Chern-Simons gravitational term coupled to a scalar field during inflation

Based on:

- N. Bartolo, G. Orlando, arXiv:1706.04627
- Ongoing project

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Chern-Simons gravity

$$\mathcal{L} = \sqrt{g}\left[\frac{1}{2}M_{Pl}^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial^\mu\chi - V(\chi)\right] + f(\chi)\,\epsilon^{\mu\nu\rho\sigma} R_{\mu\nu}{}^{\kappa\lambda} R_{\rho\sigma\kappa\lambda}\,.$$

Peculiarities of the Chern-Simons term:

- $\epsilon^{\mu\nu\rho\sigma} R_{\mu\nu}{}^{\kappa\lambda} R_{\rho\sigma\kappa\lambda}$ is a surface term, we need the coupling $f(\chi)$.
- P broken, T broken, CPT preserved.

$\longrightarrow$ Standard GR restored if at a certain time χ decades.
$\longrightarrow$ Search remnant signatures of Chern-Simons gravity from inflation.
(χ can be both the inflaton field ϕ or a spectator field in multi-field models)

- Key observable: chirality of primordial GW $\longrightarrow$ $\Theta = \frac{P_\gamma^R - P_\gamma^L}{P_\gamma^R + P_\gamma^L}$.

$$\gamma_R = \frac{1}{\sqrt{2}}(\gamma_+ - i\,\gamma_\times)\,, \qquad \gamma_L = \frac{1}{\sqrt{2}}(\gamma_+ + i\,\gamma_\times)\,.$$

Linear analysis

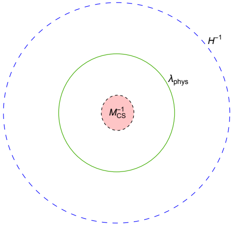
- $\tau \equiv$ conformal time

Quadratic action of GW

$$S|_{\gamma\gamma} = \sum_{s=L,R} \int d\tau \, \frac{d^3k}{(2\pi)^3} \, A^2_{\gamma,s} \left[\, |\gamma_s'(\tau,k)|^2 - k^2 |\gamma_s(\tau,k)|^2 \, \right] \, ,$$

$$A^2_{\gamma,s} = \frac{M_{Pl}^2}{2} a^2 \left(1 - \lambda_s \frac{k_{phys}}{M_{CS}} \right) \, , \qquad M_{CS} = M_{Pl}^2 / 8\dot{f}(\chi) \, .$$

- M_{CS}: Chern-Simons mass, characteristic energy scale.
- $\lambda_R = +1$, $\lambda_L = -1 \Longrightarrow$ When $k_{phys} > M_{CS}$, γ_R shows instabilities.



- We consider physical scales that at the beginning of inflation were inside Hubble radius, but outside the Chern-Simons scale.

$\longrightarrow H/M_{CS} \ll 1$.

Equations of motion for GW

- Field redefinition: $\mu_s = A_{\gamma,s}\gamma_s$.

$$\mu''_{R/L} + k^2 \left(1 - \frac{2}{k^2\tau^2} \pm \mathcal{V}\right) \mu_{R/L} = 0 \,.$$

$$\mathcal{V} = \frac{1}{k\tau}\frac{H}{M_{CS}}\mathcal{A}\,,$$

$$\mathcal{A} = \frac{1}{(1 \mp k_p/M_{CS})^2}\left\{\left[1 - \xi + \frac{1}{2}\omega - \frac{1}{2H}\frac{1}{\tau}\xi\right]\left(1 \mp \frac{k_p}{M_{CS}}\right) + \mp \frac{k_p}{2M_{CS}}\left[\frac{1}{2} + \xi + \frac{1}{2}\xi^2\right]\right\}.$$

$$k_p = k/a\,, \qquad \xi = \frac{\dot{M}_{CS}}{M_{CS}H}\,, \qquad \omega = \frac{\ddot{M}_{CS}}{M_{CS}H^2}\,.$$

- $H/M_{CS} \ll 1$ (previous slide) $\longrightarrow$ $\mathcal{V}$ vanishes *vs* standard gravity.

Example: M_{CS} = constant case

- E.o.m.'s become

$$\mu''_{R/L} + k^2 \left(1 - \frac{2}{k^2\tau^2} \pm \frac{1}{k\tau}\frac{H}{M_{CS}}\right)\mu_{R/L} = 0\ .$$

- It is Whittaker equation, we can find analytic solutions.

Super-horizon power-spectra (M. Satoh, 2010)

$$P^L_\gamma = \frac{P_\gamma}{2} e^{-\frac{\pi}{4}H/M_{CS}}\ ,$$
$$P^R_\gamma = \frac{P_\gamma}{2} e^{+\frac{\pi}{4}H/M_{CS}}\ .$$

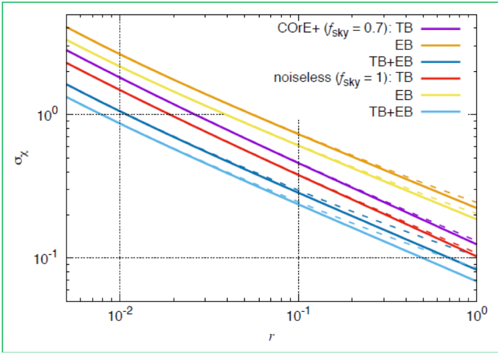
$$\Theta = \frac{P^R_\gamma - P^L_\gamma}{P^R_\gamma + P^L_\gamma} = \frac{\pi}{2}\frac{H}{M_{CS}}\ .$$

- $H/M_{CS} \ll 1 \longrightarrow \Theta \ll 1$.

$P_\gamma = \frac{4}{k^3}\frac{H^2}{M_{Pl}^2}$

Forecasts from CMB data

- *TB* and *EB* correlators are sensitive to GW chirality.



M. Gerbino et. al. (2016)

- Impossible to observe small chirality given the current experimental constraint on $r = P_T/P_S$, $r < 0.07\,(95\%CL) \longrightarrow \sigma_\Theta \geq 0.3$.

Higher order statistics

- At linear level we get no measurable imprints.
- Need for a non-linear analysis.
- In JCAP07(2017)034 *"Parity breaking signatures from a Chern-Simons coupling during inflation: the case of non-Gaussian gravitational waves"* N. Bartolo, G. Orlando, we treated the bispectrum statistics of the model.

In-In formalism

$$\langle \delta_a(\vec{k}_1)\delta_b(\vec{k}_2)\delta_c(\vec{k}_3)\rangle(t) = -i\int_{t_0}^{t} dt' \langle 0|\left[\delta_a^I(\vec{k}_1,t)\delta_b^I(\vec{k}_2,t)\delta_c^I(\vec{k}_3,t)\ ,\ H_{int}^I(t')\right]|0\rangle\ .$$

- This formalism is based on quantum interaction picture. It allows to compute/estimate non-Gaussian cosmological correlators starting from free fields (i.e. using linear solutions).

Main results

- Amplitudes of $\langle\gamma\gamma\gamma\rangle$ and $\langle\gamma\delta\chi\delta\chi\rangle$ are suppressed by H/M_{CS}.
- Amplitude of $\langle\gamma\gamma\delta\chi\rangle$ bispectrum shows a different behaviour:

$$\langle\gamma_R(\vec{k}_1)\gamma_R(\vec{k}_2)\delta\chi(\vec{k}_3)\rangle = (2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\frac{\pi}{64}\left(\sum_{i\neq j}\Delta_T(k_i)\Delta_T(k_j)\right)\times$$
$$\times\frac{\dot{\chi}}{H}\left(H^2\frac{\partial^2 f(\chi)}{\partial^2\chi}\right)\frac{(k_1+k_2)k_1k_2}{\sum_i k_i^3}\cos\theta(1-\cos\theta)^2\,,$$

$$\langle\gamma_L(\vec{k}_1)\gamma_L(\vec{k}_2)\delta\chi(\vec{k}_3)\rangle = -\langle\gamma_R(\vec{k}_1)\gamma_R(\vec{k}_2)\delta\chi(\vec{k}_3)\rangle\,,\qquad \langle\gamma_L(\vec{k}_1)\gamma_R(\vec{k}_2)\delta\chi(\vec{k}_3)\rangle = 0\,.$$

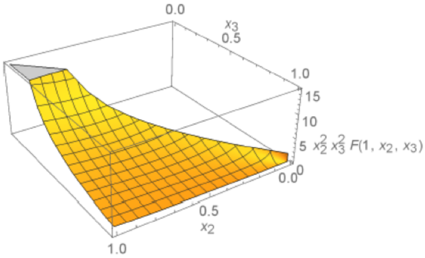
- The amplitude of this bispectrum is proportional to the second order derivative of $f(\chi)$ $\longrightarrow$ No suppression.

Squeezed shape function

- The shape function gives the dependence of the bispectrum on the three momenta k_i.

$$F(k_1^{\gamma}, k_2^{\gamma}, k_3^{\delta\chi}) = \left(\sum_{i\neq j} \frac{1}{k_i^3 k_j^3}\right) \frac{(k_1+k_2)k_1 k_2}{\sum_i k_i^3} \cos\theta(1-\cos\theta)^2,$$

$\cos\theta = \frac{k_3^2 - k_2^2 - k_1^2}{2k_1 k_2}$.



$$x_2 = k_2/k_1\,, \quad x_3 = k_3/k_1\,.$$

- The shape function is maximum in the squeezed limit when the momentum of $\delta\chi$ is much smaller than the momenta of the two γ's ($k_3 \ll k_1 \simeq k_2$).

Squeezed modulation (1)

- The squeezed limit of a generic 3-points function gives the correlation of 2 short modes with 1 long mode. This induces a modulation on the short modes.

Squeezed modulation of the tensor power spectra

$$P_\gamma^{R/L}(q)|_{\delta\chi} = P_\gamma^{R/L}(q)|_{(0)} + \delta\chi(Q)\frac{\langle\gamma_{R/L}(q_1)\gamma_{R/L}(q_2)\delta\chi(Q)\rangle'}{P_{\delta\chi}(Q)}\,,$$

$$Q \to 0\,, \qquad q_1 \simeq q_2 = q\,.$$

- Physical interpretation: after horizon crossing the scalar long mode freezes at a certain amplitude, inducing a local modification of the spatial curvature. The short modes evolve in this modified background space and inherits a specific modulation in their power spectrum statistics.
- R and L squeezed bispectra differ for a minus sign.
$\rightarrow$ Contribution to the GW chirality

Squeezed modulation (2)

Cumulative effect of soft modes

$$\mathcal{M}(q,\vec{x})_{R/L} = \int_{Q_{min}<|\vec{Q}|<Q_L} d^3Q\, e^{i\vec{Q}\cdot\vec{x}}\, \delta\chi(Q) \frac{\langle \gamma_{R/L}(q)\gamma_{R/L}(q)\delta\chi(Q)\rangle'}{P_{\delta\chi}(Q)}\,.$$

Local amplification of chirality

$$P_\gamma^{R/L}(q;\vec{x}) = \frac{P_T(q)}{2}\left[1 \pm \mathcal{A}(\vec{x})\right]\;, \qquad \Theta = \mathcal{A}(\vec{x})\,.$$

- $\mathcal{A}(\vec{x})$ follows a Gaussian distribution with mean 0 and variance

$$\sigma^2 = 32\pi^2 \epsilon_\chi \left(\frac{H^2 f''(\chi_0)}{M_{Pl}}\right)^2 \int_{Q_{min}}^{Q_L} dQ\, Q^2\, P_{\delta\chi}(Q)\,.$$

$\epsilon_\chi = \dot{\chi}_0^2/2H^2M_{Pl}^2\,.$

- χ has to be a spectator field. Squeezed bispectra are re-absorbed in single clock-inflation, consequence of gauge freedom. (see, e.g., *Creminelli et. al., 2013; Pajer et. al., 2013*).

Summary and Conclusion

- Chern-Simons gravity coupled to a scalar field χ can be tested during inflationary epoch.
- At linear level the model can not be constrained through CMB experiments. Chirality produced is low if we avoid instabilities.
- Non-linearities produce a model dependent local squeezed amplification of chirality. χ has to be different from the inflaton field.
- In progression: toy model building, parameter space.
- What about interferometers?
  - Coplanar interferometers (like LISA) are not sensitive to chirality of GW. We need a more complicated geometry (*Smith and Caldwell, 2016*).
  - Inflationary GW are red-tilted. We need a small scale amplification mechanism.
  - Idea: study the effect of chiral GW production while χ is decaying (Next step).

Back-up slide, example of a toy model

- Assume $\rho_\chi \ll \rho_\phi$, also take $\epsilon_H = -\frac{\dot{H}}{H^2} \simeq \epsilon_\phi$. $\quad (\epsilon_\chi \ll \epsilon_\phi)$, $\quad \eta_\chi \simeq 0$.
- Both ϕ and χ have vacuum fluctuations on a quasi De-Sitter space. $P_\chi, P_\phi \simeq H^2/2k^3$.

$$\frac{\zeta_\chi}{\zeta_\phi} = \frac{\delta\rho_\chi}{\delta\rho_\phi} \simeq \frac{V'(\chi)\delta\chi}{U'(\phi)\delta\phi} = \sqrt{\frac{\epsilon_\chi}{\epsilon_\phi}} \ll 1 \longrightarrow \zeta \simeq \zeta_\phi \,.$$

- Postulate $f(\chi)$ as:

$$f(\chi) = \lambda \left(\frac{\chi}{M_{Pl}}\right)^n \,,$$

$$\longrightarrow \sigma^2 = 32\pi^2 \epsilon_\chi \left(H^2 f''(\chi_0)\right)^2 \left(\frac{H}{M_{Pl}}\right)^2 \times \ln\left(\frac{Q_L}{Q_{min}}\right) \,.$$

- Take $\left(\frac{H}{M_{Pl}}\right) = 10^{-5}$, $\epsilon_\chi = 10^{-6}$, $\ln\left(\frac{Q_L}{Q_{min}}\right) = 10$, $H^2 f''(\chi_0) = 10^6$:

$$\longrightarrow \sigma^2 = 1.$$

- $\frac{H}{M_{CS}} \ll 1 \longrightarrow \left(\frac{\chi_0}{M_{pl}}\right) \ll 10^{-4}$, small field potential for χ.