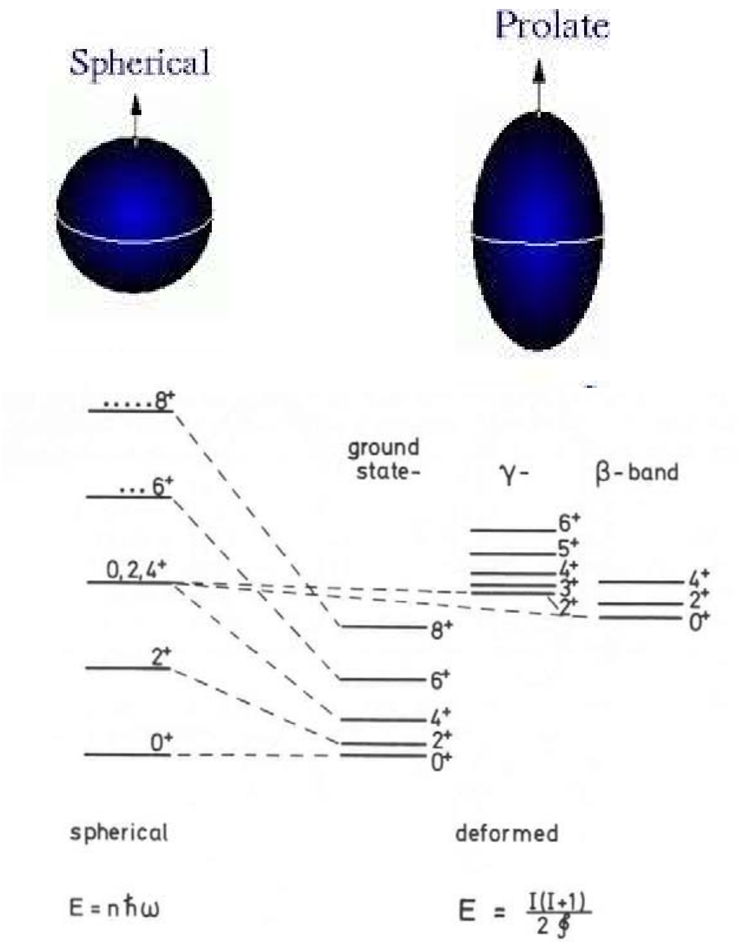
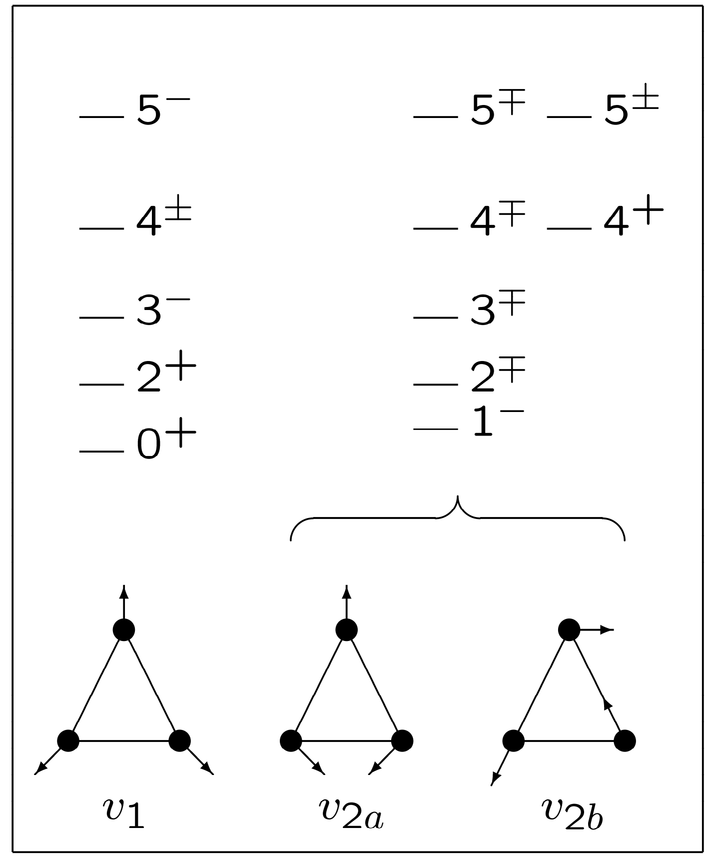
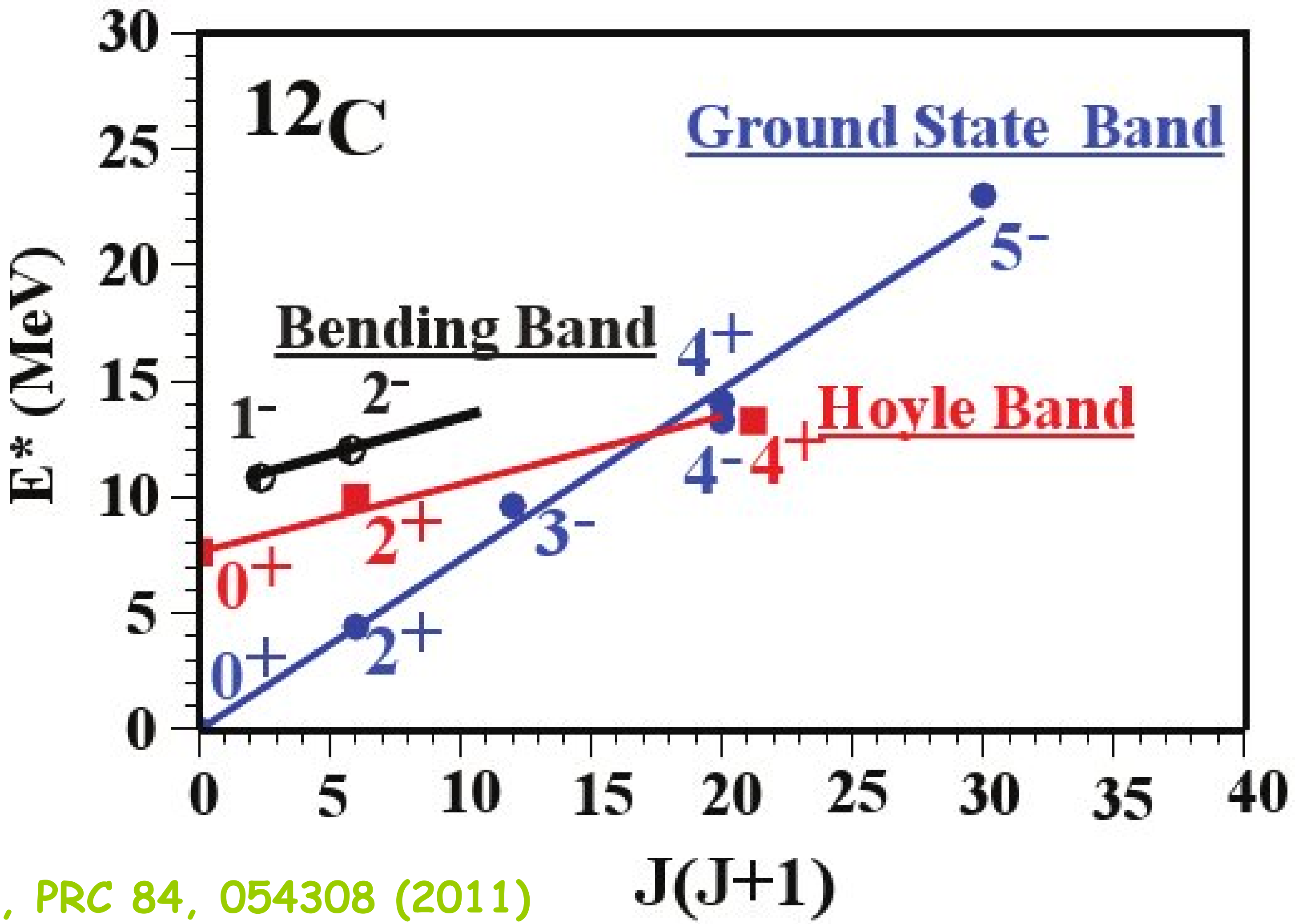
Single-Particle Levels in Cluster Potentials

- Introduction
- Algebraic Cluster Model
- Applications: ^{12}C y ^{16}O
- Cluster Shell Model
- Odd cluster nuclei
- Summary and conclusions

Nuclear Shapes







Itoh et al, PRC 84, 054308 (2011)
Freer et al, PRC 86, 034320 (2012)
Zimmerman et al, PRL 110, 152502 (2013)

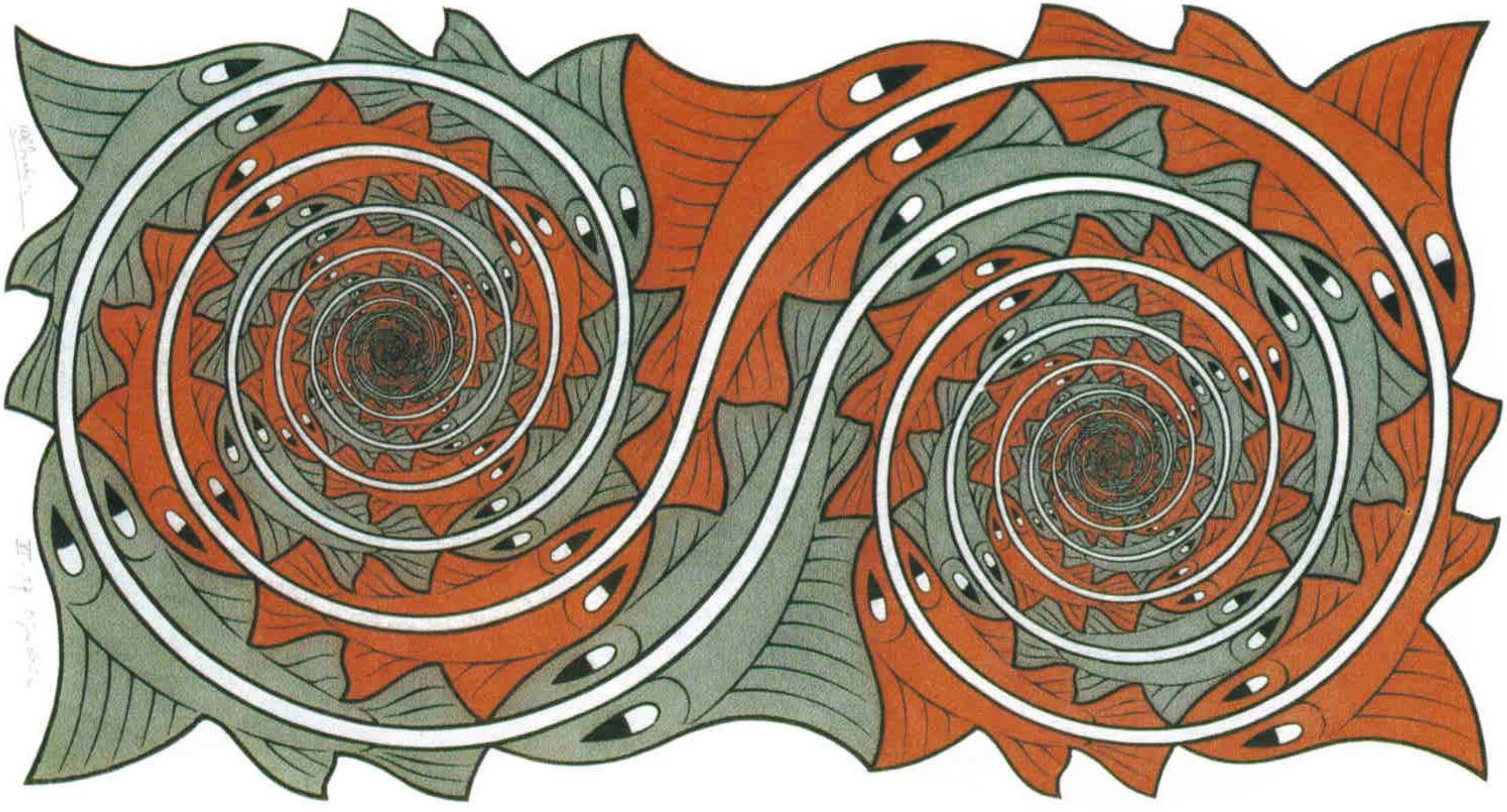
Marín-Lámbarri et al,
PRL 113, 012502 (2014)

Recent Theoretical Work

- Alpha-cluster model (Wheeler 1937, Brink 1966, Robson, 1978)

- AMD (Kanada-Enyo, PTP, 2007)
- FMD model (Chernykh et al, PRL, 2007)
- BEC-like cluster model (Funaki et al, PRC, 2009)
- Ab initio no-core shell model (Roth et al, PRL, 2011)
- Lattice EFT (Epelbaum et al, PRL, 2011, 2012)
- No-core symplectic model (Dreyfuss et al, PLB, 2013)
- SACM (Cseh et al)
- Algebraic Cluster Model (2000, 2002, 2014, 2017)

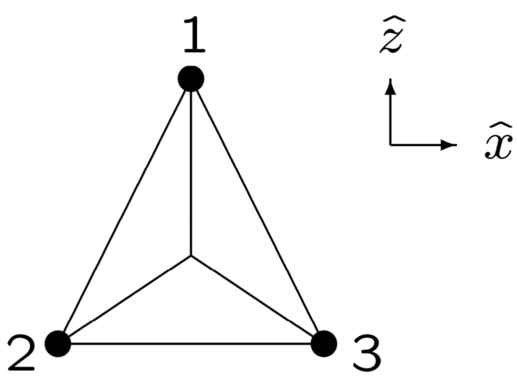
- Recent reviews: Freer & Fynbo, PPNP 78, 1 (2014), Freer, Horiuchi, Kanada-En'yo, Lee & Meissner, RMP 90, 035004 (2018)

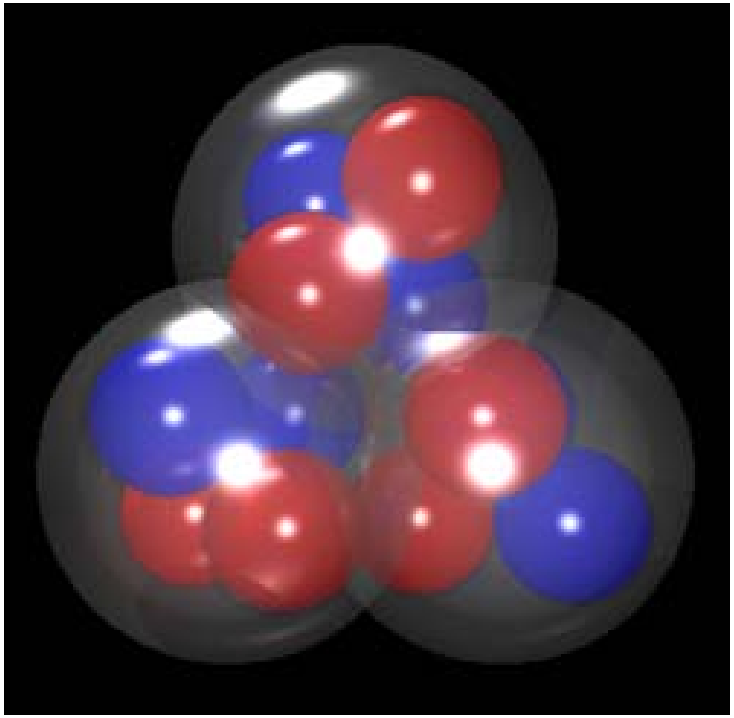


Three-Body System

$$\rho(\vec{r}) = \frac{Ze}{3}\left(\frac{\alpha}{\pi}\right)^{3/2} \sum_{i=1}^{3} e^{-\alpha(\vec{r}-\vec{r}_i)^2}$$

$$\vec{r}_1 = (\beta, 0, 0)$$
$$\vec{r}_2 = (\beta, 2\pi/3, \pi)$$
$$\vec{r}_2 = (\beta, 2\pi/3, 0)$$





Algebraic Cluster Model (ACM)

6 relative degrees of freedom: Jacobi vectors

$$\vec{\rho}_1 = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) \ , \qquad \vec{\rho}_2 = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)$$

Introduce 2 dipole bosons and a scalar boson

$$b_1^\dagger \ , \ b_2^\dagger \ , \ s^\dagger$$

One- and two-body S_3 invariant Hamiltonian

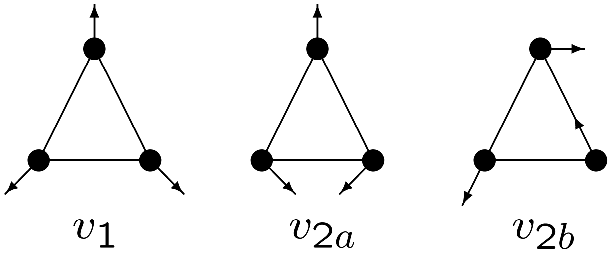
$$\begin{aligned} H \ = \ & \epsilon_0 \, s^\dagger \tilde{s} - \epsilon_1 \sum_i b_i^\dagger \cdot \tilde{b}_i + u_0 \, s^\dagger s^\dagger \, \tilde{s}\tilde{s} \\ & - u_1 \sum_i s^\dagger b_i^\dagger \cdot \tilde{b}_i \tilde{s} + v_0 \sum_i \left[b_i^\dagger \cdot b_i^\dagger \, \tilde{s}\tilde{s} + \text{h.c.} \right] \\ & + \sum_L \sum_{iji'j'} v_{iji'j'}^{(L)} \, [b_i^\dagger \times b_j^\dagger]^{(L)} \cdot [\tilde{b}_{i'} \times \tilde{b}_{j'}]^{(L)} \end{aligned}$$

Mixing between different oscillator shells

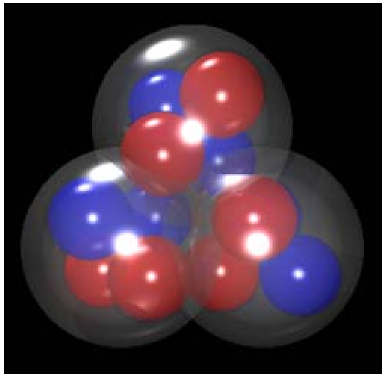
Oblate Top: Triangle

$$
\begin{aligned}
H_{\text{vib}} &= \xi_1 \, (s^\dagger s^\dagger - b_1^\dagger \cdot b_1^\dagger - b_2^\dagger \cdot b_2^\dagger) \, (\text{h.c.}) \\
&\quad + \xi_2 \left[(b_1^\dagger \cdot b_1^\dagger - b_2^\dagger \cdot b_2^\dagger) \, (\text{h.c.}) + 4 (b_1^\dagger \cdot b_2^\dagger)(\text{h.c.}) \right]
\end{aligned}
$$

Equilibrium shape:
equilateral triangle

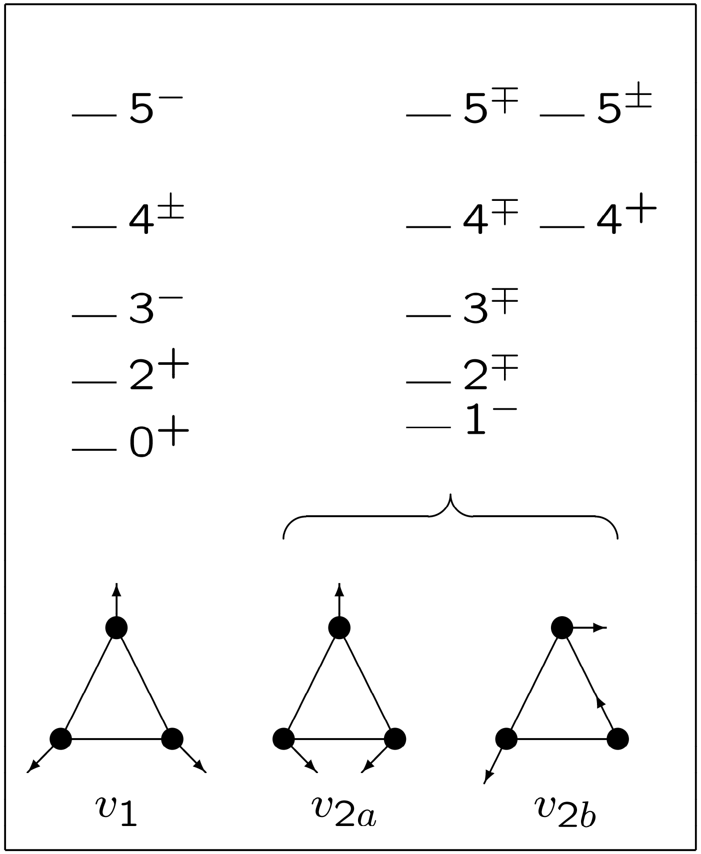


$$
\begin{aligned}
E_{\text{vib}} &\approx \omega_1 (v_1 + \frac{1}{2}) + \omega_2 (v_2 + 1) \\
\omega_1 &= 4N\xi_1 \\
\omega_2 &= 2N\xi_2
\end{aligned}
$$



Bijker, Iachello & Leviatan, *AP* 236, 69 (1994)
Bijker & Iachello, *AP* 298, 334 (2002)

Energy Spectrum



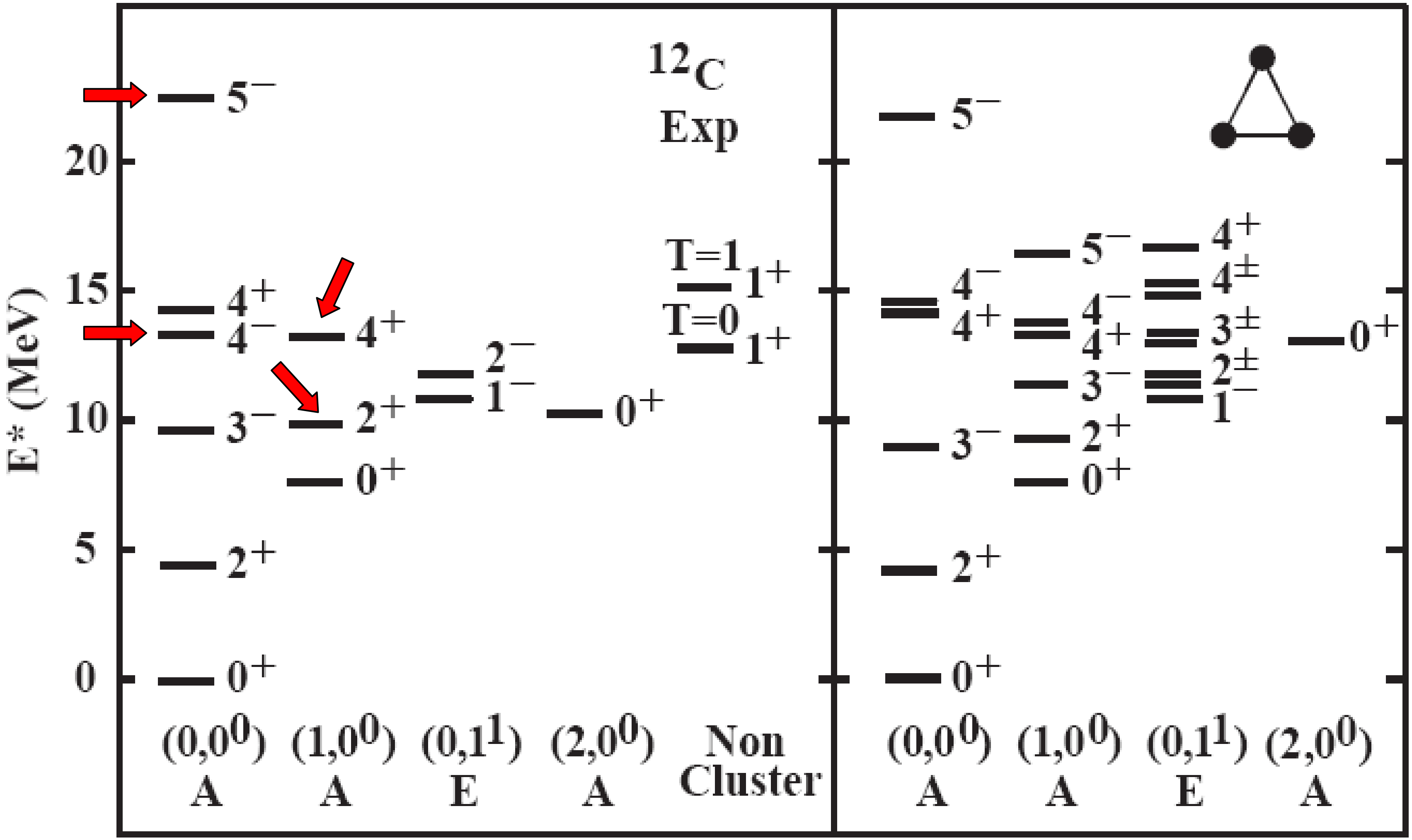
Ground state and Hoyle band (breathing vibration)

$$L^P = 0^+, 2^+, 3^-, 4^\pm, 5^-, \ldots$$

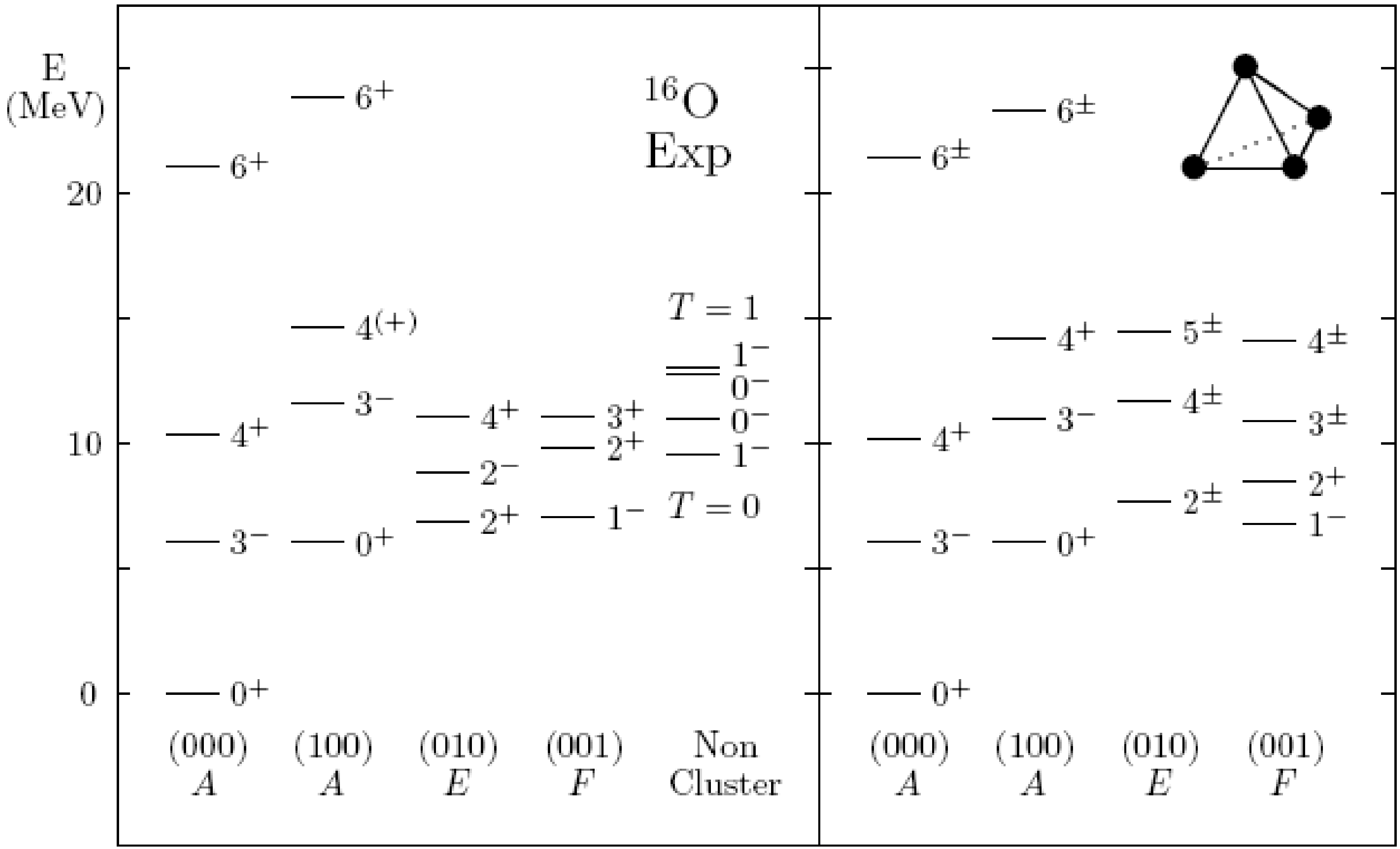
Bending vibration

$$L^P = 1^-, 2^\mp, 3^\mp, \ldots$$

Fingerprint of triangular shape with D_{3h} symmetry



12C
Exp
E* (MeV)
20
15
10
5
0
5−
4+
4−
3−
2+
0+
4+
2+
0+
2−
1−
0+
T=1
1+
T=0
1+
(0,0^0) A
(1,0^0) A
(0,1^1) E
(2,0^0) A
Non Cluster
5−
4−
4+
3−
2+
0+
5−
4−
4+
3−
2+
0+
4+
4±
3±
2±
1−
0+
(0,0^0) A
(1,0^0) A
(0,1^1) E
(2,0^0) A



Bijker & Iachello, PRL 112, 152501 (2014)
NPA 957, 154 (2017)

Electric Transitions

$$\rho(\vec{r}) = \frac{Ze}{k}\left(\frac{\alpha}{\pi}\right)^{3/2} \sum_{i=1}^{k} e^{-\alpha(\vec{r}-\vec{r}_i)^2}$$

$$\mathcal{F}_L(q) = c_L\, j_L(q\beta)\, e^{-q^2/4\alpha}$$

$$\langle r^2 \rangle^{1/2} = \sqrt{\beta^2 + 3/2\alpha}$$

$$B(EL; 0^+ \to L^P) = \frac{(Ze)^2\, c_L^2\, \beta^{2L}}{4\pi}$$

$$c_L^2 = \begin{cases} \frac{2L+1}{2}\left[1 + P_L(-1)\right] & 2\alpha\text{-cluster} \\ \frac{2L+1}{3}\left[1 + 2P_L(-\frac{1}{2})\right] & 3\alpha\text{-cluster} \\ \frac{2L+1}{4}\left[1 + 3P_L(-\frac{1}{3})\right] & 4\alpha\text{-cluster} \end{cases}$$

$$Q_{2^+} = \begin{cases} -\frac{4}{7}Ze\beta^2 & 2\alpha\text{-cluster} \\ +\frac{2}{7}Ze\beta^2 & 3\alpha\text{-cluster} \end{cases}$$

	β (fm)	α (1/fm^2)
^{4}He	0.00	0.56
^{8}Be	1.82	0.56
^{12}C	1.74	0.56
^{16}O	2.07	0.56

$\langle r^2 \rangle^{1/2}$	ACM (fm)	Exp. (fm)
^{4}He	1.674	1.674 ± 0.012
^{8}Be	2.448	
^{12}C	2.389	2.468 ± 0.012
^{16}O	2.639	2.710 ± 0.015

Electric Transitions

		ACM	Exp.	GFMC	
^{8}Be	$B(E2;2_1^+\to 0_1^+)$	14		20.0 ± 0.8	e^2fm^4
	$B(E2;4_1^+\to 2_1^+)$	20	$21\pm 2.3^*$	27.2 ± 1.5	e^2fm^4
	$B(E4;4_1^+\to 0_1^+)$	153			e^2fm^8
^{12}C	$B(E2;2_1^+\to 0_1^+)$	8.4	7.6 ± 0.4		e^2fm^4
	$B(E3;3_1^-\to 0_1^+)$	73	103 ± 17		e^2fm^6
	$B(E4;4_1^+\to 0_1^+)$	44			e^2fm^8
^{16}O	$B(E3;3_1^-\to 0_1^+)$	215	205 ± 10		e^2fm^6
	$B(E4;4_1^+\to 0_1^+)$	425	378 ± 133		e^2fm^8
	$B(E6;6_1^+\to 0_1^+)$	9626			e^2fm^{12}

Parameter Free:

Consequence of Symmetry

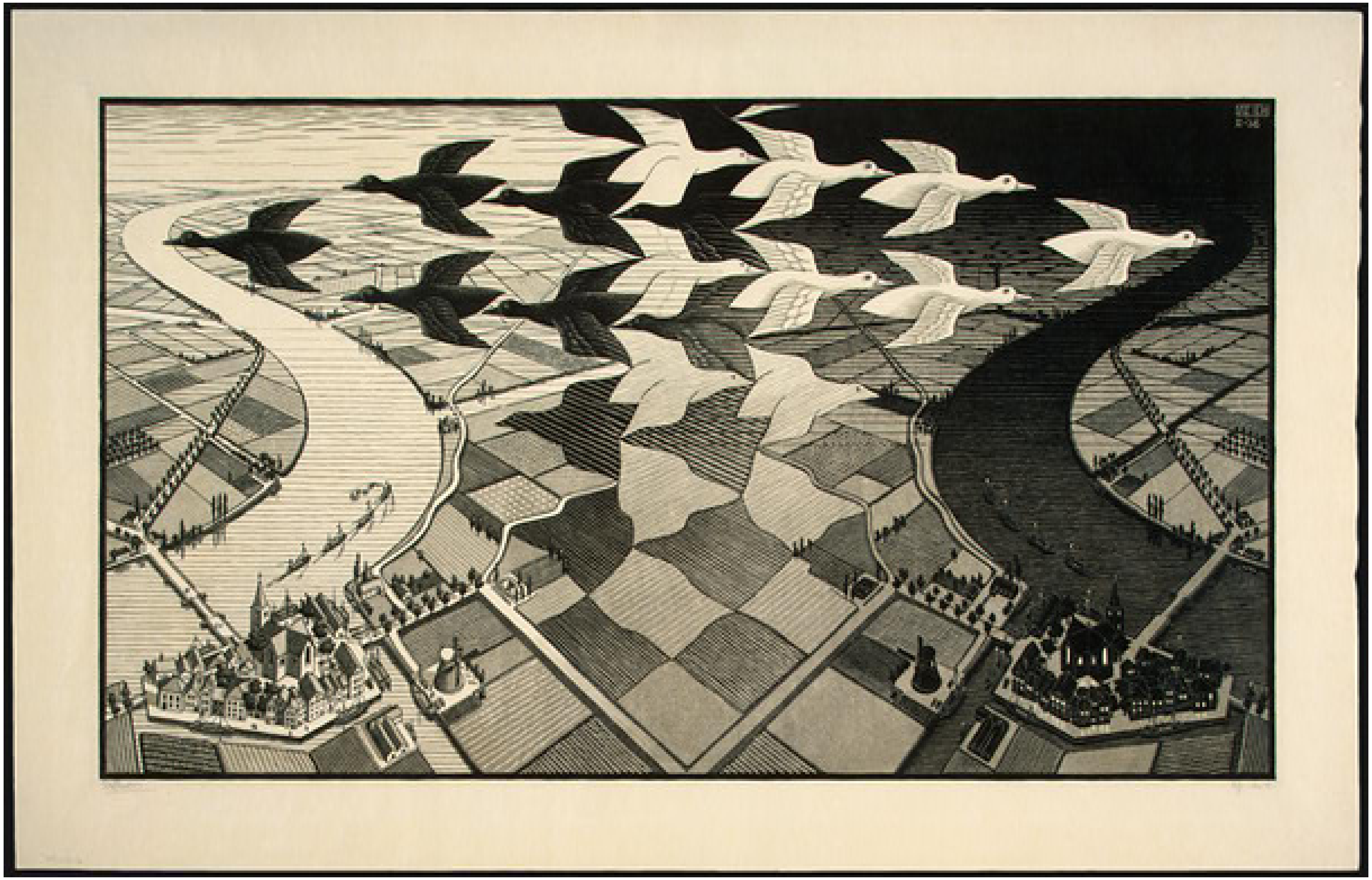
$Q_{2_1^+}$	ACM (efm^2)	Exp. (efm^2)	GFMC (efm^2)
^{8}Be	-7.6		-9.1 ± 0.2
^{12}C	$+5.2$	$+5.3\pm 4.4$	

* Estimated value GFMC

Datar et al, PRL 111, 062502 (2013)

Algebraic Cluster Model

	2α	3α	4α
ACM	$U(4)$	$U(7)$	$U(10)$
Point group	$\mathcal{Z}_2$	$\mathcal{D}_{3h}$	$\mathcal{T}_d$
Geom. conf.	Linear	Triangle	Tetrahedron
Model	Rotor	Oblate top	Spherical top
Vibrations	1	3	6
Rotations	2	3	3
G.s. band	0^+	0^+	0^+
	2^+	2^+	
		3^-	3^-
	4^+	$4^\pm$	4^+
		5^-	
	6^+	$6^{\pm +}$	$6^\pm$
Large Eλ electric transitions!			



Odd Cluster Nuclei

- What are the signatures of α-clustering in odd-mass nuclei?
- First step: Cluster Shell Model (CSM)
- Splitting of sp levels in cluster potentials
- Second step: Algebraic Cluster-Fermion Model (ACFM)

Cluster Shell Model

Cluster density

$$\rho(\vec{r}) = \frac{Ze}{k}\left(\frac{\alpha}{\pi}\right)^{3/2}\sum_{i=1}^{k}\exp\left[-\alpha\left(\vec{r}-\vec{r}_i\right)^2\right]$$

$$= \frac{Ze}{k}\left(\frac{\alpha}{\pi}\right)^{3/2} e^{-\alpha(r^2+\beta^2)} 4\pi \sum_{\lambda\mu} i_\lambda(2\alpha\beta r) Y_{\lambda\mu}(\theta,\phi) \sum_{i=1}^{k} Y^*_{\lambda\mu}(\theta_i,\phi_i)$$

$$\vec{r}_i = (\beta, \theta_i, \phi_i)$$

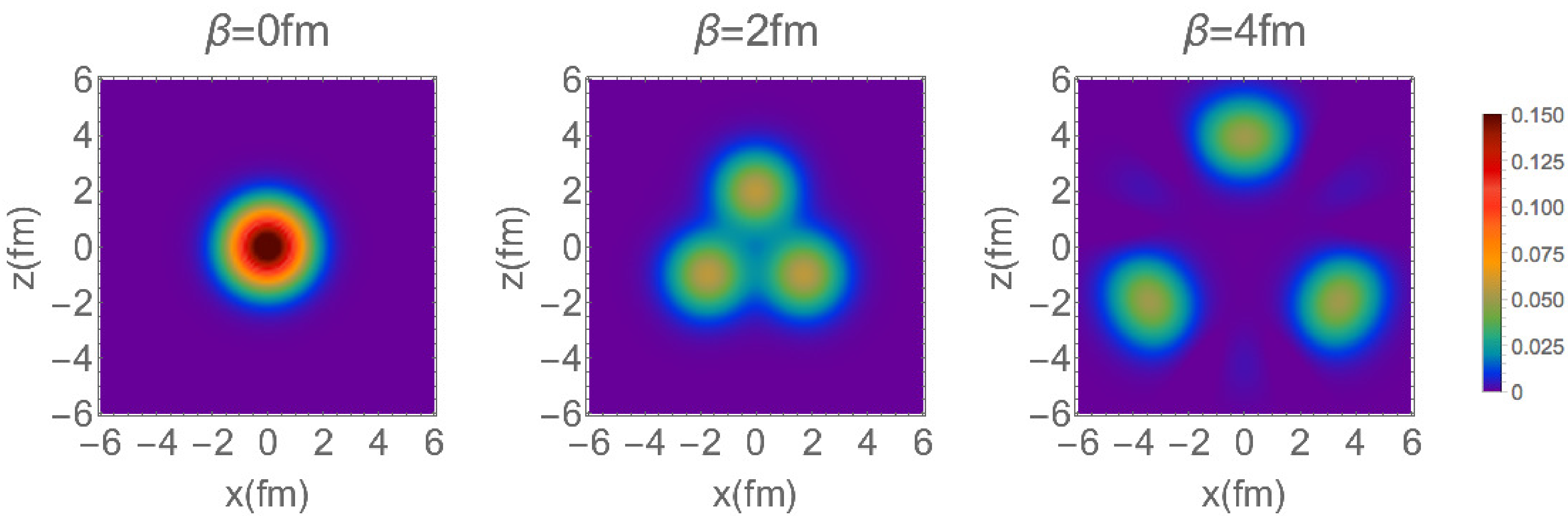
Cluster potential

$$H = \sum_i \frac{\vec{p}_i^{\,2}}{2m} + V(\vec{r}) + V_{\text{so}}(\vec{r}) + V_{\text{C}}(\vec{r})$$

Della Rocca, Bijker & Iachello
NPA 966, 158 (2017)

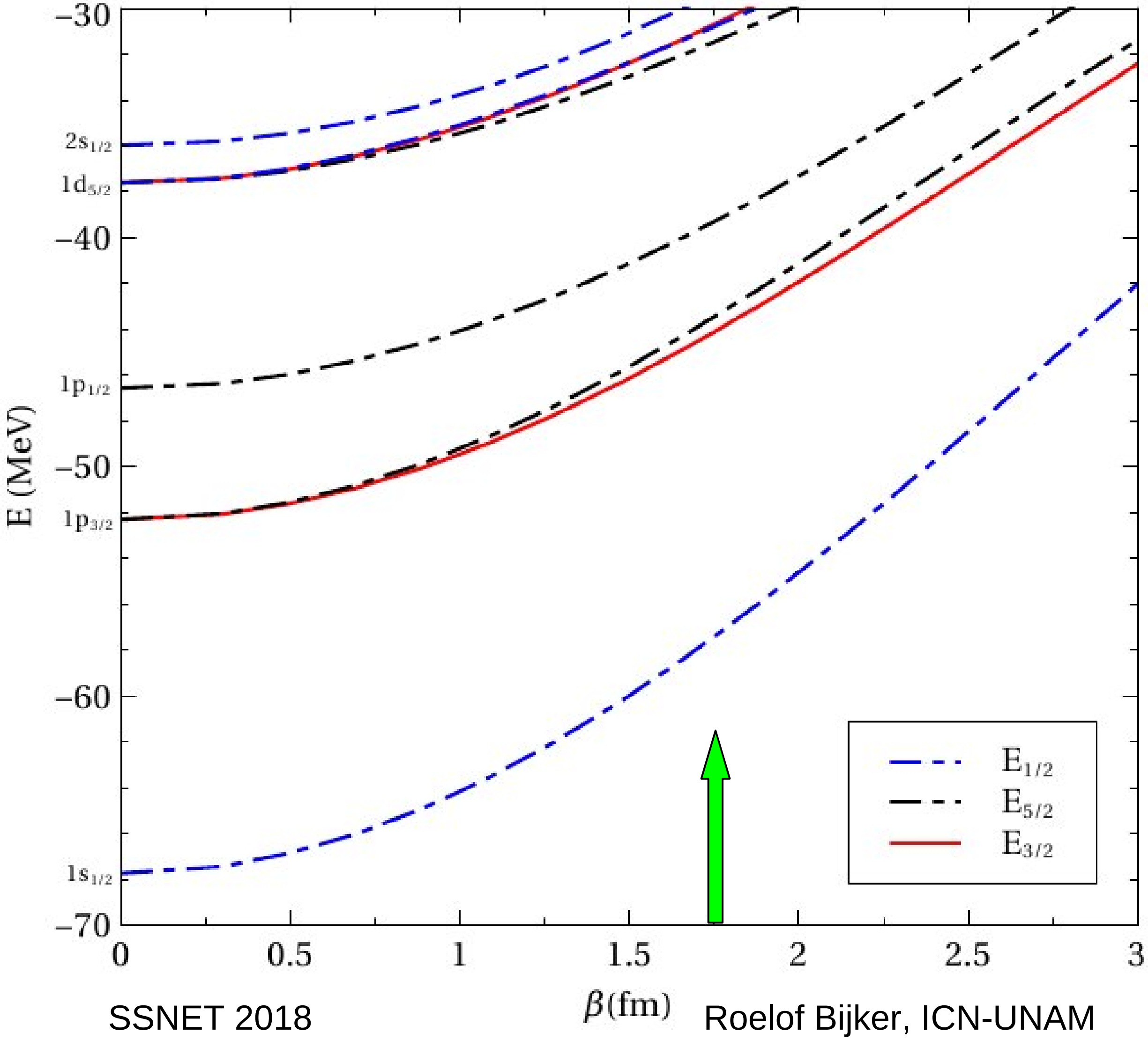
Adrian Horacio Santana Valdés
Master Thesis, UNAM (2018)

Densitities of 3-α cluster



Della Rocca, Bijker & Iachello, NPA 966, 158 (2017)

Triangular Symmetry

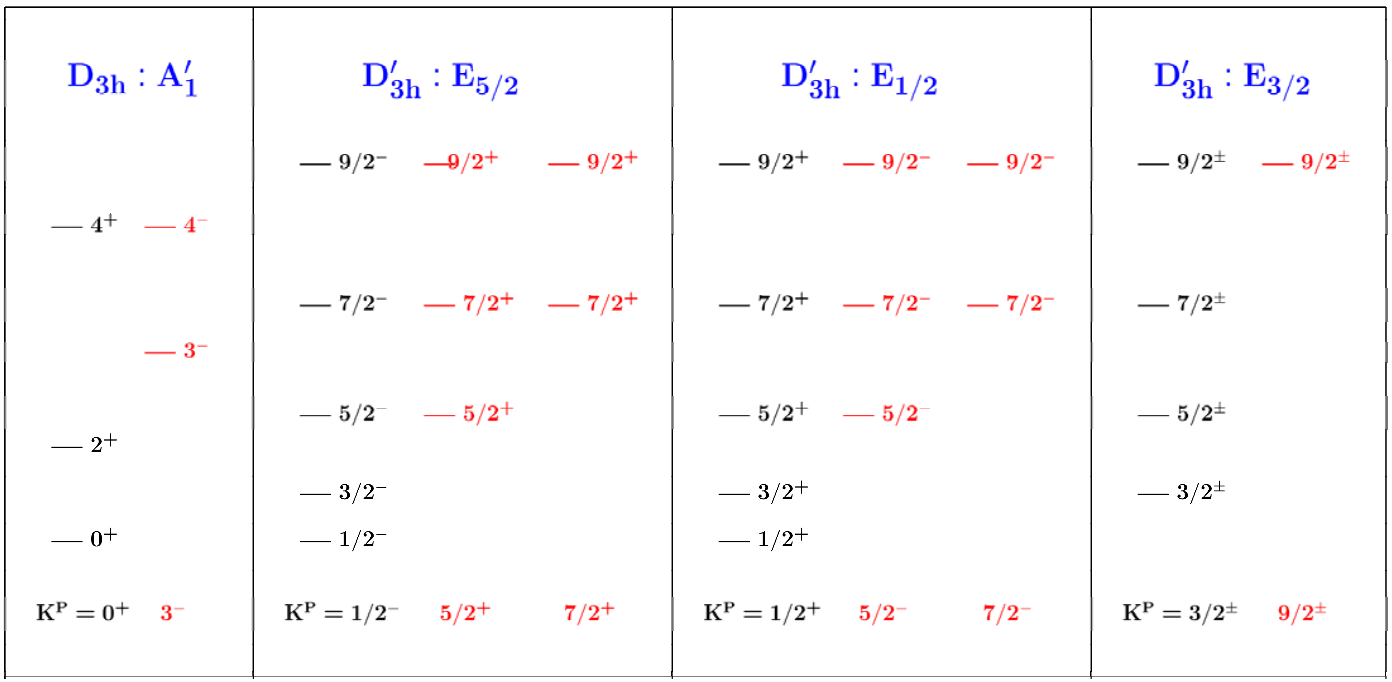


Symmetry: $\mathcal{D}_{3h}$

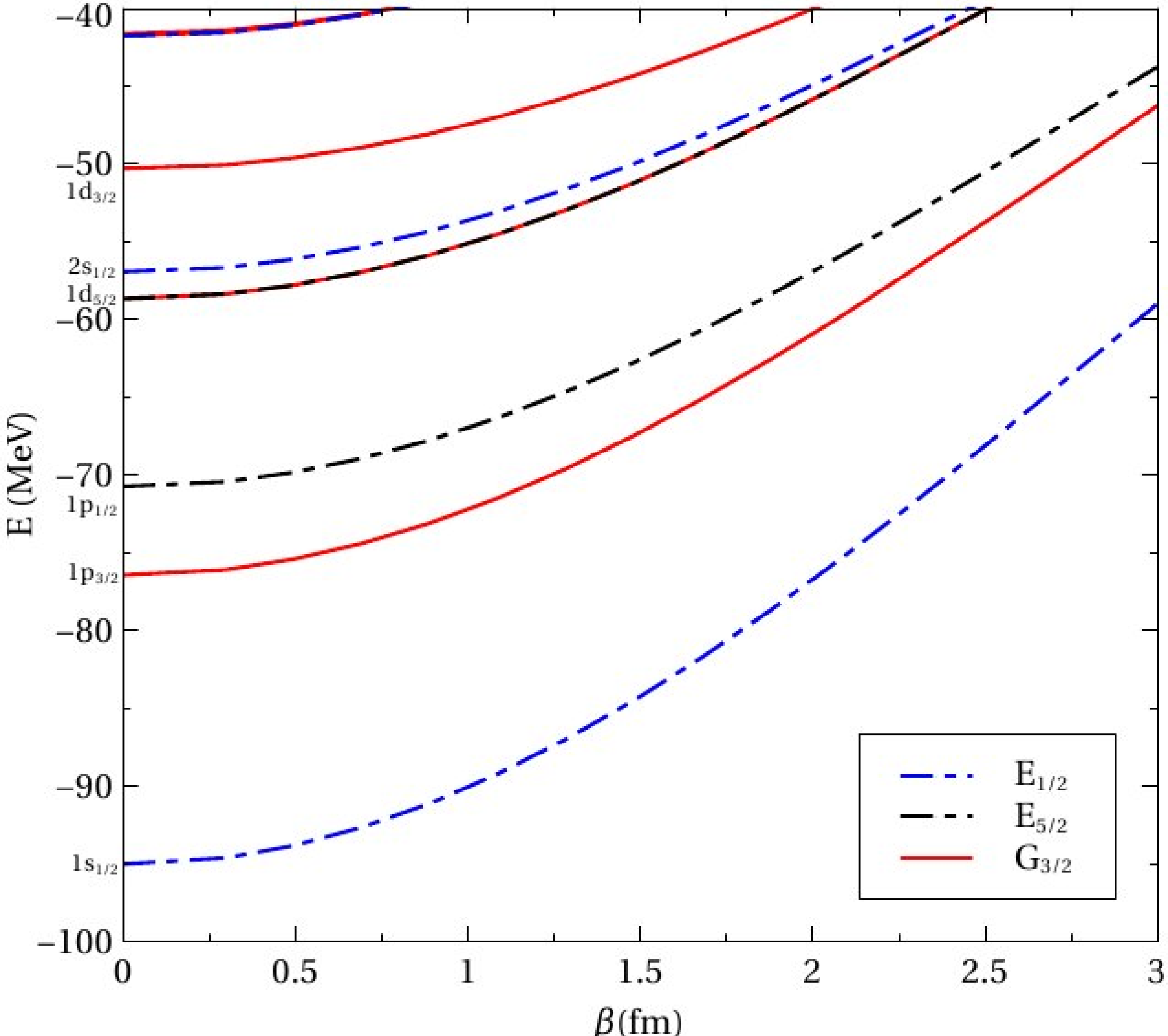
	$E_{1/2}$	$E_{5/2}$	$E_{3/2}$
Deg	2	2	2
$s_{1/2}$	1		
$p_{1/2}$		1	
$p_{3/2}$		1	1
$d_{3/2}$	1		1
$d_{5/2}$	1	1	1

Odd-Cluster Nucleus

$A_1' \otimes E_{5/2} = E_{5/2}$ $A_1' \otimes E_{1/2} = E_{1/2}$ $A_1' \otimes E_{3/2} = E_{3/2}$



Tetrahedral Symmetry

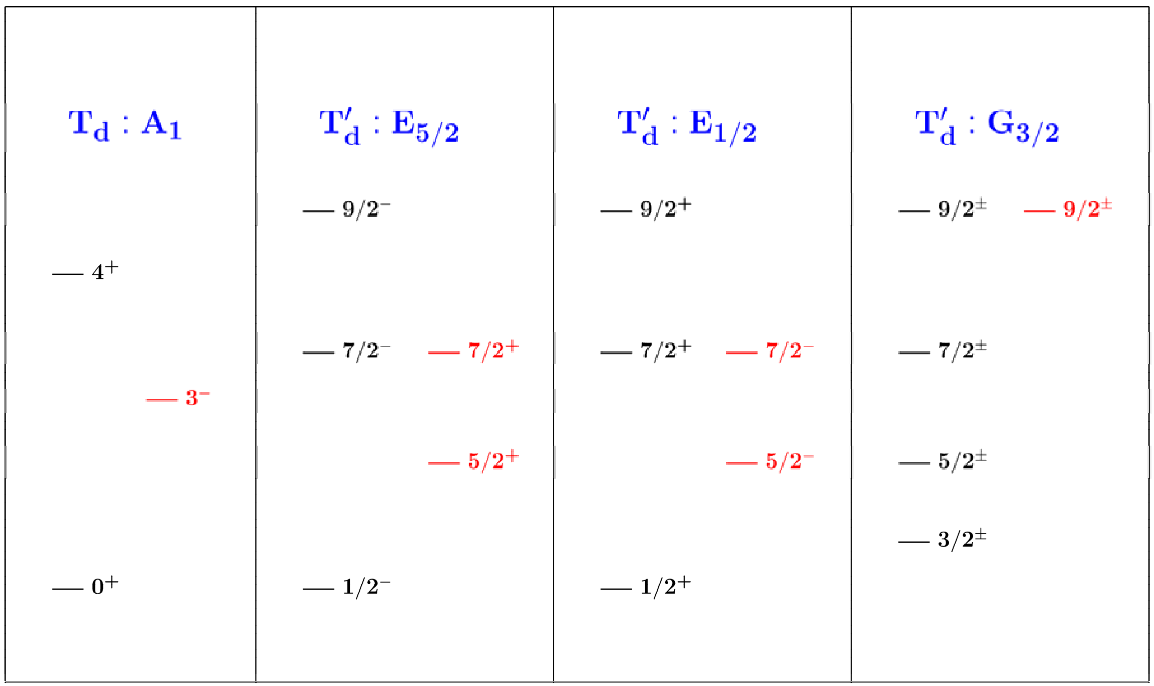


Symmetry: $\mathcal{T}_d$

	$E_{1/2}$	$E_{5/2}$	$G_{3/2}$
Deg	2	2	4
$s_{1/2}$	1		
$p_{1/2}$		1	
$p_{3/2}$			1
$d_{3/2}$			1
$d_{5/2}$		1	1

Odd-Cluster Nucleus

$A_1 \otimes E_{5/2} = E_{5/2}$ $A_1 \otimes E_{1/2} = E_{1/2}$ $A_1 \otimes G_{3/2} = G_{3/2}$



Work in progress …

Summary and Conclusions

- Algebraic Cluster Model
- Discrete and continuous symmetries
- Special solutions: spherical and deformed oscillators, oblate top, spherical top
- Rotational bands: fingerprints of geometric configurations of alpha particles
- Applications in molecular, nuclear, hadron physics

Alpha-Cluster Nuclei

- Oblate top with triangular symmetry for ^{12}C
- Ground state band: triangular
- Hoyle band: bent-arm, triangular?
- Search for negative parity states 3^-, 4^-
- Shape-phase transitions: non-rigid configuration
- Spherical top with tetrahedral symmetry for ^{16}O

- Odd-mass cluster nuclei (work in progress)
- Signatures of geometric symmetries

Tagami, Shimizu & Dudek,
PRC 87, 054306 & PRC 98, 024304

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