more on [the theoretical uncertainties of] the signal

i.e. towards refined hydrodynamic simulations[1,2]

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[1] Supported by the SNF under grant 200020-168988.

[2] Based on collaborations with Jacopo Ghiglieri and Greg Jackson.

introduction

we are interested in measuring the $T_{\mu\nu}$ correlator

in a stationary state, gravitational waves are emitted with rate

$$\frac{\mathrm{d}e_{\rm GW}}{\mathrm{d}t\,\mathrm{d}^3\mathbf{k}} = \frac{2\,C^{\rm TT}_{\Delta}(k)}{\pi^2 m^2_{\sf Pl}}\;,$$

where the transverse-traceless correlator reads ($\mathbf{k} \equiv k\,\mathbf{e}_z$)

$$C^{\rm TT}_{\Delta}(k) \;\equiv\; \int_{(t,\mathbf{x})} e^{ik(t-z)} \left\langle \frac{1}{2}\bigl\{ T^{xy}(t,\mathbf{x})\,,\; T^{xy}(0) \bigr\} \right\rangle\;.$$

at low frequencies $T^{\mu\nu}$ is given by hydrodynamics

with p = pressure, w = enthalpy density, u^μ = flow velocity, the energy-momentum tensor reads

$$T^{ij} \approx T^{ij}_{\text{ideal}} \,, \quad T^{ij}_{\text{ideal}} \equiv p g^{ij} + w u^i u^j \,.$$

$\Rightarrow$ problem: if the equation of state possesses a phase transition, the solution becomes singular within a finite period of time[3]

[3] e.g. L.D. Landau and E.M. Lifshitz, *Fluid Mechanics*.

as a regulator, viscous corrections can be included

$\Rightarrow T^{ij} = T^{ij}_{\text{ideal}} + \Delta T^{ij}$ where[4]

$$\Delta T^{ij} = -\eta\Big(u^{i;j} + u^{j;i} - \frac{2g^{ij}u^{\gamma}_{;\gamma}}{3}\Big) - \zeta g^{ij}u^{\gamma}_{;\gamma} + S^{ij} \; ,$$

here η, ζ are the shear and bulk viscosities, and S^{ij} is noise with

$$\big\langle S^{xy}(\mathcal{X}) S^{xy}(\mathcal{Y}) \big\rangle = 2\eta T \, \delta(\mathcal{X} - \mathcal{Y}) \; .$$

$\Rightarrow$ fluctuation-dissipation theorem requires that noise be present; as a consequence, even without flow,

$$\lim_{k\to 0} C^{\text{TT}}_{\Delta}(k) = 2\eta T \; .$$

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[4] E.M. Lifshitz and L.P. Pitaevskii, *Statistical Physics, Part 2*, §88-89.

paradox: in a weakly coupled system η is "large"

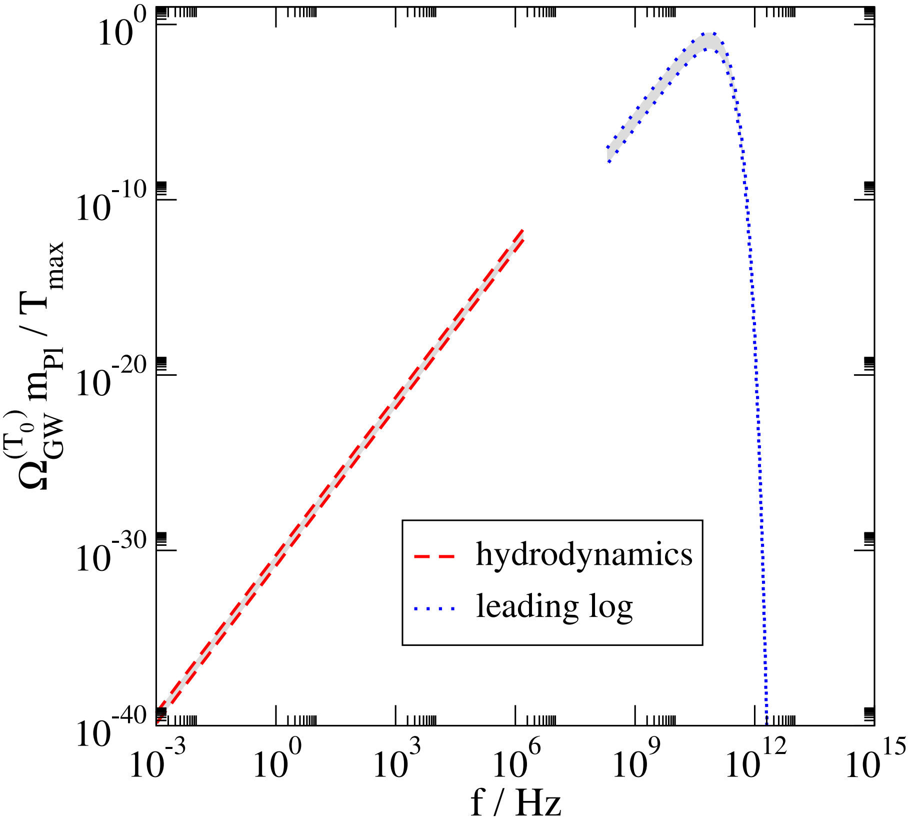
if there were *no* interactions, particles would fly freely, and long-range correlations would persist $\Rightarrow$ *large* fluctuations

in kinetic theory, $\eta \sim 1/\sigma$, where σ is a cross section related to kinetic equilibration

at $T \gg 100$ GeV,[5] $\eta \simeq \dfrac{16T^3}{g_1^4 \ln(5T/m_{\mathrm{D1}})} \simeq 400\, T^3$.

[5] S. Jeon, *Hydrodynamic transport coefficients in relativistic scalar field theory*, hep-ph/9409250; P.B. Arnold, G.D. Moore and L.G. Yaffe, *Transport coefficients in high temperature gauge theories. 1. Leading log results*, hep-ph/0010177; *Transport coefficients in high temperature gauge theories. 2. Beyond leading log*, hep-ph/0302165.

nevertheless the noise contribution is invisible to LISA



main stuff

we now move on to a "2-component system": $T^{\mu\nu} + \phi$

$T^{\mu\nu}$ is modified by a contribution from an "order parameter":

$$T^{\mu\nu}_{\text{ideal}} \equiv w\, u^\mu u^\nu + p\, g^{\mu\nu} + \phi^{,\mu}\phi^{,\nu} - \frac{g^{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}}{2}\,,$$

$$p \equiv p_0(T) - V(\phi, T)\,, \quad w \equiv T\partial_T p$$

$\Rightarrow V(\phi, T)$ may contain a first order transition

equation of motion for ϕ involves friction[6]

the dissipative coefficient γ couples ϕ to the thermal background:

$$
\begin{aligned}
T^{\mu\nu}_{;\mu} &= 0 \, , \\
\phi^{;\mu}_{;\mu} - \gamma \, u^\mu \phi_{,\mu} - \partial_\phi V &= 0 \, .
\end{aligned}
$$

$\Rightarrow$ finite wall velocity, entropy production, and equilibration

[6] J. Ignatius, K. Kajantie, H. Kurki-Suonio and ML, *The growth of bubbles in cosmological phase transitions*, astro-ph/9309059; M. Hindmarsh, S.J. Huber, K. Rummukainen and D.J. Weir, *Shape of the acoustic gravitational wave power spectrum from a first order phase transition*, 1704.05871.

then equation of motion for ϕ should also include noise

close to equilibrium, e.o.m. should be completed into

$$\phi^{;\mu}_{;\mu} - \gamma\, u^{\mu} \phi_{,\mu} - \partial_{\phi} V + \xi \;=\; 0 \;,$$

where ξ is a noise term, whose magnitude is completely fixed:

$$\big\langle \xi(\mathcal{X}) \xi(\mathcal{Y}) \big\rangle \;=\; 2\gamma T\, \delta(\mathcal{X} - \mathcal{Y}) \;.$$

contribution of γ to gravitational wave production

consider now $\delta T^{xy} = \phi^{,x}\phi^{,y}$ and average over fluctuations

$$\Rightarrow \ \delta C_{\Delta}^{\mathrm{TT}}(k) = \frac{2T^2}{\gamma}\int_{\mathbf{p}} \frac{p_x^2 p_y^2}{(p^2+m^2)^2}\left(1+\frac{\gamma^2}{p^2+m^2}\right) .$$

on a fine space-time lattice ($\frac{1}{a_s} \gg \ m, \ \gamma$),

$$\delta C_{\Delta}^{\mathrm{TT}}(k) \approx \ \frac{2T^2}{\gamma}\int_{\mathbf{p}} \frac{\tilde{p}_x^2 \tilde{p}_y^2}{\tilde{p}^4} \ = \ \frac{T^2}{\gamma}\times\frac{\Gamma^2\left(\frac{1}{24}\right)\Gamma^2\left(\frac{11}{24}\right)(\sqrt{3}-1)}{288\pi^3 a_s^3} .$$

$\Rightarrow \delta C_{\Delta}^{\mathrm{TT}}$ scales as $\frac{T^2}{\gamma}$, has a flat shape, and can be tuned at will

summary: why include fluctuations in hydrodynamics?

$\Rightarrow$ their tunable absolute magnitude ($\sim T^2/\gamma$) and known shape may help to "calibrate" the measurement algorithm

$\Rightarrow$ "spherical bubbles don't radiate", but in reality bubbles aren't quite spherical; noise is necessary for probing their instability[7]

$\Rightarrow$ in many models, noise generates first order transitions[8]

$\Rightarrow$ in principle, noise automatically takes care of nucleations[9]

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[7] e.g. P.Y. Huet, K. Kajantie, R.G. Leigh, B.H. Liu and L.D. McLerran, *Hydrodynamic stability analysis of burning bubbles in electroweak theory and in QCD*, hep-ph/9212224.

[8] e.g. B.I. Halperin, T.C. Lubensky and S.-K. Ma, *First-Order Phase Transitions in Superconductors and Smectic-A Liquid Crystals*, PRL 32 (1974) 292.

[9] e.g. G.D. Moore, K. Rummukainen and A. Tranberg, *Nonperturbative computation of the bubble nucleation rate in the cubic anisotropy model*, hep-lat/0103036.

epilogue: sometimes one can get inspiration from QCD

the theory and practice of hydrodynamic fluctuations have recently become a hot topic for heavy ion collision hydrodynamics[10]

[10] e.g. P. Kovtun and L.G. Yaffe, *Hydrodynamic fluctuations, long time tails, and supersymmetry*, hep-th/0303010; P. Kovtun, G.D. Moore and P. Romatschke, *The stickiness of sound: An absolute lower limit on viscosity and the breakdown of second order relativistic hydrodynamics*, 1104.1586; J.I. Kapusta, B. Müller and M. Stephanov, *Relativistic Theory of Hydrodynamic Fluctuations with Applications to Heavy Ion Collisions*, 1112.6405; Y. Akamatsu, A. Mazeliauskas and D. Teaney, *Bulk viscosity from hydrodynamic fluctuations with relativistic hydro-kinetic theory*, 1708.05657.