

## Nuclear structure and dynamics from *ab initio* theory

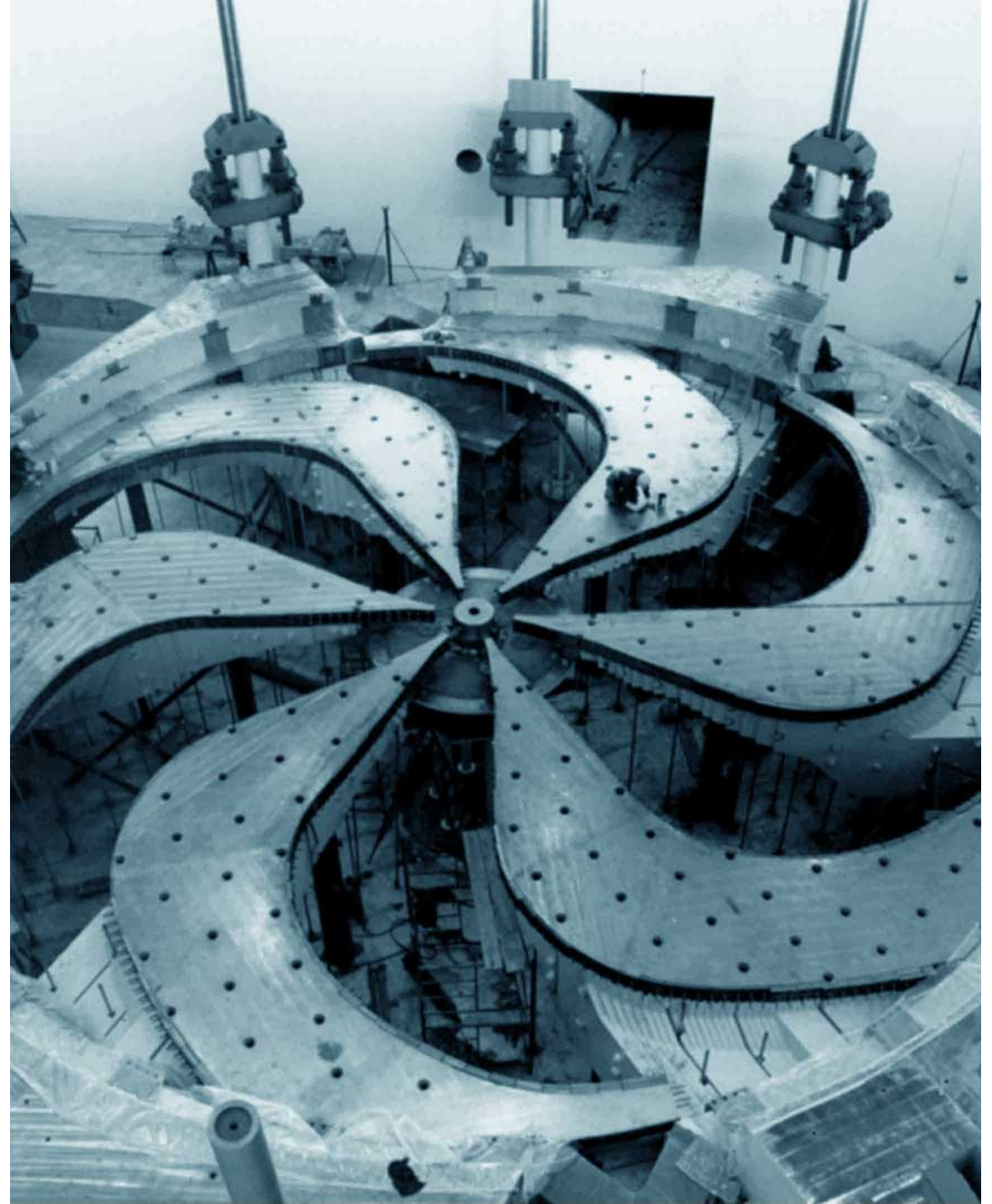
Shapes and Symmetries in Nuclei: from  
Experiment to Theory (SSNET'18 Conference)

Gif-sur-Yvette, November 5th – 9th, 2018

Petr Navratil

TRIUMF

Collaborators: S. Quaglioni (LLNL), G. Hupin (Orsay),  
M. Vorabbi, A. Calci (TRIUMF), P. Gysbers (UBC/TRIUMF),  
M. Gennari (U Waterloo), J. Dohet-Eraly (ULB),  
F. Raimondi (Saclay), W. Horiuchi (U Hokkaido)

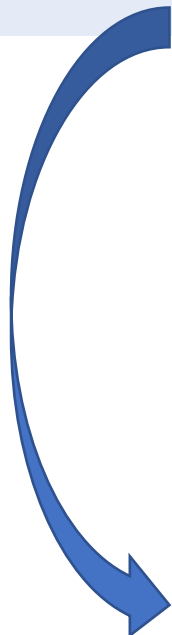


## Outline

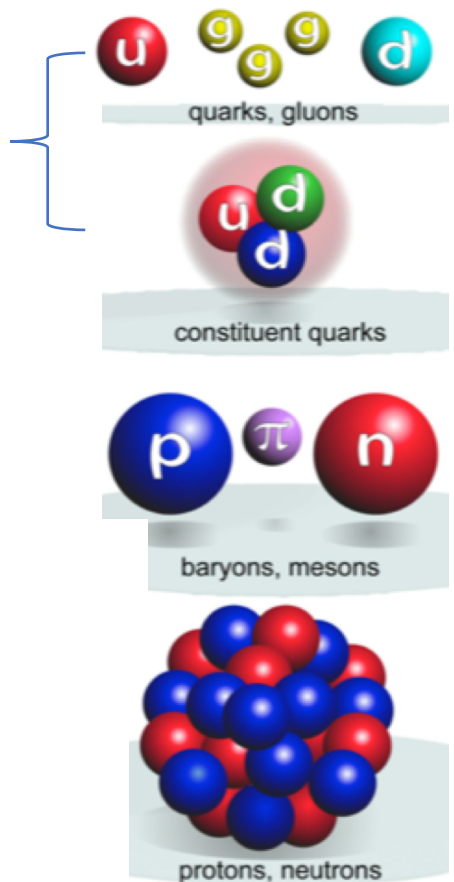
- *Ab initio* calculations in nuclear physics
- New chiral NN N<sup>4</sup>LO + 3N
  - Beta decays of light nuclei in NCSM
  - Microscopic optical potentials from NCSM densities
  - Kinetic density from NCSM
- No-Core Shell Model with Continuum (NCSMC)
  - N-<sup>4</sup>He scattering and polarized D+T fusion
  - Structure of <sup>7</sup>Be and <sup>7</sup>Li considering binary breakup thresholds

# First principles or *ab initio* nuclear theory

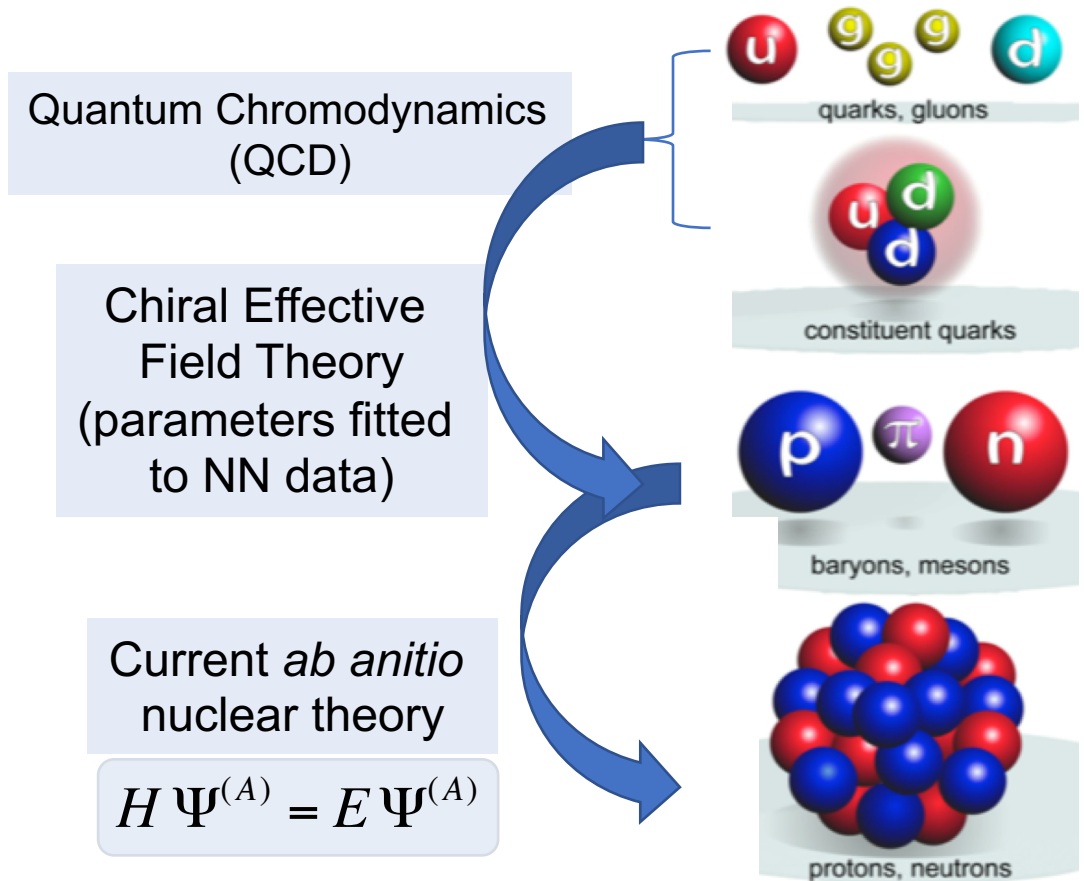
Quantum Chromodynamics  
(QCD)



Genuine *Ab Initio*



# First principles or *ab initio* nuclear theory – what we do at present



- *Ab initio*
  - ✧ Degrees of freedom: Nucleons
  - ✧ All nucleons are active
  - ✧ Exact Pauli principle
  - ✧ Realistic inter-nucleon interactions
    - ✧ Accurate description of NN (and 3N) data
  - ✧ Controllable approximations

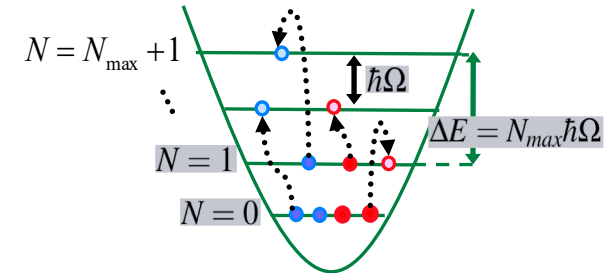
## Conceptually simplest *ab initio* method: No-Core Shell Model (NCSM)

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NCSM

- Basis expansion method
  - Harmonic oscillator (HO) basis truncated in a particular way ( $N_{\max}$ )
  - Why HO basis?
    - Lowest filled HO shells match magic numbers of light nuclei (2, 8, 20 –  $^4\text{He}$ ,  $^{16}\text{O}$ ,  $^{40}\text{Ca}$ )
    - Equivalent description in relative-coordinate and Slater determinant basis
- Short- and medium range correlations
- Bound-states, narrow resonances



$${}^{(A)} \Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^{HO}(\vec{\eta}_1, \vec{\eta}_2, \dots, \vec{\eta}_{A-1})$$

$${}^{(A)} \Psi_{SD}^A = \sum_{N=0}^{N_{\max}} \sum_j c_{Nj}^{SD} \Phi_{SDNj}^{HO}(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A) = \Psi^A \varphi_{000}(\vec{R}_{CM})$$

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journal homepage: [www.elsevier.com/locate/ppnp](http://www.elsevier.com/locate/ppnp)

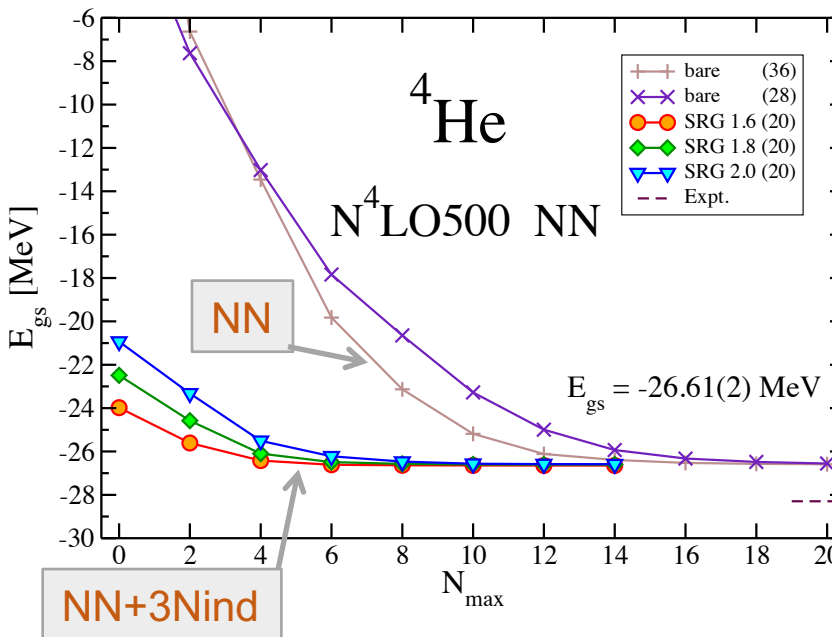
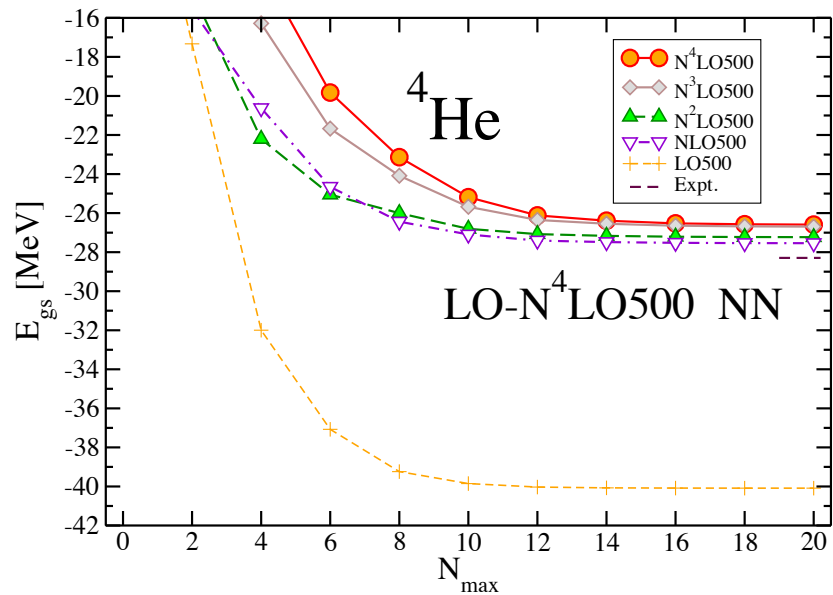
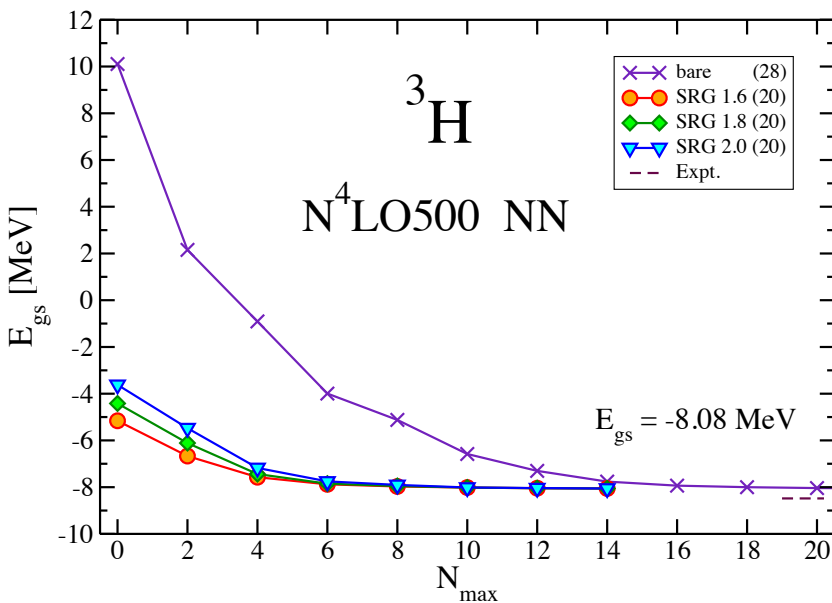
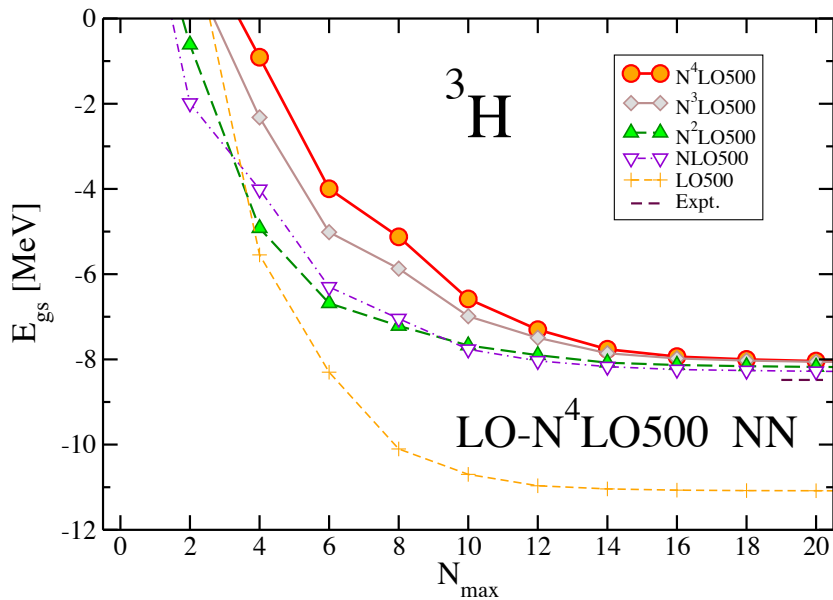


Review

*Ab initio* no core shell model

Bruce R. Barrett<sup>a</sup>, Petr Navrátil<sup>b</sup>, James P. Vary<sup>c,\*</sup>

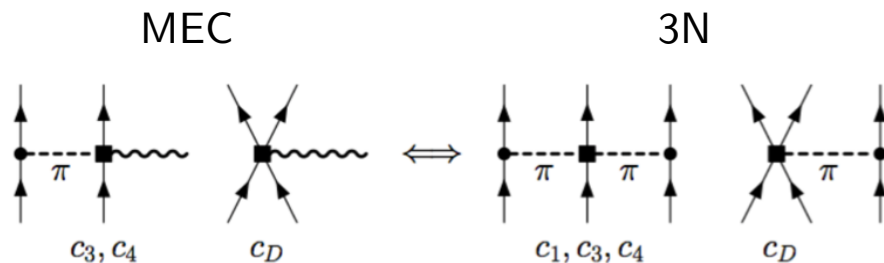
# $^3\text{H}$ and $^4\text{He}$ with chiral EFT interactions up to $\text{N}^4\text{LO}$





## Applications to $\beta$ decays in p-shell nuclei and beyond

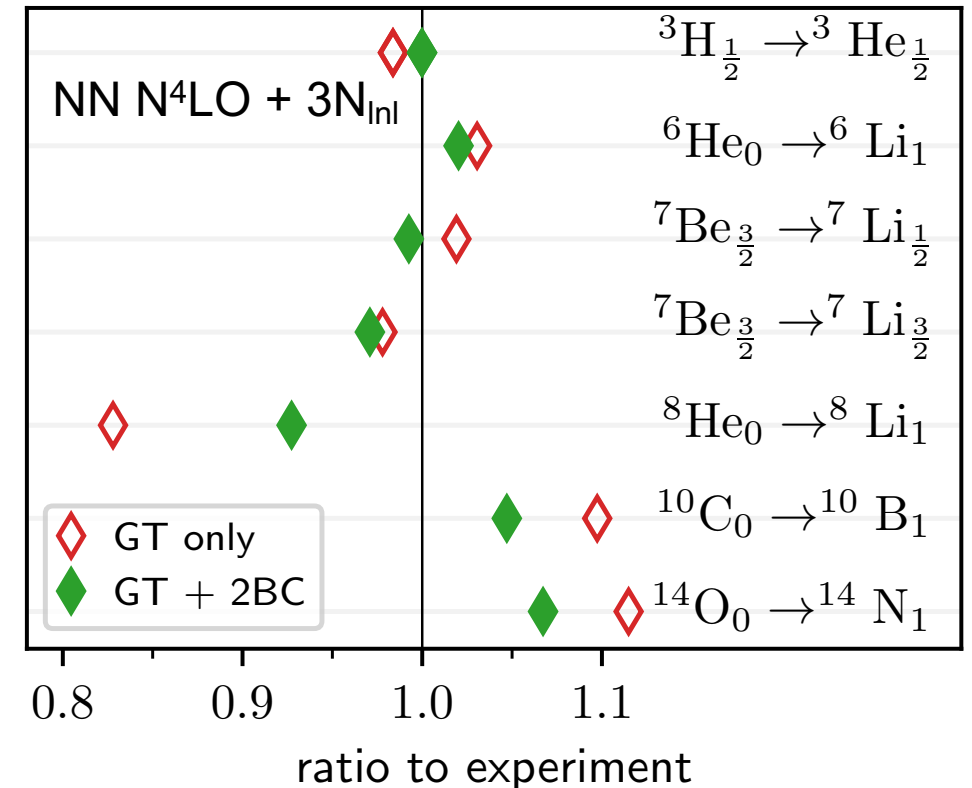
- Does inclusion of the MEC explain  $g_A$  quenching?
- In light nuclei correlations present in *ab initio* (NCSM) wave functions explain almost all of the quenching compared to the standard shell model
  - MEC inclusion overall improves agreement with experiment
- The effect of the MEC inclusion is greater in heavier nuclei
- SRG evolved matrix elements used in coupled-cluster and IM-SRG calculations (up to  $^{100}\text{Sn}$ )



Hollow symbols – GT

Filled symbols – GT+MEC

Both Hamiltonian and operators SRG evolved  
Hamiltonian and current consistent parameters





# Microscopic optical potentials from NCSM densities

Microscopic optical potentials derived from *ab initio* translationally invariant nonlocal one-body densities

Michael Gennari\*

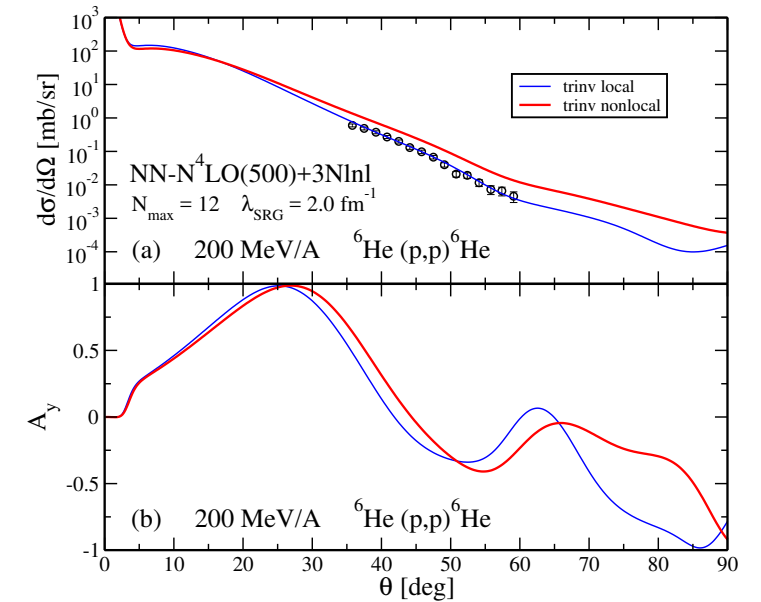
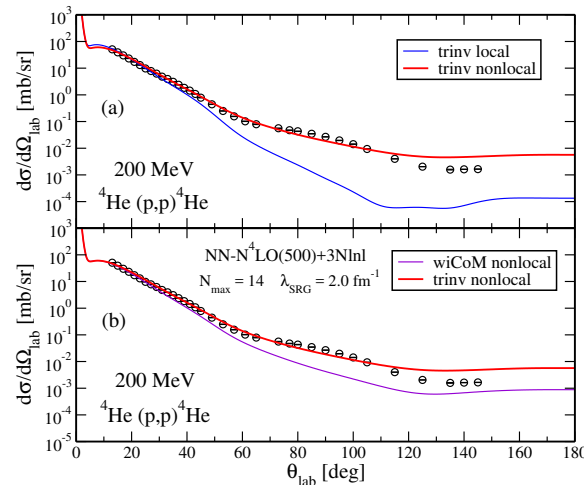
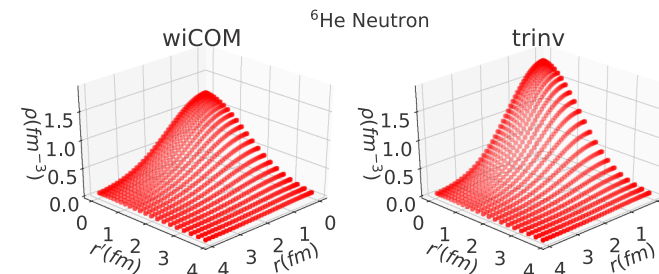
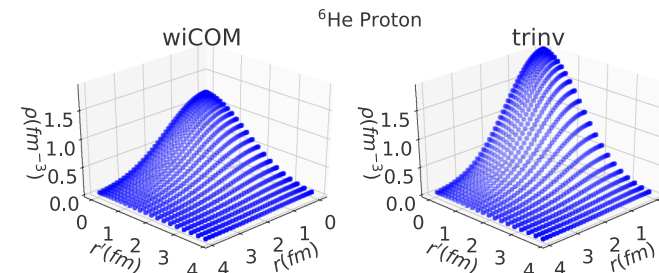
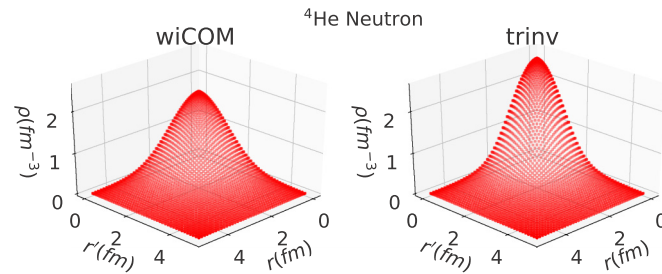
University of Waterloo, 200 University Avenue West Waterloo, Ontario N2L 3G1, Canada  
and TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

Matteo Vorabbi,† Angelo Calci, and Petr Navrátil‡

TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

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- Translationally-invariant non-local densities from NCSM calculations with chiral NN  $N^4\text{LO} + 3\text{N } N^2\text{LO}$  interactions
- High-energy proton-nucleus scattering with microscopic optical potentials from chiral  $N^4\text{LO}$  NN interaction and NCSM densities

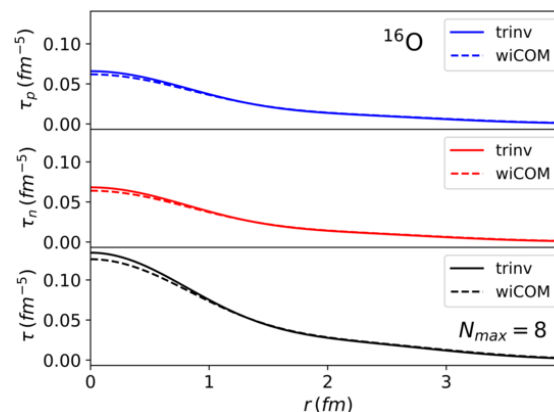
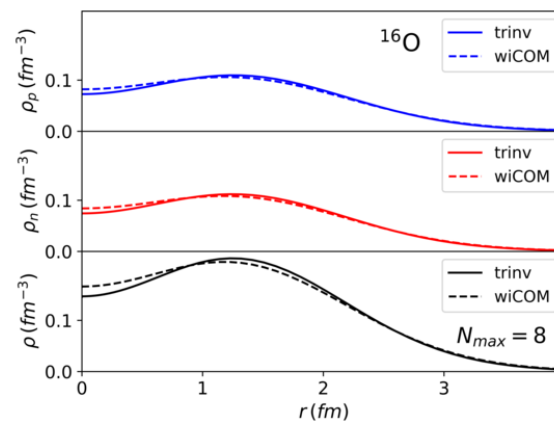
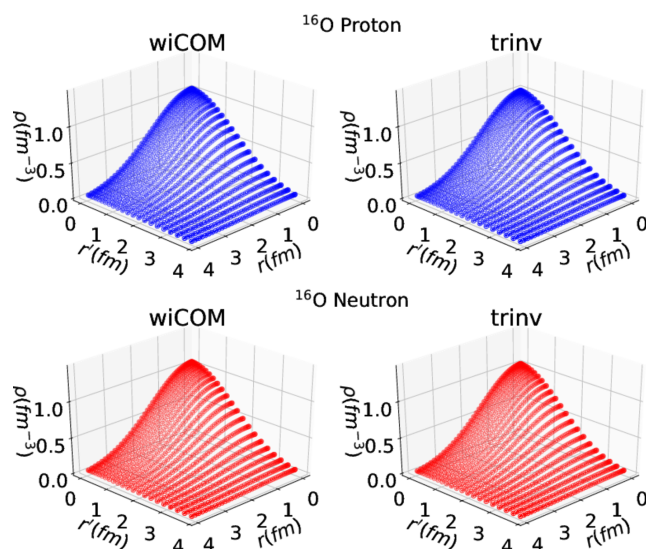


# Nuclear kinetic density from NCSM wave functions

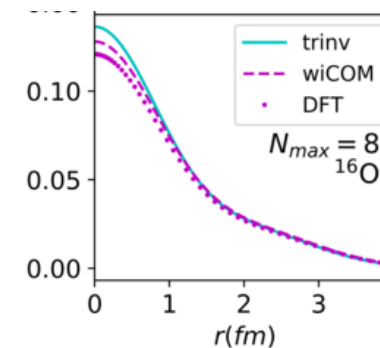
- DFT calculations include kinetic density
  - Might contain center-of-mass contamination
- Can be calculated for light nuclei in NCSM
  - Translationally invariant

$$\tau_{\mathcal{N}}(\vec{r}) = \left[ \vec{\nabla} \cdot \vec{\nabla}' \rho_{\mathcal{N}}(\vec{r}, \vec{r}') \right]_{\vec{r}=\vec{r}'}$$

$$\tau_{DFT}(\vec{r}) = \left( 1 - \frac{1}{A} \right) \tau_{wiCOM}(\vec{r})$$

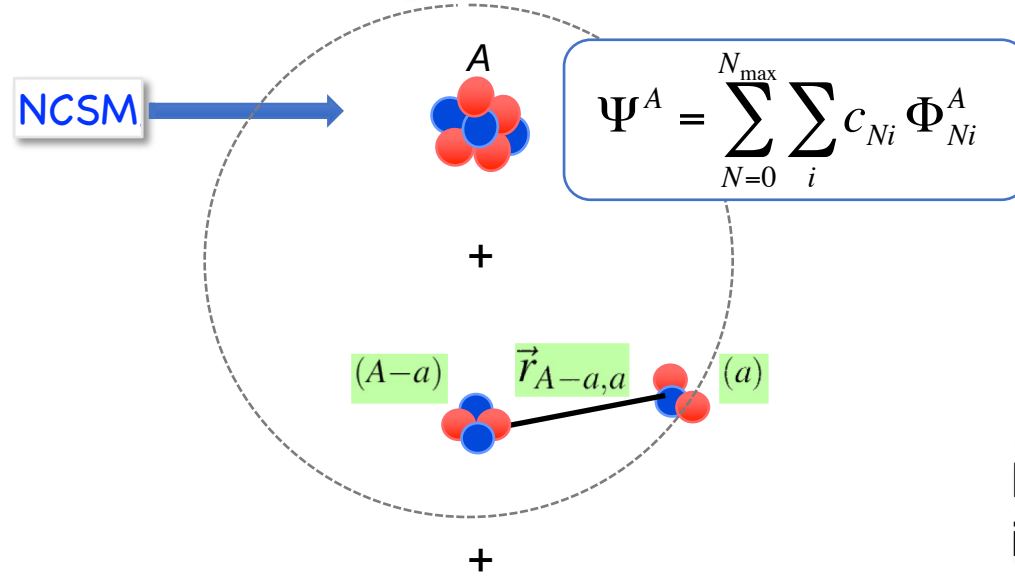


Nucleus	$N_{max}$	$\langle T_{int} \rangle$	$\langle T_{wiCOM} \rangle$	$\langle T_{DFT} \rangle$
$^4\text{He}$	14	51.91	66.91	50.18
$^6\text{He}$	12	78.26	93.26	77.72
$^8\text{He}$	10	116.30	131.30	114.89
$^{12}\text{C}$	8 IT	219.84	234.84	215.27
$^{16}\text{O}$	8 IT	301.69	316.69	296.90

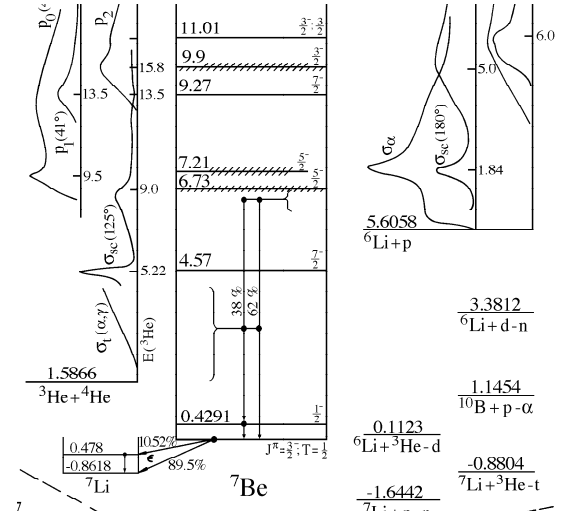


# Extending no-core shell model beyond bound states

Include more many nucleon correlations...



...using the Resonating Group Method (RGM) ideas

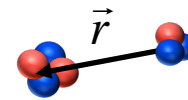


## Unified approach to bound & continuum states; to nuclear structure & reactions

- No-core shell model (NCSM)
  - $A$ -nucleon wave function expansion in the harmonic-oscillator (HO) basis
  - short- and medium range correlations
  - Bound-states, narrow resonances
- NCSM with Resonating Group Method (NCSM/RGM)
  - cluster expansion, clusters described by NCSM
  - proper asymptotic behavior
  - long-range correlations
- Most efficient: *ab initio* no-core shell model with continuum (NCSMC)



NCSM



NCSM/RGM

NCSMC

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{matrix} (A) \\ \text{Nucleus} \\ \lambda \end{matrix} \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{matrix} (A-a) & (a) \\ \text{Cluster} & \text{Cluster} \\ \nu \end{matrix} \right\rangle$$

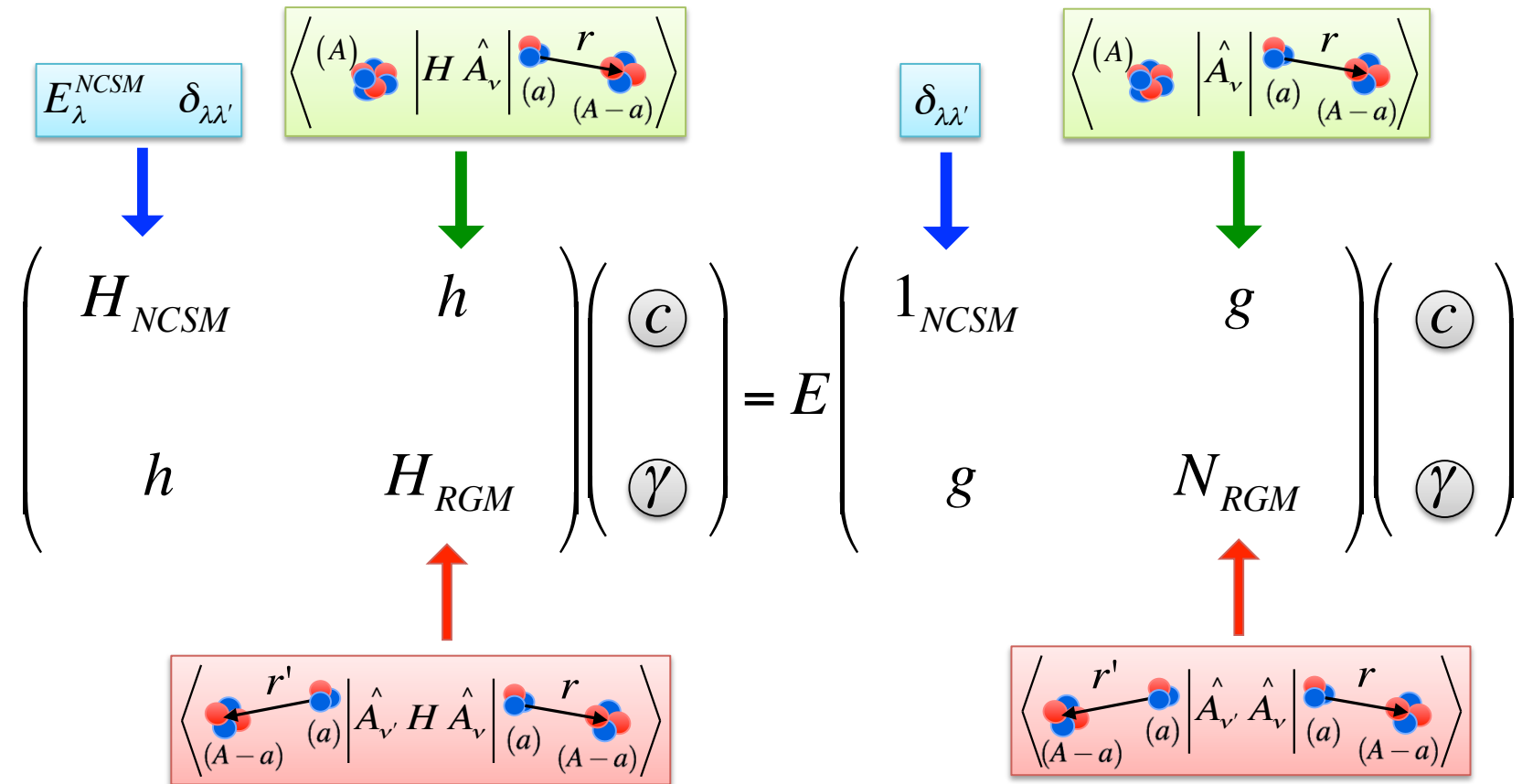
Unknowns

S. Baroni, P. Navratil, and S. Quaglioni,  
PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).

## Coupled NCSMC equations

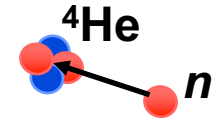
$$H \Psi^{(A)} = E \Psi^{(A)}$$

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{matrix} (A) \\ \text{cluster} \end{matrix}, \lambda \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{matrix} (A-a) & (a) \\ \text{cluster} & \text{cluster} \end{matrix}, \nu \right\rangle$$

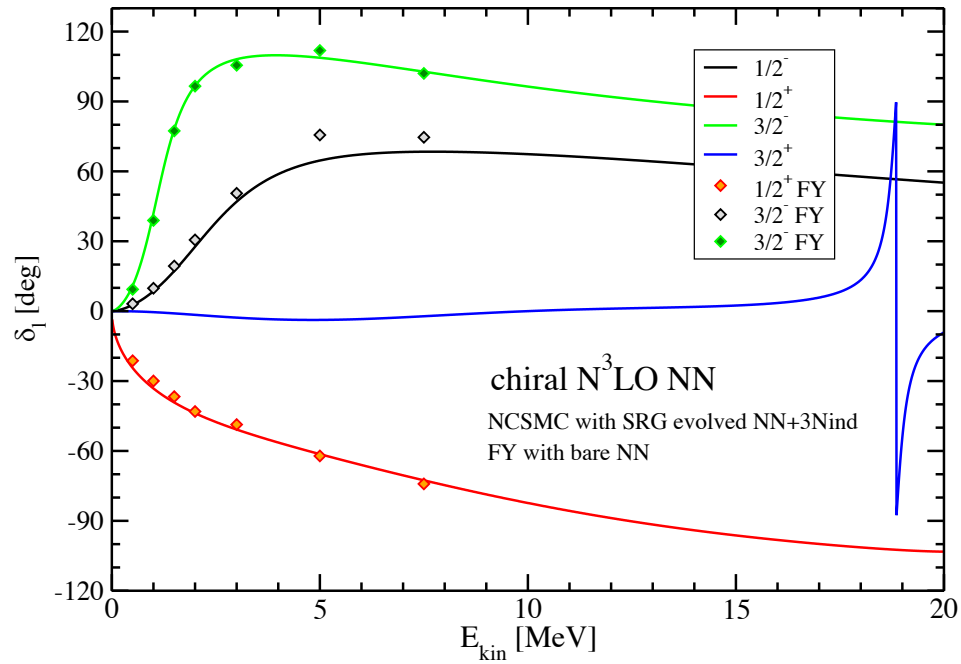


Solved by Microscopic R-matrix theory on a Lagrange mesh – efficient for **coupled channels**

# n-<sup>4</sup>He scattering within NCSMC



*n*-<sup>4</sup>He scattering phase-shifts for chiral NN



FY: Faddeev-Yakubovsky method - Rimantas Lazauskas

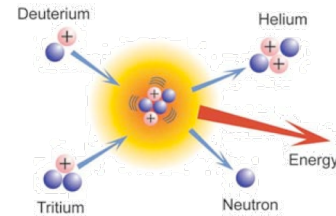
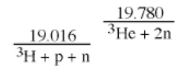
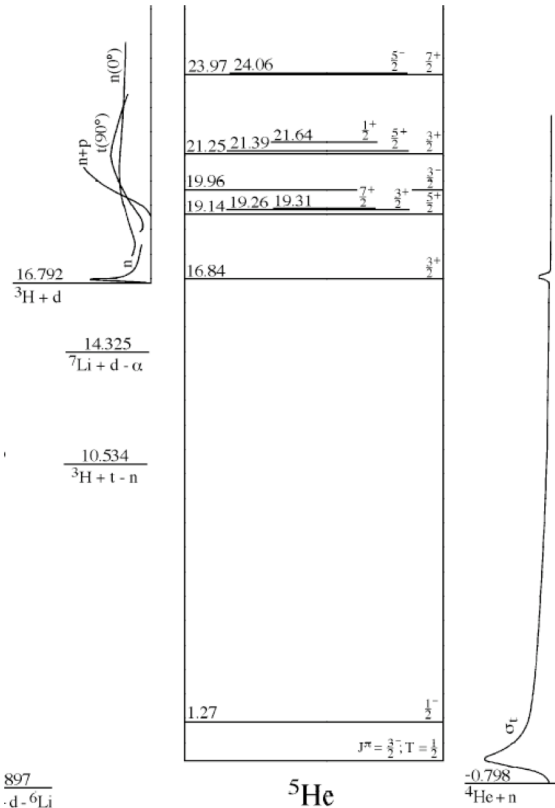
**Invited Comment**

**Unified *ab initio* approaches to nuclear structure and reactions**

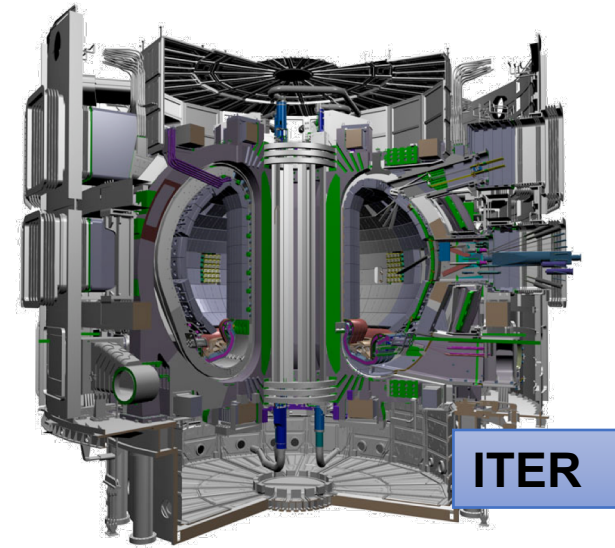
Petr Navrátil<sup>1</sup>, Sofia Quaglioni<sup>2</sup>, Guillaume Hupin<sup>3,4</sup>, Carolina Romero-Redondo<sup>5</sup> and Angelo Calci

# Deuterium-Tritium fusion

- The  $d+^3\text{H}\rightarrow n+^4\text{He}$  reaction
  - The most promising for the production of fusion energy in the near future
  - Used to achieve inertial-confinement (laser-induced) fusion at NIF, and magnetic-confinement fusion at ITER
  - With its mirror reaction,  $^3\text{He}(d,p)^4\text{He}$ , important for Big Bang nucleosynthesis

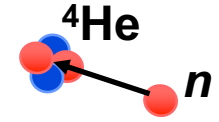


Resonance at  $E_{cm}=48$  keV ( $E_d=105$  keV) in the  $J=3/2^+$  channel  
 Cross section at the peak: 4.88 b  
**17.64 MeV energy released:**  
**14.1 MeV neutron and 3.5 MeV alpha**

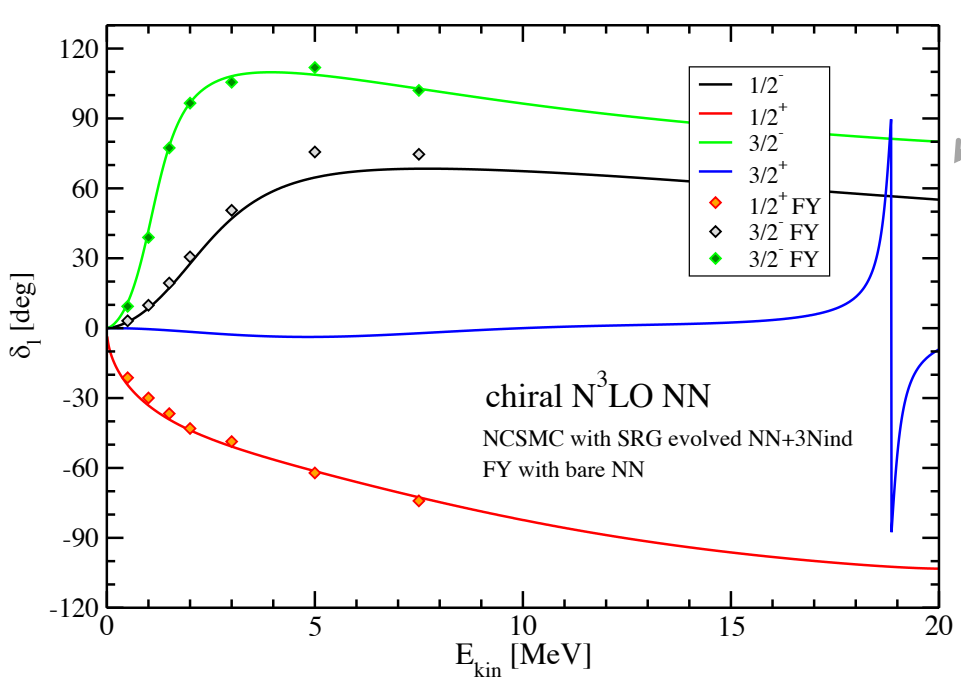


**ITER**

# $n$ - $^4\text{He}$ scattering and $^3\text{H}+d$ fusion within NCSMC

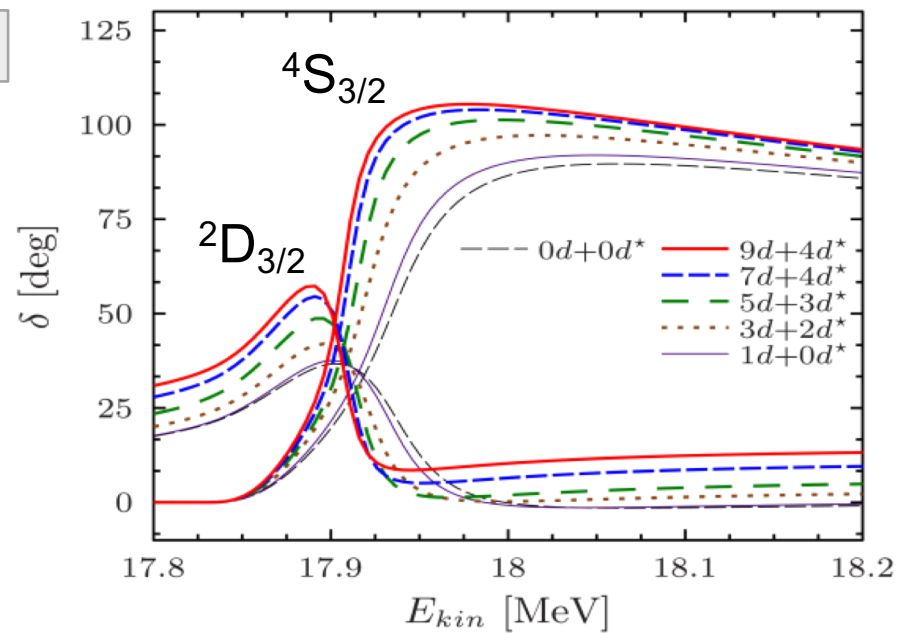


$n$ - $^4\text{He}$  scattering phase-shifts for chiral NN



$^4\text{He}+n$

$^4\text{He}+n \rightarrow ^3\text{H}+d$

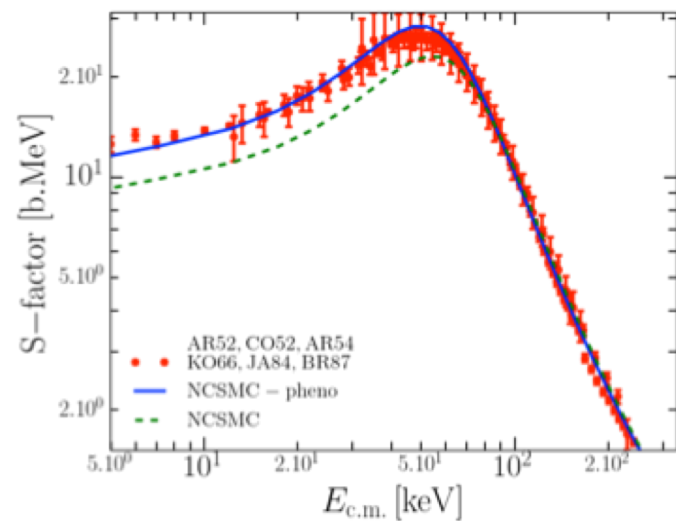


FY: Faddeev-Yakubovsky method - Rimantas Lazauskas

The  $d$ - $^3\text{H}$  fusion takes place through a transition of  $d+^3\text{H}$  is  $S$ -wave to  $n+^4\text{He}$  in  $D$ -wave: Importance of the **tensor** and **3N** force

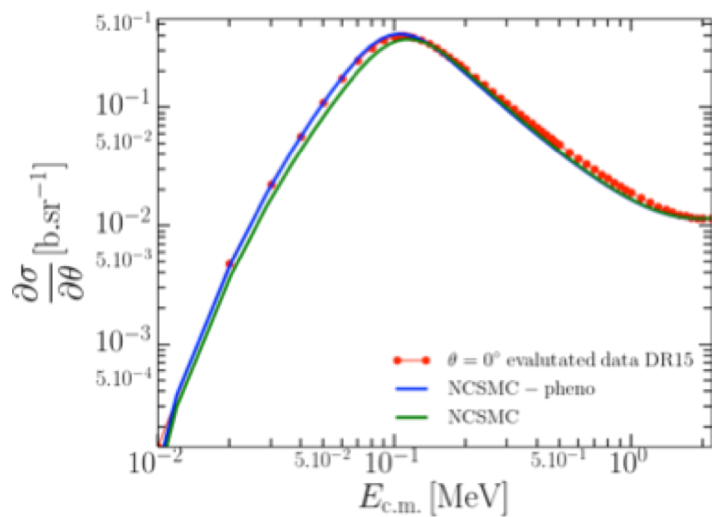


### $^3\text{H}(d,n)^4\text{He}$ with chiral NN+3N500 interaction



$$S(E) = E\sigma(E) \exp[2\pi\eta(E)]$$

$$\eta(E) = Z_{A-a}Z_a e^2 / \hbar v_{A-a,a}$$

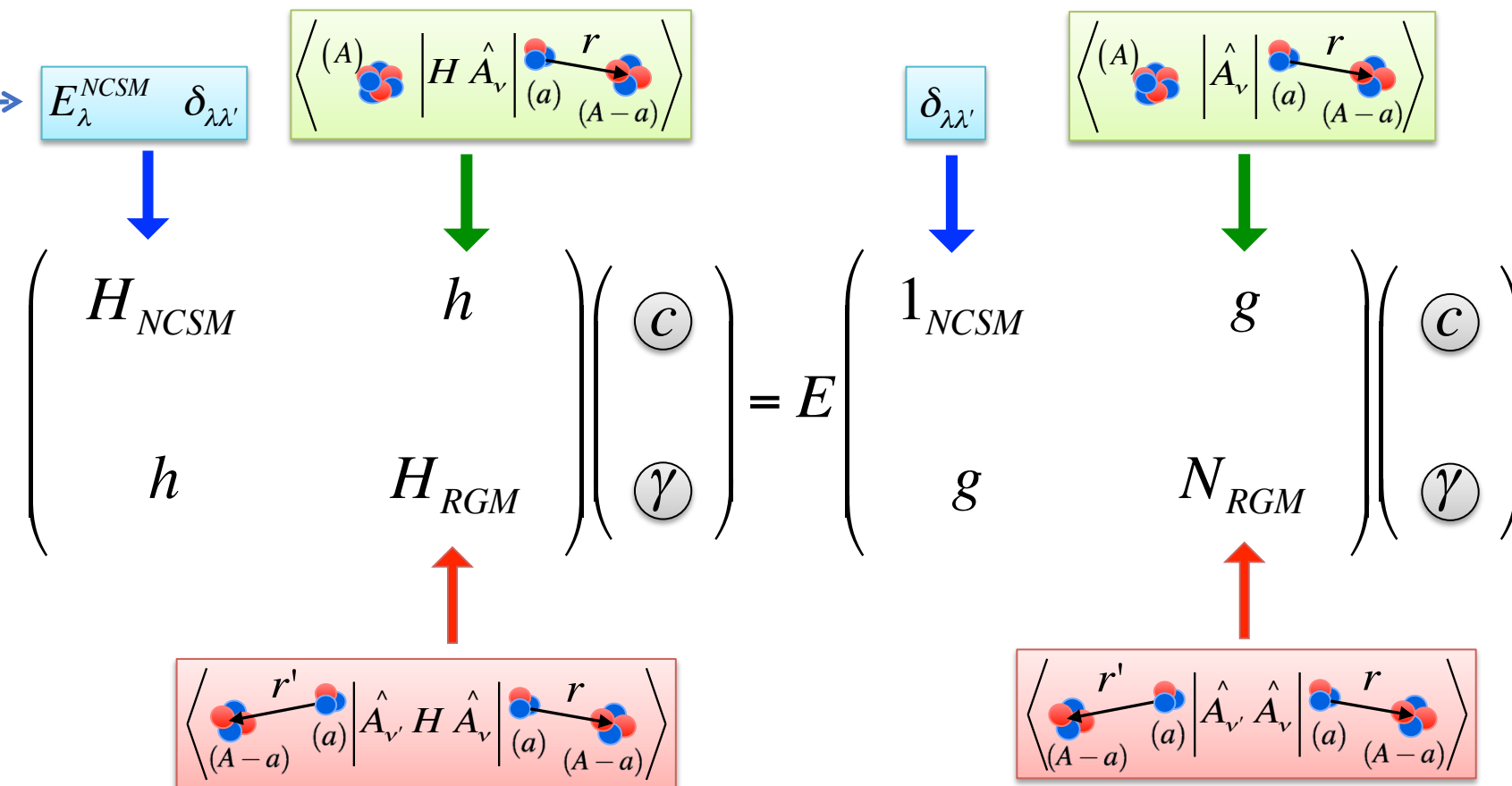


## NCSMC phenomenology

$$H \Psi^{(A)} = E \Psi^{(A)}$$

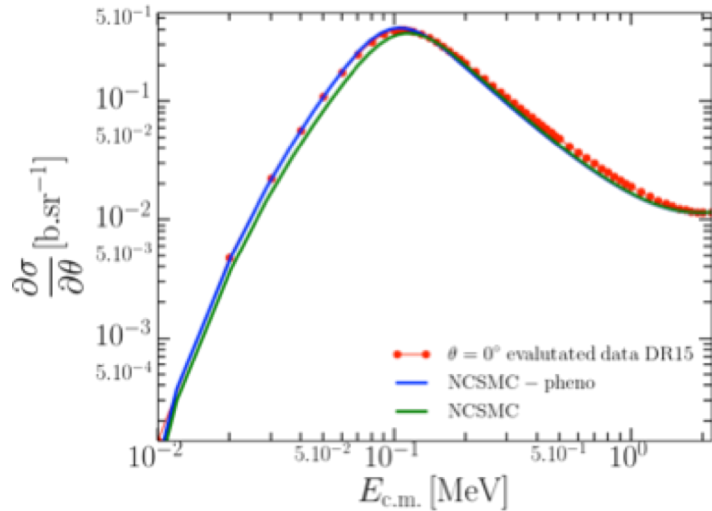
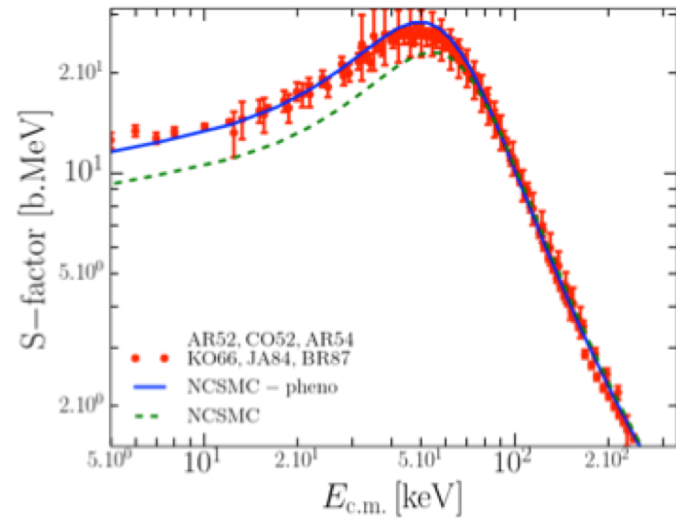
$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{matrix} (A) \\ \text{orbital} \end{matrix}, \lambda \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{matrix} (A-a) \\ \text{orbital} \end{matrix}, \nu \right\rangle$$

$E_{\lambda}^{NCSM}$  energies treated as adjustable parameters



### $^3\text{H}(d,n)^4\text{He}$ with chiral NN+3N500 interaction

Polarized fusion

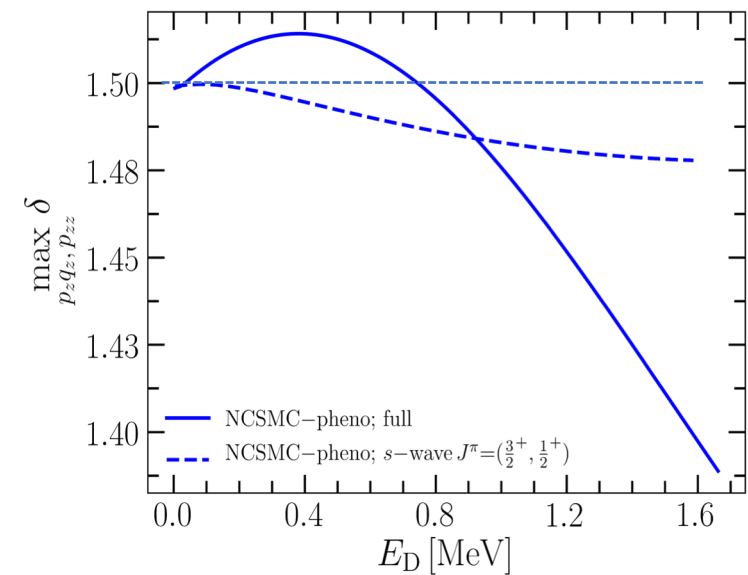
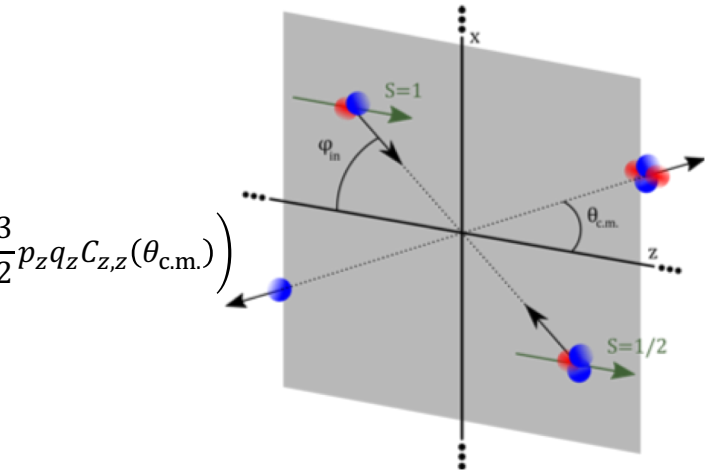
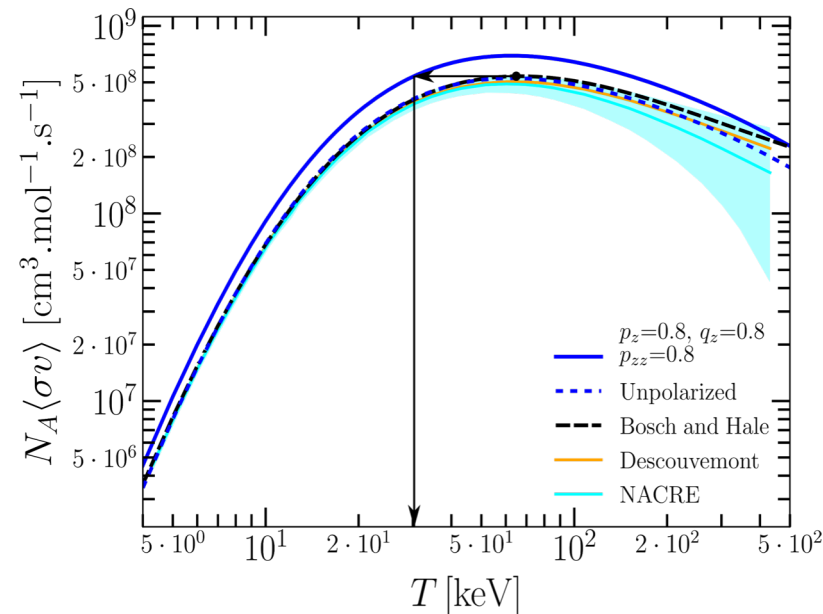


$$S(E) = E\sigma(E) \exp[2\pi\eta(E)]$$

$$\eta(E) = Z_{A-a}Z_a e^2 / \hbar v_{A-a,a}$$

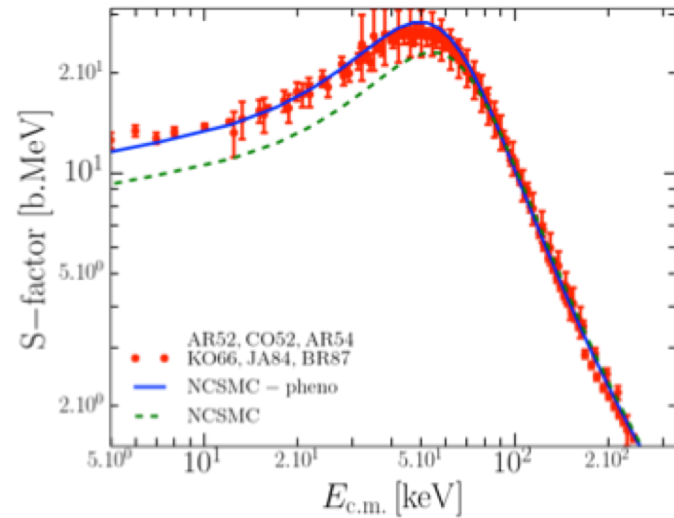
$$\frac{\partial\sigma_{pol}}{\partial\Omega_{c.m.}}(\theta_{c.m.}) = \frac{\partial\sigma_{unpol}}{\partial\Omega_{c.m.}}(\theta_{c.m.}) \left( 1 + \frac{1}{2}p_{zz}A_{zz}^{(b)}(\theta_{c.m.}) + \frac{3}{2}p_z q_z C_{z,z}(\theta_{c.m.}) \right)$$

$$\langle\sigma v\rangle = \sqrt{\frac{8}{\pi\mu(k_b T)^3}} \int_0^\infty S(E) \exp\left(-\frac{E}{k_b T} - \sqrt{\frac{E_g}{E}}\right) dE,$$



### $^3\text{H}(d,n)^4\text{He}$ with chiral NN+3N500 interaction

Polarized fusion

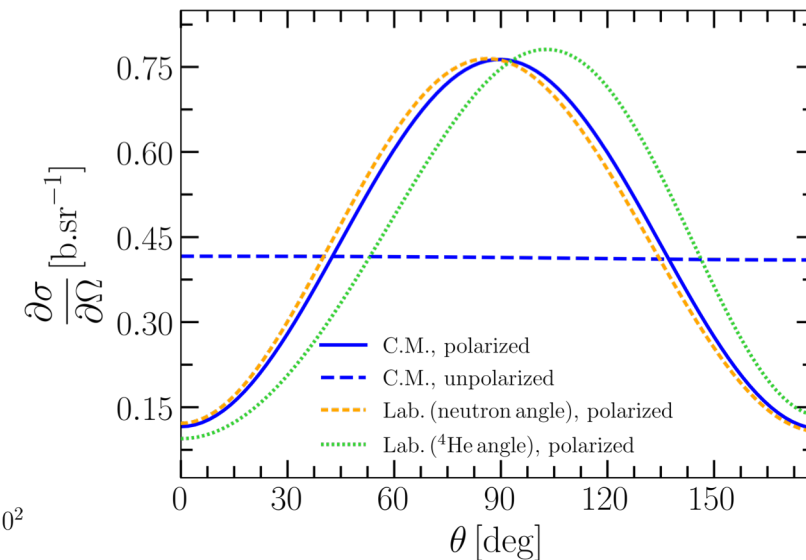
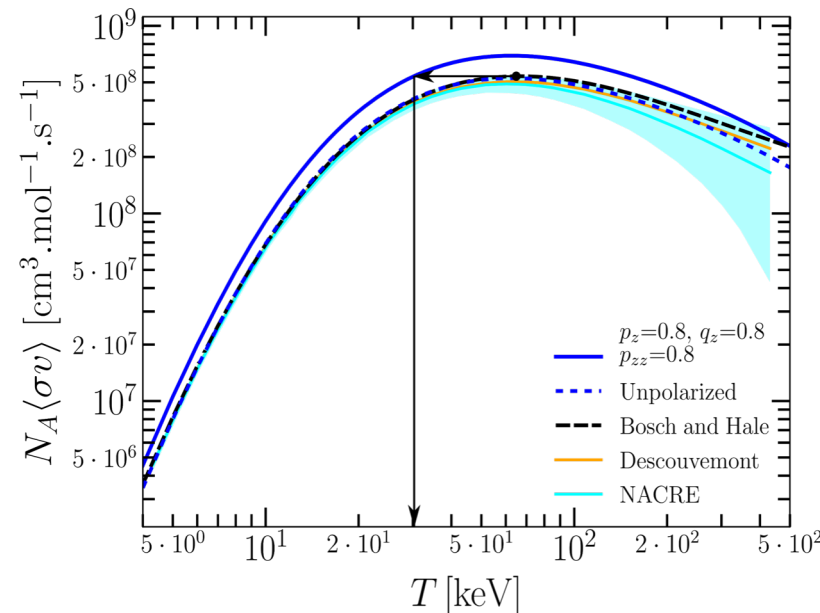
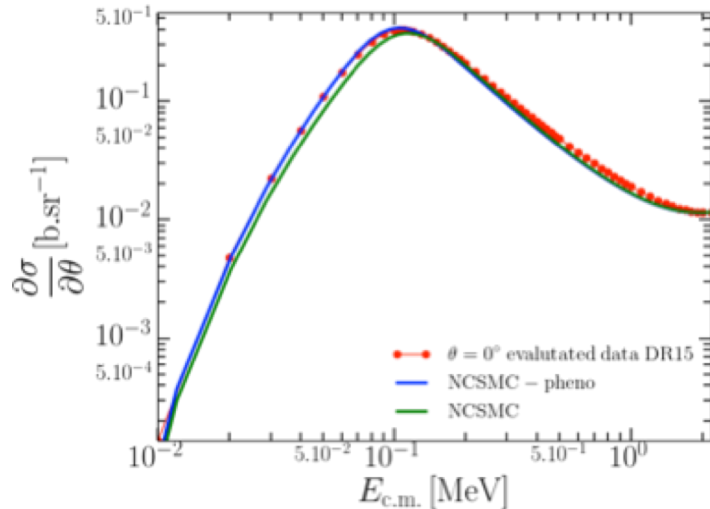
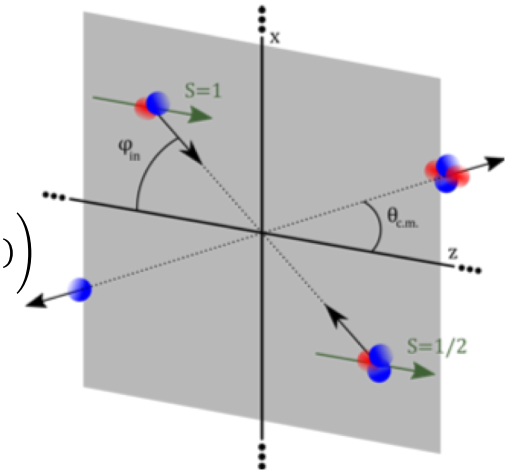


$$S(E) = E\sigma(E) \exp[2\pi\eta(E)]$$

$$\eta(E) = Z_{A-a}Z_a e^2 / \hbar v_{A-a,a}$$

$$\frac{\partial\sigma_{pol}}{\partial\Omega_{c.m.}}(\theta_{c.m.}) = \frac{\partial\sigma_{unpol}}{\partial\Omega_{c.m.}}(\theta_{c.m.}) \left( 1 + \frac{1}{2}p_{zz}A_{zz}^{(b)}(\theta_{c.m.}) + \frac{3}{2}p_z q_z C_{z,z}(\theta_{c.m.}) \right)$$

$$\langle\sigma v\rangle = \sqrt{\frac{8}{\pi\mu(k_b T)^3}} \int_0^\infty S(E) \exp\left(-\frac{E}{k_b T} - \sqrt{\frac{E_g}{E}}\right) dE,$$



# **Structure of ${}^7\text{Be}$ and ${}^7\text{Li}$ considering binary breakup thresholds**

**NCSMC with SRG evolved chiral NN**

# ${}^7\text{Be}$ system

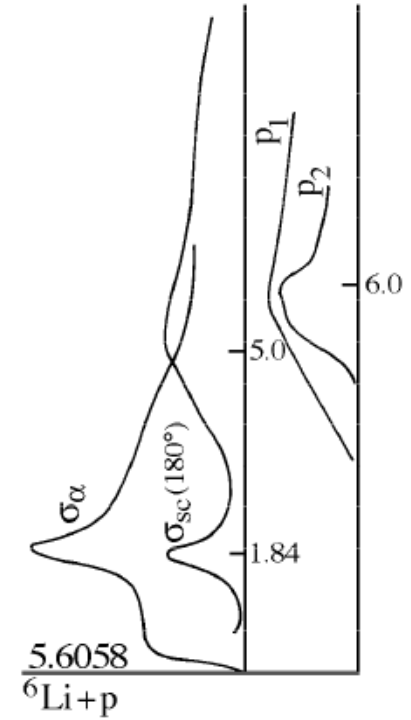
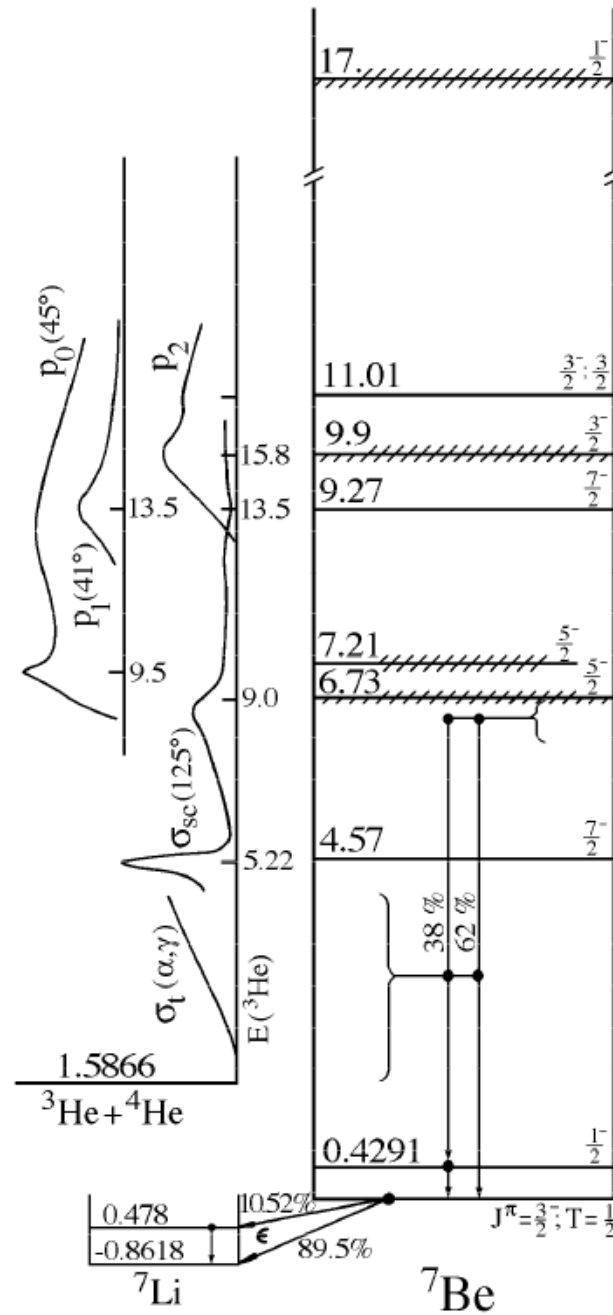
## Analyzed mass partitions

- ${}^3\text{He} + {}^4\text{He}$
- $p + {}^6\text{Li}$

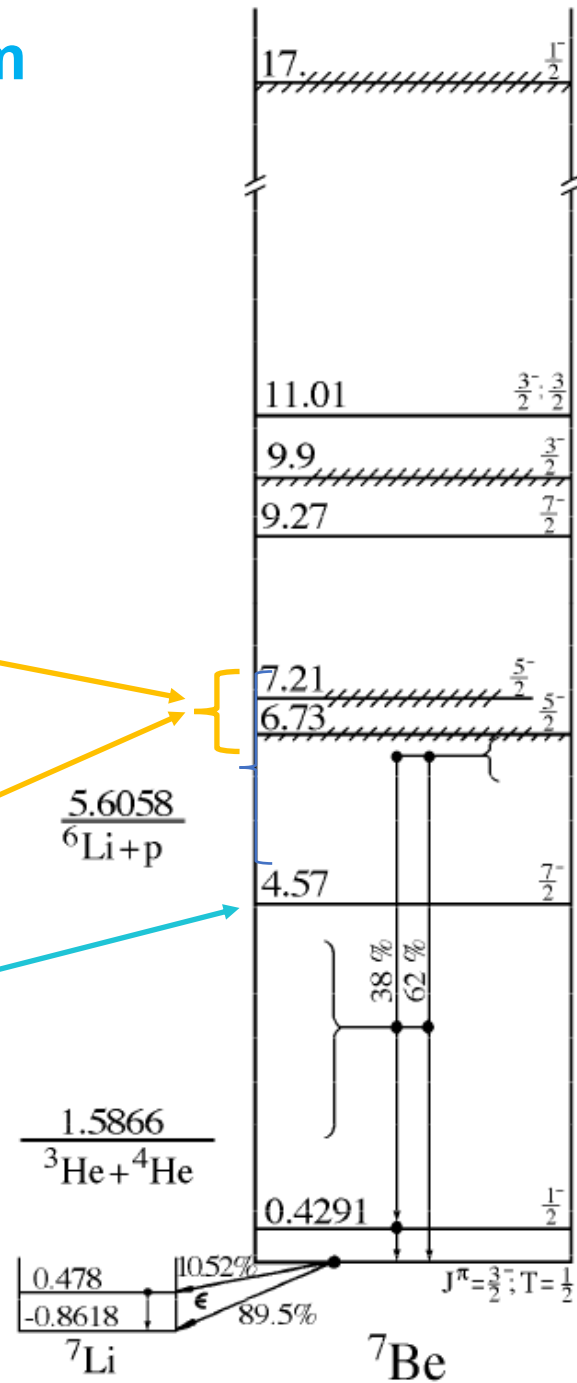
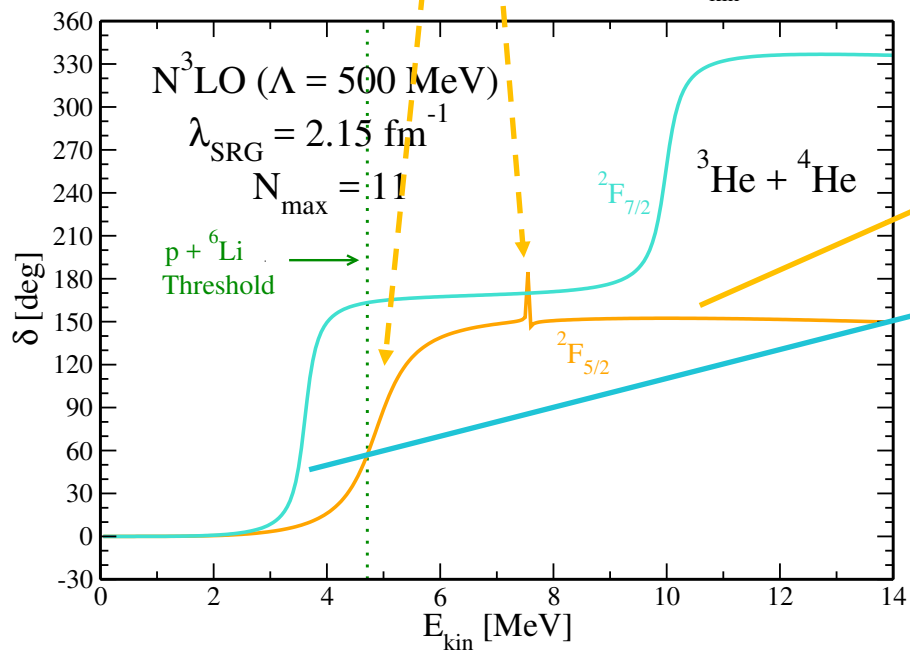
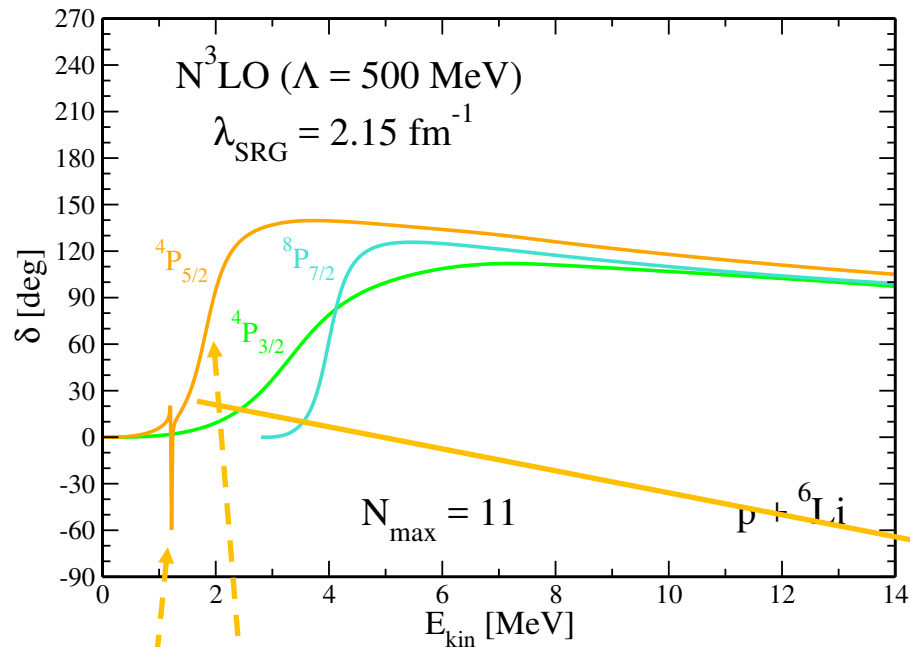
Exp.	$J^\pi = 3/2^-$
E [MeV]	-37.60

${}^3\text{He} + {}^4\text{He}$	$J^\pi = 3/2^-$	$J^\pi = 1/2^-$
$E_{\text{bound}}$	-1.519	-1.256
E [MeV]	-36.98	-36.71

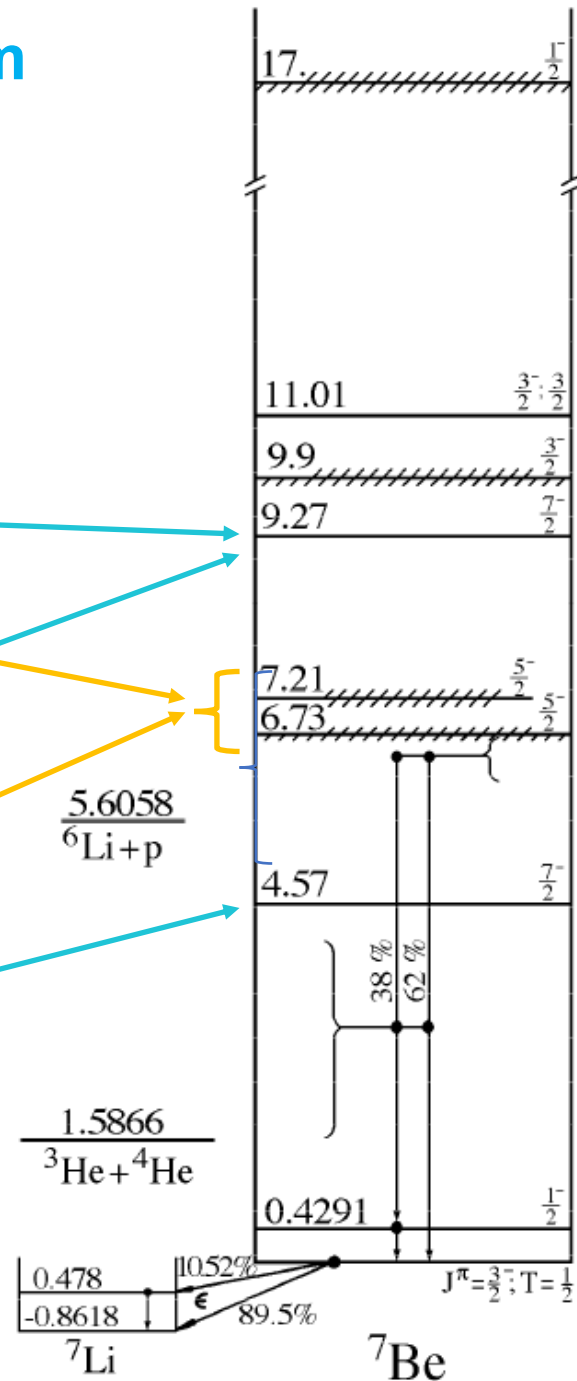
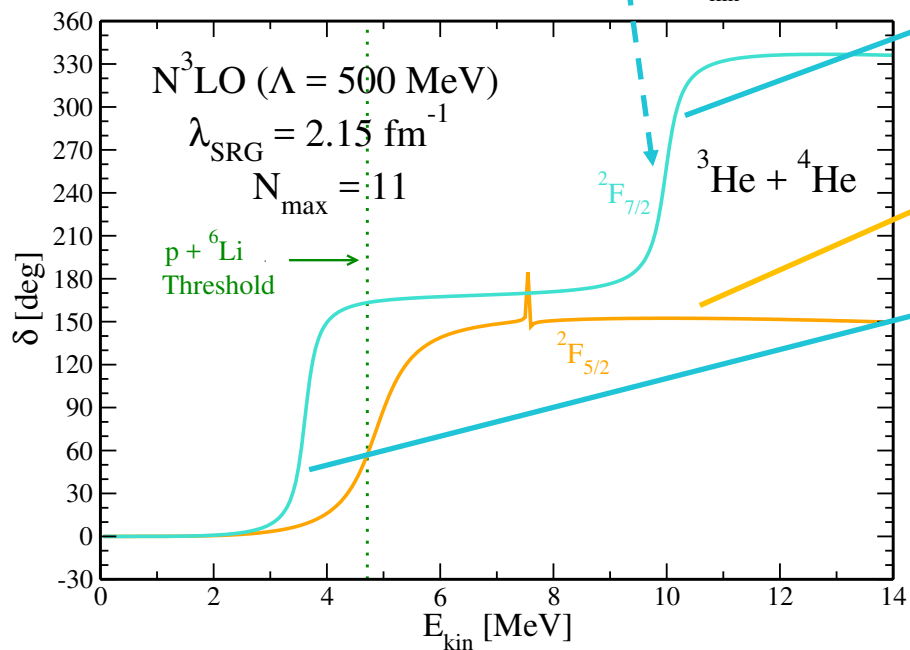
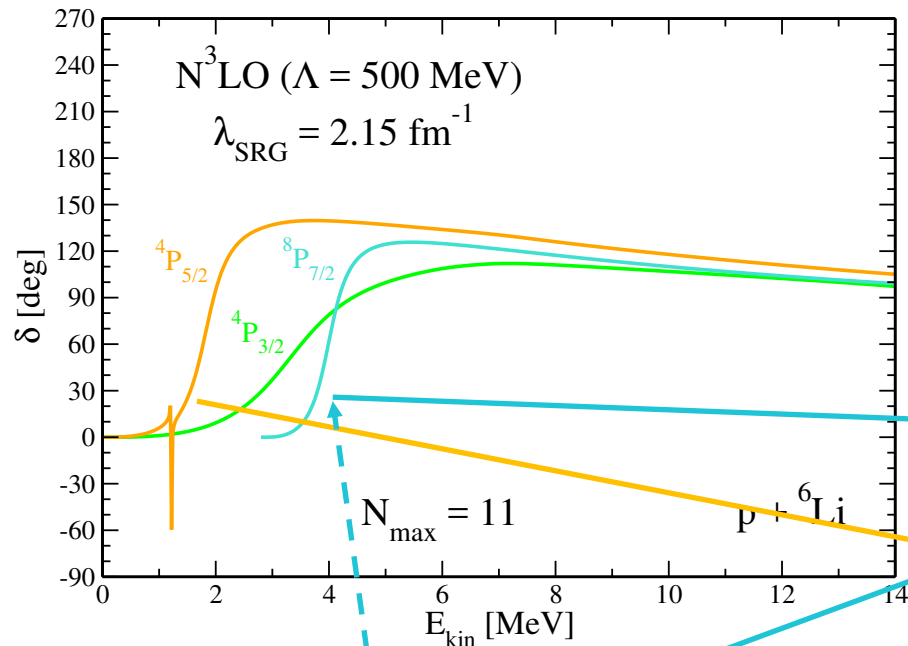
$p + {}^6\text{Li}$	$J^\pi = 3/2^-$	$J^\pi = 1/2^-$
$E_{\text{bound}}$	-5.729	-5.389
E [MeV]	-36.47	-36.13



# ${}^7\text{Be}$ – Reproducing the energy spectrum

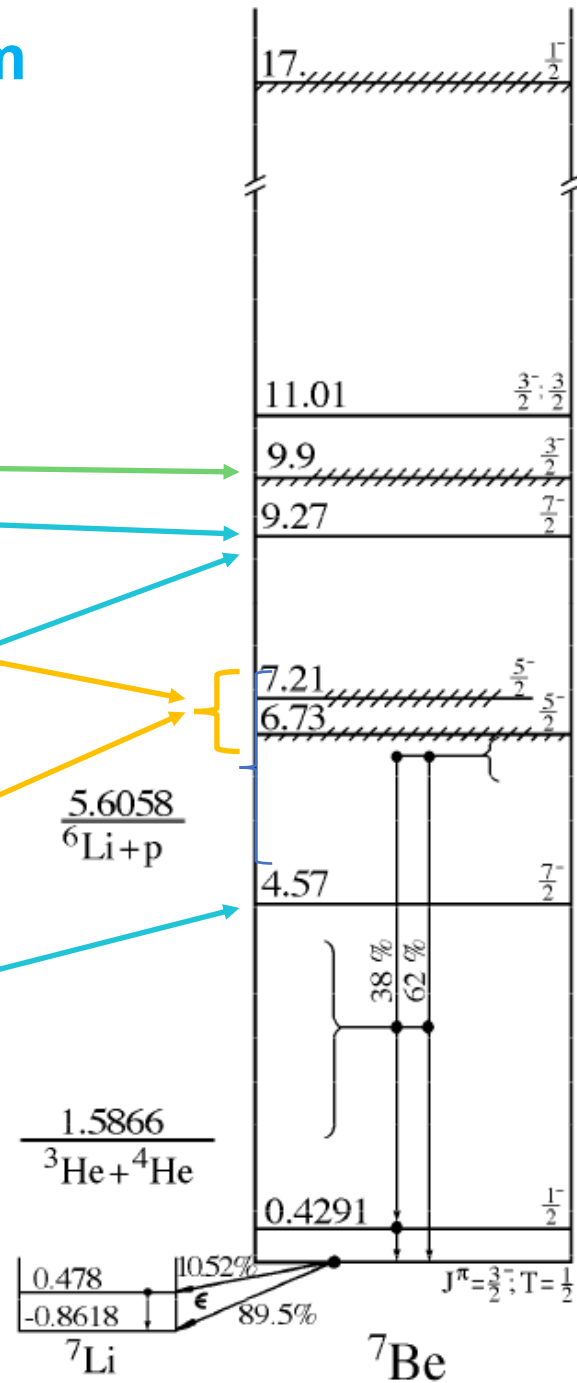
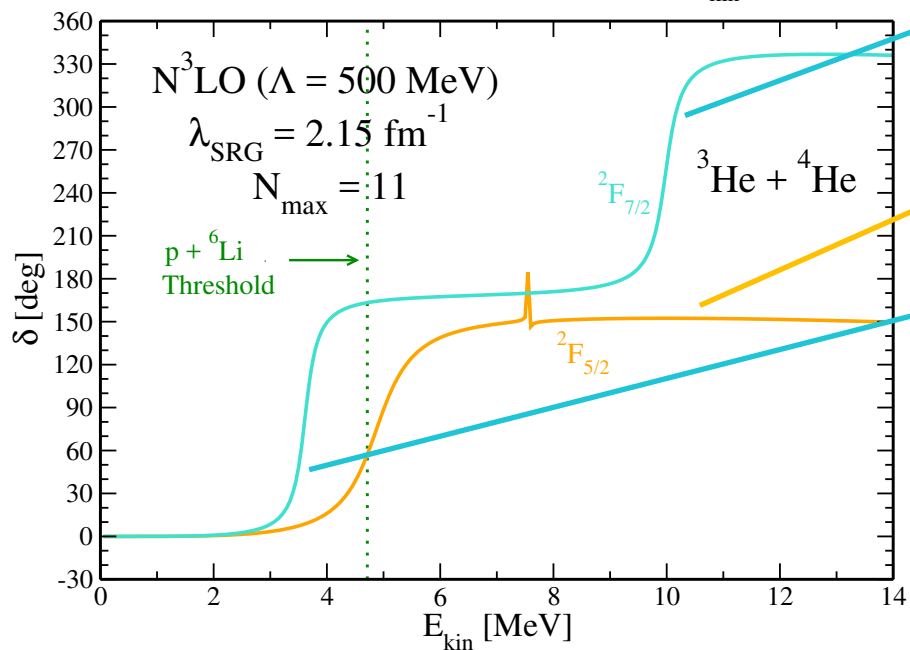
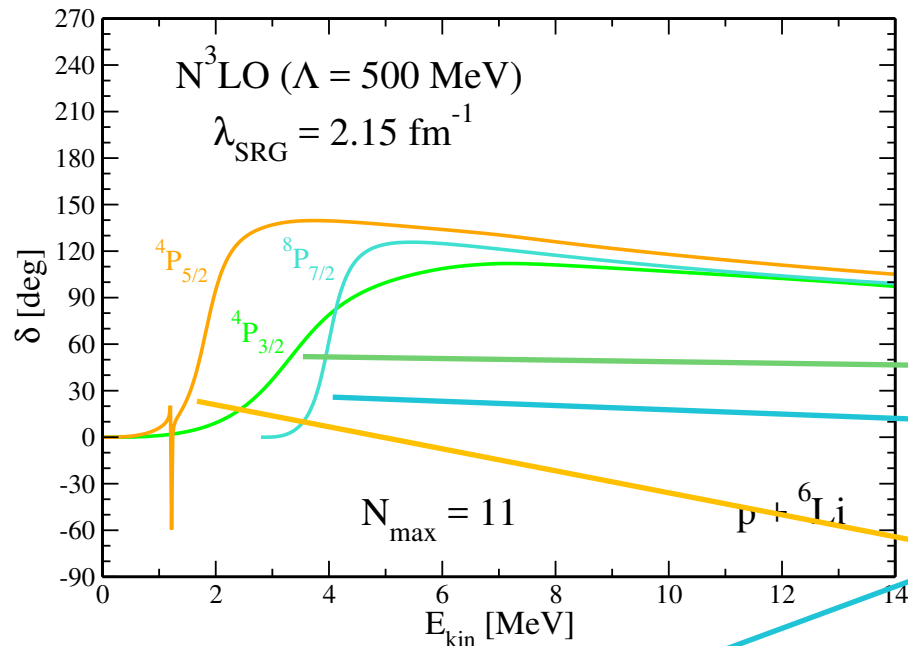


# ${}^7\text{Be}$ – Reproducing the energy spectrum

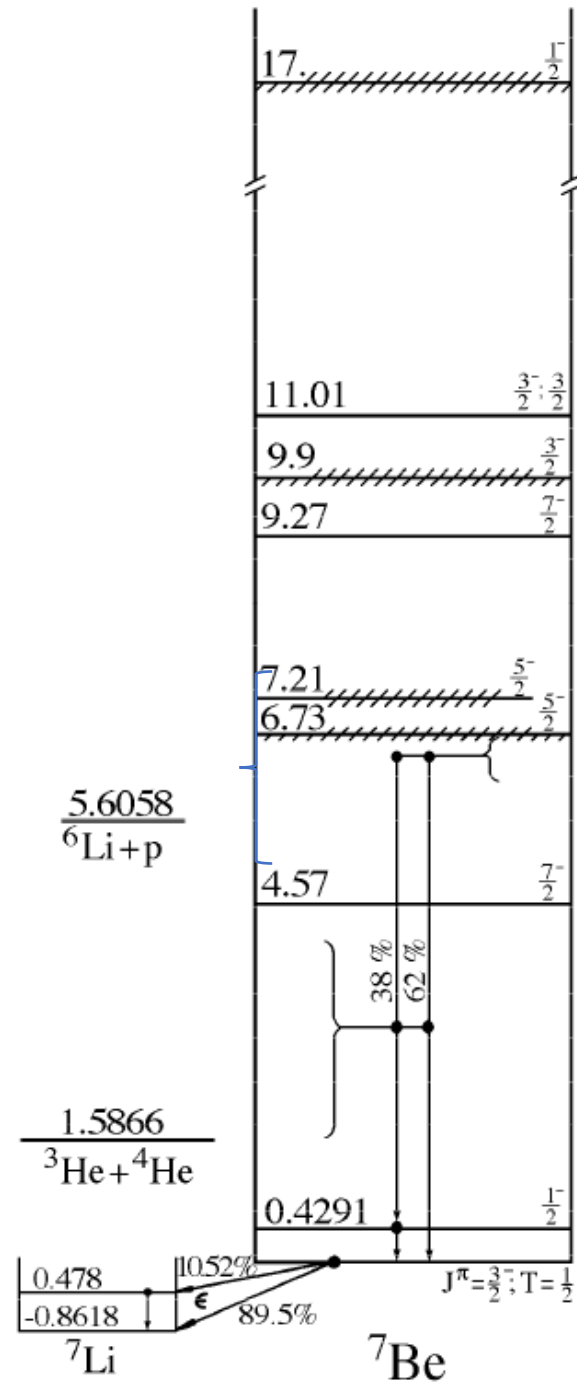
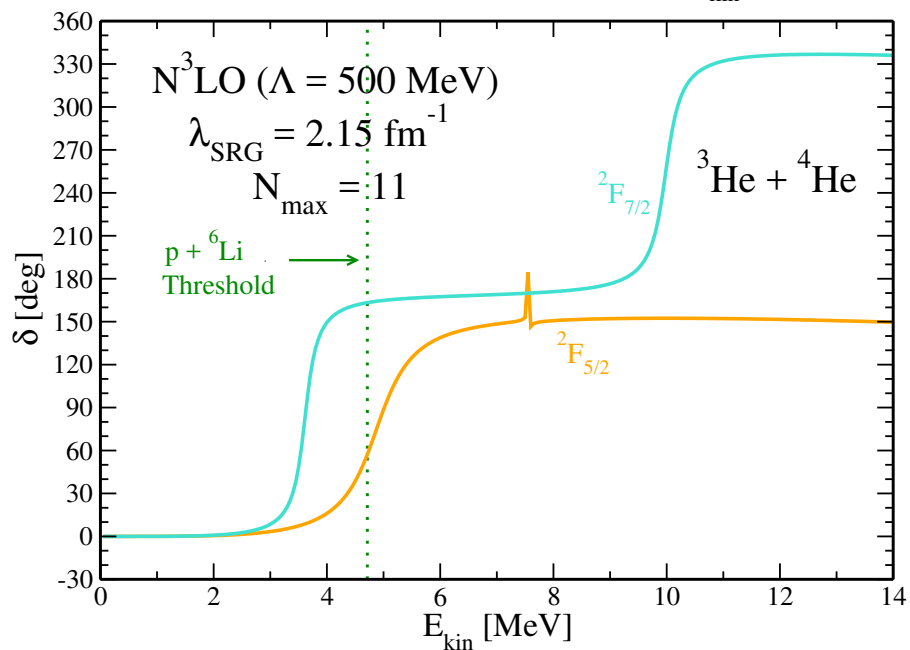
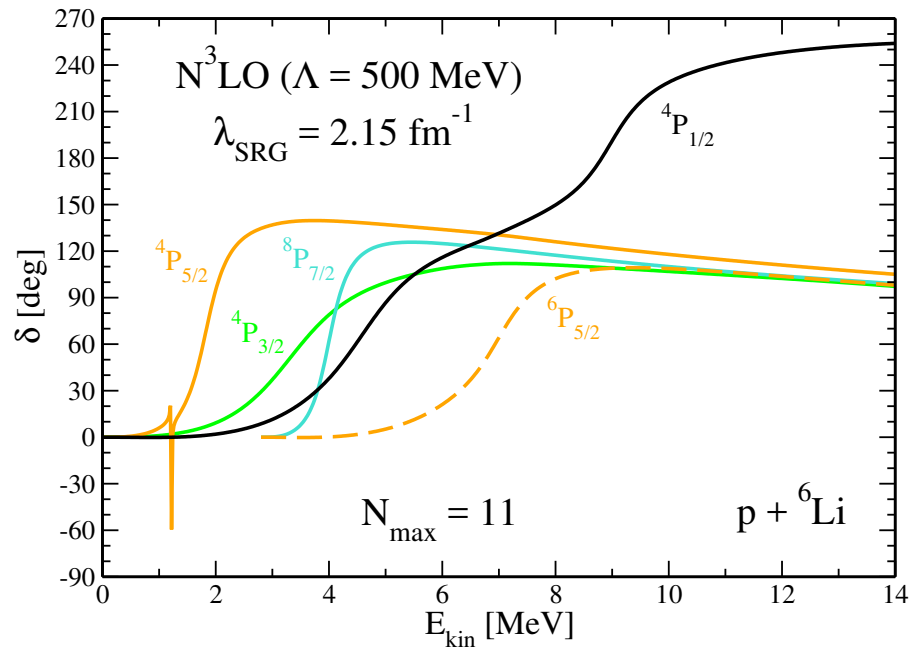




# ${}^7\text{Be}$ – Reproducing the energy spectrum



# ${}^7\text{Be}$ – New negative-parity states



# ${}^7\text{Li}$ system

## Analyzed mass partitions

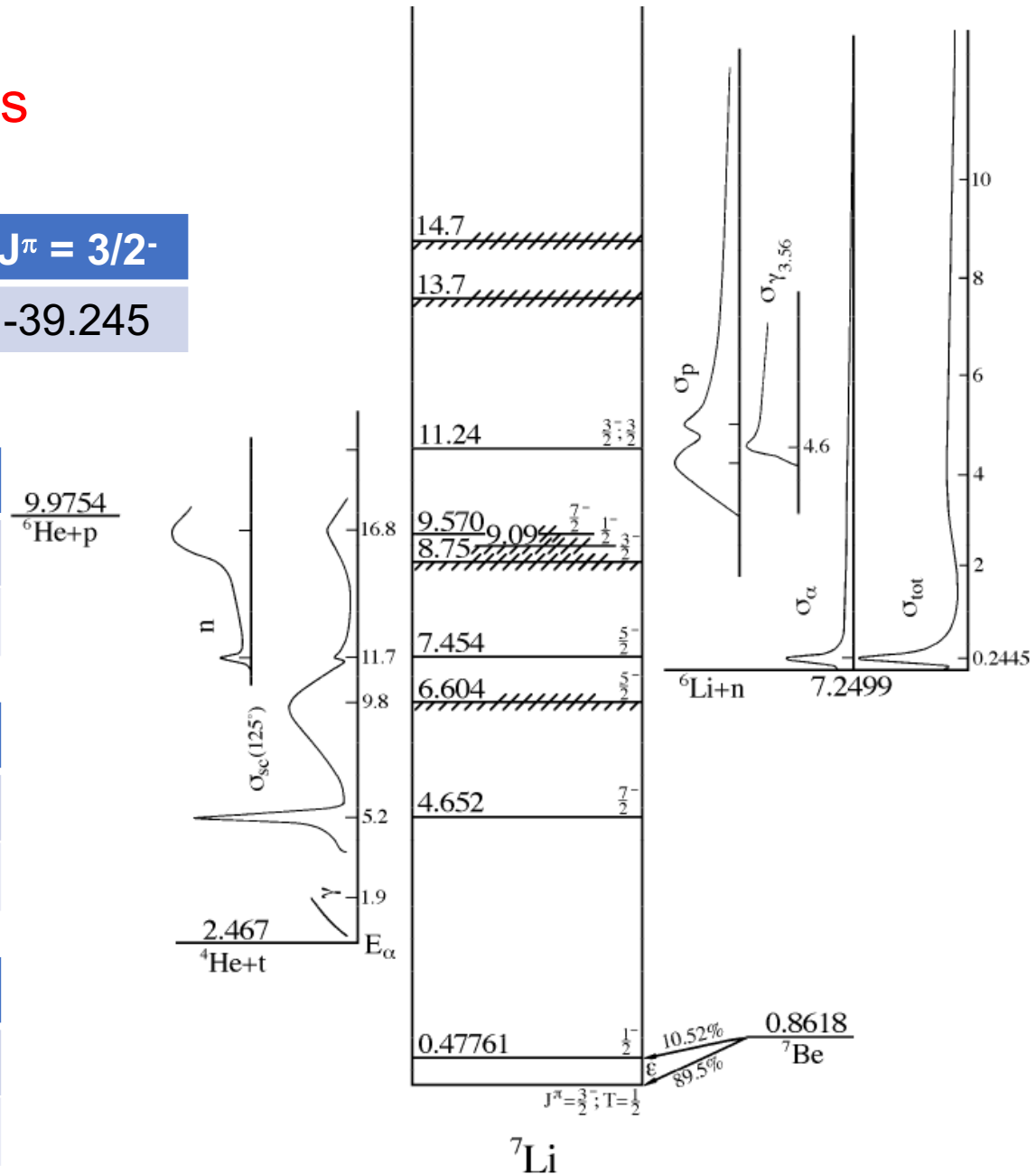
- ${}^3\text{H} + {}^4\text{He}$
- $n + {}^6\text{Li}$
- $p + {}^6\text{He}$

Exp.	$J^\pi = 3/2^-$
E [MeV]	-39.245

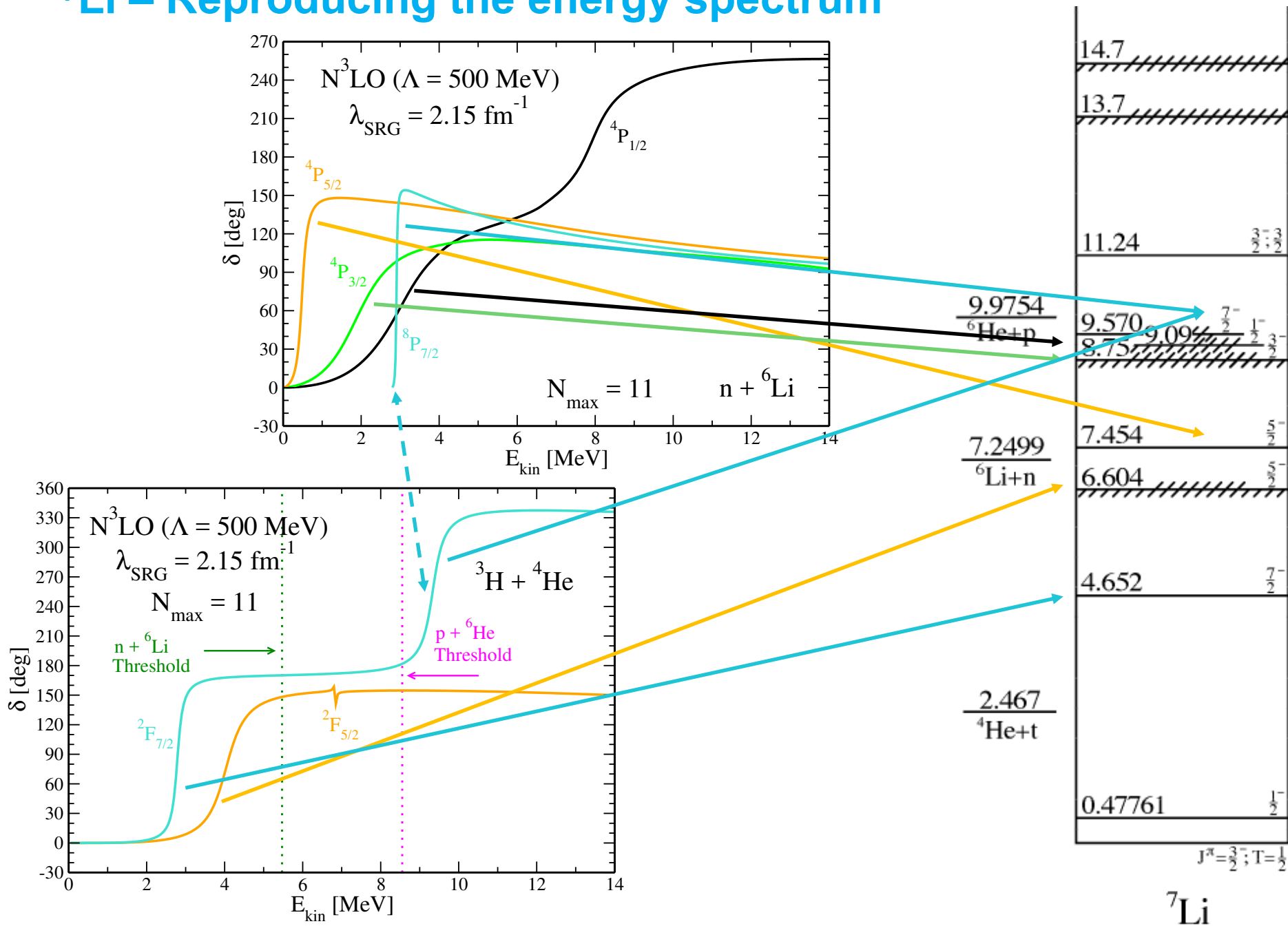
${}^3\text{H} + {}^4\text{He}$	$J^\pi = 3/2^-$	$J^\pi = 1/2^-$
$E_{\text{bound}}$	-2.432	-2.153
E [MeV]	-38.65	-38.37

$n + {}^6\text{Li}$	$J^\pi = 3/2^-$	$J^\pi = 1/2^-$
$E_{\text{bound}}$	-7.381	-7.048
E [MeV]	-38.13	-37.79

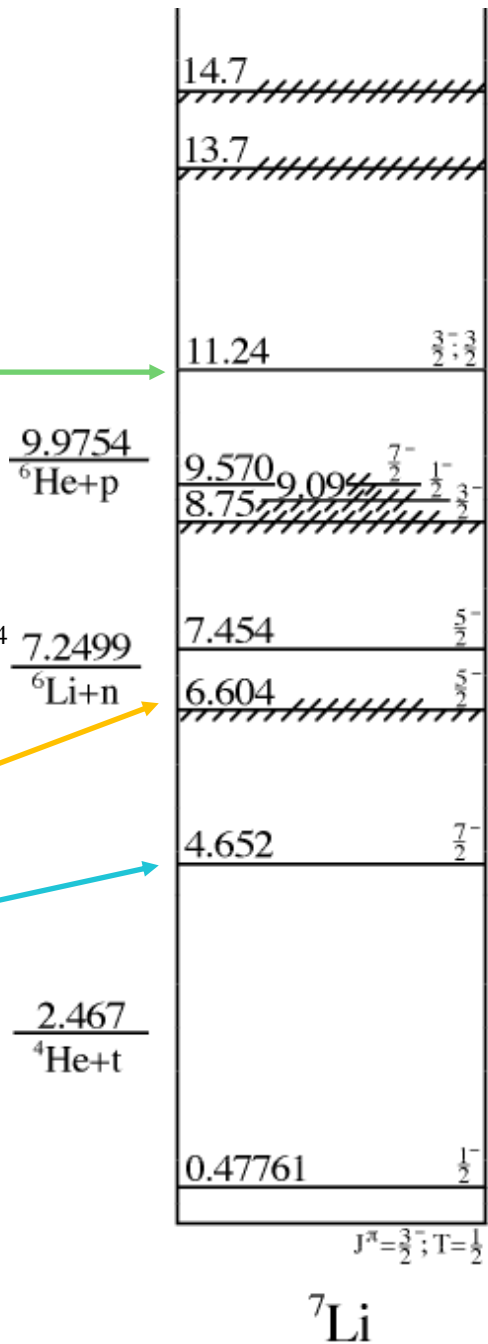
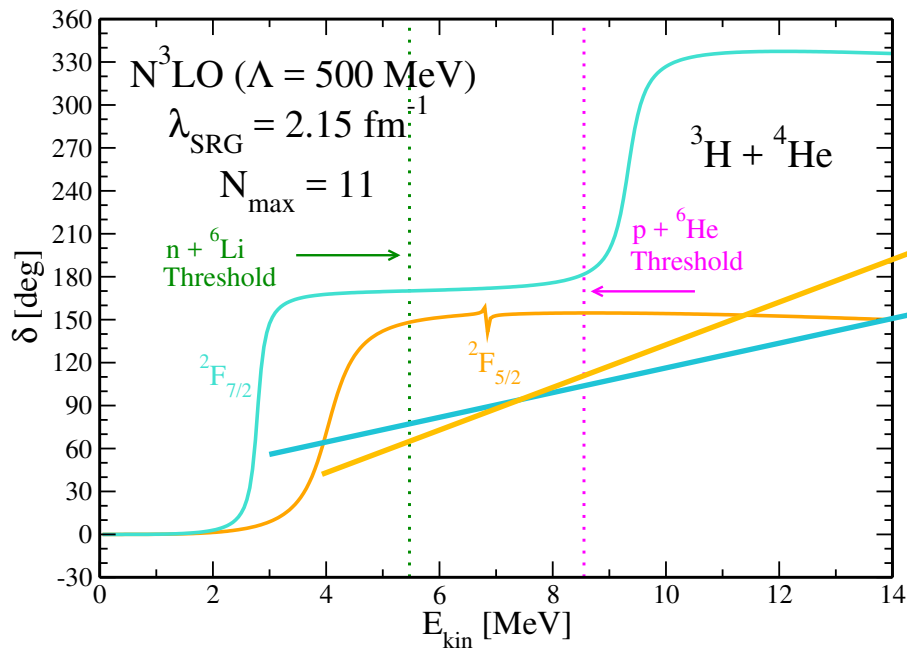
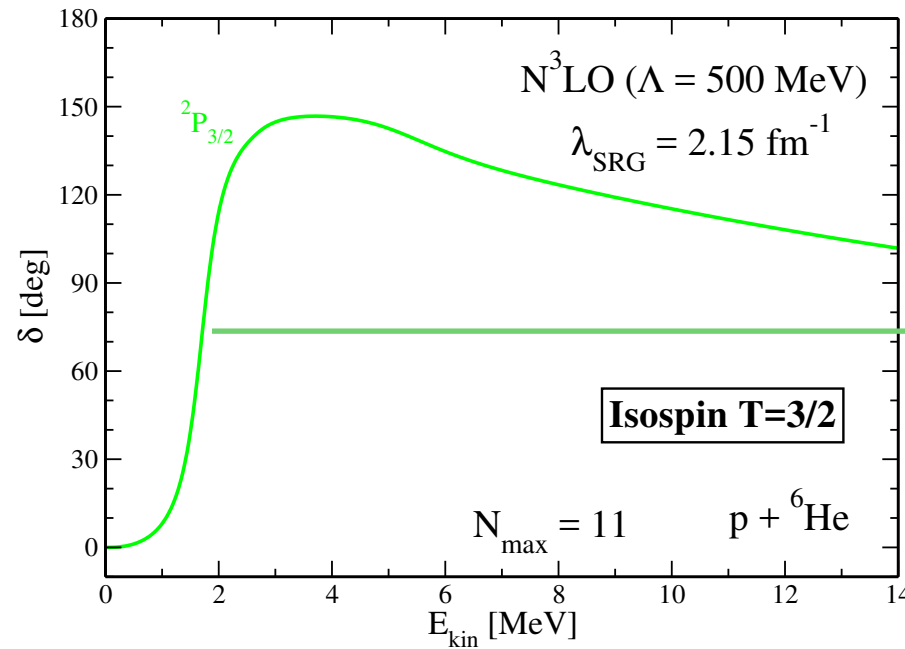
$p + {}^6\text{He}$	$J^\pi = 3/2^-$	$J^\pi = 1/2^-$
$E_{\text{bound}}$	-10.40	-10.06
E [MeV]	-38.06	-37.73



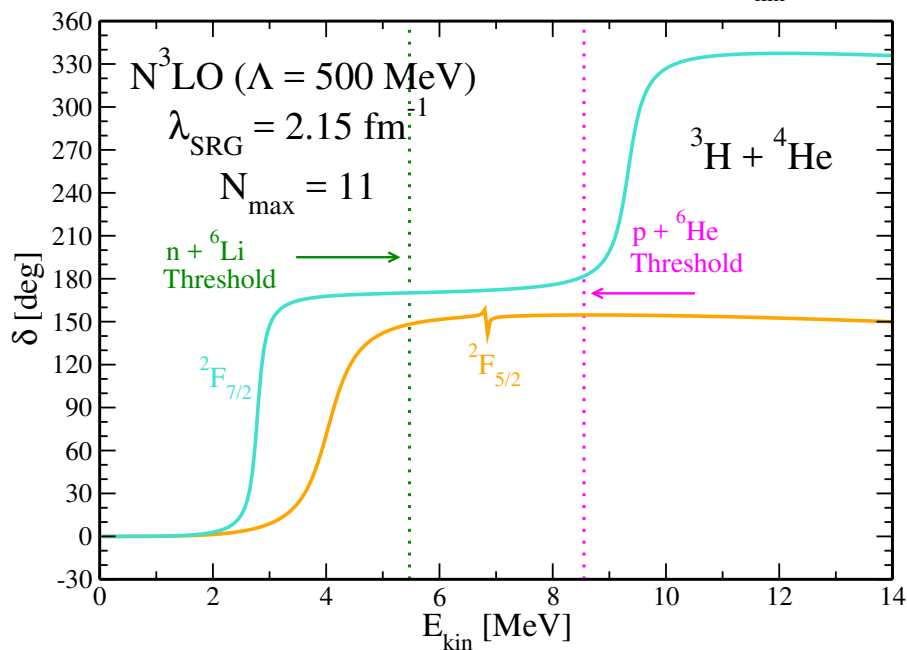
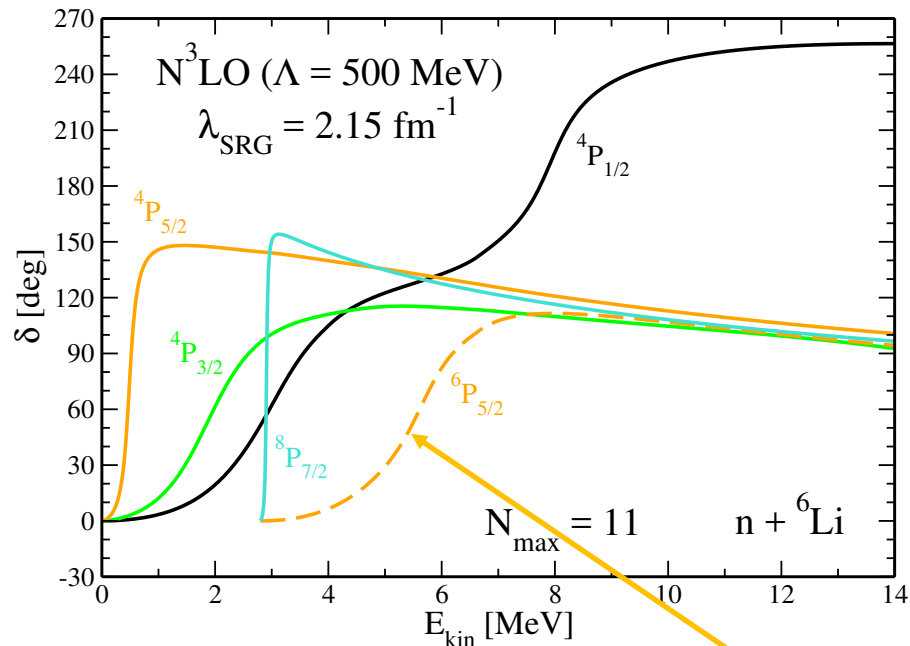
# ${}^7\text{Li}$ – Reproducing the energy spectrum



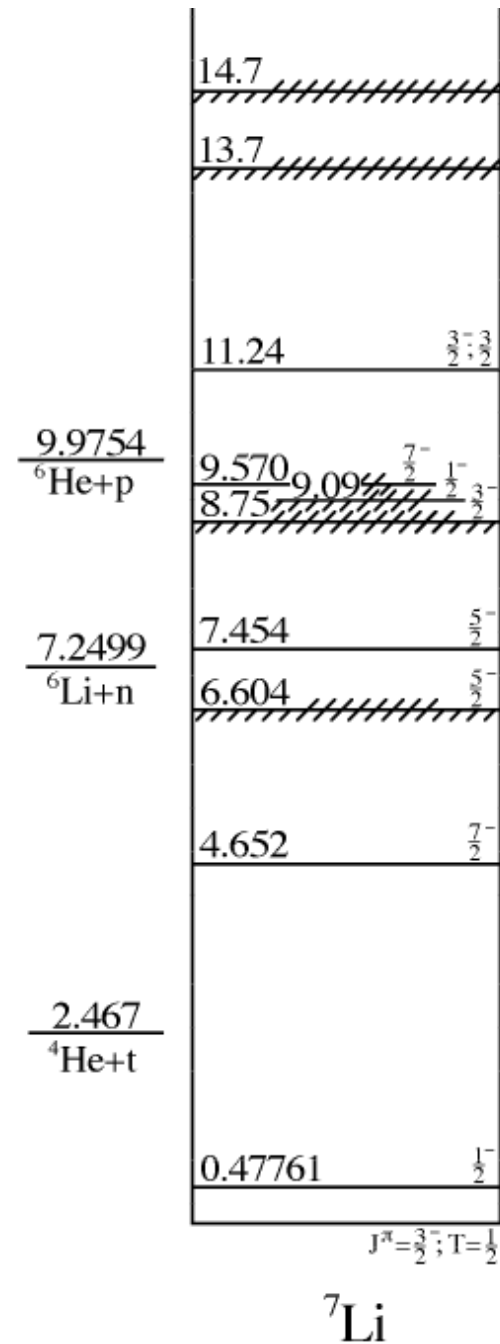
# ${}^7\text{Li}$ – Reproducing the energy spectrum



# ${}^7\text{Li}$ – New negative-parity states

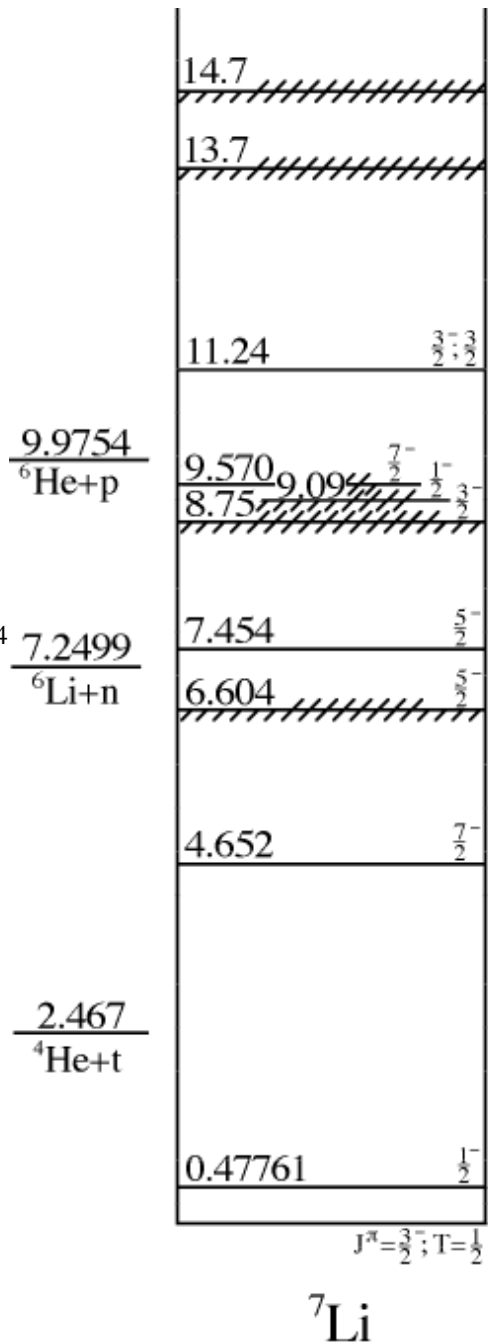
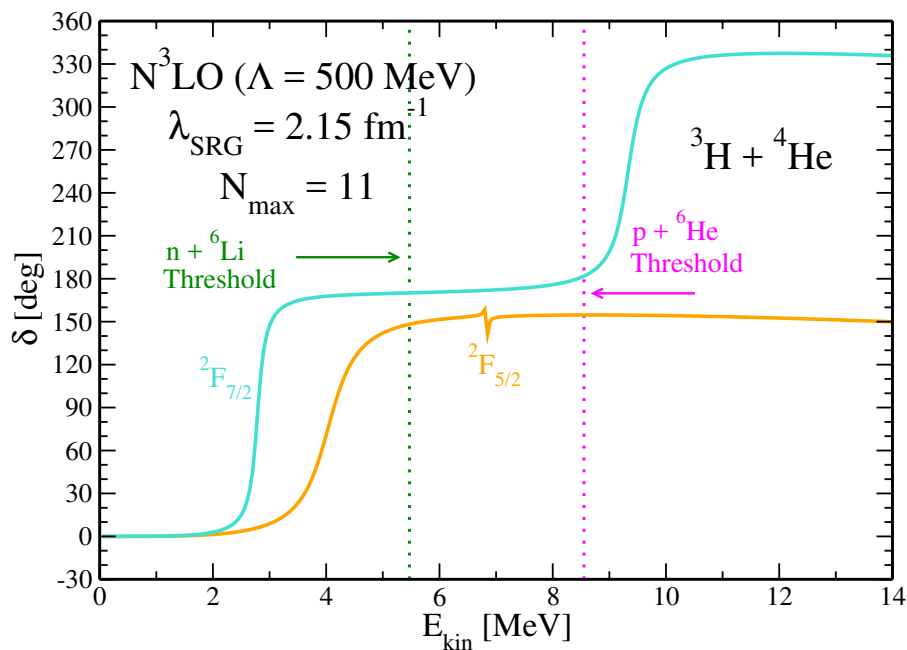
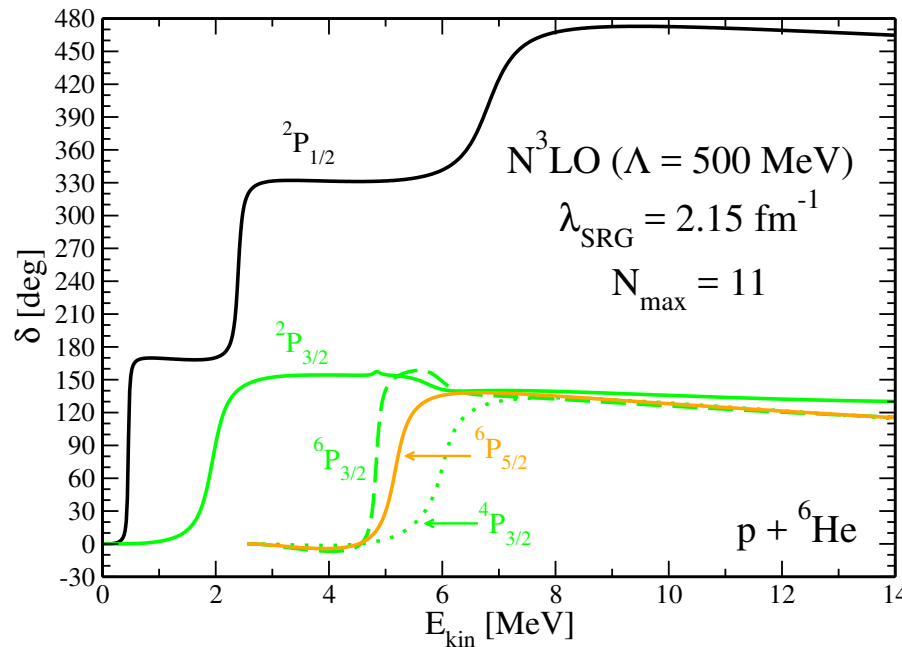


NEW



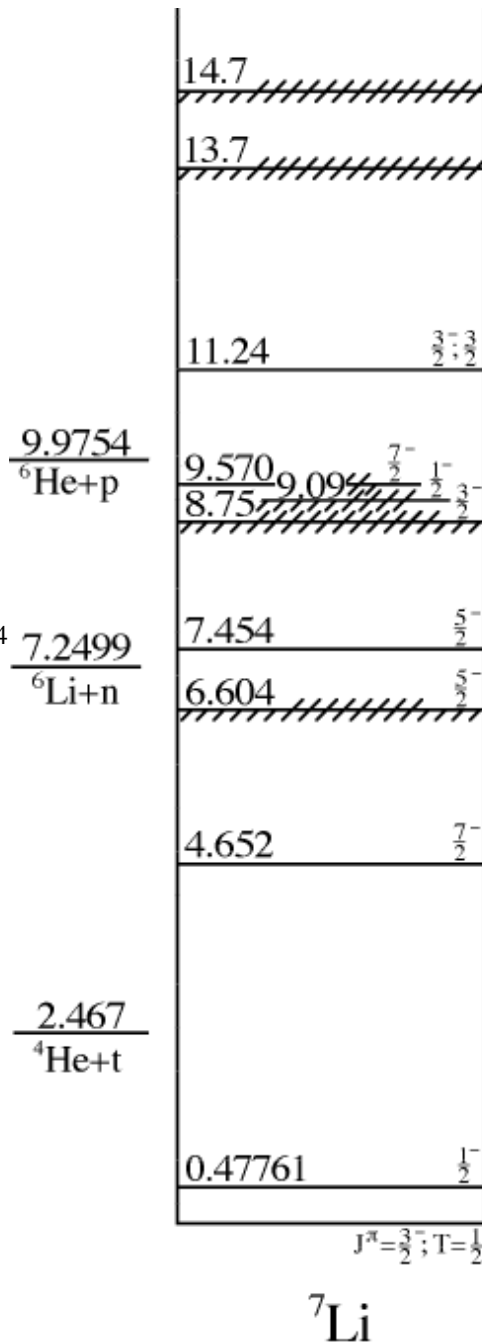
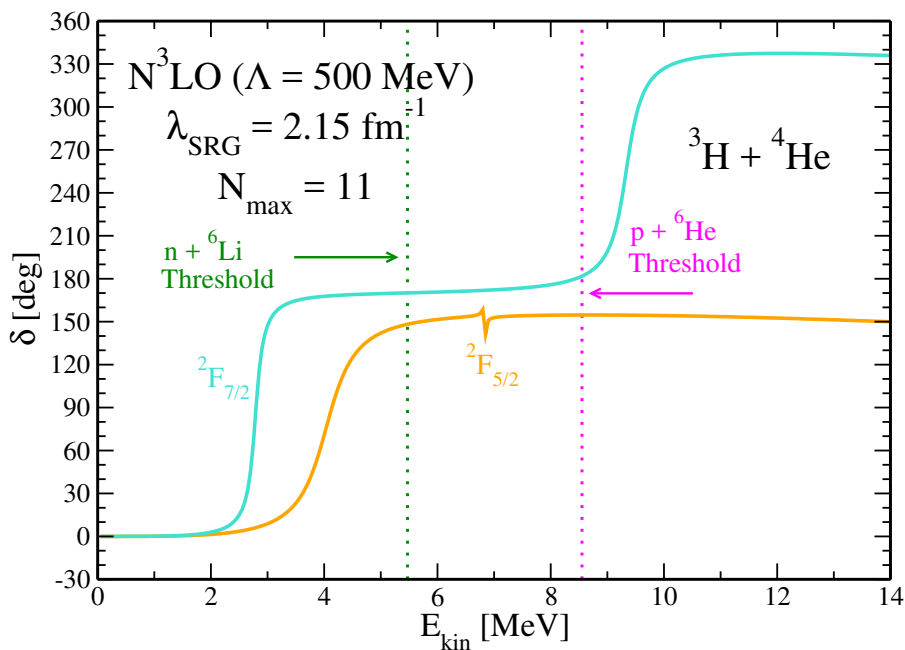
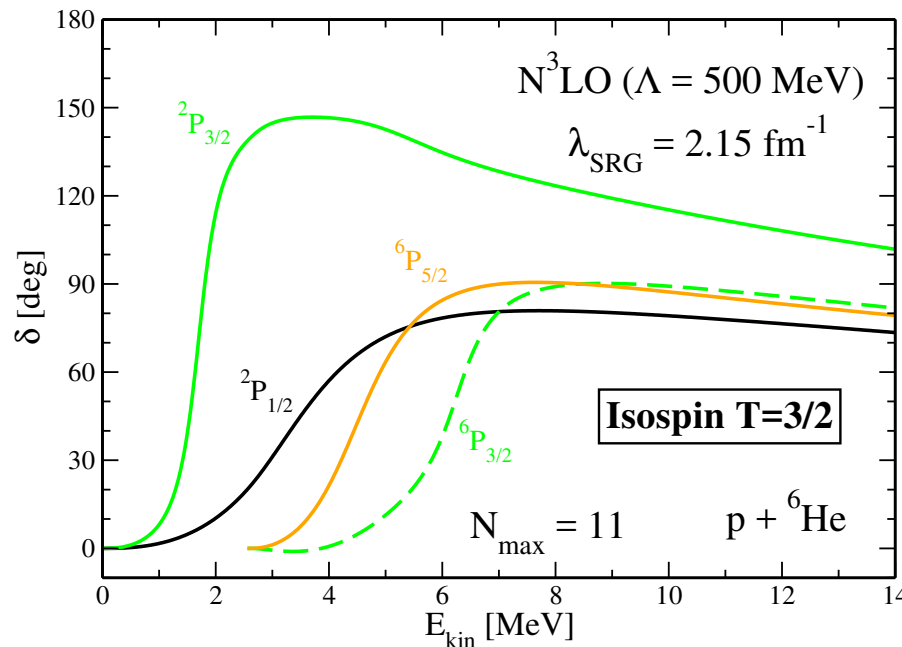
# ${}^7\text{Li}$ – New negative-parity states

5 new  $T=1/2$  resonances



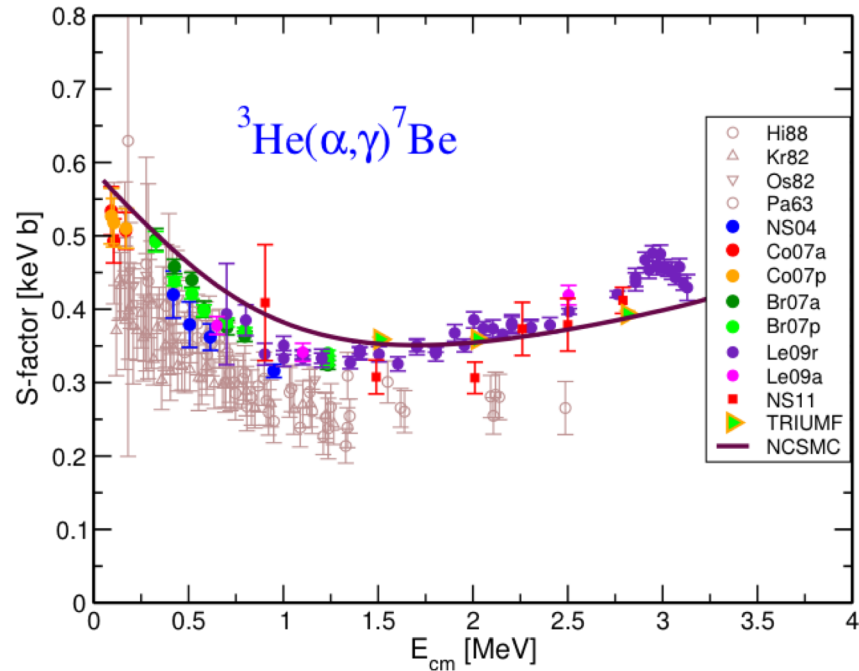
# ${}^7\text{Li}$ – New negative-parity states

3 new  $T=3/2$  resonances





# S-factor for ${}^3\text{He}(\alpha,\gamma){}^7\text{Be}$ and ${}^3\text{H}(\alpha,\gamma){}^7\text{Li}$ reactions

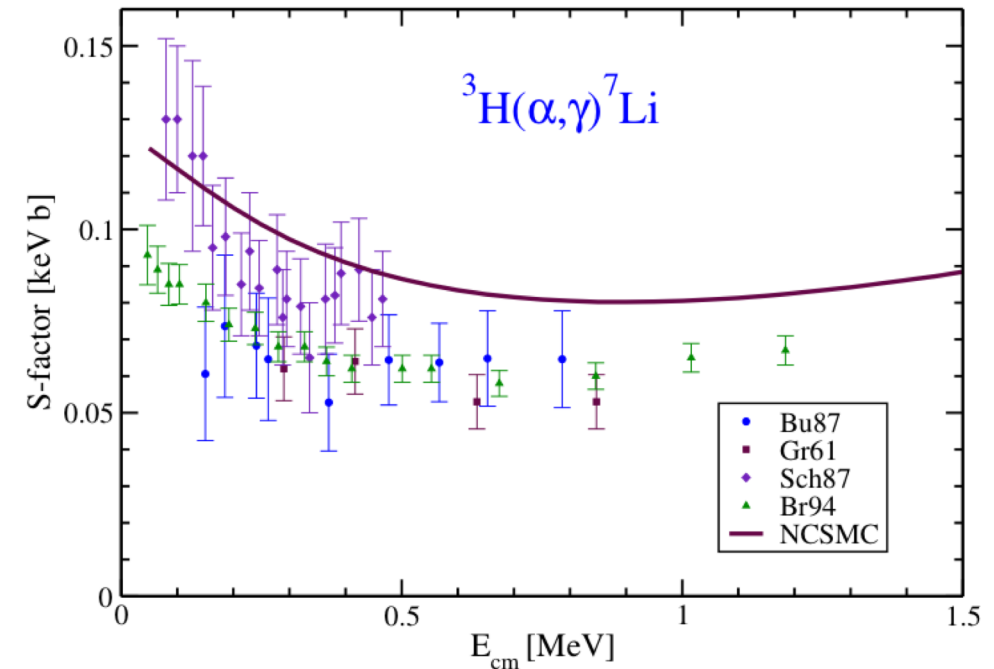


Cross section and S-factor

$$\sigma(E) = S(E)E^{-1} \exp[-2\pi\eta(E)]$$

Sommerfeld parameter

$$\eta(E) = \frac{Z_1 Z_2 e^2}{\hbar v}$$



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${}^3\text{He}(\alpha,\gamma){}^7\text{Be}$  and  ${}^3\text{H}(\alpha,\gamma){}^7\text{Li}$  astrophysical S factors from the no-core shell model with continuum



Jérémy Dohet-Eraly<sup>a,\*</sup>, Petr Navrátil<sup>a</sup>, Sofia Quaglioni<sup>b</sup>, Wataru Horiuchi<sup>c</sup>,  
Guillaume Hupin<sup>b,d,1</sup>, Francesco Raimondi<sup>a,2</sup>

## Conclusions

- *Ab initio* calculations of nuclear structure and reactions with predictive power becoming feasible beyond the lightest nuclei
- *Ab initio* structure calculations can even reach (selected) medium & medium-heavy mass nuclei
- These calculations make connections between the low-energy QCD, many-body systems, and nuclear astrophysics

Thank you!  
Merci!

