A new symmetry of shapes, shells and clusters

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Introduction and summary 1

60 years ago, i.e. 1958:
Elliott: spherical shell model > q deform. rotat.
Wildermuth et al: sph. shell m. > cluster model
Common intersection: SU(3) dynam. symmetry.

Single major shell.

Introduction and summary 2

Many major shells
(symmetry-adapted approaches from
the spherical shell model viewpoint):

Shell model: symplectic (Sp(3,R)) model.
Coll. model: contracted symplectic model
Cluster mod: semimicroscopic algebraic cl. m.

Common intersection: dynamical symmetry

$U_s(3) \times U_x(3) > U(3) > SU(3) > SO(3)$

(J. Cs. Phys. Conf. Ser. 580, 012046 (2015)).

Introduction and summary 3

The dynamical symmetry of the intersection is identical with the multichannel dynamical symmetry (MUSY) of the cluster configurations.
(Shell or quartet state: 1-cluster configuration.)

Detailed spectra of different configurations in different energy windows.
Considerable predictive power.

I. Historical background: $SU^{ST}(4)$, $SU(3)$

II. Quartets

III. Clusters

IV. Quadrupole deformation

V. Unifying symmetry

VI. Conclusion

Semimicroscopic algebraic approach

Microscopic model space (Pauli-principle included), phenomenological operators.

Fully algebraic description: group-symmetry:
not only the basis states but the operators as well.

$SU^{ST}(4)$

Supermultiplet-theory

E.P. Wigner, Phys. Rev. 51, 106 (1937).

Eisenbud, Garvey, Wigner: Nuclear Structure(1958)

The disadvantage of the supermultiplet theory…

I. SU(3)

J.P. Elliott, Proc. Roy. Soc. A245,128;562 (1958).

SU(3) shell model.
In fact $U^{ST}(4)$xSU(3) shell model.
Harmonic oscillator + QQ interaction.

Description of the spectra of light nuclei.
Shell picture of the rotation and deformation;
SU(3) symm. and quadr.def. are uniq. related.

Further SU(3) 1958

From shell model to cluster model:
Wildermuth-Kanellopoulos *Nucl. Phys. 7, 150 (1958)*
Harm. osc. appr. $H_{SM} = H_{CM}$
Bayman-Bohr: SU(3); *Nucl. Phys. 9, 596 (1958/59).*

A quadrupole collective or a cluster band is picked up from the spherical shell model basis by their special SU(3) symmetry.

Cluster-shell duality:

The GS of ^{16}O, ^{20}Ne etc. is a pure SM configuration and at the same time a pure cluster configuration.

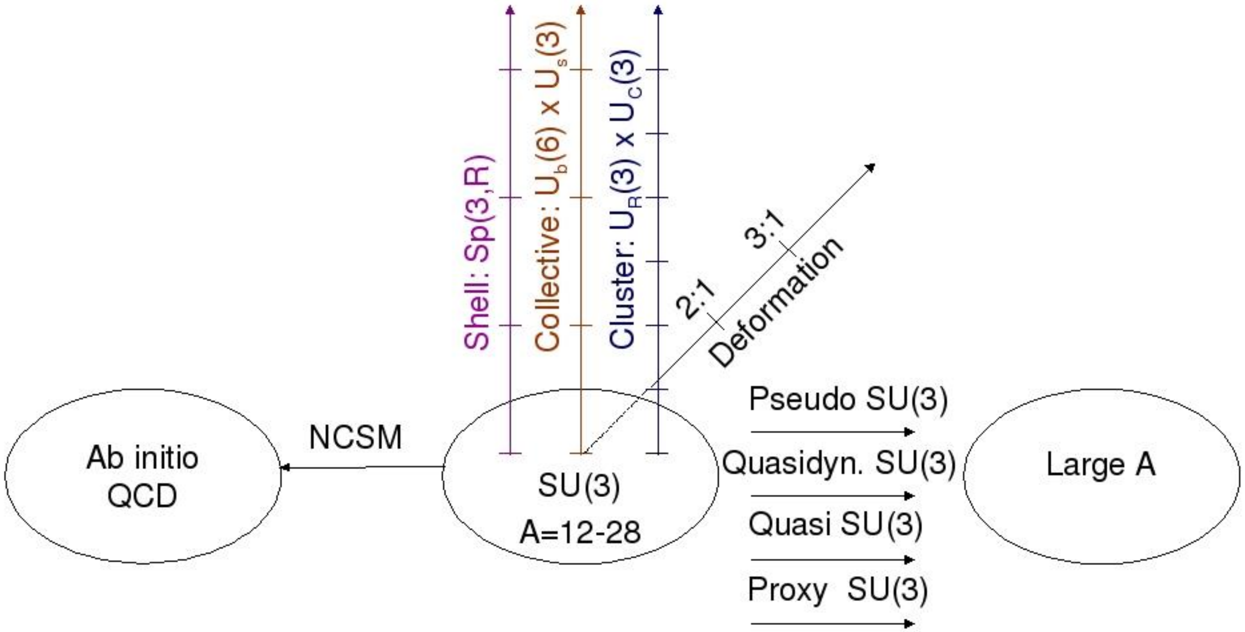
Recent discoveries:

1. Other states (SD, HD,…) as well.
2. Rediscovery in a different formalism.
   (Renaissance of the shell-like clustering.)

Soon afterwords

The SU(3) symmetry breaks down.

Much effort from different aspects.



Shell: Sp(3,R)
Collective: U_b(6) x U_s(3)
Cluster: U_R(3) x U_C(3)
2:1
3:1
Deformation
Ab initio
QCD
NCSM
SU(3)
A=12-28
Pseudo SU(3)
Quasidyn. SU(3)
Quasi SU(3)
Proxy SU(3)
Large A

Generalizations

- Many major shells
- No-core
- Mass number
- Large deformation
- Building blocks: pairs, quartets

II. Quartets

Many major shells: no-core approach.
Huge shell model space: truncatition.

Importance of quarteting:

- short-range attractive nucl.-nucl. force:
  occupy the same single-particle orbital.
- Pauli-principle: 2p+2n.

Ground state, binding energy.

70-ies: Quartet excitations

Quartet: 2p+2n in a single-particle orb. > excite

A. Arima, V. Gillet, J. Ginocchio,Phys. Rev. Lett. 25 (1970) 1043.

Quartet-symmetry: permut. [4], $U^{ST}(4)$ [1,1,1,1]

M. Harvey, Nucl. Phys. A202 (1973) 191.

2p+2n may sit in different shells;

any number of major shell excitation.

Shell-like models: semi-algebraic: only S and T.

Interacting boson models:
Fully algebraic, without shell-model connection.

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Fully algebraic description (including spectrum generation) with well-defined shell content?

Algebraic models for shell-like quarteting

(J. Cseh, Phys. Lett. B743 (2015) 213.)

Quartet: as defined by Arima et al (no internal structure)

Formalism: U(3) as introduced by Elliott,

Phenomenologic Algebraic Quartet Model (PAQM).

Quartet: as defined by Harvey (in terms of nucleons)

Formalism: $U^{ST}(4) \otimes U(3)$

Semimicroscopic Algebraic Quartet Model (SAQM).

Algebra-chain:

$$U(3) \supset SU(3) \supset SO(3) \supset SO(2)$$
$$\left|[n_1, n_2, n_3], (\lambda, \mu), K, \quad L, \qquad M\right\rangle$$

1. Complete set of basis states
2. Dynamical symmetry:

   H in terms of invariant operators

   Eigenvalue-problem: analytical solution

$$\hat{H} = (h\omega)\hat{n} + a\hat{C}_{SU3}^{(2)} + b\hat{C}_{SU3}^{(3)} + d\,\frac{1}{2\theta}\hat{L}^2,$$

$$B(E2, I_i \to I_f) = \frac{2I_f + 1}{2I_i + 1}\alpha^2 \left|\left\langle(\lambda,\mu)KI_i, (1,1)2 \,\|\, (\lambda,\mu)KI_f\right\rangle\right| C^{(2)}(\lambda,\mu)$$

SAQM is an effective model:

1) ^{16}O: positive-parity spectrum: self-consistency argument, no-parameter model.
2) SAQM
3) Symmetry-adapted no-core shell model

   (SA-NCSM) T. Dytrych et al:

   work is in progress to test how far the effective model is in line with the microscopic content of the ab initio model.

III. Clusters

Quartet state: a special shell model configuration,
single-center problem.

Cluster state: a molecule-like configuration,
two- (or more) center problem.

Interrelation: symmetry.
Overlap: small, large, finite, sometimes 100%.

Semimicroscopic Algebraic Cluster Model

J. Cseh, Phys. Lett. B281 (1992) 173;

J. Cseh, G. Lévai, Ann. Phys. (NY) (1994) 165.

Microscopic model space (like in fully micr. models), algebraic methods (like in the model by R. Bijker).

Internal structure of clusters:

Elliott-model with $U_{C_i}^{ST}(4) \otimes U_{c_i}(3)$ algebraic structure.

J.P. Elliott, Proc. Roy. Soc. 245 (1958) 128; 562.

Relative motion:

(truncated) vibron model with $U_R(4)$ algebraic structure

F. Iachello, Phys. Rev. C23 (1981) 2778.

Binary clusterization:

$$U_{C_1}^{ST}(4) \otimes U_{c_1}(3) \otimes U_{C_2}^{ST}(4) \otimes U_{c_2}(3) \otimes U_R(4).$$

Operators: group generators, matrix elements, algebraic.

Model space: only Pauli-allowed states,

as in the Microscopic Cluster Model with U(3) basis.

(H. Horiuchi, T. Hecht, Y. Suzuki, K. Kato,…)

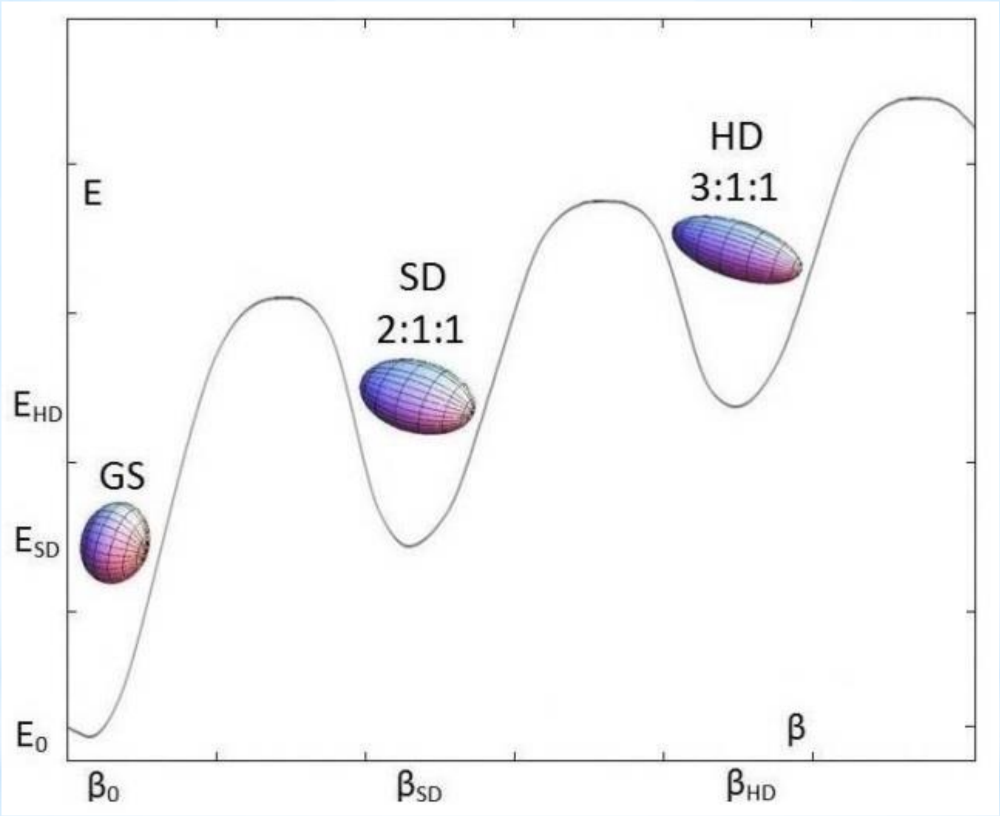
IV. Quadrupole deformation

Spherical, superdeformed, hyperdeformed,…
shapes with ratios of major axes

1:1:1 , 2:1:1 , 3:1:1, …

turn out to be exceptianally stable.

One way of seeing it is to investigate the energy-surface as a function of the quadrupole deformation.



E
HD
3:1:1
SD
2:1:1
E_{HD}
GS
E_{SD}
E_0
β
β_0
β_{SD}
β_{HD}

An alternative way:

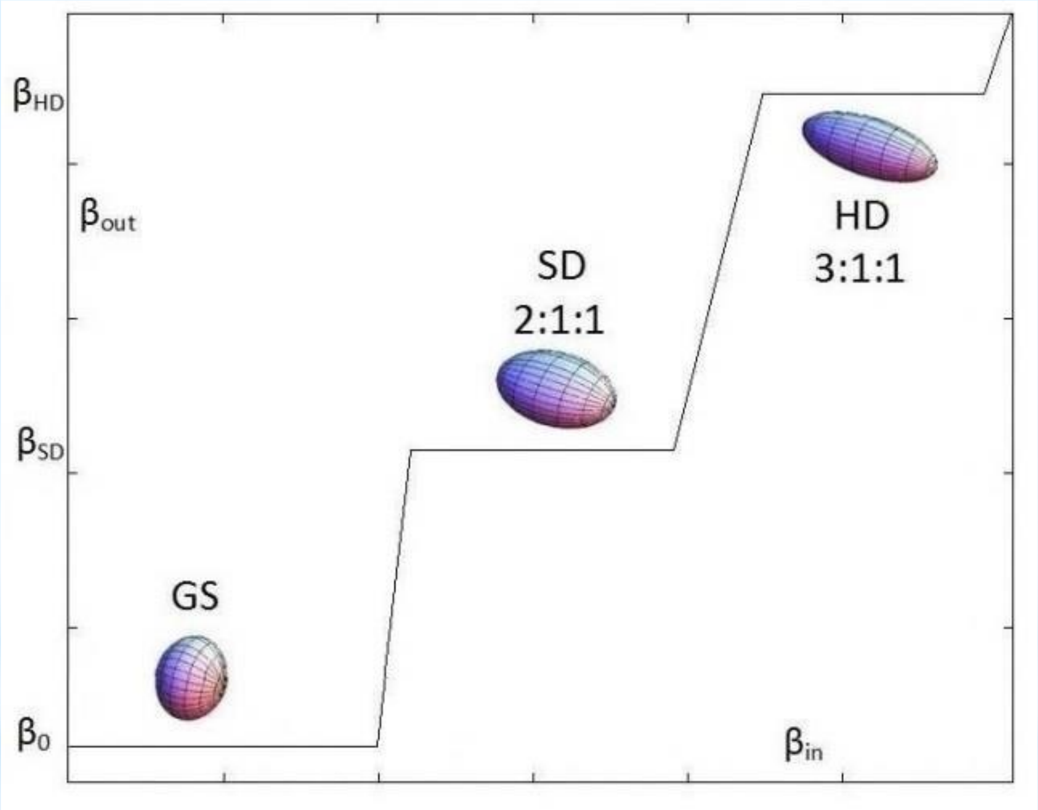
stability and self-consistency of the (quasidynamical) SU(3) symmetry (qudrupole deformation).

Scenario:

quadrupole deformation > Nilsson-calculation >

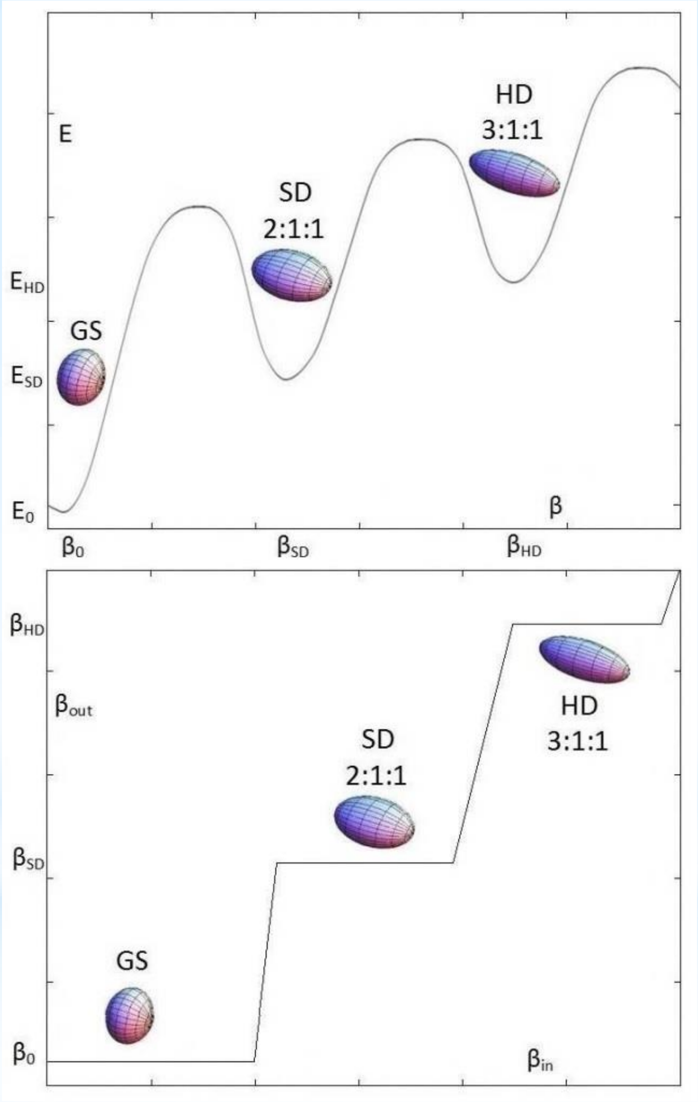
quasi-dynamical SU(3) > quadrupole deformation

Self-consistency + stability.

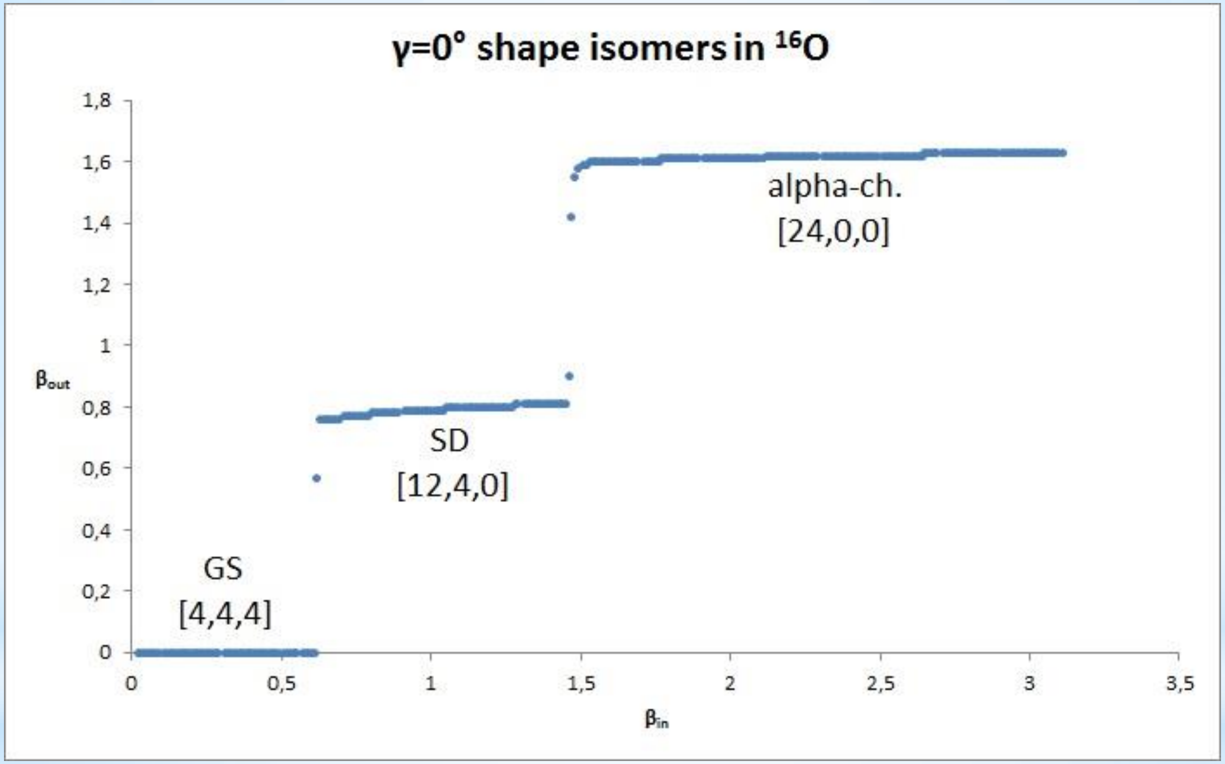

βHD
βout
βSD
β0
βin
GS
SD
2:1:1
HD
3:1:1

Energy-minima and stabil deformation (symmetry) are in good agreement. (Nice to see that different methods give the same results.)

Symmetry: selection rule, connection to cluster-configuration, connection to reaction channels.



E
HD
3:1:1
SD
2:1:1
GS
E_{HD}
E_{SD}
E_0
β
$β_0$
$β_{SD}$
$β_{HD}$
$β_{HD}$
$β_{out}$
HD
3:1:1
SD
2:1:1
$β_{SD}$
GS
$β_0$
$β_{in}$


γ=0° shape isomers in ¹⁶O
1,8
1,6
1,4
1,2
1
0,8
0,6
0,4
0,2
0
βout
0
0,5
1
1,5
2
2,5
3
3,5
βin
alpha-ch.
[24,0,0]
SD
[12,4,0]
GS
[4,4,4]

Horizontal platos:

A) Appearance of shape isomers.

B) Validity of SU(3) symmetry.

Work in progress:

How much the distribution of the wavefunction of the SA-NCSM with realistic interaction (ab initio method) is in line withe the results of the symmetry-based semimicroscopic approach.

GS, SD, alpha-chain states.

L-S coupling, Wigner's $U^{ST}(4)$

space part:

$$U_s(3) \otimes U_x(3) \supset U(3) \supset O(3)$$

V. Multichannel dynamical symmetry (MUSY)-1

Connection between the shell, collective and cluster models for multi major-shell problems:

(No-core) Symplectic shell model
(G. Rosensteel, D. Rowe, PRL 38 (1977) 10;
T. Dytrych et al. J. Phys. G 35 (2008) 123101.)

Contracted symplectic model
(D.J. Rowe, G. Rosensteel, Phys. Rev. C 25 (1982) 3236(R);
O. Castanos, J. P. Draayer, Nucl. Phys. A 491 (1989) 349.)

Semimicroscopic algebraic cluster model
(J. Cseh, G. Lévai, Ann. Phys. 230 (1994) 165.)

Multichannel dynamical symmetry-2

It connects different cluster configurations.
A composite symmetry of a composite system.
(J. Cseh, Phys. Rev. C 50 (1994) 2240.)

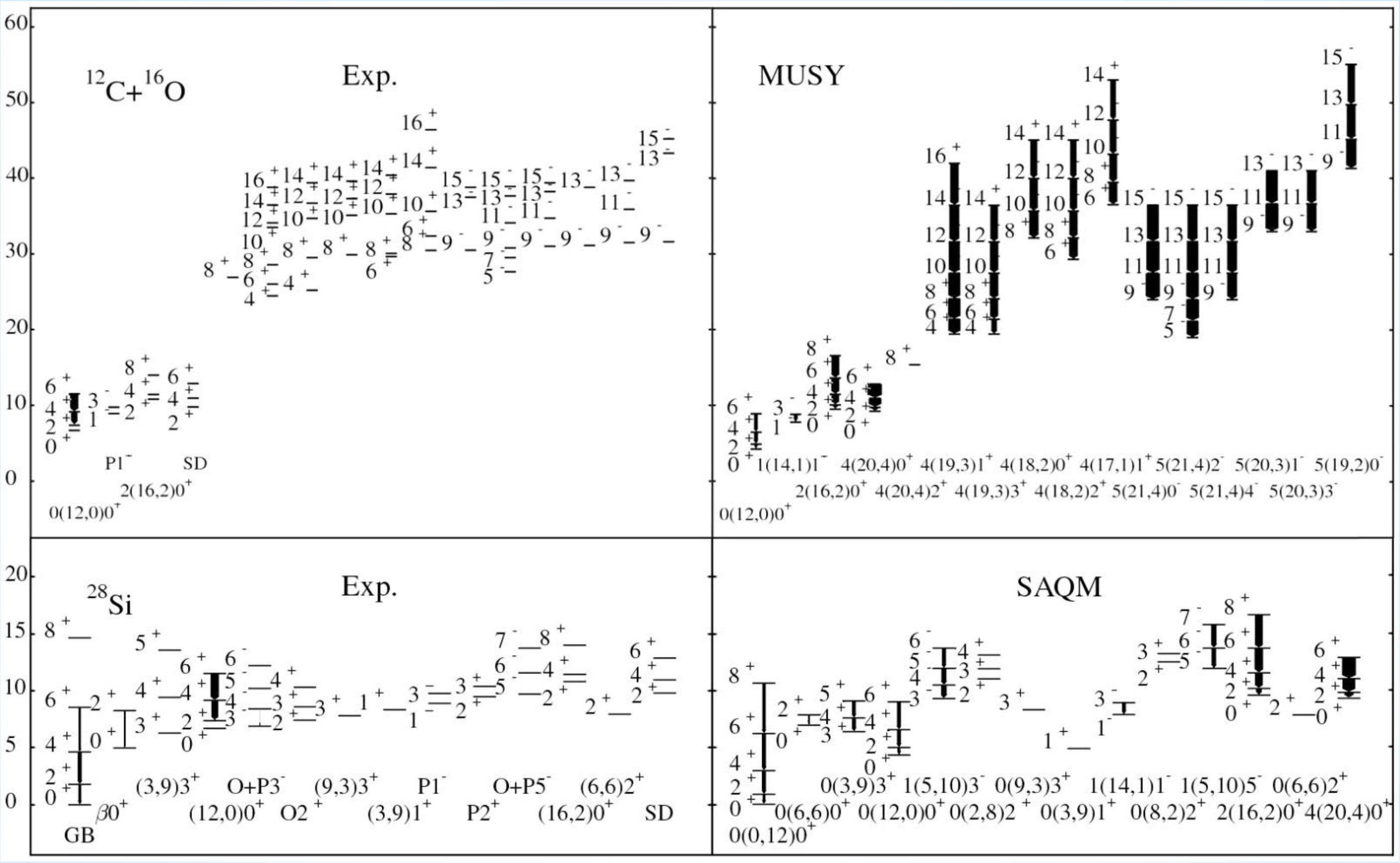
Formulated within the semimicroscopic algebraic cluster model (SACM).
(J. Cseh, Phys. Lett. B 281 (1992) 173.)

The (no-core) shell (or quartet) configuration is a special 1-cluster configuration: MUSY.
(J. Cseh, Phys. Lett. B743 (2015) 213.)

Practical aspect of MUSY

Unified description of different configurations.
J. Cseh, G. Riczu, Phys. Lett. B 757 (2016) 312.

Experimental spectra, predictions.

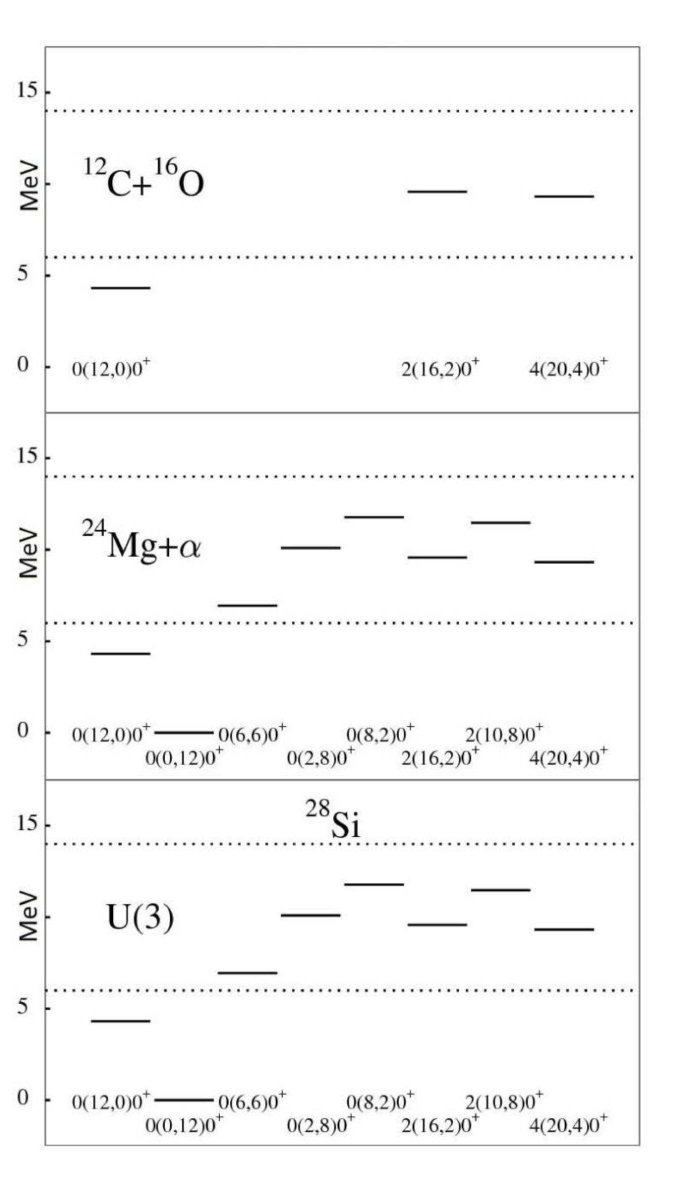


(J. Cseh, G. Riczu, Phys. Lett. B 757 (2016) 312.)

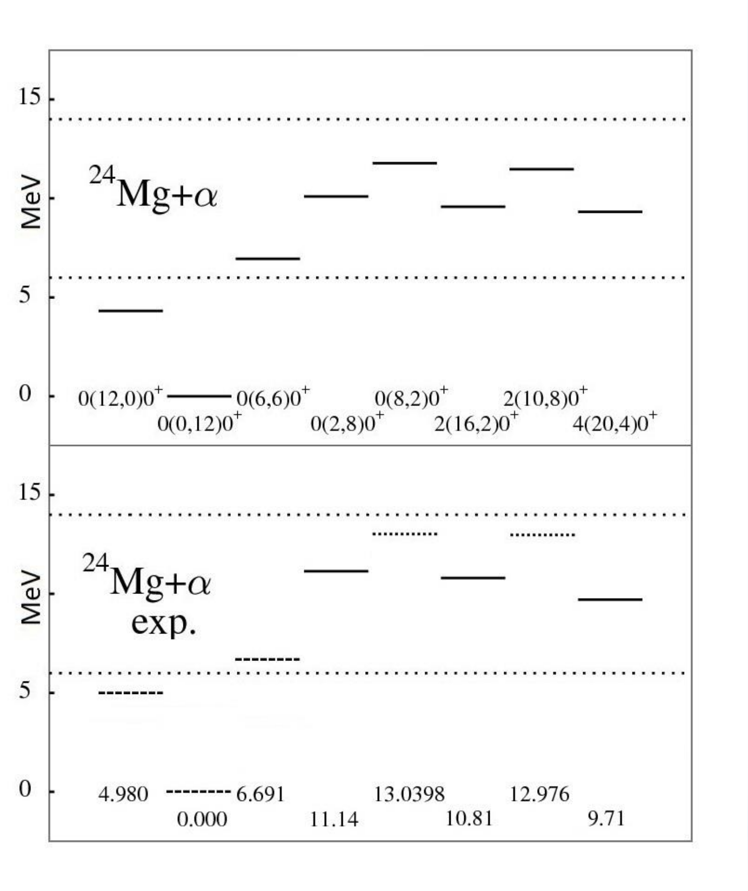
100% overlap of shell (quartet) and cluster states:

$0(0,12)\ 0^+$, GS
$0(12,0)\ 0^+$,
$1(14,1)\ 0^-$,
$2(16,0)\ 2^+$,
$4(20,4)\ 0^+$, SD.

$$\hat{H} = (h\omega)\hat{n} + a\hat{C}_{SU3}^{(2)} + b\hat{C}_{SU3}^{(3)} + d\,\frac{1}{2\theta}\hat{L}^2$$



$^{12}C+^{16}O$
$^{24}Mg+\alpha$
^{28}Si
U(3)
MeV
0
5
15
0(12,0)0+
0(0,12)0+
0(6,6)0+
0(2,8)0+
0(8,2)0+
2(16,2)0+
2(10,8)0+
4(20,4)0+



P. Adsley et al, Phys. Rev. C **95**, 024319 (2017)

VI. Conclusions

Semimicroscopic Algebraic

Quartet Model (SAQM)

Cluster Model (SACM)

Method for shape isomers

Unifying symmetry

VI. Conclusions

The multichannel dynamical symmetry (MUSY)

connects the shell (quartet), collective and cluster models of multi major-shell excitations.

Describes the detailed spectra of different configuartions in different energy window in a unified way.

It gives several shape isomers.

It has a considerable predictive power.

Dear Organizers,

Many thanks for the beautiful conference!

Thank you for your attention!