

Unification of gauge and Yukawa couplings

presented by **Alan S. Cornell**

National Institute for Theoretical Physics;
School of Physics, University of the Witwatersrand



Based on M. Khojali *et al.*, arXiv:1706.02313[hep-ph].

What do we know about the Higgs?

The discovery of a Higgs boson allows a direct probe of the EWSB sector of the SM.

Its mass has been precisely measured, and is close to the EW scale itself, which has materialised the issue of naturalness.

The couplings follow the SM expectations: being proportional to mass.

The uncertainties are still large!

Note that coupling measurements are always subject to model assumptions.

On the theoretical side, recall that mass terms for scalar fields are not protected by any quantum symmetry, therefore any new physics sector that couples to it will feed into the value of the mass.

Recall that

$$\mathcal{L}_{Higgs} = (D_\mu \phi)^\dagger (D^\mu \phi) + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

“wrong sign”

It well describes the symmetry breaking,
but no dynamical insight!

Do we still need BSM?



We have a pretty good idea of the mechanism

But we don't know how to protect it:

A Feynman diagram showing a dashed line entering from the left, connecting to a circular loop. The loop has two arrows indicating a clockwise direction. The loop then connects to another dashed line exiting to the right.
$$\delta m_h^2 \sim \frac{g^2}{16\pi^2} M_{NPhys}^2$$

Having measured a light Higgs brings the naturalness problems to the real world!

A natural Higgs = A special type of scalar

Supersymmetry: All scalars are special, associated to a fermion by a space-time symmetry

$$\text{Scalar} \leftrightarrow \text{Fermion}$$

Compositeness: Special scalars are bound states of more fundamental fermions

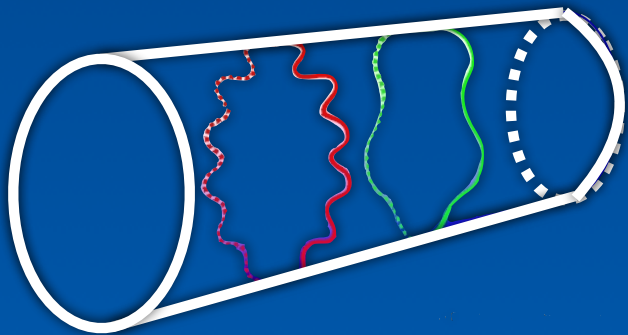
$$\phi \sim \langle \psi \psi \rangle$$

Scalars as gauge bosons: In extra dimensions, the extra polarisations of gauge bosons are special 4D scalars!

$$\phi \sim A_5, A_6, \dots A_D$$

Extra dimensions in nut-shell

Think of an additional space dimension wrapped on itself.



XD fields \rightarrow tower of KK states

Frequencies \rightarrow KK masses

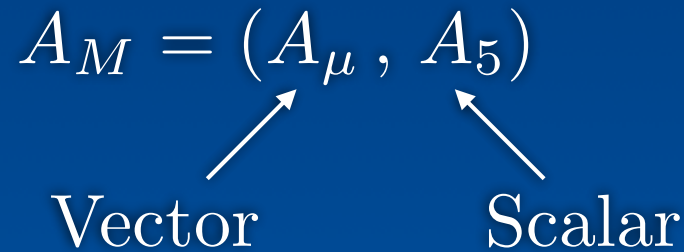
Geometry \rightarrow KK parities

For us 4D beings the 5D fields appear as massive 4D fields.

Extra dimension gauge fields

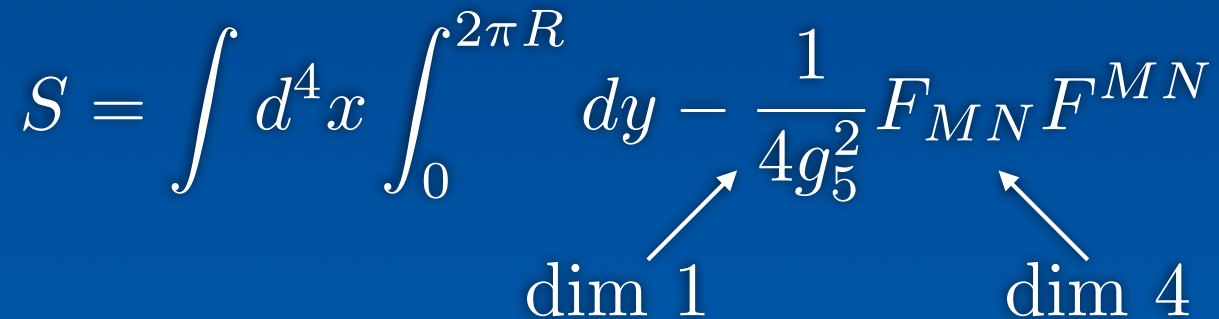
$$A_M = (A_\mu, A_5)$$

Vector Scalar



$$S = \int d^4x \int_0^{2\pi R} dy - \frac{1}{4g_5^2} F_{MN} F^{MN}$$

dim 1 dim 4




$$\frac{2\pi R}{g_5^2} = \frac{1}{g^2}$$

For the zero mode

A generic loop factor will
grow with the energy

An XD Higgs mechanism

$$\begin{aligned} F_{MN}F^{MN} &= F_{\mu\nu}F^{\mu\nu} - 2F_{5\mu}F^{5\mu} \\ &= F_{\mu\nu}F^{\mu\nu} - 2(\partial_5 A_\mu)^2 - 2(\partial_\mu A_5)^2 \\ &\quad + 4(\partial_5 A_\mu)(\partial^\mu A_5) \end{aligned}$$


A mixing between massive vectors and scalars is present

In the orbifold projection vector and scalar have opposite parities.

$A_M^+ = (A_\mu^+, A_5^-)$ Massless vector + massive vectors

$A_M^- = (A_\mu^-, A_5^+)$ Massless scalar + massive vectors

XD gauge theories have a built-in Higgs mechanism via special scalars

Gauge symmetries can be broken by orbifold projection (parity assignments)

An $SU(3)$ toy model

$$A_M(-y) = P \cdot A_M(y) \cdot P \qquad P = \begin{pmatrix} -1 & & \\ & -1 & \\ & & 1 \end{pmatrix}$$

$SU(2)$

$U(1)$

$$A_M^+ : \begin{pmatrix} \frac{1}{2}W_3 & \frac{1}{\sqrt{2}}W^+ & 0 \\ \frac{1}{\sqrt{2}}W^- & -\frac{1}{2}W_3 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \frac{1}{2\sqrt{3}} \begin{pmatrix} B & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & -2B \end{pmatrix}$$

$$A_M^- : \qquad \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H_0 \\ H^- & H_0^* & 0 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H_0 \\ H^- & H_0^* & 0 \end{pmatrix}$$

Can we identify
this with the
Higgs doublet?

$$U(1) \text{ charge: } \frac{1}{2\sqrt{3}} (1 - (-2)) = \frac{\sqrt{3}}{2}$$

$$g' = \sqrt{3}g \Rightarrow \sin^2 \theta_W = \frac{3}{4}$$

Does not match the SM value

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H_0 \\ H^- & H_0^* & 0 \end{pmatrix}$$

Can we identify
this with the
Higgs doublet?

Easy solution: add a bulk $U(1)_X$ gauge symmetry
and tune the g_X coupling

$$U(1) \text{ charge: } \frac{1}{2}g' = \frac{\sqrt{3}}{2}g + \frac{1}{2}g_X$$

Antoniadis, Benakli, Quiros: hep-th/0108005

The Hosotani mechanism

The Higgs vev can be geometrised

Hosotani, 1983

EWSB is induced by giving a vev to the Higgs, which is a gauge boson.

The vev can be removed by a suitable gauge transformation:

$$\Omega(y) = e^{i\alpha T_H y/R}$$

$$H_0 \rightarrow H_0 - \partial_y \left(\alpha \frac{y}{R} \right) = H_0 - \frac{\alpha}{R}$$

The periodicity condition is also affected:

$$\Omega(0)\phi(0) = \Omega(2\pi R)\phi(2\pi R)$$

$$\phi(0) = e^{i2\pi\alpha T_H} \phi(2\pi R)$$

$$\phi = \sum_n \tilde{\phi} e^{i(n+\alpha)y/R}$$

$$m_n = \frac{|n + \alpha|}{R}$$

The spectrum is shifted

$$m_0 = \frac{|\alpha|}{R}$$

Zero modes pick up a mass

How about fermions?

Consider a fundamental of $SU(3)$:

$$\psi = \begin{pmatrix} \psi_D \\ \psi_S \end{pmatrix} \quad U(1) \text{ charges : } \begin{pmatrix} 1/6 \\ -1/3 \end{pmatrix}$$

$$\bar{\psi} D_5 \gamma^5 \psi \Rightarrow \frac{g}{\sqrt{2}} \bar{\psi}_D H \psi_S + h.c.$$

$$y_f = \frac{g}{\sqrt{2}} : \quad \text{Yukawa couplings are related to gauge couplings}$$

The $SU(3)$ model fails

Incorrect prediction of the Weinberg angle

$$\sin^2 \theta_W = \frac{3}{4}$$

Yukawas related to the gauge couplings:

$$m_f = m_W$$

Grossmann, Neubert: [hep-ph/9912408](#)

It is easy to reduce the fermion masses by use of localisation, but hard to enhance.

The value of the top mass is a serious issues.

Many solutions attempted:

- Looking for other gauge groups: G_2, \dots

Csaki, Grojean, Murayama: hep-ph/0210133

- Embedding the top in higher representations.

Cacciapaglia, Csaki, Park: hep-ph/0510366

- Adding localised couplings/fields.

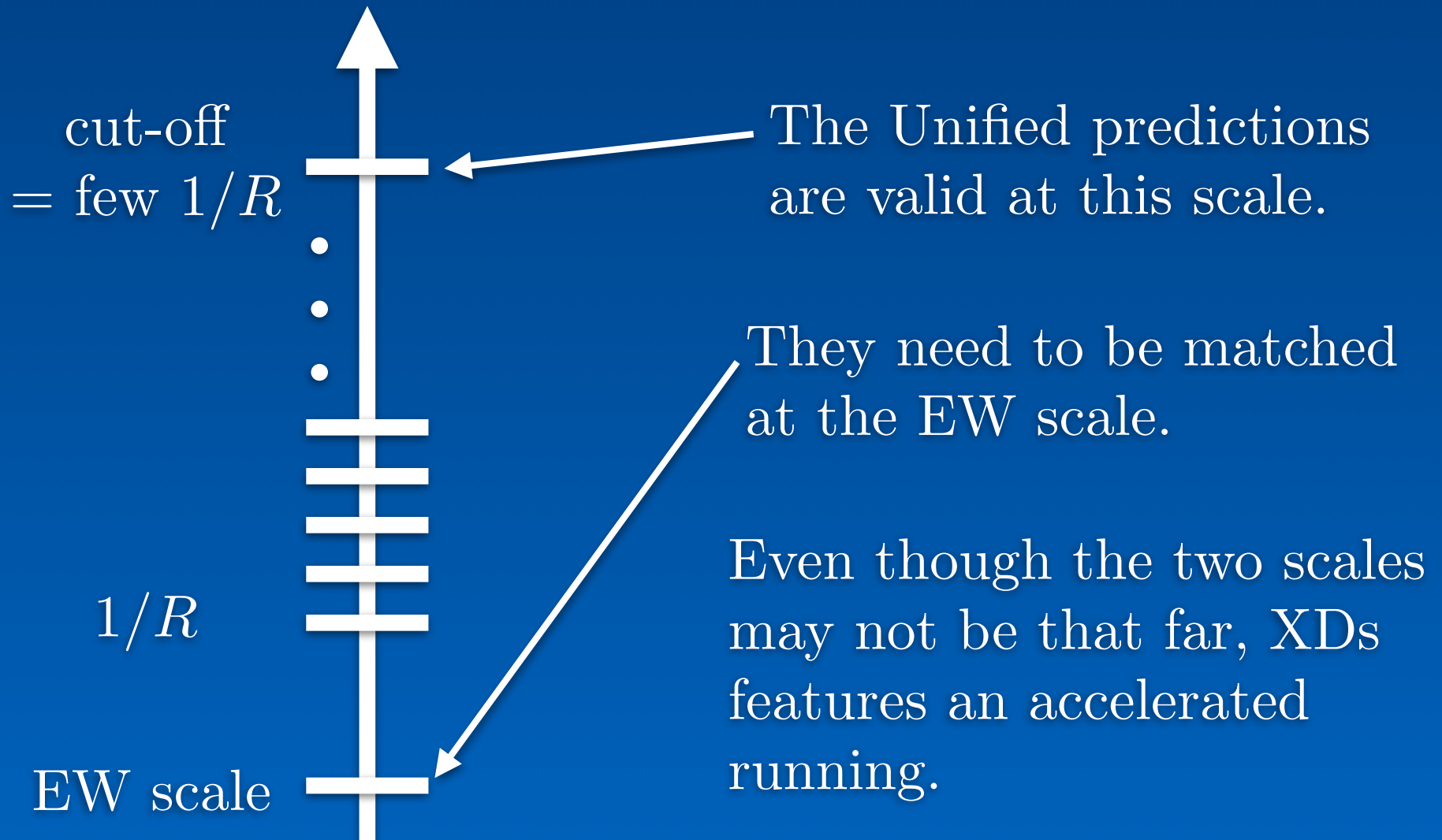
Scrucca, Serone, Silvestrini: hep-ph/0304220

- Changing the geometry of the space

Contino, Nomura, Pomarol: hep-ph/0306259

- ...

One fact has been overlooked so far: couplings run!

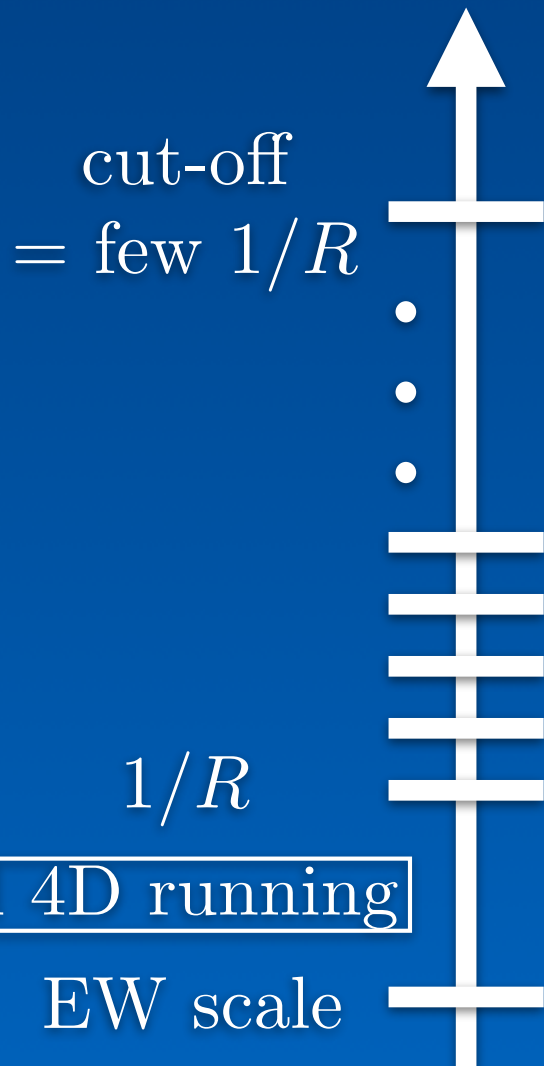


At each threshold
new states enter
the running.

The collective effect
of the resonances
produces a fast linear running
(cf. 4D log running)

This fact is well known
for the gauge couplings
(see XD GUT models).

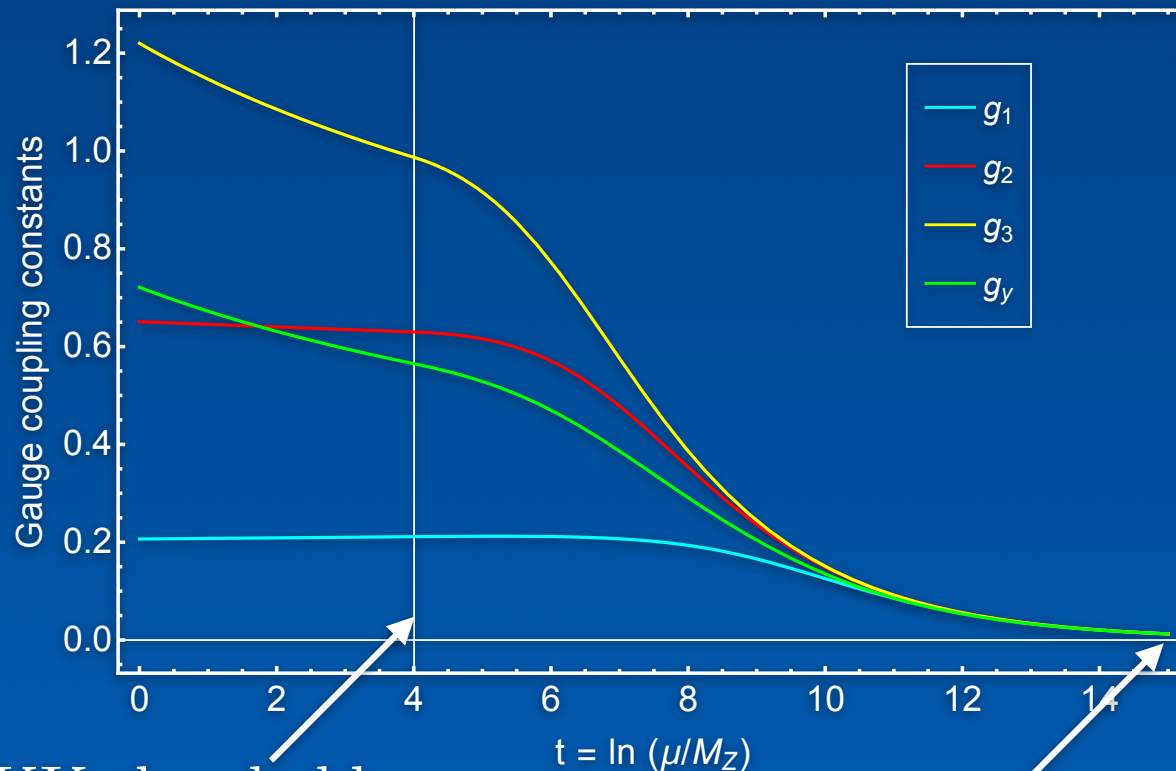
But the Yukawas
are also gauge couplings here!



Reconsider the simple $SU(3)$ model

Cacciapaglia, AC, Deandrea, Khojali: hep-ph/1706.02313

4D running



First KK threshold

$SU(2)$ and $U(1)$ unify

Running of the normalised gauges and Yukawas for the $SU(3)$ GHU model, for $1/R = 5$ TeV. The first KK mode enters at $t_{\text{KK}} \sim 4.0$, while the Unification scale corresponds to $t \sim 9.3$.

Note that the value of the couplings at Unification:

$$g_2 \sim 0.1$$

Number of KK modes below Unification:

$$n_{KK} \sim m_Z R e^{10} \sim 400$$

Naive dimensional analysis cut-off:

$$\Lambda R \sim \frac{8\pi}{g_{Unif}^2} \sim 2000$$

“top” Yukawa at the EW scale:

$$y_t \sim 0.6$$

For $g_i = \{g', g, g_s\}$:

$$16\pi^2 \frac{dg_i}{dt} = b_i^{SM} g_i^2 + (S(t) - 1) b_i^{GHU} g_i^3$$

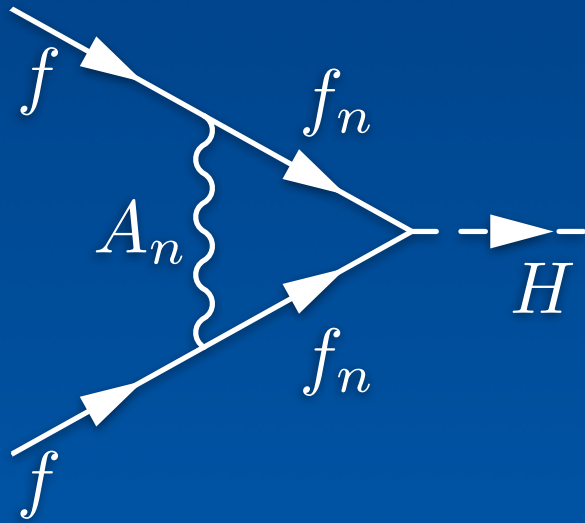
Encodes linear 5D runnings:

$$S(t) = \begin{cases} \mu R = m_Z R e^t & \text{for } \mu > 1/R \\ 1 & \text{for } m_Z < \mu < 1/R \end{cases}$$

$$b_i^{SM} = \left[\frac{41}{6}, -\frac{19}{6}, -7 \right] \quad b_i^{GHU} = \left[-\frac{17}{6}, -\frac{17}{2}, -\frac{17}{2} \right]$$

Running of the Yukawa

Complication: it depends on other couplings



We simplify by assuming that couplings of the KK modes follow the running of the gauge/Yukawa couplings

$$16\pi^2 \frac{dy_t}{dt} = \beta_t^{SM} + (S(t) - 1) \beta_t^{GHU}$$

$$\beta_t = y_t \left[c_t y_t^2 + \sum_i d_i g_i^2 \right]$$

The numerical values are:

$$c_t^{SM} = \frac{9}{2} \qquad d_i^{SM} = \left[-\frac{5}{12}, -\frac{9}{4}, -8 \right]$$

$$c_t^{GHU} = \frac{21}{2} \qquad d_i^{GHU} = \left[-\frac{35}{24}, -\frac{39}{8}, -4 \right]$$

Imposing unification relations:

$$g' = \sqrt{3}g, \quad y_t = \frac{g}{\sqrt{2}}$$

$$\beta_t = (-4g^2 - 4g_s^2) \frac{g}{\sqrt{2}}$$

Conclusions

- Gauge-Higgs Unification models have been prematurely abandoned
- Running of the gauge and Yukawa couplings needed to properly match the theory to the EW scale
- We show in a $SU(3)$ toy model that the tree level tensions can be softened by the running
- More work needs to be done towards a realistic model: Higgs mass?

Acknowledgements

We acknowledge support from the CNRS PICS project no. 07552 and the Campus France PHC PROTEA 2017 project no. 38192RC.

The work of AC and MK is also supported by the National Research Foundation of South Africa.

GC and AD also acknowledge partial support from the Labex-LIO (Lyon Institute of Origins) under grant ANR-10-LABX-66 and FRAMA (FR3127, Fédération de Recherche “André Marie Ampère”).

AD is partially supported by the “Institut Universitaire de France”.