

# Theory of the Inclusive Decays $\bar{B} \rightarrow X_{s,d}\ell\ell$

TOBIAS HURTH

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LIO International Conference on Flavour Physics

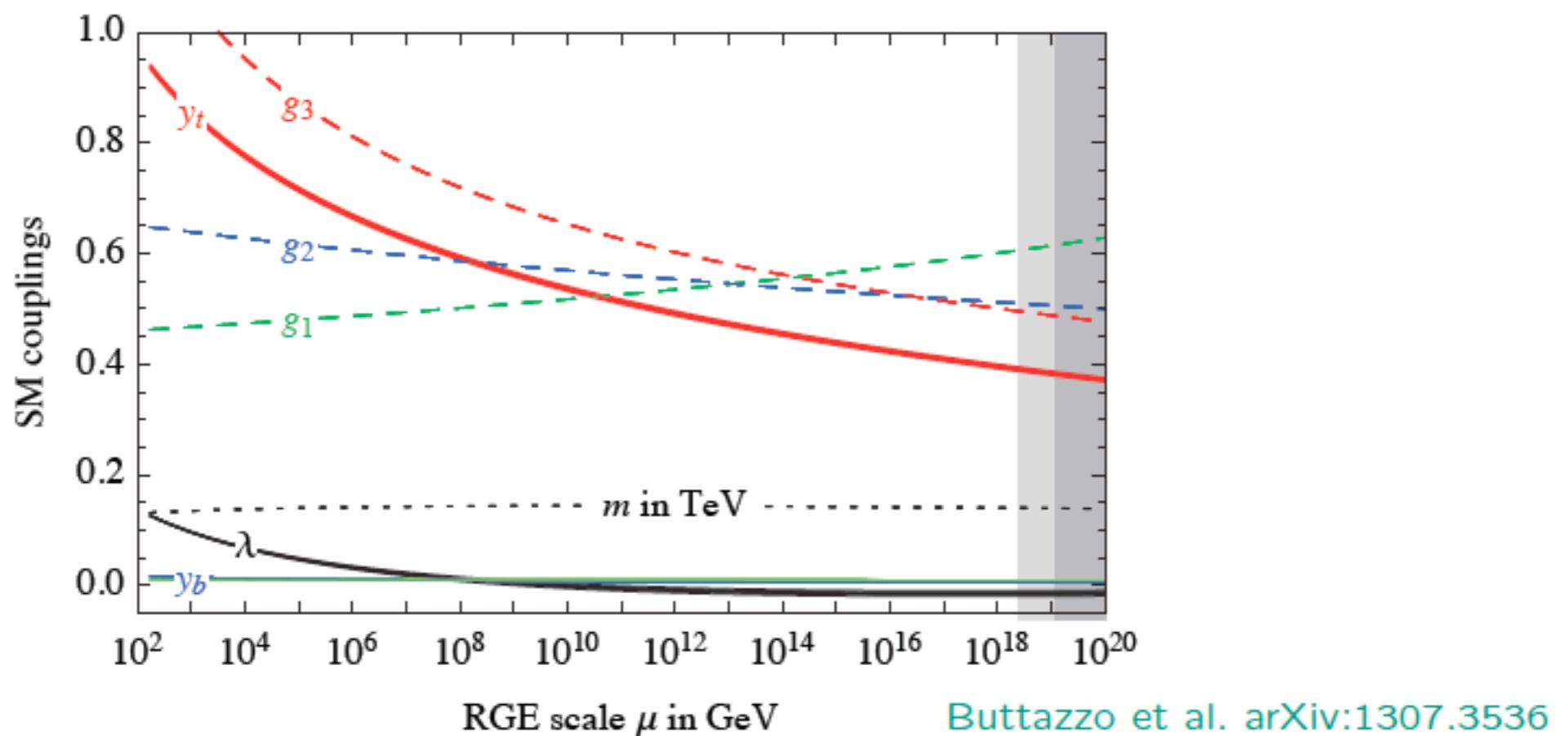
Lyon IPNL 18.-20.April 2018

# Prologue from my 2015 talk

# Self-consistency of the SM

Do we need new physics beyond the SM ?

- It is possible to extend the validity of the SM up to the  $M_P$  as weakly coupled theory.



High-energy extrapolation shows that the Yukawa couplings, weak gauge couplings and the Higgs self coupling remain perturbative in the entire energy domain between the electroweak and Planck scale (no Landau poles !).

- Renormalizability implies no constraints on the free parameters of the SM Lagrangian.

## Experimental evidence beyond SM

- Dark matter (visible matter accounts for only 4% of the Universe)
- Neutrino masses (Dirac or Majorana masses ?)
- Baryon asymmetry of the Universe (new sources of CP violation needed)

### Caveat:

Answers perhaps wait at energy scales which we do not reach with present experiments.

# Motivation: “From Flavour to New Physics”

## Error of Branching ratio $\bar{B} \rightarrow X_s \ell^+ \ell^-$

$BF$ (%) (stat,syst)	0.7/ab	5/ab	50/ab
[1.0,3.5]	29 (26,12)	13 (9.7,8.0)	6.6 (3.1,5.8)
[3.5,6.0]	24 (21,12)	11 (7.9,8.0)	6.4 (2.6,5.8)
$\geq 14.4$	23 (21,9)	10 (8.1,6.0)	4.7 (2.6,3.9)

## Error of Normalized Forward-Backward-Asymmetry

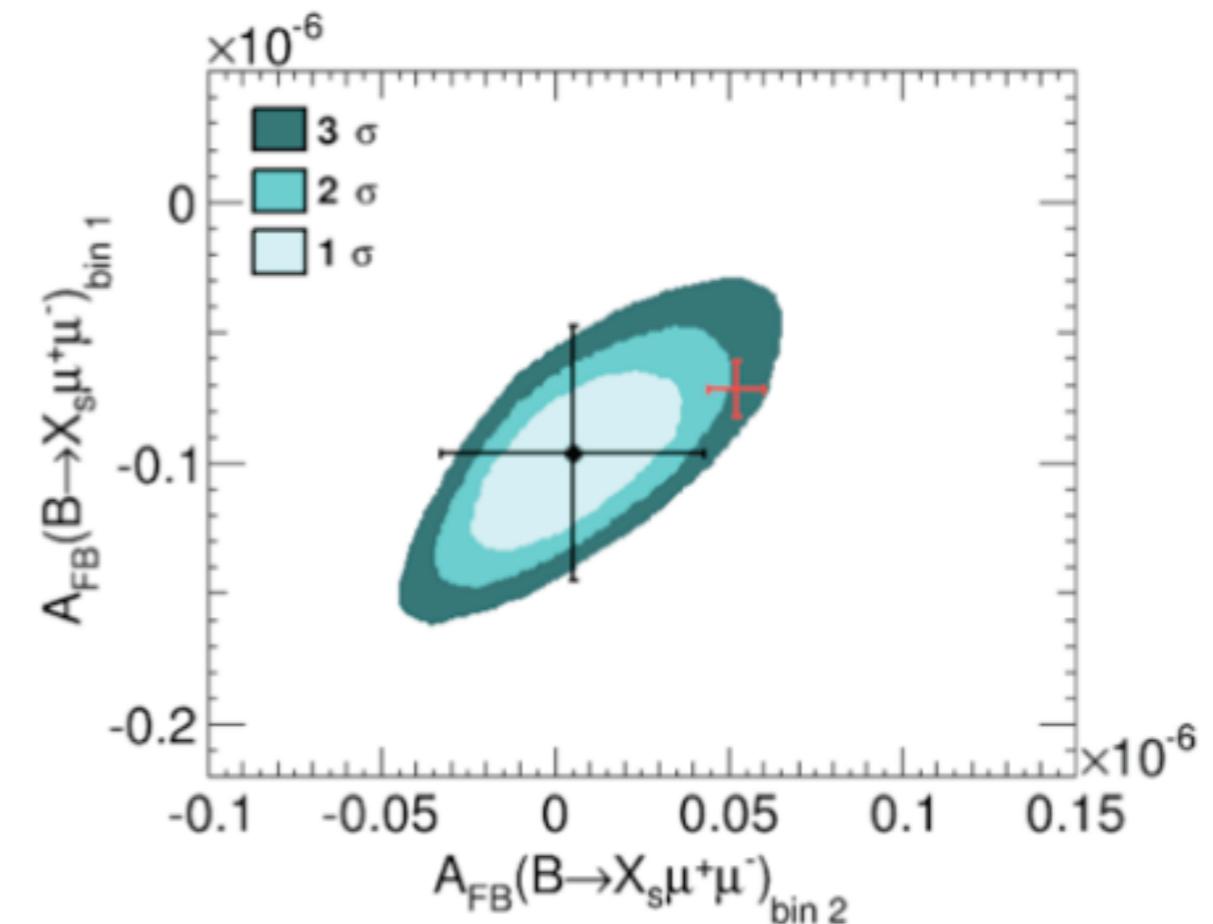
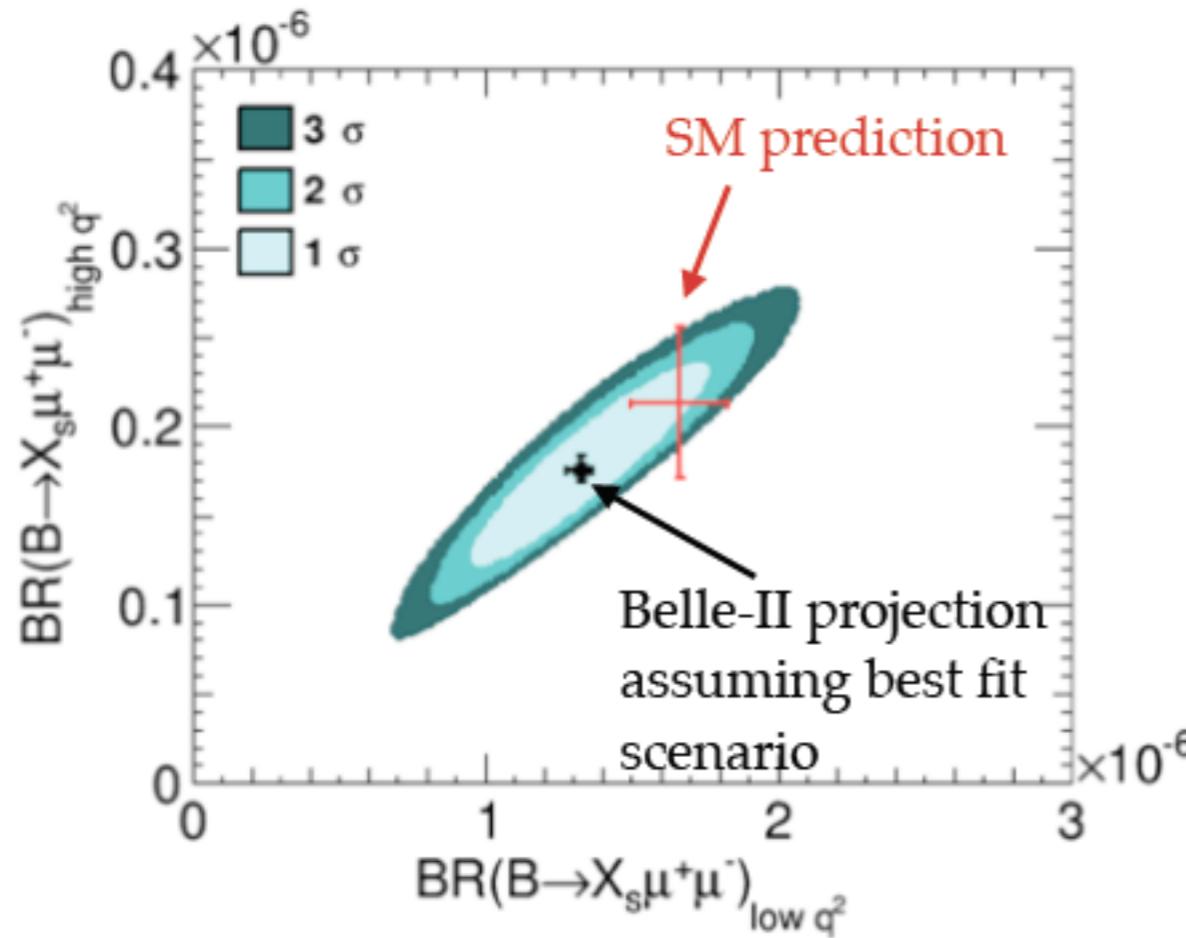
$AFBn$ (%) (stat,syst)	0.7/ab	5/ab	50/ab
[1.0,3.5]	26 (26,2.7)	9.7 (9.7,1.3)	3.1 (3.1,0.5)
[3.5,6.0]	21 (21,2.7)	7.9 (7.9,1.3)	2.6 (2.6,0.5)
$\geq 14.4$	19 (19,1.7)	7.3 (7.3,0.8)	2.4 (2.4,0.3)

$B \rightarrow (\pi, \rho) \ell^+ \ell^-$ , semi-inclusive  $\bar{B} \rightarrow X_d \ell^+ \ell^-$  at 50/ab  
 (uncertainties like  $\bar{B} \rightarrow X_s \ell^+ \ell^-$  at 0.7/ab)

# Crosscheck of LHCb anomalies with inclusive modes

Hurth,Mahmoudi,Neshatpour,arXiv:1410.4545

if SM deviations in  $R_K$  and  $P'_5$  persist until Belle-II



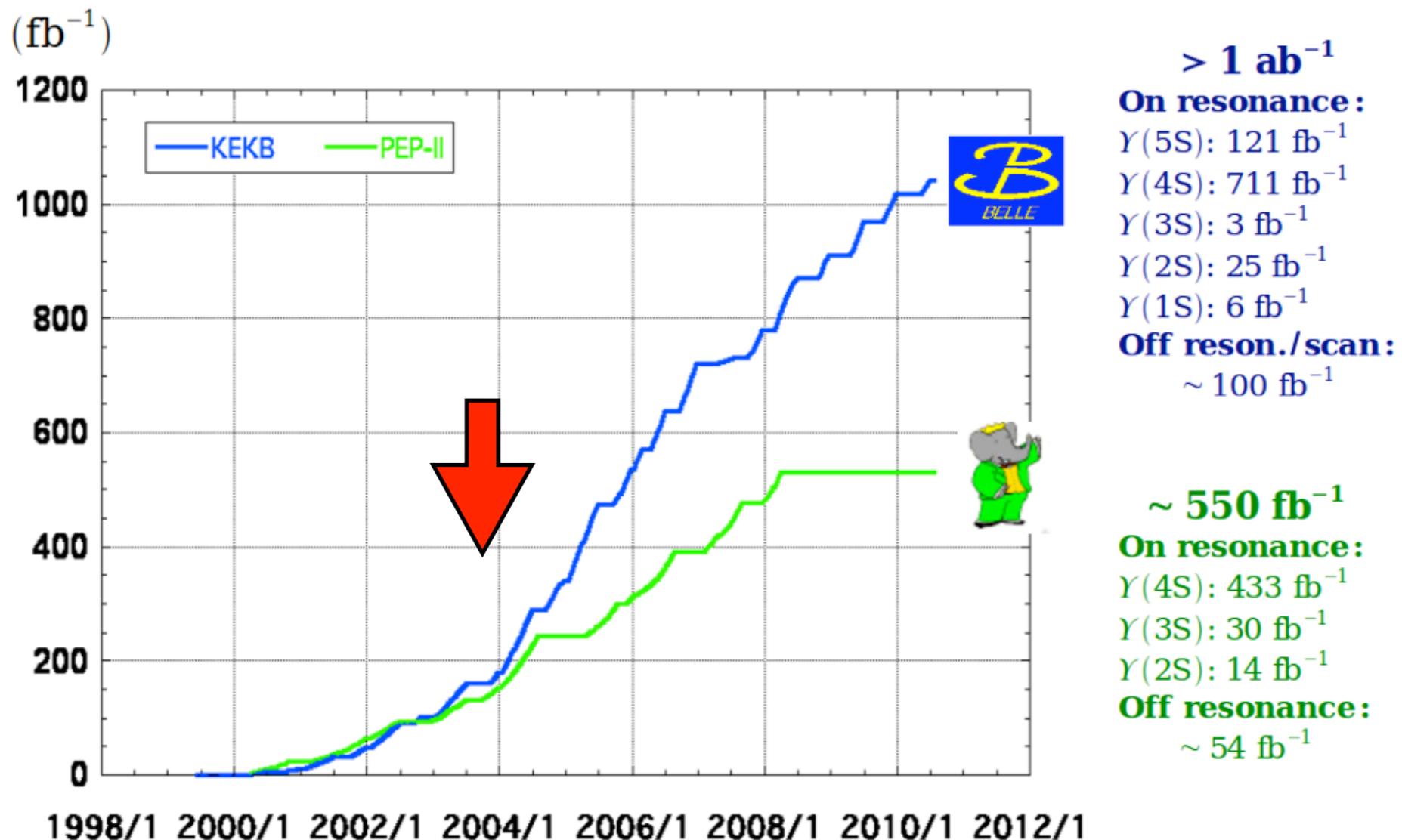
If NP then the effect of  $C_9$  and  $C'_9$  are large enough to be checked at Belle-II with theoretically clean modes.

# Experiment

- "Latest" Belle measurement of branching ratio is based on less than 30% of the total luminosity

Belle hep-ex/0503044 (!!!) (based  $152 \times 10^6 B\bar{B}$  events)

## Integrated luminosity of B factories

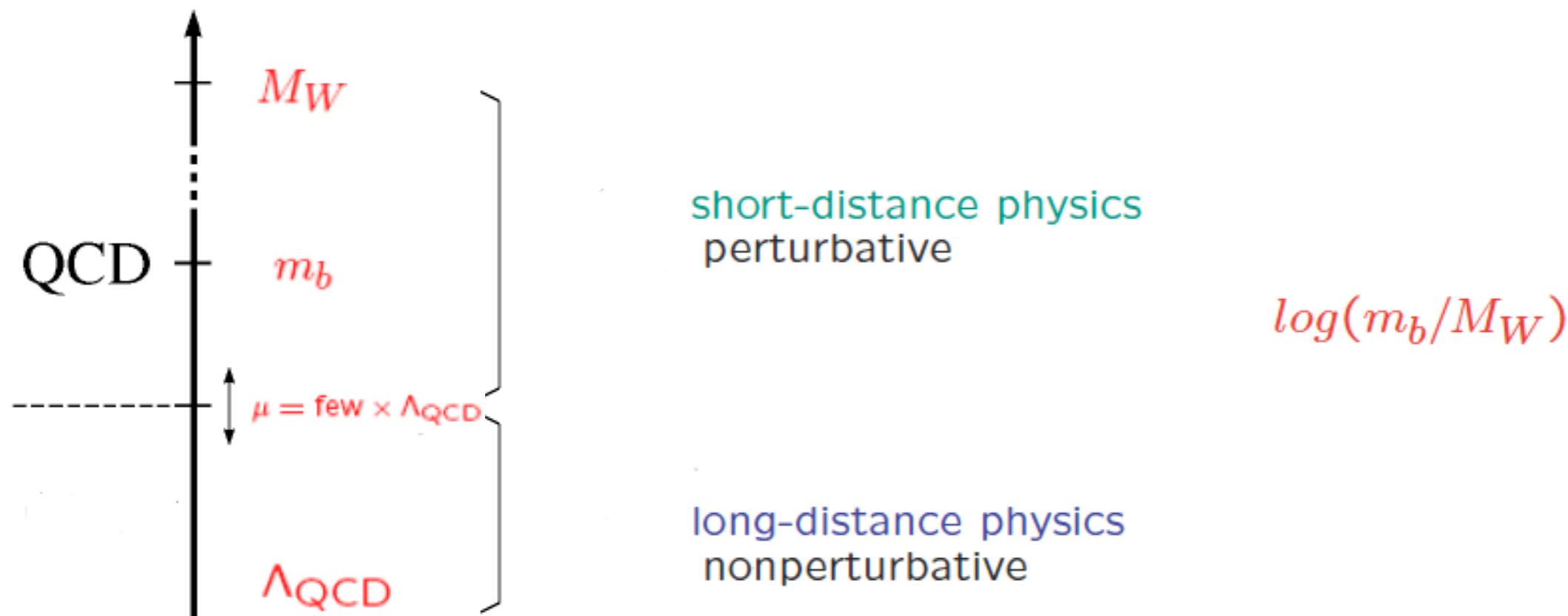


New Babar analysis on dilepton spectrum arXiv:1312.3664

New Belle analysis on AFB arXiv:1402.7134

# Theoretical Tools

# Theoretical tools for flavour precision observables



**Factorization theorems: separating long- and short-distance physics**

- Electroweak effective Hamiltonian:  $H_{eff} = -\frac{4G_F}{\sqrt{2}} \sum C_i(\mu, M_{heavy}) \mathcal{O}_i(\mu)$
- $\mu^2 \approx M_{New}^2 \gg M_W^2$ : 'new physics' effects:  $C_i^{SM}(M_W) + C_i^{New}(M_W)$

**How to compute the hadronic matrix elements  $\mathcal{O}_i(\mu = m_b)$  ?**

## Inclusive modes $B \rightarrow X_s \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$

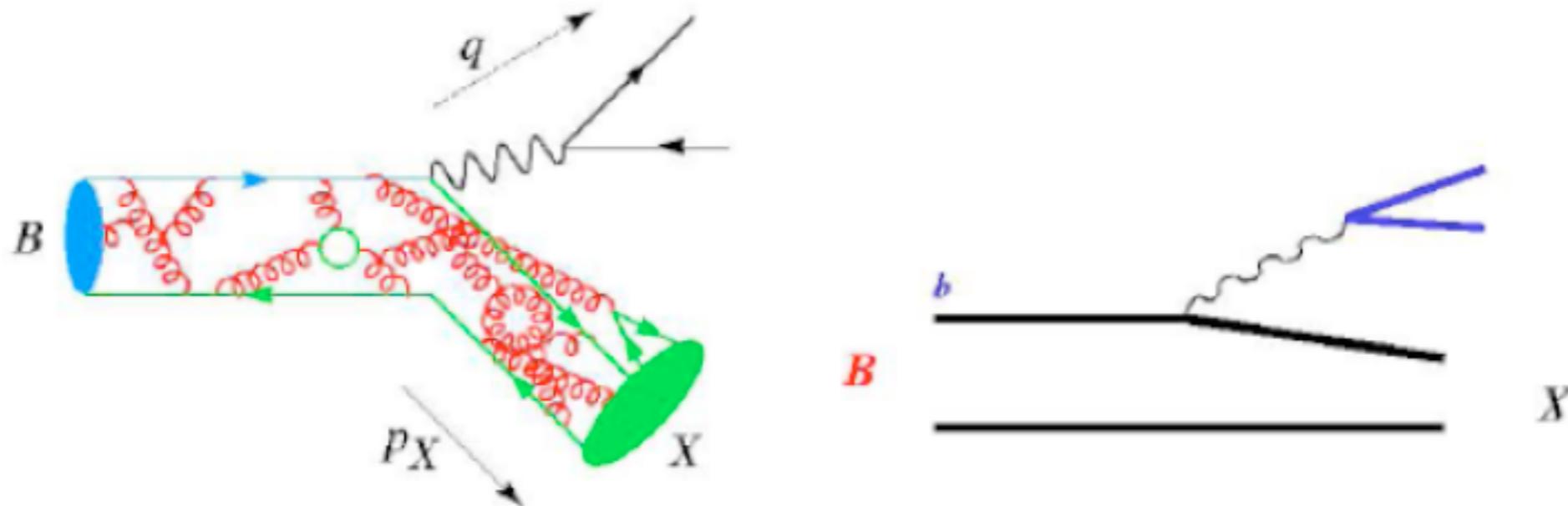
How to compute the hadronic matrix elements  $\mathcal{O}_i(\mu = m_b)$  ?

Heavy mass expansion for inclusive modes:

$$\Gamma(\bar{B} \rightarrow X_s \gamma) \xrightarrow{m_b \rightarrow \infty} \Gamma(b \rightarrow X_s^{\text{parton}} \gamma), \quad \Delta^{\text{nonpert.}} \sim \Lambda_{QCD}^2 / m_b^2$$

No linear term  $\Lambda_{QCD}/m_b$  (perturbative contributions dominant)

Chay, Georgi, Grinstein 1990



## Inclusive modes $B \rightarrow X_s \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$

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An old story:

- If one goes beyond the leading operator ( $\mathcal{O}_7, \mathcal{O}_9$ ):  
breakdown of local expansion

A new dedicated analysis:

naive estimate of non-local matrix elements leads to 5% uncertainty.

Benzke, Lee, Neubert, Paz, arXiv:1003.5012



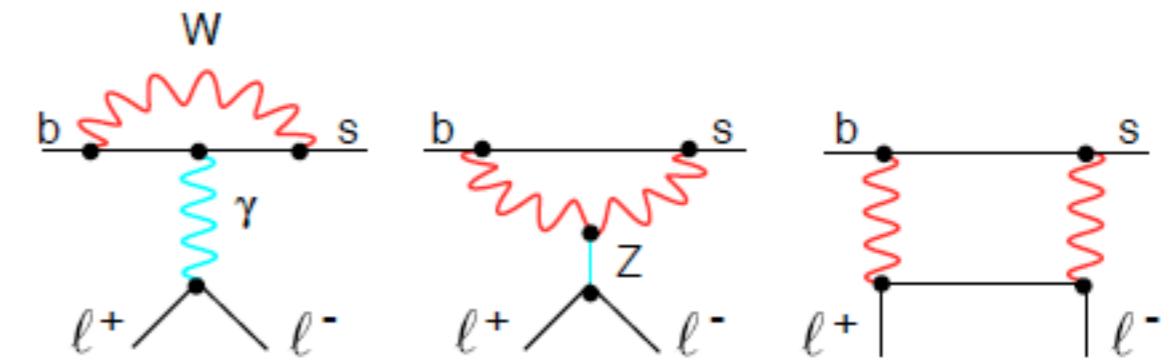
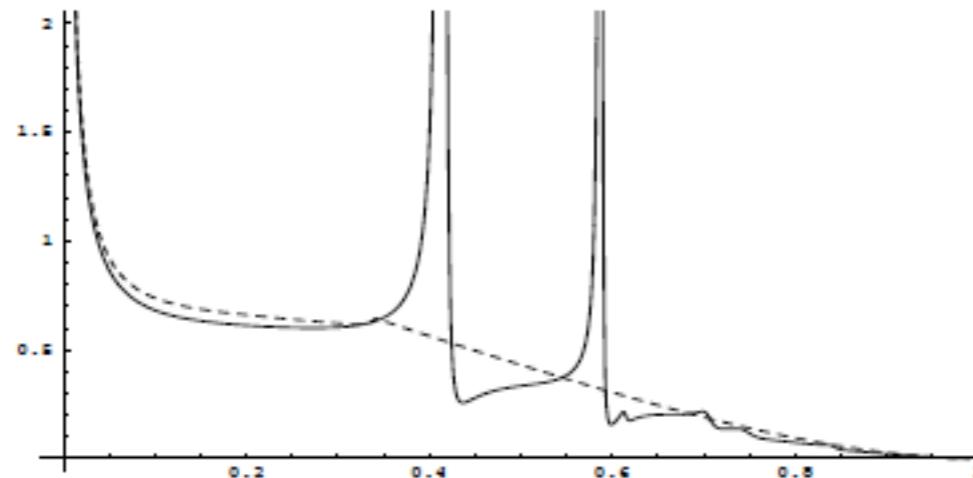
Analysis in  $B \rightarrow X_s \ell \ell$  in this talk; Benzke, Fickinger, Hurth, Turczyk

# Perturbative contributions

## Review of previous calculations for $B \rightarrow X_s l^+ l^-$

- On-shell- $c\bar{c}$ -resonances  $\Rightarrow$  cuts in dilepton mass spectrum necessary :  $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$  and  $14.4\text{GeV}^2 < q^2 \Rightarrow$  perturbative contributions dominant

$$\frac{d}{ds} BR(\bar{B} \rightarrow X_s l^+ l^-) \times 10^{-5}$$



$$\hat{s} = q^2/m_b^2$$

- NNLL prediction of  $\bar{B} \rightarrow X_s l^+ l^-$ : dilepton mass spectrum

Asatryan, Asatrian, Greub, Walker, hep-ph/0204341

Ghinculov, Hurth, Isidori, Yao, hep-ph/0312128

$$BR(\bar{B} \rightarrow X_s l^+ l^-)_{\text{Cut: } q^2 \in [1\text{GeV}^2, 6\text{GeV}^2]} = (1.63 \pm 0.20) \times 10^{-6}$$

$$BR(\bar{B} \rightarrow X_s l^+ l^-)_{\text{Cut: } q^2 > 14.4\text{GeV}^2} = (4.04 \pm 0.78) \times 10^{-7}$$

NNLL QCD corrections  $q^2 \in [1\text{GeV}^2, 6\text{GeV}^2]$

central value:  $-14\%$ ,      perturbative error:  $13\% \rightarrow 6.5\%$

- Further refinements:
  - Completing NNLL QCD corrections:  
Mixing into  $\mathcal{O}_9$  (+1%), NNLL matrixelement of  $\mathcal{O}_9$  (-4%)
  - NLL QED two-loop corrections to Wilson coefficients  
–1.5% shift for  $\alpha_{em}(\mu = m_b)$ , –8.5% for  $\alpha_{em}(\mu = m_W)$   
Bobeth,Gambino,Grbahn,Haisch,hep-ph/0312090
  - QED two-loop corrections to matrix elements  
Large collinear logarithm  $\text{Log}(m_b/m_\ell)$  which survive integration if a restricted part of the dilepton mass spectrum is considered  
+2% effect in the low- $q^2$  region for muons, for the electrons  
the effect depends on the experimental cut parameters  
Huber,Lunghi,Misiak,Wyler,hep-ph/0512066
- NNLL prediction of  $\bar{B} \rightarrow X_s \ell^+ \ell^-$ : forward-backward-asymmetry (FBA)  
Asatrian,Bieri,Greub,Hovhannisyan,hep-ph/0209006  
Ghinculov,Hurth,Isidori,Yao,hep-ph/0208088,hep-ph/0312128

$$A_{\text{FB}} \equiv \frac{1}{\Gamma_{\text{semilep}}} \left( \int_0^1 d(\cos \theta) \frac{d^2 \Gamma}{dq^2 d \cos \theta} - \int_{-1}^0 d(\cos \theta) \frac{d^2 \Gamma}{dq^2 d \cos \theta} \right)$$

( $\theta$  angle between  $\ell^+$  and  $B$  momenta in dilepton CMS)

$$A_{FB}(q_0^2) = 0 \quad \text{for} \quad q_0^2 \sim C_7/C_9 \quad q_0^2 = (3.90 \pm 0.25) \text{GeV}^2$$

- Electromagnetic corrections in high- $q^2$  and for  $A_{FB}$

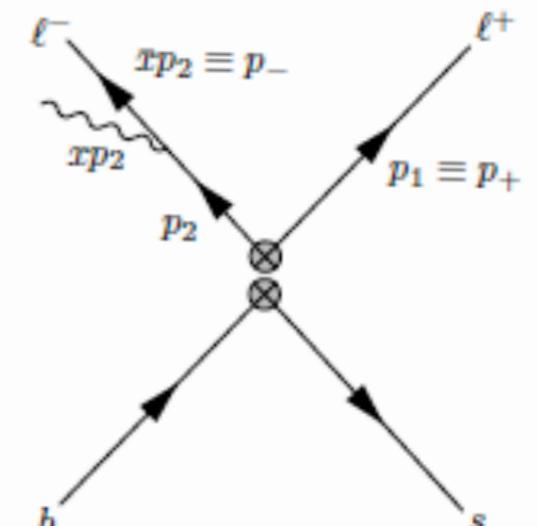
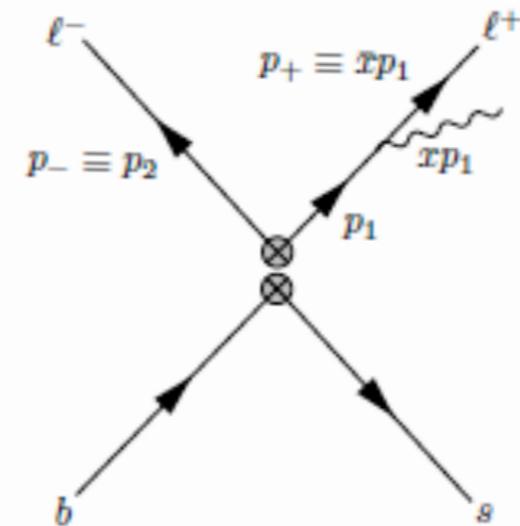
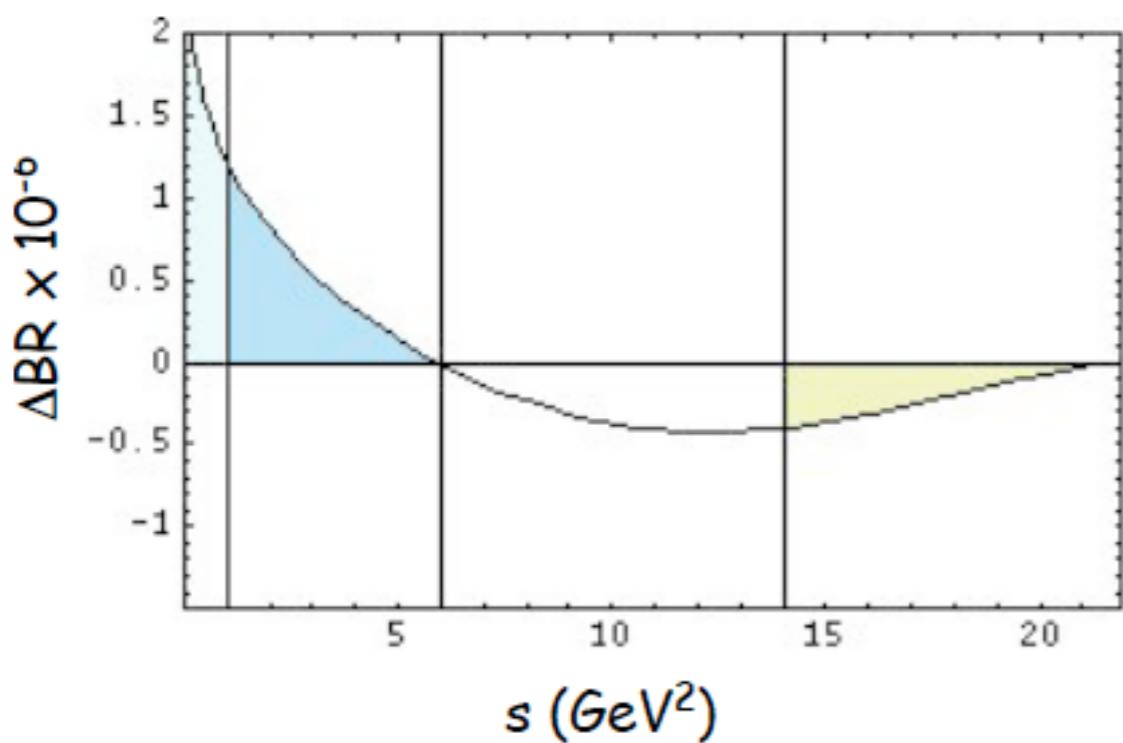
Huber,Hurth,Lunghi,Nucl.Phys.B802(2008)40

Corrections to matrix elements lead to large collinear log

$\log(m_b/m_\ell)$

$$\delta\text{BR}(B \rightarrow X_s \mu^+ \mu^-) = \begin{cases} (+2.0\%) & \text{low } q^2 \\ (-6.8\%) & \text{high } q^2 \end{cases}$$

$$\delta\text{BR}(B \rightarrow X_s e^+ e^-) = \begin{cases} (+5.2\%) & \text{low } q^2 \\ (-17.6\%) & \text{high } q^2 \end{cases}$$



# Complete angular analysis of inclusive $B \rightarrow X_s ll$

Huber,Hurth,Lunghi, arXiv:1503.04849

- Phenomenological analysis to NNLO QCD and NLO QED for all angular observables

$$\frac{d^2\Gamma}{dq^2 dz} = \frac{3}{8} [(1+z^2) H_T(q^2) + 2z H_A(q^2) + 2(1-z^2) H_L(q^2)] \quad (z = \cos \theta_\ell)$$

$$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2)$$

$$\frac{dA_{FB}}{dq^2} = 3/4 H_A(q^2)$$

- Dependence on Wilson coefficients

$H_T$  suppressed in low- $q^2$  window

$$H_T(q^2) \propto 2s(1-s)^2 \left[ |C_9 + \frac{2}{s} C_7|^2 + |C_{10}|^2 \right]$$

$$H_A(q^2) \propto -4s(1-s)^2 \operatorname{Re} \left[ C_{10} \left( C_9 + \frac{2}{s} C_7 \right) \right]$$

$$H_L(q^2) \propto (1-s)^2 \left[ |C_9 + 2 C_7|^2 + |C_{10}|^2 \right]$$

- Devide low- $q^2$  bin in two bins (zero of  $H_A$  in low- $q^2$ )

Lee,Ligeti,Stewart, Tackmann hep-ph/0612156

# New physics sensitivity

Huber,Hurth,Lunghi, arXiv:1503.04849

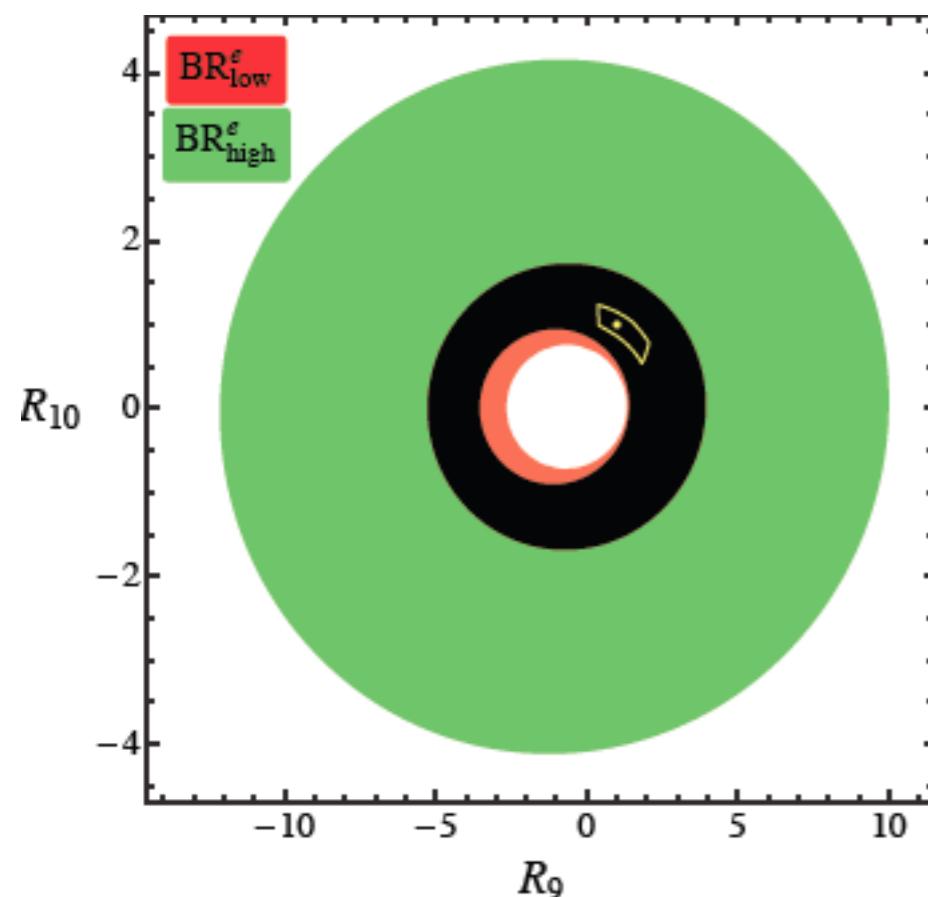
Constraints on Wilson coefficients  $C_9/C_9^{\text{SM}}$  and  $C_{10}/C_{10}^{\text{SM}}$

$$R_i = \frac{C_i(\mu_0)}{C_i^{\text{SM}}(\mu_0)}$$

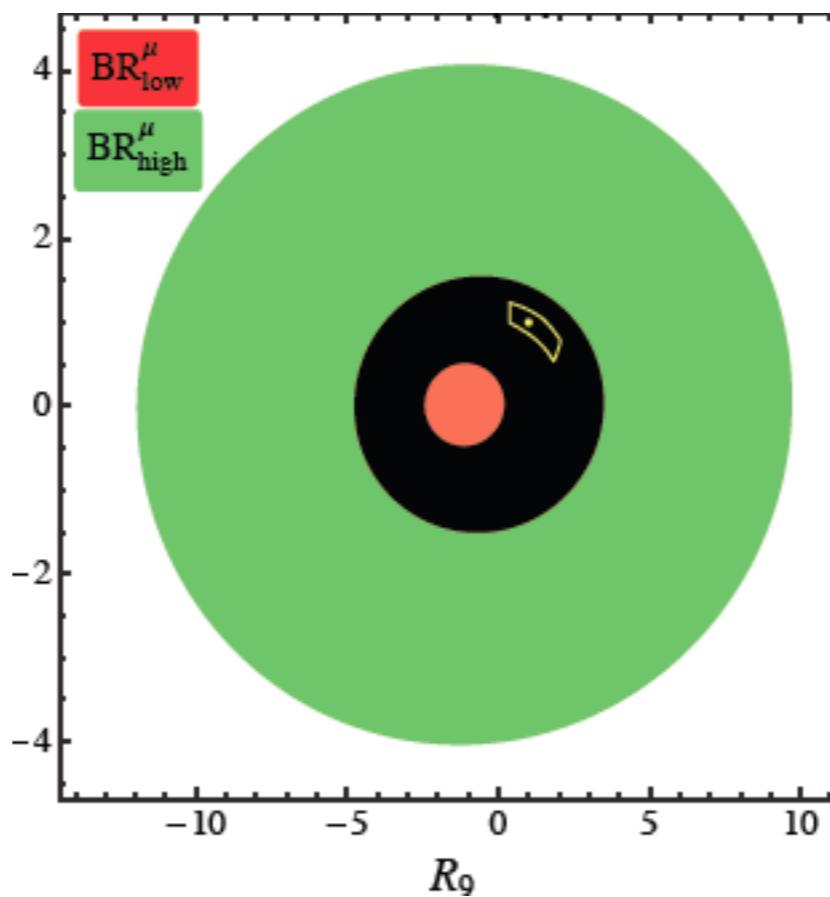
that we obtain at 95% C.L. from present experimental data  
(red low  $q^2$ , green high  $q^2$ )

that we will obtain at 95% C.L. from  $50\text{ab}^{-1}$  data at Belle-II  
(yellow)

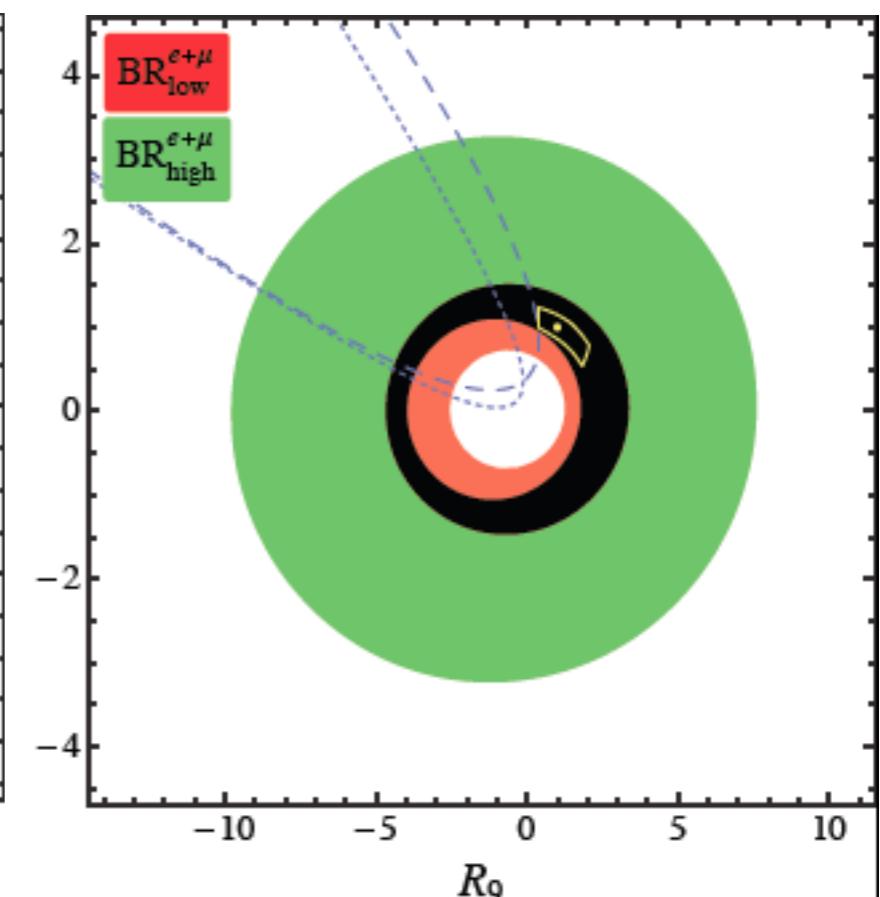
$B \rightarrow X_s ee$



$B \rightarrow X_s \mu\mu$



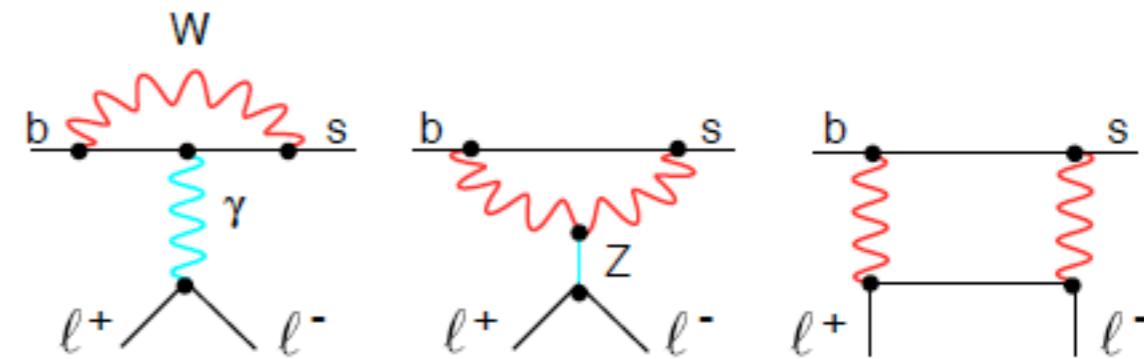
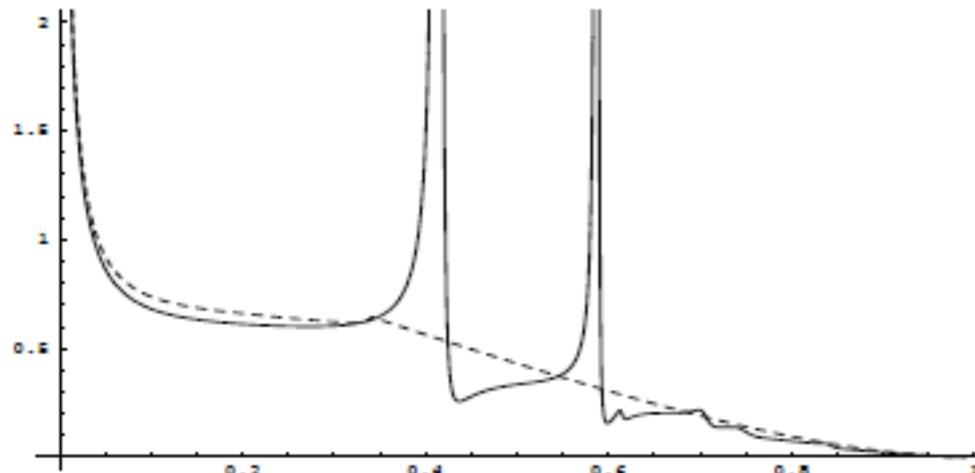
$B \rightarrow X_s \ell\ell$



## Cuts in the dilepton and hadronic mass spectra

- On-shell- $c\bar{c}$ -resonances  $\Rightarrow$  cuts in dilepton mass spectrum necessary :  
 $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$  and  $14.4\text{GeV}^2 < q^2 \Rightarrow$  perturbative contributions dominant

$$\frac{d}{ds} BR(\bar{B} \rightarrow X_s l^+ l^-) \times 10^{-5}$$



- Hadronic invariant-mass cut is imposed in order to eliminate the background like  $b \rightarrow c (\rightarrow se^+\nu)e^-\bar{\nu} = b \rightarrow se^+e^- + \text{missing energy}$ 
  - Babar,Belle:  $m_X < 1.8$  or  $2.0\text{GeV}$
  - high- $q^2$  region not affected by this cut
  - kinematics:  $X_s$  is jetlike and  $m_X^2 \leq m_b \Lambda_{QCD}$   $\Rightarrow$  shape function region
  - SCET analysis: universality of jet and shape functions found:  
the 10-30% reduction of the dilepton mass spectrum can be accurately computed using the  $\bar{B} \rightarrow X_s \gamma$  shape function  
5% additional uncertainty for  $2.0\text{GeV}$  cut due to subleading shape functions

Lee,Stewart hep-ph/0511334

Lee,Ligeti,Stewart,Tackmann hep-ph/0512191

Lee,Tackmann arXiv:0812.0001 (effect of subleading shape functions)

Bell,Beneke,Huber,Li arXiv:1007.3758 (NNLO matching QCD  $\rightarrow$  SCET)

# Nonlocal subleading contributions

# Subleading power factorization in $B \rightarrow X_s \ell^+ \ell^-$

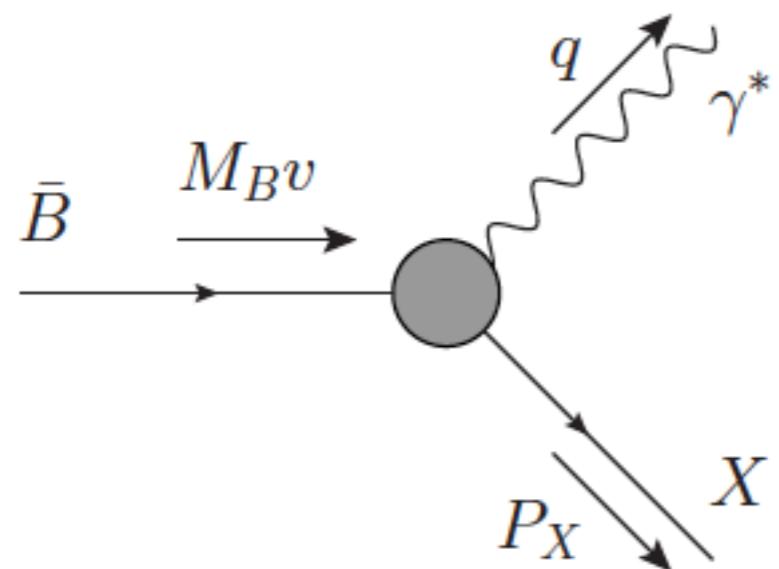
Benzke, Hurth, Turczyk, arXiv:1705.10366

## Hadronic cut

Additional cut in  $X_s$  necessary to reduce background  
affects only low- $q^2$  region.

Hadronic invariant  $m_X^2 < 1.8(2.0) \text{GeV}^2$ , jet-like  $X_s$   $E_X \sim \mathcal{O}(m_b)$

Multiscale problem  $\rightarrow$  SCET



$$M_B^2 \sim m_b^2 \gg m_X^2 \sim \Lambda_{\text{QCD}} m_b \gg \Lambda_{\text{QCD}}^2$$

$$m_X^2 = P_X^2 = (M_B - n \cdot q)(M_B - \bar{n} \cdot q)$$

**Scaling**

$$\lambda = \Lambda_{\text{QCD}} / m_b$$

## Kinematics

$B$  meson rest frame

$$q = p_B - p_X \quad 2 m_B E_X = m_B^2 + M_X^2 - q^2$$

$X_s$  system is jet-like with  $E_X \sim m_B$  and  $m_X^2 \ll E_X^2$

two light-cone components  $p_X^- p_X^+ = m_X^2$

$$\bar{n} p_X = p_X^- = E_X + |\vec{p}_X| \sim \mathcal{O}(m_B)$$

$$n p_X = p_X^+ = E_X - |\vec{p}_X| \sim \mathcal{O}(\Lambda_{\text{QCD}})$$

$$q^+ = nq = m_B - p_X^+ \quad q^- = \bar{n}q = m_B - p_X^-$$

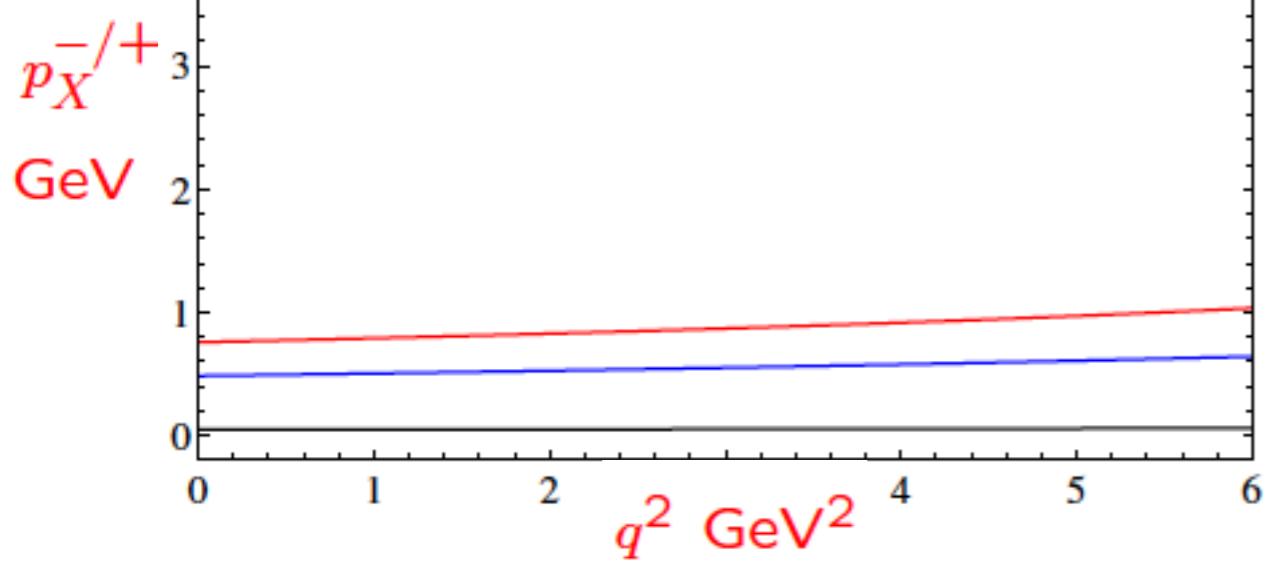
## Scaling

$$\lambda = \Lambda_{\text{QCD}} / m_b$$

$$m_X^2 \sim \lambda \Rightarrow m_b - n \cdot q \sim \lambda$$

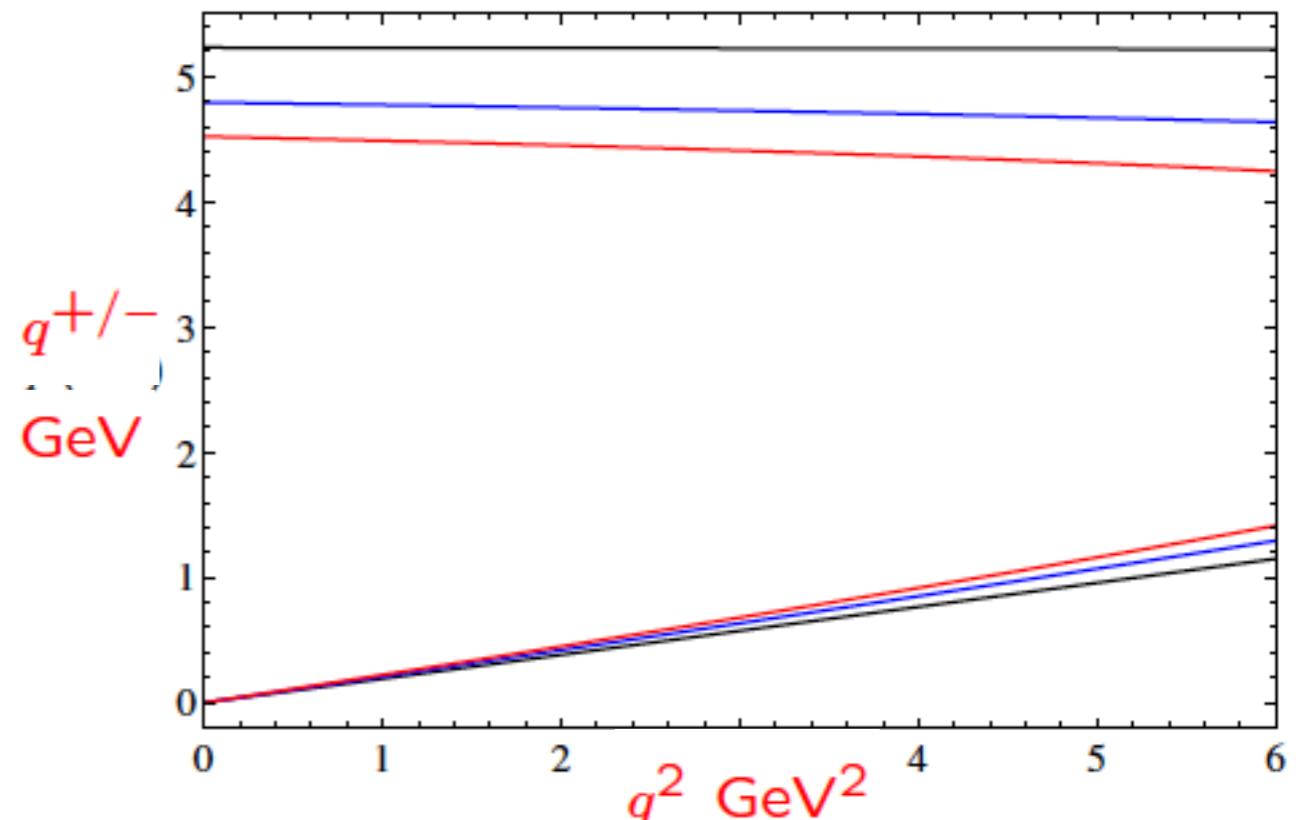
$M_x = [0.5, 1.6, 2] \text{ GeV}$  [Black,Blue,Red]

Upper lines :  $P_X^-$ , lower lines :  $P_X^+$



$M_x = [0.5, 1.6, 2] \text{ GeV}$  [Black,Blue,Red]

Upper lines :  $q^+$ , lower lines :  $q^-$



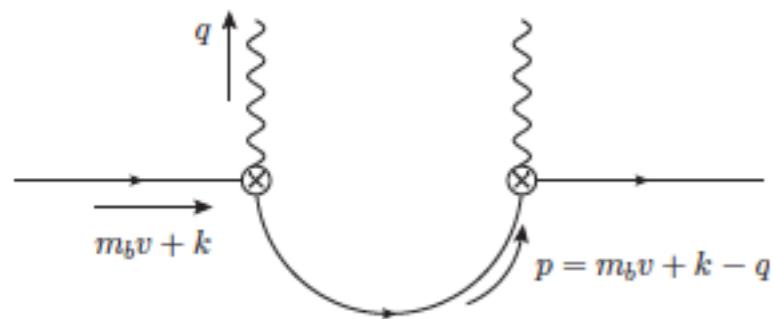
For  $q^2 < 6 \text{ GeV}^2$  the scaling of  $n p_X$  and  $\bar{n} p_X$  implies  $\bar{n} q$  is of order  $\lambda$ , means  $q$  anti-hard-collinear (just kinematics).

Stewart and Lee assume  $\bar{n} q$  to be order 1, means  $q$  is hard.

This problematic assumption implies a different matching of SCET/QCD.

## Shapefunction region

Local OPE breaks down for  $m_X^2 \sim \lambda$ :



$$\frac{1}{(m_b v + k - q)^2} = \frac{1}{m_b - n \cdot q} \left( 1 - \frac{n \cdot k}{m_b - n \cdot q} + \dots \right) \frac{1}{m_b - \bar{n} \cdot q}$$

Resummation of leading contributions into a shape function.

(scaling of  $\bar{n}q$  does not matter here; zero in case of  $B \rightarrow X_s \gamma$ )

## Factorization theorem

$$d\Gamma \sim H \cdot J \otimes S$$

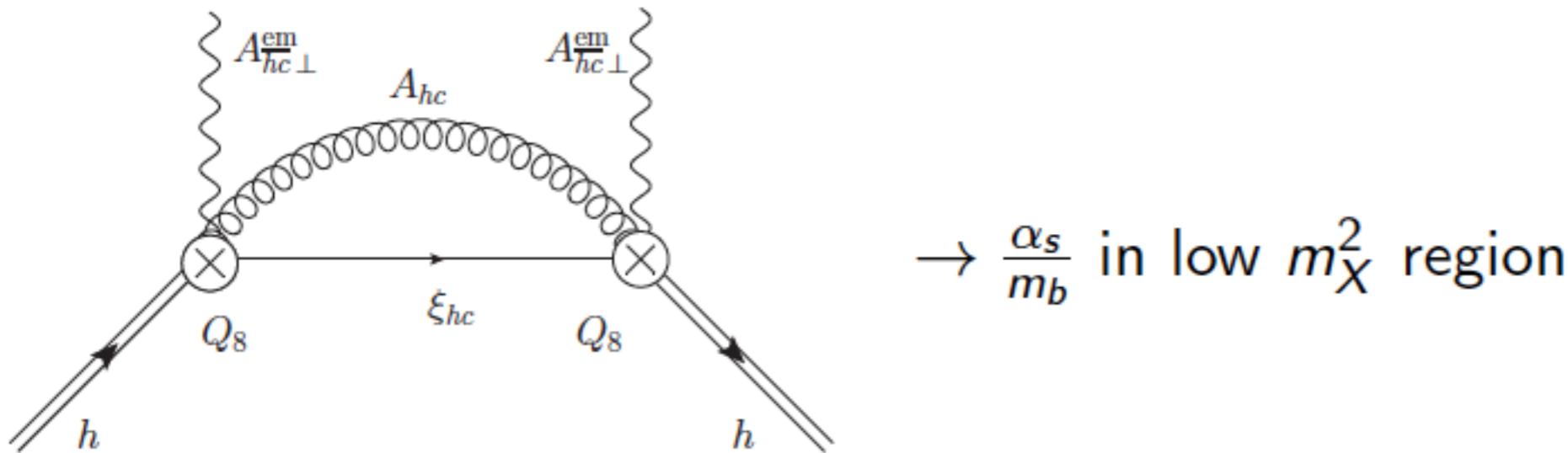
The hard function  $H$  and the jet function  $J$  are perturbative quantities.

The shape function  $S$  is a non-perturbative non-local HQET matrix element.  
(universality of the shape function, uncertainties due to subleading shape functions)

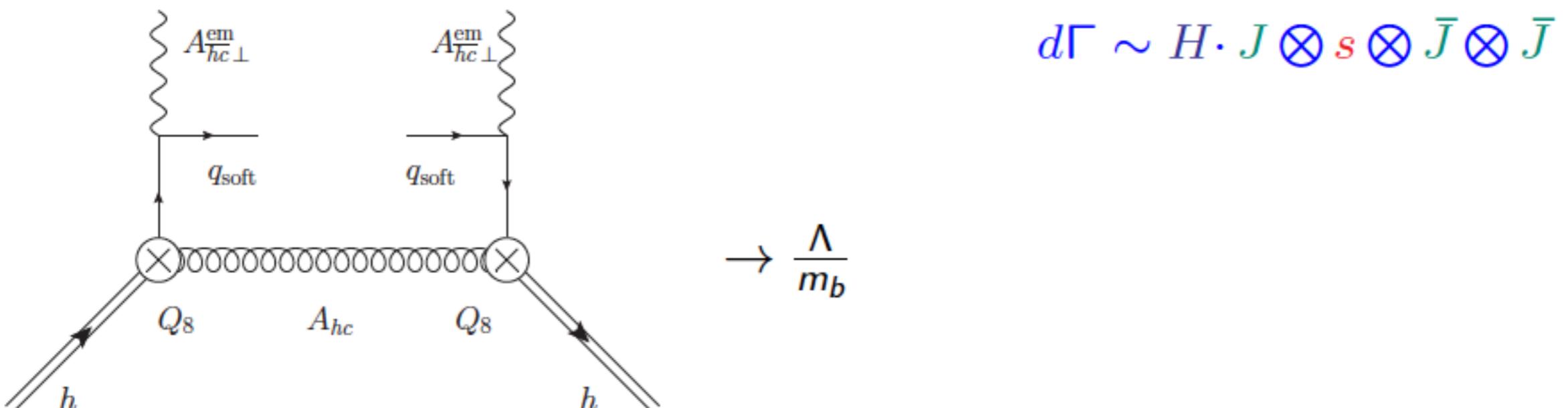
## Calculation at subleading power

Example of **direct** photon contribution which factorizes

$$d\Gamma \sim H \cdot j \otimes S$$

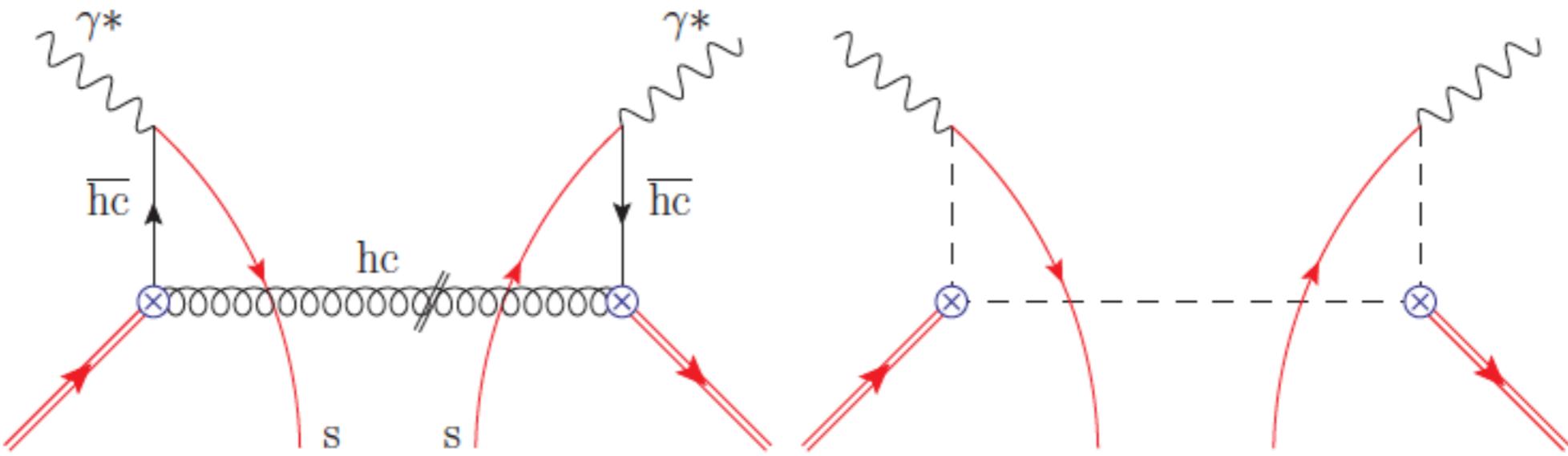


Example of **resolved** photon contribution (double-resolved) which factorizes



In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.

## Interference of $Q_8$ and $\bar{Q}_8$



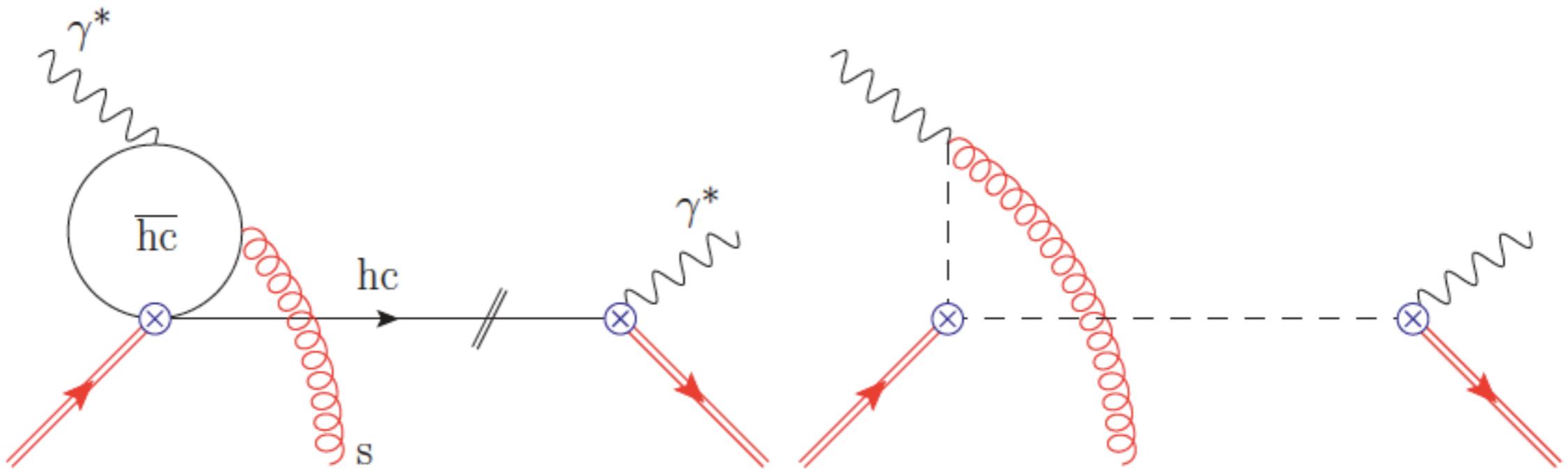
$$\frac{d\Gamma^{\text{res}}}{dn \cdot q \, d\bar{n} \cdot q} \sim \frac{e_s^2 \alpha_s}{m_b} \int d\omega \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + \bar{n} \cdot q + i\varepsilon} \int \frac{d\omega_2}{\omega_2 + \bar{n} \cdot q - i\varepsilon} g_{88}(\omega, \omega_1, \omega_2)$$

$$g_{88}(\omega, \omega_1, \omega_2) = \frac{1}{M_B} \langle \bar{B} | \bar{h}(\mathbf{t}\mathbf{n}) \dots s(\mathbf{t}\mathbf{n} + \mathbf{u}\bar{\mathbf{n}}) \bar{s}(\mathbf{r}\bar{\mathbf{n}}) \dots h(\mathbf{0}) | \bar{B} \rangle_{\text{F.T.}}$$

Shape function is non-local in two light-cone directions.

It survives  $M_X \rightarrow 1$  limit (irreducible uncertainty).

## Interference of $Q_1$ and $Q_7$



$$\frac{d\Gamma^{\text{res}}}{dn \cdot q d\bar{n} \cdot q} \sim \frac{1}{m_b} \int d\omega \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + i\varepsilon}$$

$$\frac{1}{\omega_1} \left[ \bar{n} \cdot q \left( F \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) - 1 \right) - (\bar{n} \cdot q + \omega_1) \left( F \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) - 1 \right) \right.$$

$$\left. + \bar{n} \cdot q \left( G \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) - G \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) \right) \right] g_{17}(\omega, \omega_1)$$

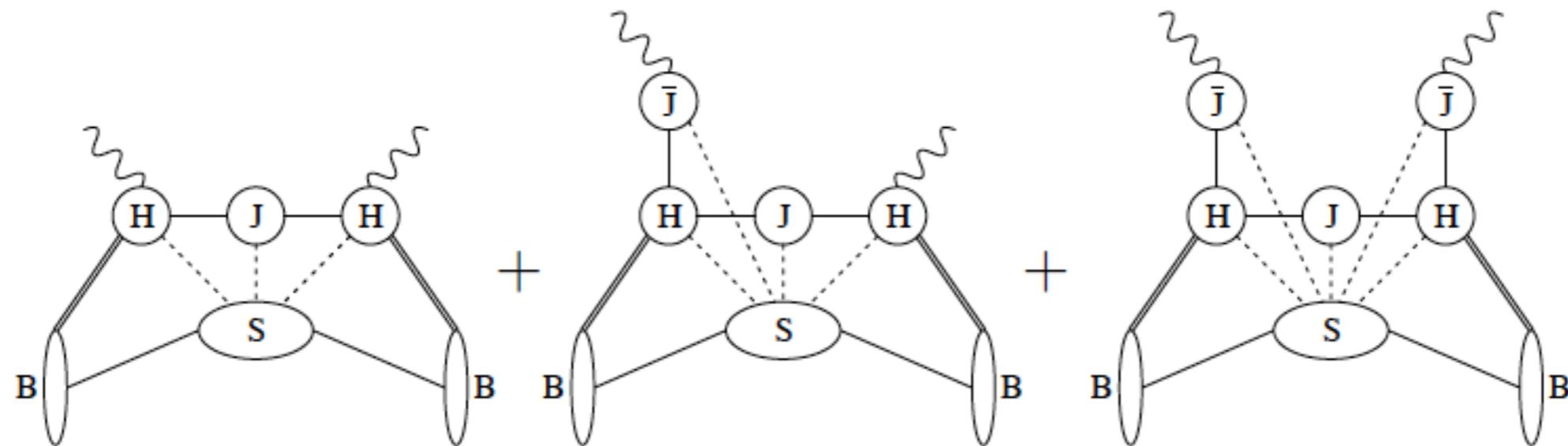
$$g_{17}(\omega, \omega_1) = \int \frac{dr}{2\pi} e^{-i\omega_1 r} \int \frac{dt}{2\pi} e^{-i\omega t} \frac{1}{M_B} \langle \bar{B} | \bar{h}(tn) \dots G_s^{\alpha\beta}(r\bar{n}) \dots h(0) | \bar{B} \rangle$$

Expansion for  $m_c \sim m_b$  leads to Voloshin term in the total rate ( $-\lambda_2/m_c^2$ ), the terms stays non-local for  $m_c < m_b$ .

## Factorization formula

In the  $m_X^2 \sim \lambda$  and  $q^2 \sim \lambda$  region we have the following factorization formula

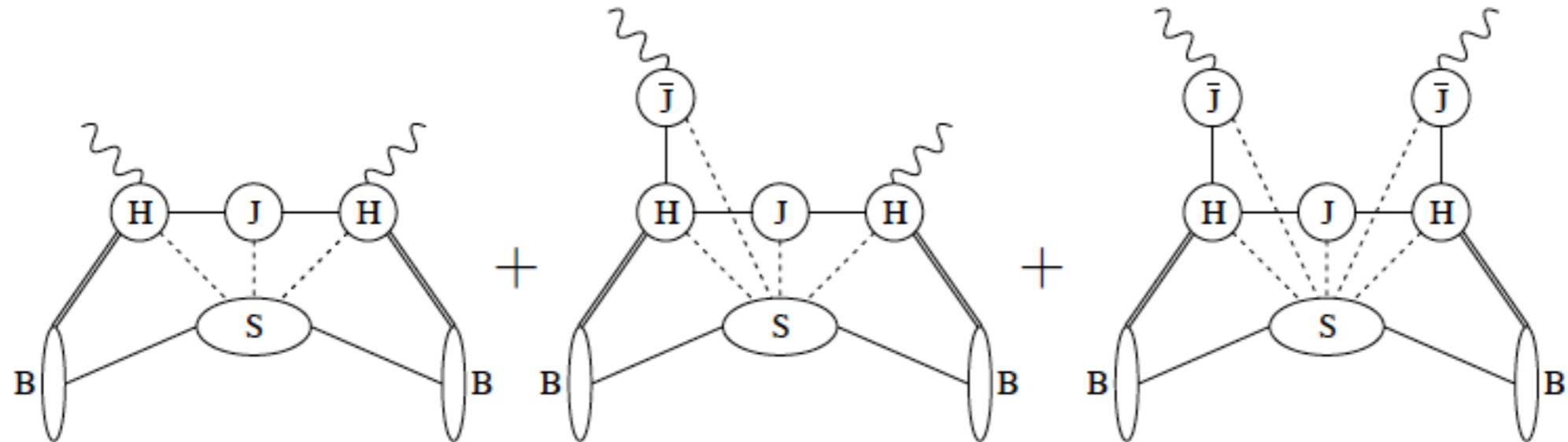
$$\begin{aligned} d\Gamma \sim & H \cdot J \otimes S + \frac{1}{m_b} \sum H \cdot j_i \otimes S + \frac{1}{m_b} \sum H \cdot J \otimes s_i \\ & + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} \otimes \bar{J} + \mathcal{O}\left(\frac{1}{m_b^2}\right) \end{aligned}$$



## Factorization formula

In the  $m_X^2 \sim \lambda$  and  $q^2 \sim \lambda$  region we have the following factorization formula

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum H \cdot j_i \otimes S + \frac{1}{m_b} \sum H \cdot J \otimes s_i \\ + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} \otimes \bar{J} + \mathcal{O}\left(\frac{1}{m_b^2}\right)$$



Subtlety in the  $Q_8$  and  $\bar{Q}_8$  contribution: convolution integral is UV divergent

- This subtlety implies that there is no complete proof of the factorization formula.
- Nevertheless one shows that scale dependence of direct and resolved contribution cancel.
- No direct analogy to the problem of IR divergent convolution integrals in power-suppressed contributions to exclusive B decays.

## Angular observables

$$\frac{d^2\Gamma}{dq^2 dz} = \frac{3}{8} \left[ (1+z^2) H_T(q^2) + 2(1-z^2) H_L(q^2) + 2z H_A(q^2) \right]$$

$$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2) \quad \frac{dA_{FB}}{dq^2} = 3/4 H_A(q^2)$$

$$d\Gamma(\bar{B} \rightarrow X_s \ell^+ \ell^-) \equiv d\Lambda_{\alpha\beta} W^{\alpha\beta}(v, q),$$

$$d\Lambda_{\alpha\beta;1/m_b} = dn \cdot q d\bar{n} \cdot q dz \frac{\alpha}{128\pi^3} (1+z^2) \frac{n \cdot q}{\bar{n} \cdot q} g_{\perp,\alpha\beta}.$$

At  $O(1/m_b)$  nonlocal powercorrections only to  $H_T(q^2)$ .

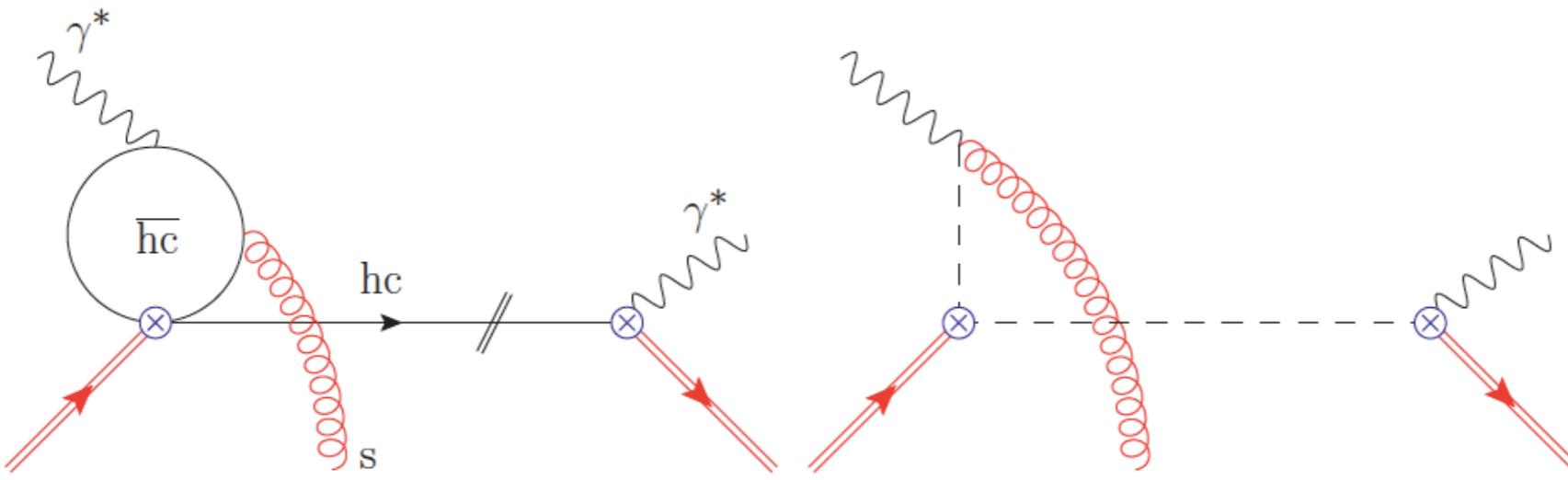
## Numerical evaluation

Benzke,Hurth,Turczyk, arXiv:1705.10366

- Subleading shape functions of resolved contributions similar to  $b \rightarrow s\gamma$
- Use explicit definition to determine properties:
  - \* PT invariance: soft functions are real
  - \* Moments of  $g_{17}$  related to HQET parameters
  - \* Vacuum insertion approximation relates  $g_{78}$  to the B meson LCDA
- Perform convolution integrals with model functions

## Numerical evaluation

$$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$$



$$d\Gamma_{17} = \frac{1}{m_b} \operatorname{Re} \left[ \hat{\Gamma}_{17} \frac{-(\lambda_t^q)^* \lambda_c^q}{|\lambda_t^q|^2} \right] \frac{\alpha}{24\pi^3} dn \cdot q d\bar{n} \cdot q \frac{(n \cdot q)^3}{\bar{n} \cdot q}$$

$$\times \operatorname{Re} \int d\omega \delta(\omega + m_b - n \cdot q) \int d\omega_1 \frac{1}{\omega_1 + i\epsilon}$$

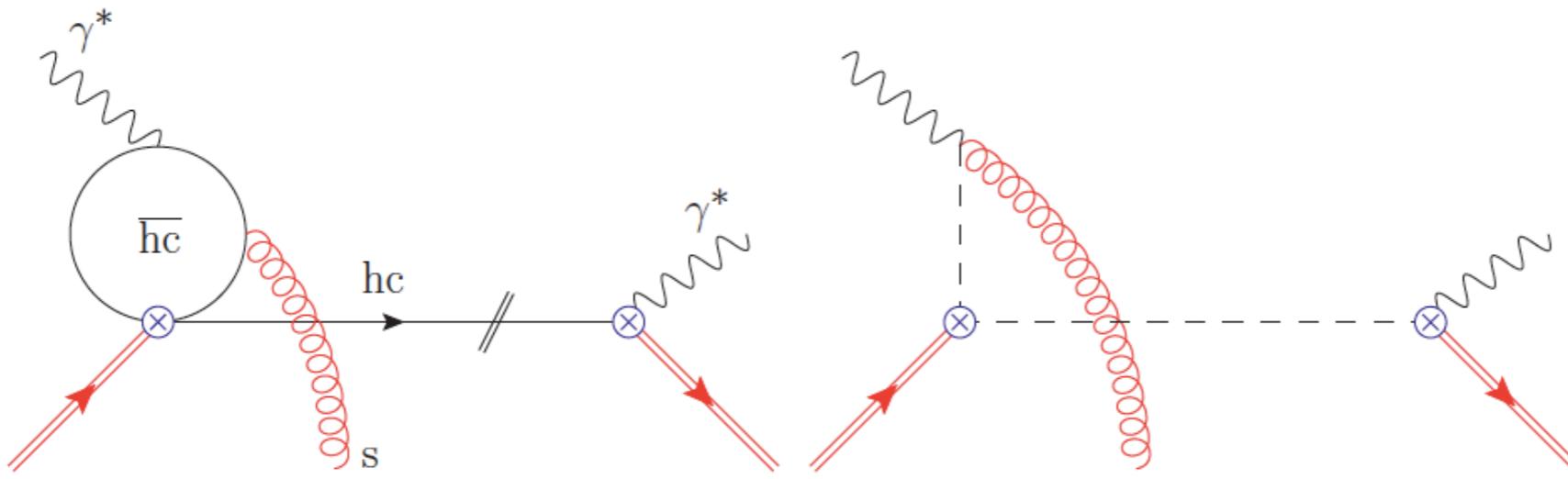
$$\begin{aligned} & \times \frac{1}{\omega_1} \left[ (\bar{n} \cdot q + \omega_1) \left( 1 - F \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) \right) - \bar{n} \cdot q \left( 1 - F \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) \right) \right. \\ & \quad \left. - \bar{n} \cdot q \left( G \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) - G \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) \right) \right] g_{17}(\omega, \omega_1, \mu), \end{aligned}$$

$$g_{17}(\omega, \omega_1, \mu) = \int \frac{dr}{2\pi} e^{-i\omega_1 r} \int \frac{dt}{2\pi} e^{-i\omega t}$$

$$\times \frac{\langle \bar{B} | (\bar{h} S_n)(tn) \not{\epsilon}(1+\gamma_5) (S_n^\dagger S_{\bar{n}})(0) i\gamma_\alpha^\perp \bar{n}_\beta (S_{\bar{n}}^\dagger g G_s^{\alpha\beta} S_{\bar{n}})(r\bar{n}) (S_{\bar{n}}^\dagger h)(0) | \bar{B} \rangle}{2M_B}$$

## Numerical evaluation

$$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$$



$$d\Gamma_{17} = \frac{1}{m_b} \operatorname{Re} \left[ \hat{\Gamma}_{17} \frac{-(\lambda_t^q)^* \lambda_c^q}{|\lambda_t^q|^2} \right] \frac{\alpha}{24\pi^3} dn \cdot q d\bar{n} \cdot q \frac{(n \cdot q)^3}{\bar{n} \cdot q}$$

$$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$$

• Limit  $m_c \rightarrow m_u = 0$

$$\begin{aligned} & \times \operatorname{Re} \int d\omega \delta(\omega + m_b - n \cdot q) \int d\omega_1 \frac{1}{\omega_1 + i\epsilon} \\ & \times \frac{1}{\omega_1} \left[ (\bar{n} \cdot q + \omega_1) \left( 1 - F \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) \right) - \bar{n} \cdot q \left( 1 - F \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) \right) \right. \\ & \left. - \bar{n} \cdot q \left( G \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) - G \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) \right) \right] g_{17}(\omega, \omega_1, \mu), \end{aligned}$$

$$\begin{aligned} g_{17}(\omega, \omega_1, \mu) &= \int \frac{dr}{2\pi} e^{-i\omega_1 r} \int \frac{dt}{2\pi} e^{-i\omega t} \\ &\times \frac{\langle \bar{B} | (\bar{h} S_n)(tn) \not{\mu}(1+\gamma_5) (S_n^\dagger S_{\bar{n}})(0) i\gamma_\alpha^\perp \bar{n}_\beta (S_{\bar{n}}^\dagger g G_s^{\alpha\beta} S_{\bar{n}})(r\bar{n}) (S_{\bar{n}}^\dagger h)(0) | \bar{B} \rangle}{2M_B} \end{aligned}$$

## Numerical evaluation

$$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$$

- Trace formalism of HQET:

$$\int_{-\infty}^{\bar{\Lambda}} d\omega g_{17}(\omega, \omega_1, \mu) = \int_{-\infty}^{\bar{\Lambda}} d\omega (g_{17}(\omega, -\omega_1, \mu))^*$$

$$d\Gamma_{17} = \frac{1}{m_b} \operatorname{Re} \left[ \hat{\Gamma}_{17} \frac{-(\lambda_t^q)^* \lambda_c^q}{|\lambda_t^q|^2} \right] \frac{\alpha}{24\pi^3} dn \cdot q d\bar{n} \cdot q \frac{(n \cdot q)^3}{\bar{n} \cdot q}$$

$$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$$

$$\times \operatorname{Re} \int d\omega \delta(\omega + m_b - n \cdot q) \int d\omega_1 \frac{1}{\omega_1 + i\epsilon}$$

- Limit  $m_c \rightarrow m_u = 0$

$$\begin{aligned} & \times \frac{1}{\omega_1} \left[ (\bar{n} \cdot q + \omega_1) \left( 1 - F \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) \right) - \bar{n} \cdot q \left( 1 - F \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) \right) \right. \\ & \left. - \bar{n} \cdot q \left( G \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) - G \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) \right) \right] g_{17}(\omega, \omega_1, \mu), \end{aligned}$$

$$\times \frac{1}{\omega_1} [ \omega_1 ] g_{17}(\omega, \omega_1, \mu)$$

## Numerical evaluation

$$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$$

- Trace formalism of HQET:  $\int_{-\infty}^{\bar{\Lambda}} d\omega g_{17}(\omega, \omega_1, \mu) = \int_{-\infty}^{\bar{\Lambda}} d\omega (g_{17}(\omega, -\omega_1, \mu))^*$

- PT invariance:  $g_{17}$  is real

$$d\Gamma_{17} = \frac{1}{m_b} \text{Re} \left[ \hat{\Gamma}_{17} \frac{-(\lambda_t^q)^* \lambda_c^q}{|\lambda_t^q|^2} \right] \frac{\alpha}{24\pi^3} dn \cdot q d\bar{n} \cdot q \frac{(n \cdot q)^3}{\bar{n} \cdot q} \quad \mathcal{O}_1^u - \mathcal{O}_{7\gamma}$$

$$\times \text{Re} \int d\omega \delta(\omega + m_b - n \cdot q) \int d\omega_1 \frac{1}{\omega_1 + i\epsilon}$$

- Limit  $m_c \rightarrow m_u = 0$

$$\begin{aligned} & \times \frac{1}{\omega_1} \left[ (\bar{n} \cdot q + \omega_1) \left( 1 - F \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) \right) - \bar{n} \cdot q \left( 1 - F \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) \right) \right. \\ & \left. - \bar{n} \cdot q \left( G \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) - G \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) \right) \right] g_{17}(\omega, \omega_1, \mu), \\ & \times \frac{1}{\omega_1} [ \omega_1 ] g_{17}(\omega, \omega_1, \mu) \end{aligned}$$

## Numerical evaluation

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- PT invariance:

$$g_{17} \text{ is real}$$

$$d\Gamma_{17} = \frac{1}{m_b} \operatorname{Re} \left[ \hat{\Gamma}_{17} \frac{-(\lambda_t^q)^* \lambda_c^q}{|\lambda_t^q|^2} \right] \frac{\alpha}{24\pi^3} dn \cdot q d\bar{n} \cdot q \frac{(n \cdot q)^3}{\bar{n} \cdot q}$$

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- Limit  $m_c \rightarrow m_u = 0$

$$\begin{aligned} & \times \frac{1}{\omega_1} \left[ (\bar{n} \cdot q + \omega_1) \left( 1 - F \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) \right) - \bar{n} \cdot q \left( 1 - F \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) \right) \right. \\ & \left. - \bar{n} \cdot q \left( G \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) - G \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) \right) \right] g_{17}(\omega, \omega_1, \mu), \end{aligned}$$

$$\times \frac{1}{\omega_1} [ \omega_1 ] g_{17}(\omega, \omega_1, \mu)$$

- Integration of  $\omega_1$ :

Interference term  $\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$  vanishes within the integrated rate

## Numerical evaluation

$$\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$$

$$\mathcal{F}^q_{17} = \frac{1}{m_b}\frac{C_1(\mu)C_{7\gamma}(\mu)}{C_\text{OPE}}e_c\,\text{Re}\bigg[\frac{-(\lambda_t^q)^*\lambda_c^q}{|\lambda_t^q|^2}\bigg]\int_{-\infty}^{+\infty}d\omega_1\,J_{17}(q_{\min}^2,q_{\max}^2,\omega_1)\,h_{17}(\omega_1,\mu)$$

## Numerical evaluation

$$\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$$

$$\mathcal{F}^q_{17} = \frac{1}{m_b}\frac{C_1(\mu)C_{7\gamma}(\mu)}{C_\text{OPE}}e_c\,\text{Re}\Big[\frac{-(\lambda_t^q)^*\lambda_c^q}{|\lambda_t^q|^2}\Big]\int_{-\infty}^{+\infty}d\omega_1\,J_{17}(q_{\min}^2,q_{\max}^2,\omega_1)\,\textcolor{red}{h}_{17}(\omega_1,\mu)$$

$$h(\omega_1,\mu):=\int_{-\infty}^{\bar{\Lambda}} d\omega\,g_{17}(\omega,\omega_1,\mu)$$

## Numerical evaluation

$$\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$$

$$\mathcal{F}^q_{17} = \frac{1}{m_b}\frac{C_1(\mu)C_{7\gamma}(\mu)}{C_\text{OPE}}e_c\,\text{Re}\left[\frac{-(\lambda_t^q)^*\lambda_c^q}{|\lambda_t^q|^2}\right]\int_{-\infty}^{+\infty}d\omega_1\,J_{17}(q_{\min}^2,q_{\max}^2,\omega_1)\,h_{17}(\omega_1,\mu)$$

$$J_{17}(q_{\min}^2,q_{\max}^2,\omega_1)=\text{Re}\frac{1}{\omega_1+\mathrm{i}\epsilon}\int_{\frac{q_{\min}^2}{M_B}}^{\frac{q_{\max}^2}{M_B}}\frac{d\overline{n}\cdot q}{\overline{n}\cdot q}\,\frac{1}{\omega_1}\qquad\qquad h(\omega_1,\mu):=\int_{-\infty}^{\bar{\Lambda}}d\omega\,g_{17}(\omega,\omega_1,\mu)\\ \Big[(\overline{n}\cdot q+\omega_1)\left(1-F\left(\frac{m_c^2}{m_b(\overline{n}\cdot q+\omega_1)}\right)\right)-\overline{n}\cdot q\left(1-F\left(\frac{m_c^2}{m_b\overline{n}\cdot q}\right)\right)\\ -\overline{n}\cdot q\left(G\left(\frac{m_c^2}{m_b(\overline{n}\cdot q+\omega_1)}\right)-G\left(\frac{m_c^2}{m_b\overline{n}\cdot q}\right)\right)\Big]\;.$$

## Numerical evaluation

$$\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$$

$$\mathcal{F}_{17}^q = \frac{1}{m_b} \frac{C_1(\mu) C_{7\gamma}(\mu)}{C_{\text{OPE}}} e_c \operatorname{Re} \left[ \frac{-(\lambda_t^q)^* \lambda_c^q}{|\lambda_t^q|^2} \right] \int_{-\infty}^{+\infty} d\omega_1 J_{17}(q_{\min}^2, q_{\max}^2, \omega_1) h_{17}(\omega_1, \mu)$$

$$J_{17}(q_{\min}^2, q_{\max}^2, \omega_1) = \operatorname{Re} \frac{1}{\omega_1 + i\epsilon} \int_{\frac{q_{\min}^2}{M_B}}^{\frac{q_{\max}^2}{M_B}} \frac{d\bar{n} \cdot q}{\bar{n} \cdot q} \frac{1}{\omega_1} h(\omega_1, \mu) := \int_{-\infty}^{\bar{\Lambda}} d\omega g_{17}(\omega, \omega_1, \mu)$$

$$\left[ (\bar{n} \cdot q + \omega_1) \left( 1 - F \left( \frac{m_c^2}{m_b(\bar{n} \cdot q + \omega_1)} \right) \right) - \bar{n} \cdot q \left( 1 - F \left( \frac{m_c^2}{m_b \bar{n} \cdot q} \right) \right) \right.$$

$$\left. - \bar{n} \cdot q \left( G \left( \frac{m_c^2}{m_b(\bar{n} \cdot q + \omega_1)} \right) - G \left( \frac{m_c^2}{m_b \bar{n} \cdot q} \right) \right) \right].$$

- One derives normalization of soft function:  $\int_{-\infty}^{\infty} d\omega_1 h_{17}(\omega_1, \mu) = 2 \lambda_2$
- $h_{17}$  should not have any significant structure (maxima or zeros) outside the hadronic range
- Values of  $h_{17}$  should be within the hadronic range

- First trial for a model function for  $h_{17}$ , a Gaussian, fulfills all needed properties.

$$h_{17}(\omega_1) = \frac{2\lambda_2}{\sqrt{2\pi}\sigma} e^{-\frac{\omega_1^2}{2\sigma^2}}$$

$\sigma = 0.5 \text{ GeV}$  as typical hadronic scale:  $\mathcal{F}_{17\text{exp}}^s \approx +1.6\%$

$\sigma = 0.1 \text{ GeV}$ :  $\mathcal{F}_{17\text{exp}}^s \approx +1.9\%$

However, convolution leads only to positive percentages !

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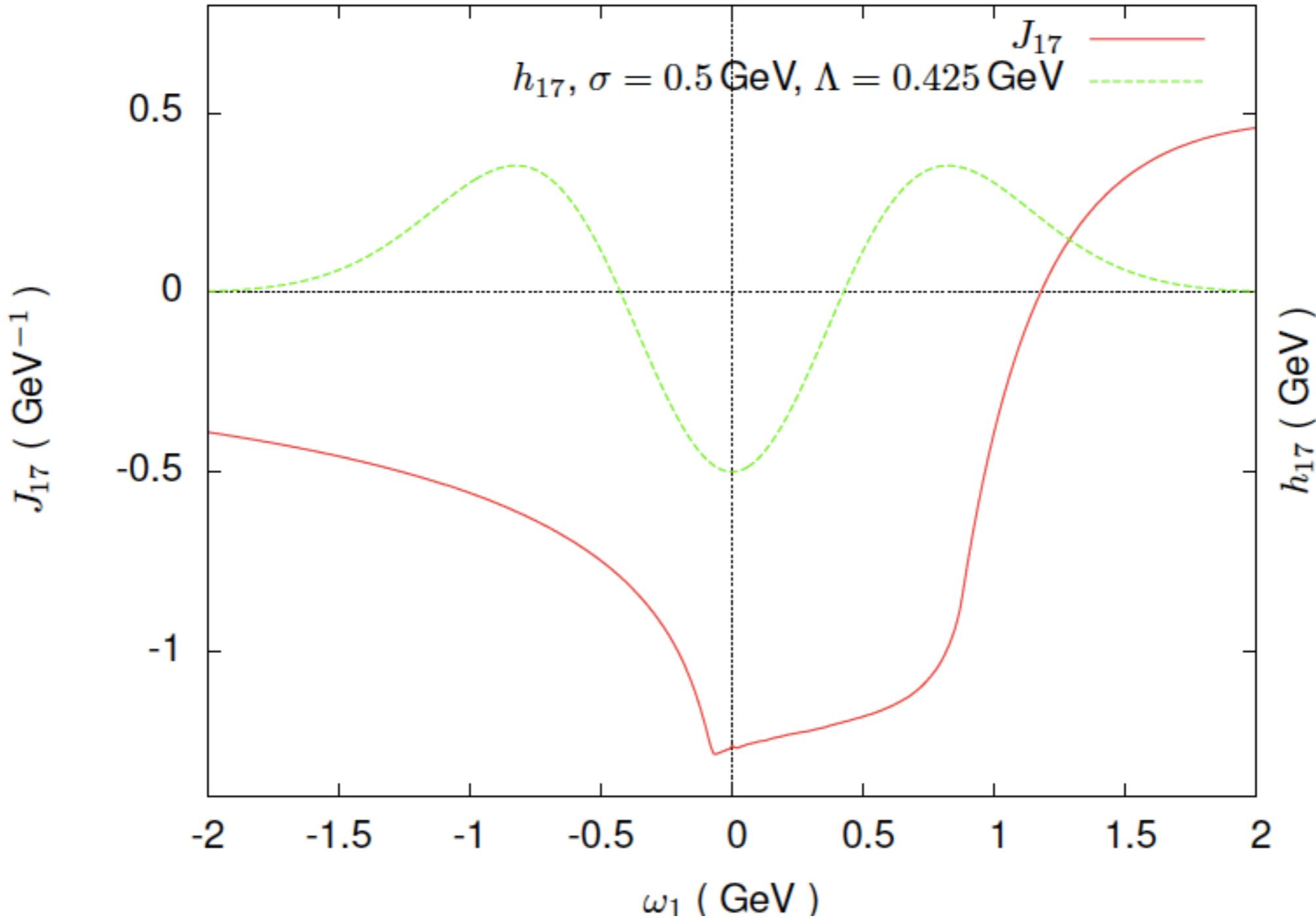
- More conservative estimate with  $h_{17}(\omega_1) = \frac{2\lambda_2}{\sqrt{2\pi}\sigma} \frac{\omega_1^2 - \Lambda^2}{\sigma^2 - \Lambda^2} e^{-\frac{\omega_1^2}{2\sigma^2}}$

With  $\Lambda$  and  $\sigma$  of order  $\Lambda_{QCD}$  all general properties of  $h_{17}$  are fulfilled.

$\sigma = 0.5 \text{ GeV}; \Lambda = 0.425 \text{ GeV}$ :  $\mathcal{F}_{17}^s = -0.5\%$

$\Lambda = 0.575 \text{ GeV}$ :  $\mathcal{F}_{17}^s = +3.4\%$

$\mathcal{F}_{17}^s \in [-0.5, +3.4]\%$ ,  $\mathcal{F}_{17}^d \in [-0.6, +4.1]\%$



$$\mathcal{F}_{17}^s \in [-0.5, +3.4] \%, \quad \mathcal{F}_{17}^d \in [-0.6, +4.1] \%$$

- Relation to the Voloshin term: Voloshin 1997, Buchalla,Isidori,Rey 1997

We can rederive the leading Voloshin term under the following assumptions:

One starts with a narrow enough Gaussian as shape function, so that one can expand the jet function around  $\omega_1 = 0$  assuming  $\Lambda_{\text{QCD}} m_b / m_c^2$  to be small ( $(m_b \omega_1) / m_c^2$  corresponds to  $t = k \cdot q / m_c^2$  in Buchalla et al.):

$$\begin{aligned} [\dots] &= \omega_1^2 \bar{n} \cdot q \left[ \frac{1}{2\bar{n} \cdot q^2} - \frac{2m_c^2}{\bar{n} \cdot q^2} \frac{1}{4m_c^2 - m_b \bar{n} \cdot q} \sqrt{\frac{4m_c^2 - m_b \bar{n} \cdot q}{m_b \bar{n} \cdot q}} \arctan \frac{1}{\sqrt{\frac{4m_c^2 - m_b \bar{n} \cdot q}{m_b \bar{n} \cdot q}}} \right] \\ &= -\frac{m_b \omega_1^2}{12m_c^2} F_V(r), \quad r = q^2 / (4m_c^2) \end{aligned}$$

However:

Voloshin term significantly underestimates the possible charm contributions.

## Final result

Benzke,Hurth,Turczyk, arXiv:1705.10366

Our final estimates of the resolved contributions to the leading order:  
(normalized to OPE result)

$$\mathcal{F}_{17}^s \in [-0.5, +3.4] \%, \quad \mathcal{F}_{17}^d \in [-0.6, +4.1] \%,$$

$$\mathcal{F}_{78}^{d,s} \in [-0.2, -0.1] \%, \quad \mathcal{F}_{88}^{d,s} \in [0, 0.5] \%$$

$$\mathcal{F}_{1/m_b}^d \in [-0.8, +4.5], \quad \mathcal{F}_{1/m_b}^s \in [-0.7, +3.8]$$

$\mathcal{F}_{19}$ :  $O(1/m_b^2)$  but  $|C_{9/10}| \sim 13|C_{7\gamma}|$   
(work in progress)

## Power corrections in the inclusive mode

- For  $q$  anti-hard-collinear we have identified a new type of subleading power corrections.
- In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.
- They constitute an irreducible uncertainty because they survive the  $M_X \rightarrow 1$  limit.
- If  $q$  was hard then these resolved contributions would not exist

**Nonlocal power corrections of  $O(1/m_b^2)$  numerically relevant**

**$M_X$  cut effects in the low- $q^2$  region with  $q^2$  anti-hard-collinear**

(work in progress)

# Summary

- Phenomenological analysis to NNLO QCD and NLO QED for all angular observables available
- Electromagnetic effects due to energetic photons are large and calculated analytically and crosschecked against Monte Carlo generator events
- Nonlocal power corrections are under control and calculated to  $O(1/m_b)$
- Theory predictions for inclusive  $\bar{B} \rightarrow X_d \ell^+ \ell^-$  (including QED and power corrections) will be soon available (work in progress)
- Inclusive  $\bar{B} \rightarrow X_s \ell \ell$  has a complementary role in new physics search to  $\bar{B} \rightarrow X_s \gamma$  and  $B \rightarrow K^{(*)} \ell \ell$



## ACTIVITIES 2018

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# Mainz Institute for Theoretical Physics

### SCIENTIFIC PROGRAMS

#### Probing Physics Beyond the SM with Precision

Ansgar Denner U Würzburg, Stefan Dittmaier U Freiburg,  
Tilman Plehn Heidelberg U

**February 26 - March 9, 2018**

#### Bridging the Standard Model to New Physics with the Parity Violation Program at MESA

Jens Erler UNAM, Mikhail Gorshteyn, Hubert Spiesberger JGU  
**April 23 - May 4, 2018**

#### Modern Techniques for CFT and AdS

Bartłomiej Czech IAS Princeton, Michał P. Heller  
MPI for Gravitational Physics, Alessandro Vichi EPFL

**May 22 - 30, 2018**

#### The Dawn of Gravitational Wave Science

Luis Lehner Perimeter Inst., Rafael A. Porto ICTP-SAIFR,  
Riccardo Sturani IIP Natal, Salvatore Vitale MIT  
**June 4 - 15, 2018**

#### The Future of BSM Physics

Gian Giudice CERN, Giulia Ricciardi U Naples Federico II,  
Tobias Hurth, Joachim Kopp, Matthias Neubert JGU  
**June 4 - 15, 2018, Capri, Italy**

#### Probing Baryogenesis via LHC and Gravitational Wave Signatures

Germano Nardini U Bern, Carlos E.M. Wagner  
U Chicago / Argonne Nat. Lab., Pedro Schwaller JGU  
**June 18 - 29, 2018**

#### From Amplitudes to Phenomenology

Fabrizio Caoletti IPPP Durham,  
Bernhard Mistlberger, Giulia Zanderighi CERN  
**August 13 - 24, 2018**

#### String Theory, Geometry and String Model Building

Philip Candelas, Xenia de la Ossa, Andre Lukas U Oxford,  
Daniel Waldram Imperial College London,  
Gabriele Honecker, Duco van Straten JGU  
**September 10 - 21, 2018**

### TOPICAL WORKSHOPS

#### The Evaluation of the Leading Hadronic Contribution to the Muon Anomalous Magnetic Moment

Carlo Carloni Calame INFN Pavia, Massimo Passera INFN Padua,

Luca Trentadue U Parma, Graziano Venanzoni INFN Pisa

**February 19 - 23, 2018**

#### Applied Newton-Cartan Geometry

Eric Bergshoeff U Groningen, Niels Obers NBI Copenhagen,  
Dam Thanh Son U Chicago

**March 12 - 16, 2018**

#### Challenges in Semileptonic B Decays

Paolo Gambino U Turin, Andreas Kronfeld Fermilab,  
Marcello Rotondo INFN-LNF Frascati, Christoph Schwanda ÖAW Vienna  
**April 9 - 13, 2018**

#### Tensions in the LCDM Paradigm

Cora Dvorkin Harvard, Silvia Galli IAP Paris,  
Fabio Iocco ICTP-SAIFR, Federico Marinacci MIT

**May 14 - 18, 2018**

#### The Proton Radius Puzzle and Beyond

Richard Hill U Kentucky / Fermilab, Gil Paz Wayne State U, Randolph Pohl JGU  
**July 23 - 27, 2018**

#### Scattering Amplitudes and Resonance Properties

from Lattice QCD

Maxwell T. Hansen CERN, Sasa Prelovsek U Ljubljana / U Regensburg,  
Steve Sharpe U Washington, Georg von Hippel, Hartmut Wittig JGU

**August 27 - 31, 2018**

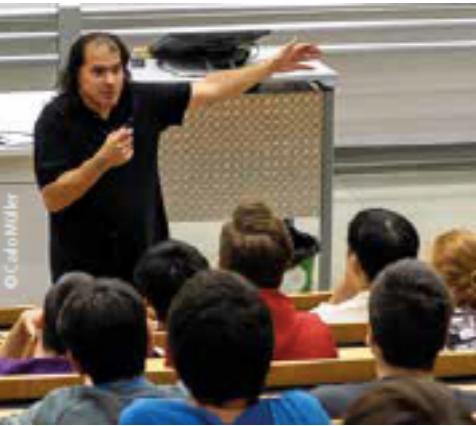
#### Quantum Fields – From Fundamental Concepts to Phenomenological Questions

Astrid Eichhorn Heidelberg U, Roberto Percacci SISSA Trieste,  
Frank Saueressig U Nijmegen

**September 26 - 28, 2018**

### MITP SUMMER SCHOOL 2018

Johannes Henn, Matthias Neubert, Stefan Weinzierl, Felix Yu JGU  
**July 2018**



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with the Parity Violation

Jens Erler UNAM, Michael Ramsey-Musolf JGU

**April 23 - May 4, 2018**

#### Moduli Physics and Beyond

Florian Bechtle U Edinburgh, Fabio Cribiori INFN Roma 1, S. Saitta INFN Roma 1, Antonio Strumia INFN Roma 1, Giacomo Vassalli INFN Roma 1, Alessandro Zoccaro INFN Roma 1

**May 21 - June 1, 2018**

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and Gravitational Wave Signatures

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#### From Amplitudes to Phenomenology

Fabrizio Caola IPPP Durham,

Bernhard Mistlberger, Giulia Zanderighi CERN

**August 13 - 24, 2018**

#### String Theory, Geometry and String Model Building

Philip Candelas, Xenia de la Ossa, Andre Lukas U Oxford,

Daniel Waldram Imperial College London,

Gabriele Honecker, Duco van Straten JGU

**September 10 - 21, 2018**

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# Extra

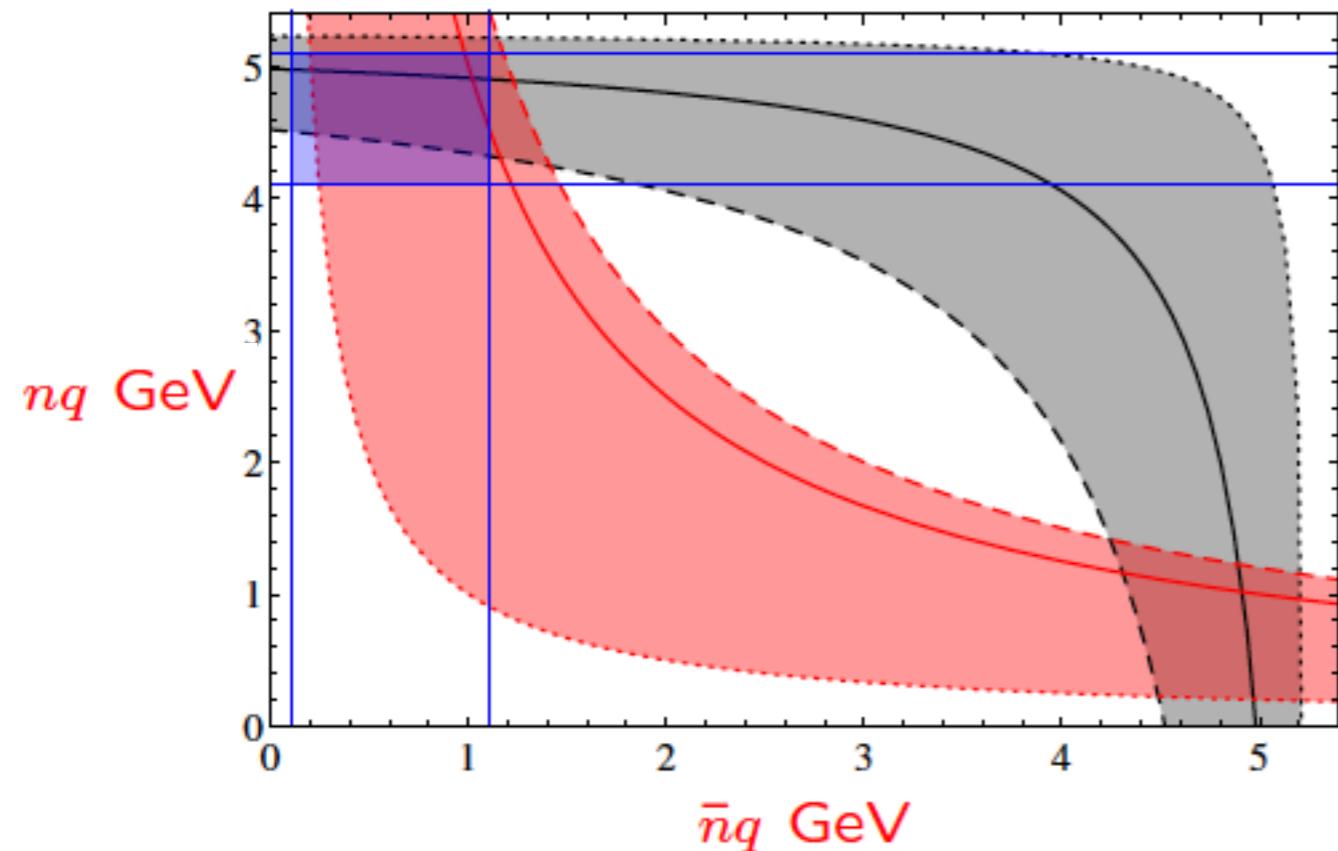
## Allowed regions

low- $q^2$

Red:  $q^2 = [1, 5, 6] \text{ GeV}^2$  [Dotted, Solid, Dashed]

Black:  $M_x = [0.495, 1.25, 2] \text{ GeV}$  [Dotted, Solid, Dashed]

Blue: anti-hard-collinear component scaling

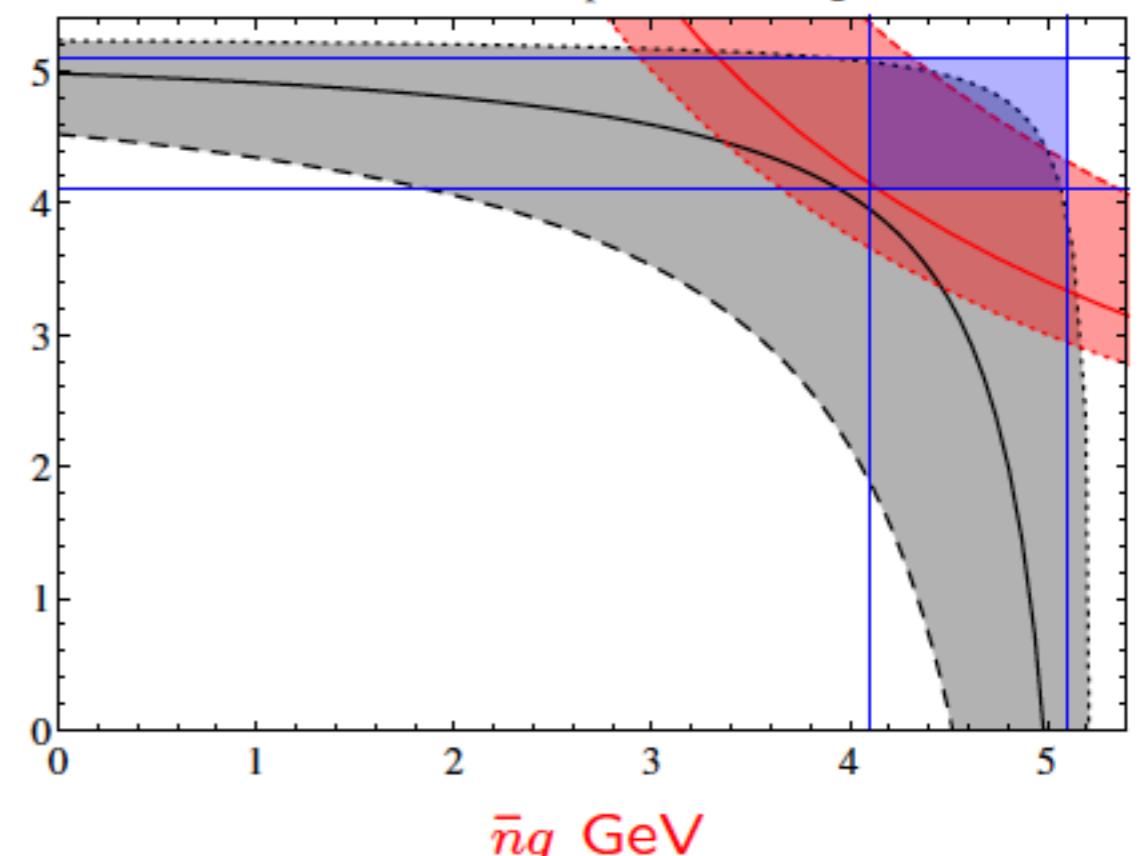


high- $q^2$

Red:  $q^2 = [15, 17, 22] \text{ GeV}^2$  [Dotted, Solid, Dashed]

Black:  $M_x = [0.495, 1.25, 2] \text{ GeV}$  [Dotted, Solid, Dashed]

Blue: hard component scaling



## Scaling

$$\lambda = \Lambda_{\text{QCD}} / m_b$$

$$m_X^2 \sim \lambda \Rightarrow m_b - n \cdot q \sim \lambda$$

# Theory predictions

- Most important input parameters

$$m_b^{1S} = (4.691 \pm 0.037) \text{GeV}, \quad \overline{m}_c(\overline{m}_c) = (1.275 \pm 0.025) \text{GeV}$$

$$|V_{ts}^* V_{tb} / V_{cb}|^2 = 0.9621 \pm 0.0027, \quad BR_{b \rightarrow c e \nu}^{\text{exp.}} = (10.51 \pm 0.13) \%$$

- Perturbative expansion (NNLO QCD + NLO QED)       $\alpha_s$        $\kappa = \alpha_{\text{em}}/\alpha_s$

$$\begin{aligned} A &= \kappa [A_{LO} + \alpha_s A_{NLO} + \alpha_s^2 A_{NNLO} + \mathcal{O}(\alpha_s^3)] \\ &+ \kappa^2 [A_{LO}^{\text{em}} + \alpha_s A_{NLO}^{\text{em}} + \alpha_s^2 A_{NNLO}^{\text{em}} + \mathcal{O}(\alpha_s^3)] + \mathcal{O}(\kappa^3) \end{aligned}$$

$$\text{LO} = \alpha_{\text{em}}/\alpha_s, \quad \text{NLO} = \alpha_{\text{em}}, \quad \text{NNLO} = \alpha_{\text{em}} \alpha_s$$

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$$\text{LO} = \alpha_{\text{em}}/\alpha_s, \quad \text{NLO} = \alpha_{\text{em}}, \quad \text{NNLO} = \alpha_{\text{em}} \alpha_s$$

- Electromagnetic effects due to energetic photons are large and calculated analytically and crosschecked against Monte Carlo generator events

Size of logs depend on experimental set-up

We assume no photons are included in the definition of  $q^2$  (di-muon channel at Babar/Belle, di-electron at Belle)

Babar's di-electron channel: Photons that are emitted in a cone of 35 mrad angular opening are included in  $q^2$

Monte Carlo techniques needed to estimate this effect !

$$q^2 = (p_{\ell^+} + p_{\ell^-})^2 \quad \text{vs.} \quad q^2 = (p_{\ell^+} + p_{\ell^-} + p_{\gamma, \text{coll}})^2$$

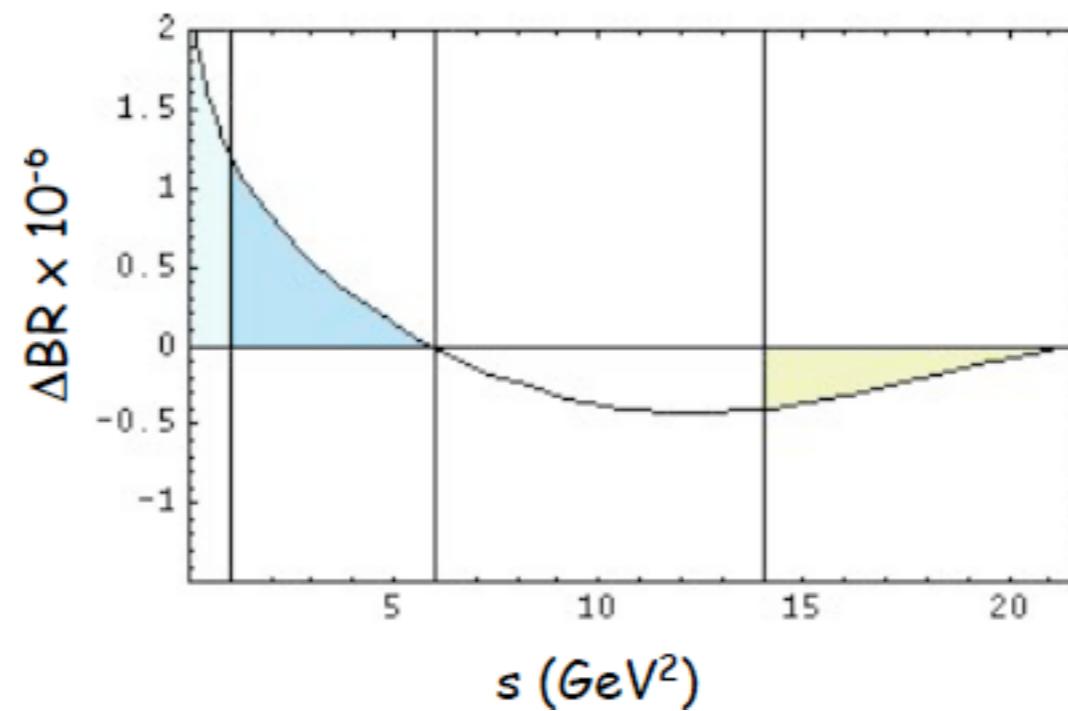
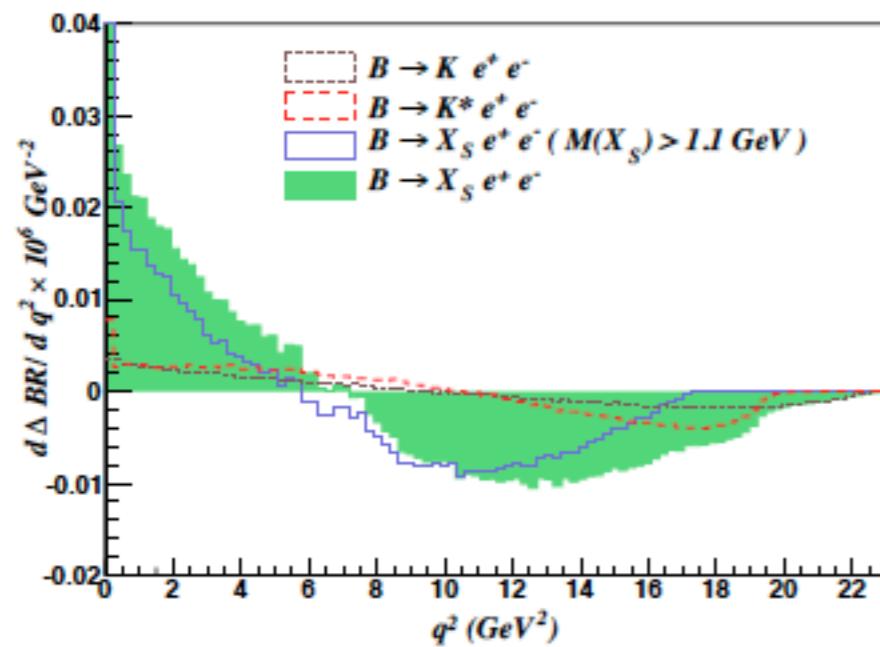
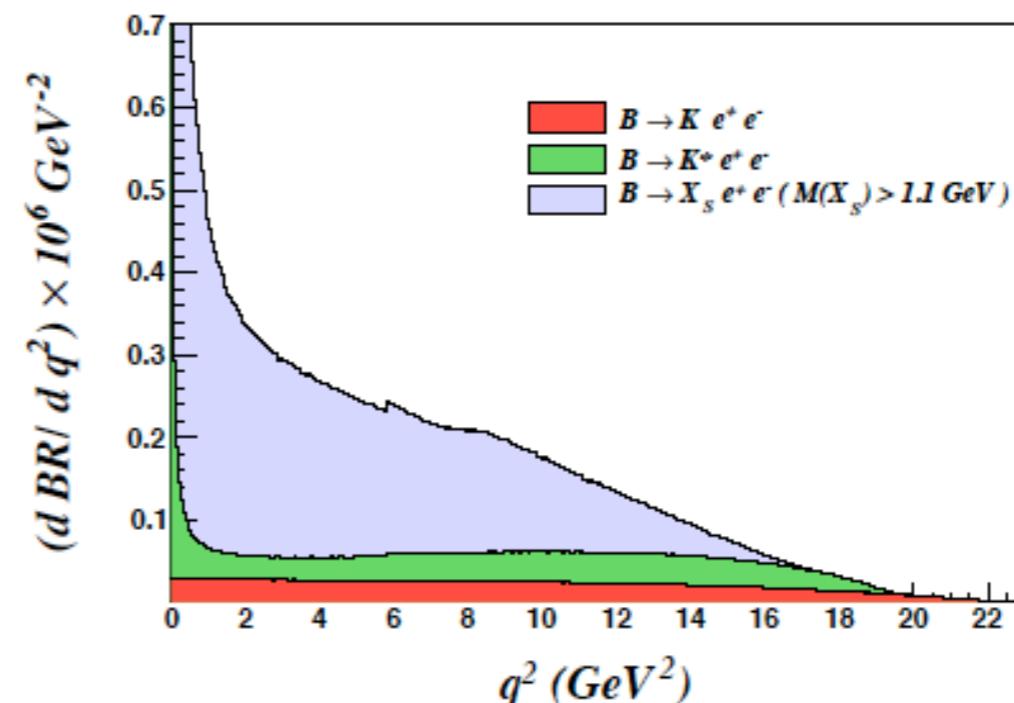
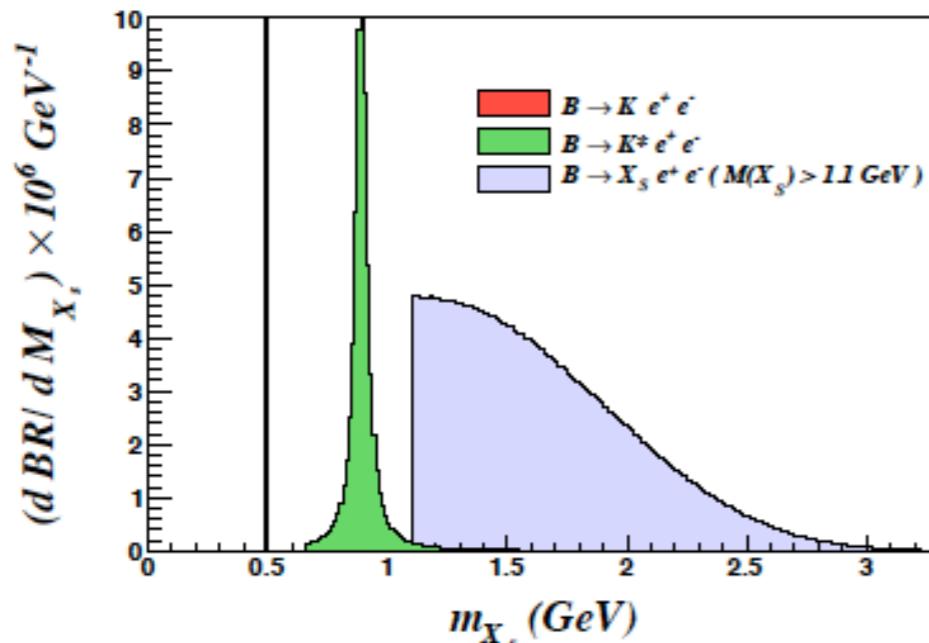
# Monte Carlo analysis

Huber,Hurth,Lunghi, arXiv:1503.04849

(event generator EVTGEN, hadronization JETSET, EM radiation PHOTOS)

$$\frac{[\mathcal{B}_{ee}^{\text{low}}]_{q=p_{e+}+p_{e-}+p_{\gamma\text{coll}}}}{[\mathcal{B}_{ee}^{\text{low}}]_{q=p_{e+}+p_{e-}}} - 1 = 1.65\%$$

$$\frac{[\mathcal{B}_{ee}^{\text{high}}]_{q=p_{e+}+p_{e-}+p_{\gamma\text{coll}}}}{[\mathcal{B}_{ee}^{\text{high}}]_{q=p_{e+}+p_{e-}}} - 1 = 6.8\%$$



## Results

Low- $q^2$  ( $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$ )

$$BR(B \rightarrow X_s ee) = (1.67 \pm 0.10) 10^{-6}$$

$$BR(B \rightarrow X_s \mu\mu) = (1.62 \pm 0.09) 10^{-6}$$

Babar:  $BR(B \rightarrow X_s \ell\ell) =$

$$= (1.60 (+0.41 - 0.39)_{stat} (+0.17 - 0.13)_{syst} (\pm 0.18)_{mod}) 10^{-6}$$

good agreement with SM

## Results

High- $q^2$ , Theory:  $q^2 > 14.4 \text{ GeV}^2$ , Babar:  $q^2 > 14.2 \text{ GeV}^2$

$$BR(B \rightarrow X_{see}) = (0.220 \pm 0.070) 10^{-6}$$

$$BR(B \rightarrow X_{s\mu\mu}) = (0.253 \pm 0.070) 10^{-6}$$

Babar:  $BR(B \rightarrow X_s \ell\ell) =$

$$(0.57 (+0.16 - 0.15)_{stat} (+0.03 - 0.02)_{syst}) 10^{-6}$$

$2\sigma$  higher than SM

Significant higher values predicted in Greub et al. due to missing power and QED corrections and different cut Greub,Pilipp,Schupbach,arXiv:0810.4077  
(but perfect agreement if we use their prescriptions)

## Further results in units of $10^{-6}$

$$H_L[1, 3.5]_{ee} = 0.64 \pm 0.03$$

$$H_L[3.5, 6]_{ee} = 0.50 \pm 0.03$$

$$H_L[1, 6]_{ee} = 1.13 \pm 0.06$$

$$H_T[1, 3.5]_{ee} = 0.29 \pm 0.02$$

$$H_T[3.5, 6]_{ee} = 0.24 \pm 0.02$$

$$H_T[1, 6]_{ee} = 0.53 \pm 0.04$$

$$H_A[1, 3.5]_{ee} = -0.103 \pm 0.005$$

$$H_A[3.5, 6]_{ee} = +0.073 \pm 0.012$$

$$H_A[1, 6]_{ee} = -0.029 \pm 0.016$$

$$H_L[1, 3.5]_{\mu\mu} = 0.68 \pm 0.04$$

$$H_L[3.5, 6]_{\mu\mu} = 0.53 \pm 0.03$$

$$H_L[1, 6]_{\mu\mu} = 1.21 \pm 0.07$$

$$H_T[1, 3.5]_{\mu\mu} = 0.21 \pm 0.01$$

$$H_T[3.5, 6]_{\mu\mu} = 0.19 \pm 0.02$$

$$H_T[1, 6]_{\mu\mu} = 0.40 \pm 0.03$$

$$H_A[1, 3.5]_{\mu\mu} = -0.110 \pm 0.005$$

$$H_A[3.5, 6]_{\mu\mu} = +0.067 \pm 0.012$$

$$H_A[1, 6]_{\mu\mu} = -0.042 \pm 0.016$$

Total error  $\mathcal{O}(5 - 8\%)$ . Still dominated by scale uncertainty.

## Further refinement

Normalization to semileptonic  $B \rightarrow X_u \ell \nu$  decay rate **with the same cut** reduces the impact of  $1/m_b$  corrections in the high- $q^2$  region significantly.

Ligeti,Tackmann arXiv:0707.1694

Theory prediction for ratio

$$R(s_0)_{ee} = (2.25 \pm 0.31) 10^{-3}$$

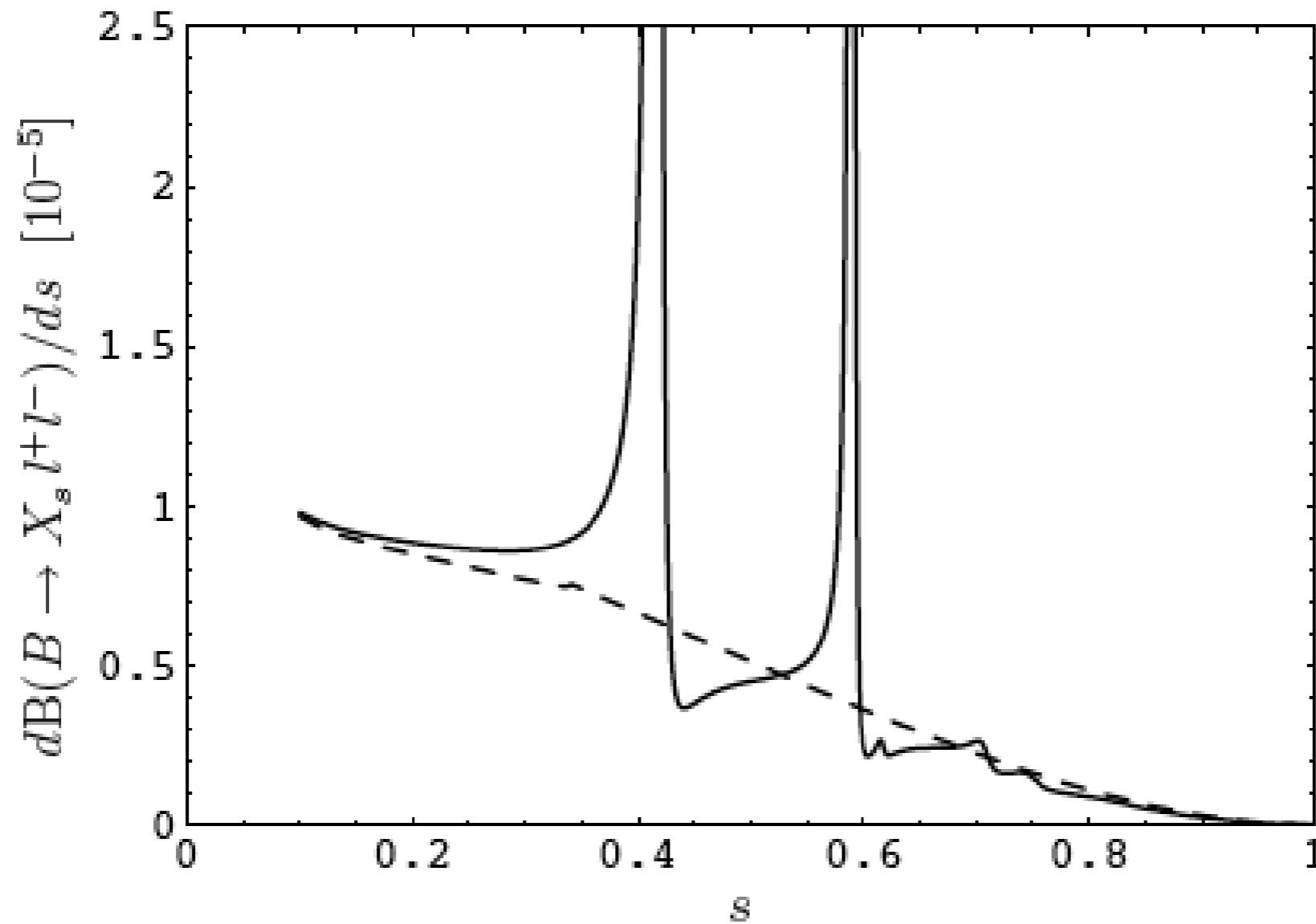
$$R(s_0)_{\mu\mu} = (2.62 \pm 0.30) 10^{-3}$$

Largest source of error are CKM elements ( $V_{ub}$ )

Note: Additional  $O(5\%)$  uncertainty due to nonlocal power corrections  $O(\alpha_s \Lambda / m_b)$

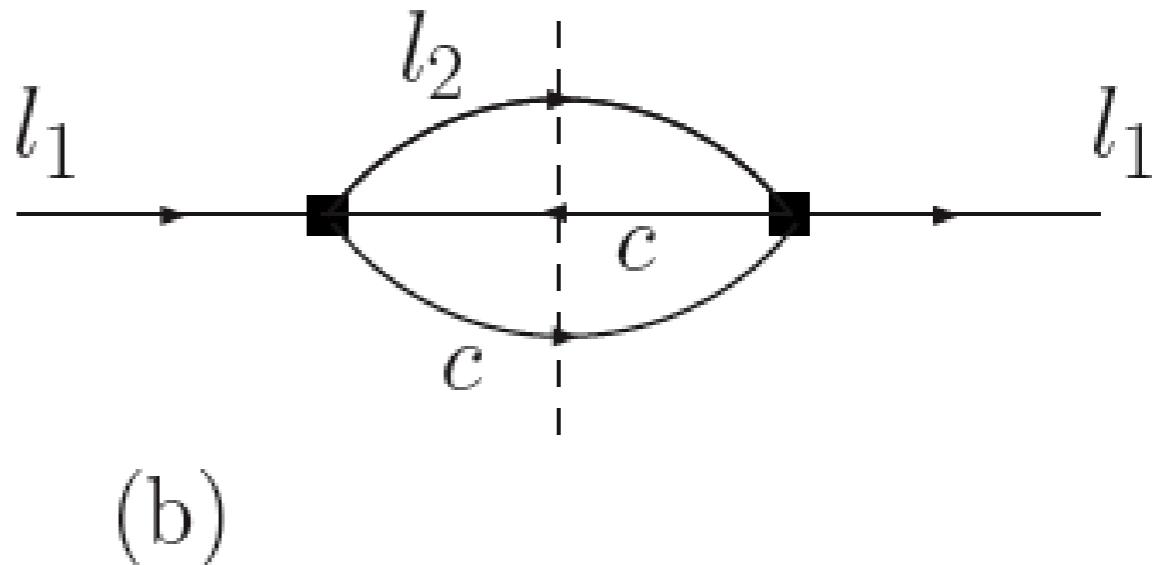
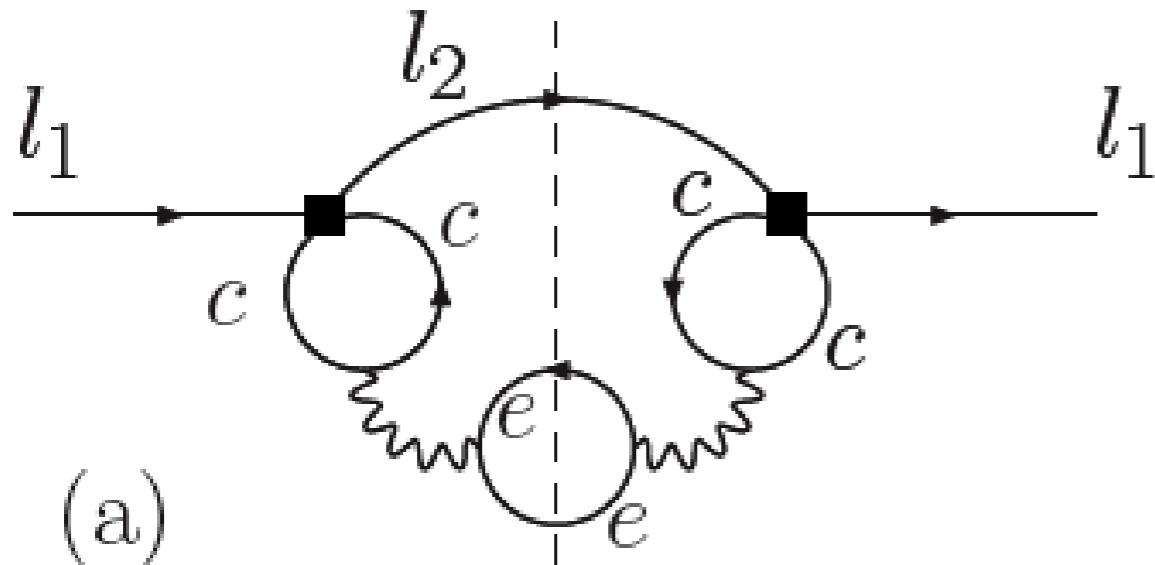
# Quark-hadron duality violated in $\bar{B} \rightarrow X_s \ell^+ \ell^-$ ? BBNS, arXiv:0902.4446

Within integrated branching ratio the resonances  $J/\psi$  and  $\psi'$  exceed the perturbative contributions by two orders of magnitude.



## Quark-hadron duality violated in $\bar{B} \rightarrow X_s \ell^+ \ell^-$ ? BBNS, arXiv:0902.4446

Within integrated branching ratio the resonances  $J/\psi$  and  $\psi'$  exceed the perturbative contributions by two orders of magnitude.



The rate  $l_1 \rightarrow l_2 e^+ e^-$  (a) is connected to the integral over  $|\Pi(q^2)|^2$  for which global duality is **NOT** expected to hold.

In contrast the inclusive hadronic rate  $l_1 \rightarrow l_2 X$  (b) corresponds to the imaginary part of the correlator  $\Pi(q^2)$ .

## Subtlety in the high- $q^2$ region

Locally: breakdown of OPE in  $\Lambda_{QCD}/m_b$  in the high- $s$  ( $q^2$ ) endpoint  
Partonic contribution vanishes in the limit  $s \rightarrow 1$ , while the  $1/m_b^2$  corrections in  $R(s)$  tend towards a nonzero value.

Theoretically: s-quark propagator in the correlator of OPE:

$$S_s(k) = \frac{k + i \not{D}}{k^2 + 2ik \cdot D - \not{D} \not{D} + i\varepsilon} .$$

Endpoint region of the  $q^2$  spectrum in  $\bar{B} \rightarrow X_s l^+ l^-$  different from endpoint region of the photon spectrum of  $\bar{B} \rightarrow X_s \gamma$ :

$q^2 \approx m_b^2 \approx M_B^2 \Rightarrow k \sim \Lambda, \quad k^2 \sim \Lambda^2 \Rightarrow$  complete breakdown of OPE  
no partial all-orders resummation possible, shape-function irrelevant  
[Buchalla, isidori](#)

Practically: for integrated high-s ( $a^2$ ) spectrum one finds an effective expansion ( $s_{\min} \approx 0.6$ ): [Ghinculov, Hurth, Isidori, Yao, hep-ph/0312128](#)

$$\int_{s_{\min}}^1 ds R(s) = \left[ 1 - \frac{1.6\lambda_2}{m_b^2(1 - \sqrt{s_{\min}})^2} + \frac{1.8\rho_1 + 1.7f_1}{m_b^3(1 - \sqrt{s_{\min}})^3} \right] \times \int_{s_{\min}}^1 ds R(s)|_{m_b \rightarrow \infty}$$