Configuration assignments for the extensive level scheme of ^{167}Lu.

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Outline

- The Cranked Nilsson-Strutinsky (CNS) and Cranked Nilsson-Strutinsky Bogoliubov (CNSB) formalisms.
- Typical features of high-spin bands, $E - E_{rld}$ diagrams.
- The extensive level scheme ^{167}Lu.
- Interpretation of the observed bands in ^{167}Lu - evolution with spin, band crossings observed.
- Conclusions.

Cranked Nilsson-Strutinsky (CNS) -Bogoliubov (CNSB) models.

Modified oscillator or Nilsson Hamiltonian:

$$H_{MO} = -\frac{\hbar^2}{2M}\Delta + V_{HO}(\varepsilon_2,\gamma) + V(\varepsilon_4) - \kappa\hbar\,\overset{0}{\omega}_0\left(2\vec{l}_t\cdot\vec{s} + \mu\left(l_t^2 - \langle l_t^2\rangle_N\right)\right).$$

Cranking Hamiltonian: $H^{\omega} = \overbrace{\underbrace{H_{MO} - \omega j_x}_{\text{CNS}} - \Delta(P^\dagger + P) - \lambda\hat{N}}^{\text{CNSB}}$

The cranking Hamiltonian is diagonalized

$\Longrightarrow$ Single (quasi)-particle energies, $e_i(E_i)$, spin projections $(j_x)_i$ etc.

Total quantities (macroscopic-microscopic formalism):

$$E_{tot} = E_{rld} + E_{shell}(+E_{pair}); \quad E_{shell} = \sum_{occ} e_\nu - \left\langle \sum_{occ} e_\nu \right\rangle \quad I = \sum_{occ}\langle j_x\rangle$$

Rotating liquid drop energy:

$$E_{\rm rld}(Z,N,I,\varepsilon_i) = E_{\rm ld}(Z,N,\varepsilon_i) + \frac{\hbar^2 I(I+1)}{2\mathcal{J}_{\rm rig.}(Z,N,\varepsilon_i)}$$

$E_{\rm ld}$: Lublin-Strasbourg drop (LSD model) [or FRLDM]

$\mathcal{J}_{\rm rig.}$: diffuse surface: $r_0 = 1.16$ fm, $a = 0.6$ fm

T. Bengtsson and I.R., NPA 436, 14 (1986), B.G. Carlsson and I.R., PRC 74, 011302 (2006)

B.G. Carlsson *et al.* PRC 78, 034316 (2008), Hai-Liang Ma, B.G. Carlsson, I.R. and H. Ryde, PRC 90, 014316 (2014)

Cranked Nilsson-Strutinsky (CNS) -Bogoliubov (CNSB) models.

Removal of virtual crossings - diabatic configurations.

CNS: Mesh in deformation space: $(\varepsilon_2, \gamma, \varepsilon_4)$.

- Unique features to label orbitals (N_{osc}-shell, high-j - low-j, (pseudo-spin partners)) and thus to fix conf's.
- Minimization in $(\varepsilon_2, \gamma, \varepsilon_4)$-space (each configuration at each spin value).

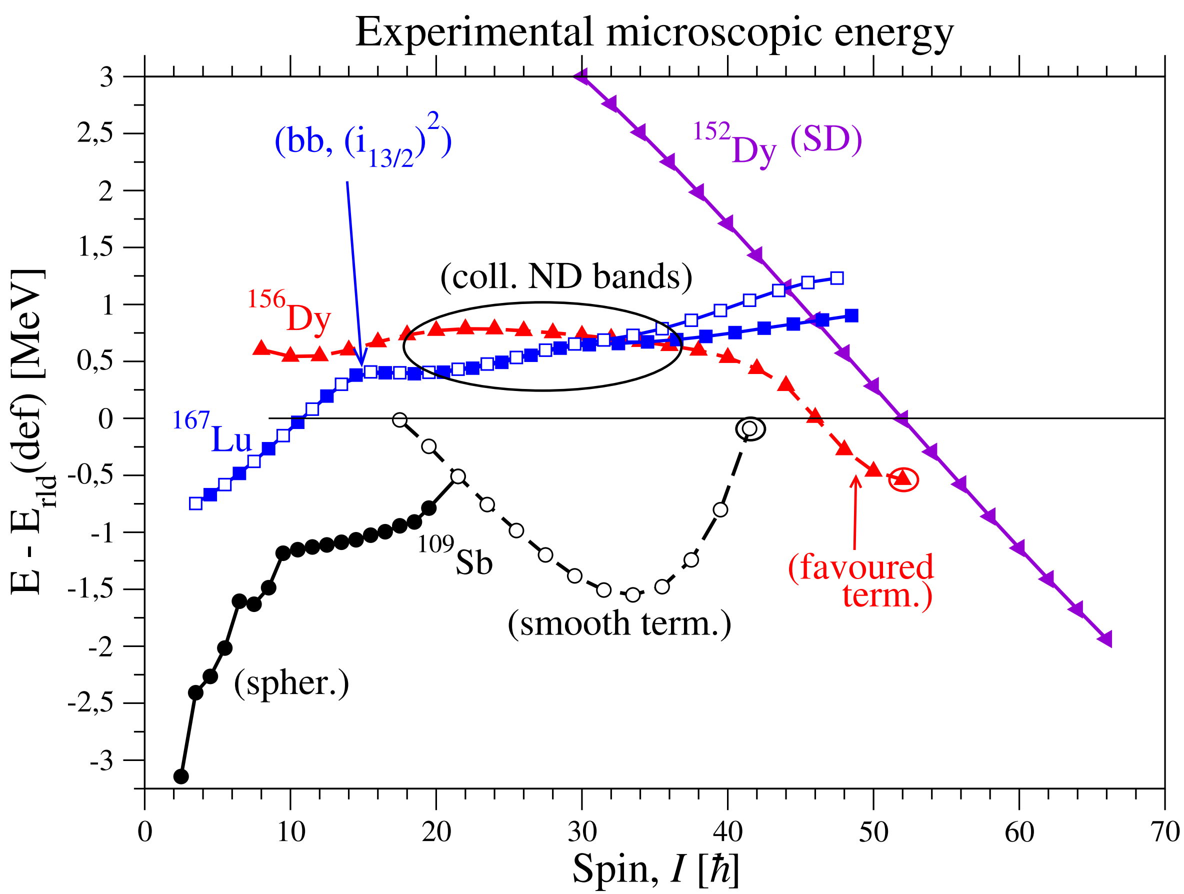
CNSB: Mesh in def. and pairing space: $(\varepsilon_2, \gamma, \varepsilon_4, \lambda_p, \lambda_n, \Delta_p, \Delta_n)$

- Minimization in $(\varepsilon_2, \gamma, \varepsilon_4, \lambda_p, \lambda_n, \Delta_p, \Delta_n)$ space
- Particle number projection.
- Removal of virtual interactions for fixed parameters

$\Rightarrow$ Possible to draw smooth PES's in the full (ε_2, γ)-plane.

Conf's: $(\pi, \alpha)_p (\pi, \alpha)_n$

Energy relative to rotating liquid drop energy.



Best way to get an understanding of nuclear level schemes, $E - E_{rld}$ plots:

- Excitation energy.
- Moments of inertia $\mathcal{J}^{(1)}$ (slope) and $\mathcal{J}^{(2)}$ (curvature).
- Band crossings.
- Signature splitting.

Comparison CNS - CNSB, ^{167}Lu .

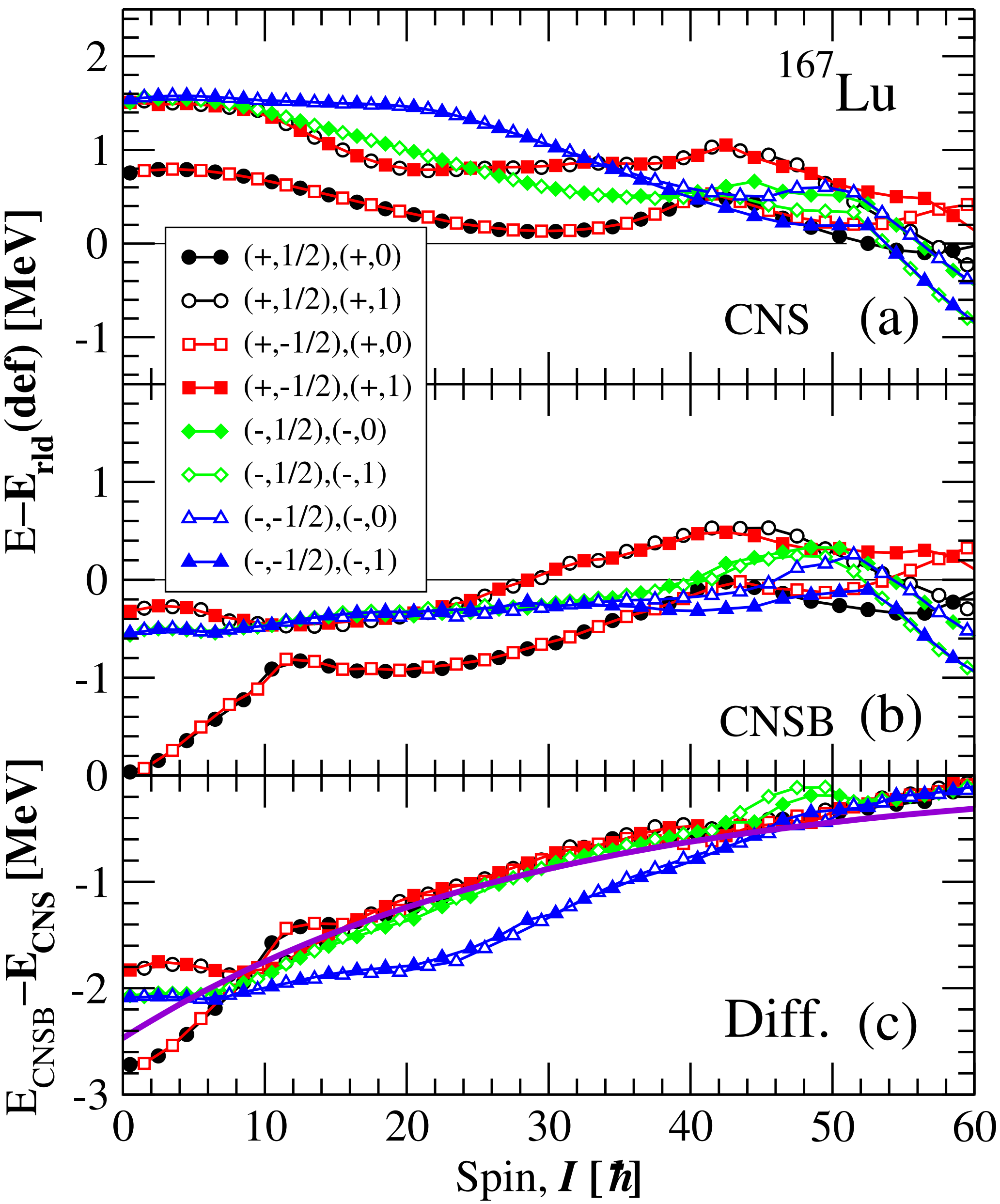
No pairing and pairing included.

Difference shows a rather smooth trend approaching zero at high spin.
$\Rightarrow$ Experiment down to rather low spin is well described by CNS if average pairing is subtracted (moment of inertia renormalized):

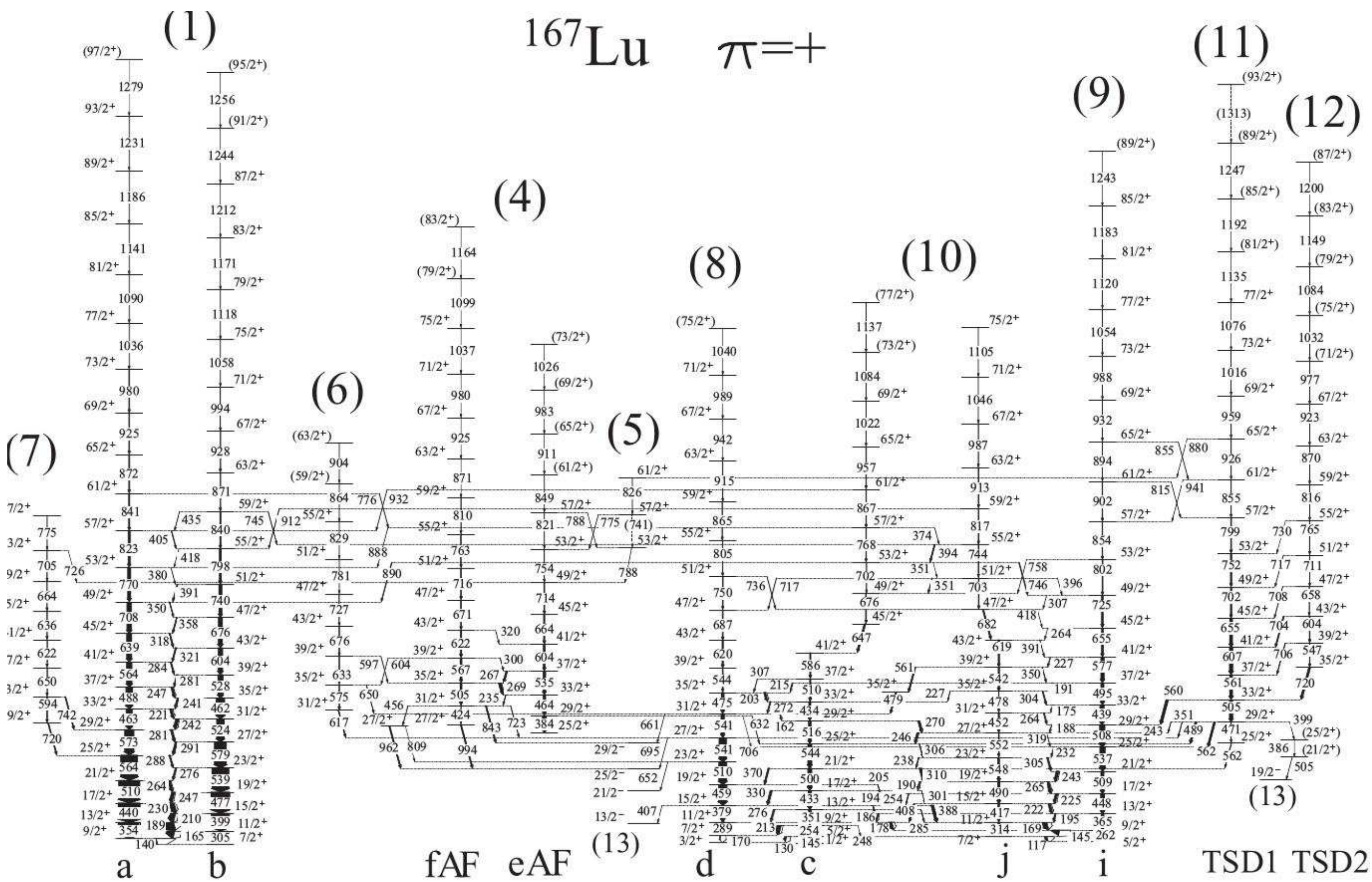
$$\langle E_{pair} \rangle = -2.47 \cdot \exp\left(\frac{I}{29}\right) \ \text{MeV}$$

Somewhat stronger pairing for $(\pi, \alpha)_p = (-, -1/2)$ conf's

No paired crossings at high spin



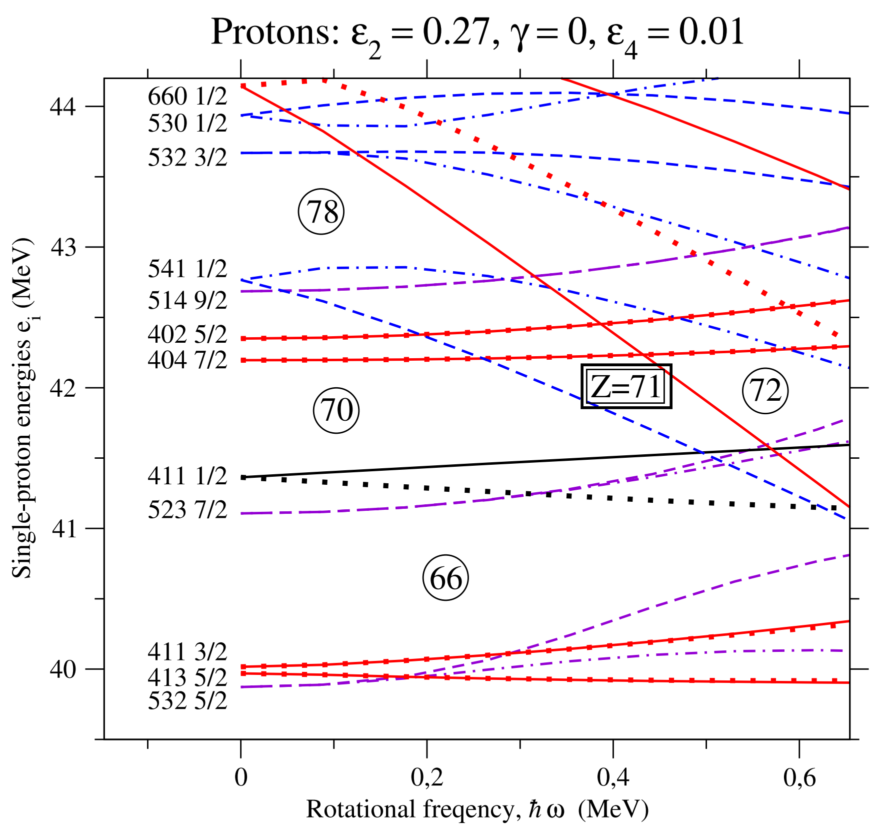
Observed positive parity bands, ^{167}Lu.

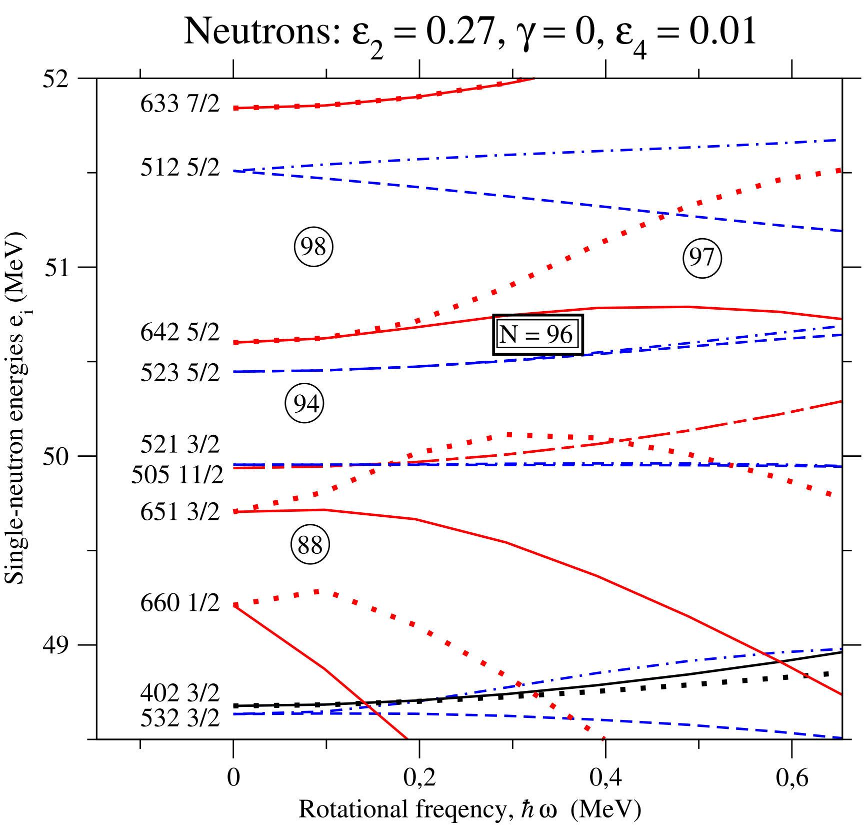


Assignments mainly based on paired band crossings and number of aligned particles (number of quasiparticles).

D.G. Roux, W.C. Ma *et al.*, Phys. Rev. C 92, 064313 (2015).

Single-particle routhians at ground state.





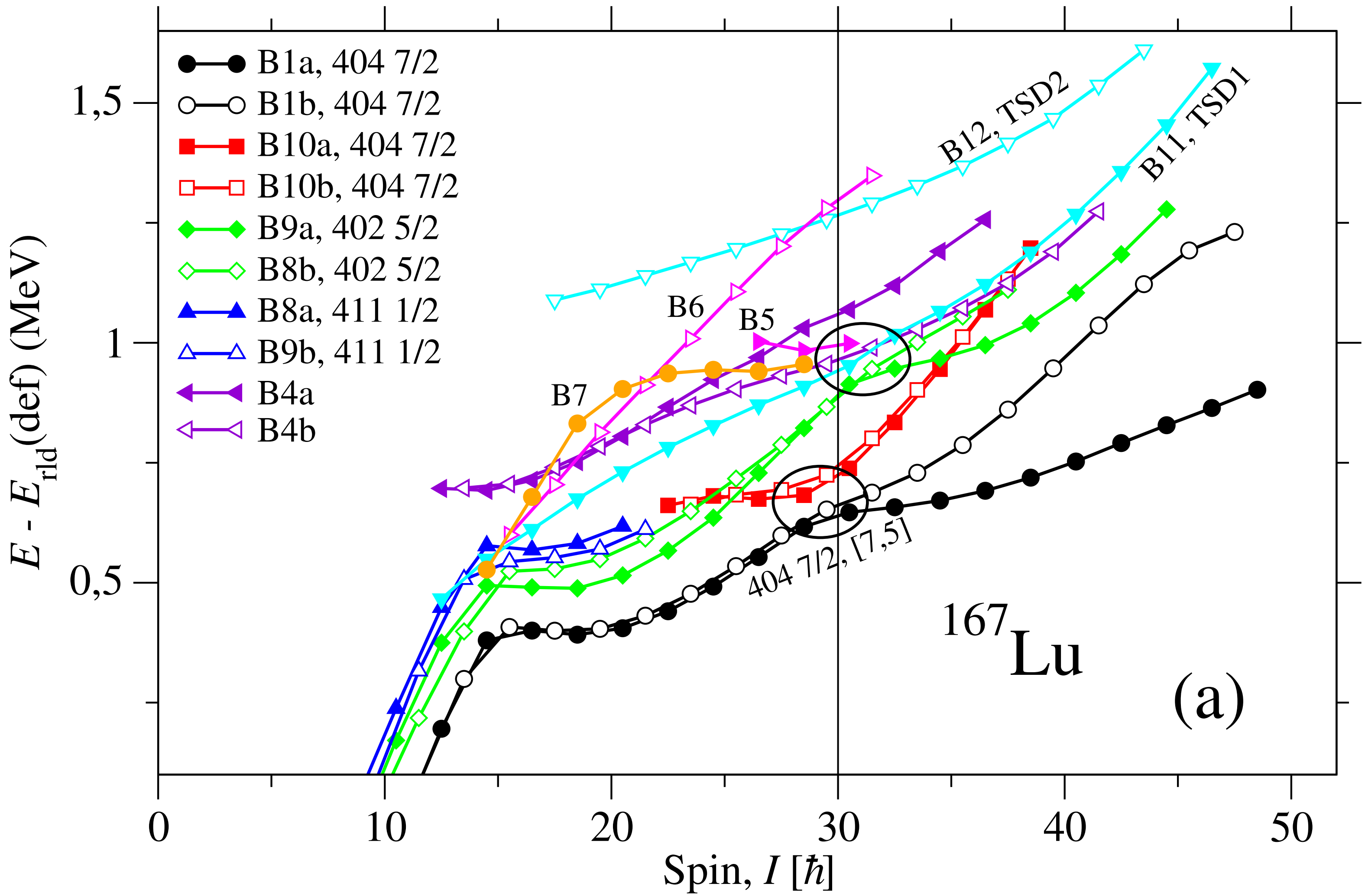
Bands built on:

- 404 7/2, 402 5/2: dg
- 411 1/2: sd
- 523 7/2, 514 9/2: $h_{11/2}$
- 541 1/2: fh

Bands of the type:

- $(fh)^{10}\ (i_{13/2})^4$
- $(fh)^9\ (i_{13/2})^5$

$\pi = +$ bands relative to E_{rld}.

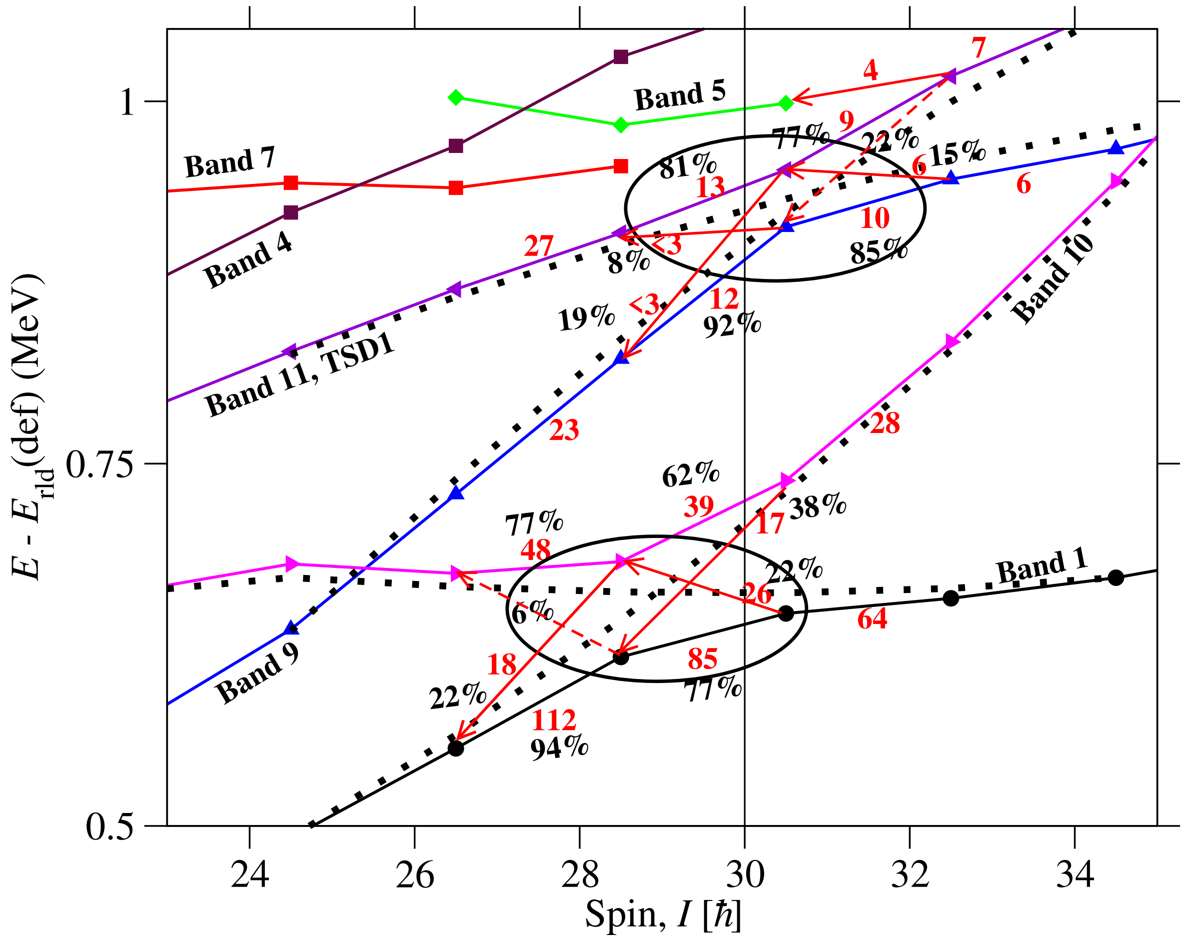


Low-spin bands: 404 7/2, 402 5/2, 411 1/2

Three q-p band: B4

TSD1 and TSD2 (wobbling)

Band crossings, $\pi = +, \alpha = 1/2$ bands.

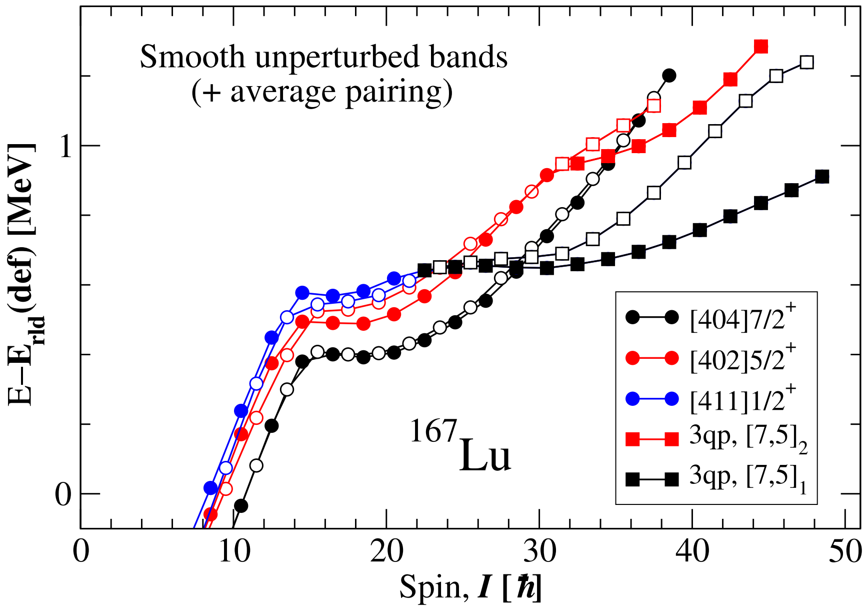


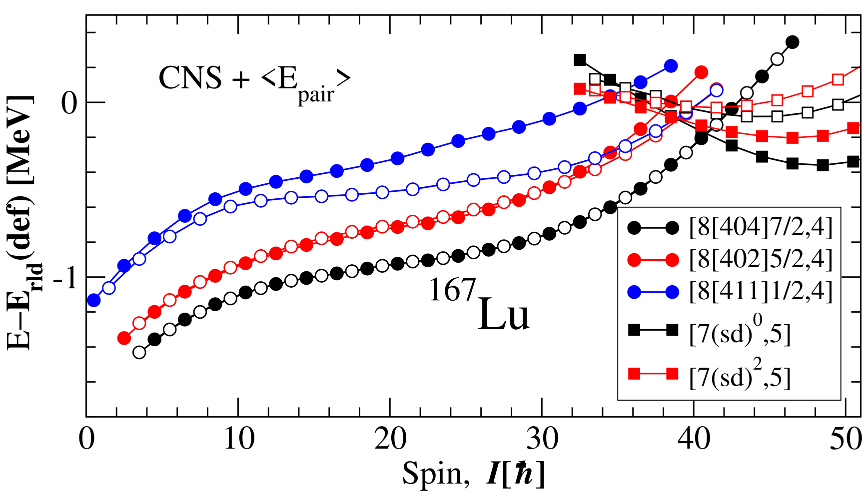
Two-band mixing calculations leading to smooth unperturbed bands:

Clear case for yrast crossing: Bands 1 and 10 (404 7/2 and 3qp band); $V_{int} = 30$ keV; ± 7 keV

Questionable; Band 9 (402 5/2) and band 11 (TSD1); $V_{int} = 24$ keV; ± 10 keV

Proton $N_{osc} = 4$ bands, smooth exp. and calc. CNS.



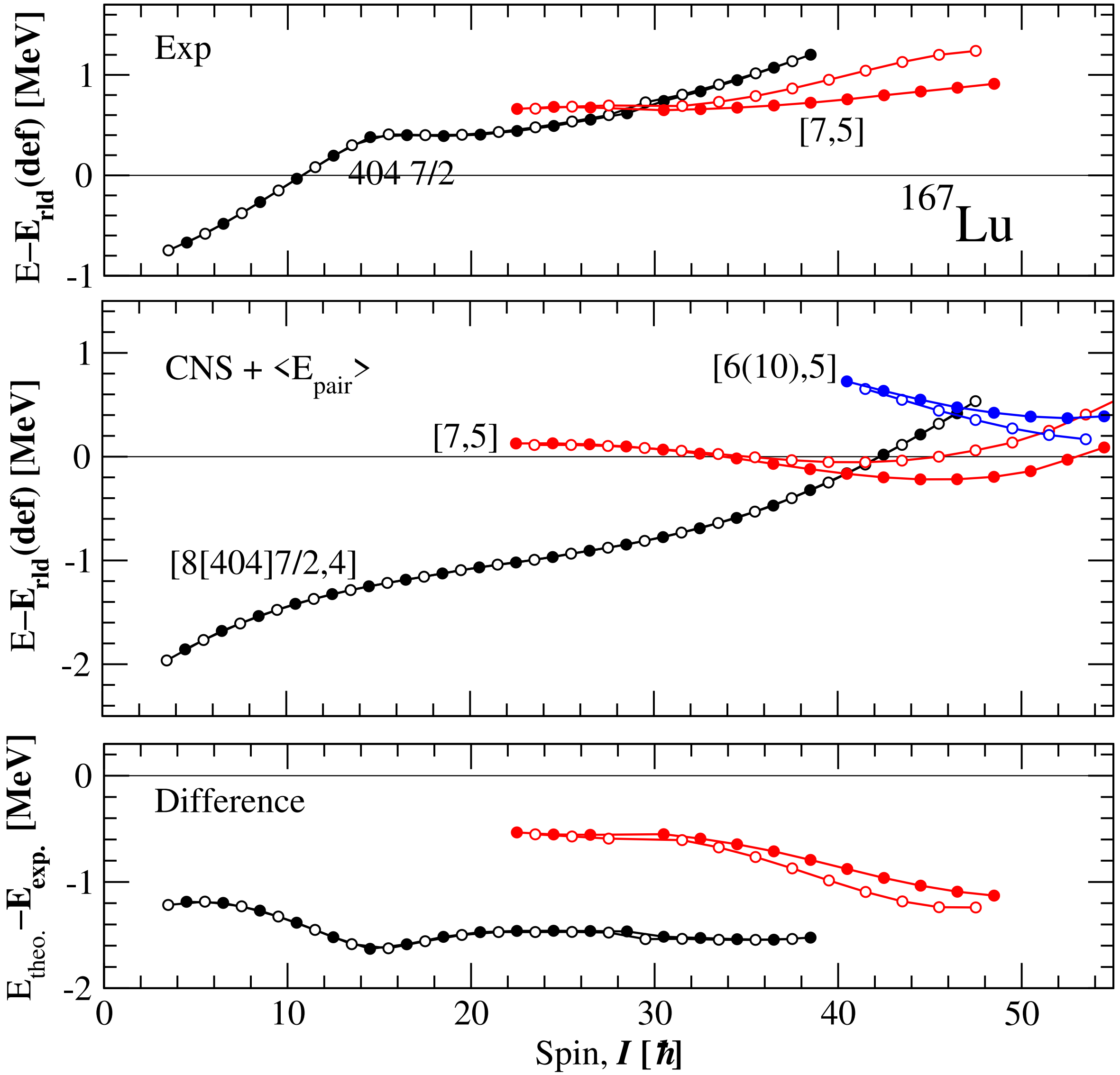


Same general structure in exp. and calc.

Small change in s.p. parameters to lower the 411 1/2 band.

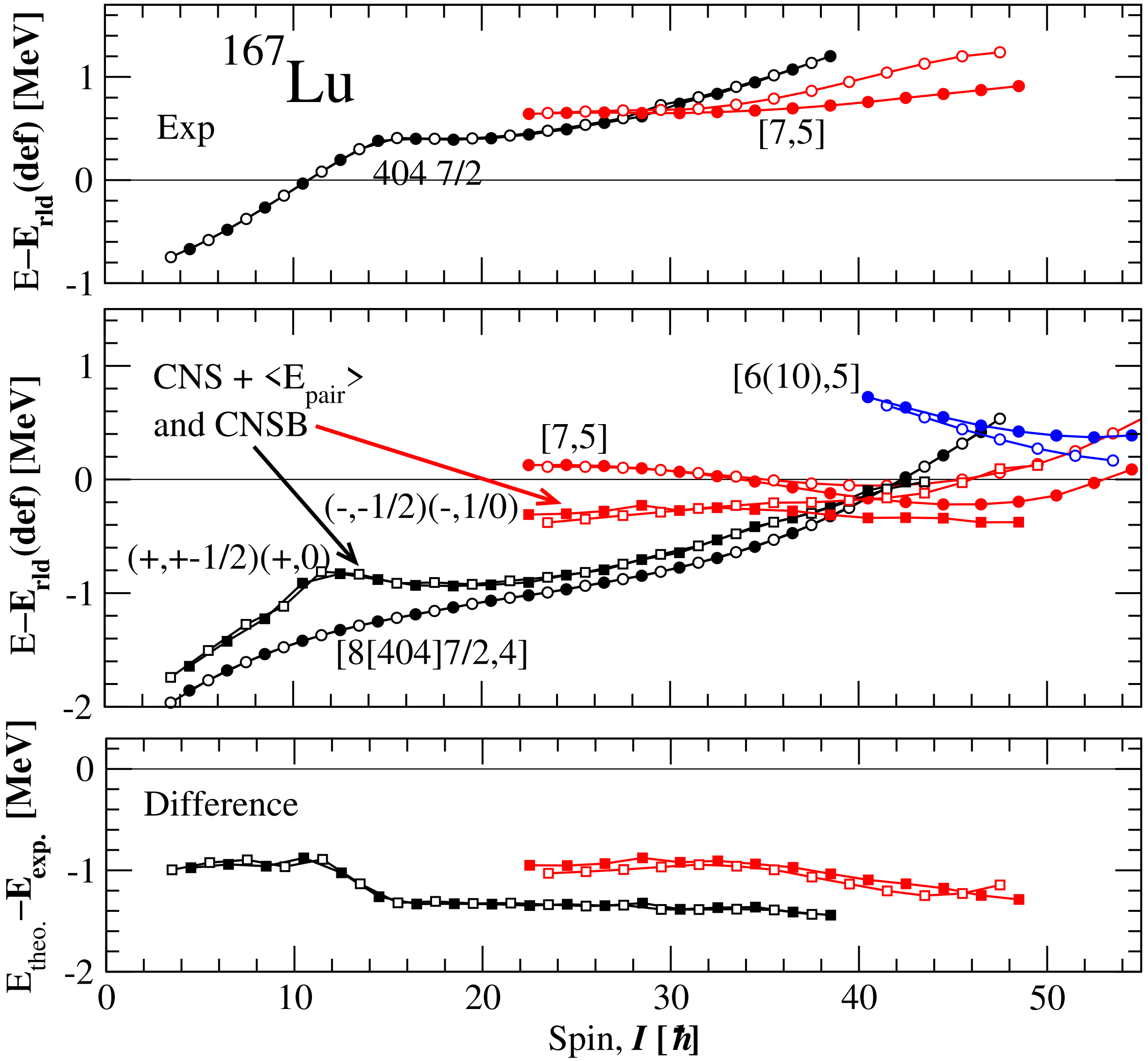
Stronger pairing for 3 qp band, $[7.5]$ $\Longrightarrow$ band crossing at lower spin.

Yrast states, $\pi = +$.



Large scattering of differences, cf. full CNSB calculations.

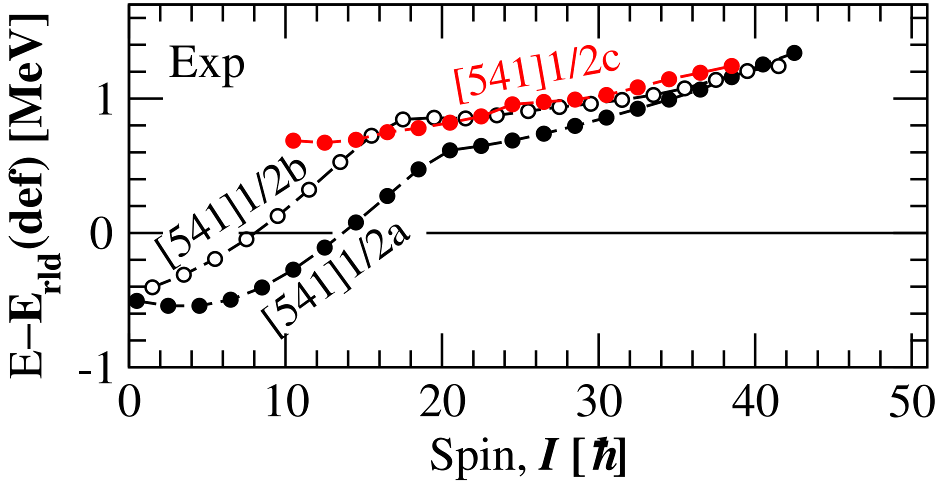
Yrast states, $\pi = +$, CNS vs. CNSB.



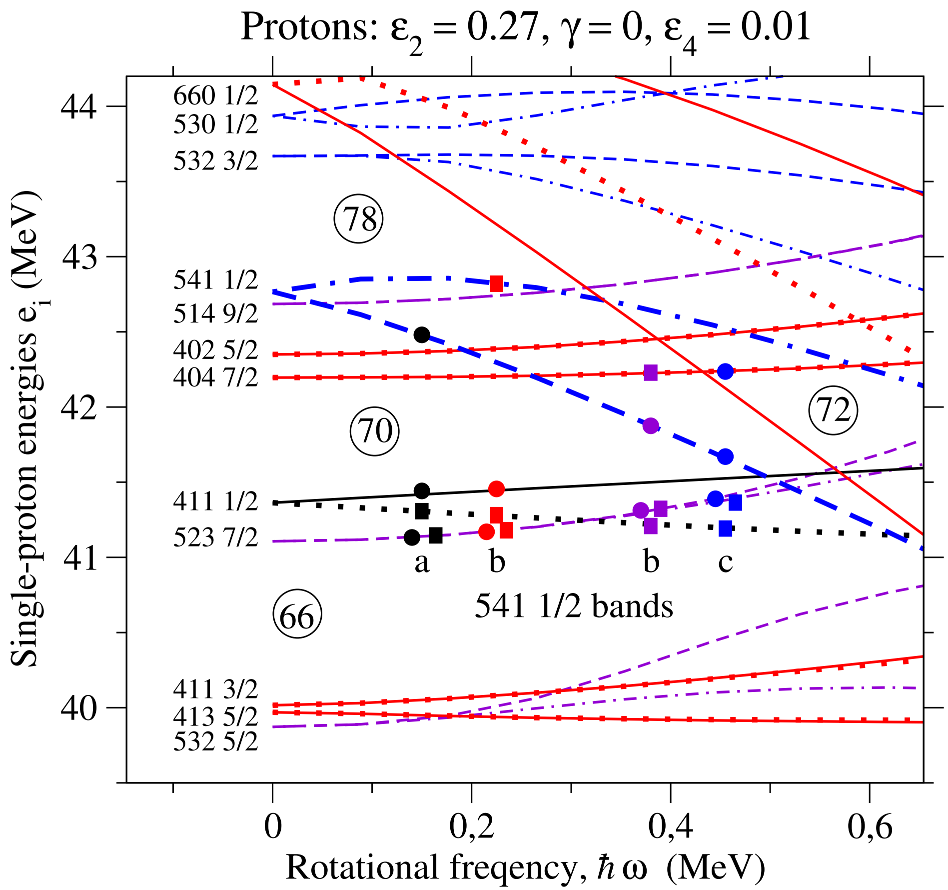
Differences within

± 0.25 MeV!

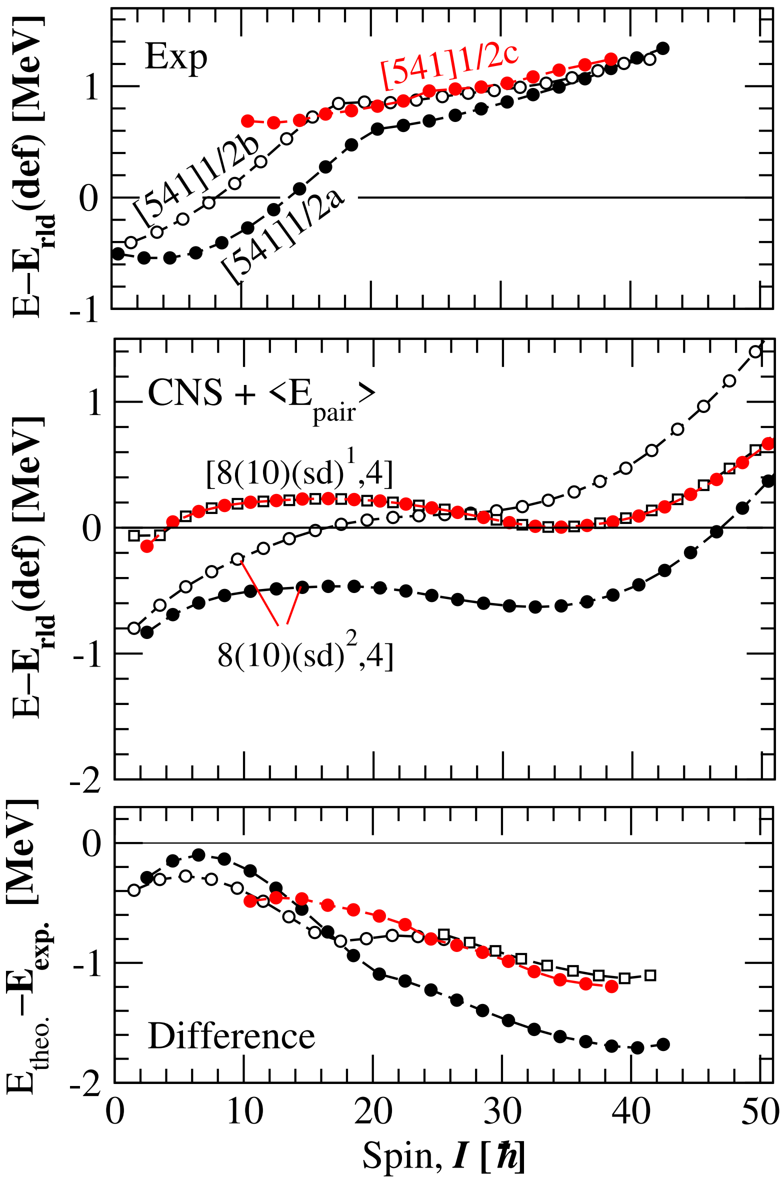
541 1/2 bands (from fh shells).

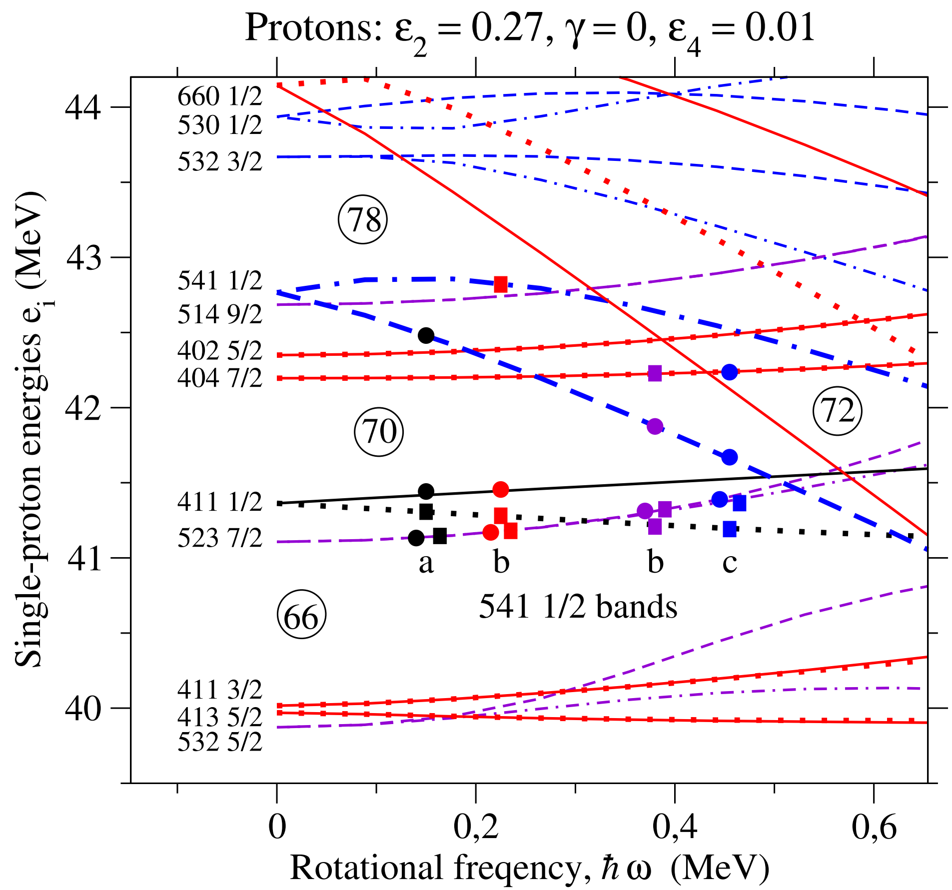


Three bands related 541 1/2 orbital (from $h_{9/2}f_{7/2}$ (fh) shells above the $Z = 82$ gap).
Orbitals 13,14 $\longrightarrow$ 541 1/2 a,b,c



Exp. vs. calculation for 541 1/2 bands.

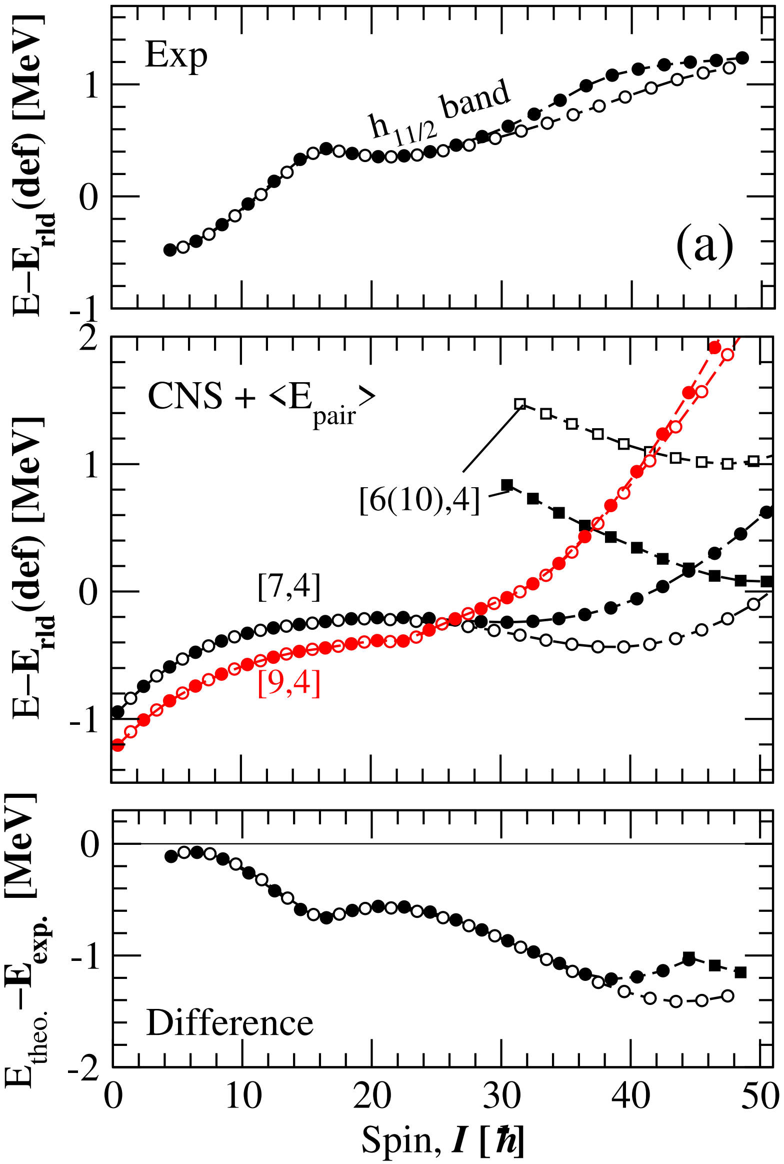


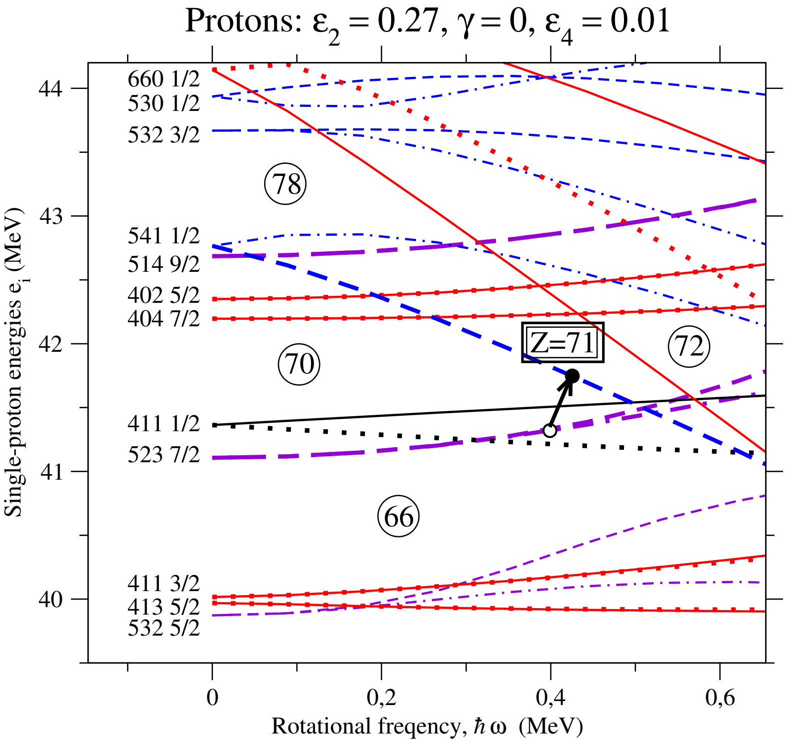


With the 404 7/2 orbital closer to the 411 1/2 orbital, bands b,c would come closer to band a.

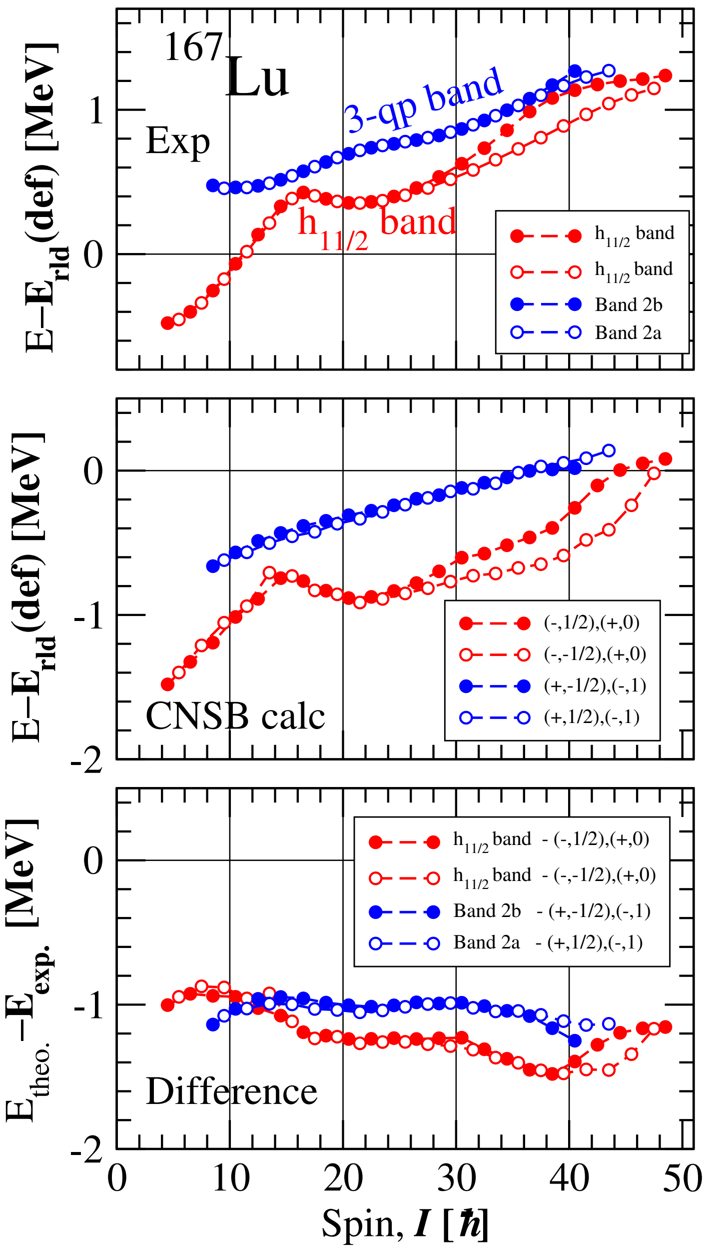
Decoupling will lower the calc. energies at low spin.

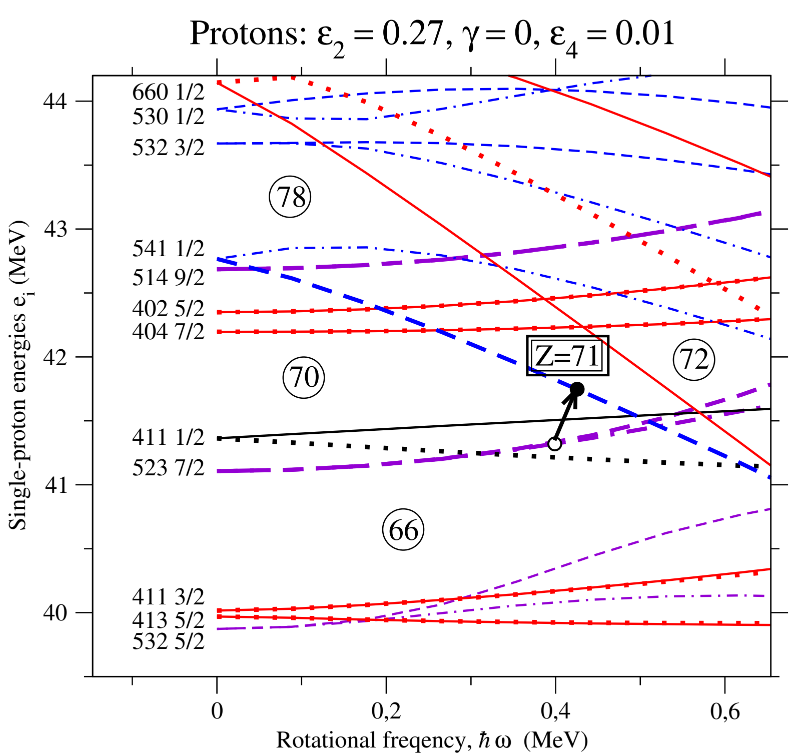
$h_{11/2}$ band vs. CNS calc.





$h_{11/2}$ and $\pi = -$ 3qp band vs. CNSB calc.





Conclusions

- Combination of CNS and CNSB powerful method to analyze nuclear level schemes.
- Assignments to observed rotational bands in ^{167}Lu.
- Unpaired CNS calculations useful because of possibility to make detailed configuration assignments (instructive at all spin values).
- Improved and modified understanding compared with assignments in experimental paper (PRC 92, 064313). Those assignments are mainly based on 'paired' band crossings.
- Such paired band crossings appear to be important only below $I \lesssim 30$ (AB and BC crossing for ^{167}Lu), cf. original article by Bengtsson and Frauendorf.
- High spin bands are mainly formed from the gradual alignment of the spin vectors of the valence particles.
- Rotational bands best characterized by the number of particles in j-shells or groups of j-shells, e.g. $\pi(h_{11/2})^8$ conf's in ^{167}Lu yrast at low spin while $\pi(h_{11/2})^6$ become more favoured in energy at high spin.

Observed $\mathcal{J}^{(2)}$ for 3 qp and TSD bands, ^{167}Lu.