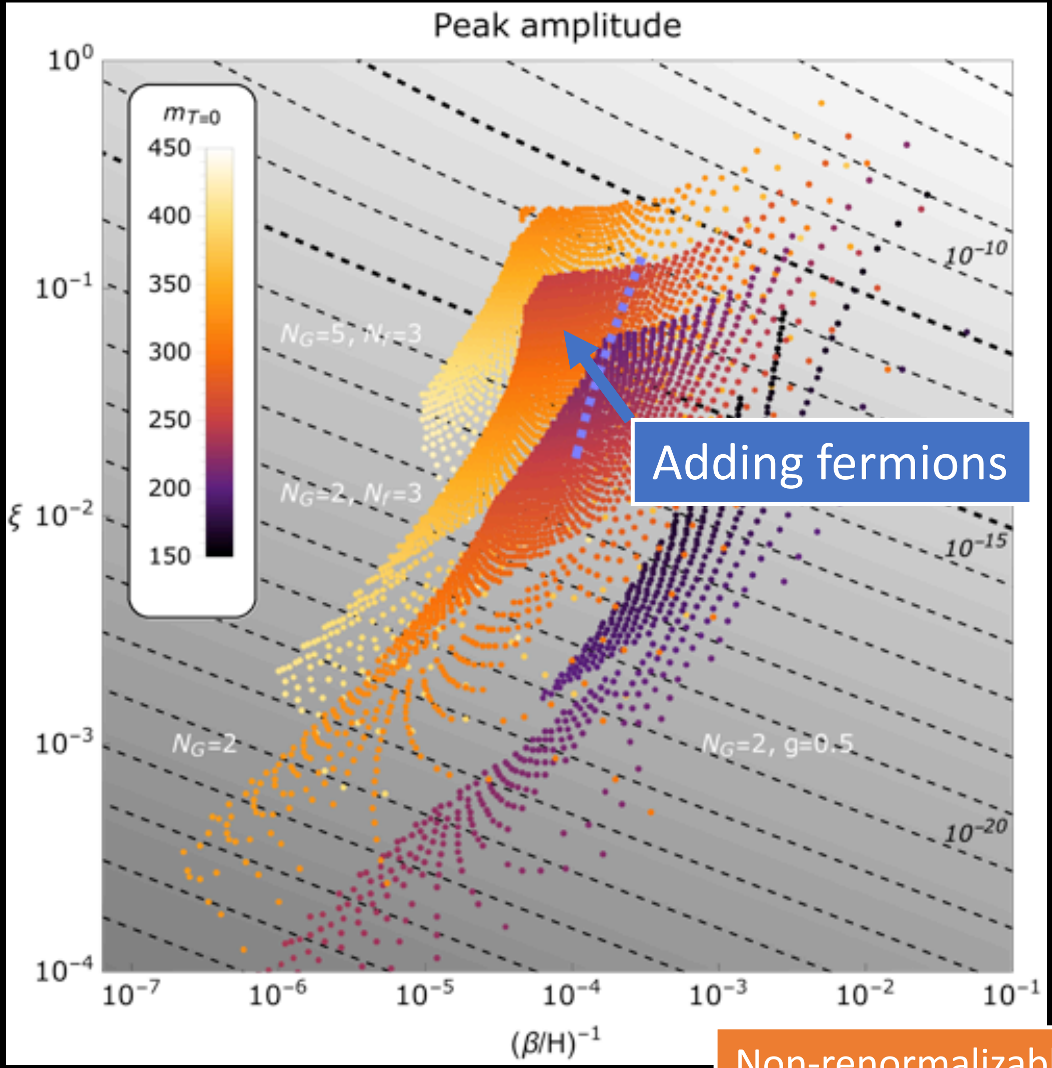


Renormalizable



Non-renormalizable

Some qualitative lessons

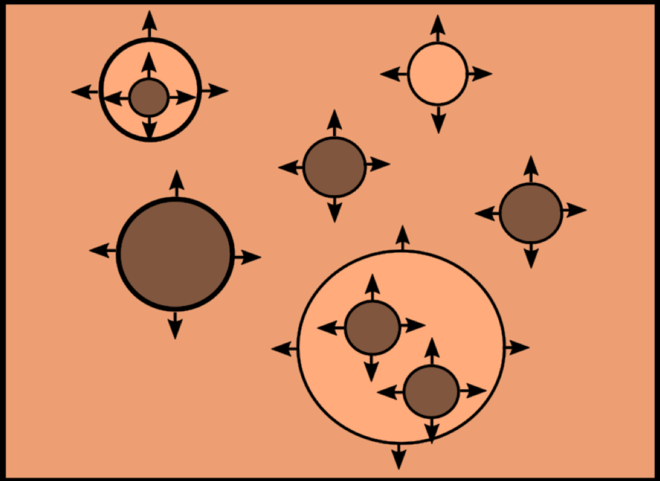
- Most thermal parameters are sensitive to the ratio **v/Λ**

- Extra fermions (N_f x y^2), and a larger gauge group (N_G),
  - Enhance the amplitude of the signal
  - Sensitive to heavier scalars (effective zero temperature mass)

- The effective non-renormalizable potential yields better detection prospects @LISA

DC, G. White [arXiv:1803.05438, JHEP]

Exotic spectra from simultaneous PT



$$\Omega_{GW} = \sum_i w_i \, \Omega_{GW}^{(i)} \left(\Omega_0^{(i)}, f^{(i)}, \Upsilon_i, v_w^{(i)}, \frac{\beta^{(i)}}{H}, T_N^{(i)} \right)$$

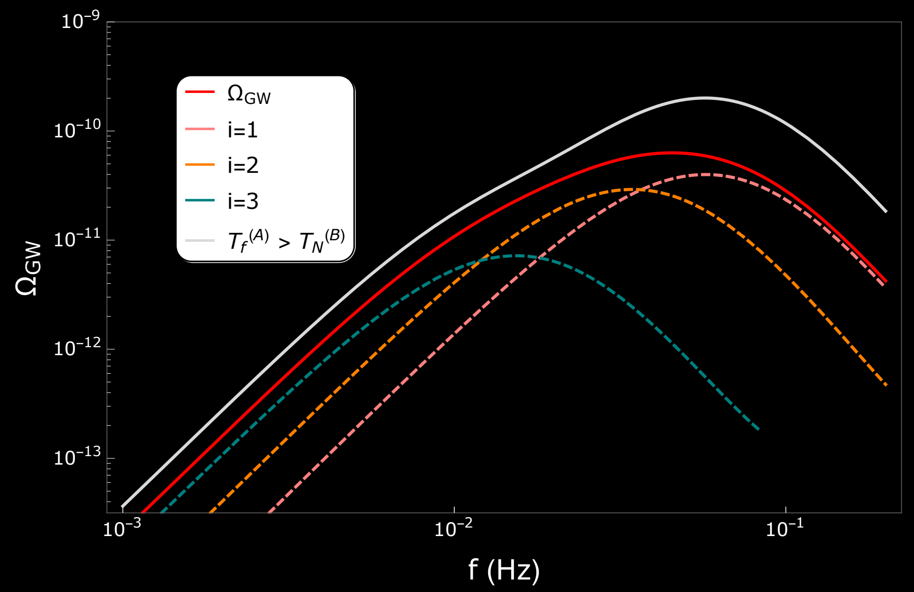
1 $(0,0) \to (v_A, 0)$ "First transition"

2 $(v_A, 0) \to (v_A, v_B)$ "Bubble in a bubble"

3 $(0,0) \to (v_A, v_B)$ "Immediate transition"

Observations:

- First PT does not reach $f_1(T) = 1$
- Transitions 2) and 3) correspond to different thermal parameters



To conclude,

- Gravitational waves are new independent probes of particle physics!

- Degeneracy is introduced in the thermal parameters, and again in the GW spectra
- However, some general qualitative lessons can be learned

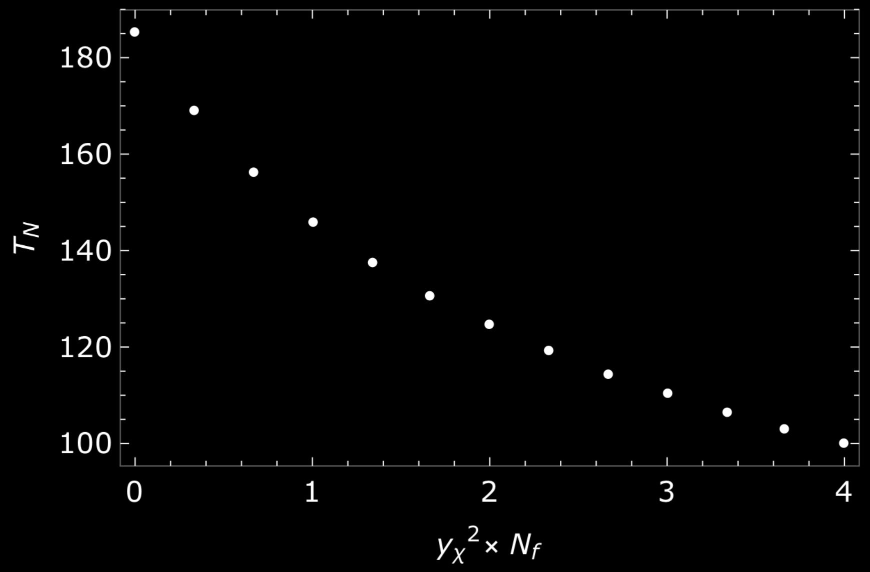
- More exotic spectra may result from simultaneous PT

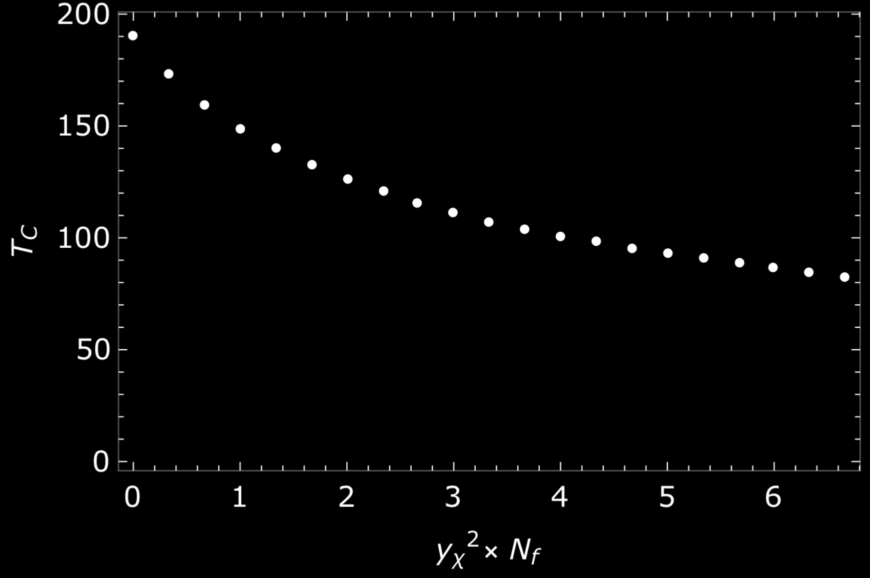
- Things to do:
  - Beyond the high-T approximation
  - Beyond one loop
  - HDOs and more general effective potentials

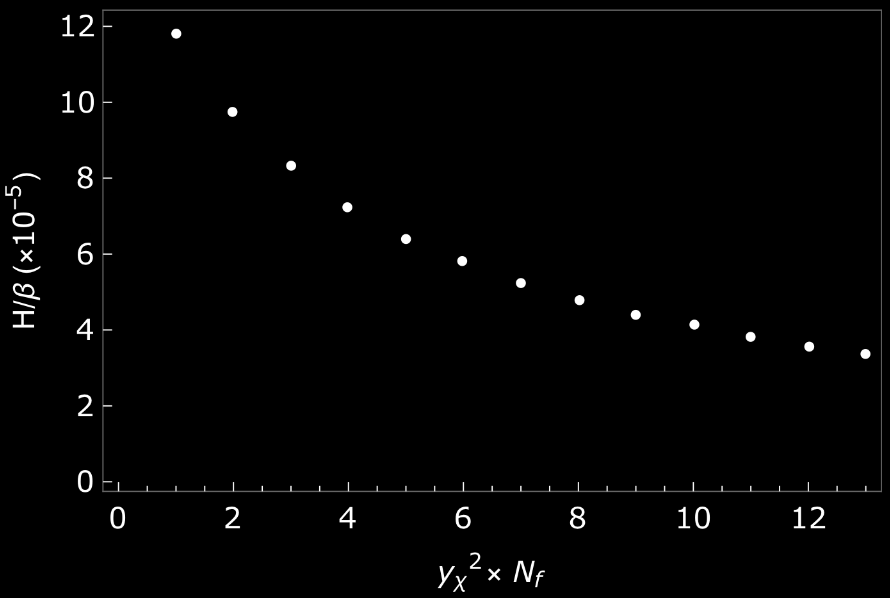
Thank you!

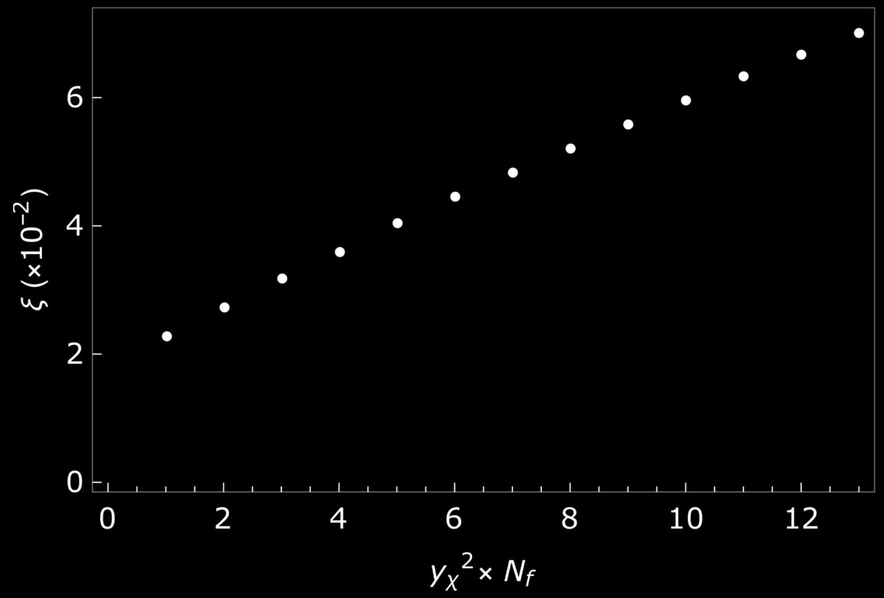
Extra slides

Fermions


T_N
180
160
140
120
100
0
1
2
3
4
$y_\chi^2 \times N_f$


T_C
200
150
100
50
0
0
1
2
3
4
5
6
$y_\chi^2 \times N_f$


H/β ($\times 10^{-5}$)
12
10
8
6
4
2
0
0
2
4
6
8
10
12
$y_\chi^2 \times N_f$


ξ ($\times 10^{-2}$)
6
4
2
0
0
2
4
6
8
10
12
$y_\chi^2 \times N_f$