

# Fusion $\mu$ -catalysée en plasmas ultradense

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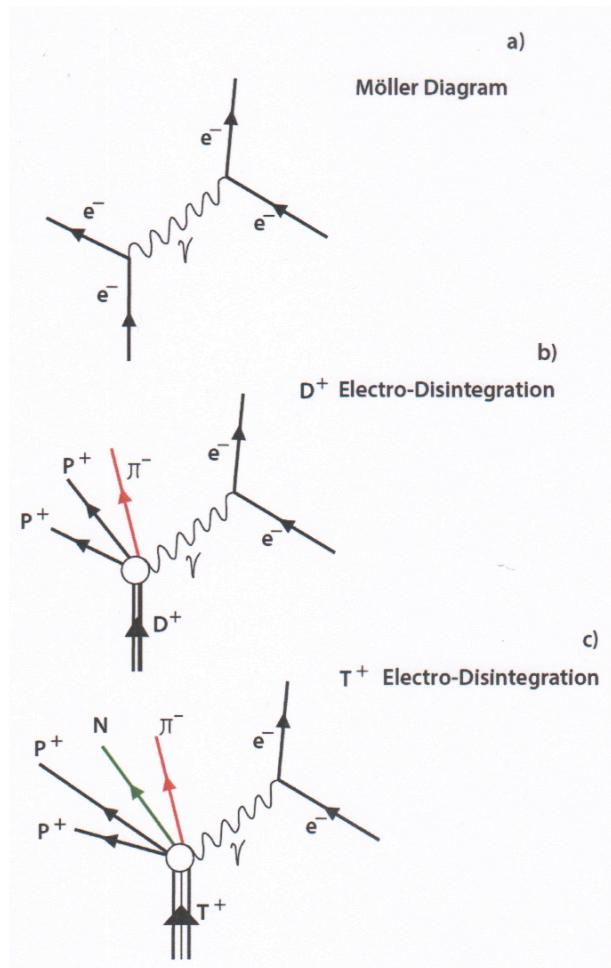


Figure 1: (color online) (a) Electron-electron covariant Möller diagram.  
 (b) D-Electron-disintegration and (c) T-Electron-disintegration. In (b) and (c)  
 $\pi^-$  appears when REB energy  $E_b \geq 150$  MeV.

- Ralentissement des  $\pi^-$ ( $\mu^-$ ) à quelque MeV en **plasmas ultradenses** :
  - WDM,  $5.64 \times 10^{23}/\text{cc}$ , 1.75 eV
  - FIS,  $10^{26}/\text{cc}$ , 0.1 et 1 keV
- Formation d'ions moléculaires hydrogénoides
- Problématique du collage ('Stincking')
- Taux de production d'énergie rapportée au coût de production du  $\pi^-$ .

## ANTICIPATION

- 134 DT fusion en WDM sur 500 psec
- 20 DT fusion en FIS sur 20 psec

## Longueurs pertinentes

$$n_e \sim 10^{26} \text{ e} - \text{cm}^{-3} \quad T \sim 1 \text{ keV}$$

$$a_{ii} = \left( \frac{4}{3} \pi n_i \right)^{-1/3} \sim 1.33 \times 10^{-9} \text{ cm}$$

Longueur de Debye  $\sim 2.35 \times 10^{-9} \text{ cm}$

Rayon de Bohr (electron)  $= 5.29 \times 10^{-9} \text{ cm}$

Rayon de Bohr (pion)  $= 1.94 \times 10^{-11} \text{ cm}$

$\Pi$ -D and  $\Pi$ -T atoms hardly affected by electron Debye-screening.

## RECOMBINAISON RADIATIVE $\Pi^- - D + (T^+)$ ( $R_2$ )

$$\sigma_0 = (2^7 \pi e^2 h / 3m_\pi'^2 c^3 g^2) e^{(-4/g)\tan^{-1}g} (1+g^2)^{-2} (1-e^{-2\pi/g})^{-1}$$

$g$  is the ratio of the initial translational energy of the electron to the energy of ionization of the *exoH* atom.  
 For low velocities the cross becomes infinite:

$$\sigma_0 \sim (2^7 \pi e^2 h / 3e_1^4 m_\pi'^2 c^3) g^{-2} \sim (V_\pi/c)^{-4} \quad e_i = 2.718\dots$$

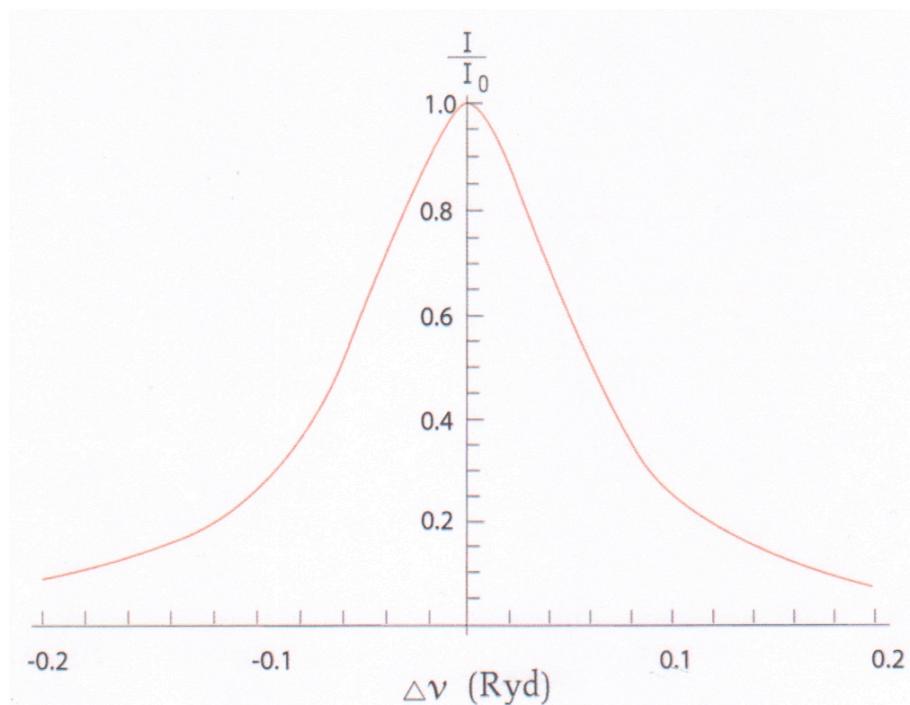
For high velocities it is given by

$$\sigma_0 \sim (64e^2 h / 3m_\pi'^2 c^3) g^{-5},$$

where

$$\begin{aligned} g &= \frac{1}{2} \cdot \frac{139.5 \times 10^6 (V_\pi/c)^2}{\frac{273 \times 27.2}{2n^2}} \\ &= 18787 n^2 \left( \frac{V_\pi}{c} \right)^2 \end{aligned}$$

J. R. Oppenheimer. Zeit. f. Physik **41**, 268 (1927);  
 Phys. Rev. **31**, 349 (1928).



X-Ray (Stark) line profiles emitted by D/T  $\mu$  exoatoms in FIS plasmas ( $N_e = 10^{26}/cc$ ,  $T = 1$  keV).  
Energies are in Ryd = 2812.35 eV (muon a.u.).  
Lyman  $\alpha$  with unperturbed  $\lambda = 6.146\text{\AA}$ .

Taux de Recombinaison D/T ions -  $\mu \gamma^\mu (sec^{-1}) = N_{D/T} \bar{\sigma}_0 v$   
in terms of exoatom main quantum number n and Maxwell  
averaged  $\bar{\sigma}_0$  taken down to  $10^{-13} V_{th}$

n	WDM ( $N_{D/T} = 5.64 \times 10^{23}/cc, T = 1.75eV$ )	FIS( $N_{D/T} = 10^{26}/cc$ )	
		100 eV	1 keV
1	$6.109 \times 10^{18}$	$2.50 \times 10^{18}$	$7.92 \times 10^{16}$
2	$3.82 \times 10^{17}$	$1.564 \times 10^{17}$	$4.95 \times 10^{15}$
3	$7.54 \times 10^{16}$	$3.09 \times 10^{16}$	$9.77 \times 10^{14}$
4	$2.39 \times 10^{16}$		
5	$9.77 \times 10^{15}$		
6	$4.71 \times 10^{15}$		

Recombinaison 3-corps  $D/T - \mu - D/T$  via la polarisabilité  $\alpha$  et le dipole magnétique de l'exoatome  $D/T - \mu$

$$\boxed{\bar{K} = (2/\mu^{1/2})[\alpha^{1/2} + c\mu_D(2/BT)^{1/2}]}$$

with

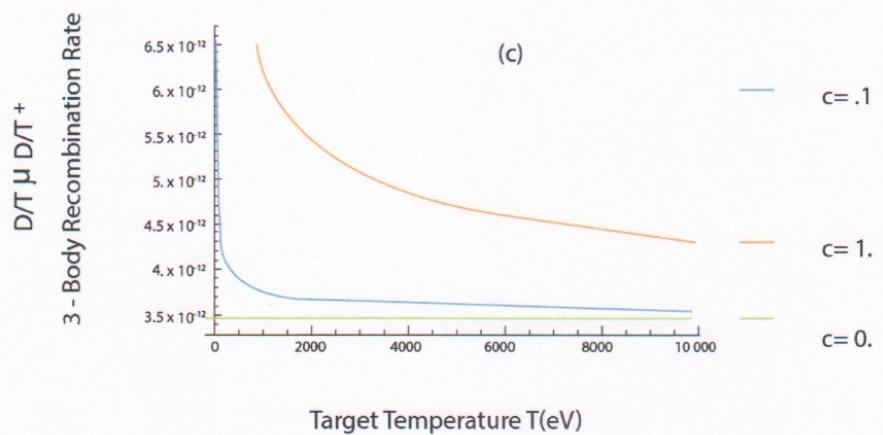
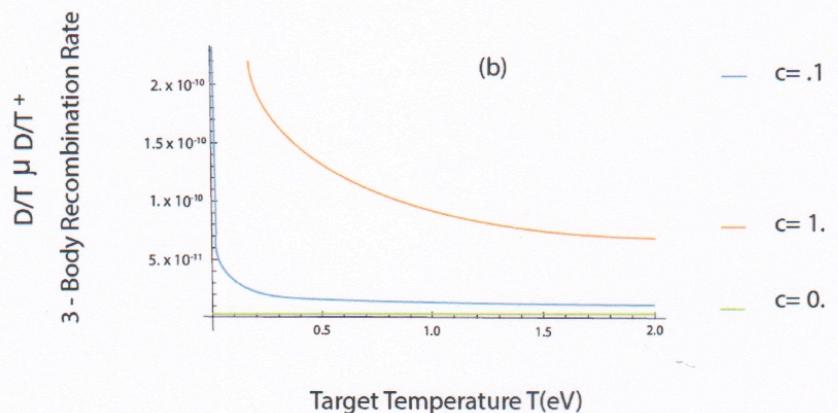
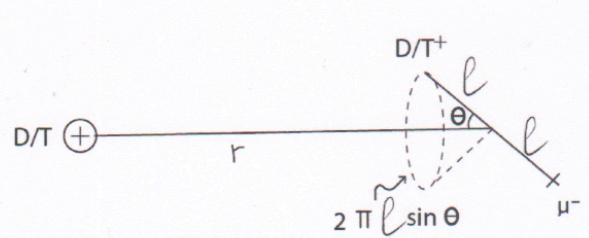
$$\alpha = \frac{1}{n} \sum_{\ell=0}^{n-1} \left[ a_0^{n\ell} + 2 \sum_{m=0}^{\ell} a_2^{n\ell} \frac{3m^2 - \ell(\ell+1)}{\ell(2\ell-1)} \right] cm^3,$$

where

$$a_0^{n\ell} = \frac{n^4}{4} [4n^2 + 14 + 7\ell(\ell+1)]$$

$$a_2^{n\ell} = \frac{-n^4\ell}{4(2\ell+3)} [3n^2 - 9 + 11\ell + \ell(\ell+1)]$$

and dipole moment  $\mu_D = \frac{ea_0^\mu}{2n} \sum_{\ell=0}^{n-1} (3n^2 - \ell(\ell+1))$  averaged over  $(n, \ell)$  levels through Holtsmark Stark-mixing, with tuning parameter  $c$  featuring either a locked dipole ( $c = 1$ ) or a rotating one ( $c = 0.1$ ).



## RECOMBINAISON 3 corps $D/T - \mu - D/T(psec^{-1})$

Three-body capture rates  $N_{D/T}\bar{K}$  ( $psec^{-1}$ ) in terms of exoatom main quantum number n and amount of dipole contribution c.

a) WDM target ( $T = 1.75eV, N_{D/T} = 5.64 \times 10^{23}/cc$ )

n	1	2	3	4	5	6
c=0.1	1.34	5.71	14.45	28.85	50.2	79.73
c=1	10.55	39.5	89.2	161	256	375.5

b) FIS target ( $T = 1keV, N_{D/T} = 10^{26}/cc$ )

n	1	2	3	4
c=0.1	64.32	374.46	1151.3	2757.0
c=1	132.64	624.95	1705.4	3601.25

c) FIS target ( $T = 100eV, N_{D/T} = 10^{26}/cc$ )

n	1	2	3	4
c=0.1	80.73	434.64	1824.44	2857.3
c=1	296.8	1226.8	3036.75	5953.83

## Analyse Dynamique

$$\frac{dN_\mu(t)}{dt} = -(2\lambda_{\frac{D}{T}\mu} + \lambda_0^\mu)N_\mu(t) + 2\lambda_{\frac{D}{T}\mu\frac{D}{T}}N_{\frac{D}{T}\mu}(t),$$

$$\frac{dN_\mu(t)}{dt} = 2\lambda_{\frac{D}{T}\mu\frac{D}{T}}N_{D/T}N_\mu(t) - 2(\lambda_{\frac{D}{T}\mu} + \lambda_0^\mu)N_{\frac{D}{T}\mu}(t)$$

expressed more compactly under the form

$$\frac{dN_\mu(t)}{dt} = -a_1 N_\mu(t) + 2b_1 N_{D/T\mu}(t),$$

$$\frac{dN_\mu(t)}{dt} = 2a_2 N_\mu(t) - 2b_2 N_{D/T\mu}(t),$$

where

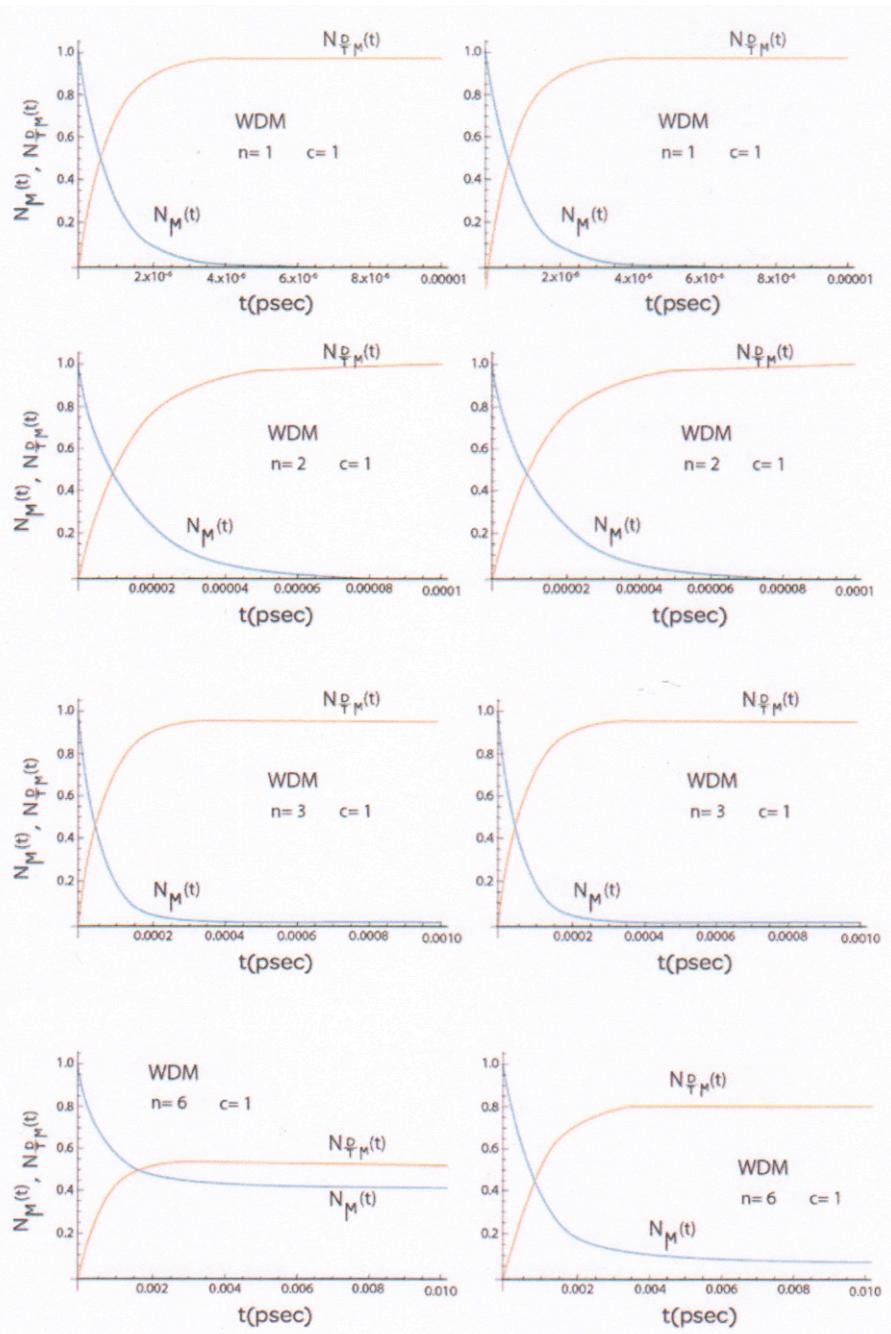
$$a_1 = 2\lambda_{D/T\mu}N_{D/T} + \lambda_0^\mu,$$

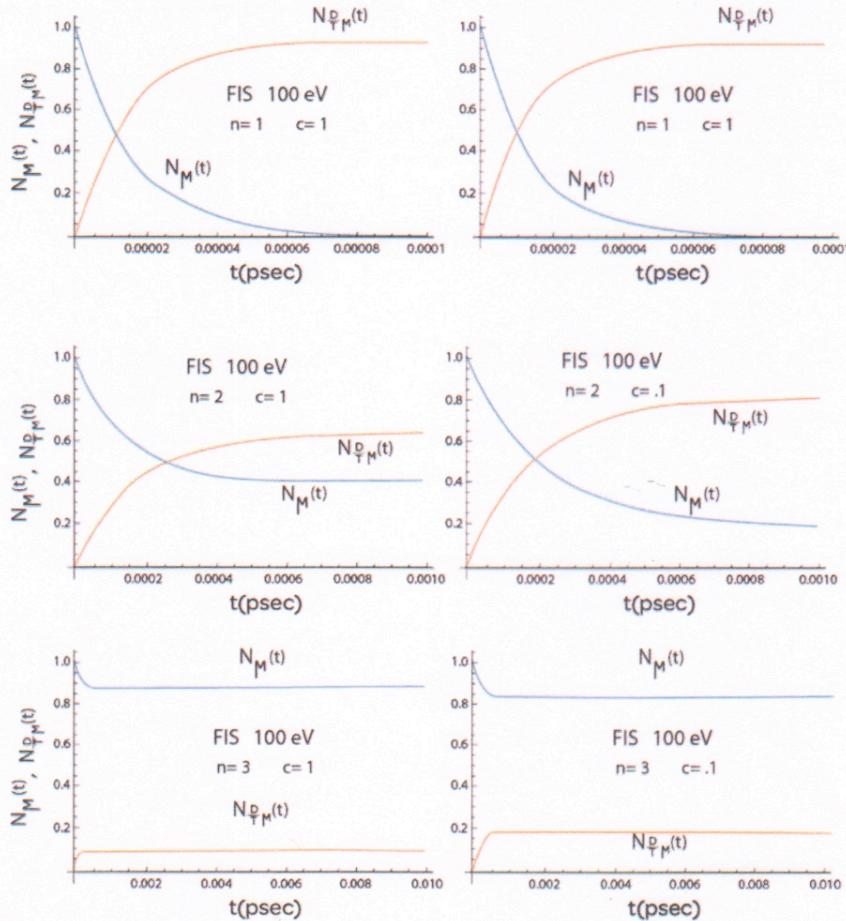
$$a_2 = \lambda_{D/T\mu}N_{D/T},$$

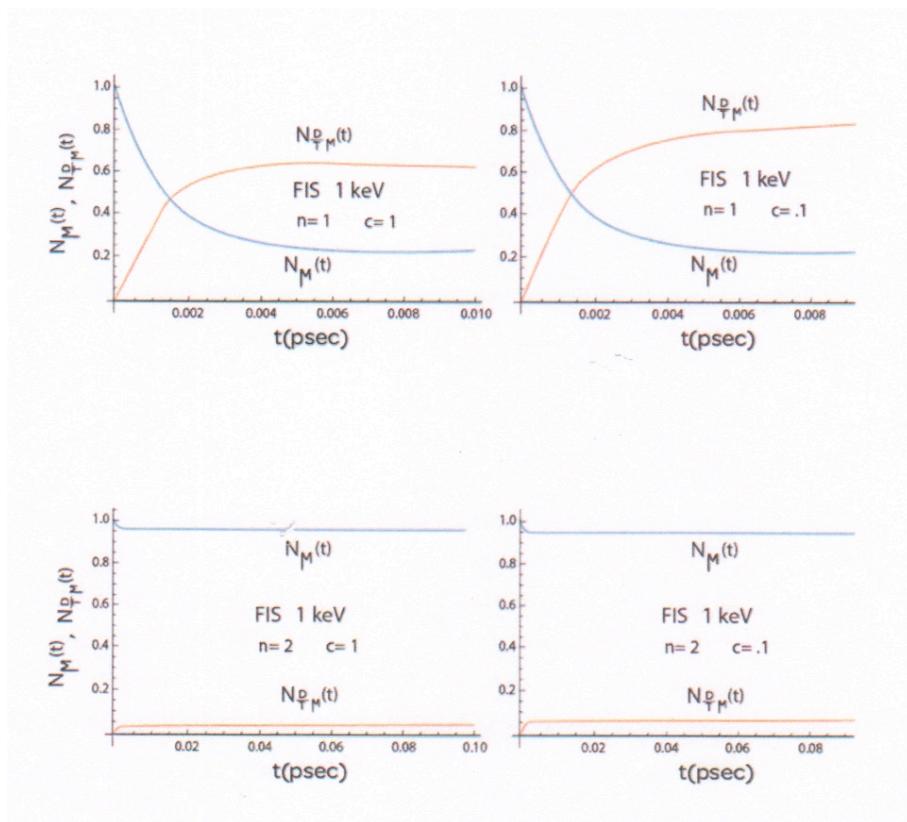
$$b_1 = \lambda_{D/T\mu D/T}$$

and

$$b_2 = b_1 + \lambda_0^\mu$$







## PROBABILITE DE REACTIVATION ( $\mu^-$ ) R=1-exp(-JR)

(a)  $He_\mu^+(n = 1)$

WDM		FIS			
JR	R	100 eV		1 keV	
		JR	R	JR	R
7.08	0.9992	200.6	1	12.28	1

(b)  $D/T\mu$

WDM (n=6)		FIS (n=3)			
JR	R	100 eV		1 keV	
		JR	R	JR	R
$2.74 \times 10^{-4}$	0	$1.34 \times 10^{-6}$	0	$2.5 \times 10^{-4}$	0

## PROBLEMATIQUE du COLLAGE $He_{\mu}^{+}$

$$\sigma_{strip}(E) = \sigma^{tr}(E) + \sigma^{ion}(E)$$

sum of charge transfer and ionization with

$$\sigma_n^{tr} = 324n^4(nv)^2 / \{[0.187 + (nv)^2][286 + (nv)^7]\}$$

and

$$\sigma_n^{ion} = 5.43n^4(nv)^{16} / \{[1.30 + (nv)^{11.8}][204 + (nv)^{6.2}]\},$$

in  $6.55 \times 10^{-22} cm^2$  for muonic atoms for a given bound state n in terms of the  $He_{\mu}^{+}$  velocity in m.a.u with respect to the in situ D/T ions.

The Jackson-Rafelski ratio (JR) then denotes the probability that the negative muon will be stripped off [1,2] during its slowing down.

It reads as

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$$JR = -N_{D/T} \int_0^{3.5 MeV} \frac{\sigma_{strip}(E)}{\frac{dE}{dx}} dE,$$

**RALENTISSEMENT**  $He_\mu^+(35MeV)$  **et**  $\mu^-(V_0 = 2.19 \times 10^8 \text{cm/sec})$

PLASMA WDM  $Ne = 5.64 \times 10^{23}/cc$

(a)  $He_\mu^-$

T(eV)	1.75	5	10	15
T <sub>stop</sub> (psec)	91.57	21.89	9.74	2.59
R( $\mu m$ )	513.7	102.43	41.62	3.02

(b)  $\mu^-$

T(eV)	1.75	5	10	15
T <sub>stop</sub> (psec)	0.372	0.175	0.0975	0.071
R( $\mu m$ )	0.56	0.234	0.12	0.083

PLASMA FIS  $Ne = 10^{26}/cc$

(a)  $He_{\mu}^+$

T(eV)	100	1000
$T_{stop}$ (psec)	9.554	0.586
$R(\mu m)$	79.45	3.247

(b)  $\mu^-$

T(eV)	100	1000
$T_{stop}$ (psec)	0.00394	0.0034
$R(\mu m)$	0.0063	0.0053

## TRANSITIONS RADIATIVES

Higher levels  $n \geq 3$  have to be treated with radiative decay down to  $n=1$ , included, according to the relationship

$$\Gamma_{rad,if}^\mu = M \Gamma_{rad,if}^H ,$$

for a meson of mass  $M$  and usual allowed dipole transitions taking place in exoatom ( $M$ ) or usual  $H$  atom, between states  $i$  and  $f$ . In this regard, it should appreciated that the apparent restriction to  $np \rightarrow 1S$  transitions, for exoatom  $\frac{D}{T}\mu$ ,

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$$\Gamma_{rad,n1 \rightarrow 10}^\mu = 206.77 \frac{2^8 n(n-1)^{2n-2}}{(n+1)^{2n+2}} \text{sec}^{-1},$$

is not a real one, because the very high Holtzmark field due to the high charge densities, guarantees a very efficient Stark mixing of  $(n, \ell)$  sublevels. Transitions are pictured on Table III.

Hydrogenic radiative decay times for exoatom  $\frac{T}{D}\mu$  in level  $n$ .

n	2	3	4	5	6
$\tau$ decay (sec $^{-1}$ )	$7.75 \times 10^{12}$	$2.91 \times 10^{11}$	$7.12 \times 10^{11}$	$141 \times 10^{10}$	$2.46 \times 10^{10}$

Energie thermonucléaire produite durant le temps de confinement  $\tau$

$$Ef = 2\lambda_{\frac{D}{T}\mu_T^D} \times Q_{DT} \int_0^\tau dt N_{\frac{D}{T}\mu}(t)$$

rapportée au coût de la production des  $\mu$

$$R_e = \frac{Ef}{N_\mu(t=0)E_\mu}$$

pris au minimum en fonction du nombre quantique n de l'exoatom  $\frac{D}{T}\mu$  et de la contribution dipolaire c

WDM (500 psec)	FIS 100 eV (1 psec)	FIS 1 keV (1psec)
c=1      c=.1	c=1    c=.1	c=1    c=.1
n=1    33.1    4.17	n=1    1.87    0.57	n=1    0.83    0.404
n=2    124    17.93	n=2    3.89    2.72	n=2    3.49    2.19
n=3    280    45.4	n=3    17.4    11	n=3    3.90    3.32
n=6    1093    246.35		

## SOMMAIRE

- $\pi^- - D^+/T^+$  états liés démontrés en plasmas FIS et WDM ( $R_2$ )
- Attachement  $D^+/T^+$  sur exoatomes assurés par recombinaison 3-corps ( $R_3$ )
- Diagnostique X des exoatomes
- Les cibles ultra-denses et de courte durée de vie domine l'annihilation pion-nucleon
- Les  $\alpha'_S$  produits de fusion ne ‘collent’ pas le catalyseur  $\pi^-(\mu^-)$ .

## **CONCLUSION**

La catalyse mesonique apparaît possible dans les plasmas ultradenses.