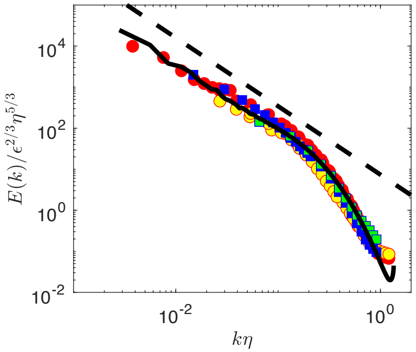
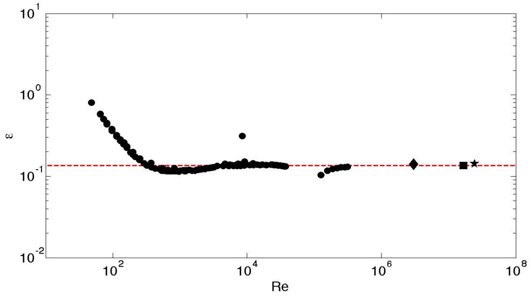
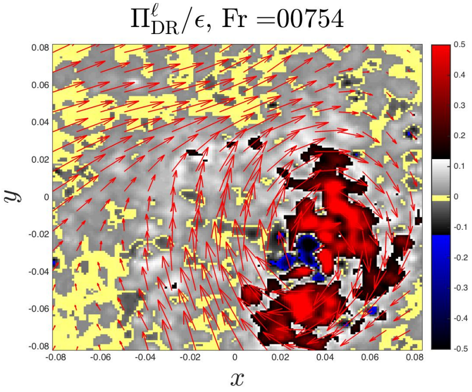
Physique de la turbulence à l'échelle de Kolmogorov:
(Enquête autour de 5 énigmes de la turbulence)

B. Dubrulle
CEA Saclay/SPEC/SPHYNX
CNRS UMR 3680







What is turbulence?

Hiroshige 1797-1858
Narito vortex

*Définition: T**urbulence** describes the state of a fluid (liquid or gas) in which velocity is in a **swirling** state.*

$$\vec{\nabla} \bullet \vec{u} = \vec{0}$$

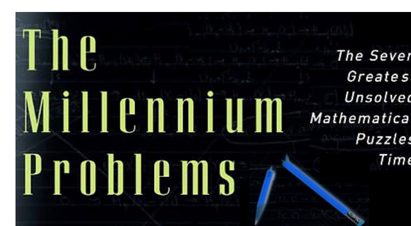
$$\partial_t \vec{u} + (\vec{u} \bullet \vec{\nabla})\vec{u} = -\frac{1}{\rho}\vec{\nabla} p + \nu \Delta \vec{u}$$

$$\mathrm{Re} = \frac{(u \nabla u)}{\nu \Delta u} = \frac{LU}{\nu}$$

Turbulence: Re>>1

Open basic question:

Navier-Stokes equations are they well-posed?
Are they any singularities?



Puzzle ♯ 0 of turbulence

Vortices in the Universe…

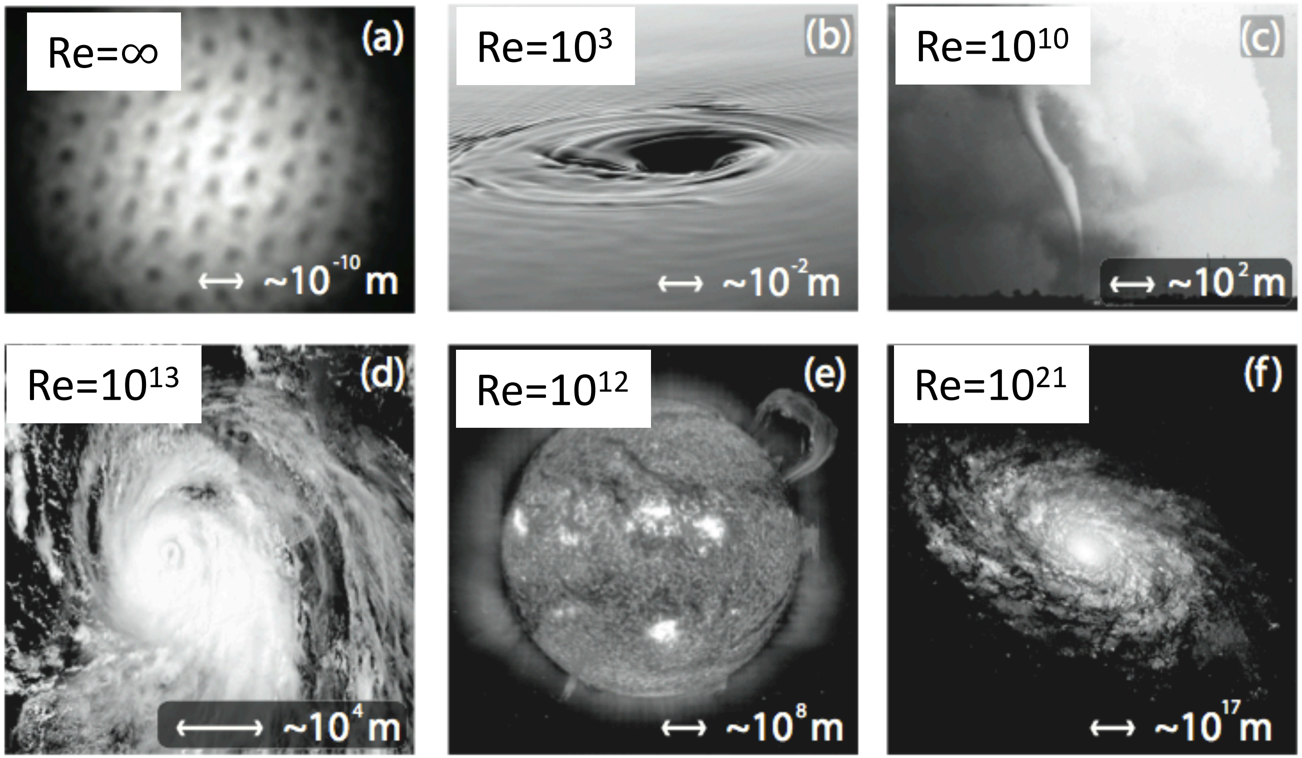
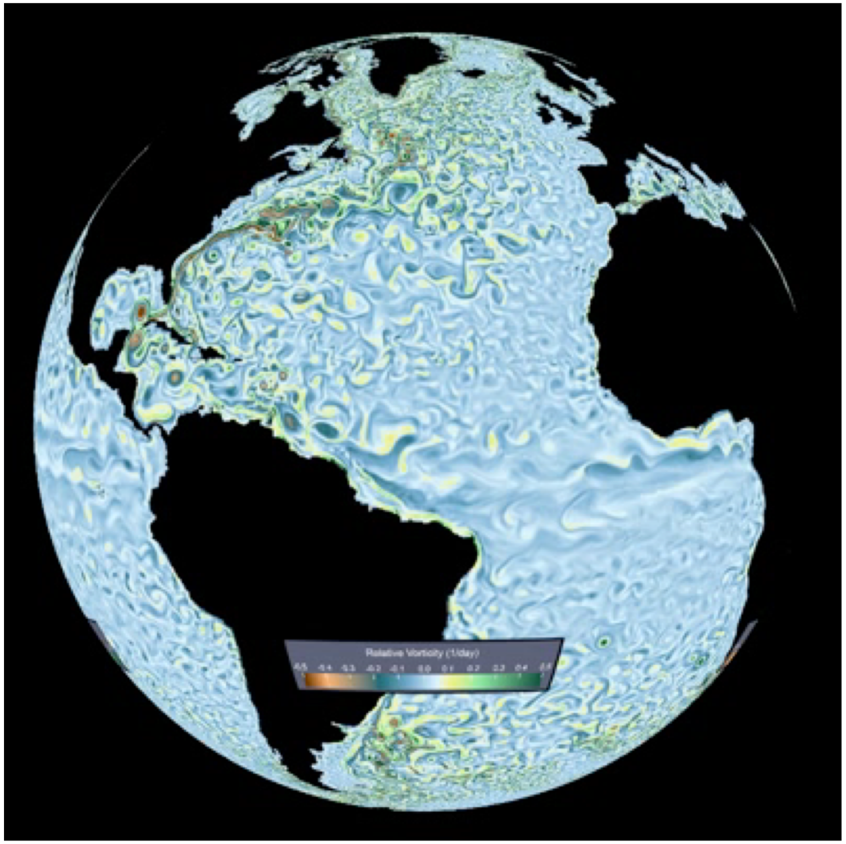
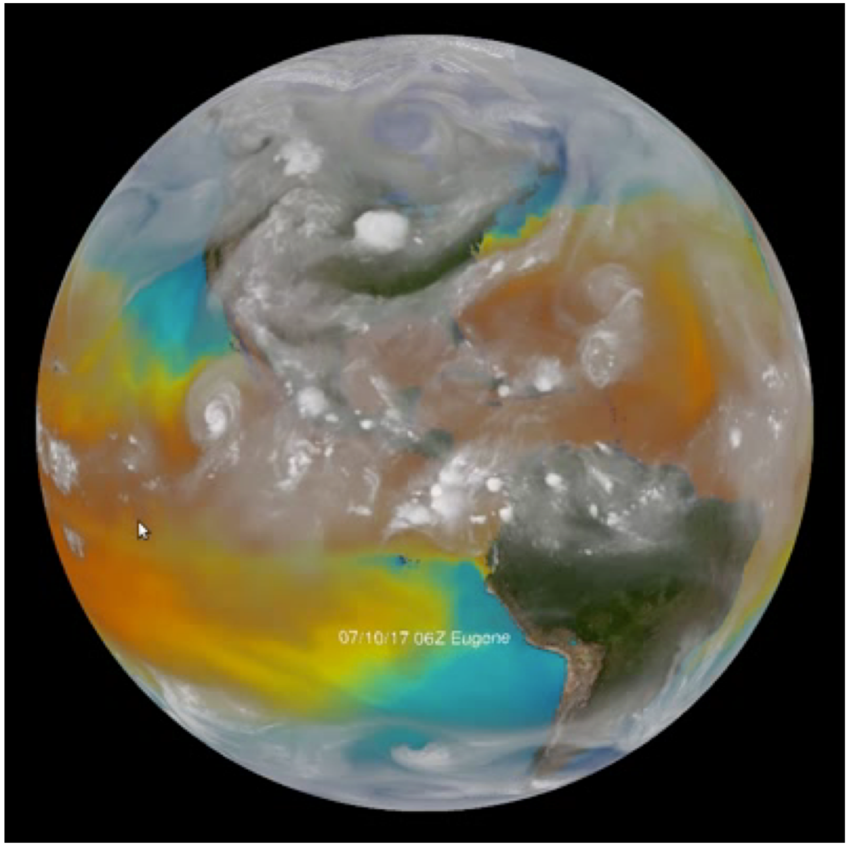


Figure 1.1: Vortices affect fluid behavior on all scales. (a) quantum vortices in a superfluid [130] (b) bathtub vortex [152] (c) tornado [109] (d) hurricane [106] (e) sun spot vortices [110] (f) spiral galaxy [105] (numbers approximate)

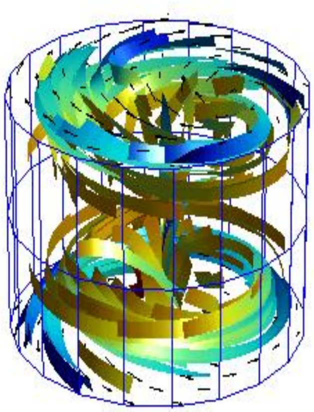
R. Fuchs, PhD thesis

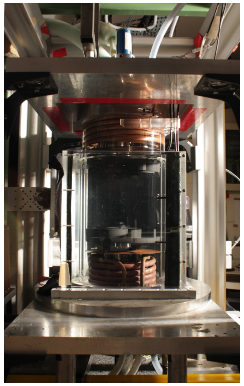
Vortices hierarchy on the Earth





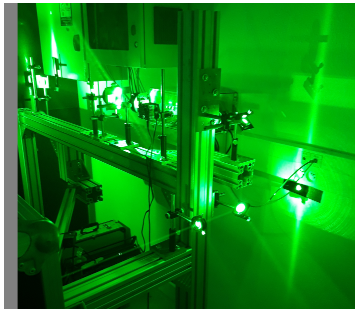
Vorticity hierarchy in the lab



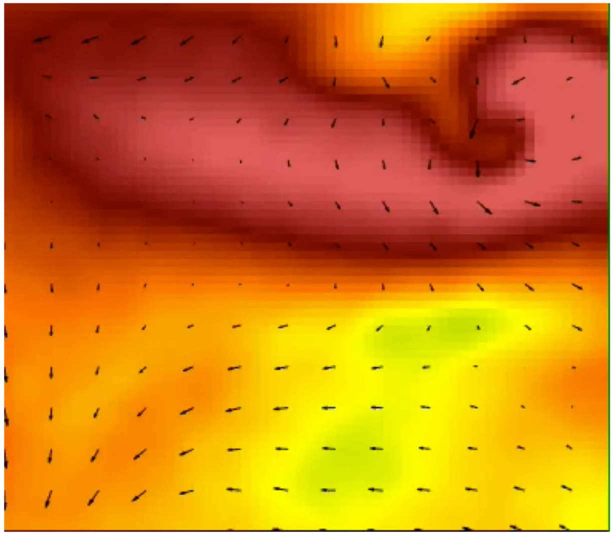


4 fast cameras

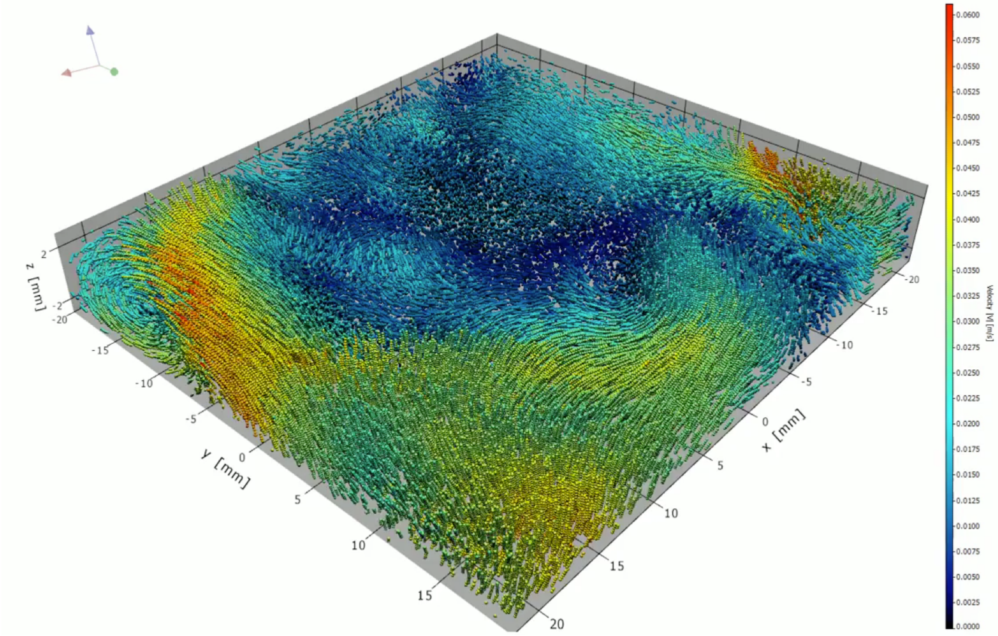
Laser



Turbulent flow in SPIV



Turbulent flow in 4D-PTV



ANR EXPLOIT: F. Daviaud, P. Debue, B. Dubrulle, J-M. Foucault, J-P. Laval, Y. Ostovan, V. Padilla, V. Valori, C. Wiertel

Self-similarity of vortices in turbulence

Seymour Narrows,
Between Vancouver and Quadra Islands

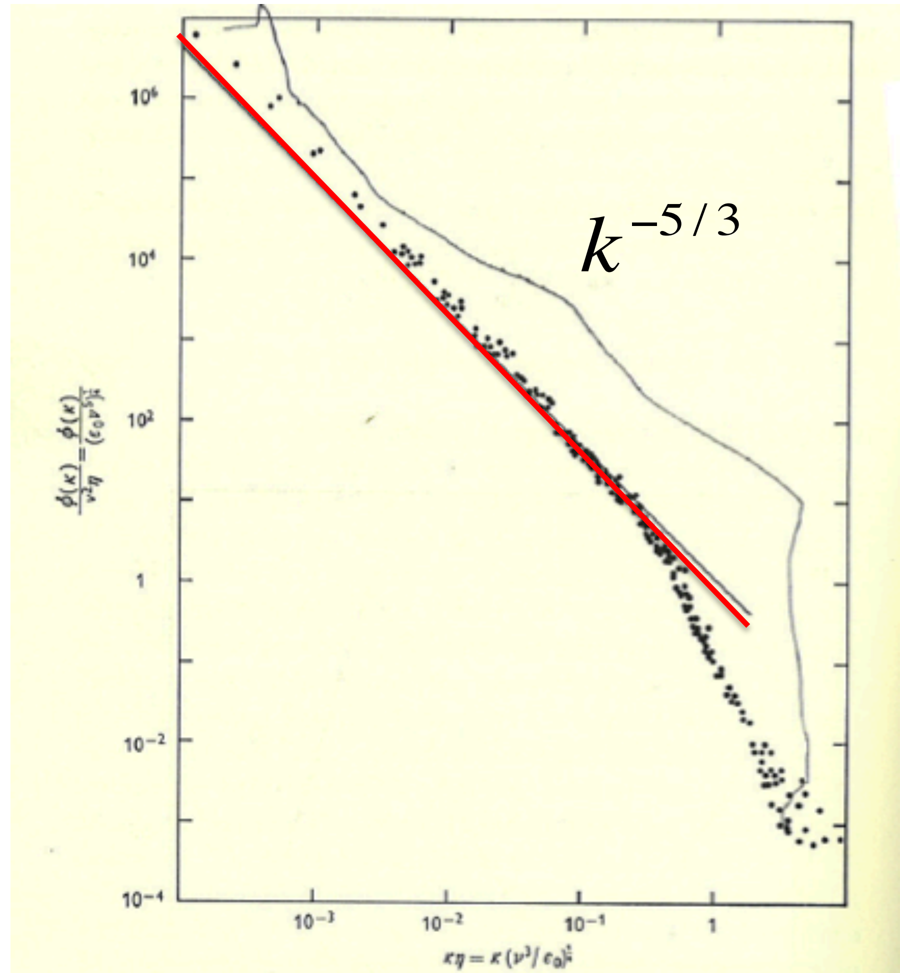


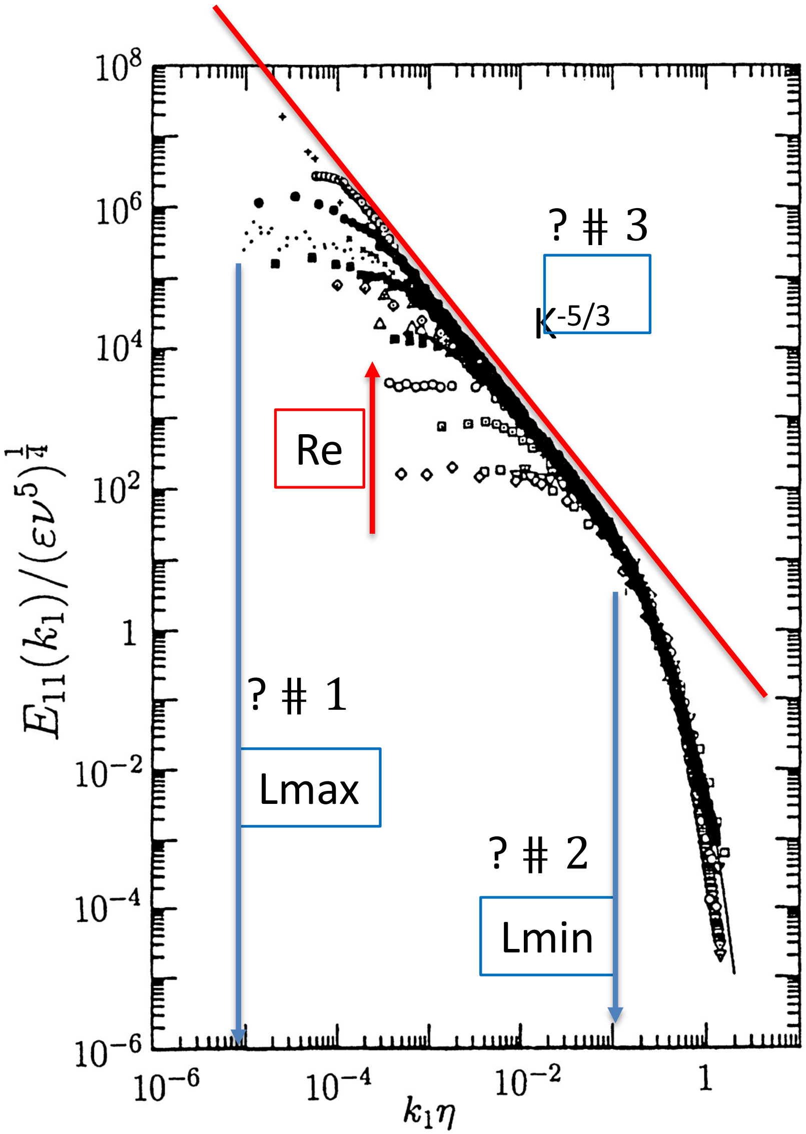
Fig. 6.2. The turbulence spectra, measured by Grant, Stewart and Moilliet (1962) and scaled according to the Kolmogorov parameters. The viscous dissipation rate ϵ_0 varied over a range of values of the order 100. The straight line represents variation as $\kappa^{-\frac{5}{3}}$. The top few points are believed to be rather high on account of the low frequency heaving motions of the ship.

3 puzzles set by the vortices hierarchy

Number of degrees of freedom

$$N = \left(\frac{Lmax}{Lmin}\right)^3$$

3 more puzzles of turbulence
Practical importance for DNS and theories



Solving # 1 and # 2 puzzles:

How are Lmax and Lmin selected?

The forcing matters

$$\vec{\nabla} \bullet \vec{u} = \vec{0}$$

$$\partial_t \vec{u} + (\vec{u} \bullet \vec{\nabla})\vec{u} = -\frac{1}{\rho}\vec{\nabla} p + \nu \Delta \vec{u}$$

Energy dissipation!

Without forcing, unique equilibrium state u=0

$$\vec{\nabla} \bullet \vec{u} = \vec{0}$$

$$\partial_t \vec{u} + (\vec{u} \bullet \vec{\nabla})\vec{u} = -\frac{1}{\rho}\vec{\nabla} p + \nu \Delta \vec{u} \quad +F$$

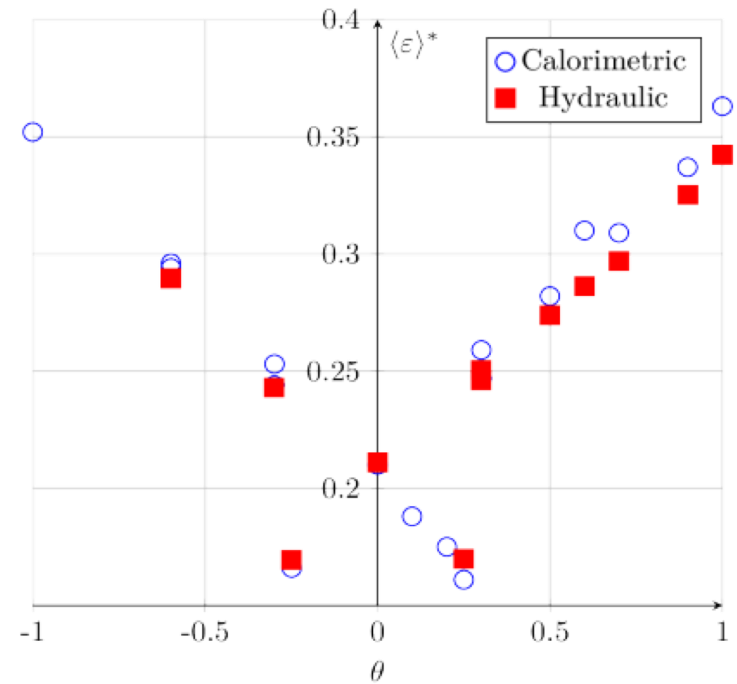
With forcing (possibly multiple) out-of-equilibrium stationary state

The dissipation matters

$$\partial_t \int \frac{u^2}{2} dx^3 = \int F.u\, dx^3 - \nu \int (\nabla u)^2\, dx^3$$

Energy balance in a turbulent flow

$$\boxed{dE = dW + dQ}$$



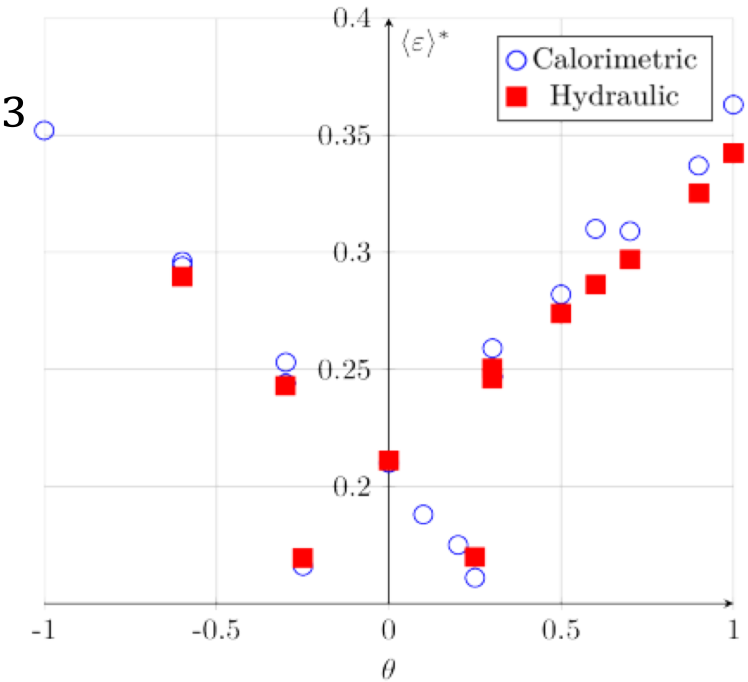
$$P_{inj} = P_{diss} \equiv \rho L^3 \varepsilon$$

Cf Joule's experiment:
Work measured by Torques applied at Shafts
=Heat flux measured By keeping T constant

Rousset et al, RSI 85, 103908 (2014);

Solution of puzzle # 1

$$\partial_t \int \frac{u^2}{2} dx^3 = \int F.u \, dx^3 - \nu \int (\nabla u)^2 \, dx^3$$



Energy balance in a turbulent flow

$$P_{inj} = P_{diss} \equiv \rho V \varepsilon$$

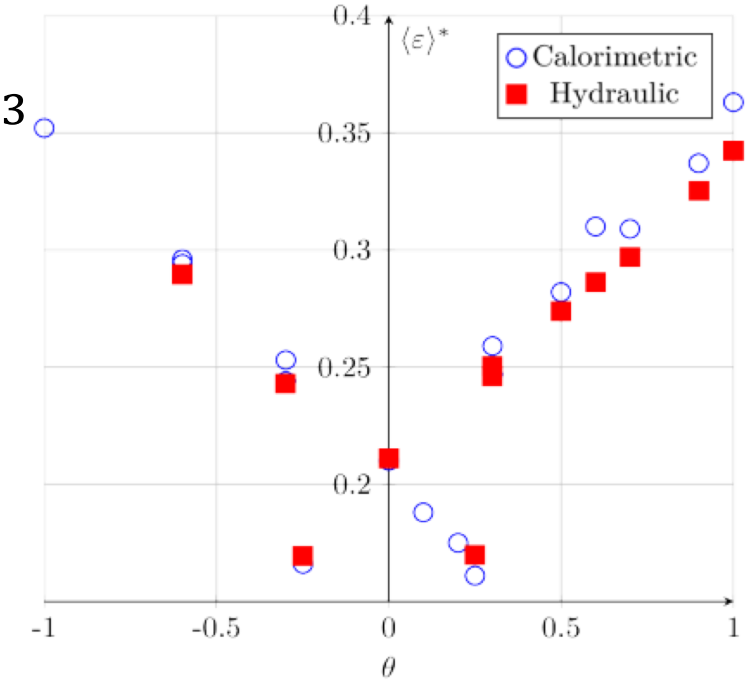
1st caracteristic scale: L: size of the stirrer

Injection scale

Solution of puzzle # 2

$$\partial_t \int \frac{u^2}{2} dx^3 = \int F.u \, dx^3 - \nu \int (\nabla u)^2 \, dx^3$$

Energy balance in a turbulent flow



$$P_{inj} = P_{diss} \equiv \rho V \varepsilon$$

Dimensional analysis

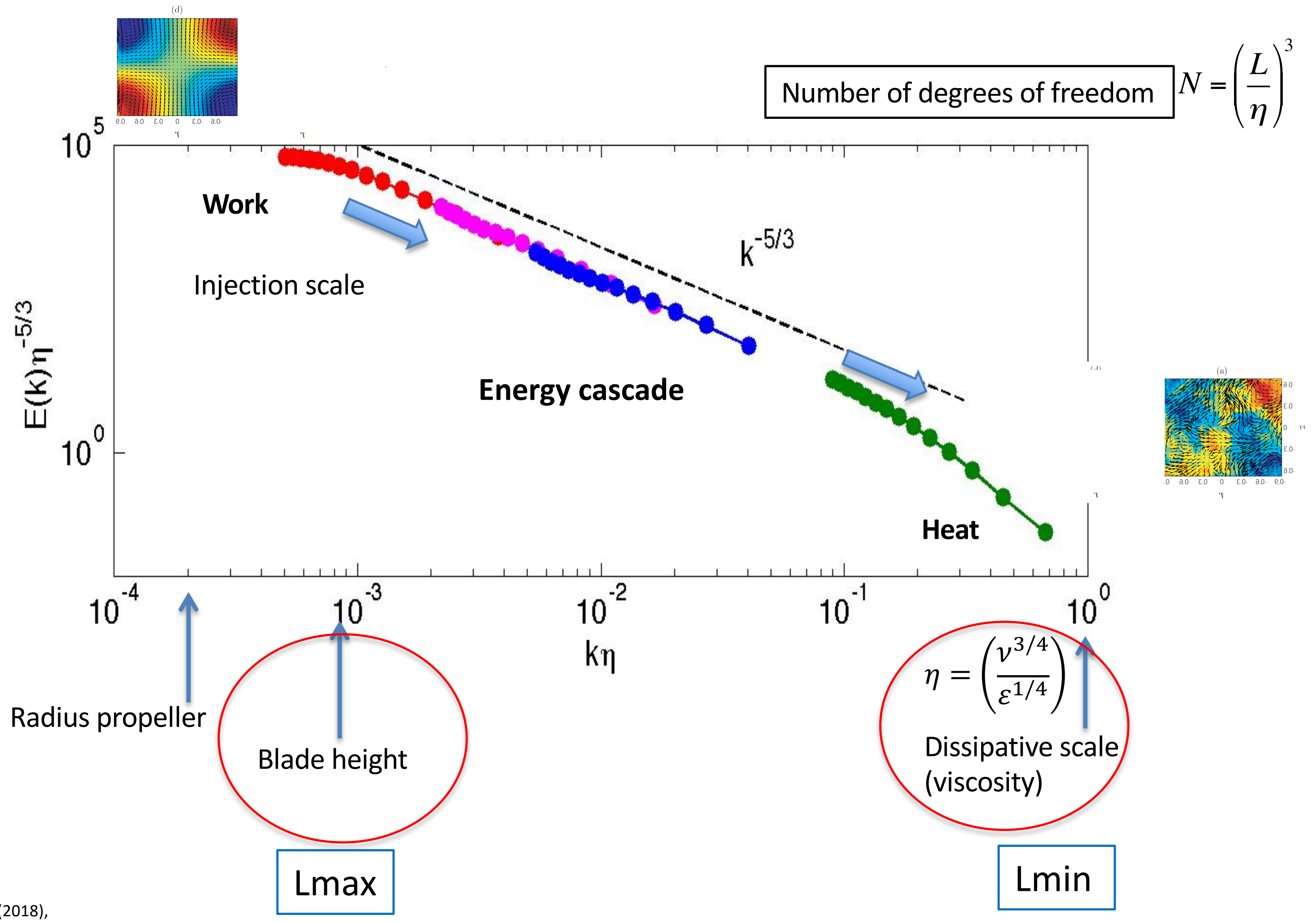
$$\varepsilon = [m^2 s^{-3}]$$
$$\nu = [m^2 s^{-1}]$$

2nd caracteristic scale

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$$

Kolmogorov scale

Preliminary phenomenological picture



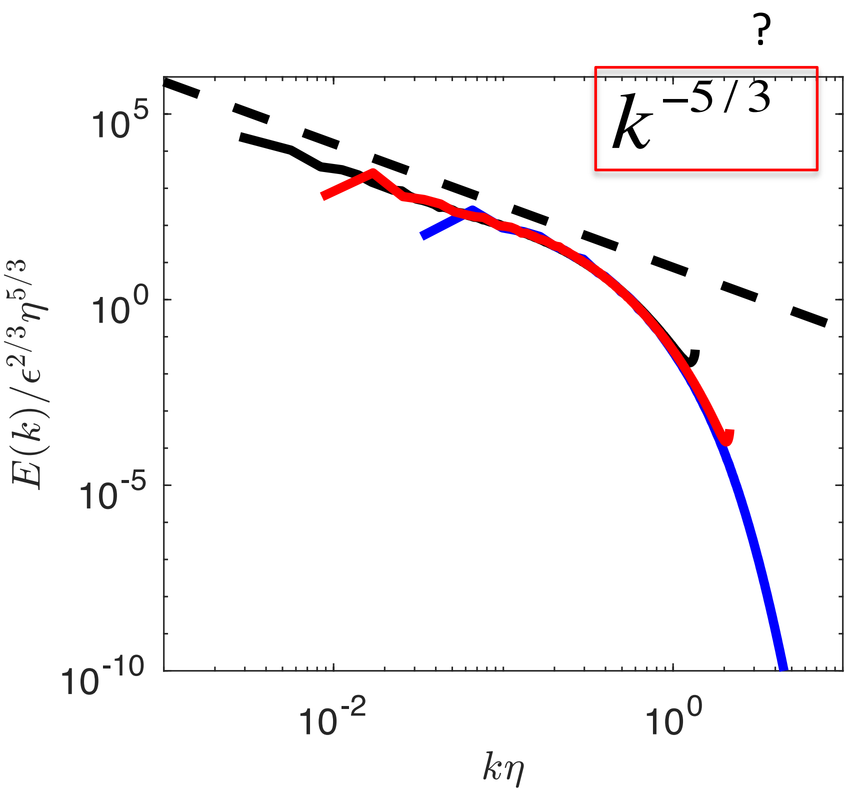
Debue et al et al., PRF (2018),

Solving # 3 puzzle:

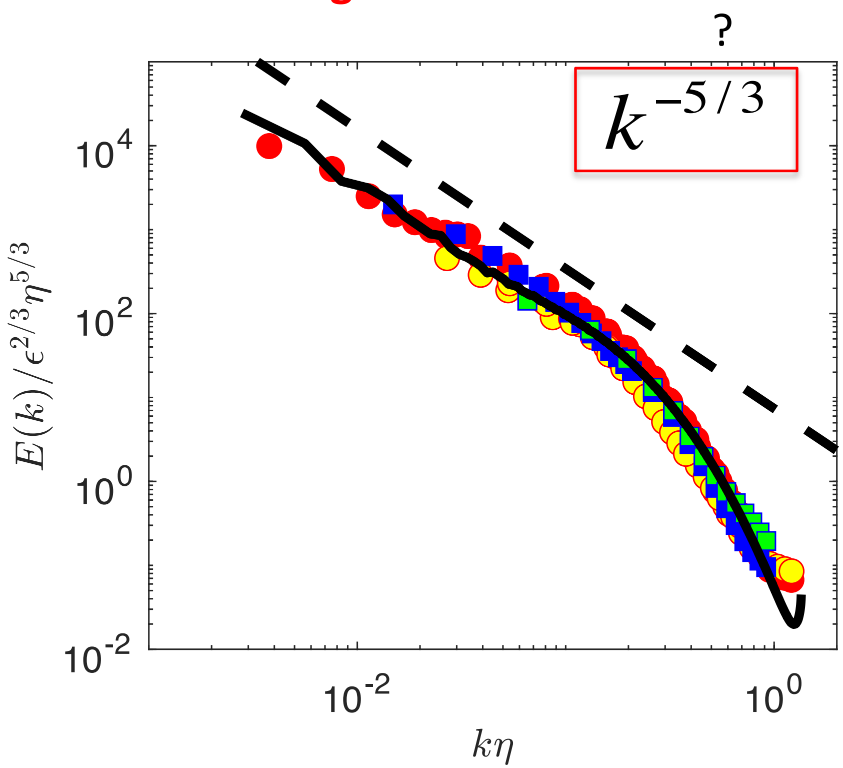
How is spectral exponent selected?

The spectral exponent is very robust but not given by dimensional analysis

Numerical simulations NSE



Experiments for a wide class of forcing conditions



By dimensional analysis

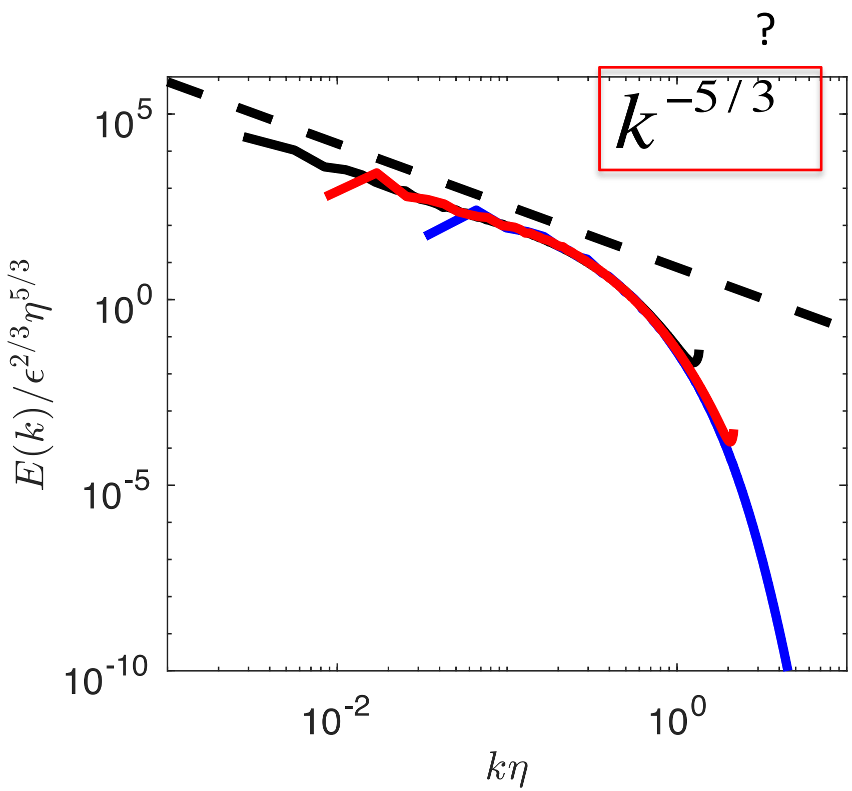
$$E(k) = [m^3 s^{-2}] \quad \Rightarrow \quad E(k) = \varepsilon^{2/3}\eta^{5/3}\,(k\eta)^{-1-2h}$$

How is the exponent h=1/3 selected?

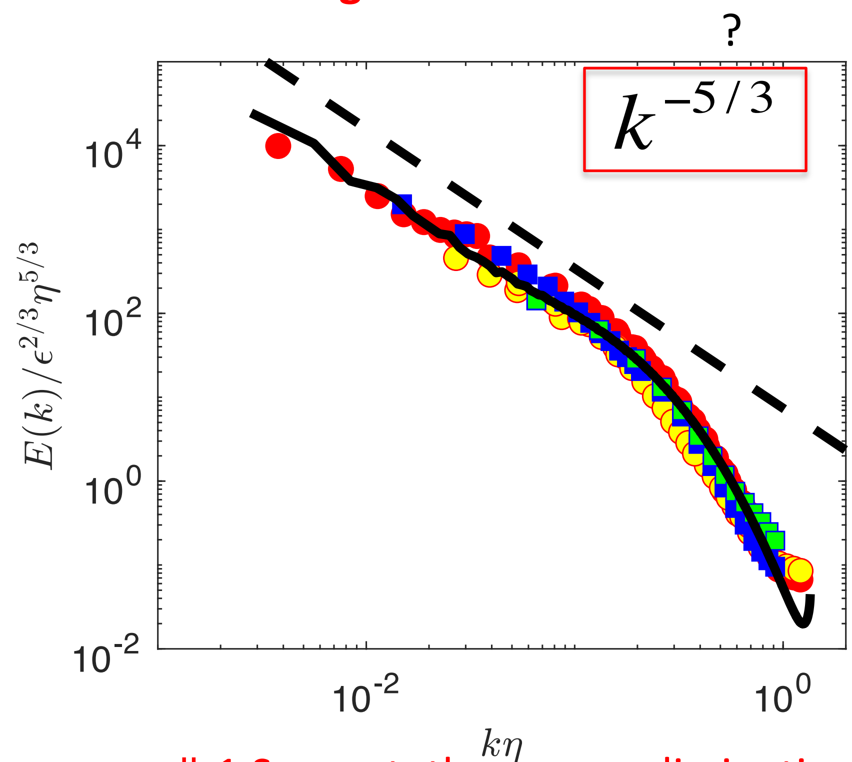
$$v(k) \approx k^{-h}$$

The spectral exponent is very robust but not given by dimensional analysis

Numerical simulations NSE



Experiments for a wide class of forcing conditions



By dimensional analysis

$$E(k) = [m^3 s^{-2}]$$

\# 1 Suspect: the energy dissipation

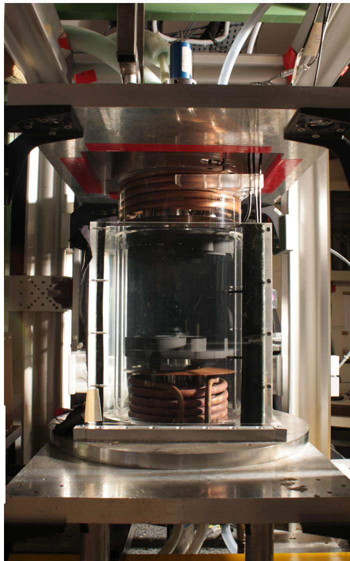
$$E(k) = \varepsilon^{2/3}\eta^{5/3}\,(k\eta)^{-1-2h}$$

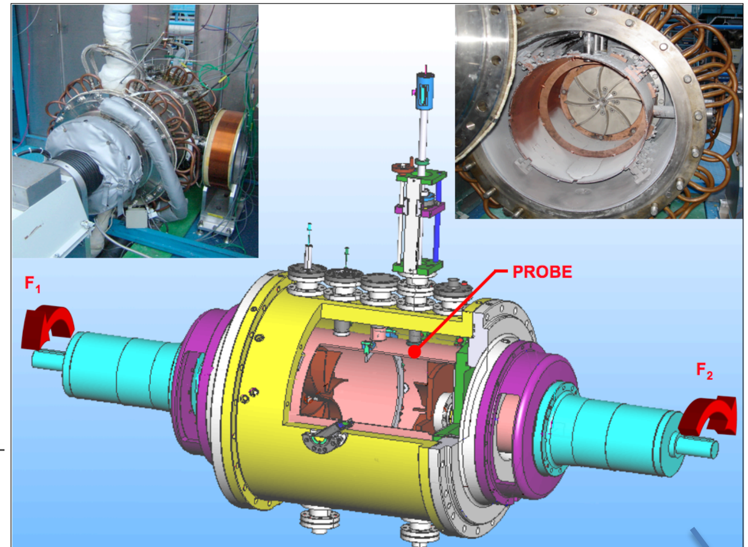
$$v(k) \approx k^{-h}$$

How is the exponent h=1/3 selected?

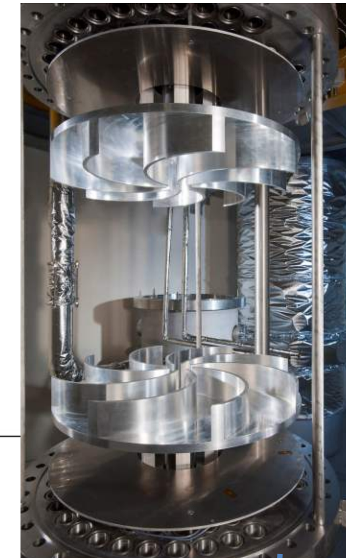
A closer look at energy dissipation

VKE

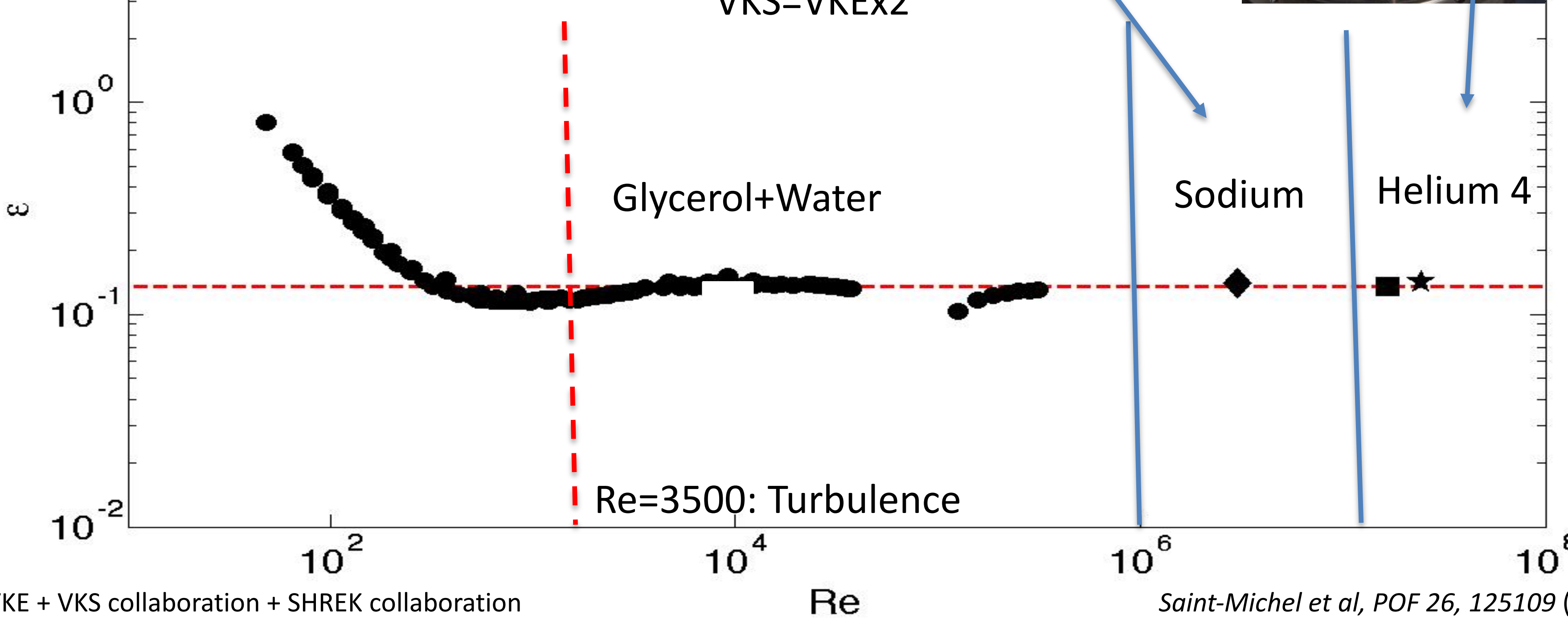




VKS=VKEx2



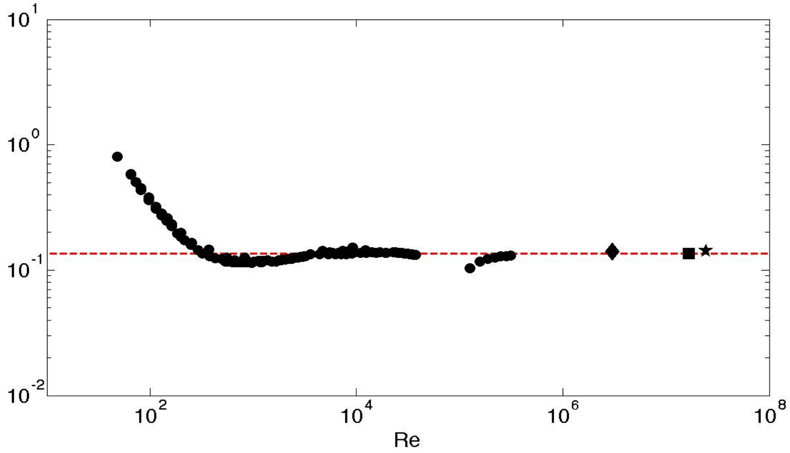
SHREK=VKEx4



VKE + VKS collaboration + SHREK collaboration

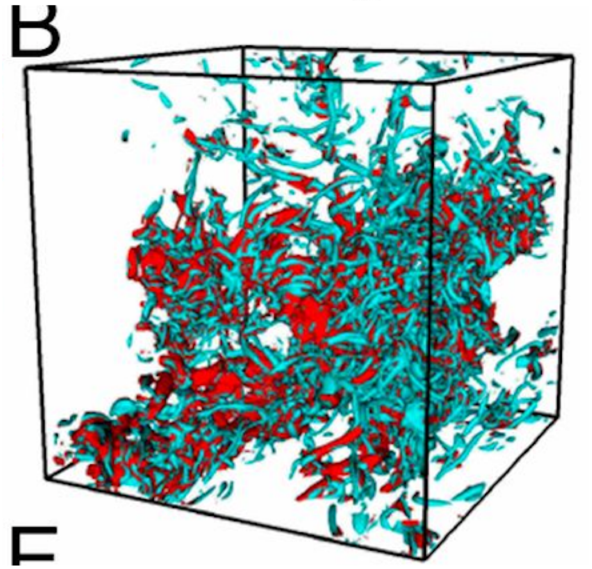
Saint-Michel et al, POF 26, 125109 (2014);

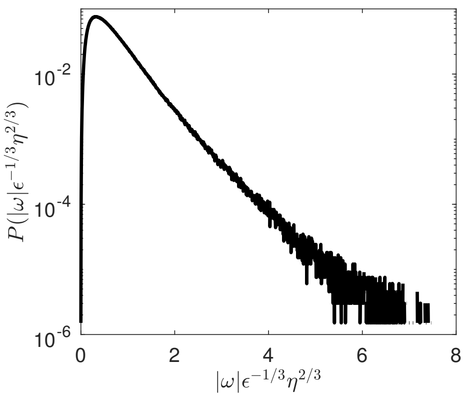
Where we meet again puzzle # 0



Non-dimensional energy dissipation per unit mass is constant at large Reynolds

Independent of viscosity?





$$\epsilon = \nu < (\nabla u)^2 >= \nu < \omega^2 > \Longrightarrow < \omega^2 > \approx \frac{\epsilon}{\nu}$$

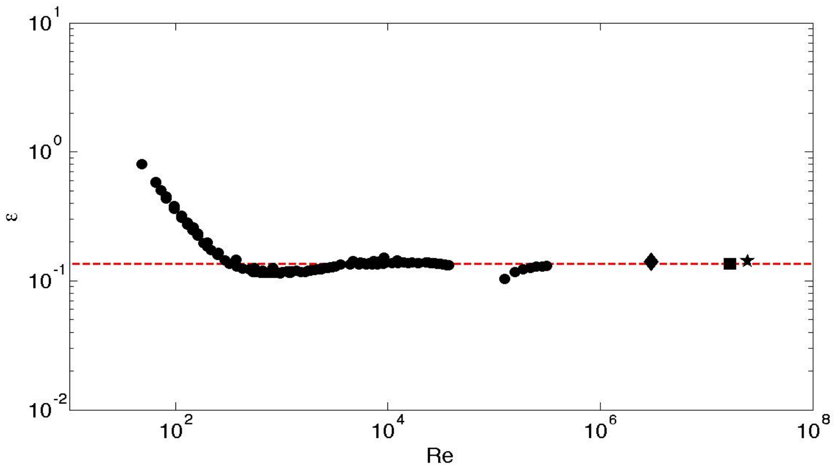
$$lim_{\nu \to 0} < \omega^2 > = \infty$$

Building of very large gradients at small scale… Singularity?

<u>**How to measure them/quantify them/understand this?**</u>

P. K. Yeung et al. PNAS 2015;112:41:12633-12638

Local energy budget for irregular fields



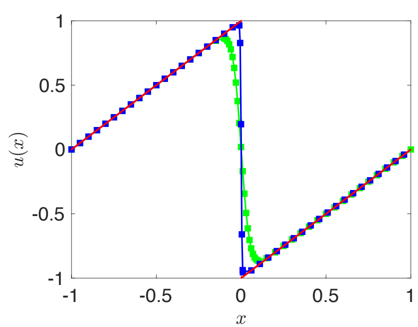
$$\frac{1}{2}\partial_t \mathbf{u}^2 + \mathrm{div}\left(\mathbf{u}\left(\frac{1}{2}\mathbf{u}^2 + p\right) - \nu\nabla\mathbf{u}\right) = \boxed{D(u)} - \nu\ \nabla\mathbf{u})^2$$

Duchon&Robert. Nonlinearity (2000),

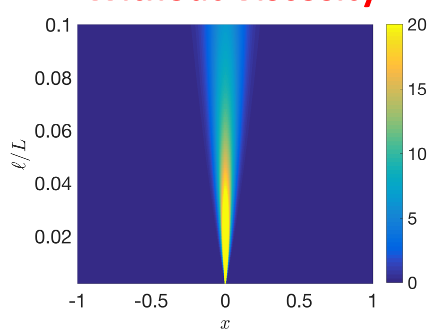
Inertial dissipation= singularity/large gradient detector!

$$D(u) = \lim_{\ell\to 0}\frac{1}{4}\int_{r\le\ell} d^3r\ \nabla\phi_\ell(r)\cdot\delta u_r|\delta u_r|^2$$

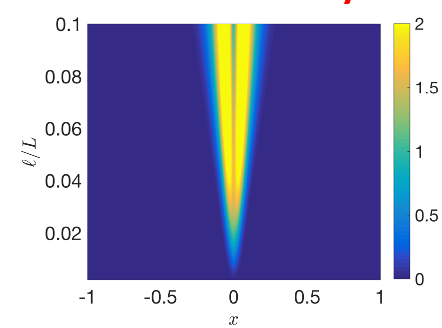
δu=u(x+r)-u(x) Velocity increment



Without Viscosity



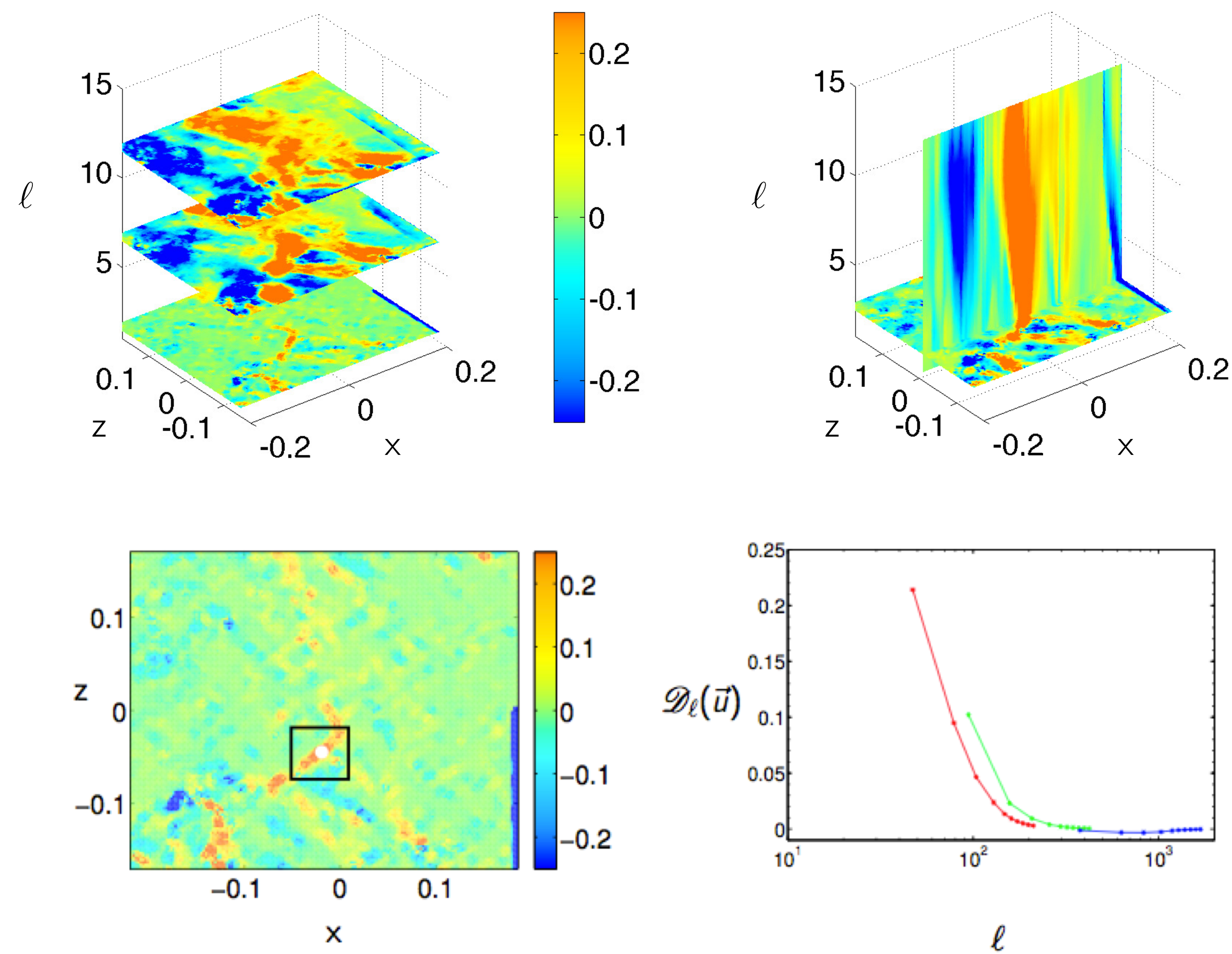
With Viscosity



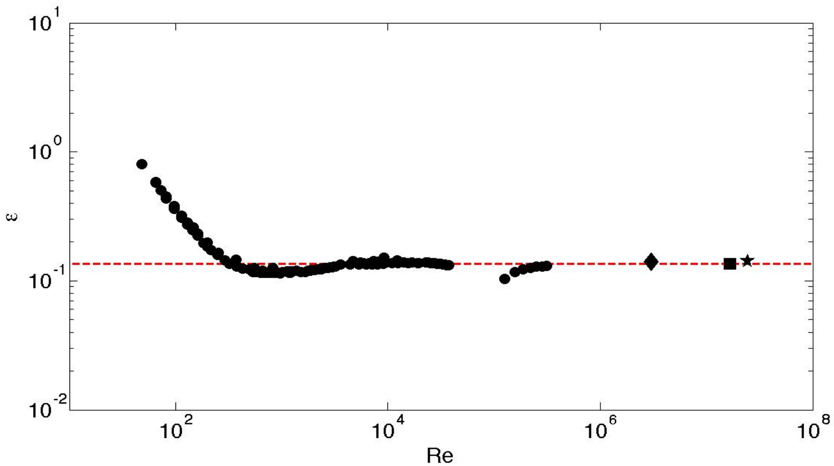
Applying the DR detector in von Karman flow

$$D_\ell(\mathbf{u}) = \frac{1}{4}\int_{\mathcal{V}} d^3r\ (\boldsymbol{\nabla} G_\ell)(\mathbf{r}) \cdot \delta\mathbf{u}(\mathbf{r})\ |\delta\mathbf{u}(\mathbf{r})|^2, \qquad G_\ell(r) = \frac{1}{N}\exp(-1/(1-(r/2\ell)^2)),$$

-Kuzzay D. et al. (2017), Nonlinearity



Solving puzzle # 3: Onsager's conjecture



"...in three dimensions a mechanism for complete dissipation of all kinetic energy, even without the aid of viscosity, is available."

L. Onsager, 1949

$$\frac{1}{2}\partial_t \mathbf{u}^2 + \mathrm{div}\left(\mathbf{u}\left(\frac{1}{2}\mathbf{u}^2 + p\right) - \nu\nabla\mathbf{u}\right) = \boxed{D(u)} - \nu\ \nabla\mathbf{u})^2$$

Duchon&Robert. Nonlinearity (2000),

Inertial dissipation= singularity/large gradient detector!

$$D(u) = \lim_{\ell\to 0}\frac{1}{4}\int_{r\le\ell} d^3r\ \nabla\phi_\ell(r)\cdot\delta u_r|\delta u_r|^2$$

$\delta u(\ell) \sim \ell^{\,h}$ In the limit of $\ell \approx 0$

$$D(u)[x] \propto \lim_{\ell\to 0} \ell^{3h-1}$$

If *h > 1/3* → Euler equation conserves energy, Dissipation in Navier-Stokes by viscosity.

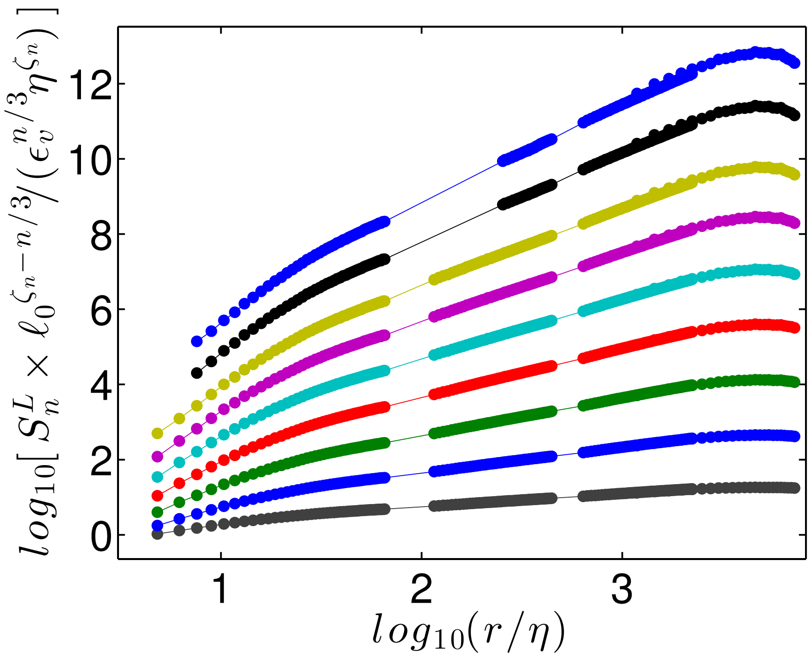
If *h ≤ 1/3* → Dissipation through irregularities (singularities)

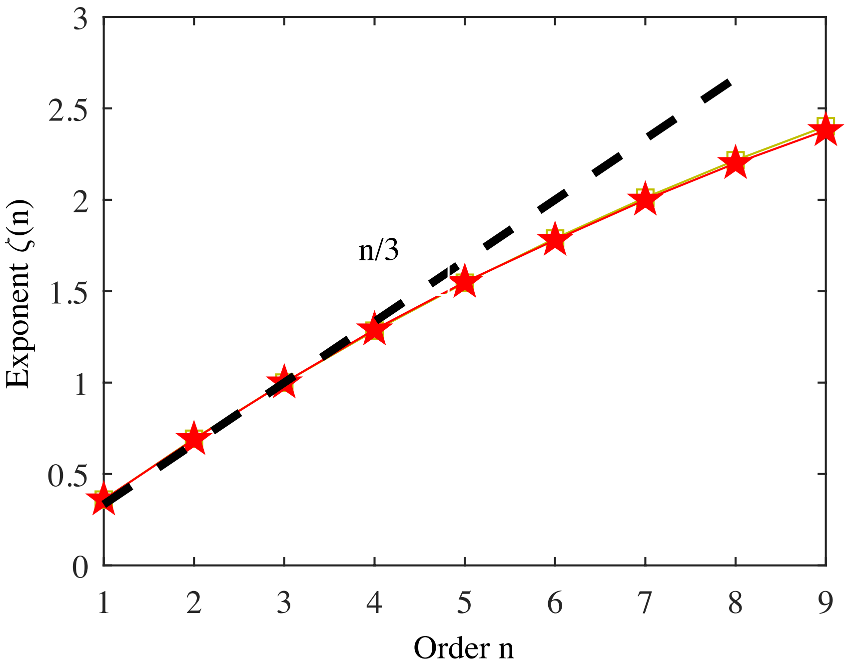
Without viscosity !

Kolmogorov: h=1/3; D=ε=cte

Kolmogorov spectrum exponent is the « critical value » !!!!
Is this the only one? -> look statistics of velocity increments

Puzzle # 4: $h = \frac{1}{3}$ *is not unique*!





$\delta u \sim \ell^{1/3} \Longrightarrow S_n(\ell) =< (\delta u)^n >= \ell^{n/3}$

Not observed!

Kolmogorov spectrum exponent h=1/3 is the « critical value » !!!!
It is not unique…

What is the physics involved?

Saw et al et al et al. JFM (2018)

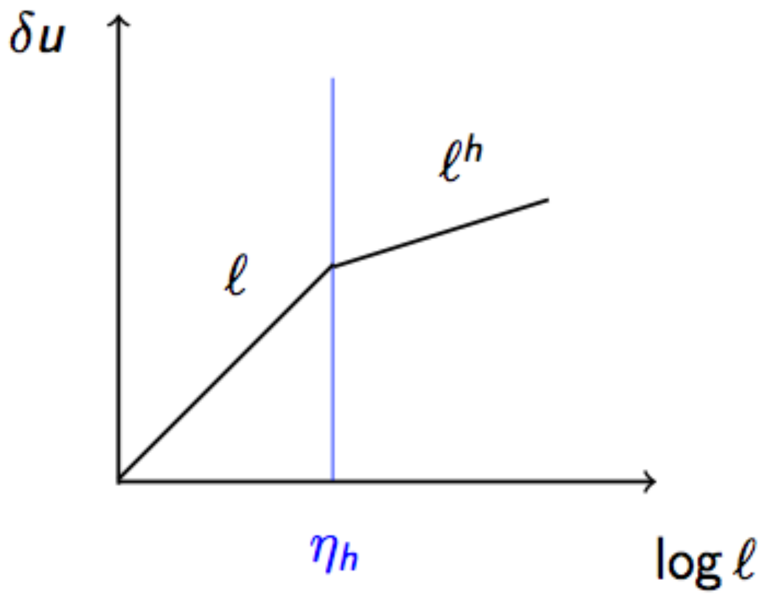
Local energy balance in turbulence

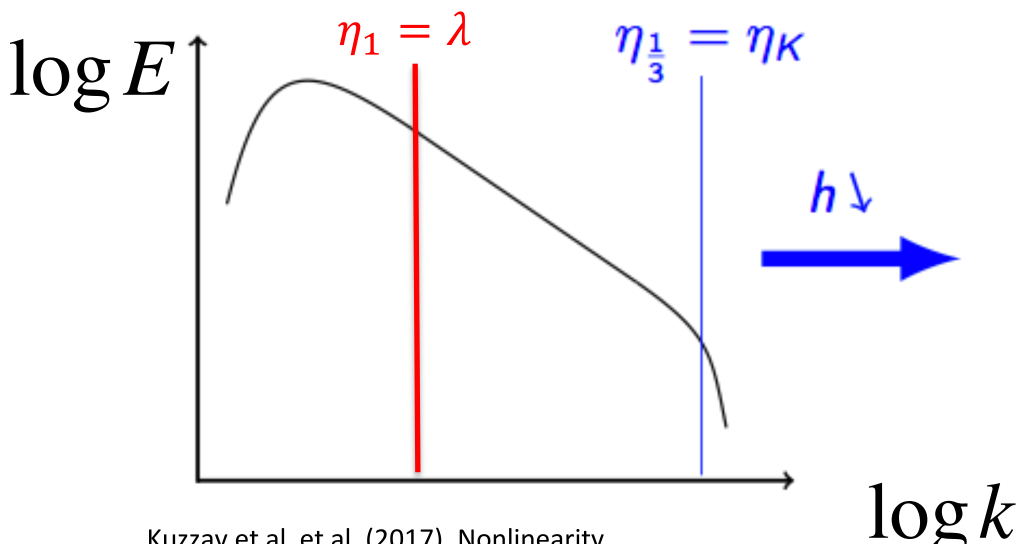
$$\partial_t E^\ell + \partial_j J_j^\ell = -\frac{1}{4}\int \nabla\phi^\ell(\xi)\cdot\delta\mathbf{u}(\delta u)^2 d\xi + \nu\partial^2 E^\ell$$
$$\equiv -D_\ell^I - D_\ell^\nu,$$

$$\delta u \approx \ell^h \Rightarrow D_\ell^I \approx \ell^{3h-1} \; and \; D_\ell^\nu \approx \nu\ell^{2h-2}$$

$$\boxed{\frac{D_\ell^I}{D_\ell^\nu} \approx \frac{\ell^{3h-1}}{\nu\ell^{2h-2}} \approx \frac{\ell^{h+1}}{\nu} \Rightarrow \eta_h \approx \mathrm{Re}^{-1/(1+h)}}$$

η_{-1}=0



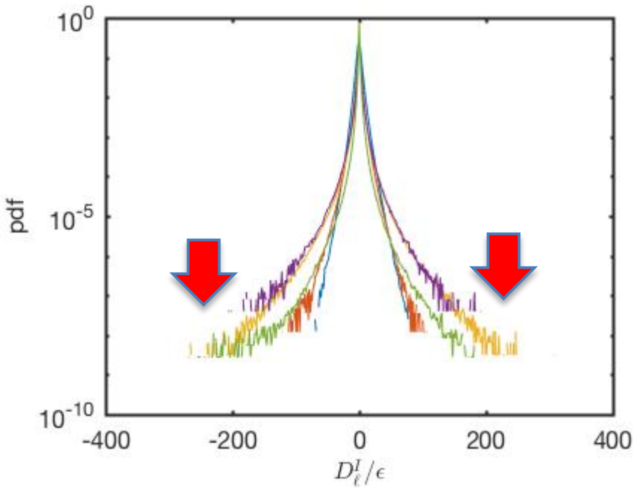


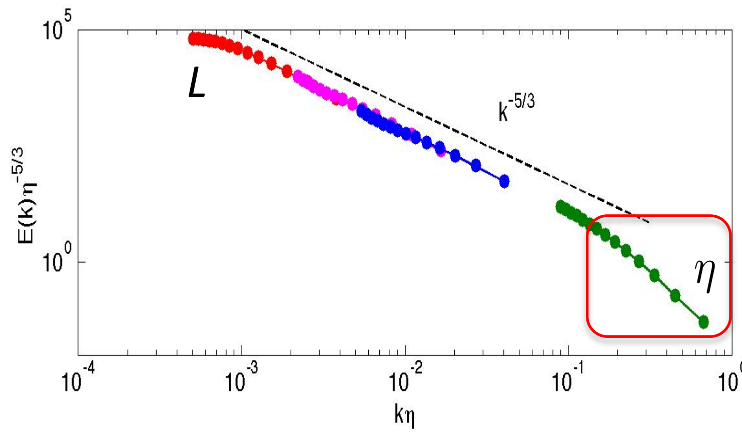
Kuzzay et al et al. (2017), Nonlinearity

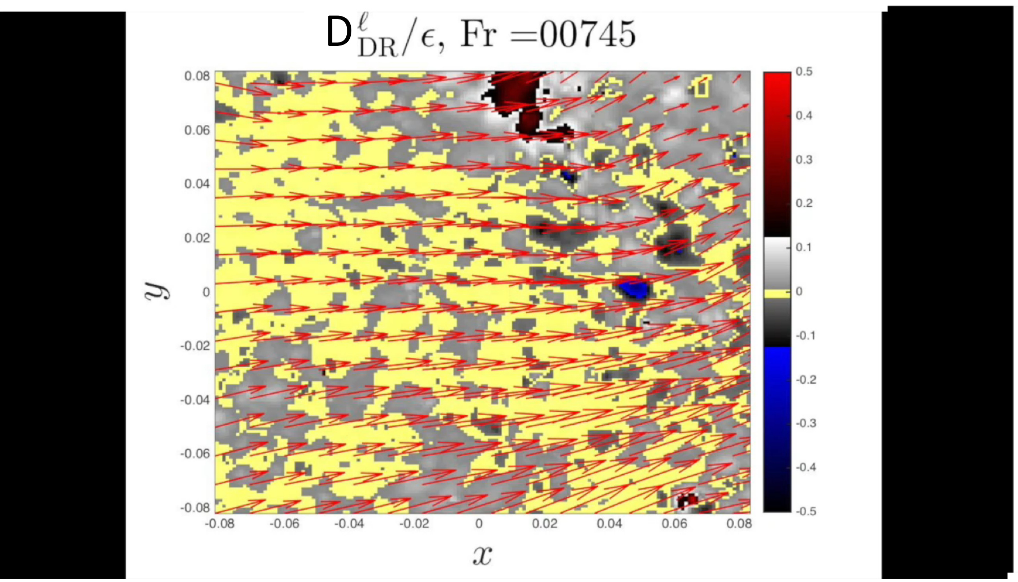
Solving puzzle # 4:
Is h=1/3 the only possible exponent?

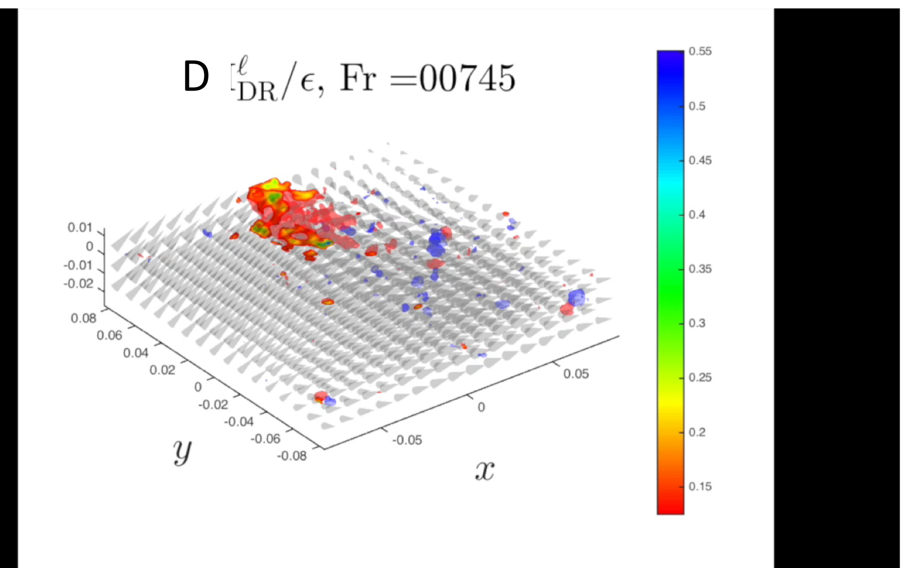
Look at the physics of turbulence at Kolmogorov scale

Large events of D(u)





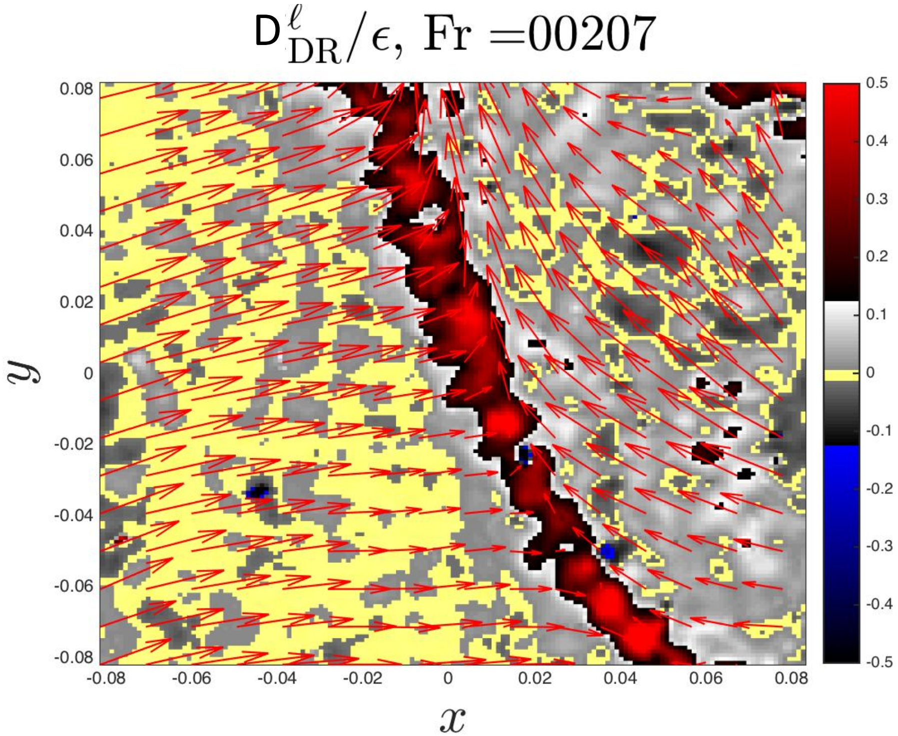




Color map: M. Farge, 1990 L'Aéronautique et l'Astronautique, ***140****, 24-33*

ANR Exploit: collaboration Saclay/LML: Debue, Shukla, Cuvier, Saw, Wiertel, Padilla, Daviaud, Dubrulle, Foucaut, Laval

2D Structure of large events of inertial dissipation



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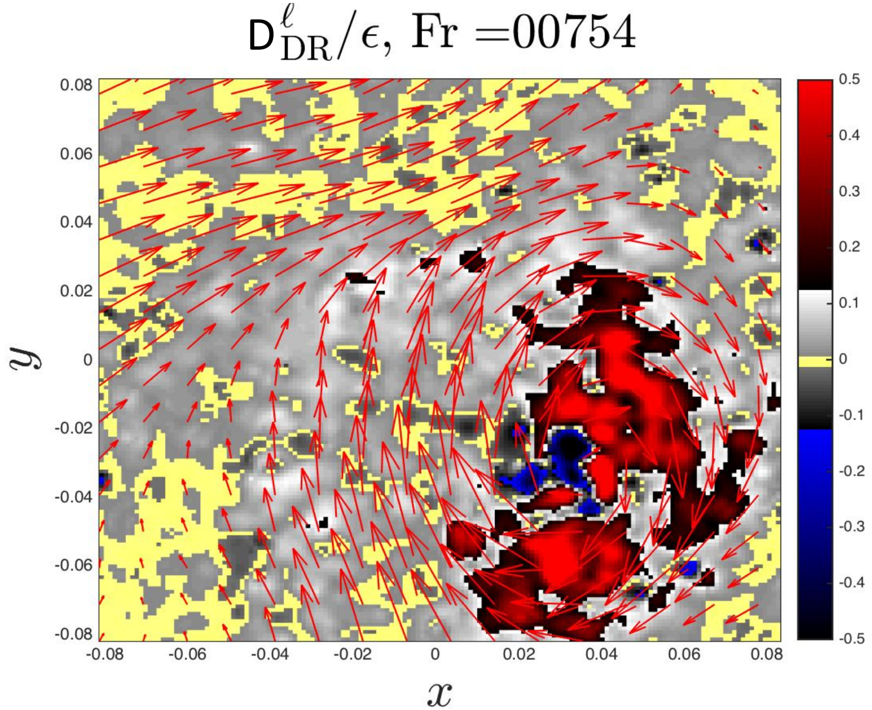
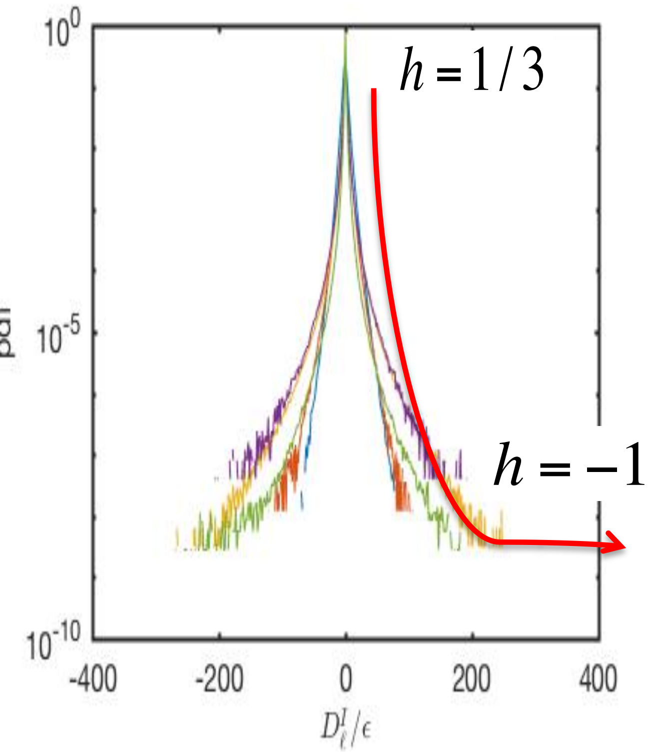


Spiral

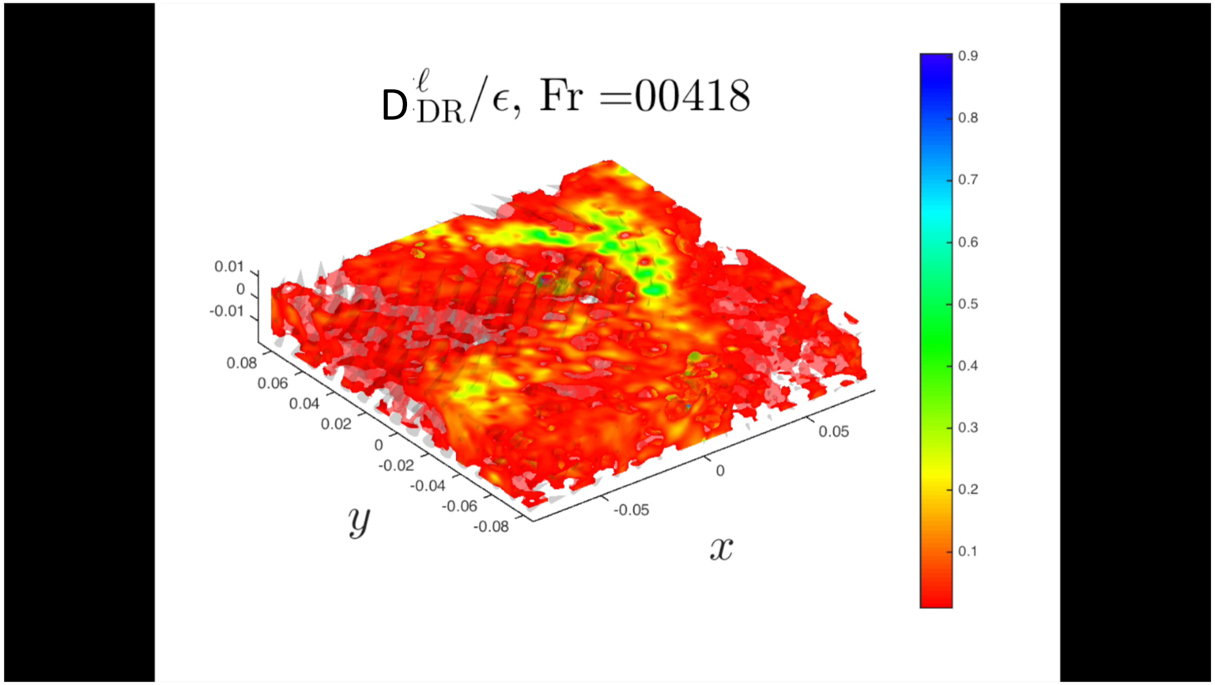
*Color map: M. Farge, 1990 L'Aéronautique et l'Astronautique, **140**, 24-33*

Saw, Kuzzay et al. (2016), Nature-Comm. 7

Structure of large events of inertial dissipation



Hierachy of h!



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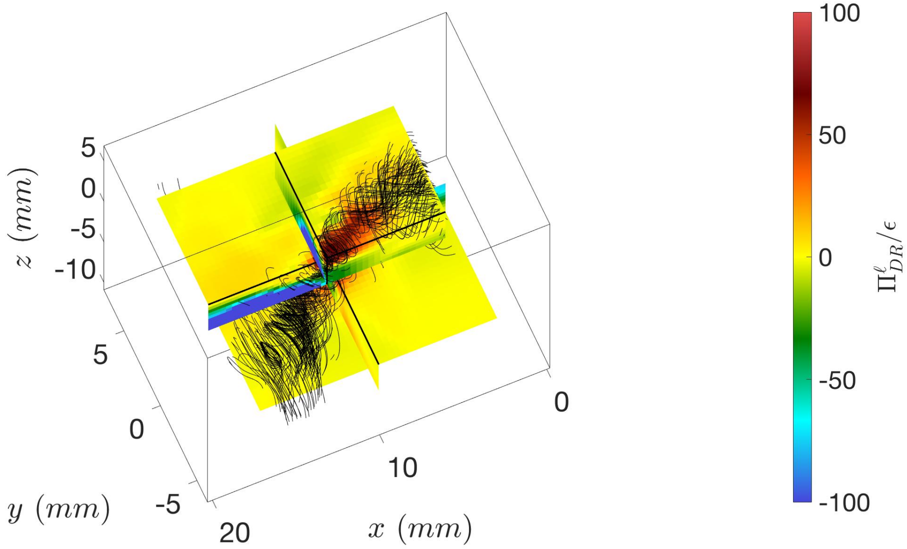
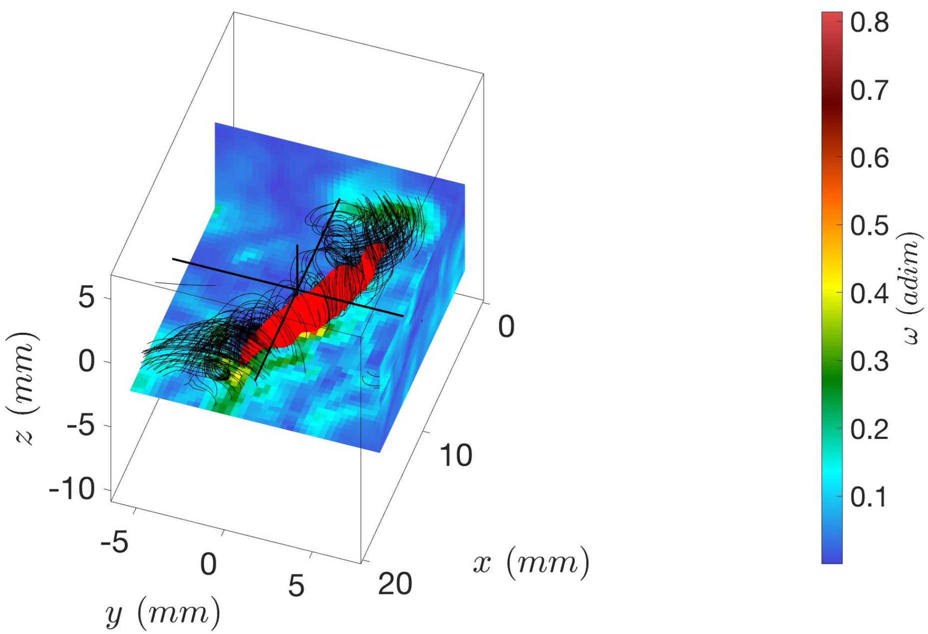
3D Structure of large events of inertial dissipation



Swirling filament

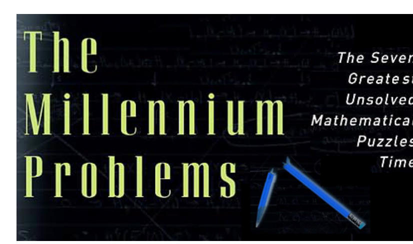
High enstrophy in the Vicinity of the event



P. Debue PhD Thesis; ANR EXPLOIT

Solving # 0 puzzle:

Are these footprints of singularities?



Open basic question:

Navier-Stokes equations are they well-posed?
Are they any singularities?

A possible suspect of NS singularity

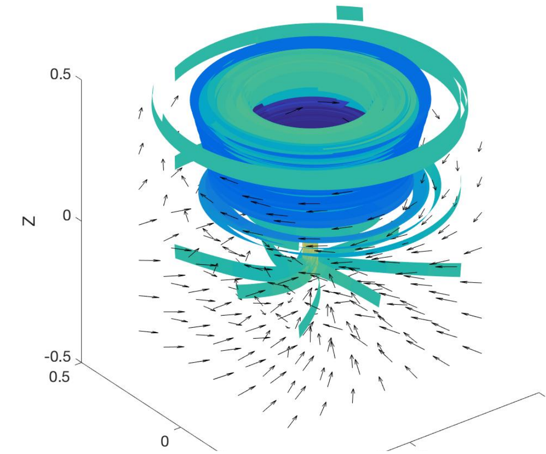
Symmetry $\quad (t, x, u) \longrightarrow (\gamma^2\, t, \gamma x, \gamma^{-1} u)(\,\nu \neq 0)$

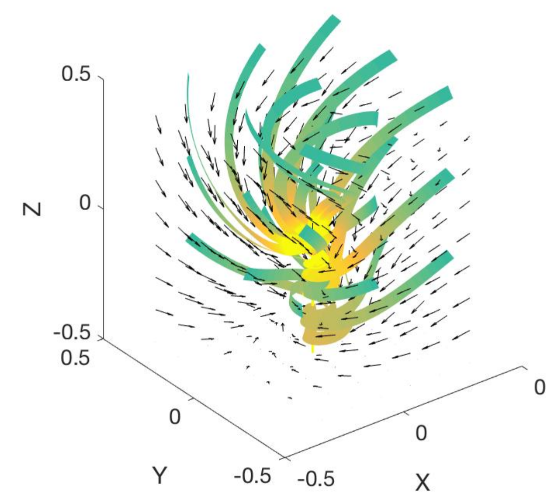
$u(\gamma^2\, t, \gamma x) = \gamma^{-1} u(t, x)$ **homogeneous solutions of NS of degree -1**

Axisymmetric case: classification started by Landau in 1944

$$
\begin{aligned}
U_r &= -\frac{1}{r}\Big\{1 + \frac{2(\gamma+1)}{(\gamma+1)\ln(\frac{1+\cos\theta}{2}) - 2} \\
&+ \frac{1-\cos\theta}{1+\cos\theta}\frac{3(\gamma+1)}{((\gamma+1)\ln(\frac{1+\cos\theta}{2}) - 2)^2}\Big\}, \qquad (1) \\
U_\theta &= \frac{1}{r}\left(\frac{1-\cos\theta}{\sin\theta}\right)\left(1 + \frac{2(\gamma+1)}{(\gamma+1)\ln(\frac{1+\cos\theta}{2}) - 2}\right) \\
U_\phi &= \begin{cases} \frac{1}{r\sin(\theta)}(b\ln(1+\cos\theta) + a) & \text{for } \gamma = -1 \\ \frac{1}{r\sin\theta}\left(\frac{b}{\ln(1+\cos\theta}+a\right) & \text{for } \gamma > -1 \end{cases}
\end{aligned}
$$

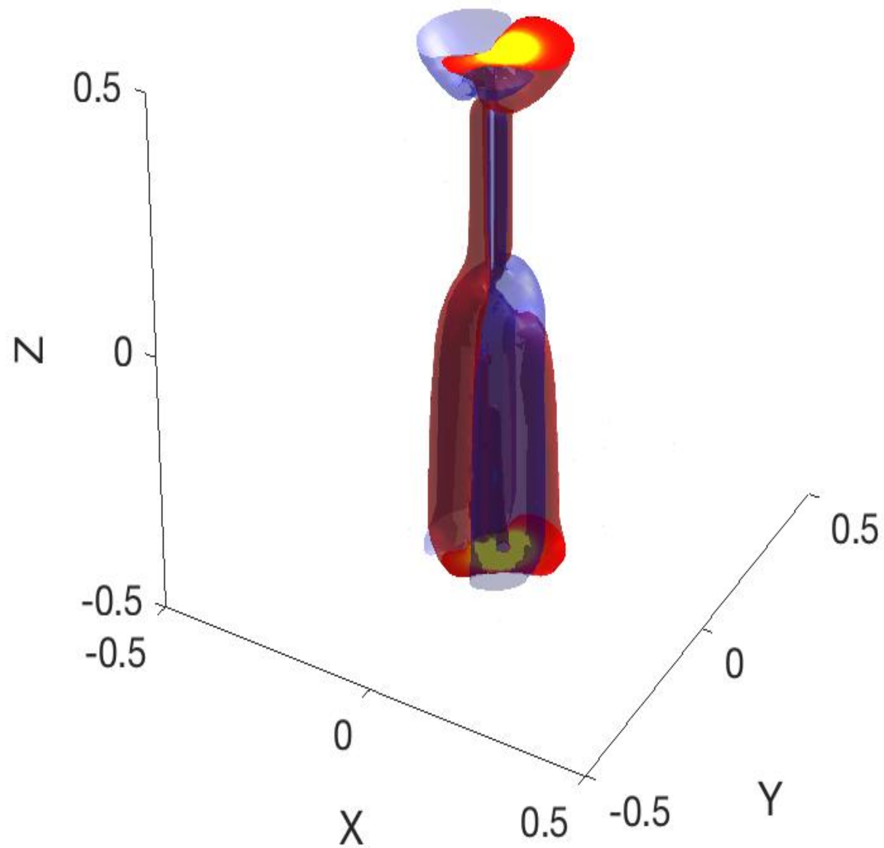
where $\gamma \geq 1$, a, et b are free parameters. Note singular behaviour for $\theta = \pi$.

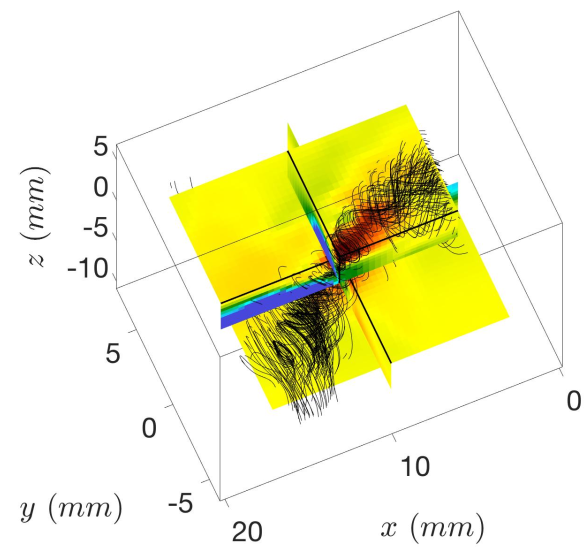




Li, Yan Yan, (2016)
H. Faller M2 Thesis
H. Faller et al, in preparation (2017)

3D footprints of the -1 degree homogeneous solution





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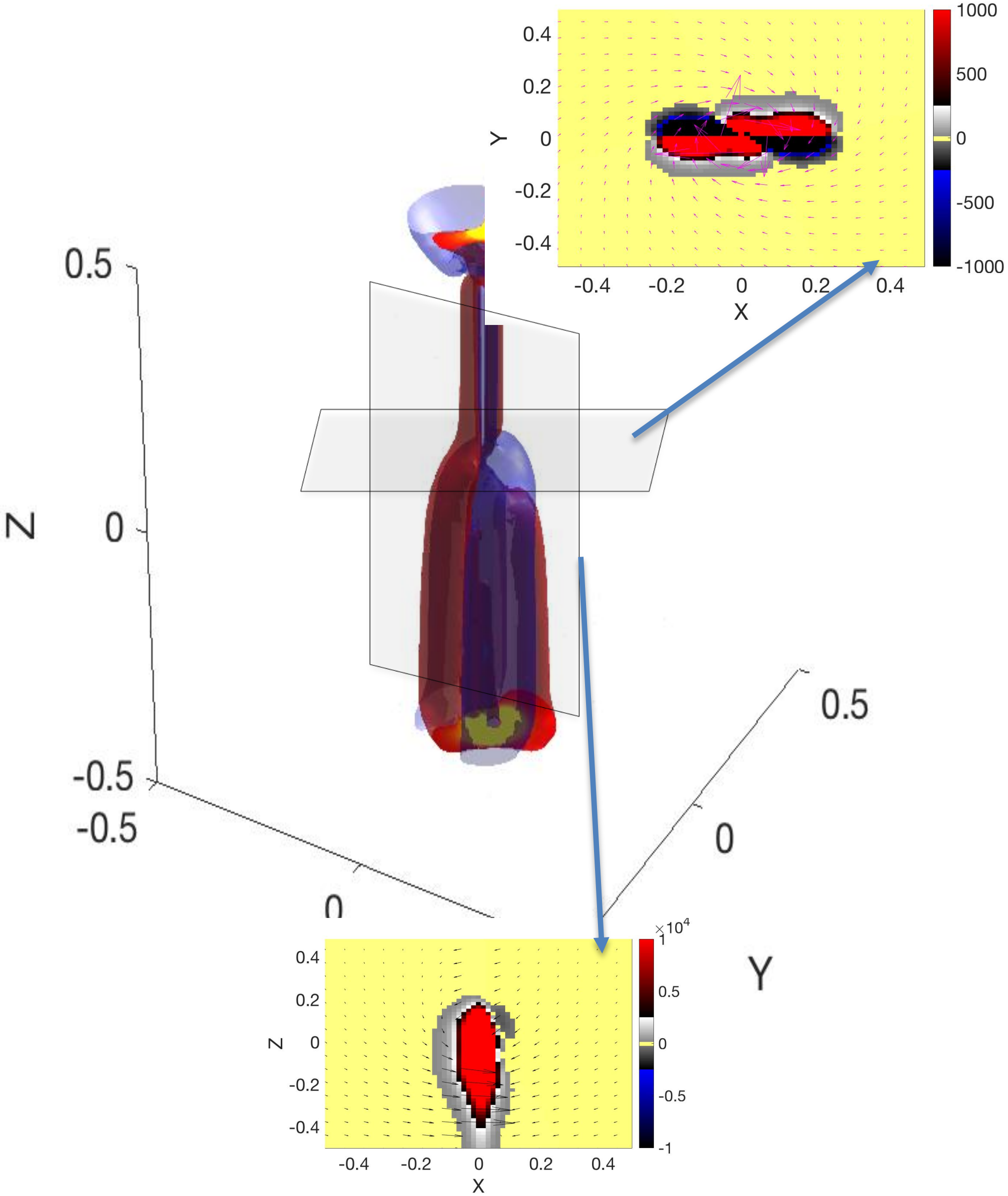
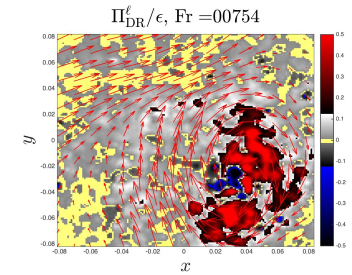
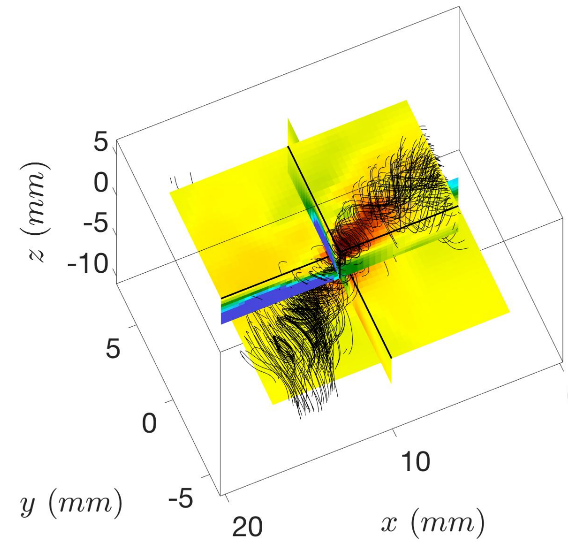
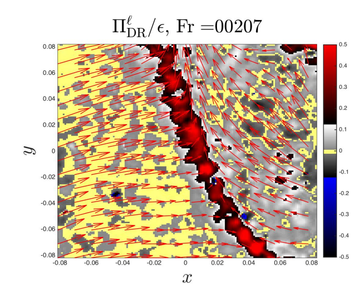
2D footprints of the -1 degree homogeneous solution









H. Faller M2 Thesis

H. Faller et al, in preparation (2017)

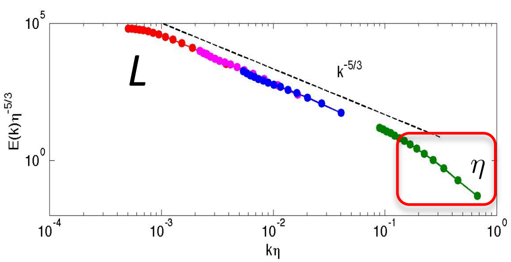
Conclusions and perspectives

- Turbulence is plagued with puzzles of fundamental importance. One of them is connected with possible quasi-singularities or singularities of the NSE.
- We have found possible footpints of such quansi singularities. They are linked with coherent structures, living below the Kolmogorov scale,

- **-> Impact on DNS of NSE!!!!**

Impact on DNS of turbulence

Re=6x10^3
L=10 cm



Resolution of events with exponent h

$$\eta_h = L\ Re^{-1/(1+h)}$$

$$N = \left(\frac{L}{\eta_h}\right)^3 \sim Re^{3/(1+h)}$$

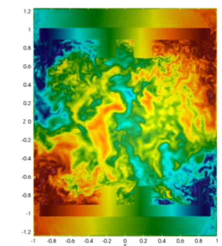
Kolmogorov

$$\eta_{1/3} = L\ Re^{-3/4}$$

$$N = \left(\frac{L}{\eta_{1/3}}\right)^3 \sim Re^{9/4}$$

η=0,4 mm

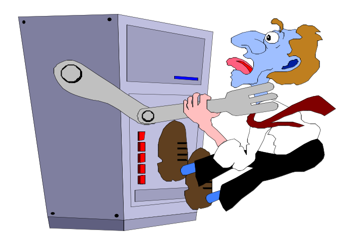
$N \sim 10^7$



MFR with h_{min}=-0.2

$$\eta_{-1/5} = Re^{-5/4} = \eta Re^{-1/2} = 5\ 10^{-3}\ mm$$

$$N = \left(\frac{L}{\eta_{-1/5}}\right)^3 \sim 8\ 10^{12}$$



Conclusions and perspectives

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- We have found possible footpints of such quansi singularities. They are linked with coherent structures, living below the Kolmogorov scale,

- **-> Impact on DNS of NSE!!!!**

- **Perspective:**

The study of the dynamics and properties of these structures is underway, using 4D-PTV
We are currently building a larger experiment to explore sub-Kolmogorov regimes, to look for more singular structures).

Prespectives: GVK experiment

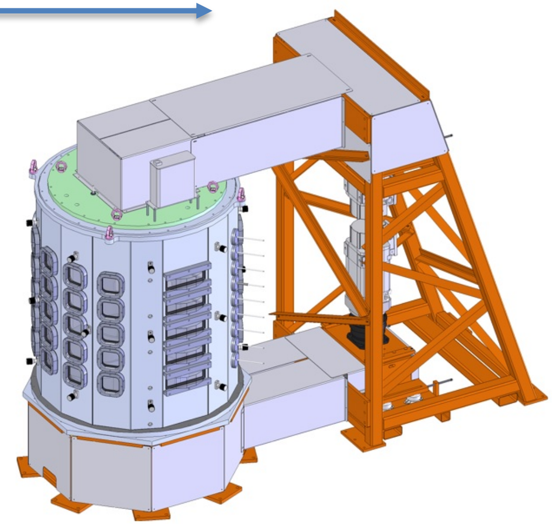
20 cm

Present experiment R=10 cm

Re=10^6
η=0,01 mm

Re=$6x10^3$
η=0,4 mm
$\Delta x \sim \eta$

1 m



GVK experiment R=50 cm

Re=10^6
η=0,05 mm

Re=$6x10^3$
η=2 mm
$\Delta x \sim \eta/5$

Possibility to explore sub-Kolmogorov scales
Detection of stronger velocity gradients and quasi-singularities

Thanks to…

F. Ravelet, B. Saint-Michel, D. Kuzzay, P. Debue, D. Faranda, E-W. Saw, A. Guittonneau , H. Faller, F. Nguyen, V. Shukla

F. Daviaud, A. Chiffaudel, J-P. Laval, J-M. Foucaut,P. Kestener, C. Nore

VKS collaboration

SHREK collaboration