

$b \rightarrow c\ell\nu$ review (theory)

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Status of $|V_{cb}|$

- V_{cb} plays an important role in the **Unitarity Triangle**.
↳ We want to overconstrain the triangle as a new physics test.
- V_{cb} goes into the prediction of ε_K via

$$\varepsilon_K \propto x |V_{cb}|^4 + \dots$$

- V_{cb} goes into the predictions of **flavor changing neutral currents**.
- The ratio

$$\left| \frac{V_{ub}}{V_{cb}} \right|$$

directly constrains **one side** of the Unitarity Triangle.

Status: HFLAV V_{cb} averages

[HFLAV, 1612.07233v3]

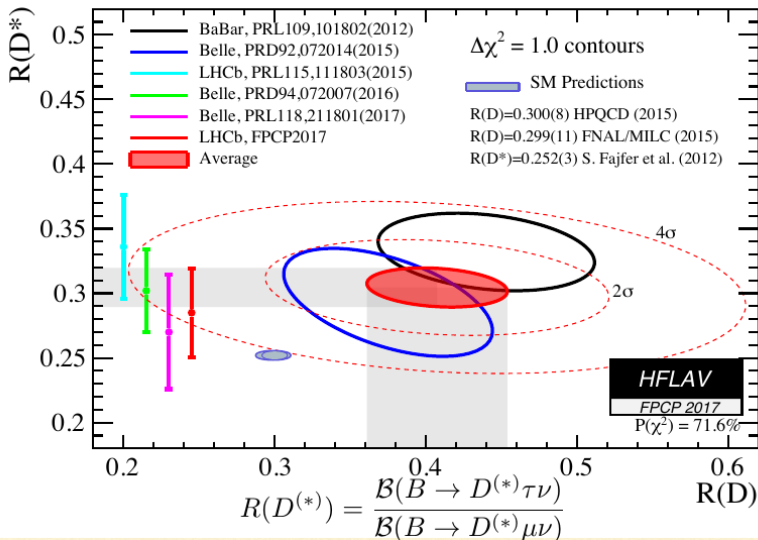
$$|V_{cb}| = (42.19 \pm 0.78) \cdot 10^{-3} \quad \text{from } B \rightarrow X_c l \nu$$

$$|V_{cb}| = (39.05 \pm 0.47_{\text{exp}} \pm 0.58_{\text{th}}) \cdot 10^{-3} \quad \text{from } B \rightarrow D^* l \nu$$

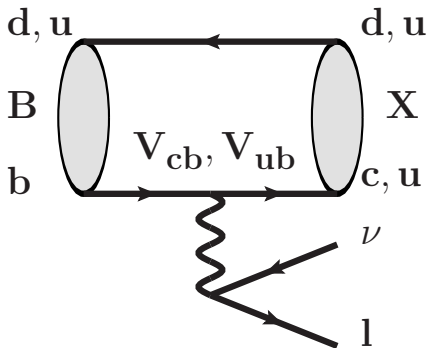
$$|V_{cb}| = (39.18 \pm 0.94_{\text{exp}} \pm 0.36_{\text{th}}) \cdot 10^{-3} \quad \text{from } B \rightarrow D l \nu$$

Status of Lepton Flavor Universality Violation in $b \rightarrow c l \nu$

[HFLAV 1612.07233v3 and update FPCP 2017]



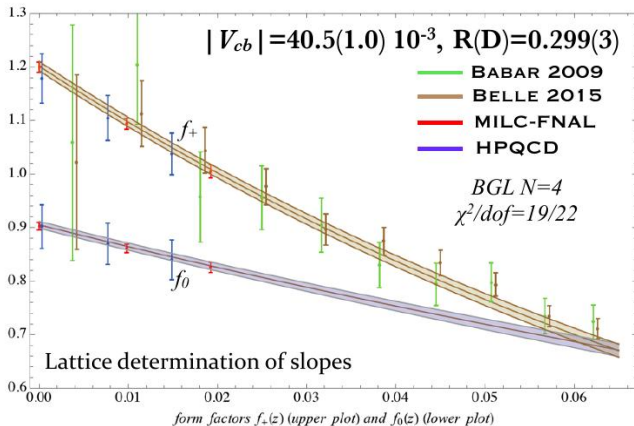
Semileptonic B decays



- Allow for the determination of V_{cb} and V_{ub} .
- **Exclusive analyses** look at specific final states, e.g., $X = D, D^*, \pi, \rho$.

Lattice + Exp fit for $B \rightarrow D l \nu$

[Bigi Gambino 1606.08030]



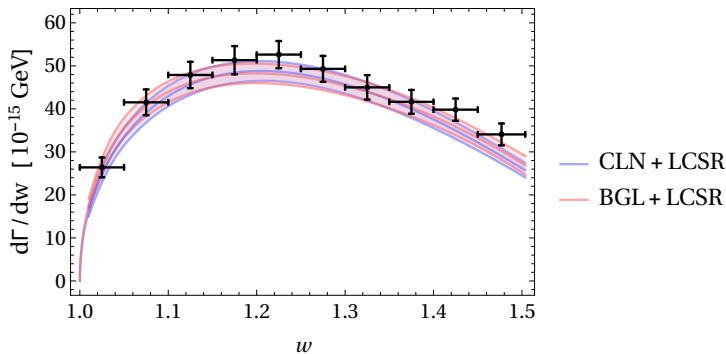
- Data + lattice beyond zero recoil allow good form factor determination.
- Including S_1 , important for $R(D)$.

Good consensus of theory predictions for $R(D)$

Ref.	$R(D)$	Deviation
Experiment [HFLAV update]	0.407(39)(24)	—
2018: Calculation of soft photon corrections		
[de Boer Kitahara Nisandzic 1803.05881]	amplify $R(D^{+(0)})$ by $\lesssim 5.5\%$ (3.6%)	
2016/17 theory results, using new lattice and exp. data:		
[Bigi Gambino 1606.08030]	0.299(3)	2.4σ
[Bernlochner Ligeti Papucci Robinson 1703.05330]	0.299(3)	2.4σ
[Jaiswal Nandi Patra 1707.09977]	0.302(3)	2.3σ
2012 theory results:		
[Fajfer Kamenik Nisandzic 1203.2654]	0.296(16)	2.3σ
[Celis Jung Li Pich 1210.8443]	$0.296\left(\frac{8}{6}\right)$ (15)	2.3σ
[Tanaka Watanabe 1212.1878]	0.305(12)	2.2σ

Recent (preliminary) Belle data for $B \rightarrow D^* l \nu$

- First time w and angular **deconvoluted distributions independent** of parametrization.
↳ Possible to use **different parametrizations**.



$$w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}}, \quad q^2 = (p_B - p_{D^*})^2$$

Model independent form factor parametrization

[Boyd Grinstein Lebed (BGL), hep-ph/9412324, hep-ph/9504235, hep-ph/9705252]

Boyd Grinstein Lebed parametrization

$$f_i(z) = \frac{1}{B_i(z)\phi_i(z)} \sum_{n=0}^{\infty} a_n^i z^n,$$
$$z = \frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}}, \quad w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}}.$$

- $0 < z < 0.056$ for $B \rightarrow D^* l \nu \Rightarrow$ truncation at $N = 2$ enough, $z^3 \sim 10^{-4}$.
- $B_i(z)$: “Blaschke factor”: removes poles.
- $\phi_i(z)$: phase space factors.
- Limit of **massless** leptons:
 - 3 form factors V_4 (vector), A_1 and A_5 (axial vector).
- **Massive** lepton $m_\tau \neq 0$: additional form factor P_1 (pseudoscalar).

Form Factor Basis for $B^{(*)} \rightarrow D^{(*)}$

	$B \rightarrow D$	$B \rightarrow D^*$
$V, 1^-$	V_1	V_4
$A, 1^+$	—	A_1, A_5
$S, 0^+$	S_1	—
$P, 0^-$	—	P_1
	$B^* \rightarrow D$	$B^* \rightarrow D^*$
$V, 1^-$	V_5	V_2, V_3, V_6, V_7
$A, 1^+$	A_2, A_6	A_3, A_4, A_7
$S, 0^+$	—	S_2, S_3
$P, 0^-$	P_2	P_3

Commonly used ratios with A_1

$$R_0 = \frac{P_1}{A_1},$$

$$R_1 = \frac{V_4}{A_1},$$

$$R_2 = \frac{w-r}{w-1} \left(1 - \frac{1-r}{w-r} \frac{A_5}{A_1} \right).$$

Unitarity Constraints

[Boyd Grinstein Lebed 1994, 1997]

- Use **dispersion relations** to relate physical **semileptonic region**

$$m_l^2 \leq q^2 \leq (m_B - m_D)^2, \quad q^2 \equiv (p_B - p_{D^*})^2,$$

to **pair-production region** beyond threshold

$$q^2 \geq (m_B + m_D)^2, \quad \text{with poles at } q^2 = m_{B_c}^2.$$

- **Constrain** form factors in **pair-production** region with pert. QCD.
- **Translate** constraint to **semileptonic region** using analyticity.

(Weak) Unitarity Conditions

- **Vector** current: $\sum_{i=0}^{\infty} (a_n^{V4})^2 \leq 1.$
- **Axial vector** current: $\sum_{i=0}^{\infty} \left((a_n^{A1})^2 + (a_n^{A5})^2 \right) \leq 1.$

Additional Theory Information on Form Factors

2 unquenched Lattice QCD (LQCD) results

$$A_1(1) = 0.906(13) \text{ [FNAL/MILC 1403.0635]} \quad A_1(1) = 0.895(26) \text{ [HPQCD 1711.11013]}$$

Average $A_1(1) = 0.904(12)$: Normalization for $|V_{cb}|$ extraction

Light Cone Sum Rules (LCSR)

[Faller Khodjamirian Klein Mannel 0809.0222]

$$A_1(w_{\max}) = 0.65(18), \quad R_1(w_{\max}) = 1.32(4), \quad R_2(w_{\max}) = 0.91(17).$$

Heavy Quark Effective Theory and QCD sum rules (HQET)

[Bernlochner Ligeti Papucci Robinson 1703.05330, Caprini Lellouch Neubert hep-ph/9712417, Luke Phys.Lett B252,447 (1990),

Neubert Rieckert Nucl. Phys. B382, 97 (1992) Neubert hep-ph/9306320, Ligeti Neubert Nir hep-ph/9209271, 9212266, 9305304]

- Important constraints for all $B^{(*)} \rightarrow D^{(*)}$ form factors.
- In the heavy quark limit $m_{c,b} \gg \Lambda_{\text{QCD}}$ all $B^{(*)} \rightarrow D^{(*)}$ form factors either vanish or are proportional to 1 Isgur-Wise (IW) function.
- NLO corrections at $O(\Lambda_{\text{QCD}}/m_{c,b}, \alpha_s)$ known, expressible with 3 subleading IW functions, which are extracted using QCDSRs.

How large are the theoretical uncertainties due to unknown NNLO corrections, $O(\alpha_s^2, \frac{\Lambda_{\text{QCD}}^2}{m_{c,b}^2}, \alpha_s \frac{\Lambda_{\text{QCD}}}{m_{c,b}})$?

- **Reliable estimate** from NLO corrections complicate:
At zero recoil several form factors **protected from NLO power corrections** through Luke's theorem [Luke Phys.Lett B252,447 (1990)]
- Protection **does not apply** to NNLO corrections.
- The form factors which are not protected by Luke's theorem do have **NLO corrections up to 60%**.

$$\frac{V_6(w)}{V_1(w)} = 1.0, \quad (\text{LO})$$

$$\frac{V_6(w)}{V_1(w)} = 1.58(1 - 0.18(w - 1) + \dots). \quad (\text{NLO})$$

Compare LQCD and HQET+QCDSR results: Difference from beyond NLO corrections

$$\begin{aligned} \left. \frac{S_1(w)}{V_1(w)} \right|_{\text{LQCD}} &\approx 0.975(6) + 0.055(18)w_1, & \left. \frac{S_1(w)}{V_1(w)} \right|_{\text{HQET}} &\approx 1.021(30) - 0.044(64)w_1 \\ \left. \frac{A_1(1)}{V_1(1)} \right|_{\text{LQCD}} &= 0.857(15), & \left. \frac{A_1(1)}{V_1(1)} \right|_{\text{HQET}} &= 0.966(28) \\ \left. \frac{S_1(1)}{A_1(1)} \right|_{\text{LQCD}} &= 1.137(21), & \left. \frac{S_1(1)}{A_1(1)} \right|_{\text{HQET}} &= 1.055(2), \quad (w_1 = w - 1) \end{aligned}$$

↳ Deviations of 5% – 13%.

Taking everything into account

NNLO corrections as large as $O(10\% - 20\%)$ are natural. They cannot be neglected for robust tests of the SM and reliable extractions of V_{cb} .

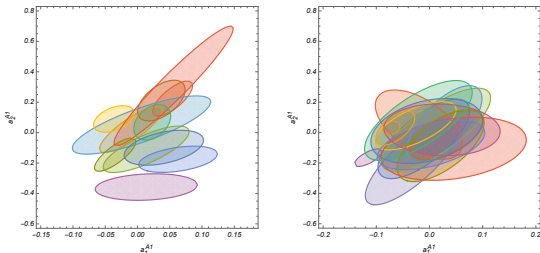
Strong Unitarity Constraints and HQET Input

- Use HQET information on further $b \rightarrow c$ channels:
 $B \rightarrow D, B^* \rightarrow D, B^* \rightarrow D^*$, to relate them to $B \rightarrow D^*$.
- Make the unitarity bounds **stronger**:

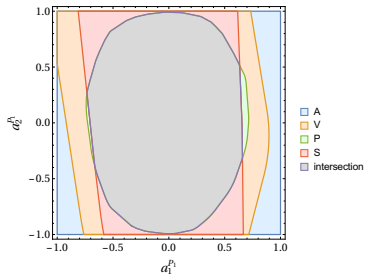
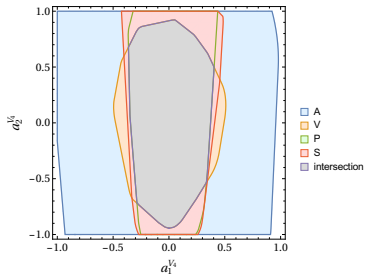
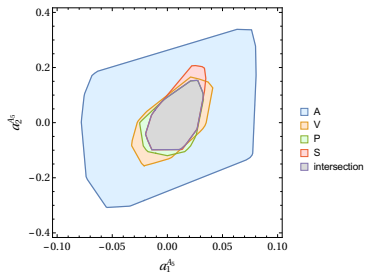
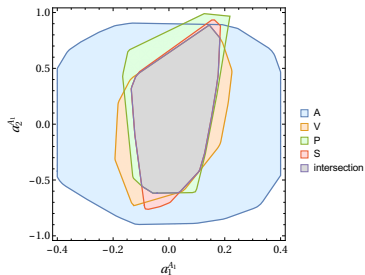
[BGL, hep-ph/9705252]

$$\sum_{i=1}^H \sum_{n=0}^{\infty} b_{in}^2 \leq 1. \quad \text{for } S, P, V, A \text{ currents}$$

- Vary QCDSR parameters + higher order corrections:
obtain many different unitarity bounds.
- Take their **envelope** as **side condition in the fit**.



Allowed regions for BGL parameters from strong unitarity constraints



Different Method: Use strong unitarity/HQET to eliminate parameters and obtain simplified parametrization

Caprini Lellouch Neubert parametrization as used in exp. analyses

$$h_{A_1}(w) = h_{A_1}(1) \left(1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3 \right),$$

$$R_1(w) = R_1(1) - 0.12(w - 1) + 0.05(w - 1)^2, \quad R_1(1) = 1.27,$$

$$R_2(w) = R_2(1) + 0.11(w - 1) - 0.06(w - 1)^2, \quad R_2(1) = 0.80.$$

- **Theoretical uncertainties** for slope and curvature of form factor ratios R_1 and R_2 are **set to zero**.
- Relation of curvature and slope of axial form factor A_1 is **fixed to central value**.
- **Uncertainties** on fixed parameters **never included** in exp. analyses.
- At **current exp. precision** these **cannot** be longer **neglected**.
- Also, inconsistent to fit $R_{1,2}(1)$ and fix other parameters: dependent on common underlying theory parameters.

Central values of V_{cb} differ by 3.6% (with LCSR) and 5.6% (wo LCSR)

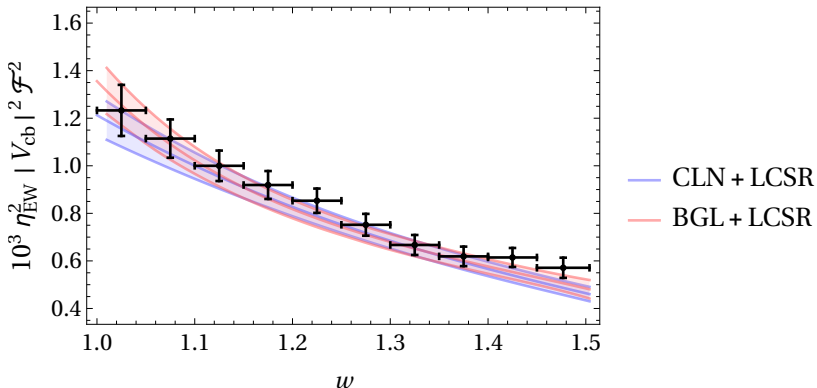
(preliminary Belle data + HFLAV average $\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \Gamma^- \bar{\nu}_l) = 0.0488 \pm 0.0010$)

[Bigi Gambino Schacht 1703.06124 and 1707.09509, "BGL weak" agreeing with Grinstein Kobach, 1703.08170]

Fit	BGL weak	BGL weak	BGL strong	BGL strong
LCSR	×	✓	×	✓
χ^2/dof	28.2/33	32.0/36	29.6/33	33.1/36
$ V_{cb} $	0.0424 (18)	0.0413 (14)	0.0415 (13)	0.0406 $\left(\begin{smallmatrix} +12 \\ -13 \end{smallmatrix}\right)$

Fit	CLN	CLN
LCSR	×	✓
χ^2/dof	35.4/37	35.9/40
$ V_{cb} $	0.0393(12)	0.0392(12)

Main reason for deviation

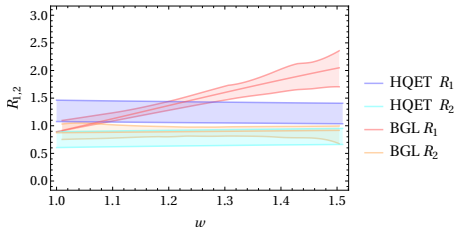


- CLN fit has **limited flexibility** of slope.
↳ CLN band **underestimates** all three **low recoil** points.
- Extrapolation near $w = 1$ **crucial**: Lattice input for V_{cb} extraction.
- CLN fit with free floating $R_{1,2}$ slopes (wo LCSR): $|V_{cb}| = 0.0415(19)$.
- **Intrinsic uncertainties** of CLN fit can no longer be neglected.

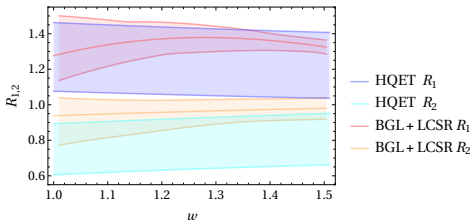
Comparison of $R_{1,2}$ fits with HQET+QCDSR

[Bigi Gambino Schacht 1703.06124, Bernlochner Ligeti Papucci Robinson 1708.07134]

“BGL strong” without LCSR.



“BGL strong” with LCSR.



- Fits for R_2 in good agreement with HQET+QCDSR. Same goes for R_1 with LCSR.
- R_1 without LCSR well compatible with HQET only at small/moderate recoil. At large w clear tension with both HQET and LCSR.
↳ Fit without LCSR appears somewhat disfavored.
- Lattice will compute A_1 and $R_{1,2}$ and settle the story.

Role of HQET relations in V_{cb} extraction

(preliminary Belle data only)

STRONG HQET INPUT	SMALL V_{cb}	Refs.
“practical” CLN:	$ V_{cb} = 38.2(1.5) \cdot 10^{-3}$	[1,5,6,7,8]
CLN+QCD sumrule errors + $B \rightarrow D$	$ V_{cb} = 38.5(1.1) \cdot 10^{-3}$	[2]
same + lattice at non-zero recoil	$ V_{cb} = 39.3(1.0) \cdot 10^{-3}$	[2]
BGL,HQET,LCSR, $B \rightarrow D$,nuisance	$ V_{cb} = 40.9(0.9) \cdot 10^{-3}$	[3]
BGL + strong unitarity	$ V_{cb} = 40.8(1.5) \cdot 10^{-3}$	[4]
BGL + weak unitarity	$ V_{cb} = 41.7(2.0) \cdot 10^{-3}$	[5,6,7,8]
NO HQET INPUT	LARGE V_{cb}	

[1] [Belle 1702.01521] [2] [Bernlochner Ligeti Papucci Robinson 1703.05330]

[3] [Jaiswal Nandi Patra 1707.09977] [4] [Bigi Gambino Schacht 1707.09509]

[5] [Bigi Gambino Schacht 1703.06124] [6] [HPQCD 1711.11013]

[7] [Bernlochner Ligeti Papucci Robinson 1708.07134] [8] [Grinstein Kobach 1703.08170]

Lepton Flavor Universality Violation:

$$\text{Anatomy of } R(D^*) \equiv \frac{\int_1^{w_{\tau,\max}} dw d\Gamma_{\tau}/dw}{\int_1^{w_{\max}} dw d\Gamma/dw}$$

Differential decay rate for $B \rightarrow D^* \tau \nu_{\tau}$

[BGL, hep-ph/9705252]

$$\begin{aligned}\frac{d\Gamma_{\tau}}{dw} &= \frac{d\Gamma_{\tau,1}}{dw} + \frac{d\Gamma_{\tau,2}}{dw} \\ \frac{d\Gamma_{\tau,1}}{dw} &= \left(1 - m_{\tau}^2/q^2\right)^2 \left(1 + m_{\tau}^2/(2q^2)\right) \frac{d\Gamma}{dw} \\ \frac{d\Gamma_{\tau,2}}{dw} &= |V_{cb}|^2 m_{\tau}^2 \times \text{kinematics} \times P_1(z)^2\end{aligned}$$

- $d\Gamma/dw$: **Measured** differential decay rate of $B \rightarrow D^* l \nu$ with $m_l = 0$, depends on axial vector form factors A_1, A_5 and vector form factor V_4 .
- P_1 : Additional **unconstrained** pseudoscalar **form factor**.
- $d\Gamma_{\tau,2}/dw$ contributes $\sim 10\%$ to $R(D^*)$.
- Common normalization/notation:

$$R_0 = \frac{P_1}{A_1} = 1 \quad \text{in heavy quark limit}$$

[BGL, hep-ph/9705252]

Standard $R(D^*)$ Calculation: Normalizing P_1 on A_1

- **NLO** HQET result for $R_0 = P_1/A_1$. [Bernlochner Ligeti Papucci Robinson 1703.05330]
- Estimate of **NNLO** uncertainty as **15%** of P_1 central value (enters quadratically).
- Our result using this method and strong unitarity bounds:
 - **with LCSR:**

$$R_{\tau,1}(D^*) = 0.232 \quad R_{\tau,2}(D^*) = 0.026, \\ R(D^*) = 0.258(5)_{(-7)}^{(+8)} = 0.258 \left(\begin{smallmatrix} +10 \\ -9 \end{smallmatrix} \right).$$

- **without LCSR:**

$$R_{\tau,1}(D^*) = 0.232, \quad R_{\tau,2}(D^*) = 0.025, \\ R(D^*) = 0.257(5)_{(-7)}^{(+8)} = 0.257 \left(\begin{smallmatrix} +10 \\ -8 \end{smallmatrix} \right).$$

More precise: BGL expansion + enforcing a constraint at

$$q^2 = 0$$

Use $N = 2$ BGL expansion

$$P_1(w) = \frac{\sqrt{r}}{(1+r)B_{0^-}(z)\phi_{P_1}(z)} \sum_{n=0}^2 a_n^{P_1} z^n,$$

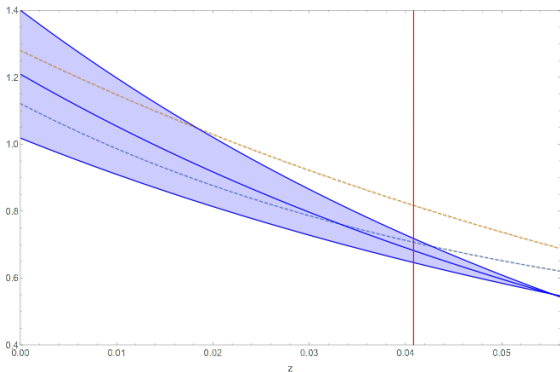
with **3 unknowns** $a_0^{P_1}$, $a_1^{P_1}$, $a_2^{P_1}$ and **3 constraints**:

- Kinematical endpoint relation $P_1(w_{\max}) = A_5(w_{\max})$, with fit result for $A_5(w_{\max})$.
- **HQET result** $P_1(1) = 1.21 \pm 0.06 \pm 0.18$.
 - **1st error**: Parametric **NLO error**.
 - **2nd error**: Estimate of the **NNLO uncertainty** as **15%** of central value.
- **Strong unitarity**.

$$\rightarrow R_{\tau,2}(D^*) = 0.028, \quad R(D^*) = 0.260(5)(6) = \mathbf{0.260(8)}.$$

Comparison of Different Normalizations for P_1

- Dashed yellow:
normalized on V_1 ,
 $R(D^*) = 0.268 \begin{pmatrix} +15 \\ -13 \end{pmatrix}$.
- Dashed blue:
normalized on A_1 ,
 $R(D^*) = 0.258 \begin{pmatrix} +10 \\ -9 \end{pmatrix}$.
- Solid blue:
zero-recoil
normalization to **IW**
function and
 $P_1(w_{\max}) = A_5(w_{\max})$,
 $R(D^*) = 0.260(8)$.



Ref.	$R(D^*)$	Deviation
Experiment [HFLAV update]	0.304(13)(7)	—
2017 theory results, using new lattice and exp. data:		
[Bernlochner Ligeti Papucci Robinson 1703.05330]	0.257(3)	3.1σ
Our result [Bigi Gambino Schacht 1707.09509]	0.260(8)	2.6σ
[Jaiswal Nandi Patra 1707.09977]	0.257(5)	3.0σ
2012 theory results:		
[Fajfer Kamenik Nisandzic 1203.2654]	0.252(3)	3.5σ
[Celis Jung Li Pich 1210.8443]	0.252(2)(3)	3.4σ
[Tanaka Watanabe 1212.1878]	0.252(4)	3.4σ

Due to accounting for **unkown NNLO** corrections, we have a **larger uncertainty** as present in the literature.

Fit to NLO HQET Parametrization

[Bernlochner Ligeti Papucci Robinson 1703.05330]

- Fit QCDSR parameters in **HQET parametrization at $O(\Lambda_{\text{QCD}}/m_{c,b}, \alpha_s)$** to $B \rightarrow D^{(*)}l\nu$ data.
- Include conservative apriori range for QCDSR parameters.
- Introduce HQET breaking in normalization through extra normalization factors which **rescale form factor results to lattice input**.
- Rescaling with normalization factors effectively includes **higher order effects** for those form factors which are calculated on the lattice.
- Of course rescaling not possible for P_1 as no lattice result available.
↳ **no NNLO error included for $P_1 \Rightarrow R(D^*) = 0.257(3)$.**

Future is Bright

Lattice

- $B \rightarrow D^* l \nu$ at **non-zero recoil** is on the way, preliminary results shown at Lattice 2017. [Vaquero Avils-Casco, DeTar, Du, El-Khadra, Kronfeld, Laiho, Van de Water 2017]
- We can test **HQET** using lattice QCD.
- This will **stabilize** the fits and **reduce** the errors of V_{cb} and $R(D^*)$.

More R observables for $b \rightarrow c \tau \nu$

$$\begin{array}{ccc} R(B_c \rightarrow J/\psi \tau \nu) & R(\Lambda_b \rightarrow \Lambda_c^{(*)} \tau \nu) & R(B^0 \rightarrow D^+ \tau \nu) \\ R(B^0 \rightarrow D^0 \tau \nu) & R(B_s \rightarrow D_s \tau \nu) & R(B \rightarrow D^{**} \tau \nu) \end{array}$$

High Luminosity: More than just higher precision

- Extend perspective to $b \rightarrow u \tau \nu$.

$$R(B \rightarrow p \bar{p} \tau \nu) \quad R(\Lambda_b \rightarrow p \tau \nu) \quad R(B \rightarrow \pi \tau \nu) \quad R(B_s \rightarrow K^* \tau \nu)$$



We will need **robust theory predictions** for **all the R 's**.

Future has already started...

$R(B_c)$

$$R(B_c)^{\text{exp}} = 0.71 \pm 0.25. \quad [\text{LHCb 1711.05623}]$$

Theorists have to work harder: $R(B_c)^{\text{SM}} = 0.25 - 0.28$ (range of models).

Theory of $\Lambda_b \rightarrow \Lambda_c^{(*)} \tau \nu$ is available

$$R(\Lambda_c)^{\text{SM}} = 0.3328 \pm 0.0102. \quad [\text{Detmold Lehner Meinel 2015}]$$

$$\Lambda_b \rightarrow \Lambda_c^* \tau \nu : \quad [\text{Böer Bordone Graverini Owen Rotondo Van Dyk 2018}]$$

There's more than just R

q^2 dependence, angular observables, τ polarization, CP violation, making use of the full decay chain.

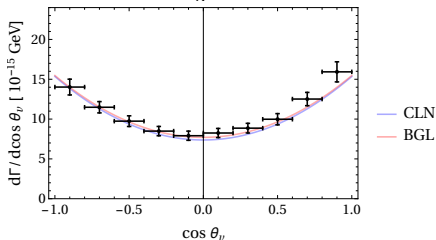
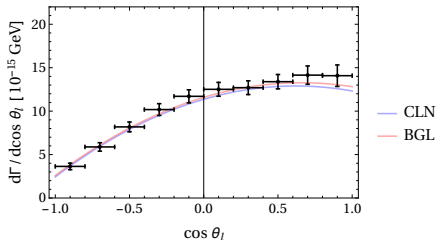
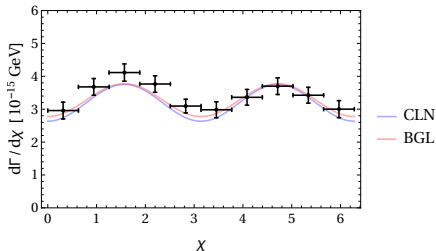
[Tanaka Watanabe 2010, Sakaki Tanaka 2013, Hagiwara Nojiri Sakaki 2014, Bordone Isidori van Dyk 2016, Ligeti Papucci Robinson 2016, Becirevic Fajfer Nisandzic Tayduganov 2016, Ivanov Koerner Tran 2017, Alonso Camalich Westhoff 2017, ...]

Conclusions

- Belle has **new data**: Deconvoluted, independent of parametrization.
- Different parametrizations imply **different theoretical assumptions** and **different** treatments of **theoretical uncertainties**.
- They give **different results** for $|V_{cb}|$, also with **strong unitarity**.
- In view of today's exp. precision, it is important to take into account **theoretical uncertainties of HQET**, including $O(10\% - 20\%)$ uncertainty from unknown corrections beyond NLO.
- This is rather accomplished using the **BGL parametrization** with side conditions than by **simplified parametrizations**.
- Reanalysis of **previous Belle** and **BaBar data** is necessary. Together with **future lattice data on slopes** this will conclusively settle the case.
- Results: $|V_{cb}| = 40.6 \left(\begin{smallmatrix} +1.2 \\ -1.3 \end{smallmatrix} \right) \cdot 10^{-3}$ (with LCSR),
 $|V_{cb}| = 41.5(1.3) \cdot 10^{-3}$ (wo LCSR),
 $R(D^*) = 0.260(8)$ (with and wo LCSR).
- The $R(D^*)$ anomaly is **persistent**, **slightly reduced to 2.6σ** .

BACK-UP

Angular Dependence

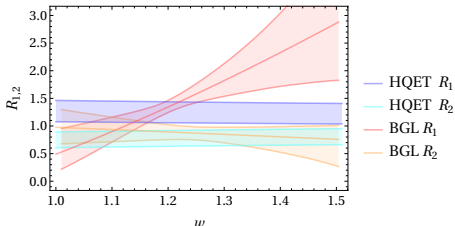


- Angular bins have **little sensitivity** to low recoil region.
- **Dilute information** of first bins in w spectrum.

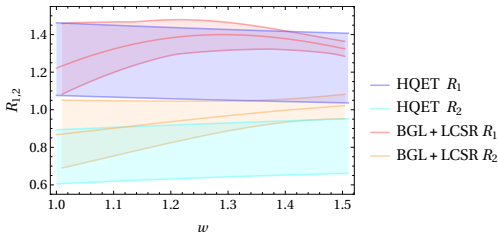
Comparison in case of weak unitarity bounds

[Bigi Gambino Schacht 1703.06124, Bernlochner Ligeti Papucci Robinson 1708.07134]

“BGL weak” without LCSR:



“BGL weak” with LCSR:



Anatomy of numerical large NLO HQET corrections

Ratios are all 1 at LO

- $B \rightarrow D^*$

$$\begin{array}{ll} \left(\frac{P_1(1)}{V_1(1)} \right)_{\text{full NLO}} = 1.21 & \left(\frac{P_1(1)}{V_1(1)} \right)_{\text{only } 1/m} = 1.24 \\ \left(\frac{V_4(1)}{V_1(1)} \right)_{\text{full NLO}} = 1.24 & \left(\frac{V_4(1)}{V_1(1)} \right)_{\text{only } 1/m} = 1.19 \end{array}$$

- $B^* \rightarrow D^*$

$$\begin{array}{ll} \left(\frac{V_3(1)}{V_1(1)} \right)_{\text{full NLO}} = 1.18 & \left(\frac{V_3(1)}{V_1(1)} \right)_{\text{only } 1/m} = 1.18 \\ \left(\frac{V_6(1)}{V_1(1)} \right)_{\text{full NLO}} = 1.58 & \left(\frac{V_6(1)}{V_1(1)} \right)_{\text{only } 1/m} = 1.54 \\ \left(\frac{V_7(1)}{V_1(1)} \right)_{\text{full NLO}} = 1.39 & \left(\frac{V_7(1)}{V_1(1)} \right)_{\text{only } 1/m} = 1.34 \end{array}$$

Future Scenario of Lattice Input

Future lattice fits	χ^2/dof	$ V_{cb} $
CLN	56.4/37	0.0407 (12)
CLN+LCSR	59.3/40	0.0406 (12)
BGL	28.2/33	0.0409 (15)
BGL+LCSR	31.4/36	0.0404 (13)

- Fits including a **hypothetical future lattice** calculation giving

slope information at **5%**: $\left. \frac{\partial \mathcal{F}}{\partial w} \right|_{w=1} = -1.44 \pm 0.07.$

- Additional theory input **stabilizes** the results.