CKM parameters from the observables in $B \to (\pi, K)\ell^+\ell^-$ decays

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based on A. Khodjamirian, A.V. Rusov, JHEP **1708** (2017) 112

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Motivation

- Recently, the ratio $|V_{td}/V_{ts}|$ has been extracted from the measured $B \to \pi \ell^+ \ell^-$ and $B \to K \ell^+ \ell^-$ partial decay widths
  [LHCb, JHEP10(2015)034]
- Actually, the ratio of $B \to \pi \ell^+ \ell^-$ and $B \to K \ell^+ \ell^-$ partial decay widths has more complicated CKM structure
- In principle, it's possible to constrain the Wolfenstein parameters of the CKM matrix from the observables in $B \to \pi \ell^+ \ell^-$ and $B \to K \ell^+ \ell^-$ decays

$\rightarrow$ One needs to accurately determine the hadronic input:

- ⋆ Form factors
- ⋆ Hadronic amplitudes of nonlocal effects

Effective Hamiltonian

We consider semileptonic $B \to P\ell^+\ell^-$ decays ($\ell = e, \mu,\ P = \pi, K$)

Effective Hamiltonian for $b \to q$ [Buchalla, Buras, Lautenbacher (1996)]

$$\mathcal{H}_{\text{eff}}^{b\to q} = \frac{4G_F}{\sqrt{2}} \left(\lambda_u^{(q)} \sum_{i=1}^{2} C_i\, \mathcal{O}_i^u + \lambda_c^{(q)} \sum_{i=1}^{2} C_i\, \mathcal{O}_i^c - \lambda_t^{(q)} \sum_{i=3}^{10} C_i\, \mathcal{O}_i \right) + h.c.$$

$$\lambda_p^{(q)} = V_{pb} V_{pq}^*, \quad p = u, c, t, \quad q = d, s$$

- $B \to K\ell^+\ell^-$: $\lambda_t^{(s)} \approx -\lambda_c^{(s)} \sim \lambda^2 \gg \lambda_u^{(s)} \sim \lambda^4$
- $B \to \pi\ell^+\ell^-$: $\lambda_t^{(d)} \sim \lambda_c^{(d)} \sim \lambda_u^{(d)} \sim \lambda^3$

Hadronic input

Form Factors

$$\langle P(p)|\bar{q}\gamma^\mu b|B(p+q)\rangle = f^+_{BP}(q^2)\,(2p^\mu + q^\mu) + \left(f^+_{BP}(q^2) - f^0_{BP}(q^2)\right)\frac{m_B^2 - m_P^2}{q^2}q^\mu$$

$$\langle P(p)|\bar{q}\sigma^{\mu\nu}q_\nu b|B(p+q)\rangle = \frac{i f^T_{BP}(q^2)}{m_B + m_P}\left[2q^2 p^\mu + \left(q^2 - (m_B^2 - m_P^2)\right)q^\mu\right]$$

Nonlocal effects via correlation functions

$$\mathcal{H}^{(p)}_{BP,\,\mu} = i\int d^4x\, e^{iqx}\langle P(p)|\mathrm{T}\Big\{j^{\mathrm{em}}_\mu(x), \Big[C_1\mathcal{O}^p_1(0) + C_2\mathcal{O}^p_2(0) + \sum_{k=3-6,8g} C_k\mathcal{O}_k(0)\Big]\Big\}|B(p+q)\rangle = \left[(p\cdot q)q_\mu - q^2 p_\mu\right]\mathcal{H}^{(p)}_{BP}(q^2)$$

Amplitude

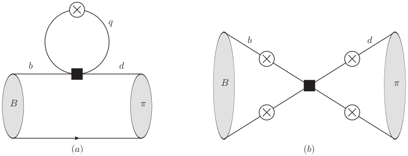
$$A(B \to P\ell^+\ell^-) = \frac{G_F}{\sqrt{2}}\frac{\alpha_{\rm em}}{\pi}\lambda_t^{(q)} f^+_{BP}(q^2)\left[\left(\bar{\ell}\gamma^\mu\ell\right) p_\mu \left(C_9 + \frac{2(m_b+m_q)}{m_B+m_P} C_7^{\rm eff}\frac{f^T_{BP}(q^2)}{f^+_{BP}(q^2)}\right)\right.$$

$$\left. + \left(\bar{\ell}\gamma^\mu\gamma_5\ell\right) p_\mu C_{10} - (\bar{\ell}\gamma^\mu\ell)p_\mu \frac{16\pi^2}{f^+_{BP}(q^2)}\left(\frac{\lambda_u^{(q)}}{\lambda_t^{(q)}}\mathcal{H}^{(u)}_{BP}(q^2) + \frac{\lambda_c^{(q)}}{\lambda_t^{(q)}}\mathcal{H}^{(c)}_{BP}(q^2)\right)\right]$$

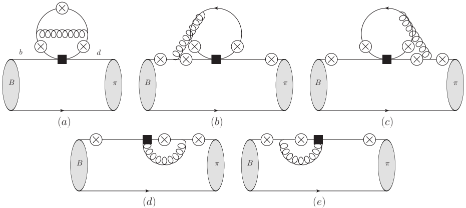
- Wilson coefficients at **NLO** [C. Bobeth, M. Misiak, and J. Urban (2000)]
- Form factors from **LCSR** [A. Khodjamirian, A.V. Rusov (2017)]
- Nonlocal hadronic amplitudes via **QCDF, LCSR** and **hadronic dispersion relations**
  - $\mathcal{H}^{(c)}_{BK}(q^2)$ [A. Khodjamirian, Th. Mannel, Y.M. Wang (2013)]
  - $\mathcal{H}^{(u)}_{B\pi}(q^2)$ and $\mathcal{H}^{(c)}_{B\pi}(q^2)$ [Ch. Hambrock, A. Khodjamirian, A.V. Rusov (2015)]

Calculation of $\mathcal{H}^{(u,c)}_{BP}(q^2)$ at $q^2 < 0$

- LO, factorizable loop and weak annihilation
  [M. Beneke, Th. Feldmann, D. Seidel (2001)]

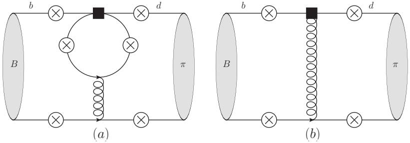


- NLO, factorizable
  [H.H.Asatryan, H.M. Asatrian, C. Greub, M. Walker (2002);
  H.M.Asatrian, K. Bieri, C. Greub, M. Walker (2004)]

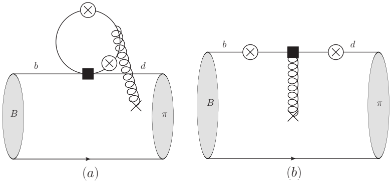


Calculation of $\mathcal{H}_{BP}^{(u,c)}(q^2)$ at $q^2 < 0$

- NLO, nonfactorizable (hard gluons)
  [M. Beneke, Th. Feldmann, D. Seidel (2001)]



- Soft gluons, nonfactorizable
  [A. Khodjamirian, Th. Mannel, A.A. Pivovarov, Y.-M. Wang (2010)]
  [A. Khodjamirian, Th. Mannel, Y.-M. Wang (2013)]



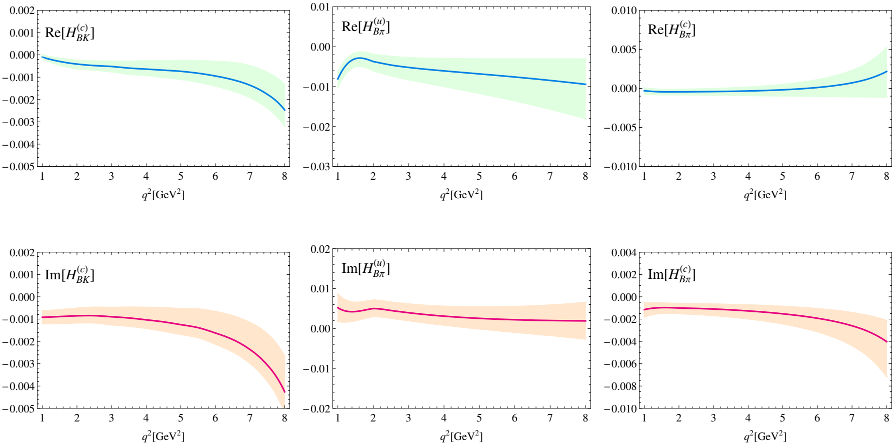
Dispersion relations for $\mathcal{H}_{BP}^{(u,c)}(q^2)$

Dispersion relations (analytic continuation of $\mathcal{H}_{BP}^{(u,c)}(q^2)$ to $q^2 > 0$):

$$\mathcal{H}_{BP}^{(u,c)}(q^2) = (q^2 - q_0^2)\left[\sum_{V=\rho,\omega,J/\psi,\psi(2S)} \frac{k_V f_V A_{BVP}^{u,c}}{(m_V^2 - q_0^2)(m_V^2 - q^2 - i m_V \Gamma_V^{\rm tot})} + \int\limits_{s_0^{u,c}}^{\infty} ds \frac{\rho_{BP}^{(u,c)}(s)}{(s - q_0^2)(s - q^2 - i\epsilon)}\right] + \mathcal{H}_{BP}^{(u,c)}(q_0^2)$$

- $A_{BVP}^{u,c} = |A_{BVP}^{u,c}| e^{i\delta_{BVP}^{u,c}}$
- $|A_{BVP}^{u,c}|$ are extracted from nonleptonic $B \to VP$ decays
- $\delta_{BVP}^{u,c}$ are extracted from the fit of the dispersion relation to $\mathcal{H}_{BP}^{(u,c)}(q^2)$ for $q^2 < 0$
- For $\rho_{BP}^{(u,c)}(s)$ one applies quark-hadron duality

Results for $\mathcal{H}_{BK}^{(c)}, \mathcal{H}_{B\pi}^{(u)}, \mathcal{H}_{B\pi}^{(c)}$



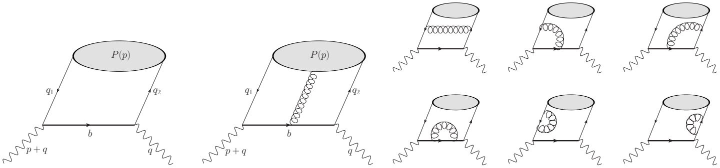
Form factors from LCSR

Starting objects – correlation functions:

$$
\begin{aligned}
F^{\mu}_{BP}(p,q) &= i\int d^4x\, e^{iqx}\langle P(p)|\,T\{\bar{q}(x)\Gamma^{\mu}b(x),\, m_b\bar{b}(0)i\gamma_5 u(0)\}|0\rangle \\
&= \begin{cases} F_{BP}(q^2,(p+q)^2)p^{\mu} + \tilde{F}_{BP}(q^2,(p+q)^2)q^{\mu}, & \Gamma^{\mu} = \gamma^{\mu} \\ F^{T}_{BP}(q^2,(p+q)^2)\left[q^2 p^{\mu} - (q\cdot p)q^{\mu}\right], & \Gamma^{\mu} = -i\sigma^{\mu\nu}q_{\nu} \end{cases}
\end{aligned}
$$

Region of light-cone dominance ($x^2 \sim 0$): $\quad q^2 \ll m_b^2, \;\; (p+q)^2 \ll m_b^2$

OPE =



LCSR: a general scheme

- Transform the OPE result to the dispersion form (in $(p+q)^2$ variable):

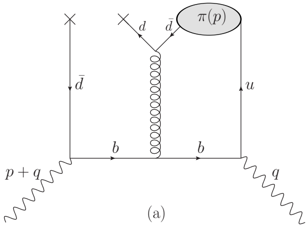
$$F_{BP}^{(T)(\mathrm{OPE})}(q^2,(p+q)^2) = \frac{1}{\pi}\int\limits_{m_b^2}^{\infty} ds \frac{\mathrm{Im} F_{BP}^{(T)(\mathrm{OPE})}(q^2,s)}{s-(p+q)^2}$$

- Matching to the hadronic dispersion relation (isolating B-meson state)
- Applying the quark hadron duality
- Applying the Borel transform
- Finally

$$f_{BP}^{+}(q^2) = \frac{e^{m_B^2/M^2}}{2m_B^2 f_B}\frac{1}{\pi}\int\limits_{m_b^2}^{s_0^B} ds\, \mathrm{Im} F_{BP}^{(\mathrm{OPE})}(q^2,s) e^{-s/M^2}$$

$$f_{BP}^{T}(q^2) = \frac{(m_B+m_P)e^{m_B^2/M^2}}{2m_B^2 f_B}\frac{1}{\pi}\int\limits_{m_b^2}^{s_0^B} ds\, \mathrm{Im} F_{BP}^{T(\mathrm{OPE})}(q^2,s) e^{-s/M^2}$$

Factorizable twist-5 and twist-6 contributions



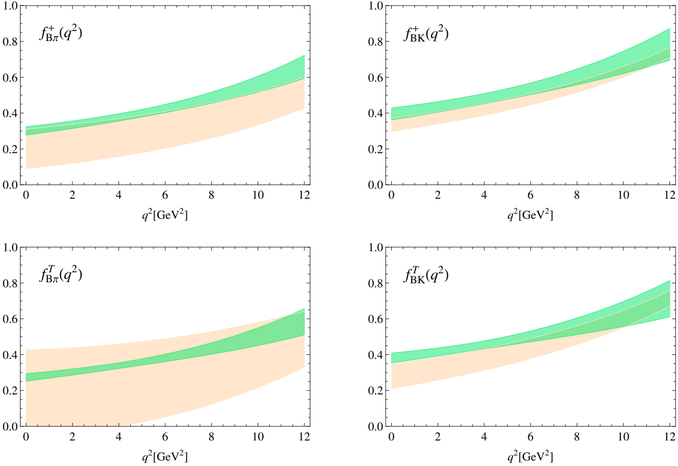
- In the framework of the factorization approximation

$$[F_{BP}(q^2)]_{\rm tw5,6} \sim \langle \bar{q} q \rangle \left(T_0^{(5)} \otimes \varphi_P^{(2)} + T_0^{(6)} \otimes \varphi_P^{(3)} \right)$$

- Calculation reveals [A.V. Rusov (2017)]

$$[f^+_{BP}(0)]_{\rm tw5,6} / f^+_{BP}(0) < 0.1\%$$

- $\Rightarrow$ Truncation up to twist-4 contributions is reliable



LCSR vs Lattice QCD

Observables in $B \to P\ell^+\ell^-$ decays

q^2-binned branching fraction

$$\mathcal{B}(\bar{B} \to P\ell^+\ell^-[q_1^2, q_2^2]) = \frac{G_F^2 \alpha_{\rm em}^2 |\lambda_t^{(q)}|^2}{192\pi^5} \Big\{ \mathcal{F}_{BP}[q_1^2, q_2^2] + \kappa_q^2\, \mathcal{D}_{BP}[q_1^2, q_2^2]$$
$$+2\kappa_q \Big(\cos\xi_q\, \mathcal{C}_{BP}[q_1^2, q_2^2] - \sin\xi_q\, \mathcal{S}_{BP}[q_1^2, q_2^2] \Big) \Big\} \tau_B$$

q^2-binned direct CP-asymmetry

$$\mathcal{A}_{BP}[q_1^2, q_2^2] = \frac{\mathcal{B}(\bar{B} \to P\ell^+\ell^-[q_1^2, q_2^2]) - \mathcal{B}(B \to \bar{P}\ell^+\ell^-[q_1^2, q_2^2])}{\mathcal{B}(\bar{B} \to P\ell^+\ell^-[q_1^2, q_2^2]) + \mathcal{B}(B \to \bar{P}\ell^+\ell^-[q_1^2, q_2^2])}$$
$$= \frac{-2\kappa_q \sin\xi_q\, \mathcal{S}_{BP}[q_1^2, q_2^2]}{\mathcal{F}_{BP}[q_1^2, q_2^2] + \kappa_q^2\, \mathcal{D}_{BP}[q_1^2, q_2^2] + 2\kappa_q \cos\xi_q\, \mathcal{C}_{BP}[q_1^2, q_2^2]}$$

$$\frac{\lambda_u^{(q)}}{\lambda_t^{(q)}} = \frac{V_{ub} V_{uq}^*}{V_{tb} V_{tq}^*} \equiv \kappa_q\, e^{i\xi_q}, \quad q = d, s$$

Binned parts of the decay width

$$\mathcal{F}_{BP}[q_1^2, q_2^2] = \frac{1}{q_2^2 - q_1^2} \int_{q_1^2}^{q_2^2} dq^2 \, p_{BP}^3 |f_{BP}^+(q^2)|^2 \left(|c_{BP}(q^2)|^2 + |C_{10}|^2 \right)$$

$$\mathcal{D}_{BP}[q_1^2, q_2^2] = \frac{1}{q_2^2 - q_1^2} \int_{q_1^2}^{q_2^2} dq^2 \, p_{BP}^3 \, |h_{BP}(q^2)|^2$$

$$\begin{pmatrix} \mathcal{C}_{BP}[q_1^2, q_2^2] \\ \mathcal{S}_{BP}[q_1^2, q_2^2] \end{pmatrix} = \frac{1}{q_2^2 - q_1^2} \int_{q_1^2}^{q_2^2} dq^2 \, p_{BP}^3 \, |f_{BP}^+(q^2) c_{BP}(q^2) h_{BP}(q^2)| \begin{pmatrix} \cos \delta_{BP}(q^2) \\ \sin \delta_{BP}(q^2) \end{pmatrix}$$

$$c_{BP}(q^2) = C_9 + \frac{2(m_b + m_q)}{m_B + m_P} C_7^{\rm eff} \frac{f_{BP}^T(q^2)}{f_{BP}^+(q^2)} + 16\pi^2 \frac{\mathcal{H}_{BP}^{(c)}(q^2)}{f_{BP}^+(q^2)}$$

$$h_{BP}(q^2) = 16\pi^2 \left(\mathcal{H}_{BP}^{(c)}(q^2) - \mathcal{H}_{BP}^{(u)}(q^2) \right), \quad \delta_{BP}(q^2) = \mathrm{Arg}(h_{BP}(q^2)) - \mathrm{Arg}(c_{BP}(q^2))$$

Ratio $|V_{td}/V_{ts}|$

- Without nonlocal hadronic amplitudes

$$\frac{\mathcal{B}(\bar{B} \to \pi \ell^+ \ell^- [q_1^2, q_2^2])}{\mathcal{B}(\bar{B} \to K \ell^+ \ell^- [q_1^2, q_2^2])} = \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{\mathcal{F}_{B\pi}[q_1^2, q_2^2]}{\mathcal{F}_{BK}[q_1^2, q_2^2]}$$

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- Including nonlocal effects ($\lambda_u^{(s)}$ neglected)

$$\frac{\mathcal{B}(\bar{B} \to \pi\ell^+\ell^-[q_1^2, q_2^2])}{\mathcal{B}(\bar{B} \to K\ell^+\ell^-[q_1^2, q_2^2])} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{\mathcal{F}_{B\pi}[q_1^2, q_2^2]}{\mathcal{F}_{BK}[q_1^2, q_2^2]} \left\{1 + \kappa_d^2 \frac{\mathcal{D}_{B\pi}[q_1^2, q_2^2]}{\mathcal{F}_{B\pi}[q_1^2, q_2^2]} + 2\kappa_d \left(\cos\xi_d \frac{\mathcal{C}_{B\pi}[q_1^2, q_2^2]}{\mathcal{F}_{B\pi}[q_1^2, q_2^2]} - \sin\xi_d \frac{\mathcal{S}_{B\pi}[q_1^2, q_2^2]}{\mathcal{F}_{B\pi}[q_1^2, q_2^2]}\right)\right\}$$

$$\kappa_d\, e^{i\xi_d} = \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}$$

CKM factors: way of extraction

- Switch to Wolfenstein parameters A, λ, ρ, η

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- $A = \frac{(192\pi^5)^{1/2}}{G_F \alpha_{em} \lambda^2} \left(\frac{1}{\mathcal{F}_{BK}[q_1^2, q_2^2]} \right)^{1/2} \left(\frac{\mathcal{B}_{BK}[q_1^2, q_2^2]}{\tau_B} \right)^{1/2}$

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- $\eta = \frac{1}{2\lambda^2(1-\lambda^2/2)} \left(\frac{\mathcal{F}_{BK}[q_1^2, q_2^2]}{\mathcal{S}_{B\pi}[q_1^2, q_2^2]} \right) \left(\mathcal{A}_{B\pi}[q_1^2, q_2^2] \frac{\mathcal{B}_{B\pi}[q_1^2, q_2^2]}{\mathcal{B}_{BK}[q_1^2, q_2^2]} \right)$

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- $$\frac{\mathcal{B}_{B\pi}[q_1^2, q_2^2]}{\mathcal{B}_{BK}[q_1^2, q_2^2]} = \frac{\lambda^2}{\mathcal{F}_{BK}[q_1^2, q_2^2]} \Bigg(\left[(1-\rho)^2 + \eta^2\right] \mathcal{F}_{B\pi}[q_1^2, q_2^2] + \frac{\left[\rho(1-\rho) - \eta^2\right]^2 + \eta^2}{(1-\rho)^2 + \eta^2} \left(1 - \frac{\lambda^2}{2}\right)^2 \mathcal{D}_{B\pi}[q_1^2, q_2^2] + 2\left[\rho(1-\rho) - \eta^2\right] \left(1 - \frac{\lambda^2}{2}\right) \mathcal{C}_{B\pi}[q_1^2, q_2^2] \Bigg)$$

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- $$\frac{\mathcal{B}_{B\pi}[q_1^2, q_2^2]}{\mathcal{B}_{BK}[q_1^2, q_2^2]} = \frac{\lambda^2}{\mathcal{F}_{BK}[q_1^2, q_2^2]} \Bigg(\left[(1-\rho)^2 + \eta^2\right] \mathcal{F}_{B\pi}[q_1^2, q_2^2] + \frac{\left[\rho(1-\rho) - \eta^2\right]^2 + \eta^2}{(1-\rho)^2 + \eta^2} \left(1 - \frac{\lambda^2}{2}\right)^2 \mathcal{D}_{B\pi}[q_1^2, q_2^2] + 2\left[\rho(1-\rho) - \eta^2\right] \left(1 - \frac{\lambda^2}{2}\right) \mathcal{C}_{B\pi}[q_1^2, q_2^2] \Bigg)$$

There are yet no experimental data on $\mathcal{A}_{B\pi}[q_1^2, q_2^2]$

Conclusion

- Nonlocal effects induce non-trivial dependence of $B \to (\pi, K)\ell^+\ell^-$ decays on CKM matrix elements
- The relevant hadronic input has been determined
- The new way of determination of the Wofenstein parameters from the observables in $B \to (\pi, K)\ell^+\ell^-$ decays is suggested
- More experimental data is desired

Backup

Operators basis

$$\mathcal{O}_9 = \frac{\alpha_{\rm em}}{4\pi}\left(\bar{d}_L \gamma^\mu b_L\right)\left(\bar{\ell}\gamma_\mu \ell\right) , \quad \mathcal{O}_{10} = \frac{\alpha_{\rm em}}{4\pi}\left(\bar{d}_L \gamma^\mu b_L\right)\left(\bar{\ell}\gamma_\mu \gamma_5 \ell\right) ,$$

$$\mathcal{O}_{7\gamma} = -\frac{e\, m_b}{16\pi^2}\left(\bar{d}_L \sigma^{\mu\nu} b_R\right) F_{\mu\nu},$$

$$\mathcal{O}_1^u = \left(\bar{d}_L \gamma_\mu u_L\right)\left(\bar{u}_L \gamma^\mu b_L\right), \quad \mathcal{O}_2^u = \left(\bar{d}_L^i \gamma_\mu u_L^j\right)\left(\bar{u}_L^j \gamma^\mu b_L^i\right),$$

$$\mathcal{O}_1^c = \left(\bar{d}_L \gamma_\mu c_L\right)\left(\bar{c}_L \gamma^\mu b_L\right), \quad \mathcal{O}_2^c = \left(\bar{d}_L^i \gamma_\mu c_L^j\right)\left(\bar{c}_L^j \gamma^\mu b_L^i\right),$$

$$\mathcal{O}_3 = \left(\bar{d}_L \gamma_\mu b_L\right)\sum_q \left(\bar{q}_L \gamma^\mu q_L\right), \quad \mathcal{O}_4 = \left(\bar{d}_L^i \gamma_\mu b_L^j\right)\sum_q \left(\bar{q}_L^j \gamma^\mu q_L^i\right),$$

$$\mathcal{O}_5 = \left(\bar{d}_L \gamma_\mu b_L\right)\sum_q \left(\bar{q}_R \gamma^\mu q_R\right), \quad \mathcal{O}_6 = \left(\bar{d}_L^i \gamma_\mu b_L^j\right)\sum_q \left(\bar{q}_R^j \gamma^\mu q_R^i\right),$$

$$\mathcal{O}_{8g} = -\frac{g_s m_b}{16\pi^2}\left(\bar{d}_L^i \sigma_{\mu\nu} (T^a)^{ij} b_R^j\right) G^{a\,\mu\nu}$$

$B \to P$ form factors: z-expansion (BCL parametrization)

$$f^{+,T}_{BP}(q^2) = \frac{f^{+,T}_{BP}(0)}{1 - q^2/m^2_{B^*_{(s)}}} \left\{ 1 + b^{+,T}_{1,BP} \left[z(q^2) - z(0) + \frac{1}{2}\left(z(q^2)^2 - z(0)^2 \right) \right] \right\}$$

$$z(q^2) = \frac{\sqrt{(m_B + m_P)^2 - q^2} - \sqrt{(m_B + m_P)^2 - t_0}}{\sqrt{(m_B + m_P)^2 - q^2} + \sqrt{(m_B + m_P)^2 - t_0}}, \qquad t_0 = (m_B + m_P)(\sqrt{m_B} - \sqrt{m_P})^2$$

Transition	$f^{+}_{BP}(0)$	$b^{+}_{1\,(BP)}$	Correlation
$B_s \to K$	0.336 ± 0.023	-2.53 ± 1.17	0.79
$B \to K$	0.395 ± 0.033	-1.42 ± 1.52	0.72
$B \to \pi$	0.301 ± 0.023	-1.72 ± 1.14	0.74

Transition	$f^{T}_{BP}(0)$	$b^{T}_{1\,(BP)}$	Correlation
$B_s \to K$	0.320 ± 0.019	-1.08 ± 1.53	0.74
$B \to K$	0.381 ± 0.027	-0.87 ± 1.72	0.75
$B \to \pi$	0.273 ± 0.021	-1.54 ± 1.42	0.78

Numerical results on observables

- Binned parts of the decay width

Decay mode	$\mathcal{F}_{BP}[\mathrm{GeV}^3]$	$\mathcal{D}_{BP}[\mathrm{GeV}^3]$	$\mathcal{C}_{BP}[\mathrm{GeV}^3]$	$\mathcal{S}_{BP}[\mathrm{GeV}^3]$
$B^- \to K^-\ell^+\ell^-$	$75.0^{+10.5}_{-9.7}$	—	—	—
$B^- \to \pi^-\ell^+\ell^-$	$47.7^{+6.4}_{-5.9}$	$16.1^{+2.8}_{-10.1}$	$14.3^{+7.8}_{-5.8}$	$-9.8^{+7.1}_{-7.2}$
$\bar{B}_s \to K^0\ell^+\ell^-$	$61.0^{+7.0}_{-6.8}$	$7.8^{+3.4}_{-2.5}$	$-12.9^{+2.4}_{-2.2}$	$-3.4^{+1.1}_{-2.6}$

- Our predictions (CKM matrix elements taken from global fit)

Process	$\mathcal{B}_{BP}[\mathrm{GeV}^{-2}] \times 10^{-8}$	$\mathcal{A}_{BP}$
$B^- \to K^-\ell^+\ell^-$	$4.38^{+0.62}_{-0.57} \pm 0.28$	0
$B^- \to \pi^-\ell^+\ell^-$	$0.131^{+0.023}_{-0.022} \pm 0.010$	$-0.15^{+0.11}_{-0.11}$
$\bar{B}_s \to K^0\ell^+\ell^-$	$0.154^{+0.018}_{-0.017} \pm 0.011$	$-0.04^{+0.01}_{-0.03}$

$$[q_1^2, q_2^2] = [1.0, 6.0]\,\mathrm{GeV}^2$$

Decay mode	$B^- \to K^- \ell^+ \ell^-$	$B^- \to \pi^- \ell^+ \ell^-$	$\bar{B}_s \to K^0 \ell^+ \ell^-$
Measurement or calculation	$\mathcal{B}_{BK}[1.0\,,6.0]$	$\mathcal{B}_{B\pi}[1.0\,,6.0]$	$\mathcal{B}_{B_s K}[1.0, 6.0]$
Belle	$2.72\,^{+0.46}_{-0.42} \pm 0.16$	—	—
CDF	$2.58 \pm 0.36 \pm 0.16$	—	—
BaBar	$2.72\,^{+0.54}_{-0.48} \pm 0.06$	—	—
LHCb	$2.42 \pm 0.07 \pm 0.12$	$0.091^{+0.021}_{-0.020} \pm 0.003$	—
HPQCD	3.62 ± 1.22	—	—
Fermilab/MILC	3.49 ± 0.62	0.096 ± 0.013	—
This work	$4.38^{+0.62}_{-0.57} \pm 0.28$	$0.131^{+0.023}_{-0.022} \pm 0.010$	$0.154^{+0.018}_{-0.017} \pm 0.011$

In the units of 10^{-8} GeV^{-2}

Ratio $|V_{ts}/V_{td}|$

- LHCb 2015 (exp. data on $B^+ \to (K^+, \pi^+)\mu^+\mu^-$)

$$|V_{td}/V_{ts}| = 0.24^{+0.05}_{-0.04}$$

- Fermilab Lattice and MILC (using Lattice $B \to P$ form factors):

$$|V_{td}/V_{ts}|_{\mathrm{low}-q^2} = 0.25 \pm 0.04, \quad |V_{td}/V_{ts}|_{\mathrm{high}-q^2} = 0.19 \pm 0.02$$

$$|V_{td}/V_{ts}| = 0.20 \pm 0.02$$

- PDG 2016 (from the mixing of $B_d^0 \leftrightarrow \bar{B}_d^0$ and $B_s^0 \leftrightarrow \bar{B}_s^0$):

$$|V_{td}/V_{ts}| = 0.215 \pm 0.011$$