SEMICLASSICAL ORIGIN OF ASYMMETRIC FISSION

— Nascent-Fragment Shell Effect in the Periodic-Orbit Theory —

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Introduction

Shell structure in the fission processes with the periodic-orbit theory

- Semiclassical theory of shell structure
- Prefragment shell effect — relation to classical periodic orbits

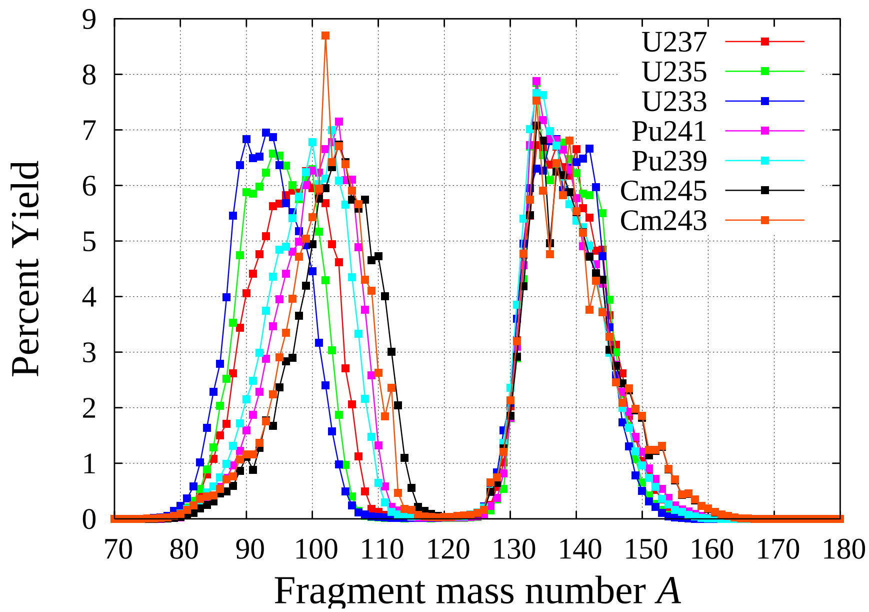
Semiclassical analysis of shell structure in the fission processes

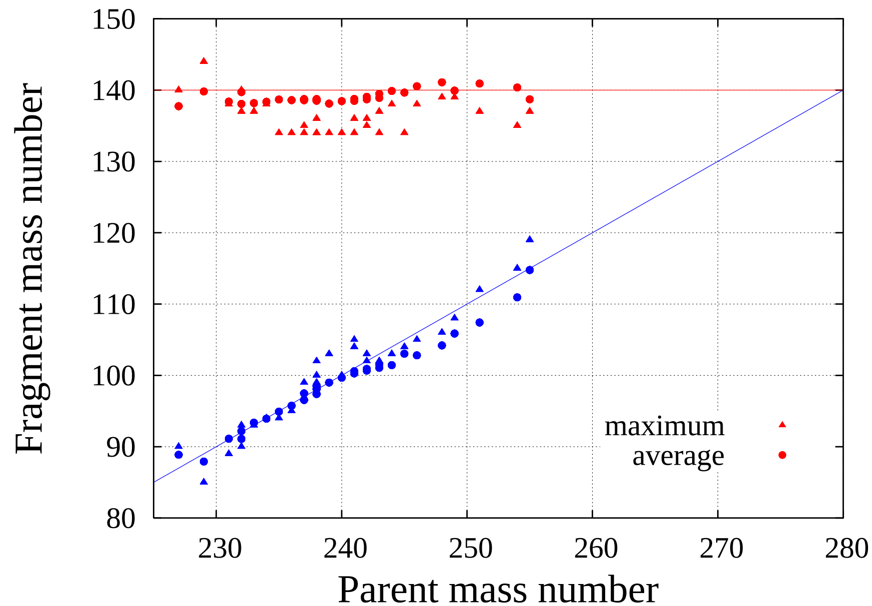
- Contribution of the truncated periodic-orbit family localized in the prefragment
- Potential energy surface as function of elongation and asymmetry
- Effect of prefragment magic numbers on the total shell energies

Summary

Introduction

❑ **Fragment mass distributions in n-induced fission of actinide nuclei**





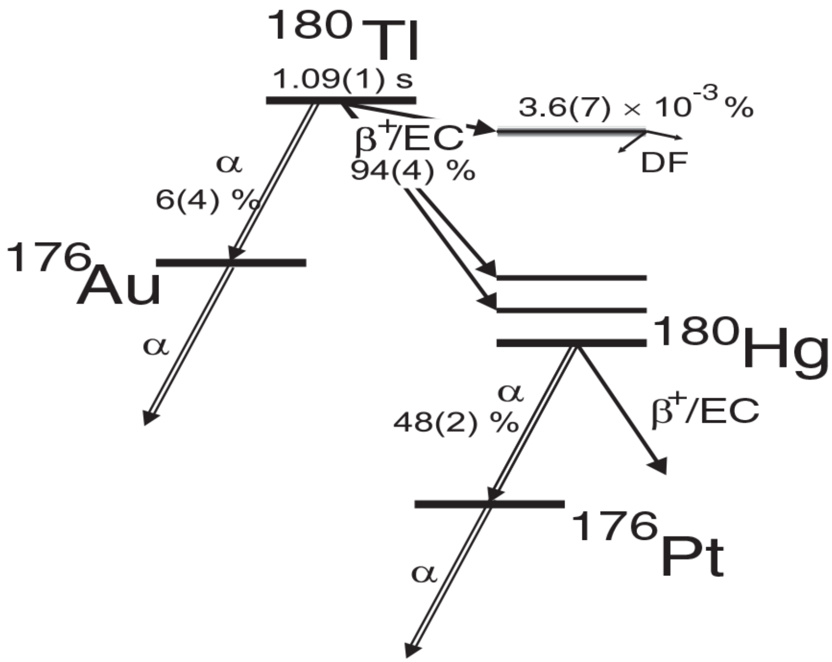
◆ Asymmetries in fragment mass distribution (quantum shell effect)

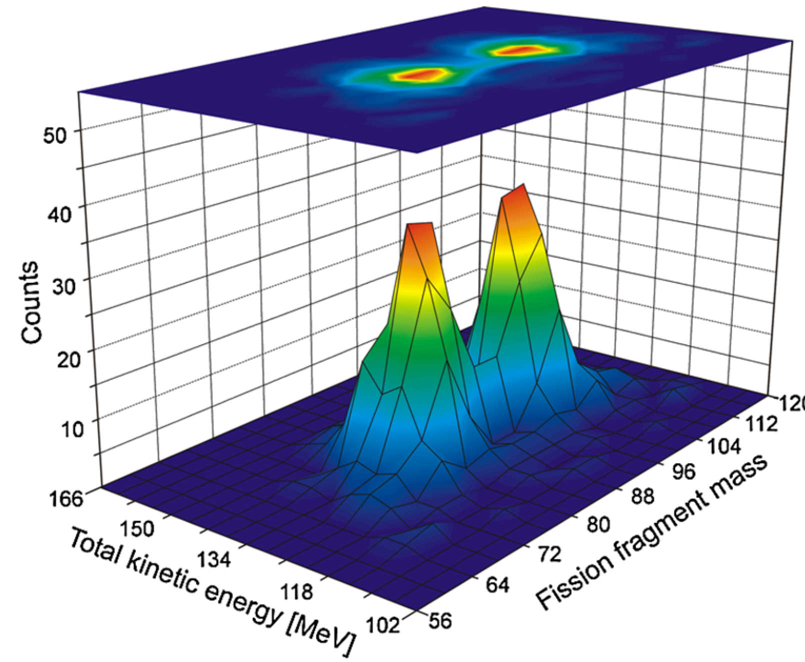
◆ Heavier fragments have $A \approx 140$ independent on the parent species
$\cdots$ Shell effect of doubly-magic ^{132}Sn ($Z = 50$, $N = 82$)

❑ **Asymmetric fission in n-deficient Hg nuclei**

Beta-delayed fission experiments for ^{180}Hg

Andreyev et al., Phys. Rev. Lett. **105**, 252502 (2010)



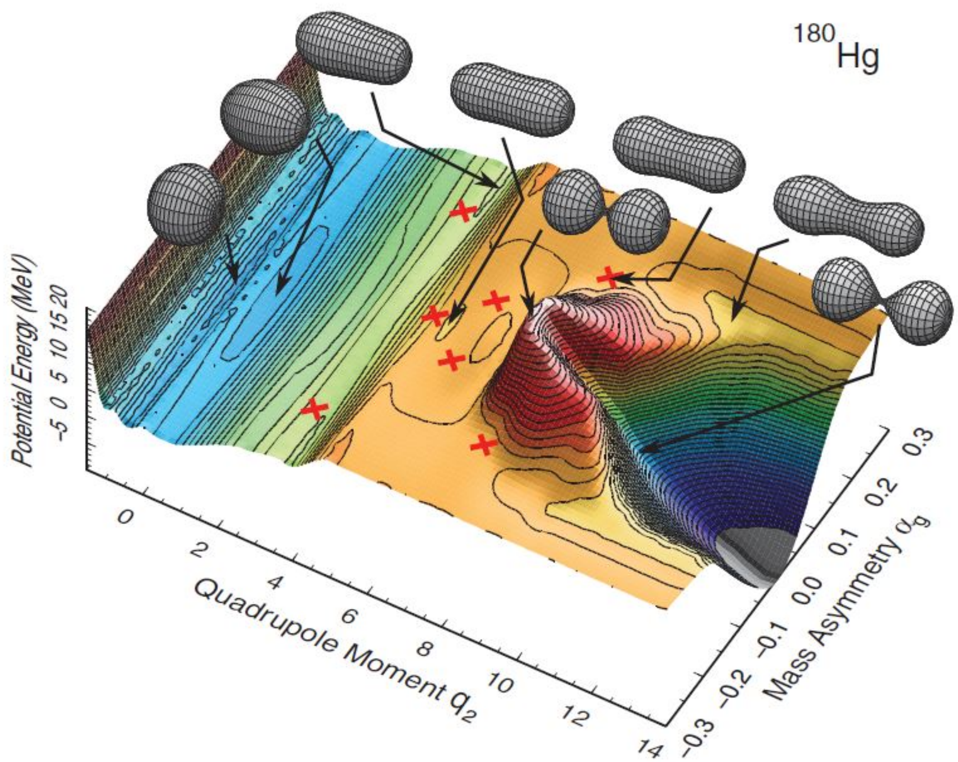


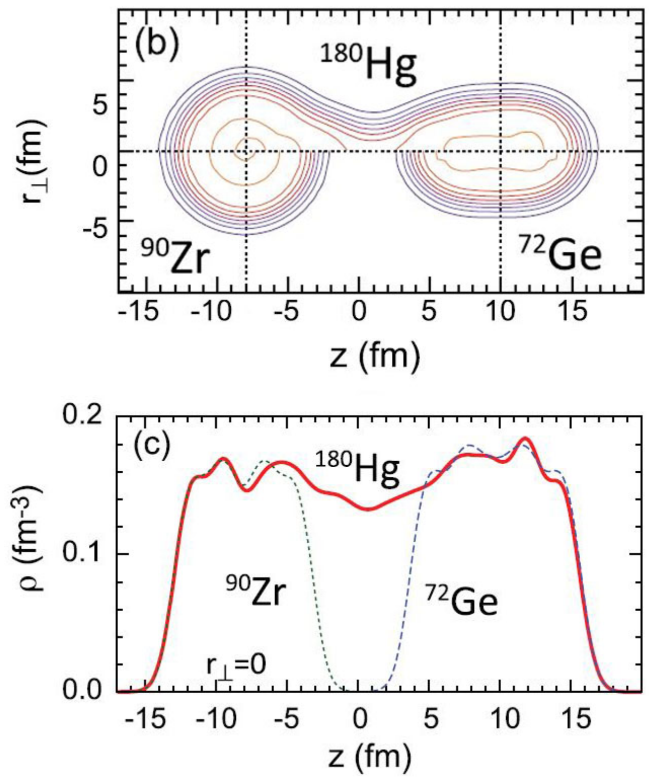
- Asymmetry (main channel: ^{100}Ru+^{80}Kr) in spite of the stability of fragments in symmetric fission (^{90}Zr+^{90}Zr: $Z = 40,\ N = 50$)
- Absence of sufficient fragment shell effects for the asymmetry
  Contrary to expectation from the systematics $\cdots$ "New Type"

❑ Theoretical approaches based on the realistic mean field models

• Fission saddles in 5-D potential energy sufrace by the shell correction method
Ichikawa et al.
Phys. Rev. C **86** (2012) 024610.

• Fully microscopic HFB with D1S effective interaction
Warda et al.
Phys. Rev. C **86** (2012) 024601.





Significance of the deformed shell effect in the fission process
What determines the shape stability of the prefragment ?

- Previous approaches
  using single-particle levels solved for the total mean-field potential
  Most of the wave functions are delocalized
- Unable to define the shell effect *exclusively associated with each of the prefragments*
- How can we do it within the mean-field approach ?

$$\Downarrow$$

Semiclassical periodic orbit theory

Shell structure in the fission processes with the periodic-orbit theory

❑ Semiclassical theory of shell structures

Single-particle level density

$$g(E)=\sum_n \delta(E-E_n)=\mathrm{Tr}\,\delta(E-\hat{H})=\frac{1}{2\pi\hbar}\int dt e^{itE/\hbar}\int d\boldsymbol{r}K(\boldsymbol{r},\boldsymbol{r};t)$$

Path integral representation of the transition amplitude K

$$K(\boldsymbol{r},\boldsymbol{r};t)=\langle\boldsymbol{r}|e^{-i\hat{H}t/\hbar}|\boldsymbol{r}\rangle=\int\mathcal{D}[\boldsymbol{r}(\tau)]\exp\left[\frac{i}{\hbar}\int_0^t\mathcal{L}(\boldsymbol{r},\dot{\boldsymbol{r}})d\tau\right]$$

Semiclassical evaluation of the integrals using stationary-phase method
$\to$ Contribution of classical periodic orbits (PO)

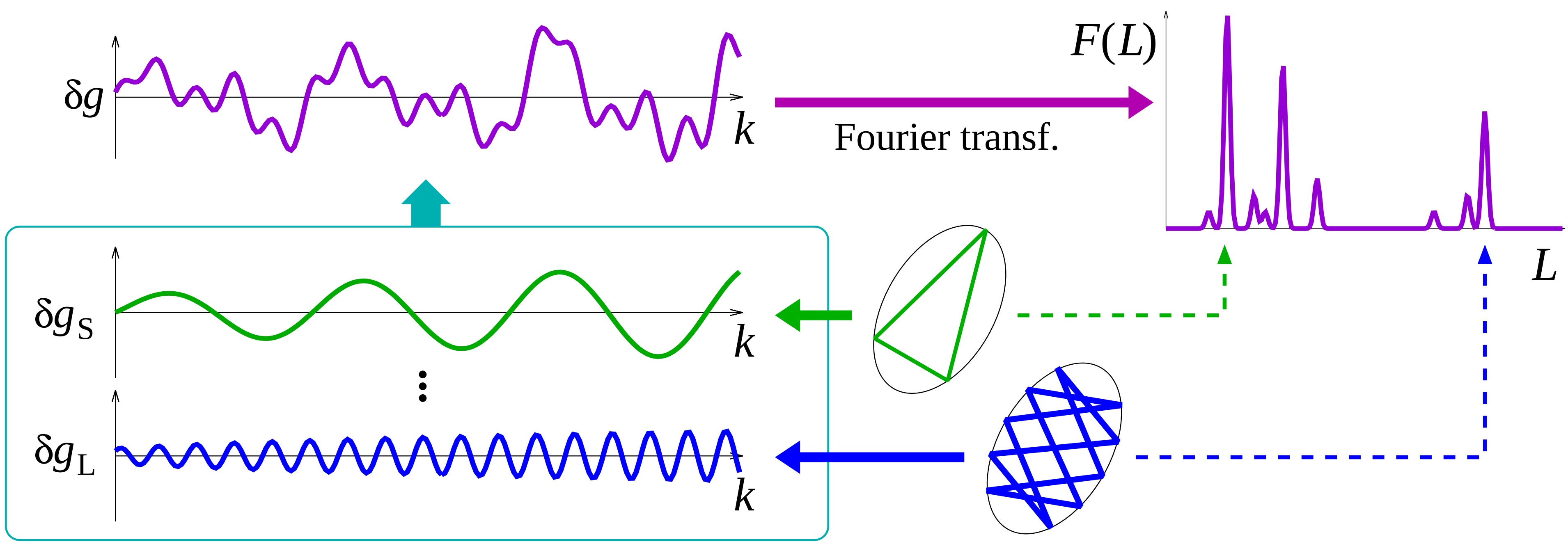
$$g(E)\simeq\bar{g}(E)+\sum_{\mathrm{PO}}A_{\mathrm{PO}}(E)\cos\left(\frac{1}{\hbar}S_{\mathrm{PO}}(E)-\frac{\pi}{2}\mu_{\mathrm{PO}}\right)$$ Trace Formula

$$S_{\mathrm{PO}}(E)=\oint_{\mathrm{PO}}\boldsymbol{p}\cdot d\boldsymbol{r}\quad\text{(action integral)}$$

Trace formula for cavity potential model

$$g(k) \simeq \bar{g}(k) + \sum_{\mathrm{PO}} A_{\mathrm{PO}}(k) \cos\left(kL_{\mathrm{PO}} - \frac{\pi}{2}\mu_{\mathrm{PO}}\right) \qquad k : \text{wave number}$$

- Structure in the level density fluctuation is build as the superposition of some regular structures associated with the classical periodic orbits
- Shorter orbit is responsible for gross structures
- Amplitude A_{PO} is related mainly to the degeneracy and the stability of the orbit

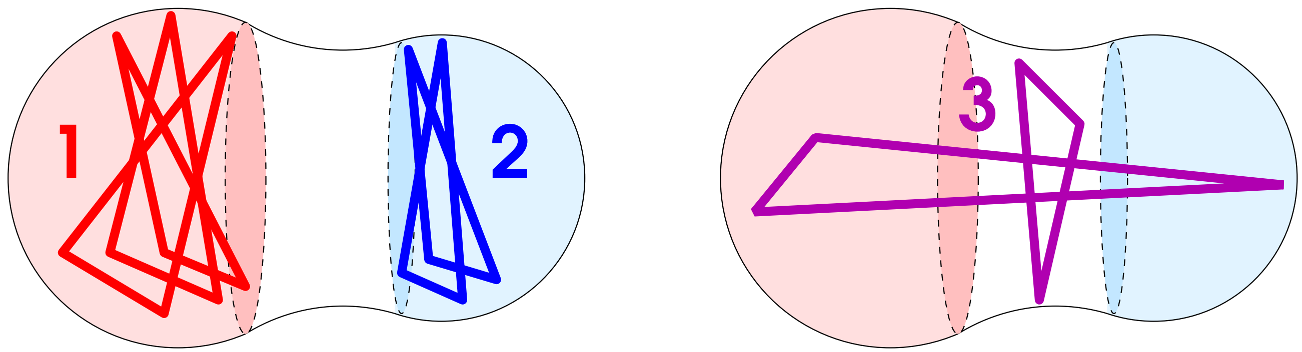


❑ Prefragment shell effect — relation to classical periodic orbits

Extraction of the prefragment contribution out of the total shell energy

Classify the periodic orbits into 3 groups :

- localized in the prefragment 1 and 2
- the others



Unambiguous definition of the prefragment shell effect by the contribution of orbits localized in the corresponding prefragment

$$\delta E(N) = \delta E_1(N) + \delta E_2(N) + \delta E_3(N)$$

Category "3" $\cdots$ less degeneracy, generally more unstable and longer

$\Rightarrow \quad \delta E_1,\ \delta E_2 \gg \delta E_3$

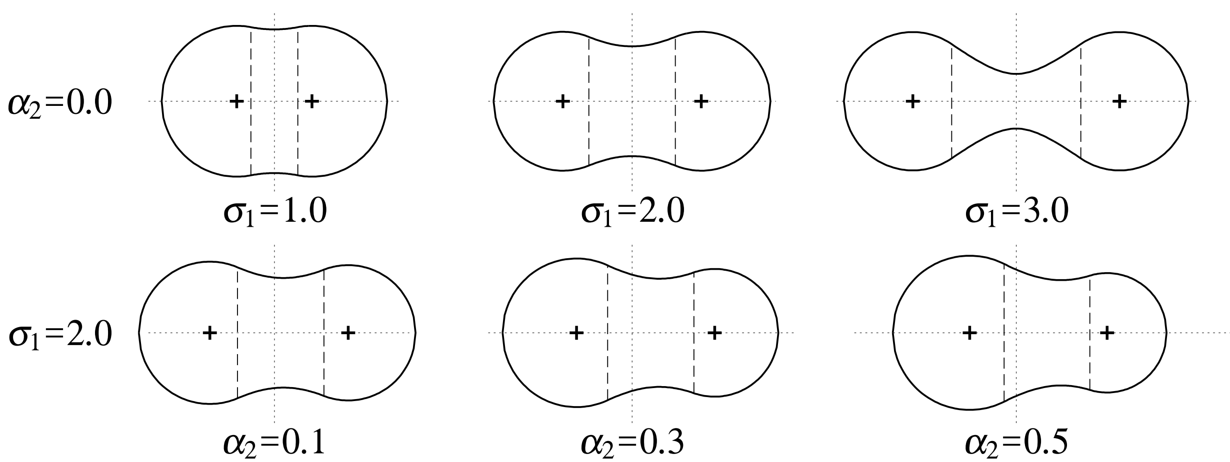
Prefragment shell effect dominate the total shell effect with neck formation

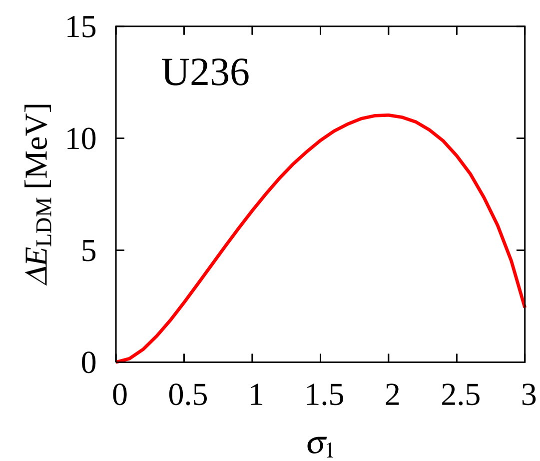
Deformed shell effect (along the fission path) $\Leftarrow$ Stabilities of prefragments

Semiclassical analysis of shell structures in the fission processes

❑ **Cavity potential model for the fission processes**

- TQS (Three Quadratic Surfaces) cavity (infinite well) model
  5 shape parameters (elongation, neck width, prefragment mass asymmetry, prefragment deformations)
- Assume sphericity of the prefragments
  Fixed neck parameter to a typical value in realistic calculations
- Analyze shell structures as functions of **elongation** σ_1 and **prefragment mass asymmetry** α_2





❑ Trace formula and Fourier analysis

$$g(k) = g_0(k) + \sum_{\mathrm{PO}} A_{\mathrm{PO}} \cos\left[\frac{1}{\hbar} S_{\mathrm{PO}}(k) - \frac{\pi}{2}\mu_{\mathrm{PO}}\right] \quad \text{(trace formula)}$$

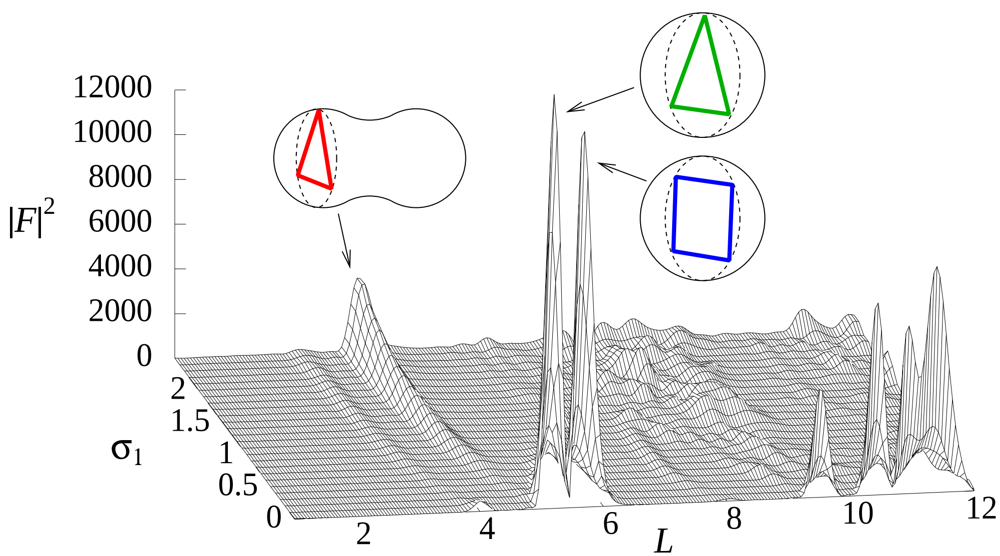
$$S_{\mathrm{PO}}(k) = \oint_{\mathrm{PO}} \boldsymbol{p}\cdot d\boldsymbol{r} = \hbar k L_{\mathrm{PO}} \quad (L_{\mathrm{PO}}\text{: orbit length})$$

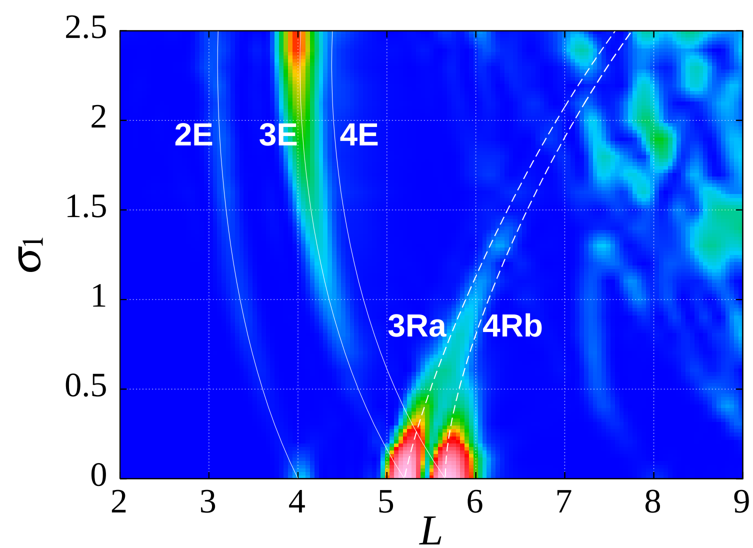
$$F(L) = \frac{2}{\sqrt{2\pi}k_c}\int dk\, g(k) e^{ikL} e^{-(k/k_c)^2/2}$$

$$\simeq F_0(L) + \sum_{\mathrm{PO}} A_{\mathrm{PO}} e^{-(L-L_{\mathrm{PO}})^2 k_c^2/2} \quad \text{(peaks at } L = L_{\mathrm{PO}}\text{)}$$

(energy dependence of the amplitude is ignored here for simplicity)

$F(L_{pt}) \simeq A_{pt}$ direct information on the semiclassical amplitude





❑ Prefragment contribution to the deformed shell effect

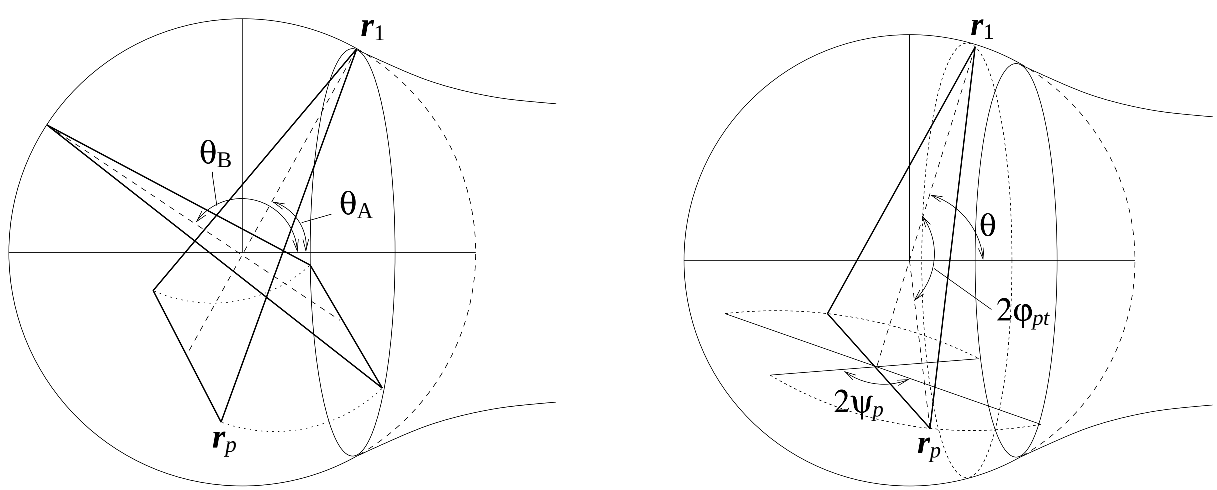
Trace formula for the truncated spherical cavity

K.A., arXiv:1806.06490, to appear in Phys. Rev. C

Regular polygon orbits (p, t) with p vertices and t turns

$\cdots$ form 3-parameter families in the spherical cavity

Restriction of the parameters in the prefragments



Reduction factor f_p from the occupation volume of the parameter space

$$f_p = \frac{1}{2\pi} \int_{\theta_\mathrm{A}}^{\theta_\mathrm{B}} \psi_p(\theta) \sin\theta d\theta$$

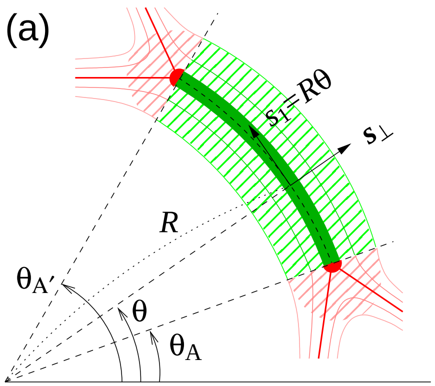
$A_{pt} = f_p A_{pt}^{(\mathrm{sph})}$ → considerable underestimation of the quantum results

Consideration of the end-point corrections (marginal orbits)

$$A_{pt} = f_p A_{pt}^{(\mathrm{sph})} + A_{pt}^{(\mathrm{marginal})} \;\rightarrow\; \text{quantum results are nicely reproduced}$$

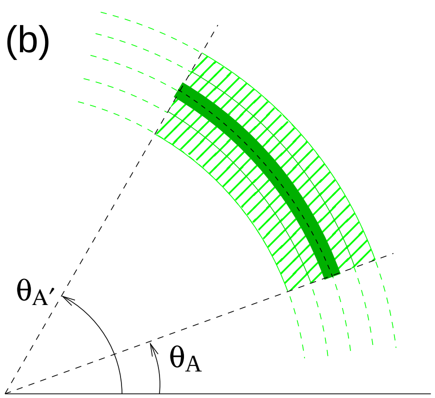
Periodic orbit: stationary point of the action along the closed trajectory $S(\boldsymbol{r}) = \oint_{\boldsymbol{r}}^{\boldsymbol{r}} \boldsymbol{p} \cdot d\boldsymbol{r}$

(a)



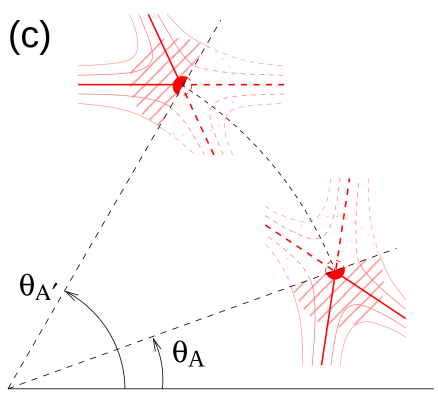
truncated family

(b)

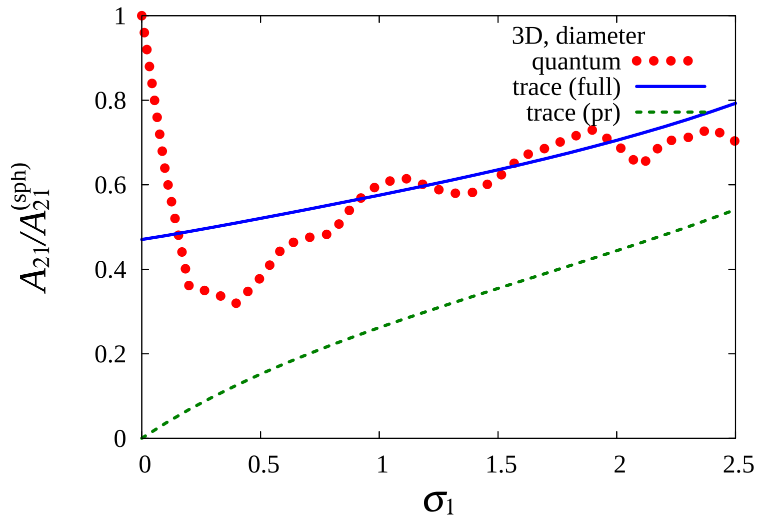


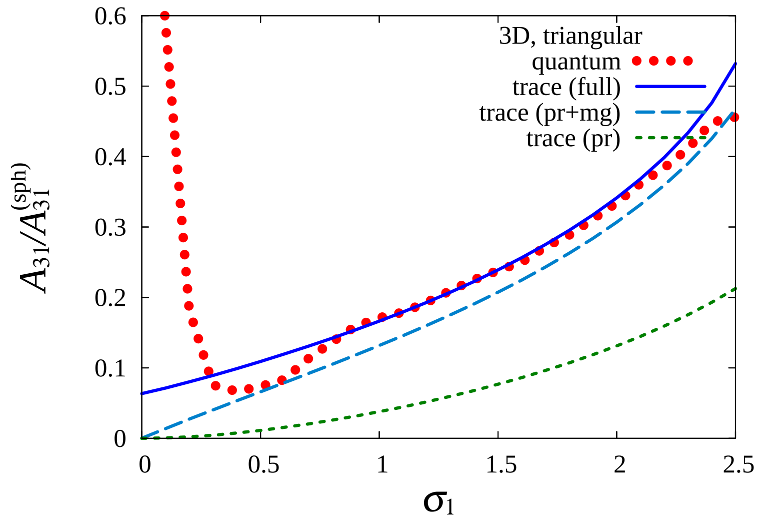
principal part

(c)



marginal parts



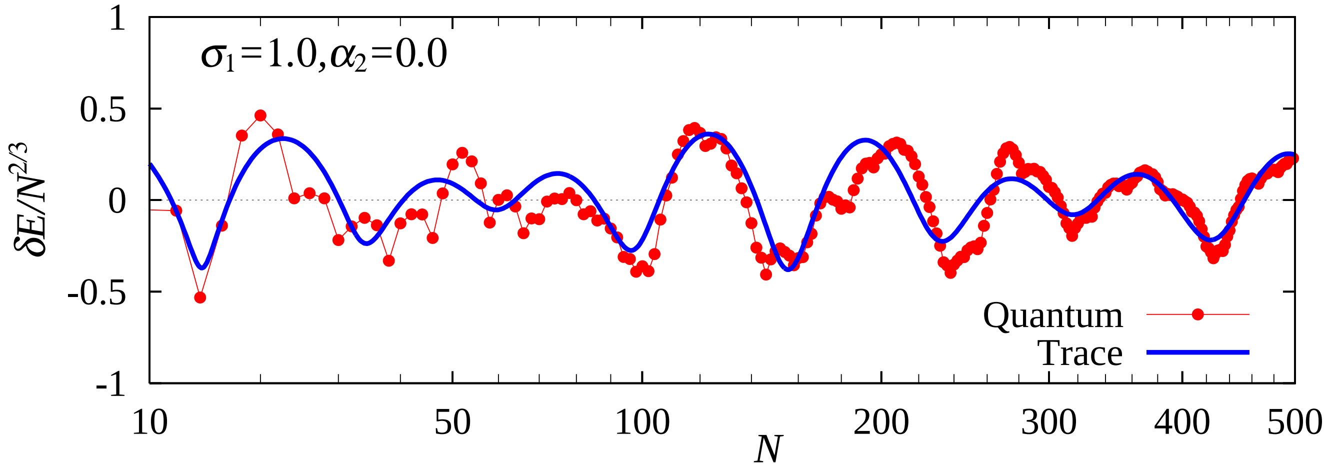


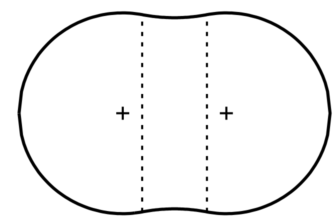
❑ Semiclassical shell energy compared with quantum

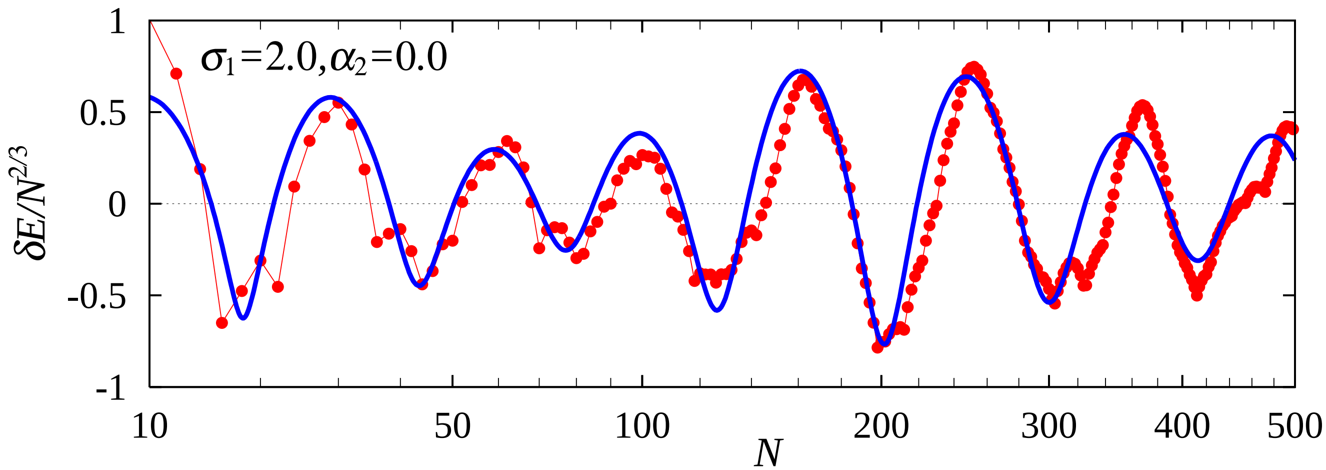
Symmetric shapes

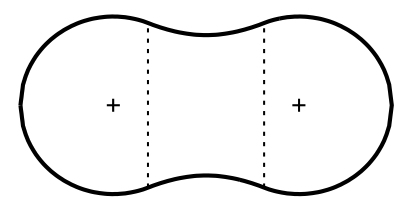
∗ Trace formula nicely reproduces the quantum results by taking only the prefragment orbits into account

∗ Enhancement of shell effect with neck formation



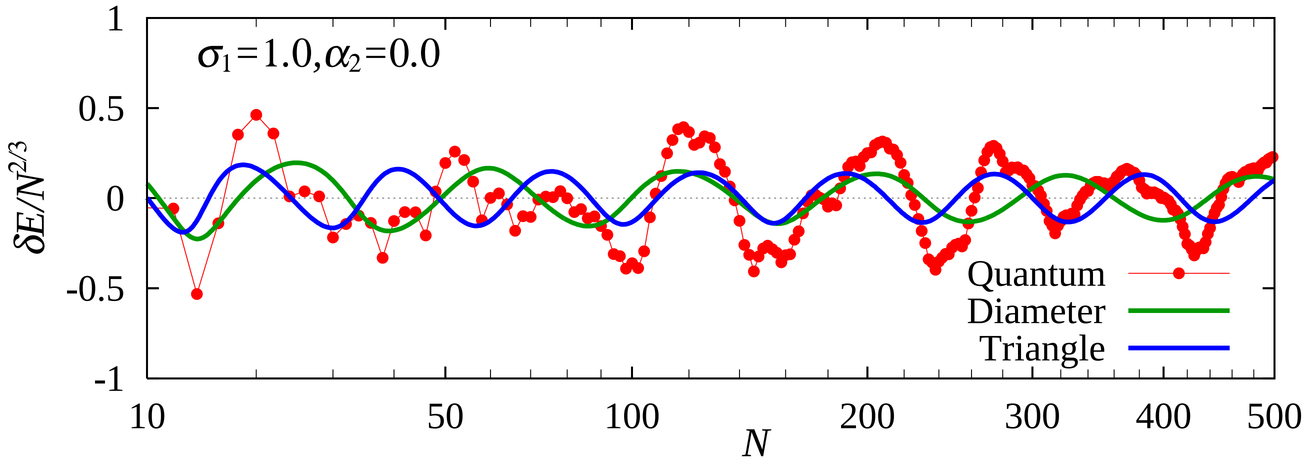


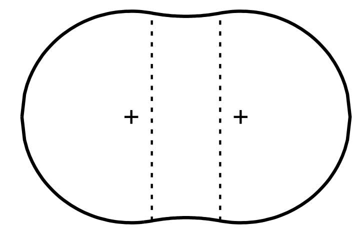


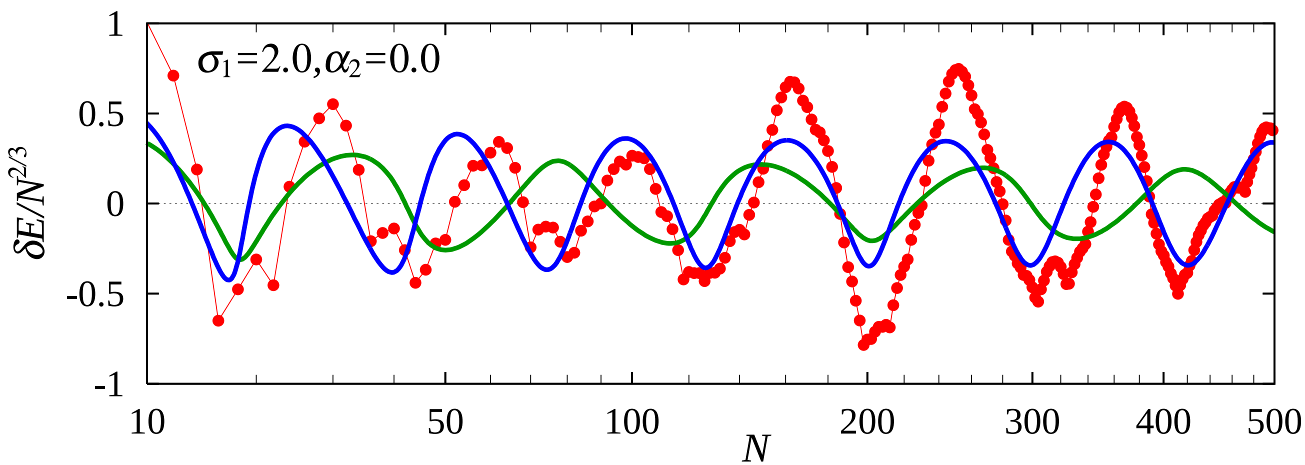


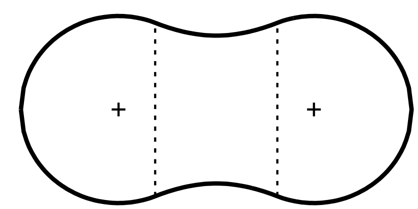
$*$ Modulations in shell oscillations

Interference between diameter and triangle orbit contributions



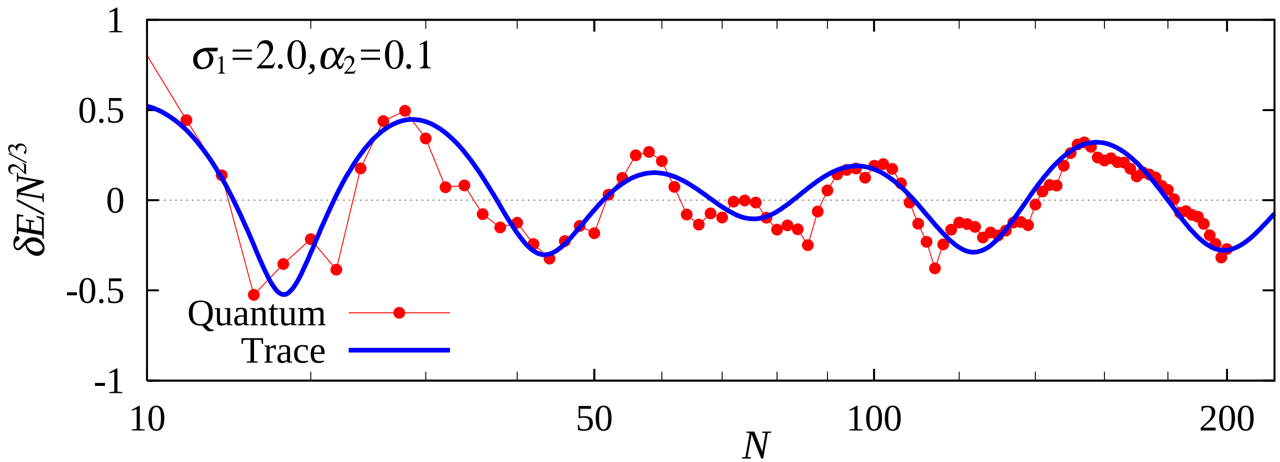


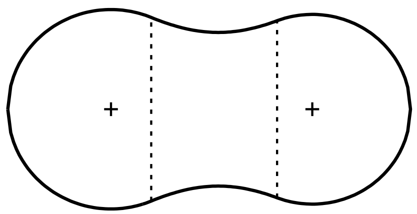


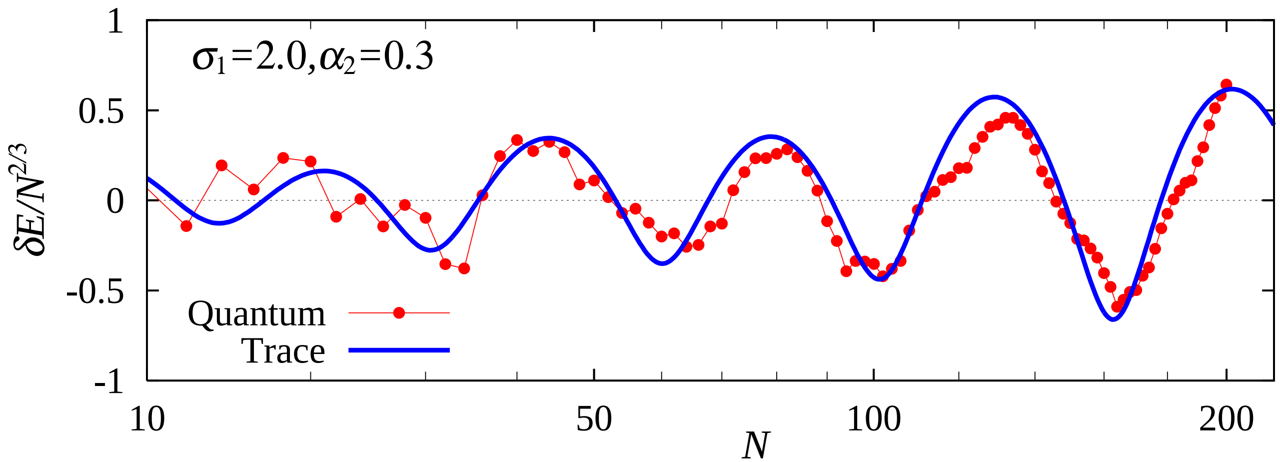


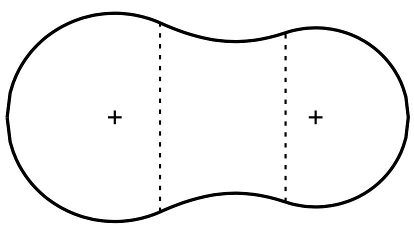
Asymmetric shapes

* Asymmetry switched on for $\sigma_1 = 2.0$ (around fission saddle)
* Quantum results are also nicely reproduced



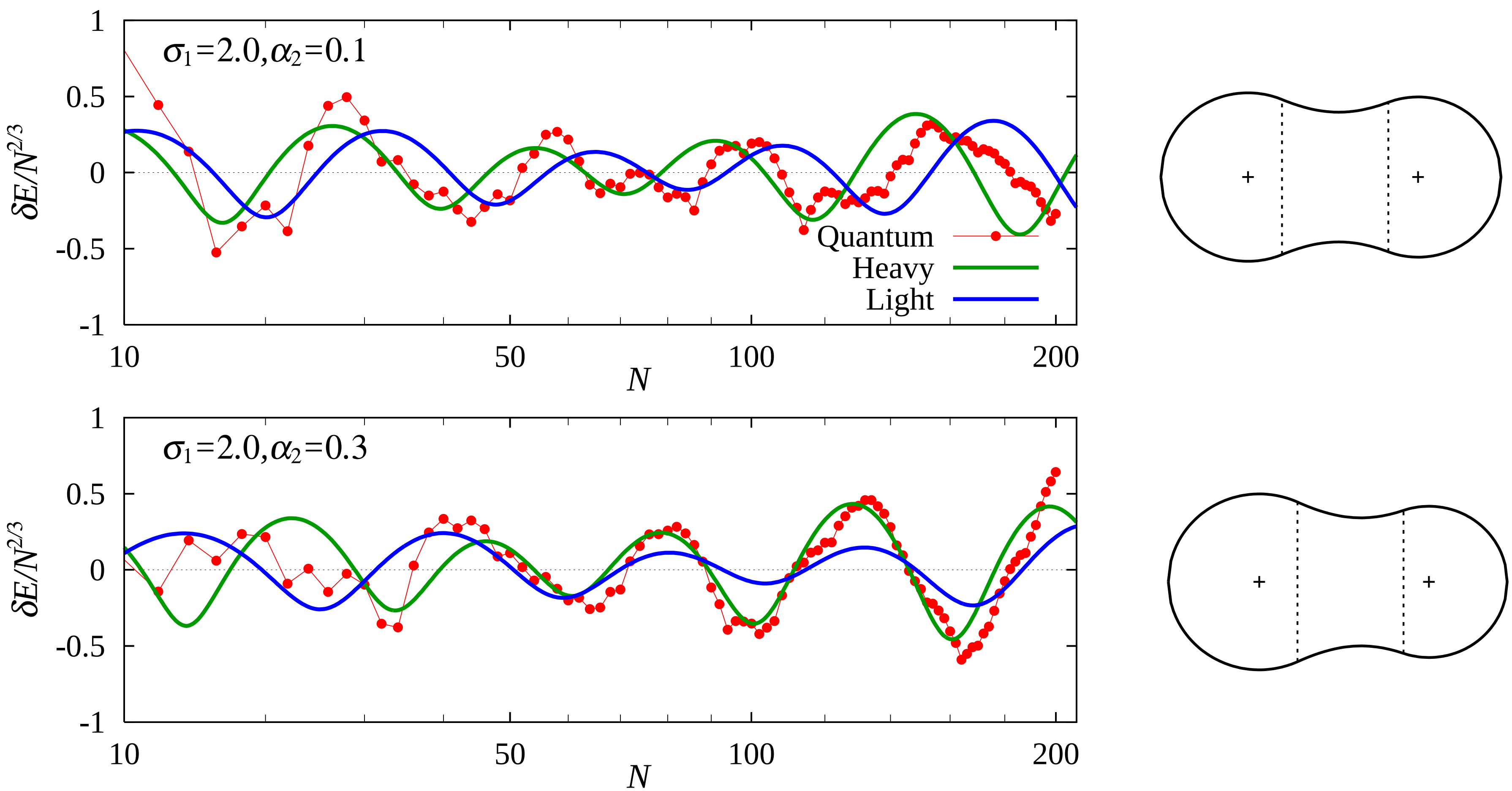






∗ Periodic orbits have different lengths in the two prefragments

⇒ Interference between heavy and light prefragments

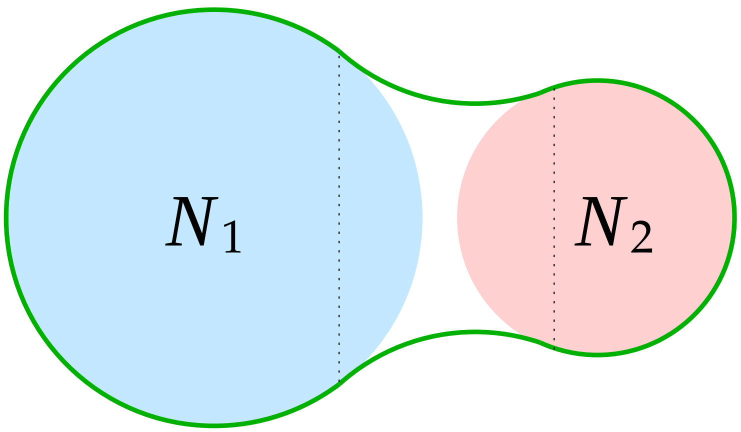


❑ Effect of prefragment magics on asymmetric fission

- Dominant contribution of triangle orbit $(p,t)=(3,1)$

$$\delta E_i(N) \approx \mathcal{A}_{31}(k_F, R_i)\cos\Big(k_F L_{31}(R_1) - \pi\mu_{31}/2\Big)$$
$$\approx w_{31}^{(i)}(\sigma_1, \alpha_2, k_F)\delta E^{\rm sph}(N_i, R_i), \quad w_{31}^{(i)} = \mathcal{A}_{31}^{(i)}/\mathcal{A}_{31}^{(\rm sph)}$$

Deformed shell energy in terms of two spherical fragments

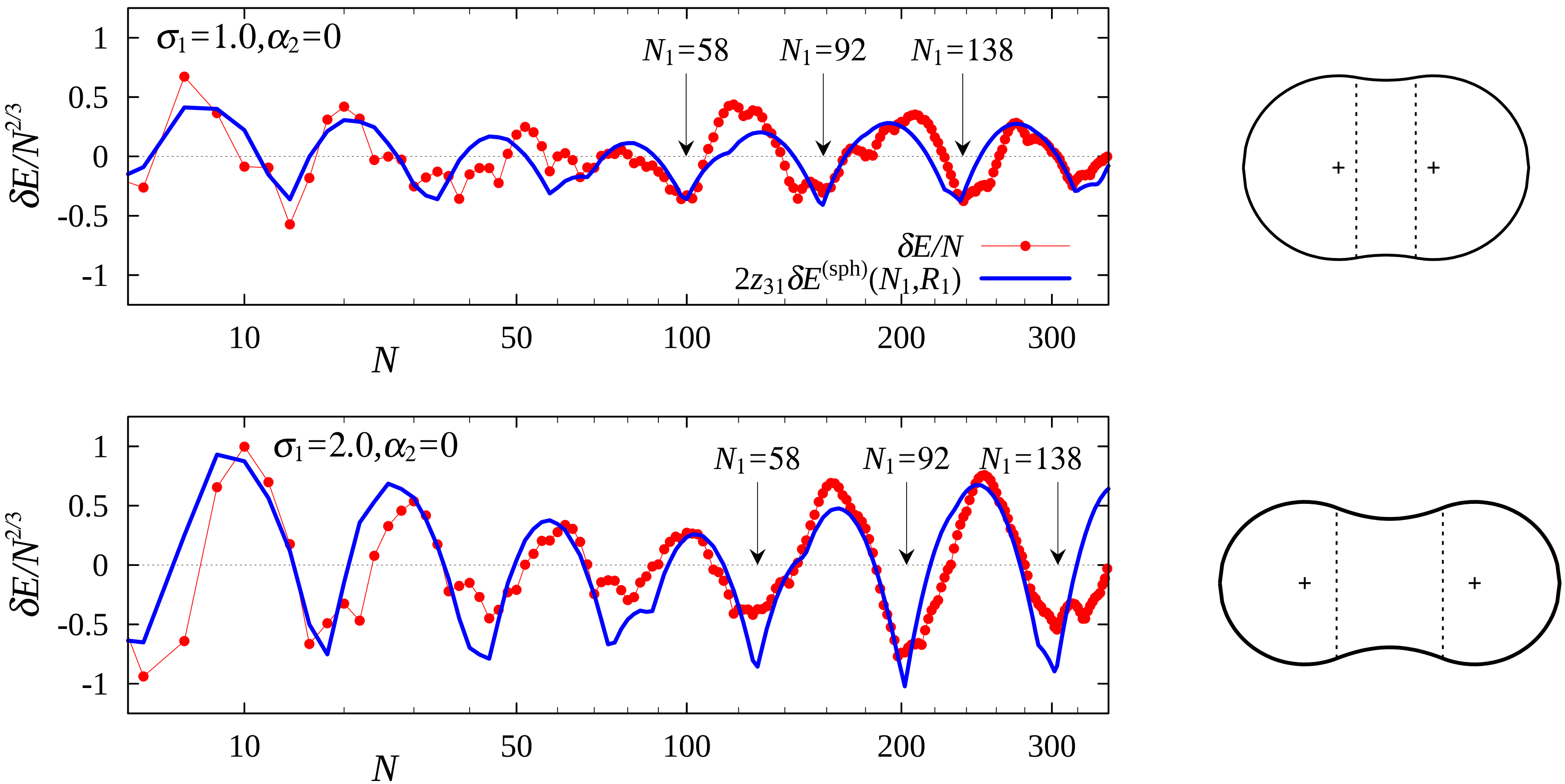


Prefragment particle numbers $N_i \approx \left(\frac{R_i}{R_0}\right)^3 N$

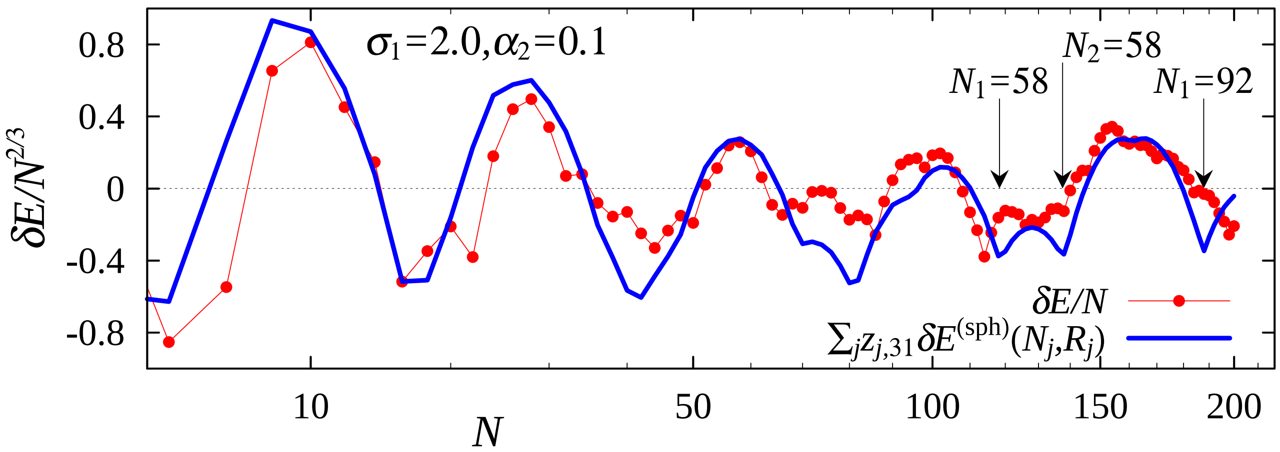
Coefficients $w_{31}^{(i)}$ are obtained by the semiclassical formula

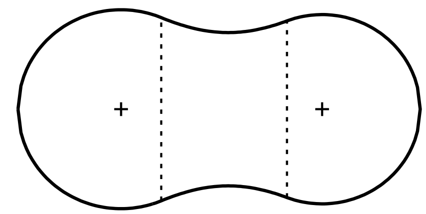
- Spherical magic numbers : $\cdots, 34, 58, 92, 138, \cdots$

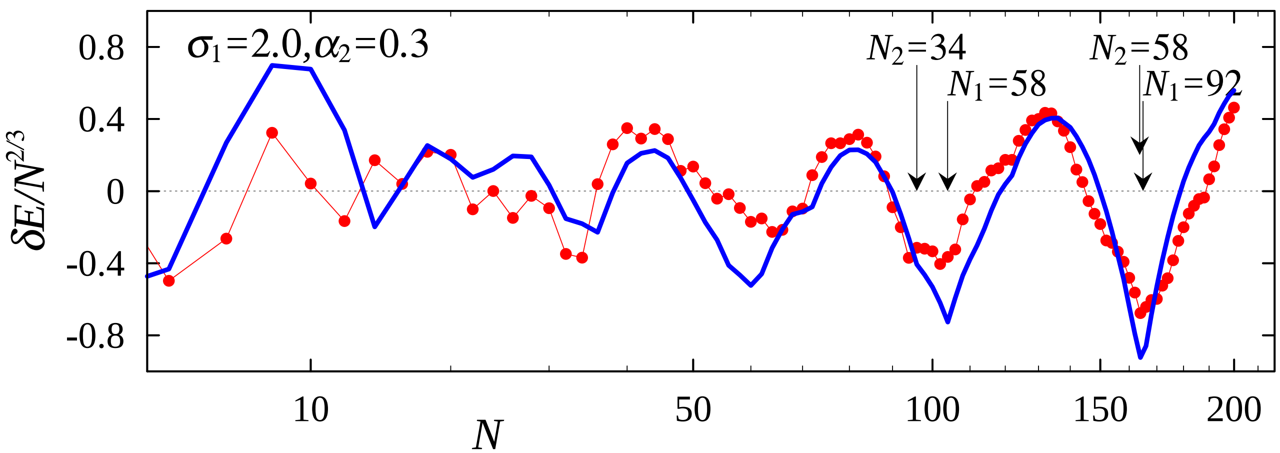
Symmetric shapes

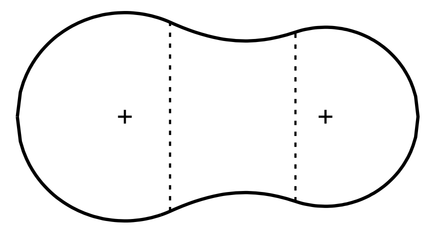


Asymmetric shapes









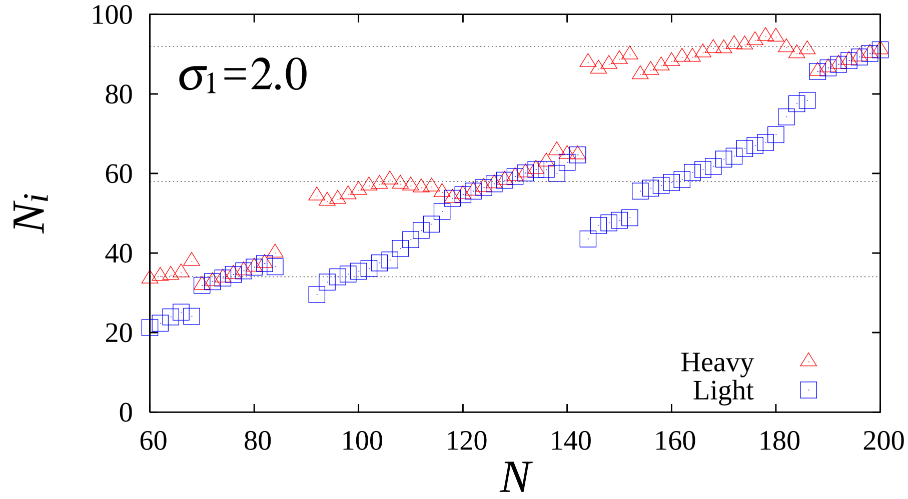
* Prefragment orbits dominate the shell effect as developing neck
* Strong correlation between energy minima and prefragment magicks
* Large shell effect at “double magick” for two prefragments

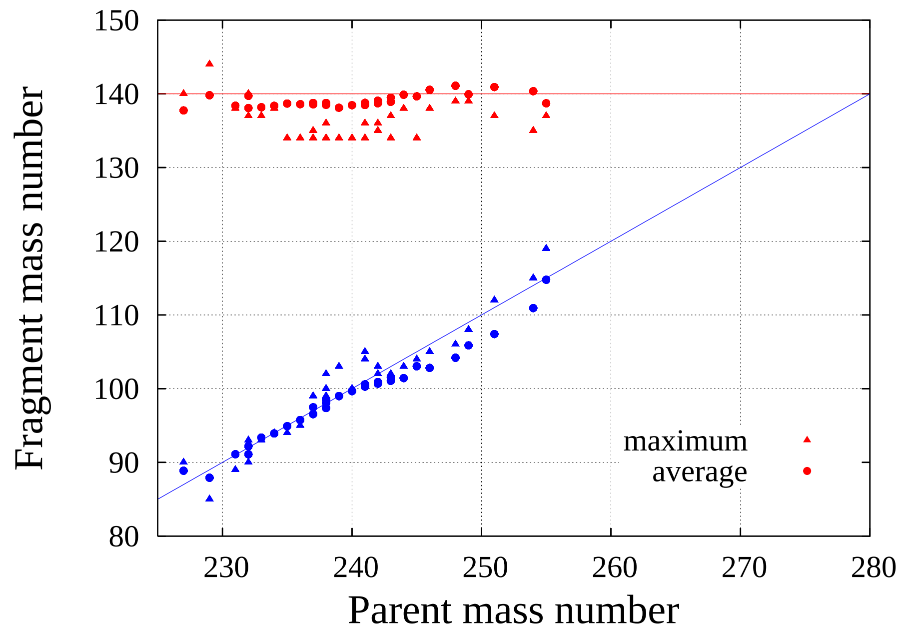
❑ Effect of prefragment magics on the fission deformations

Asymmety α_2 which minimizes the shell energy for given elongation σ_1
$\Rightarrow$ prefragment particle numbers N_1, N_2 reduced from the prefragment radii R_1, R_2

$$N_i \approx \left(\frac{R_i}{R_0}\right)^3 N$$

- N_1 keeps spherical magic
- Good comparison with the fragment mass distribution in acitinide region





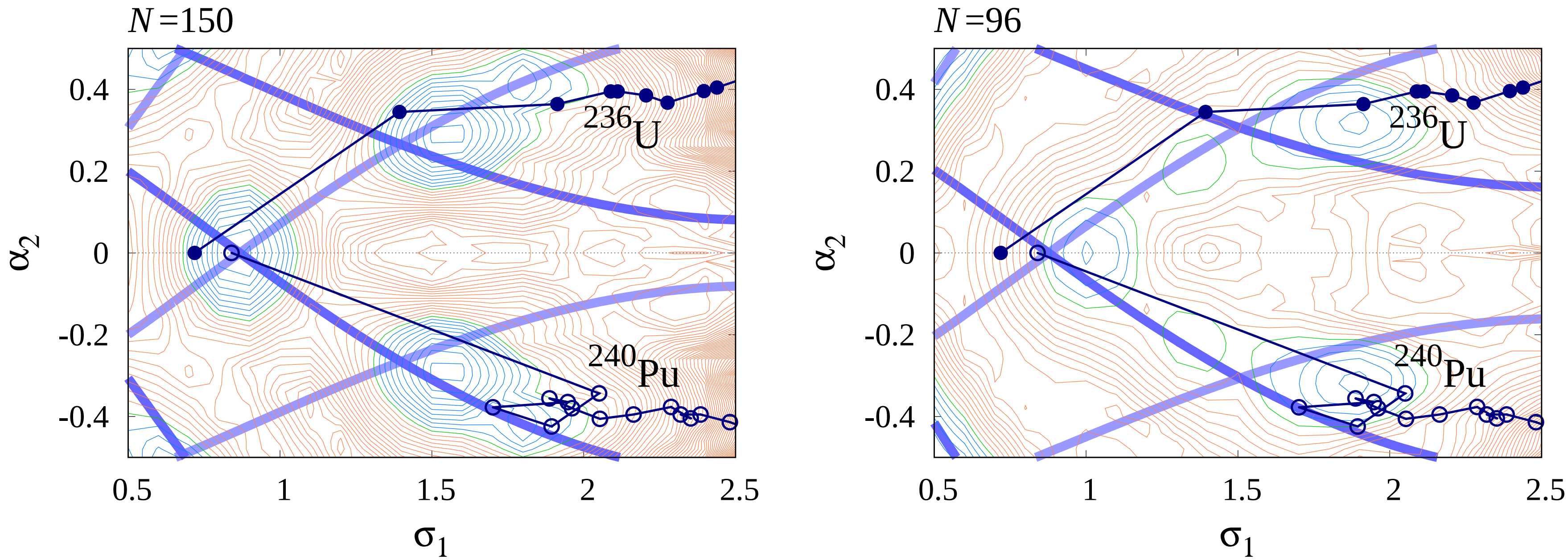
❑ Constant-action curve and the prefragment magics

The valleys in deformation space (σ_1, α_2) is described by the constant-action curve of the triangle orbits

$$k_F(N)L_{31}^{(i)}(\sigma_1, \alpha_2) - \frac{\pi}{2}\mu_{31} = (2n+1)\pi, \qquad (n = 0, 1, 2, \cdots)$$

$$L_{31}^{(i)}(\sigma_1, \alpha_2) = \frac{(2n+1+\mu_{31}/2)\pi}{k_F(N)}$$

actinide region ⇔ cavity model with $(N, Z) \approx (150, 100)$



Valleys in the potential energy surface along the constant-action curve of the triangle orbit ⇒ semiclassical origin of the fission path

SUMMARY

- ❏ Shell structures in fission processes with the TQS cavity model
- ❏ Prefragment shell effect defined by the POT
  Dominant contribution of the prefragment family with neck formation
  $\Leftarrow$ Enhancement of the prefragment shell effect mainly by the triangular orbit family
- ❏ Significance of the prefragment magics in determining the fission path in the potential energy surface $\Rightarrow$ semiclassical origin of the asymmetric fission

Future subject

- ❏ Deformations of prefragments
  Quadrupoles and Octupoles   cf. Scamps and Simenel, arXiv:1804.03337
- ❏ Application to more realistic mean-field model
  Two-center shell model, Woods-Saxon model