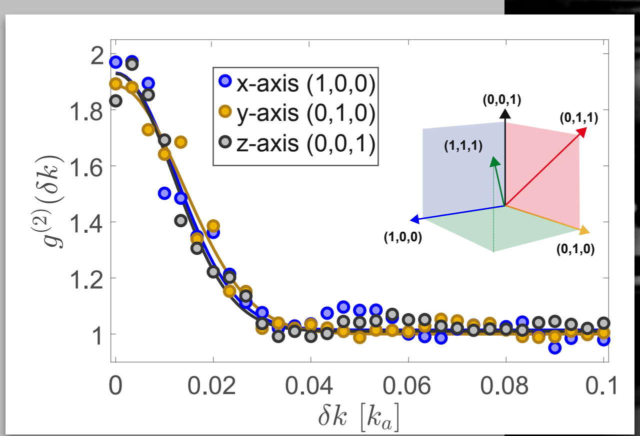
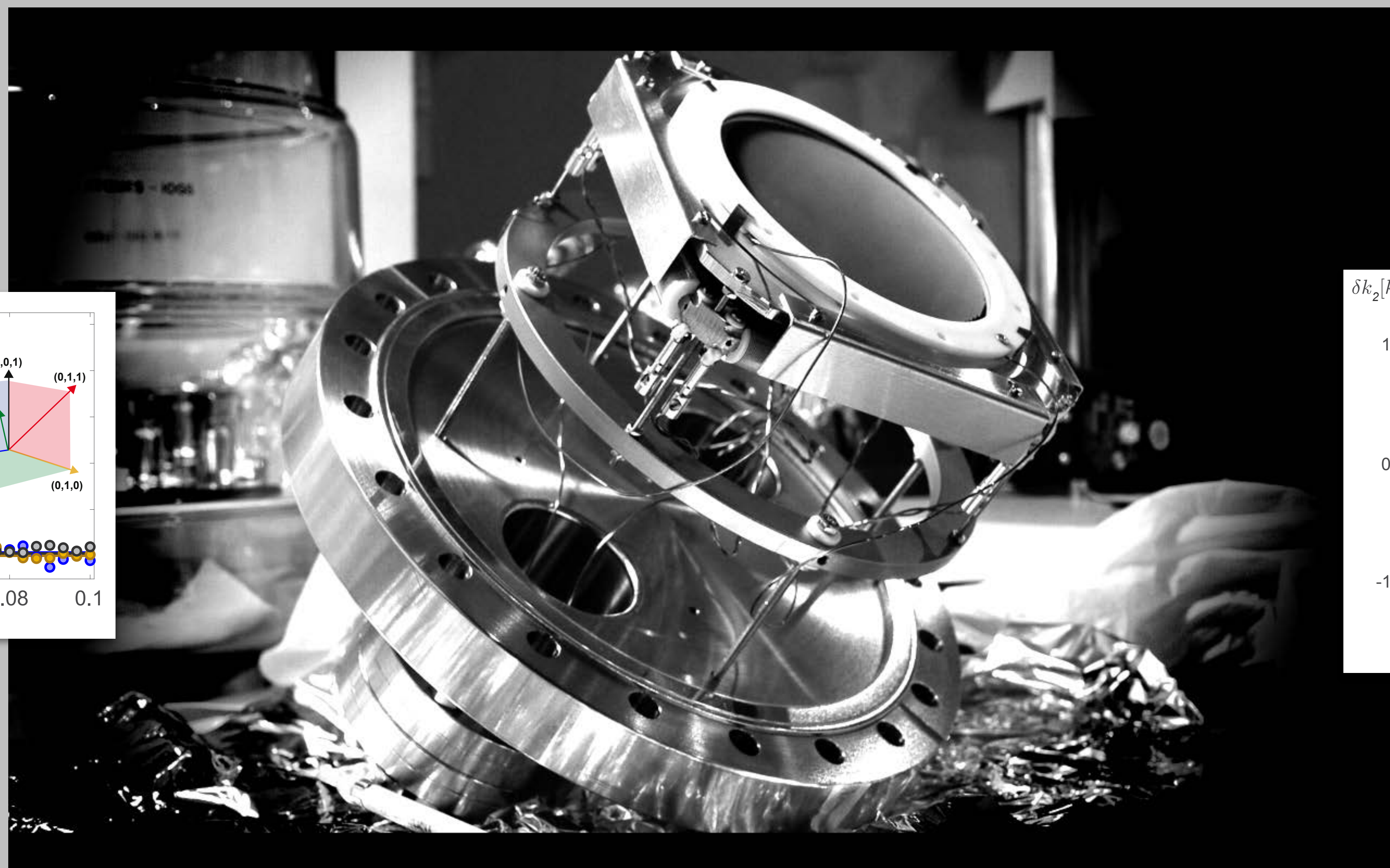
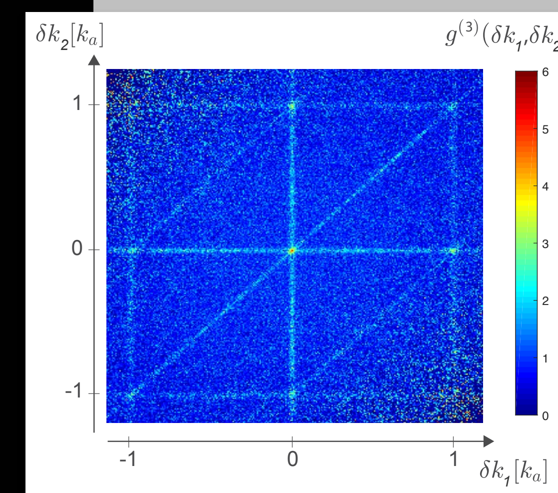
# Momentum-space atom correlations in a Mott insulator





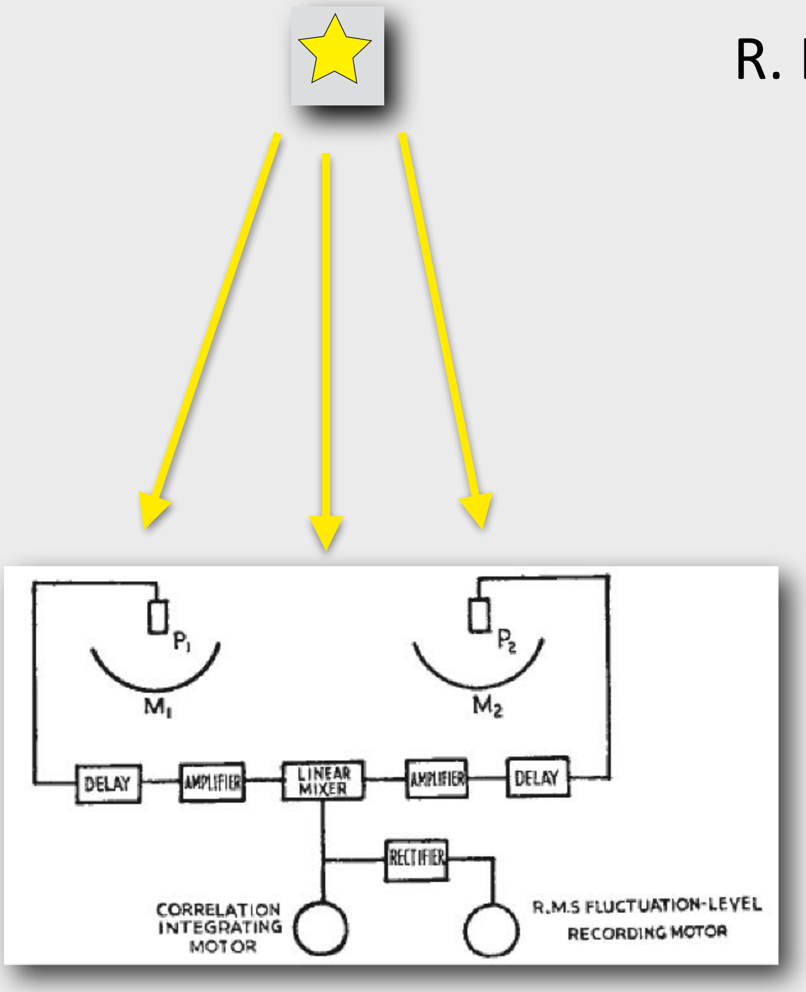


***David CLEMENT***

*Institut d'Optique - Palaiseau, France*


**Congrès général de la SFP 2019**

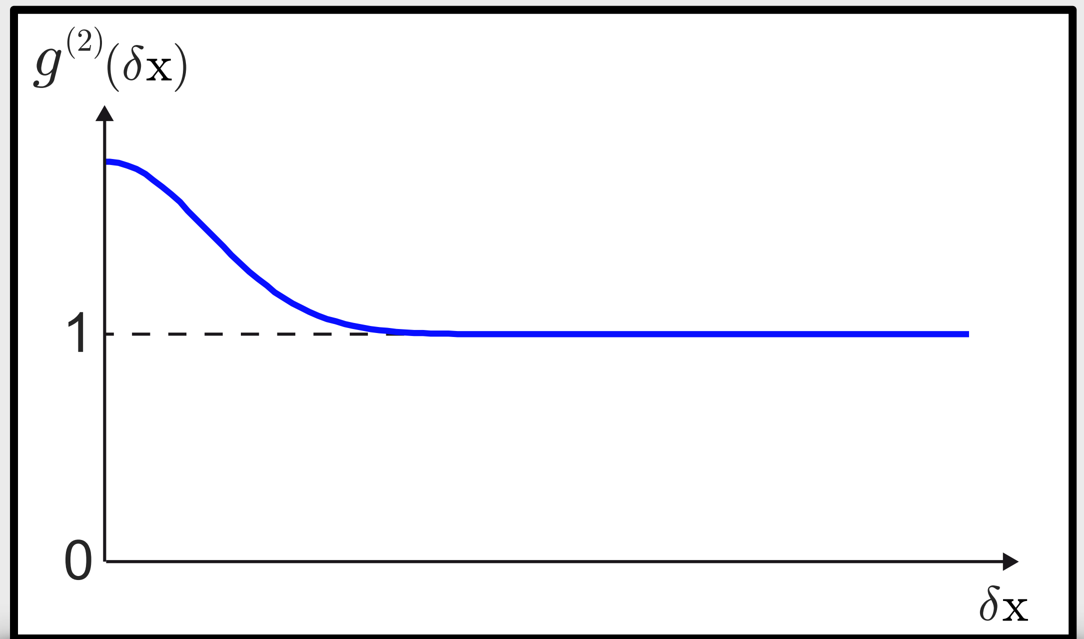
# A small detour in quantum optics: HBT experiment




R. Hanbury-Brown and R. Q. Twiss: **correlations in the light intensity**

Nature **177**, 27-29 (1956)

$$g^{(2)}(\vec{r}_1, \vec{r}_2, \tau) = \frac{\langle i(\vec{r}_1, t) i(\vec{r}_2, t+\tau)\rangle}{\langle i(\vec{r}_1, t)\rangle \langle i(\vec{r}_2, t+\tau)\rangle}$$



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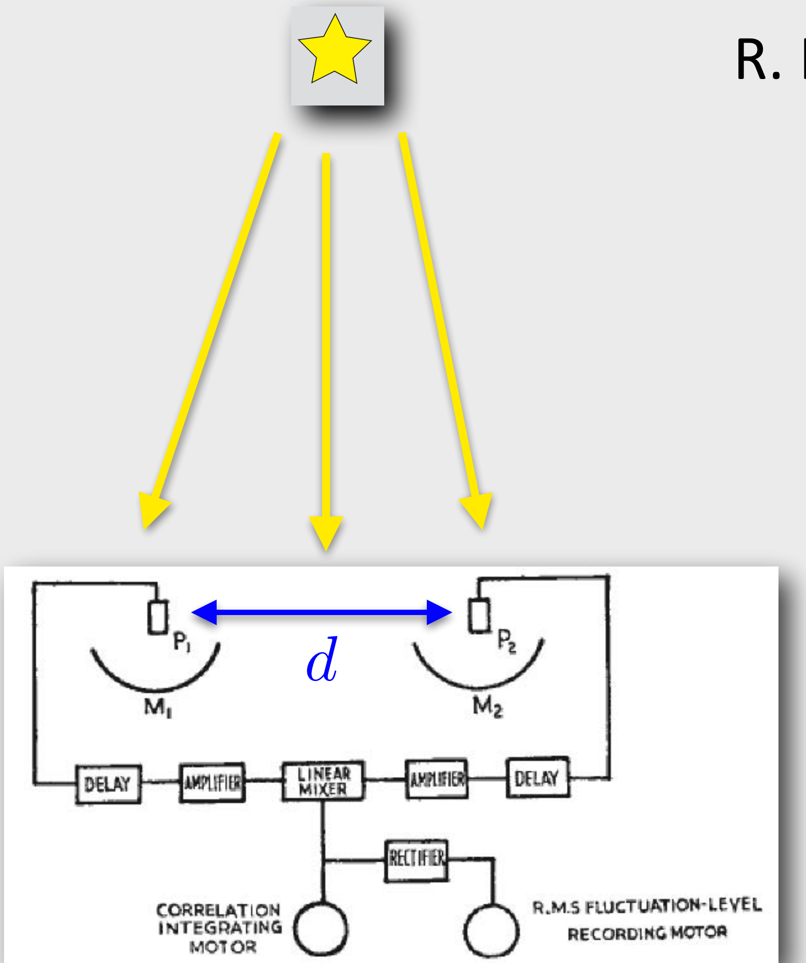
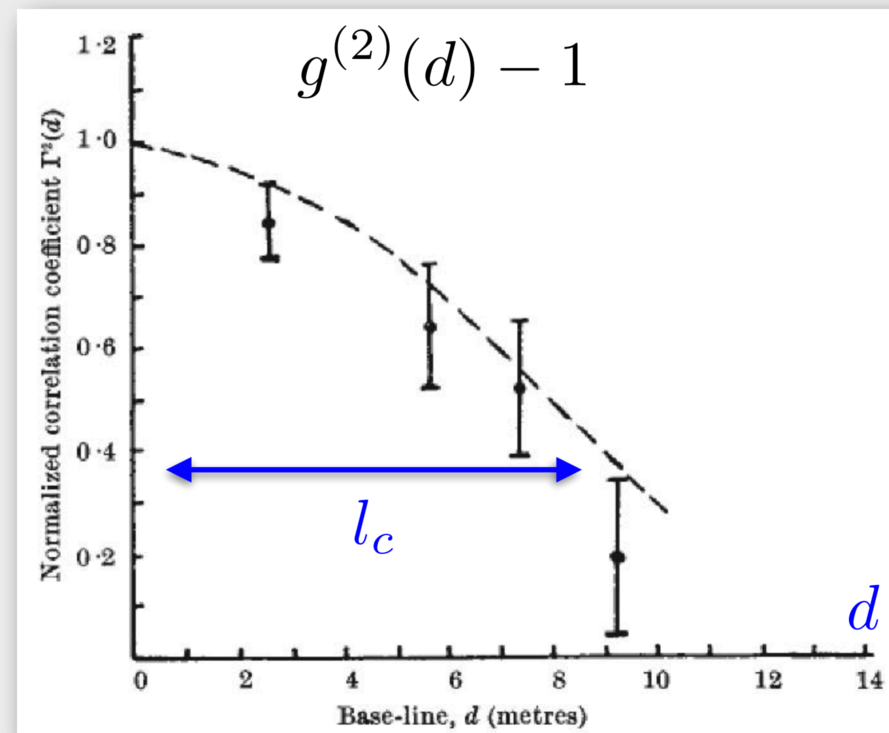


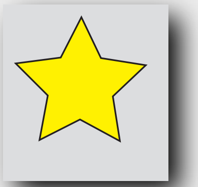

Nature **178**, 1046-1048 (1956)

**Correlation at zero distance equals 2**: $\boxed{g^{(2)}(d=0) = 2}$

Correlation length $l_c$ provides measure of the **angular diameter** $\alpha$ **of the star**: $\boxed{l_c \propto \lambda/\alpha}$

# The HBT stellar interferometer: a classical explanation

A star is an incoherent source of light:
modelled by <u>many random emitters</u>



The detected field is sum of many independent random variables:

$$\mathcal{E}(P_1, t) = \sum_{\text{star}} a_j \ \mathrm{e}^{i[\phi_j - \omega_j t - k.r_j]}$$

<u>Central limit theorem:</u> $\mathcal{E}(P_1, t)$ is a **Gaussian random variable**

$$\boxed{g^{(2)}(d) = 1 + \mid g^{(1)}(d) \mid^2}$$

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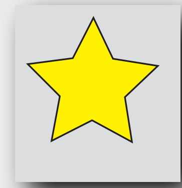

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$$\langle \mathcal{E}(P_1, t)\mathcal{E}(P_2, t) \rangle \simeq \sum_{\text{star}} \langle a_j^* a_j \rangle \; \mathrm{e}^{i(k-k').r_j}$$

<u>distribution of the **incoherent** emitters:</u>
<u>diameter of the star</u>

*(identical to Michelson interferometry)*

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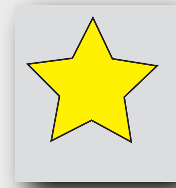

Thermal light = many modes of ideal bosons (photons) populated:

$$H = \sum_k N(k)\hbar\omega(k)\ a(k)^\dagger a(k)$$

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Described by a **Gaussian density operator**

<u>Central limit theorem:</u> $\mathcal{E}(P_1, t)$ is a **Gaussian random variable**

$$\boxed{g^{(2)}(d) = 1 + \mid g^{(1)}(d) \mid^2}$$

Wick theorem applies:

$$\boxed{\langle a_1^\dagger a_2^\dagger a_3 a_4 \rangle = \langle a_1^\dagger a_3 \rangle \langle a_2^\dagger a_4 \rangle + \langle a_1^\dagger a_4 \rangle \langle a_2^\dagger a_3 \rangle}$$

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<u>Constructive interferences: quantum statistics of ideal bosons!</u>

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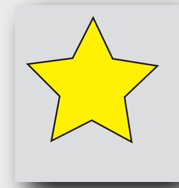

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$$\downarrow$$

$$g^{(2)}(d) = 1 + \mid g^{(1)}(d) \mid^2$$

<u>Constructive interferences: quantum statistics of ideal bosons!</u>

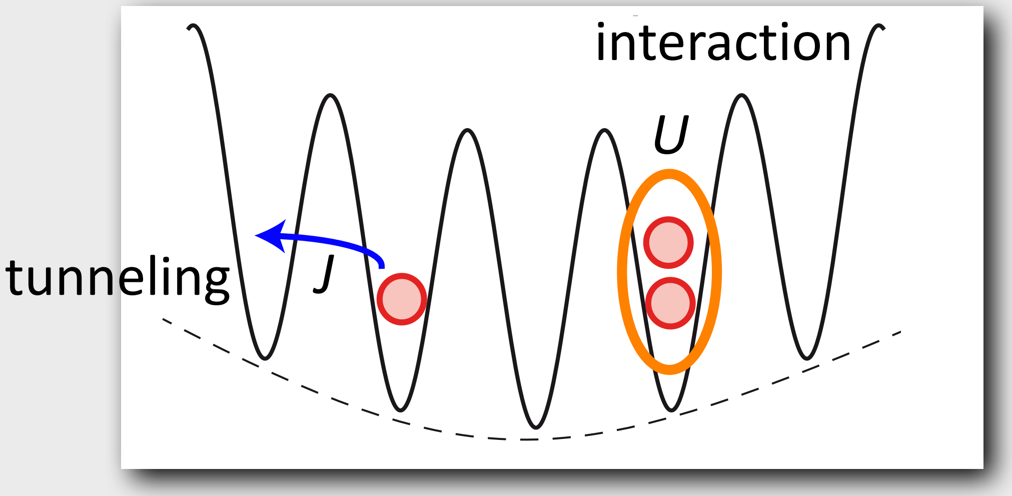
## <u>Characteristics of HBT experiments with incoherent source of non-interacting bosons:</u>

$\longrightarrow$ $g^{(2)}(d=0) = 2$ : source with **Gaussian statistics**

$\longrightarrow$ Shape of $g^{(2)}(d)$ is set by the **spatial distribution of the incoherent emitters**

# Correlations in a Mott insulator deep in the insulating regime

Bosons loaded in the lowest band of a lattice described by the **Bose-Hubbard Hamiltonian**:




$$H = -J \sum_{j,j'} b_j^\dagger b_{j'} + \frac{U}{2} \sum_j n_j (n_j - 1)$$

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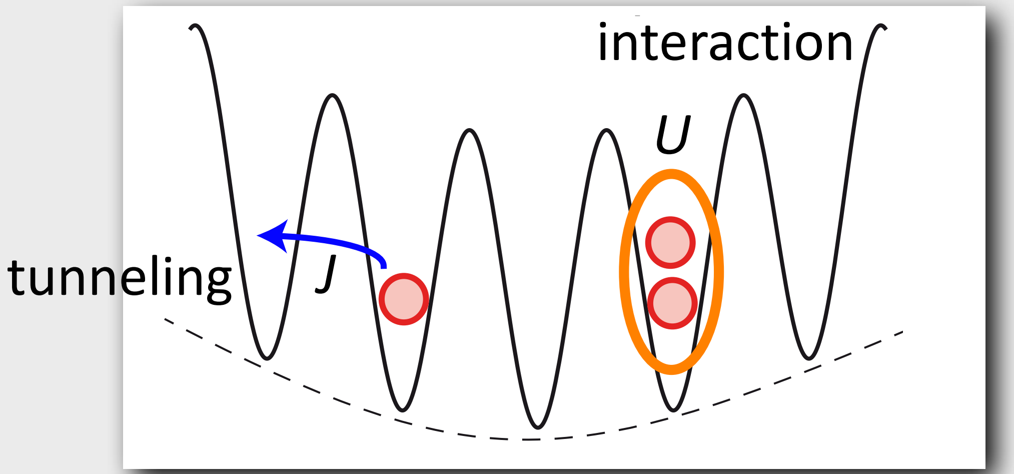
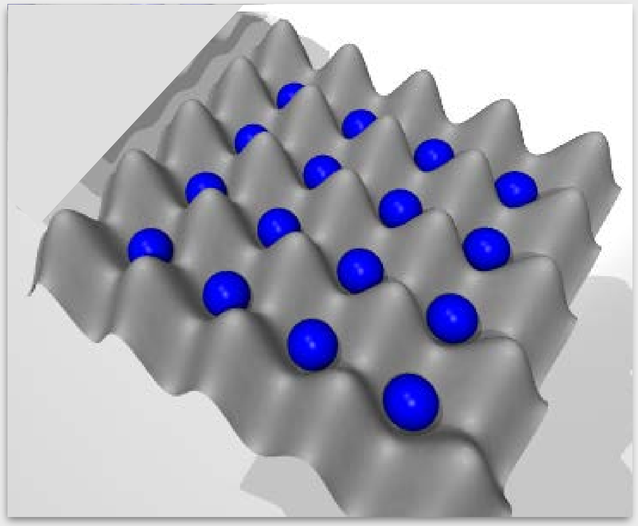



$$H = -J \sum_{j,j'} b_j^\dagger b_{j'} + \frac{U}{2} \sum_j n_j (n_j - 1)$$

In the regime $U \gg J$ the atoms are pinned in the lattice site with unity occupation: **Mott insulator**



**<u>Deep in the Mott regime:</u>**

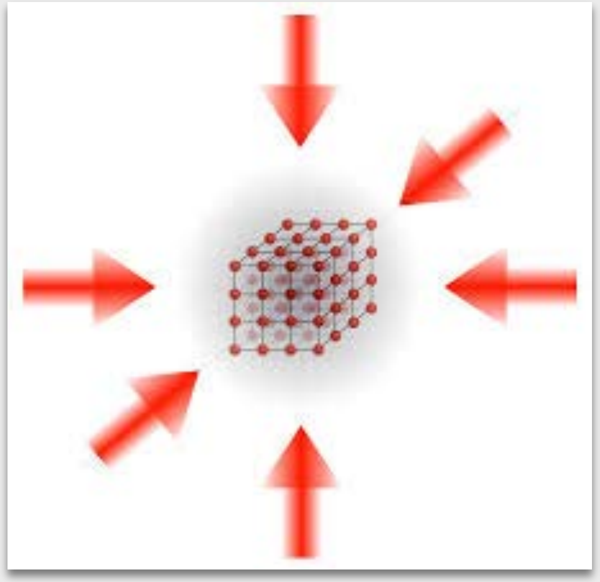
- negligible fluctuations of atom number per lattice site
- negligible site-to-site phase coherence

Interaction term is negligible: $H \simeq \sum_k \epsilon(k) a^\dagger(k) a(k)$

Similar to ideal bosons with no coherence!

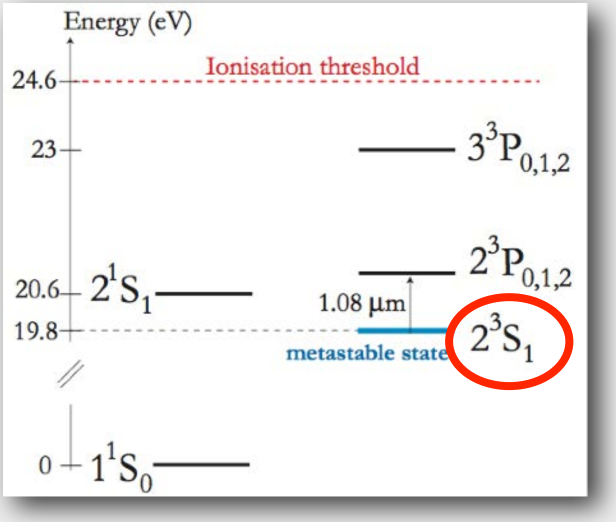
# Detection of lattice gases with the Helium detector

Adiabatic loading of Bose-Einstein Condensate into a 3D square optical lattice (wave-length 1550 nm)



3 pairs of counter-propagating laser beams forming standing waves

Bose-Einstein condensates of metastable Helium-4




Phys. Rev. A **90** 063407 (2014)
Phys. Rev. A **91** 061402(R) (2015)

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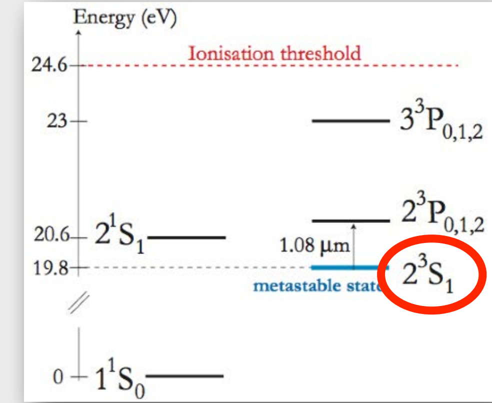
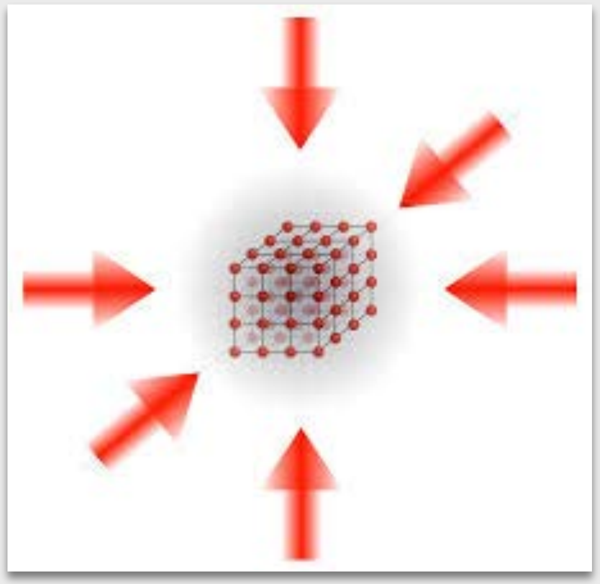
$^{4}\mathrm{He}^{*}$



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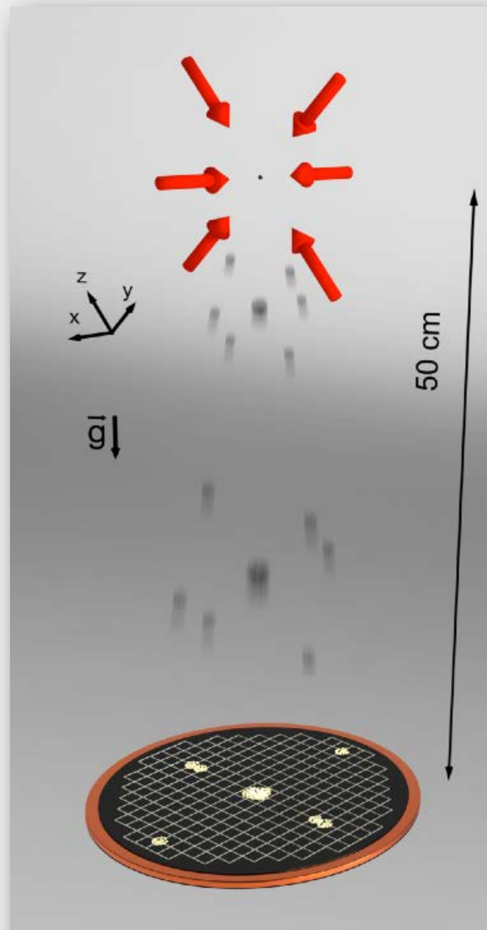
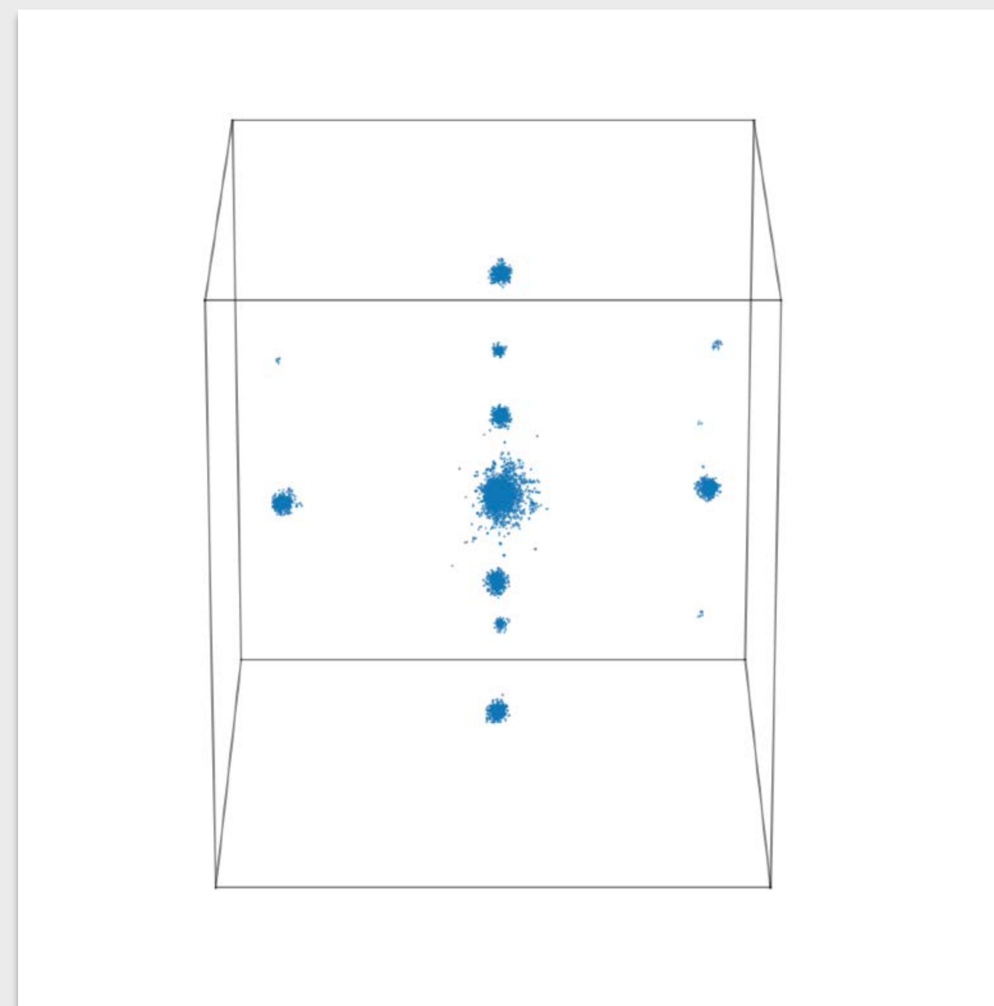


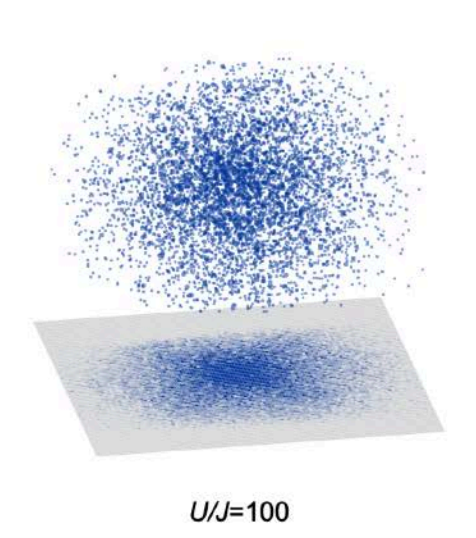

Bose-Einstein condensate when released from a 3D lattice

**3D distribution of single atoms**



Phys. Rev. A **97** 061609(R) (2018)

# Two-body correlations deep in the Mott regime




Mott insulator with $U/J = 100$

**Momentum density is featureless**
(due to absence of phase coherence)

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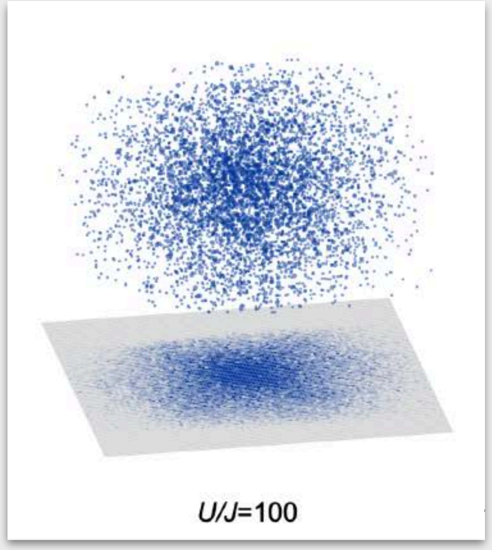


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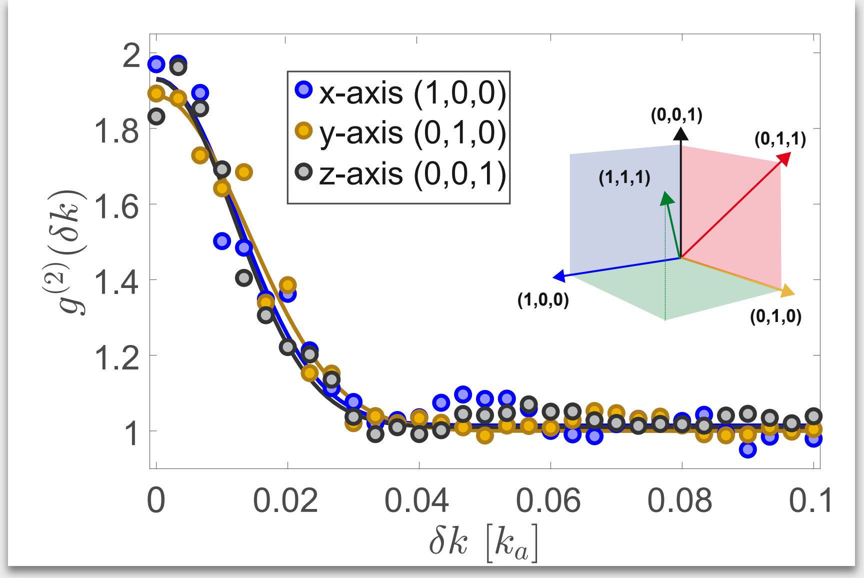
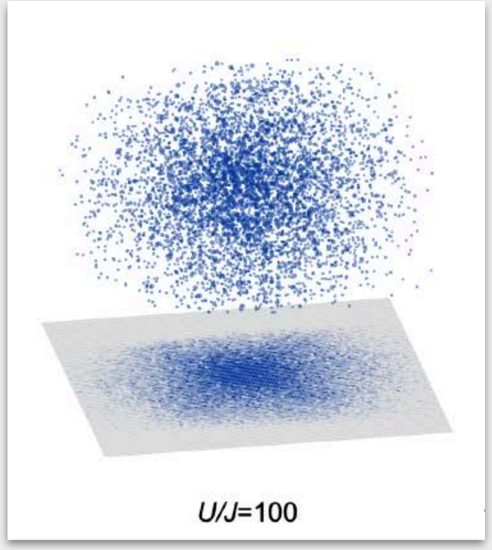
Two-body correlations in momentum-space: $$g^{(2)}(\vec{k},\vec{k'}) = \frac{\langle a^\dagger(\vec{k})a^\dagger(\vec{k'})a(\vec{k})a(\vec{k'})\rangle}{\rho(\vec{k})\rho(\vec{k'})}$$




1D cuts along a given axis: $\vec{k} - \vec{k'} = \delta k.\vec{u}$

**perfectly contrasted bosonic bunching**

# Two-body correlations deep in the Mott regime

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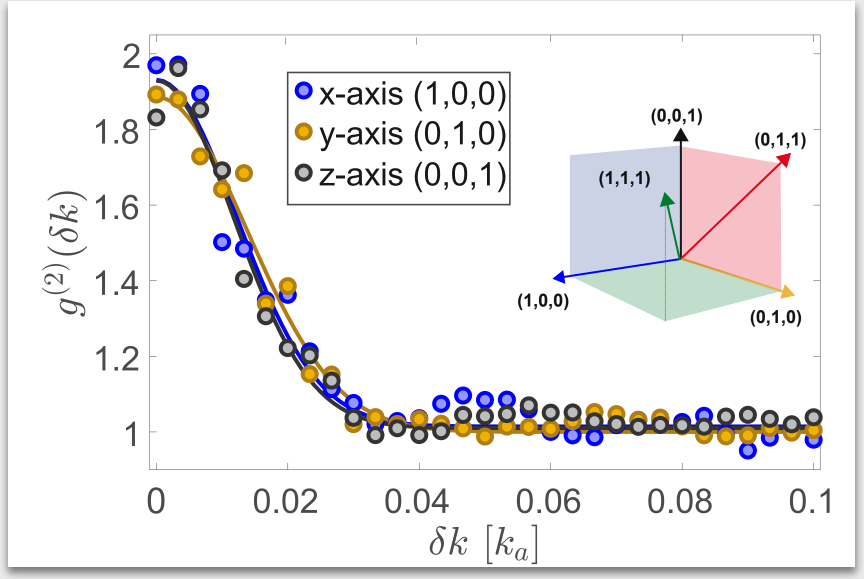


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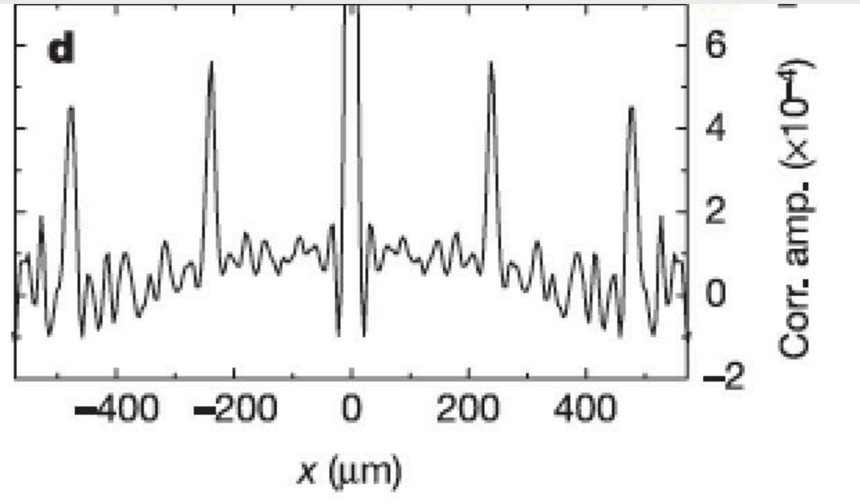
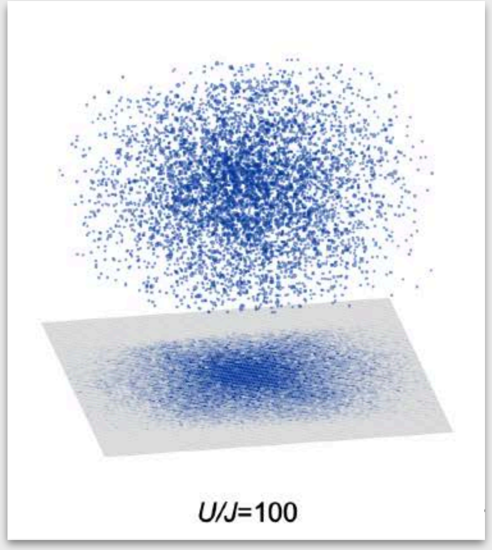
Pioneering experiment using noise correlation on atomic densities:




Nature **434**, 481 (2005)

Bunching observed in the expanded density!

# Two-body correlations deep in the Mott regime

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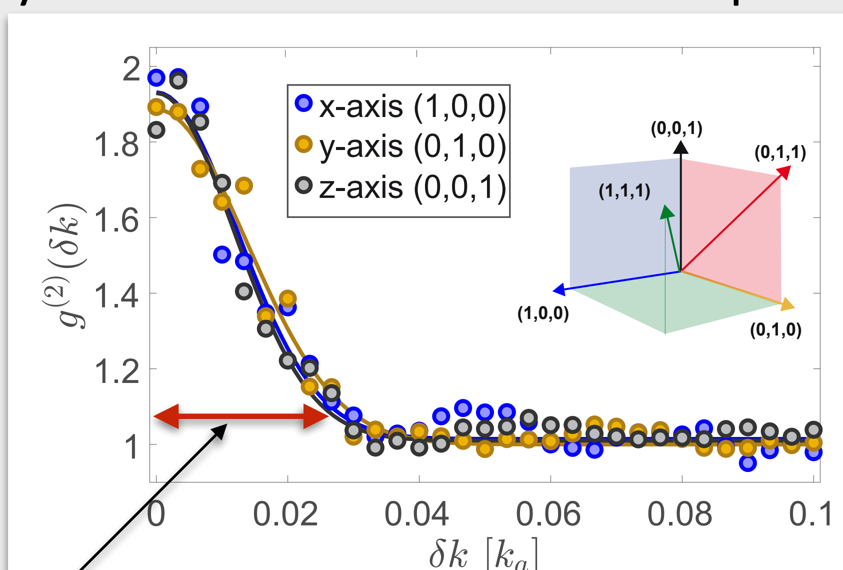
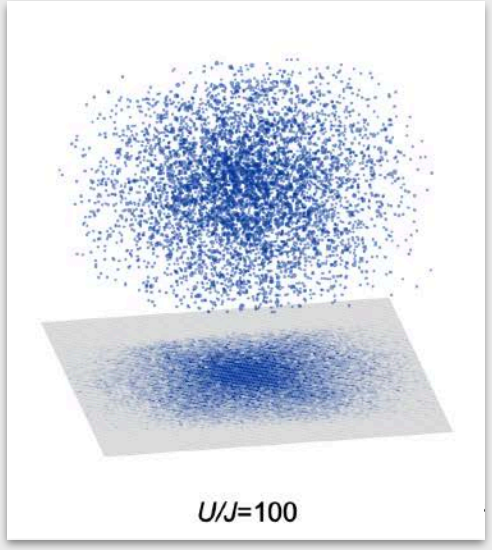



Two-body correlation length

# Two-body correlations deep in the Mott regime

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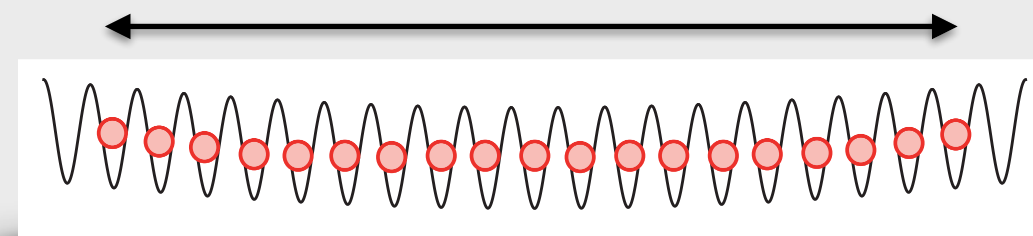
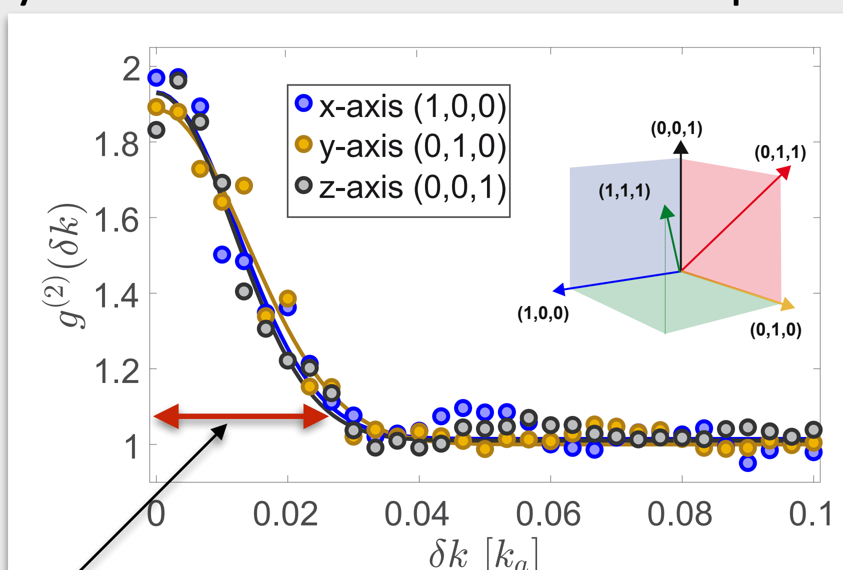
in-trap size $L$



Two-body correlations in momentum-space:

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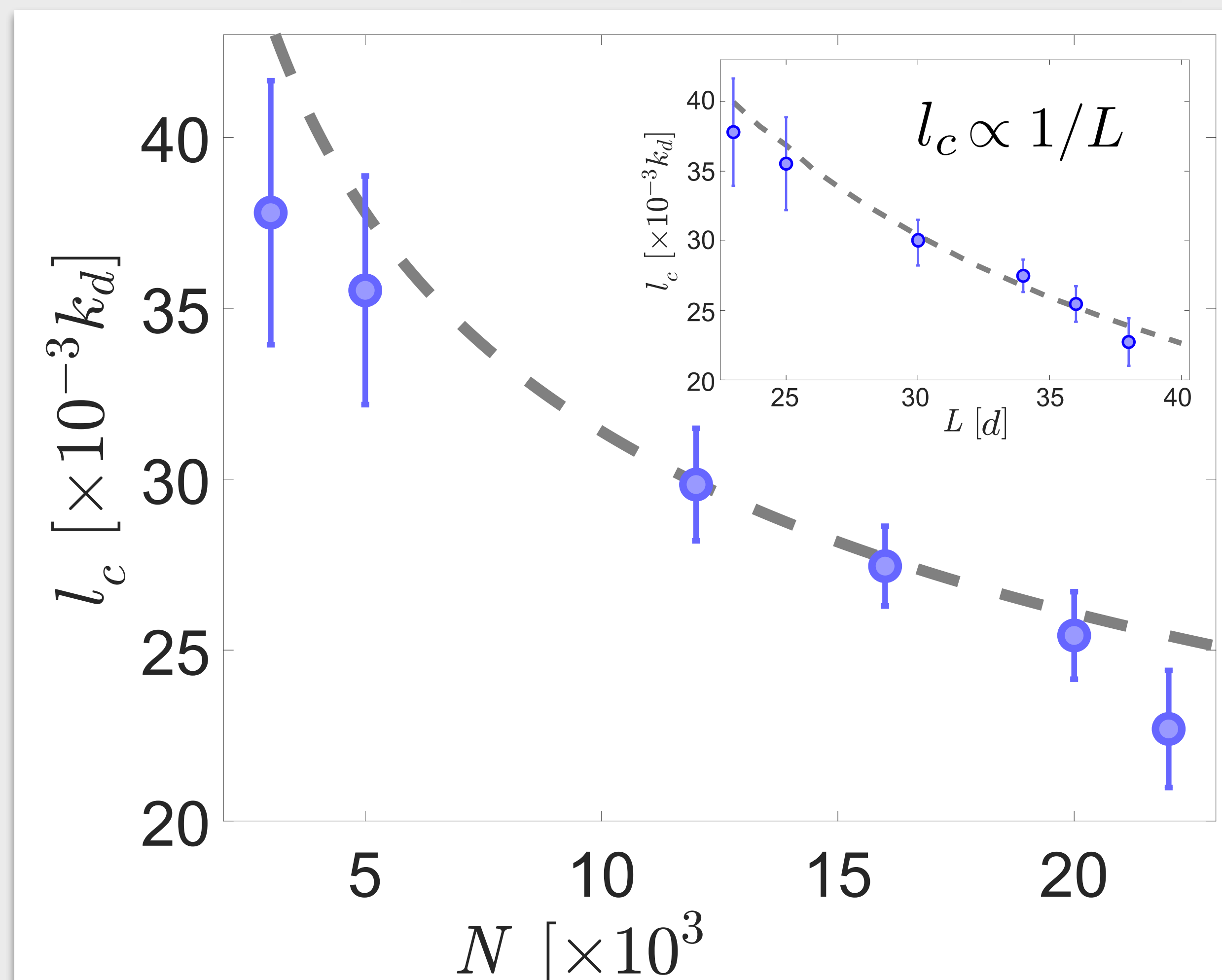
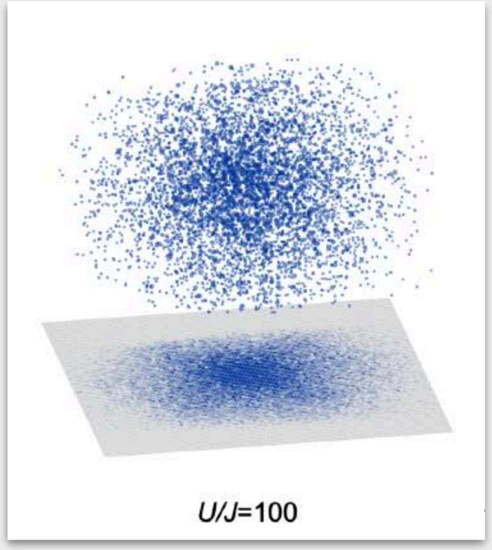



**Excellent agreement with no adjustable parameters!**

(incompressible Mott state)

dashed line: Gutzwiller solution for the experiment (3D Mott state and $\langle n_j \rangle = 1$ )

# Two-body correlations deep in the Mott regime




Mott insulator with $U/J = 100$

**Momentum density is featureless**
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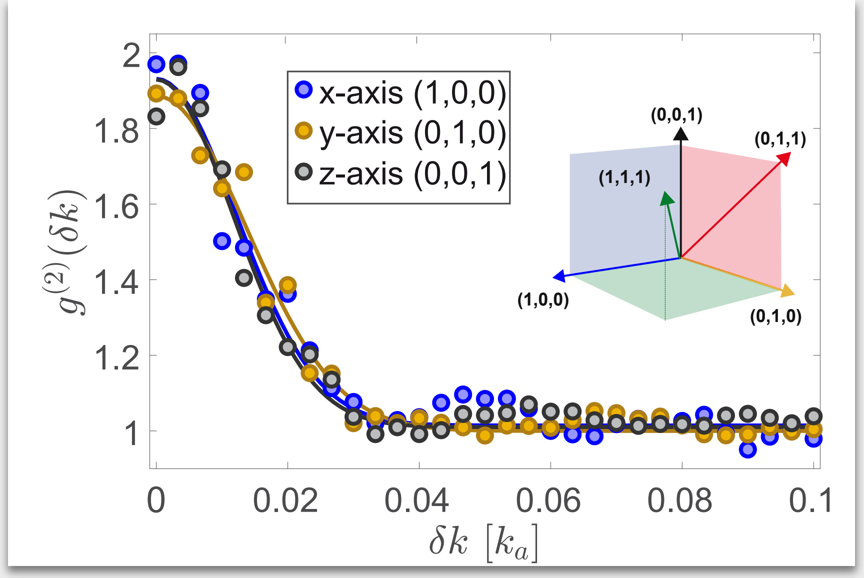
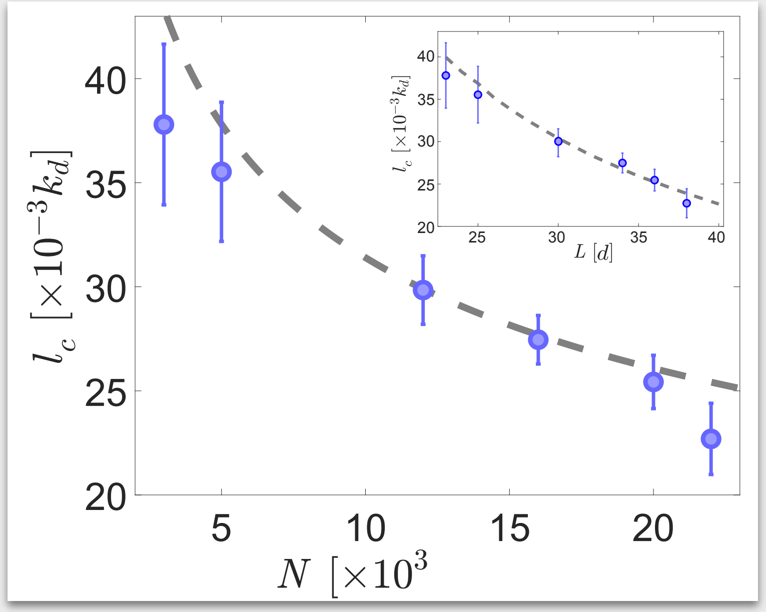
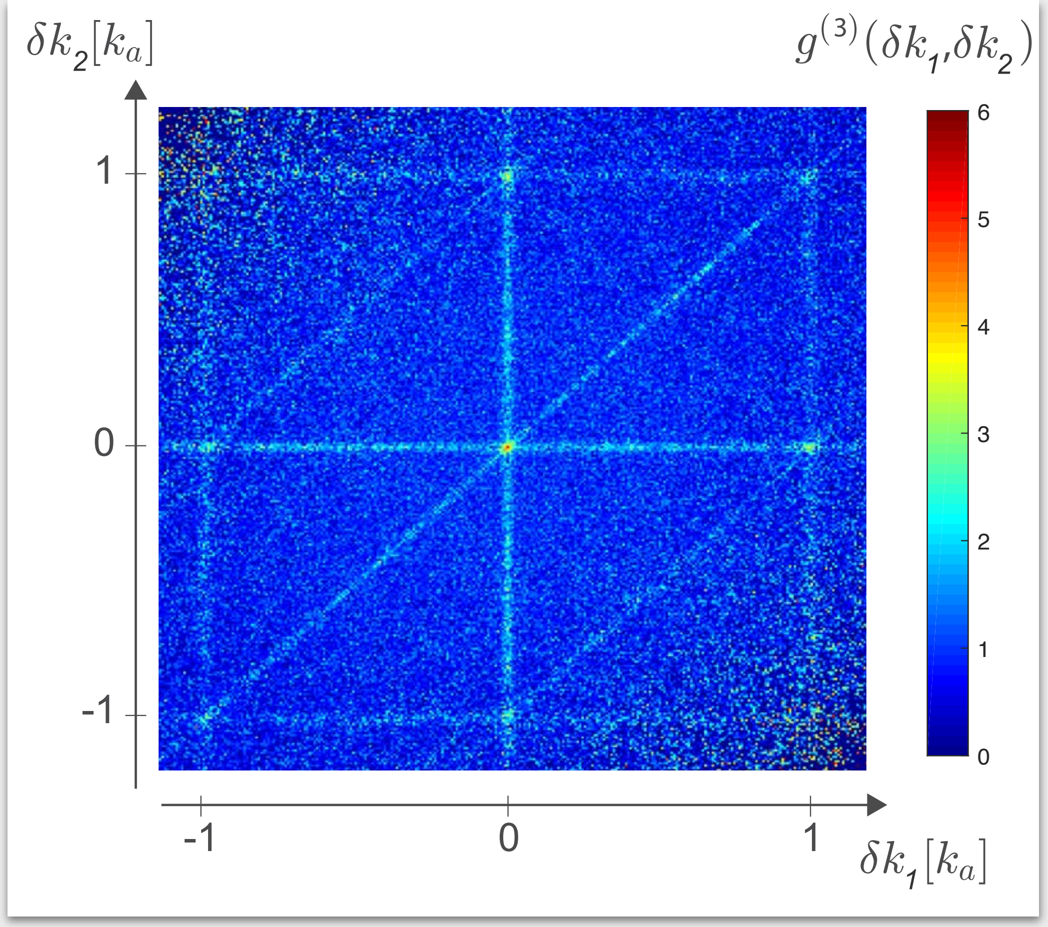
From **recorded full distribution of atoms** in 3D, we can extract higher-order correlations:

Two-body correlations in momentum-space:




Three-body correlations

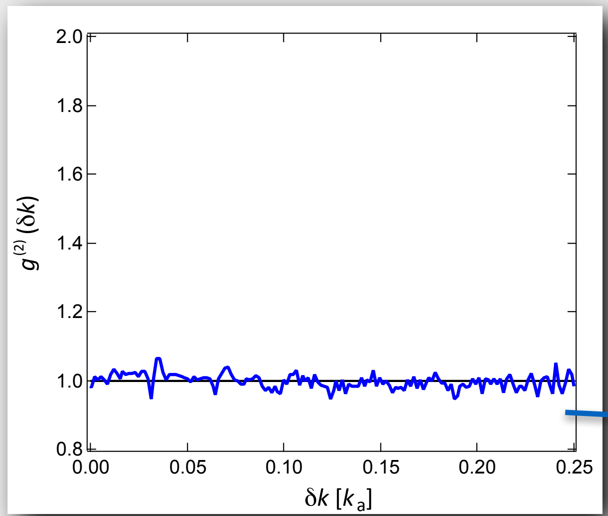



**Momentum-space correlations deep in the Mott regime are that of a Gaussian many-body ground-state**

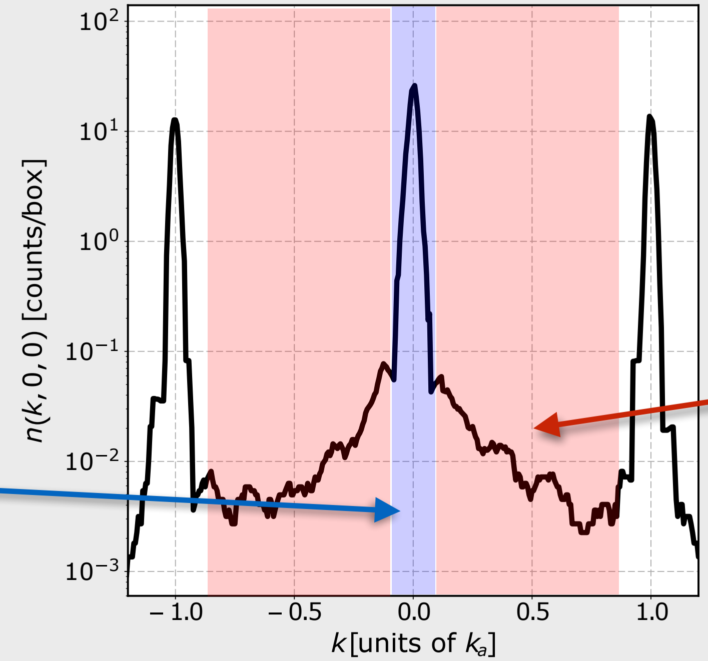
# Depletion of strongly-interacting condensate

Momentum distribution of lattice condensates:

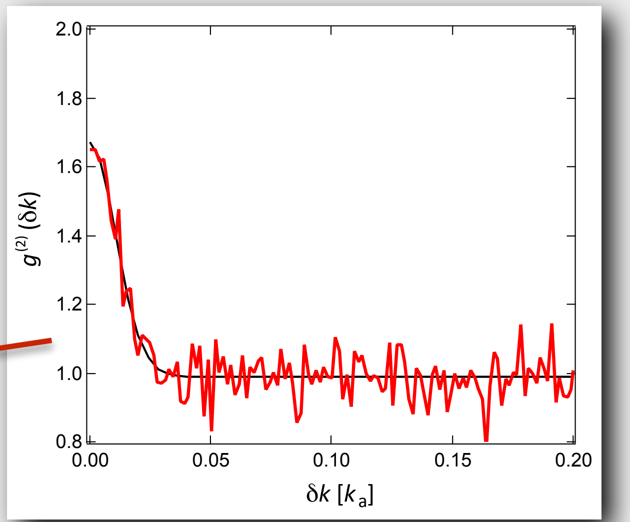
**Condensate**




*A single quantum state: no bunching*

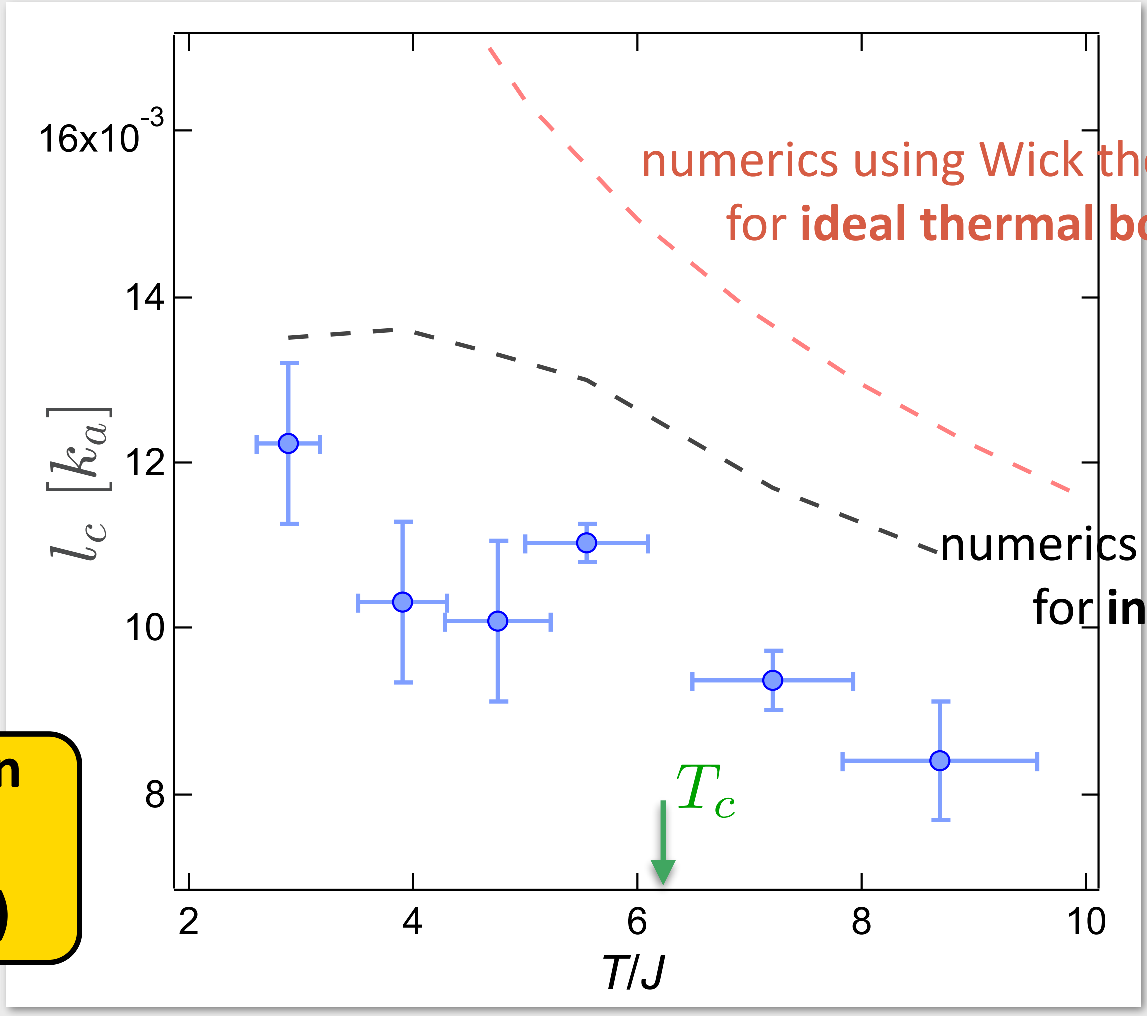



**Depletion of condensate**



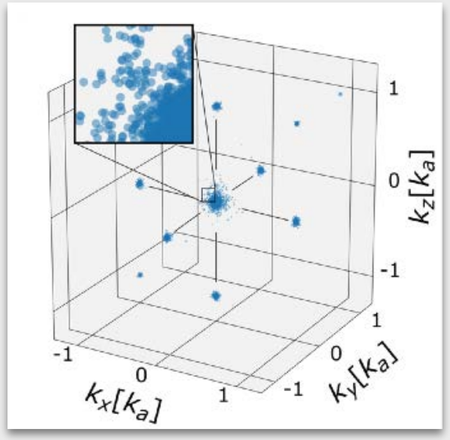

*Bosonic bunching*

$$U/J = 10$$



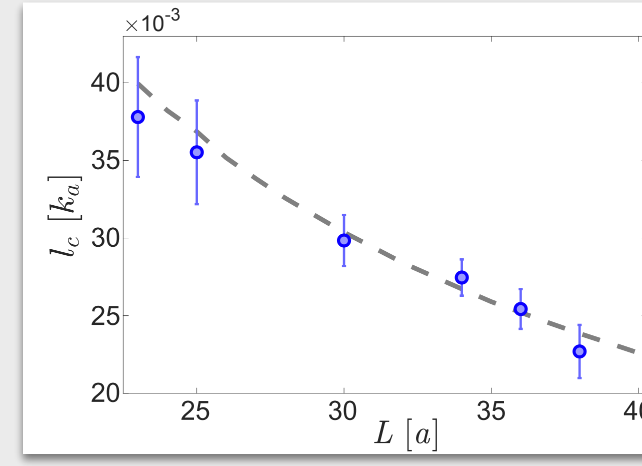

**Density operator non Gaussian for the depletion of strongly interacting condensates (U>T)**

# Conclusion



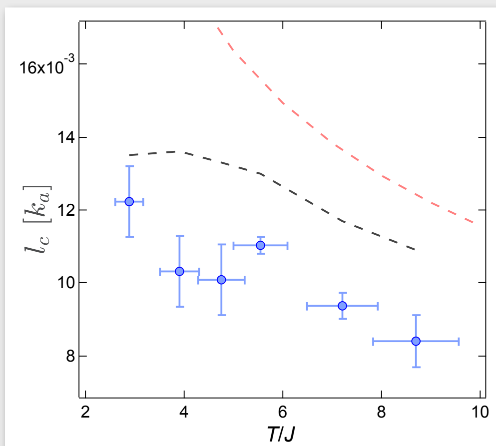

## Single-atom detection of lattice gases in 3D momentum space

H. Cayla *et al.*, Phys. Rev. A **97** 061609(R) (2018)

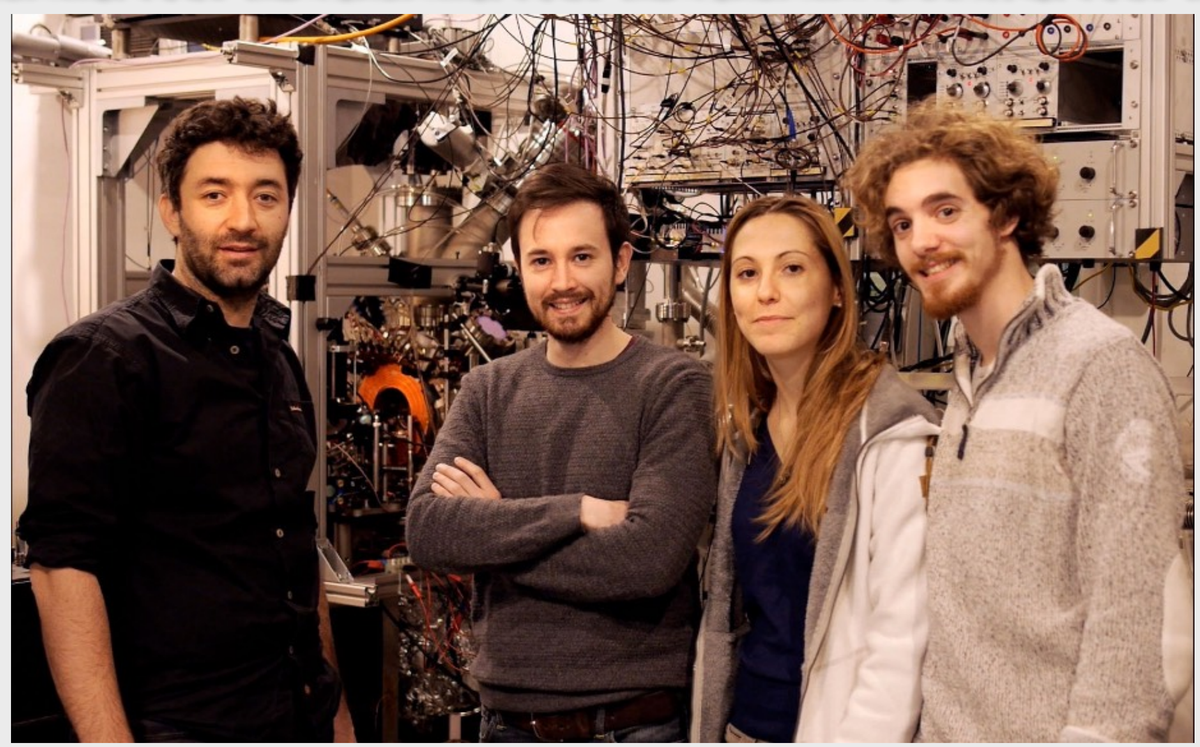



## Many-body correlations in the deep Mott regime

C. Carcy *et al.*, arXiv:1904.10995 (2019)




## Non-Gaussian correlations in interacting depleted bosons



**M. Mancini** **C. Carcy** **H. Cayla**

**A. Tenart**

**A. Aspect**

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**G. Carleo**
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