Revolving rotation or transverse wobbling?
1qp-plus-triaxial-rotor model calculations

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Three-dimensional (3D) rotation or wobbling ?

the rotational axis precesses around another axis

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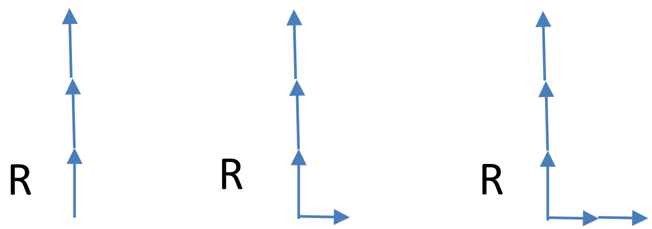
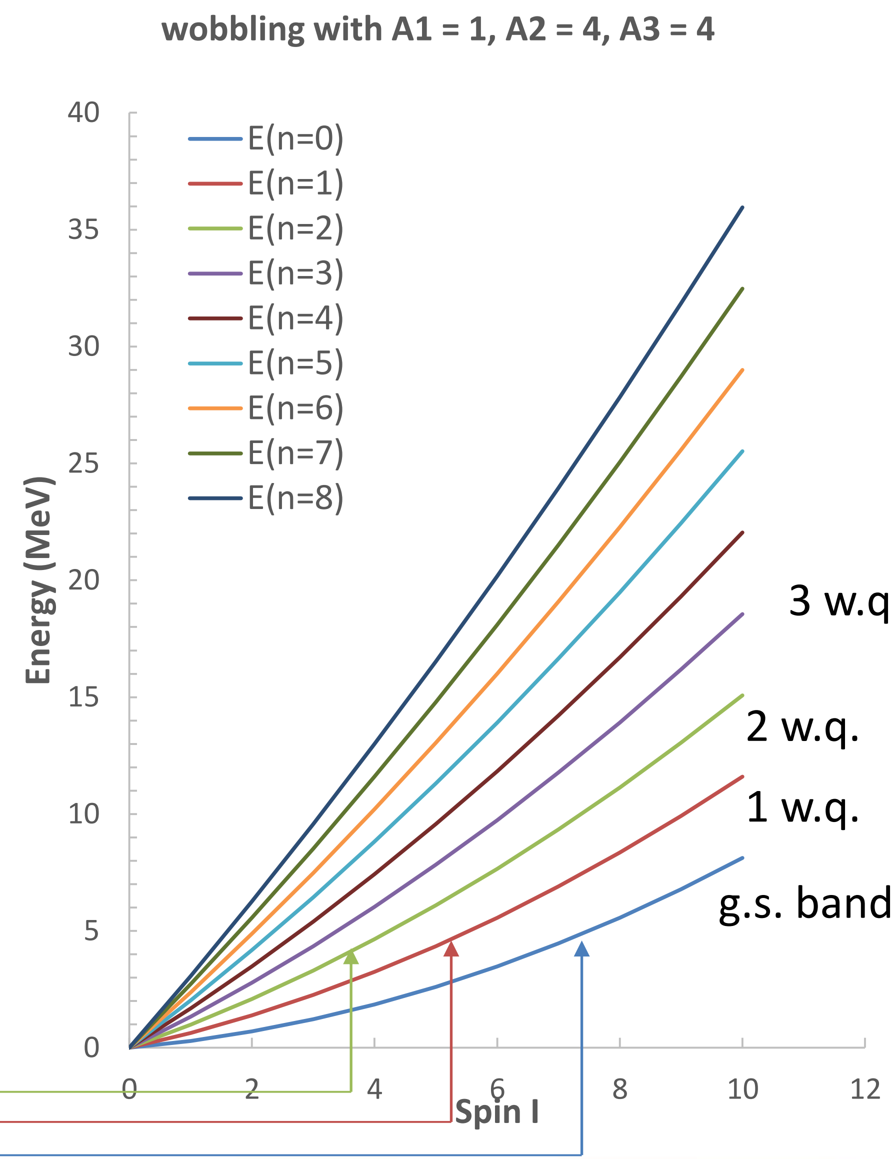
Wobbling around the axis with largest MOI in triaxial even-even nuclei

$H = A_1 I_1^2 + A_2 I_2^2 + A_3 I_3^2 \approx$
$\approx A_1 I^2 + \hbar\omega\,(n+1/2)$
Bohr & Mottelson



1 wobbling quantum

2 wobbling quanta

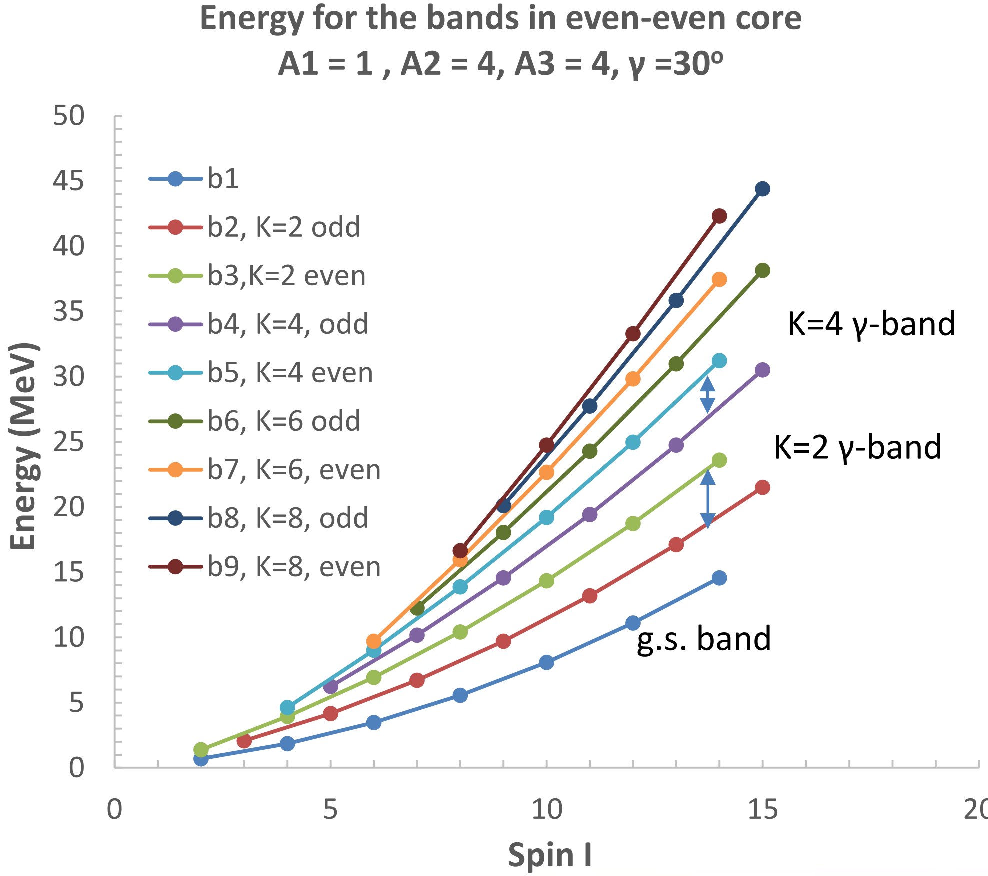


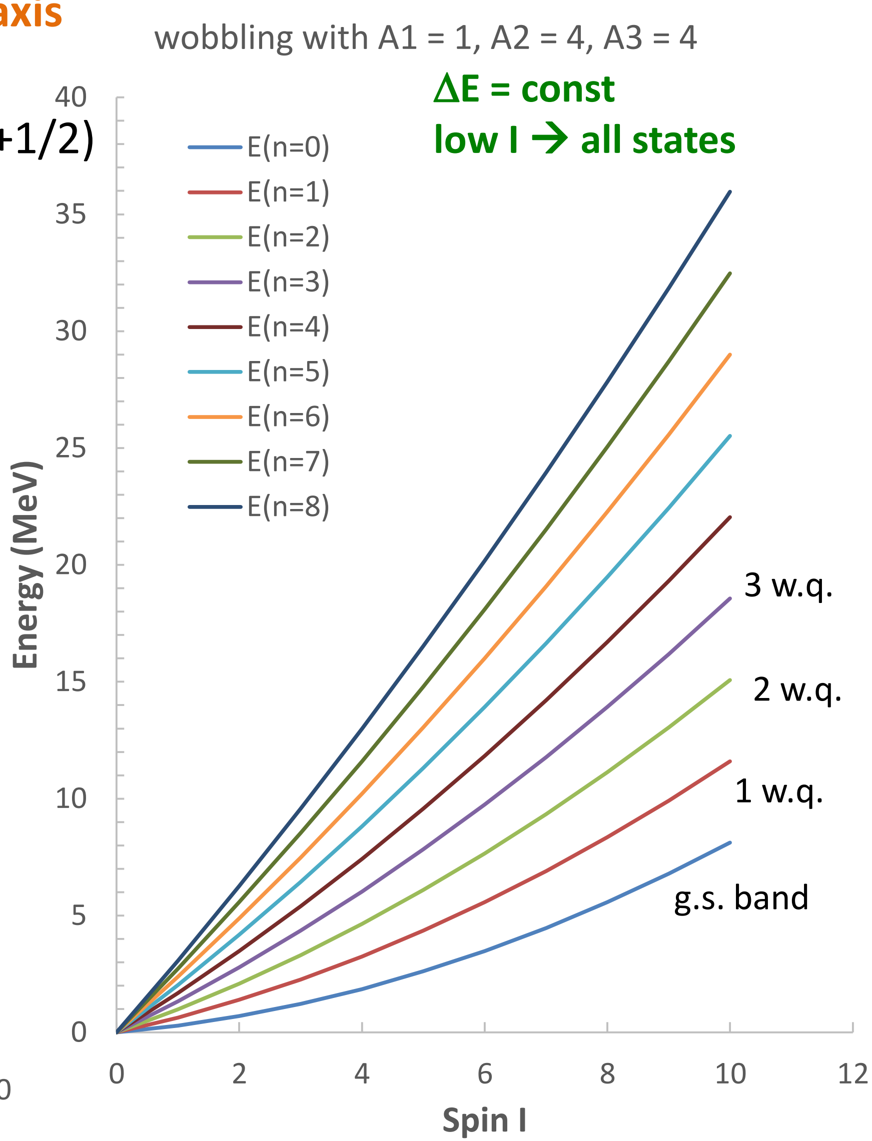
Approximation valid if $I_2^2 + I_3^2 \ll I^2$
good approximation **at high spins only**

Wobbling around the axis with largest MOI – 1-axis

$$H = A_1 I_1^2 + A_2 I_2^2 + A_3 I_3^2 = A_1 I^2 + H' \approx A_1 I^2 + \hbar\omega\,(n+1/2)$$

Bohr & Mottelson





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difference relative excitation energies
different low-spin states

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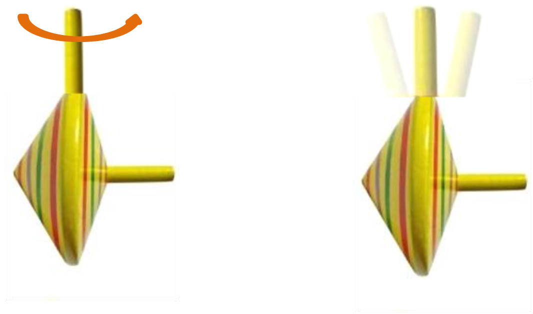
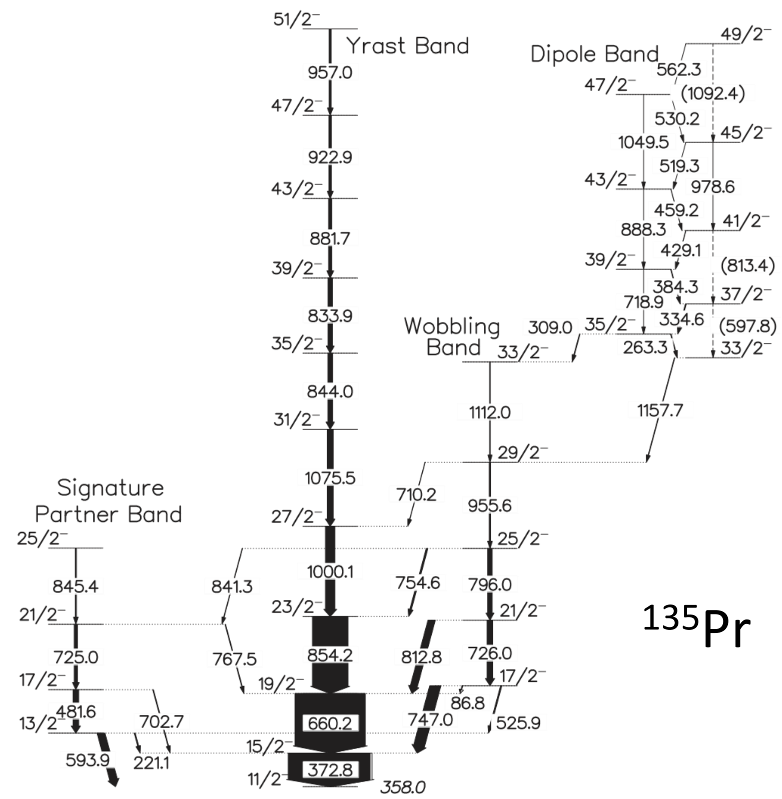
Transverse wobbling - wobbling around an axis with medium MOI

S. Frauendorf and F. Dönau, Phys. Rev. C 89, 014322 (2014).
J. T. Matta et al., Phys. Rev. Lett. 114 (2015) 082501





Where?
→ in odd nuclei
→ one qp with large spin, e.g. $h_{11/2}$
→ triaxial shape

How to identify it?
→ large mixing ratios on the linking transitions
→ decreasing wobbling energy

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Transverse wobbling - wobbling around an axis with medium MOI, 3-axis, $A_1 < A_3 < A_2$

frozen particle angular momentum along the 3-axis

S. Frauendorf and F. Dönau, Phys. Rev. C 89, 014322 (2014).

$H = A_3 (I_3 - j)^2 + A_1 I_1^2 + A_2 I_2^2 \approx A_3 (I - j)^2 + \alpha (n+1/2) + 1/2\beta(c^+c^+ + cc)$, $A_3' = A_3(1-j/I)$

$E(n,I) = A_3 (I-j)^2 + (n+1/2)\hbar\omega$

$\hbar\omega = (\alpha^2 - \beta^2)^{1/2} = 2I[(A_1-A_3')(A_2-A_3')]^{1/2}$ → decreasing with I

$$B(E2, n, I \rightarrow n, I \pm 2) = \frac{5}{16\pi} e^2 \frac{n}{I} Q_2{}^2$$

$$B(E2, n, I \rightarrow n-1, I-1) = \frac{5}{16\pi} e^2 \frac{n}{I} (\sqrt{3} Q_o x - \sqrt{2} Q_2 y)^2$$ → large

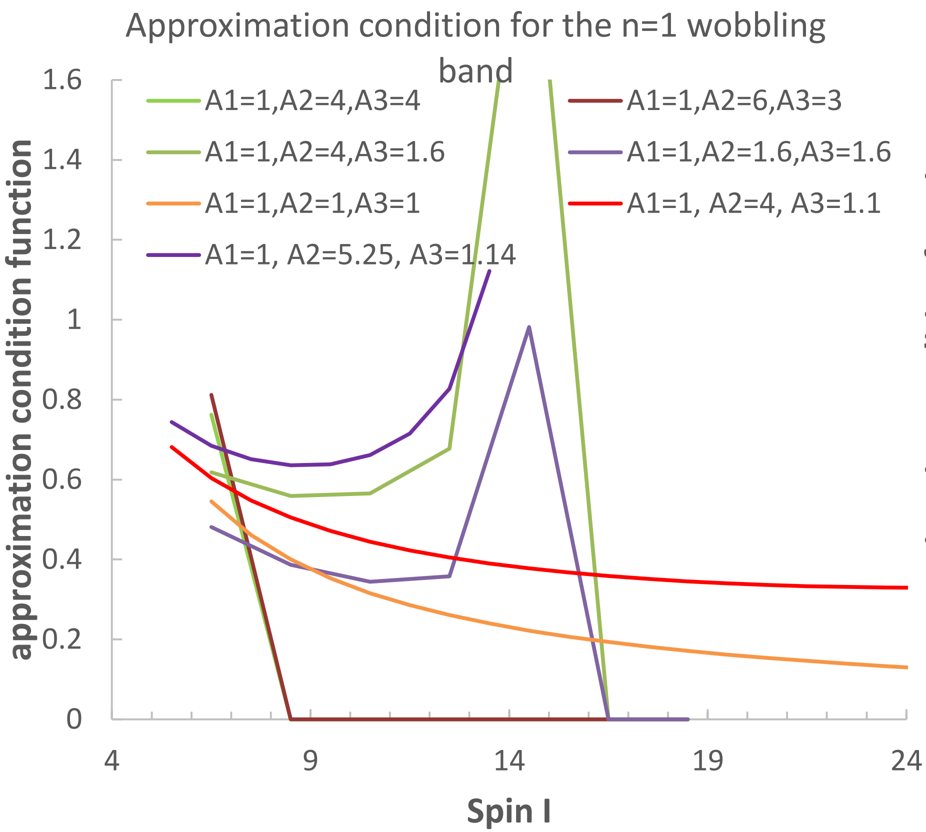
$$B(M1, n, I \rightarrow n-1, I-1) = \frac{3}{4\pi} \frac{n}{I} \left[j(g_j - g_R) x \right]^2;$$

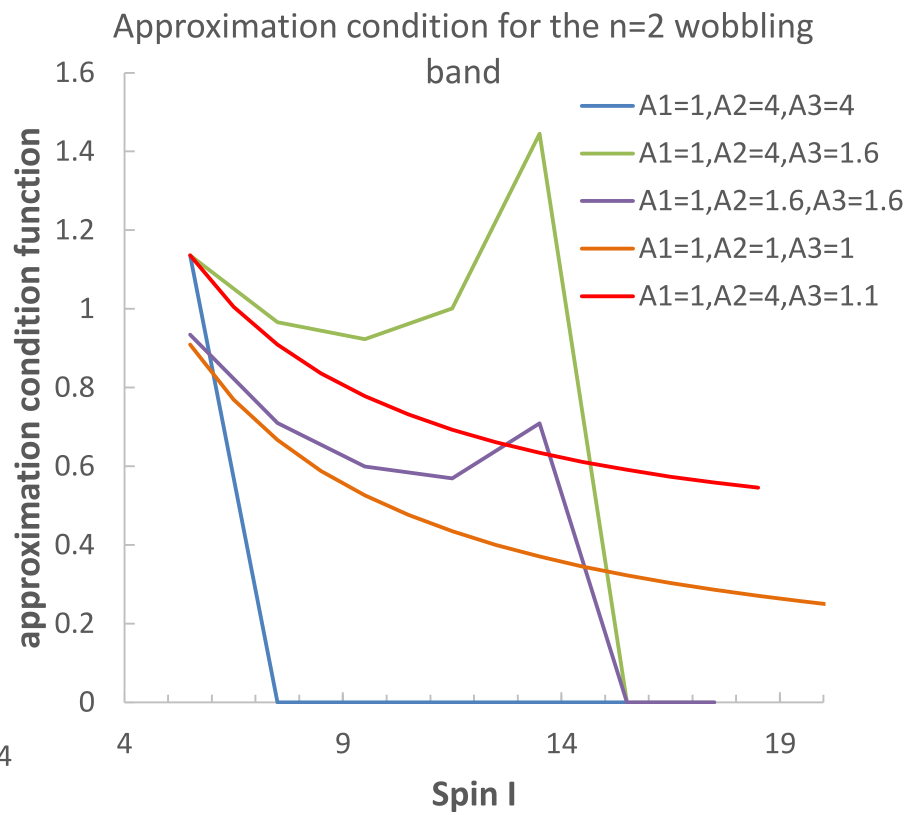
Wobbling approximation is valid if:

1) frozen particle angular momentum
2) $A_1 - A_3' = A_1 - A_3(1 - j/I) > 0$ limit at $I_{max} < j A_3/(A_3-A_1)$
3) $I_1^2 + I_2^2 << I^2$ or $(2n+1) (A_2 + A_1 - 2A_3') /[(A_1-A_3')^{1/2}(A_2 - A_3')^{1/2}] << I$
   or $f = (2n+1) (A_2 + A_1 - 2A_3') /[(A_1-A_3')^{1/2}(A_2-A_3')^{1/2}] /I << 1$,

Approximation condition for the harmonic wobbling description in PRM

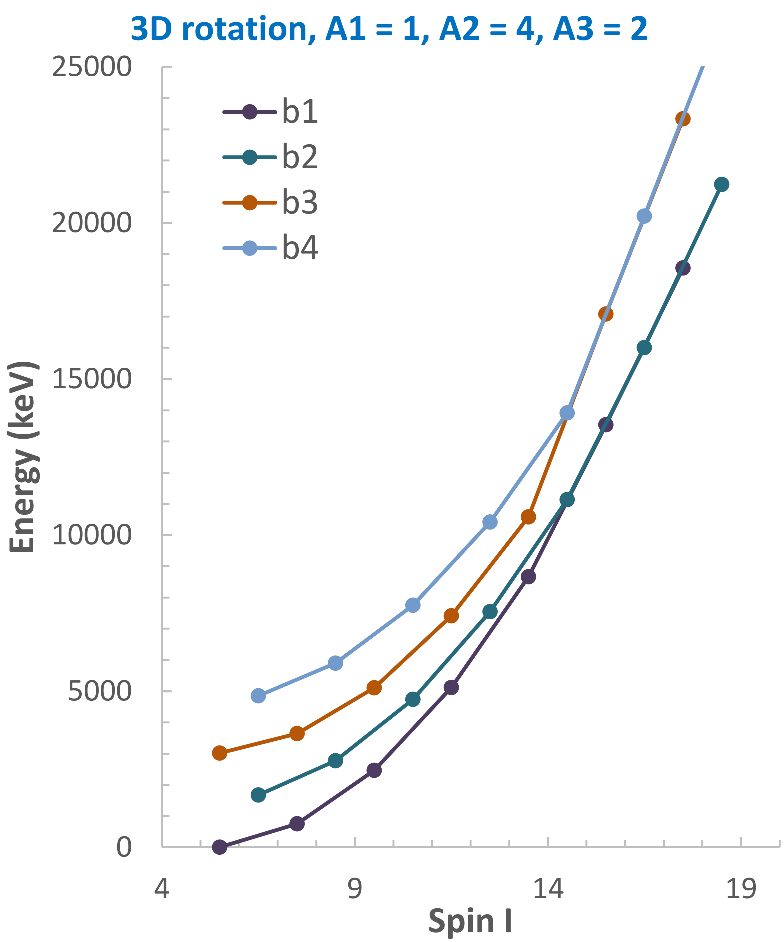
$$f = (2n+1)\,(A_2 + A_1 - 2A_3')\,/[2(A_1 - A_3')^{1/2}(A_2 - A_3')^{1/2}]\,/I << 1$$

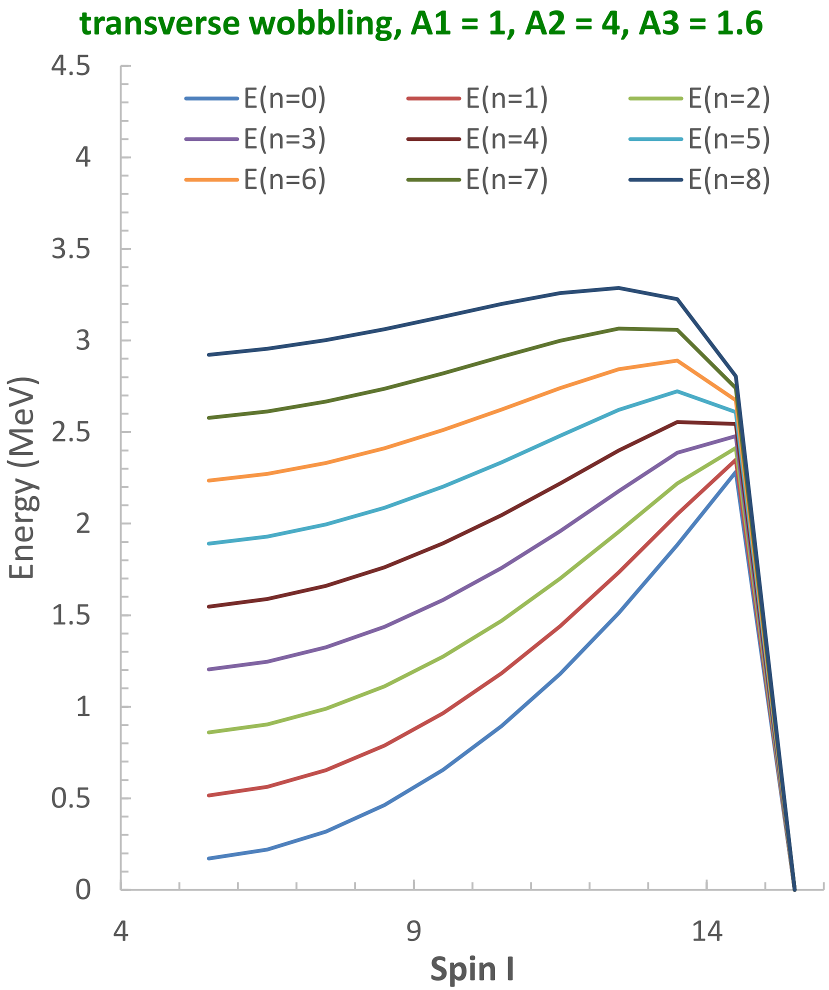




The approximation of 3D rotation as transverse wobbling is a **bad approximation**!

3D rotation and Transverse wobbling
are they the same?





pairs of bands vs regular pattern

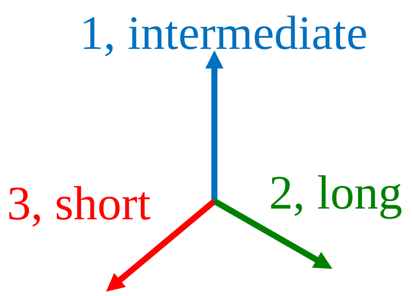
ΔE is increasing vs const

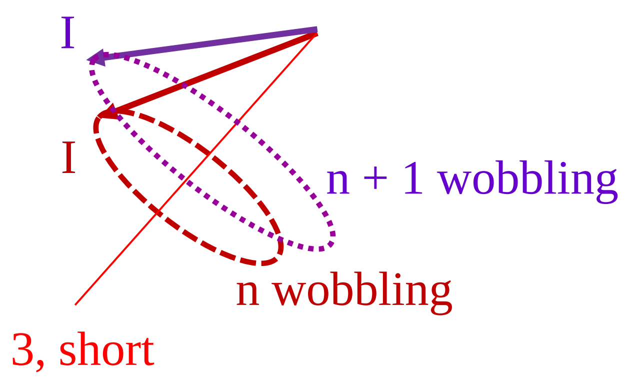
Physics : Transverse wobbling

j is frozen around the short nuclear axis

$E(n,I) = A_3\,(I - j)^2 + (n+1/2)\hbar\omega$

rotation around the short nuclear axis coupled to the excitation of n wobbling quanta





each wobbling quantum has the same energy and angular momentum

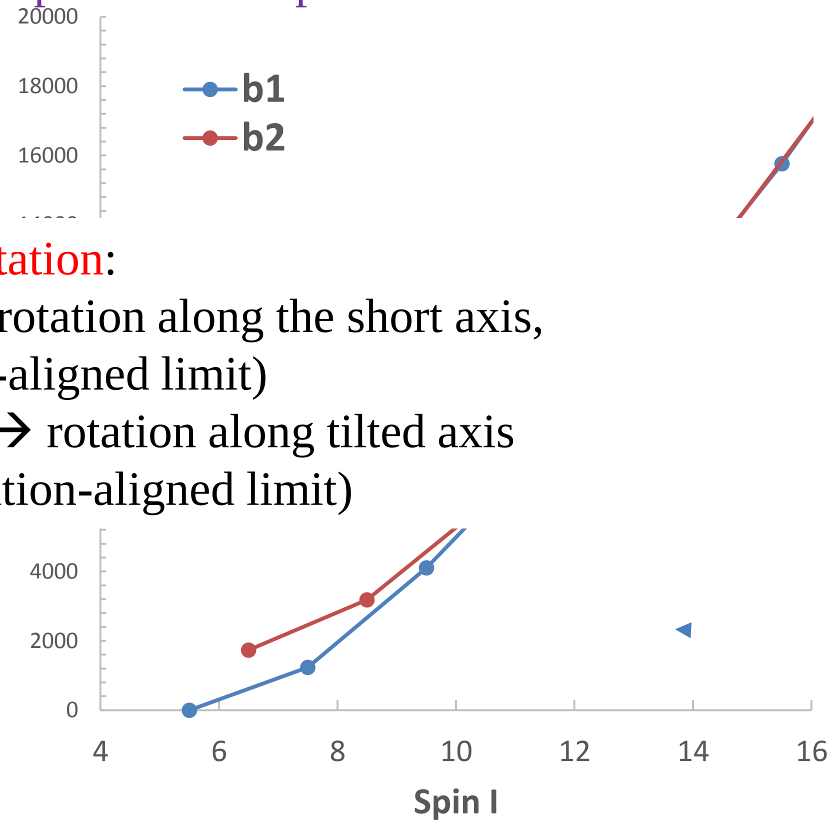
Physics : 3D rotation

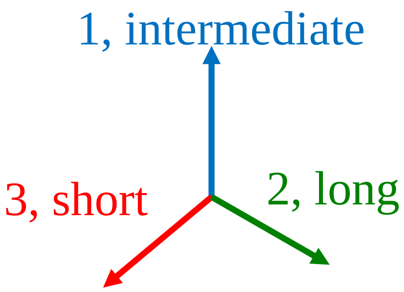
j is frozen on the short axis

Total angular momentum: intersection of sphere and ellipsoid

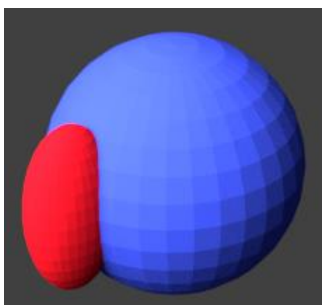
$$I^2 = I_1^2 + I_2^2 + I_3^2$$

$$E = A_3 (I_3 - j)^2 + A_1 I_1^2 + A_2 I_2^2$$



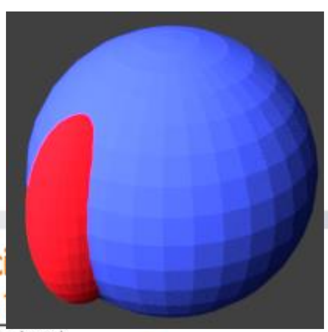


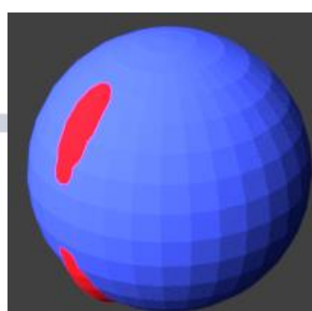
Revolving rotation:
low spins → rotation along the short axis,
(like rotation-aligned limit)
higher spins → rotation along tilted axis
(like deformation-aligned limit)

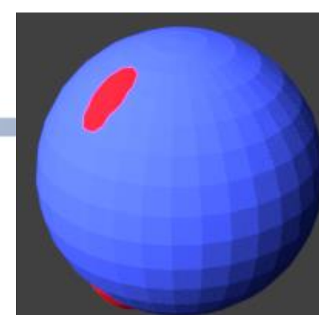


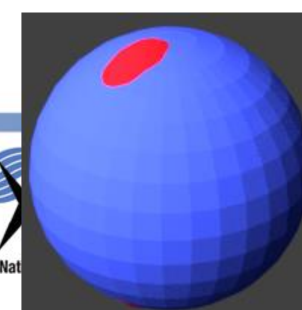
low spins
along short axis

high spins
plane of short – intermediate axes



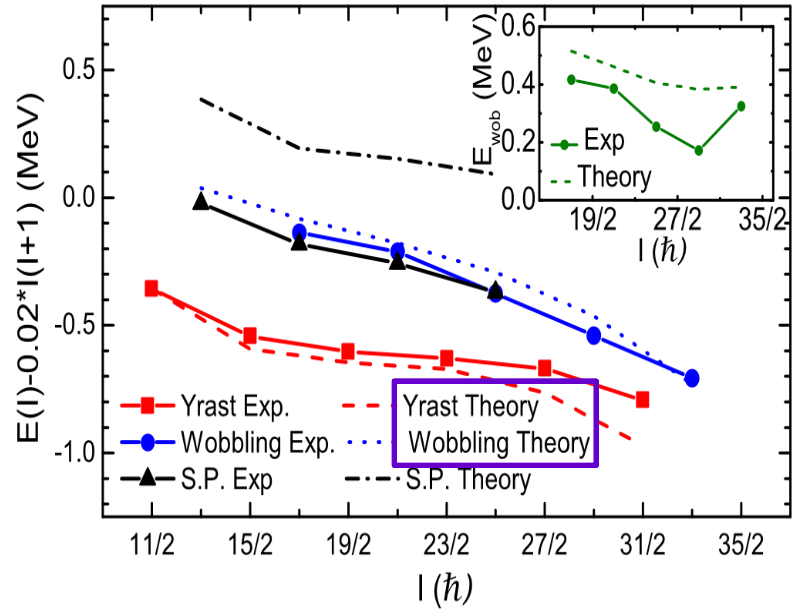






Warning!

If wobbling is considered
→ use the wobbling formulae for the energy and the transition probabilities!

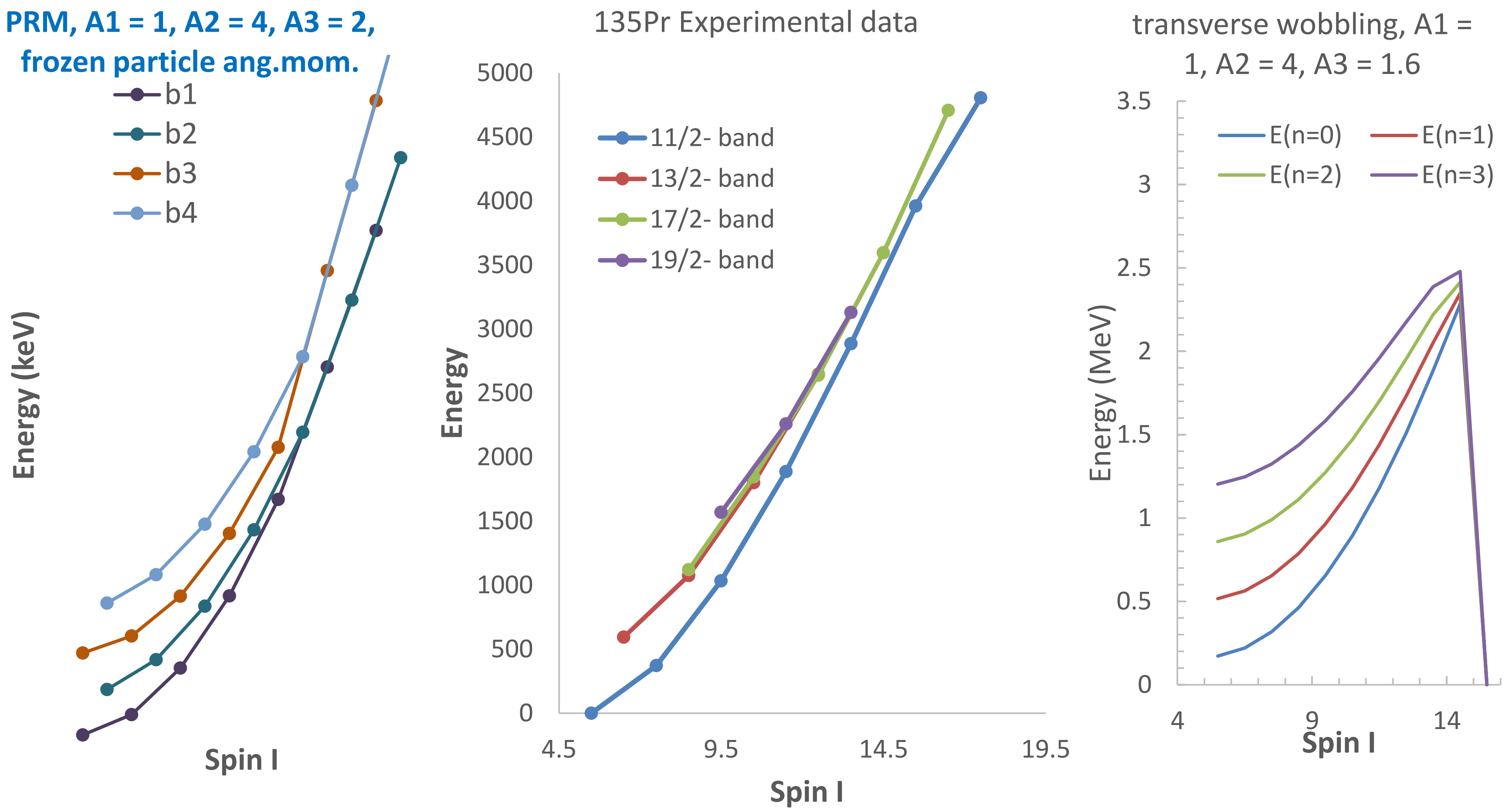


Initial I^π	Final I^π	E_γ (keV)	δ	Asymmetry	E2 Fraction (%)	$\frac{B(M1_{out})}{B(E2_{in})} \left(\frac{\mu_N^2}{e^2b^2}\right)$ Experiment	QTR	$\frac{B(E2_{out})}{B(E2_{in})}$ Experiment	QTR
$\frac{17}{2}^-$	$\frac{15}{2}^-$	747.0	-1.24 ± 0.13	0.047 ± 0.012	60.6 ± 5.1	...	0.213	...	0.908
$\frac{21}{2}^-$	$\frac{19}{2}^-$	812.8	-1.54 ± 0.09	0.054 ± 0.034	$70.3 \pm 2.$	$.164 \pm 0.014$	0.107	0.843 ± 0.032	0.488
$\frac{25}{2}^-$	$\frac{23}{2}^-$	754.6	-2.38 ± 0.37	...	85.0 ± 4.0	0.035 ± 0.009	0.070	0.500 ± 0.025	0.290
$\frac{29}{2}^-$	$\frac{27}{2}^-$	710.2	...	...	...	$\leq 0.016 \pm 0.004$	0.056	$\geq 0.261 \pm 0.014$	0.191
$\frac{13}{2}^-$	$\frac{11}{2}^-$	593.9	-0.16 ± 0.04	-0.092 ± 0.023	2.5 ± 1.2	...	...	...	...

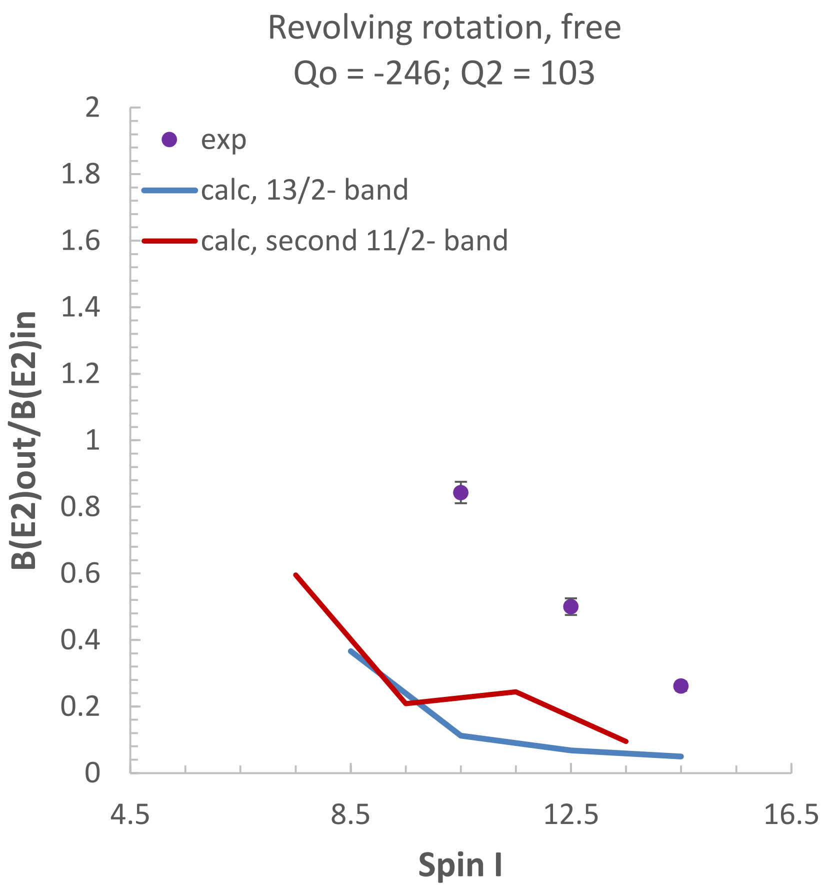
J. T. Matta et al., Phys. Rev. Lett. 114 (2015) 082501

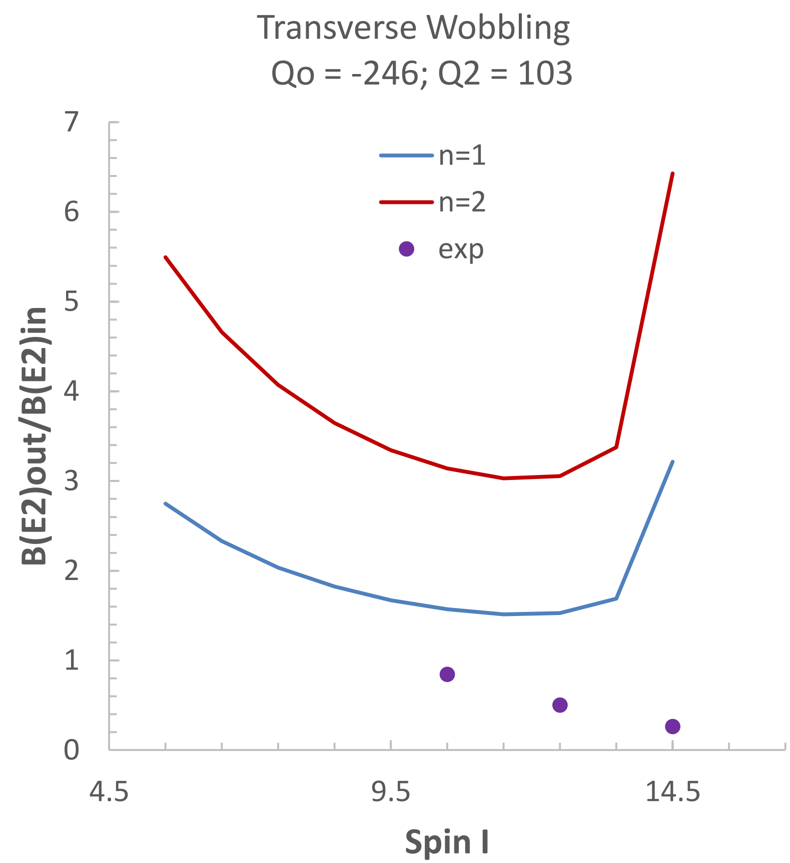
Good agreement with QTR supports interpretation as revolving rotation

Revolving rotation or Transverse wobbling?
excitation energy



Revolving rotation or Transverse wobbling?
B(E2)out/B(E2)in





$$B(E2, n, I \to n, I \pm 2) = \frac{5}{16\pi} e^2 \frac{n}{I} Q_2{}^2$$

$$B(E2, n, I \to n-1, I-1) = \frac{5}{16\pi} e^2 \frac{n}{I} (\sqrt{3}Q_o x - \sqrt{2}Q_2 y)^2$$

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Transverse wobbling:
→ B(E2)out/B(E2)in has a specific trend
→ factor of 2 increase for n=2 band

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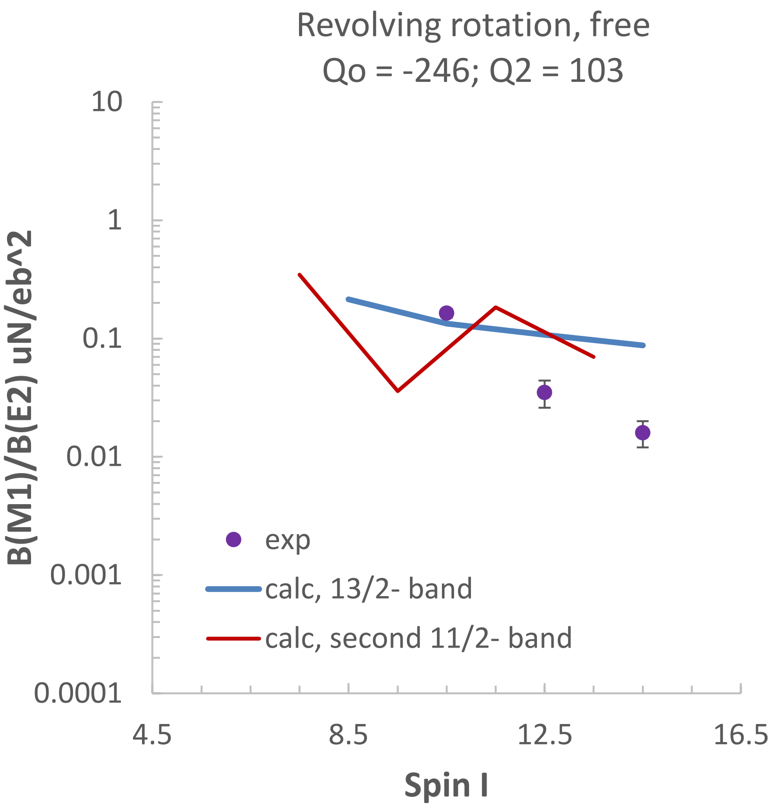
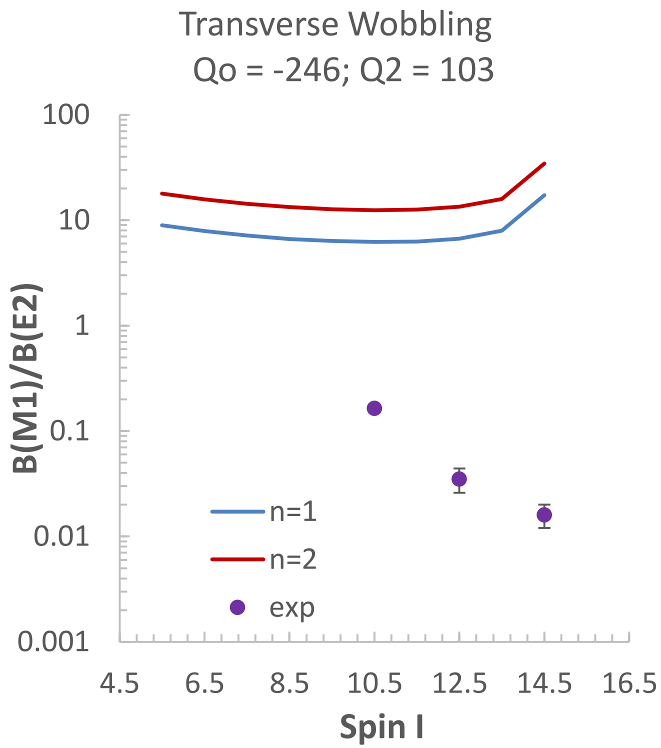
Revolving rotation or Transverse wobbling?
B(M1)out/B(E2)in





$$B(M1, n, I \rightarrow n-1, I-1) = \frac{3}{4\pi}\frac{n}{I}\left[j(g_j - g_R)x\right]^2;$$

$$B(E2, n, I \rightarrow n, I \pm 2) = \frac{5}{16\pi} e^2 \frac{n}{I}\, {Q_2}^2$$

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Transverse wobbling:
→ B(M1)out/B(E2)in has a specific trend
→ factor of 2 increase for n=2 band

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Summary

The wobbling approximation is a **bad approximation** of the 3D rotational Hamiltonian, i.e. it neglects terms that are not negligible.

Transverse wobbling and Revolving rotation → different physics

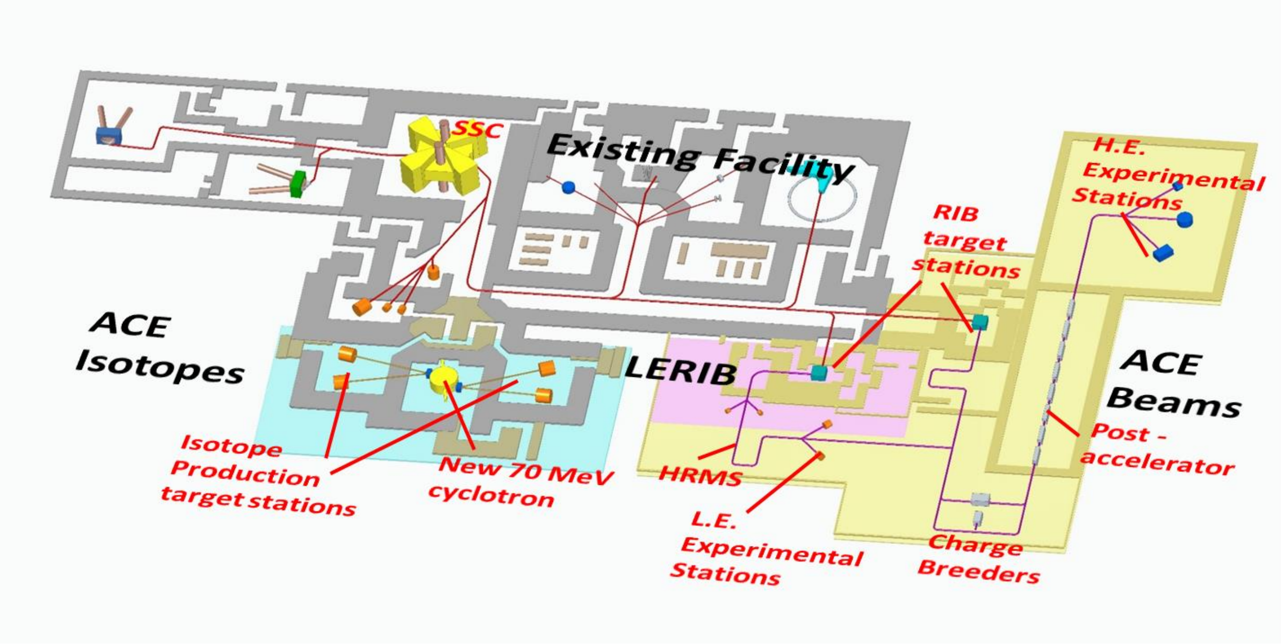
Revolving rotation → rotational axis moves from the shot to the intermediate axis, looks a bit like a transition from rotation-aligned to a deformation-aligned coupling

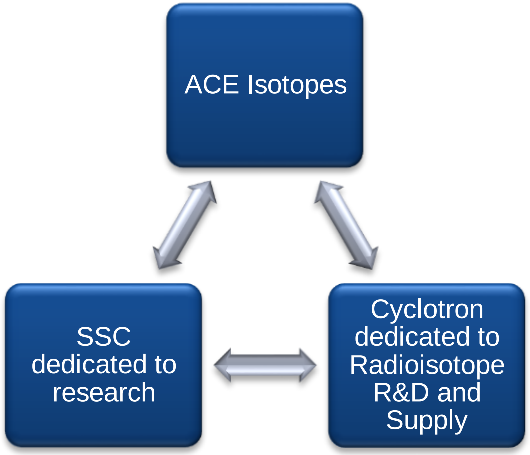
To test transverse wobbling or revolving rotation

- the trends of the reduced transition probabilities – gradual decrease with a steep increase near the top for transverse wobbling
- absolute values of the B(M1)out, B(E2)out, etc.
- distinct differences in the energies, B(E2)s, B(M1) for n=2, excited bands

experimental data of ^{135}Pr
does not match the expected trends of the transition probabilities for transverse wobbling

South African Isotope Facility (SAIF)





<u>Phase I: ACE Isotopes and LeRIB</u>

- 70MeV cyclotron: dedicated to the production of isotopes.
- SSC: dedicated to beams for research (stable and LeRIB).
- Timeline - 4 years to operations

<u>Phase 2: ACE Beams</u>

- SSC: dedicated to beams for research (stable and radioactive).
- Post-accelerated radioactive beams.
- Timeline - 8 years to operations