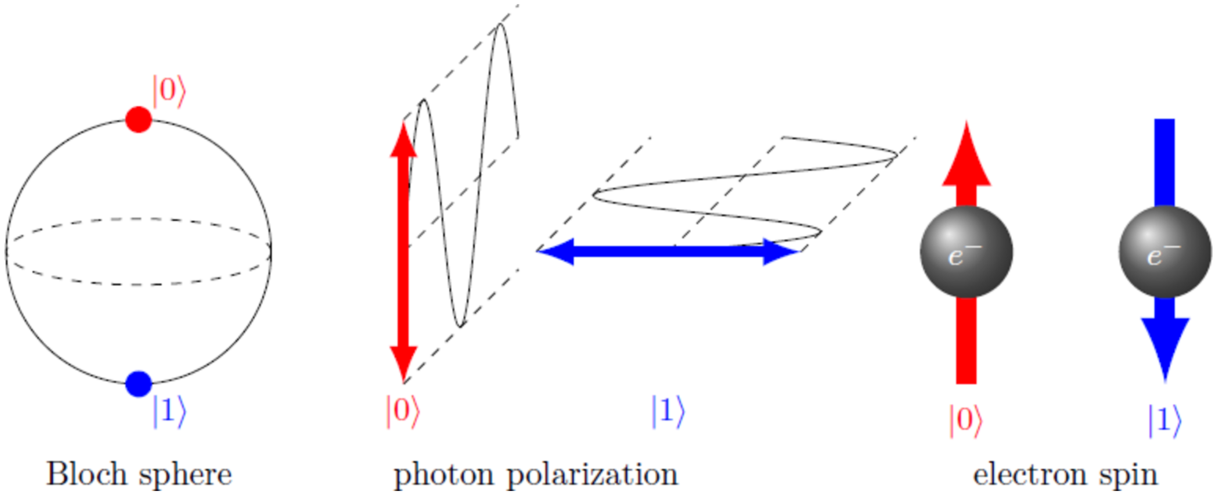
One Qubit Physics: A teacher's guide to experimentally introduce Quantum Mechanics by Polarization of light



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Single photon version for the polarisation Qubit
(the expensive one)

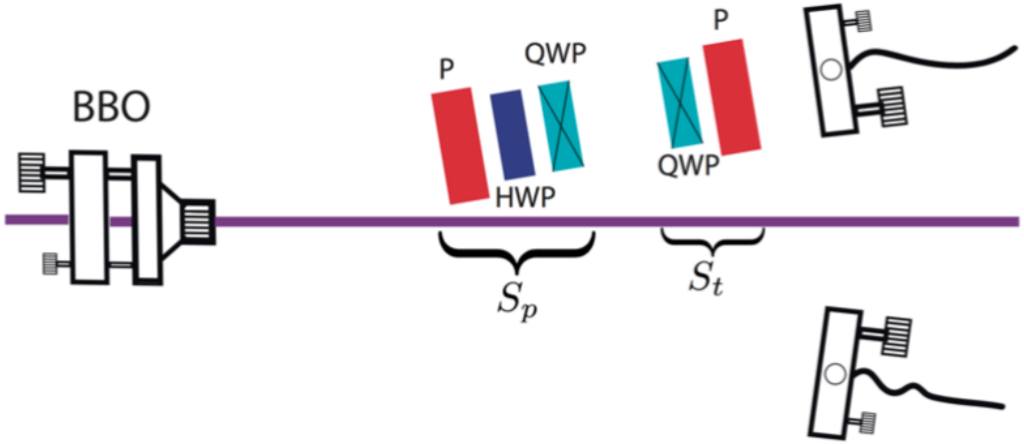
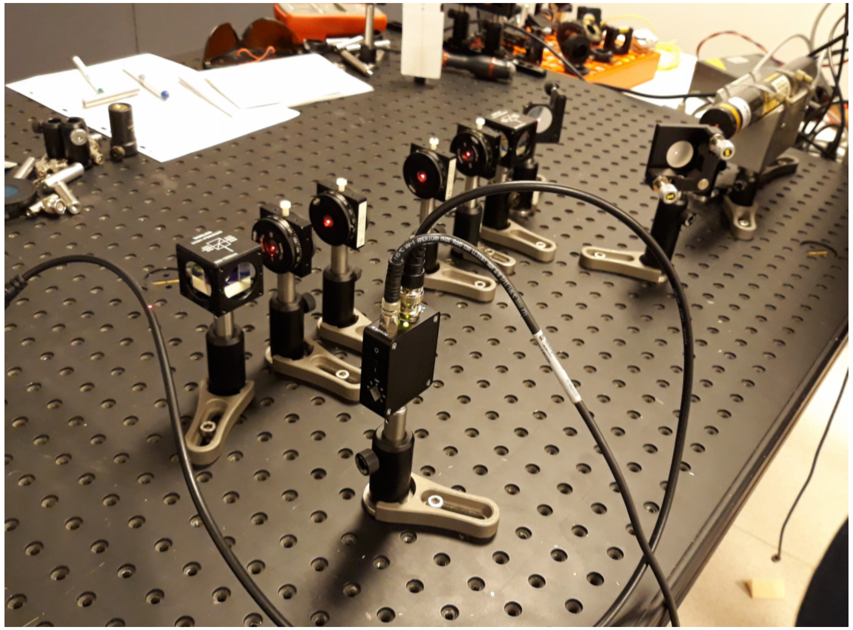


Figure : Set up for the single photon case. The pump beam insides over the BBO. Single photons are obtained out of BBO crystal undergoing Spontaneous Parametric Down Convertion. This photons are heralded and must be detected in coincidence. S_p designates the quantum state preparation system and S_t the tomographic system.

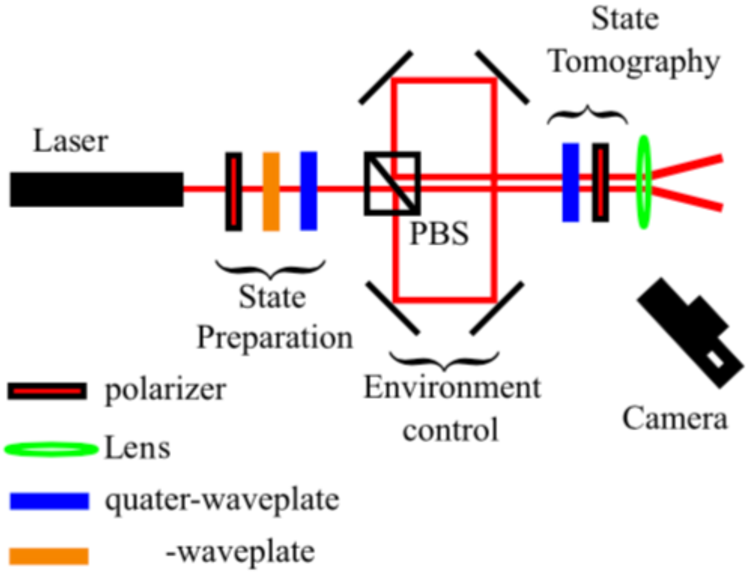
BBO non linear crystal+ single photon detectors = 2000$

An analogy using classical light to introduce The Qubit

- Simple setup



TP L3: ENS Paris 2019



An analogy using classical light to introduce The Qubit

- Simple setup

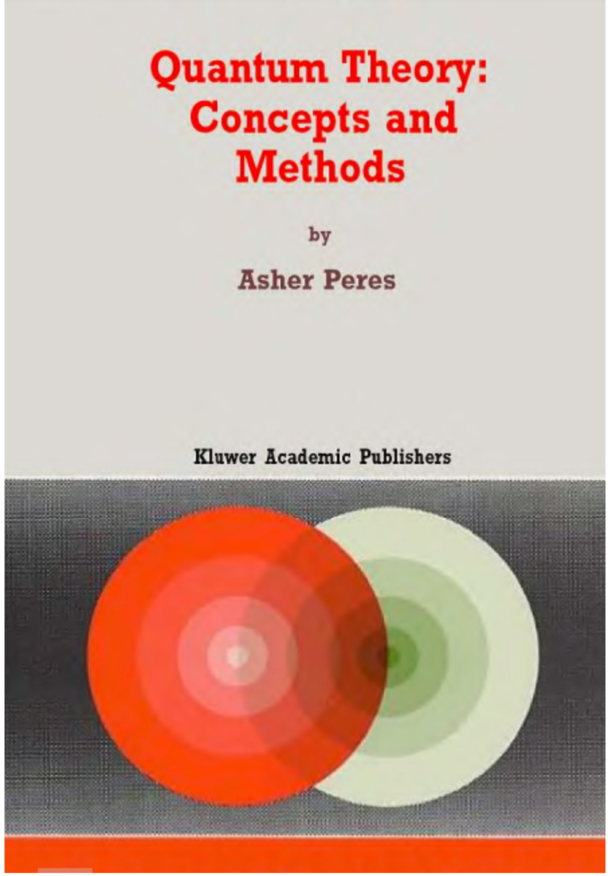
- 'Low cost'
  - Almost any laser will do! (1$)
  - Photodiodes are cheap! (1$)
  - Polarisers are cheap! (1$)
  - Mirrors are cheap! (1$)
  - Mounts can be 3D printed

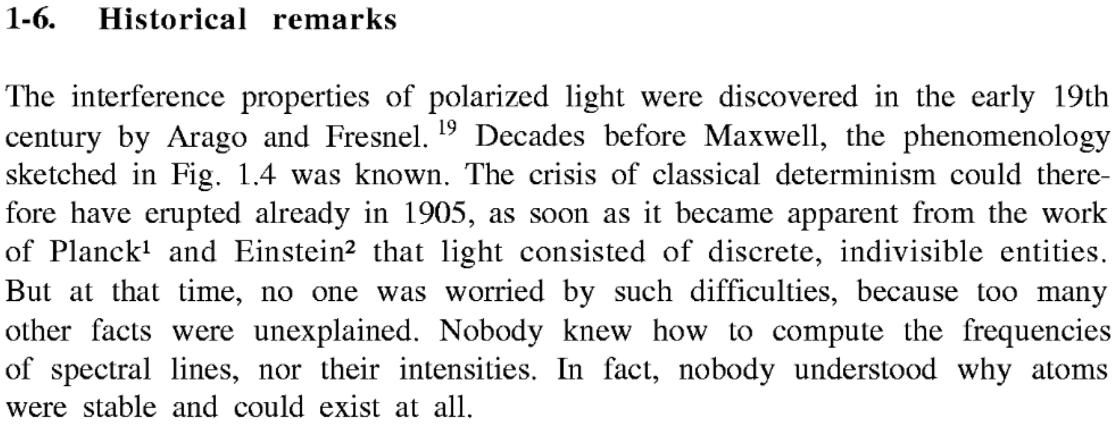
- Wave plates are not cheap but…

-Maybe already present for other TPs
-Many lab can donate wave plates 'dead for science'
-Not so expensive options available in the internet (20$)

An analogy using classical light to introduce The Qubit

- A very (very very) strong analogy





1-6. Historical remarks

The interference properties of polarized light were discovered in the early 19th century by Arago and Fresnel.[19] Decades before Maxwell, the phenomenology sketched in Fig. 1.4 was known. The crisis of classical determinism could therefore have erupted already in 1905, as soon as it became apparent from the work of Planck[1] and Einstein[2] that light consisted of discrete, indivisible entities. But at that time, no one was worried by such difficulties, because too many other facts were unexplained. Nobody knew how to compute the frequencies of spectral lines, nor their intensities. In fact, nobody understood why atoms were stable and could exist at all.

‘If you believe in the photon there is no classical explanation to the Malus law’

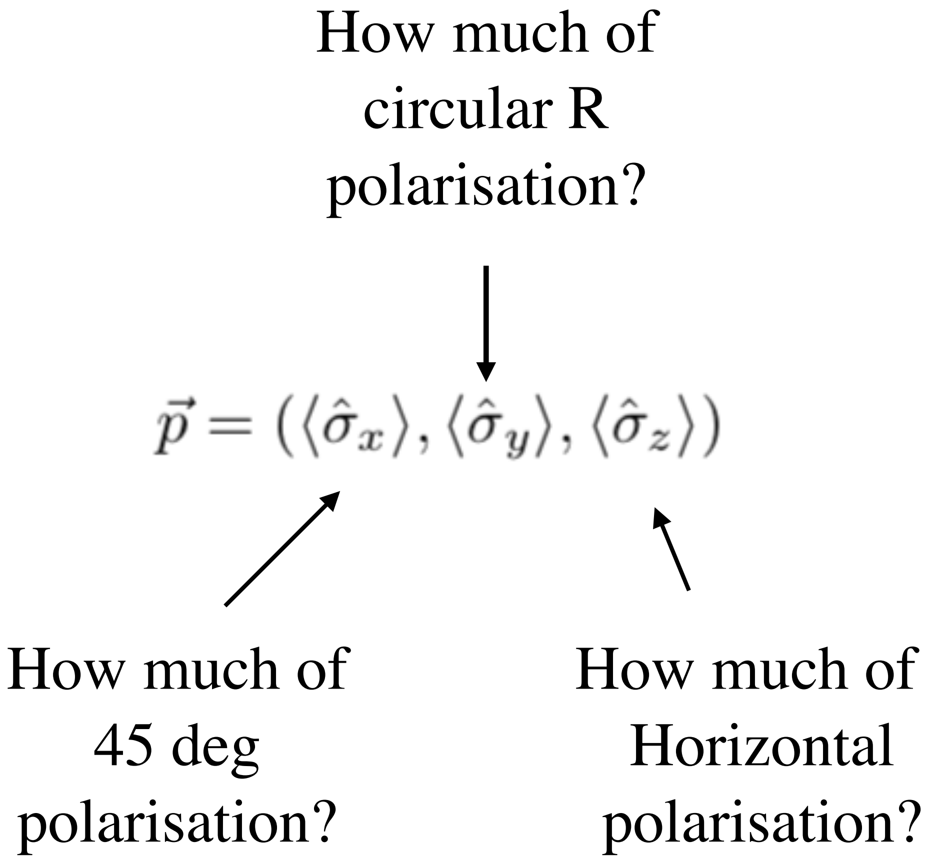
An analogy using classical light to introduce
The Qubit

- A very (very very) strong analogy
- Simple setup
- ‘Low cost’
- Conceptually rich
  - The Schrodinger equation
  - Heisenberg inequalities
  - The density matrix
  - Decoherence
  - Entanglement
  - The wave function
  - The Bloch sphere
  - Purity
  - Gates
  - The Born rule
  - Fidelity of states
  - etc…

Analogy:
Heisenberg inequalities

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

$$(\Delta A)^2(\Delta B)^2 \geq \frac{1}{4}|\langle[\hat{A},\hat{B}]\rangle|^2 + |\frac{1}{2}\langle\{\hat{A},\hat{B}\}\rangle - \langle\hat{A}\rangle\langle\hat{B}\rangle|^2$$



How much of circular R polarisation?

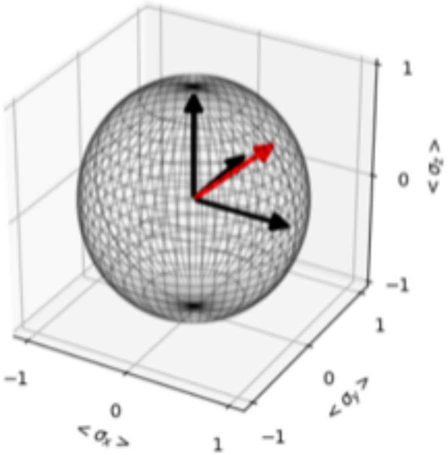
$$\vec{p} = (\langle\hat{\sigma}_x\rangle, \langle\hat{\sigma}_y\rangle, \langle\hat{\sigma}_z\rangle)$$

How much of 45 deg polarisation?

How much of Horizontal polarisation?

$$1 \geq \langle\hat{\sigma}_x\rangle^2 + \langle\hat{\sigma}_y\rangle^2 + \langle\hat{\sigma}_z\rangle^2 \equiv ||\vec{p}||^2$$

It is a sphere!
The Bloch vector lives in the Bloch sphere



Analogy:

Tomography of the density matrix

The mathematics of polarisation were developed well before QM and for one quit there are exact analogs

QM—CM

Bloch Sphere—Poincare Sphere

State vectors—Jones vectors

Density matrix—Stokes parameters

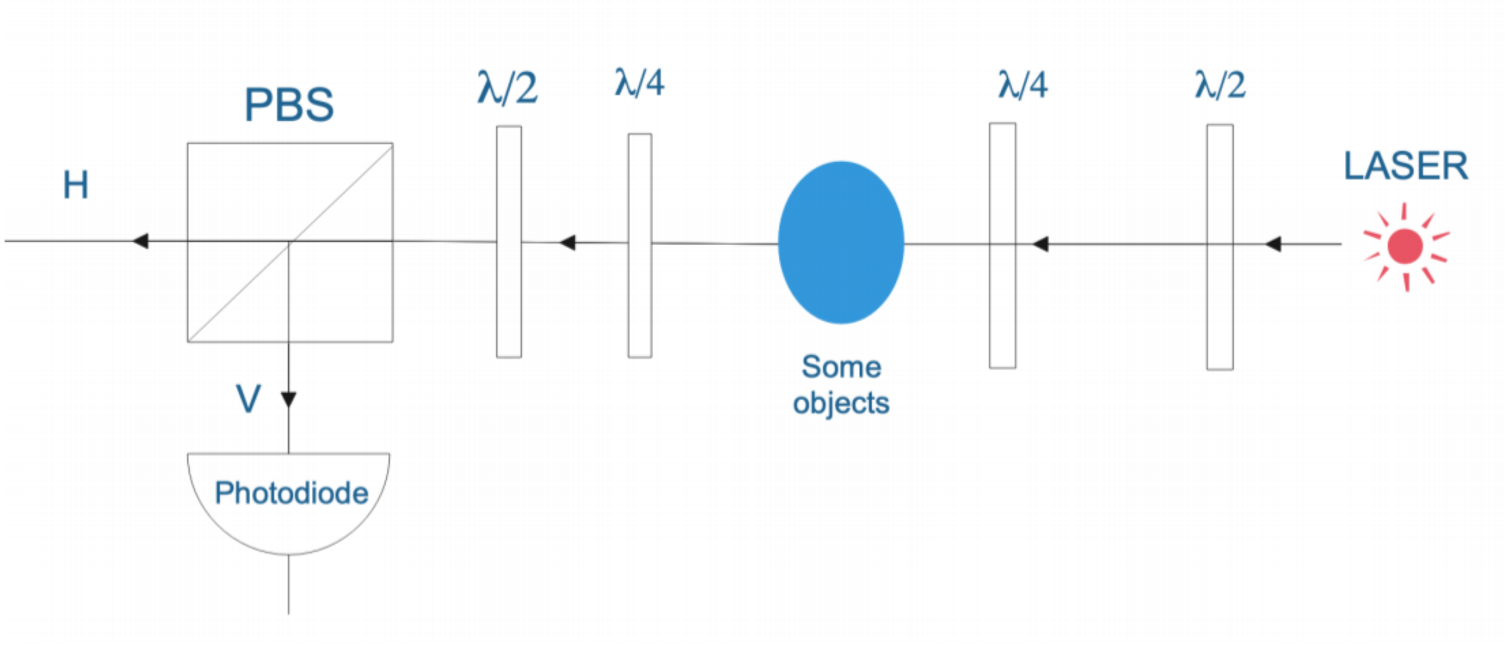
Analogy:

Tomography of the density matrix

$$\vec{p} = (\langle\hat{\sigma}_x\rangle, \langle\hat{\sigma}_y\rangle, \langle\hat{\sigma}_z\rangle)$$

$$\hat{\rho} = \frac{1}{2}(\hat{1} + \langle\hat{\sigma}_x\rangle\hat{\sigma}_x + \langle\hat{\sigma}_y\rangle\hat{\sigma}_y + \langle\hat{\sigma}_z\rangle\hat{\sigma}_z)$$

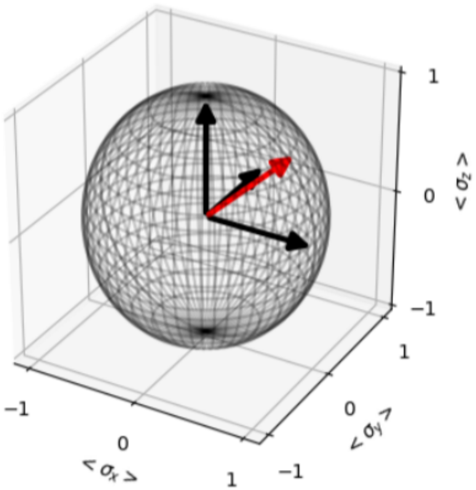
$$= \frac{1}{2}(\hat{1} + \frac{2I_{+45^\circ} - I_T}{I_T}\hat{\sigma}_x + \frac{2I_L - I_T}{I_T}\hat{\sigma}_y + \frac{2I_H - I_T}{I_T}\hat{\sigma}_z).$$



Analogy:

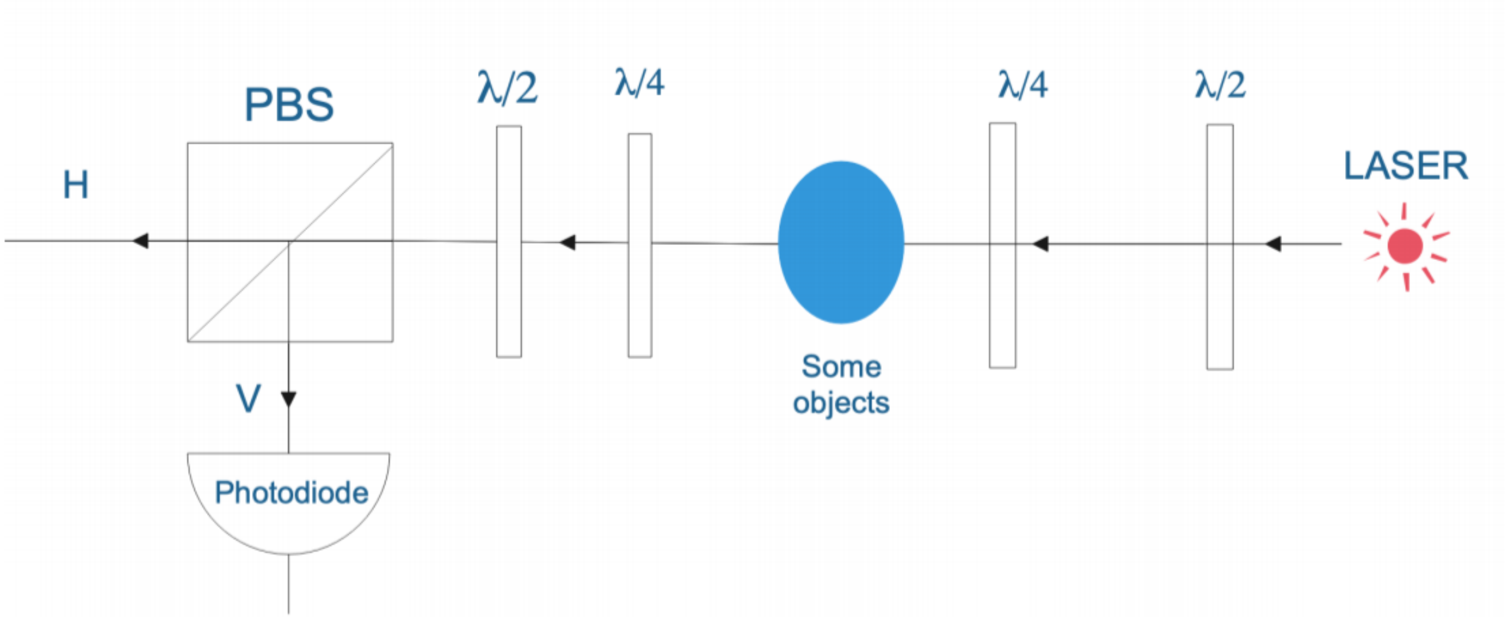
Tomography of the density matrix

	LASER
$\|H\rangle$	2.325
$\|V\rangle$	0.339
$\|45^\circ\rangle$	1.527
$\|R\rangle$	2.196
$\langle\hat{\sigma}_x\rangle$	0.746
$\langle\hat{\sigma}_y\rangle$	0.146
$\langle\hat{\sigma}_z\rangle$	0.649
$\|\vec{\boldsymbol{p}}\|$	0.999
γ	0.999



$? \to \|V\rangle$	$\lambda/4$	$\lambda/2$
$\|H\rangle$	0°	45°
$\|V\rangle$	0°	0°
$\|45^\circ\rangle$	45°	+22.5°
$\|-45^\circ\rangle$	45°	−22.5°
$\|R\rangle$	45°	0°
$\|L\rangle$	45°	45°

How to prepare $|V\rangle$ with different polarization states.



Analogy:

Tomography of the density matrix

	$\lvert H\rangle$	$\lvert 45^\circ\rangle$	$\lvert R\rangle$
$\lvert H\rangle$	2.150	1.207	1.381
$\lvert V\rangle$	0.020	1.046	0.905
$\lvert 45^\circ\rangle$	1.081	2.174	1.102
$\lvert R\rangle$	1.219	1.204	2.153
$\langle\hat{\sigma}_x\rangle$	0.982	0.071	0.208
$\langle\hat{\sigma}_y\rangle$	-0.004	0.930	-0.036
$\langle\hat{\sigma}_z\rangle$	0.124	0.069	0.884
$\lVert\vec{\boldsymbol{p}}\rVert$	0.989	0.935	0.909
γ	0.989	0.937	0.913

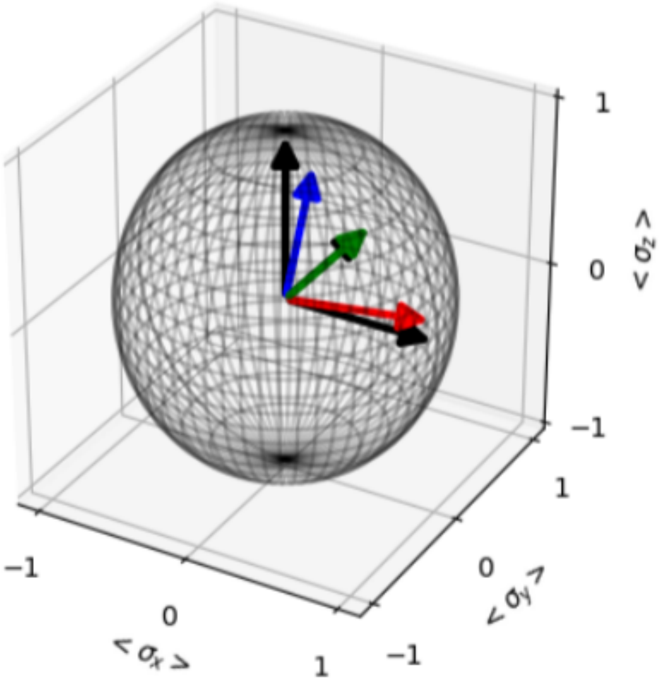
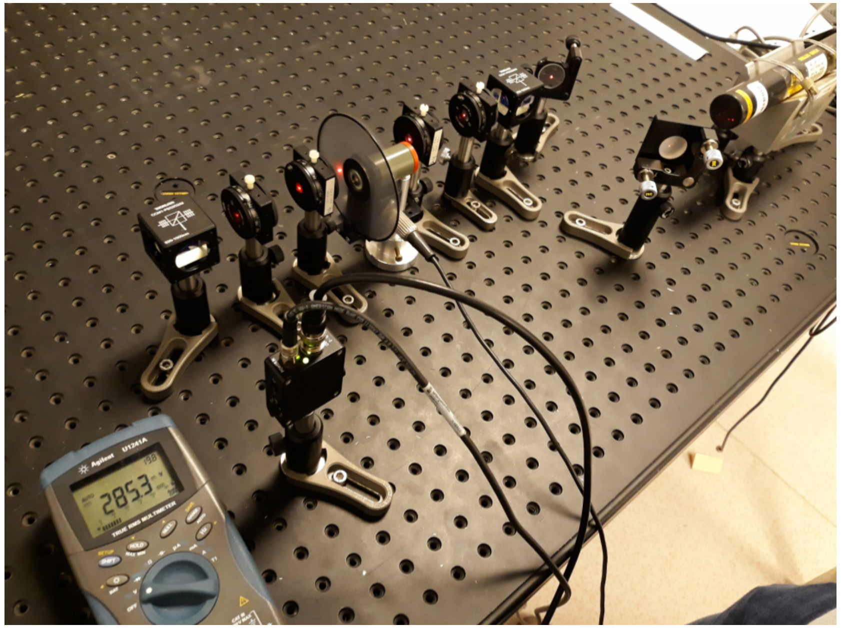
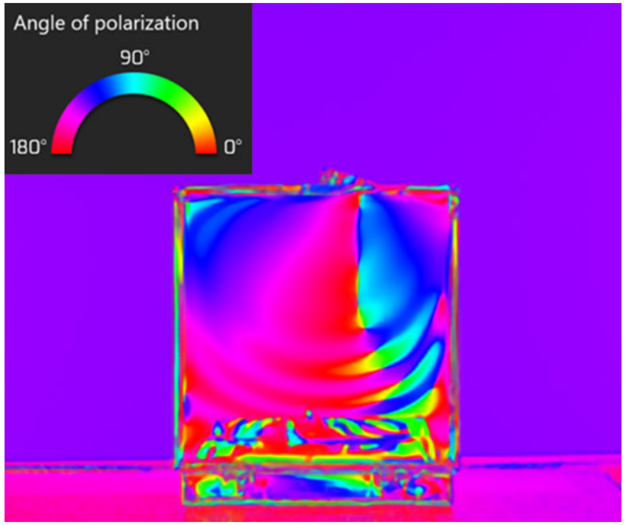


Figure 6: *Red*: $\lvert H\rangle$; *Green*: $\lvert 45^\circ\rangle$; *Blue*: $\lvert R\rangle$.

Analogy: ’Mundane decoherence’





	$\lvert H\rangle$, rotating plastic fiber	$\lvert H\rangle$, non-rotating plastic fiber
$\lvert H\rangle$	0.281	1.010
$\lvert V\rangle$	0.209	0.029
$\lvert 45°\rangle$	0.239	0.469
$\lvert R\rangle$	0.291	0.592
$\langle\hat{\sigma}_x\rangle$	0.147	0.944
$\langle\hat{\sigma}_y\rangle$	-0.024	-0.097
$\langle\hat{\sigma}_z\rangle$	0.188	0.140
$\lVert\vec{\boldsymbol{p}}\rVert$	0.240	0.959
γ	0.529	0.960

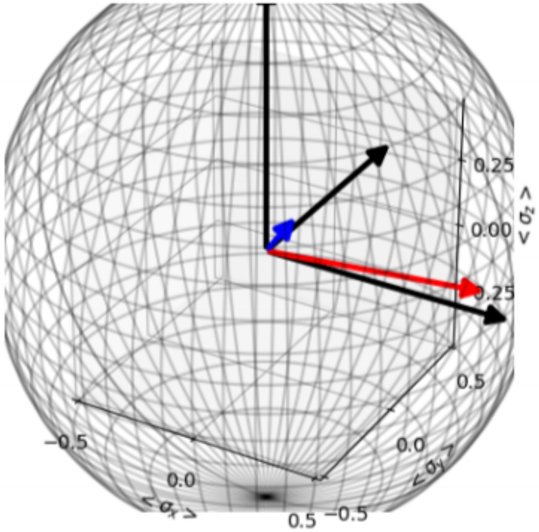


Figure 7: *Blue*: rotating ; *Red*: non-rotating.

Analogy:

The wave function for the photon is a tricky concept…

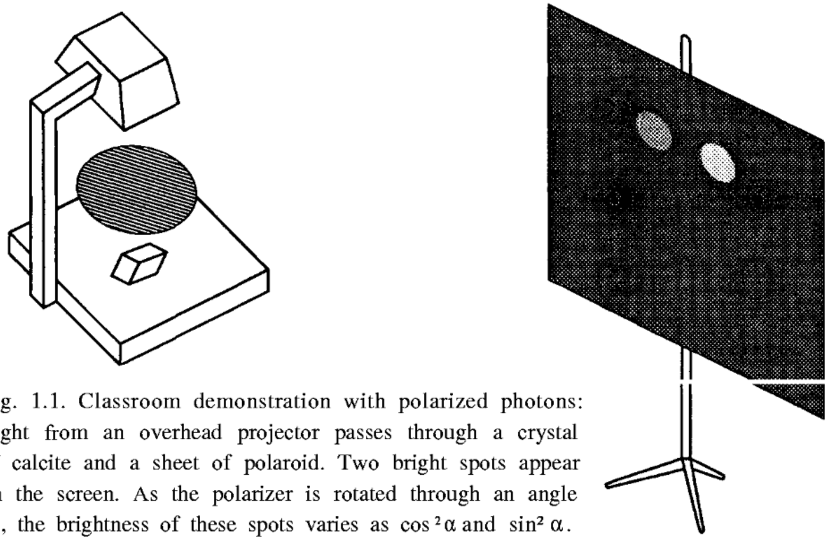
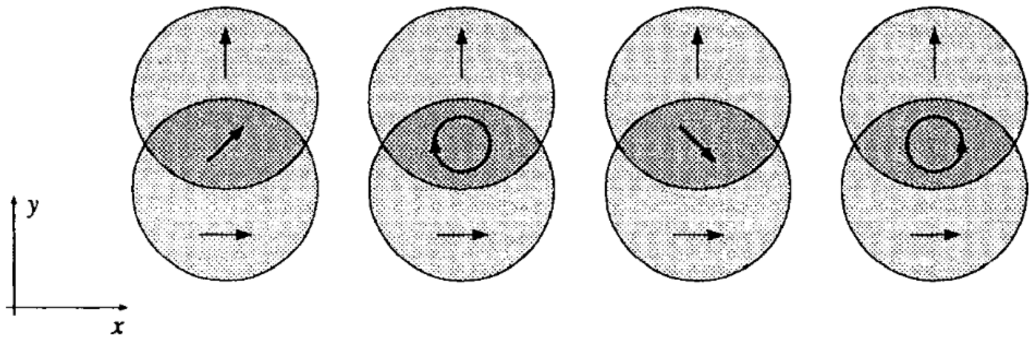


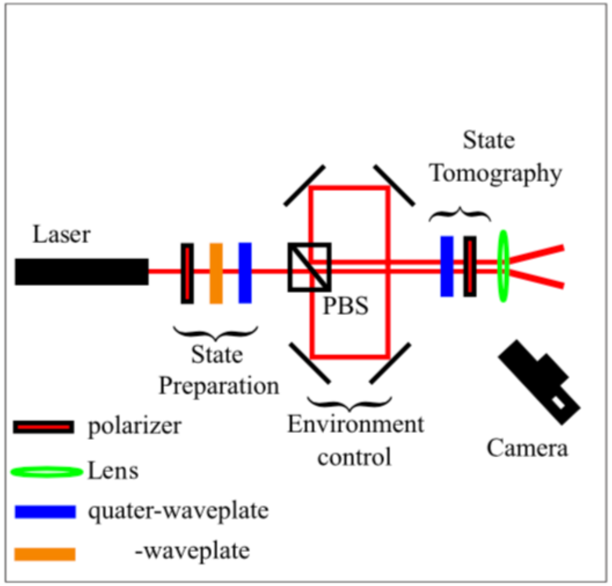
Fig. 1.1. Classroom demonstration with polarized photons: Light from an overhead projector passes through a crystal of calcite and a sheet of polaroid. Two bright spots appear on the screen. As the polarizer is rotated through an angle α, the brightness of these spots varies as $\cos^2\alpha$ and $\sin^2\alpha$.

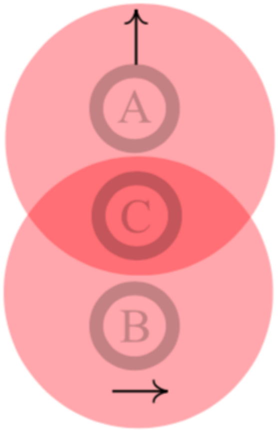


$\delta = 0$ $\delta = \pi/2$ $\delta = \pi$ $\delta = 3\pi/2$

Analogy:

Decoherence from entanglement

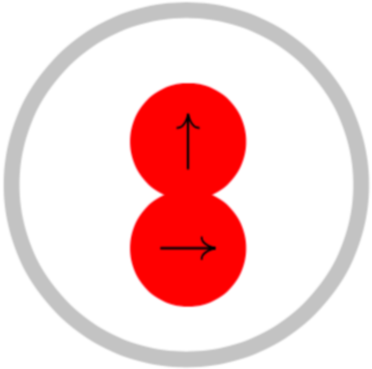




$$\begin{array}{cc} & \begin{array}{cc} |H\rangle & |V\rangle \end{array} \\ \rho_{\text{A}} = & \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{array}{c} |H\rangle \\ |V\rangle \end{array} \end{array}$$

$$\begin{array}{cc} & \begin{array}{cc} |H\rangle & |V\rangle \end{array} \\ \rho_{\text{C}} = & \begin{pmatrix} 1/2 & e^{i\delta}/2 \\ e^{-i\delta}/2 & 1/2 \end{pmatrix} \begin{array}{c} |H\rangle \\ |V\rangle \end{array} \end{array}$$

$$\begin{array}{cc} & \begin{array}{cc} |H\rangle & |V\rangle \end{array} \\ \rho_{\text{B}} = & \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{array}{c} |H\rangle \\ |V\rangle \end{array} \end{array}$$



$$\begin{array}{cc} & \begin{array}{cc} |H\rangle & |V\rangle \end{array} \\ \rho_{mix} \sim & \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{array}{c} |H\rangle \\ |V\rangle \end{array} \end{array}$$

Thank you

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Thank you.

PS: A ~25 pages TP guide is available
bringing together many concepts

theory + experimental realisation

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The Schödinger equation

If polarization and position remain factorized all allong the evolution (ie: there is no entanglement growing between the degrees of freedom) one is left with the pure state for polarization

$$|\psi(x)\rangle = \alpha|H\rangle + \beta e^{i\delta}e^{ik_0\Delta(x)}|V\rangle = \alpha e^{ik_H x}|H\rangle + \beta e^{i\delta}e^{ik_V x}|V\rangle$$

where we have used the global phase freedom to write the equality.
This state is the most general one and satisfies the differential equation

$$i\hbar\partial_x|\psi(x)\rangle = \hat{p}|\psi(x)\rangle,$$

where we have introduced a momentum operator for the photon in the crystal that is

$$\hat{p} = \begin{pmatrix} \hbar k_H & 0 \\ 0 & \hbar k_V \end{pmatrix}$$

If one forces the change of variables $x = ct$,

$$i\hbar\partial_t|\psi(t)\rangle = c\hat{p}|\psi(t)\rangle.$$

This is a Schrödinger like equation for the polarization of the photon inside a birefringent crystal that reproduces the correct equations of motion. The operator $c\hat{p}$ plays the role of the Hamiltonian, and reminds us of the relativistic energy for a masless particle $E = cp$.