# Recent applications of equation of motion phonon method

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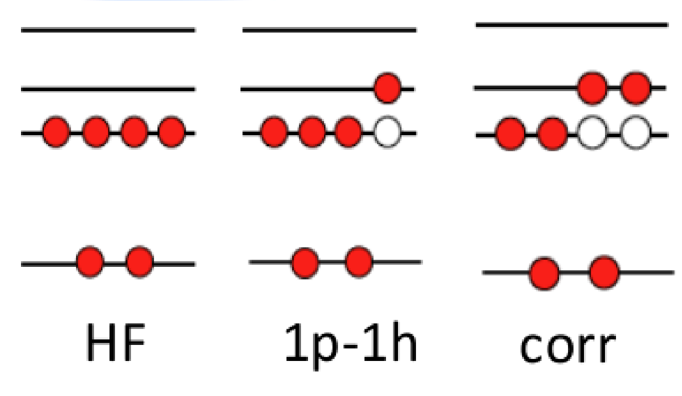
**Shapes and Symmetries in Nuclei: from Experiment to Theory (SSNET'18),**
**Gif-sur-Yvette, November 2018**

# Equation of Motion Phonon Method




**Equation of Motion Phonon Method** (EMPM):

**Features**:

- nuclear **ground state** properties
- energy **spectra**
- **collective** excitations
- **wide**-range **applicability** (across the nuclear chart)
- exact treatment of **Pauli principle** (unlike the methods based on RPA)
- applicable on any nuclear Hamiltonian but usually **realistic Hamiltonian** is adopted

**Applications**:

- EMPM first developed for **even-even nuclei** *Phys. Rev. C 85 014313 (2012), Phys. Rev. C 90 014310 (2014), Phys. Rev. C 92 054315 (2015)*
- **quasiparticle** formulation of **EMPM** for open-shell nuclei *Phys. Rev. C 93 044314 (2016)*
- **EMPM** extended to **even-odd** nuclei *Phys. Rev. C 94 061301 (2016), Phys. Rev. C 95 034327 (2017)*
- extension of **EMPM** to **hypernuclei** *in progress…*

# EMPM

**Hilbert space** – divided into subspaces

$$\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \ldots \oplus \mathcal{H}_n$$

**HF** – Hartree-Fock state (nucleons occupy lowest single-particle levels)
**1p-1h** = 1**particle** – 1**hole** excitation of HF
**2p-2h** = 2**particle** – 2**hole** excitation of HF
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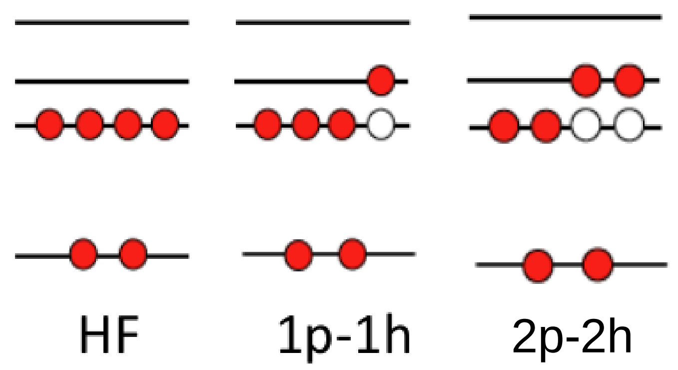
.
**np-nh** = n**particle** – n**hole** excitation of HF




Instead of multiple **particle-hole** excitations we can excite multiple **TDA phonons**

**Tamm-Dancoff** (TDA) phonons

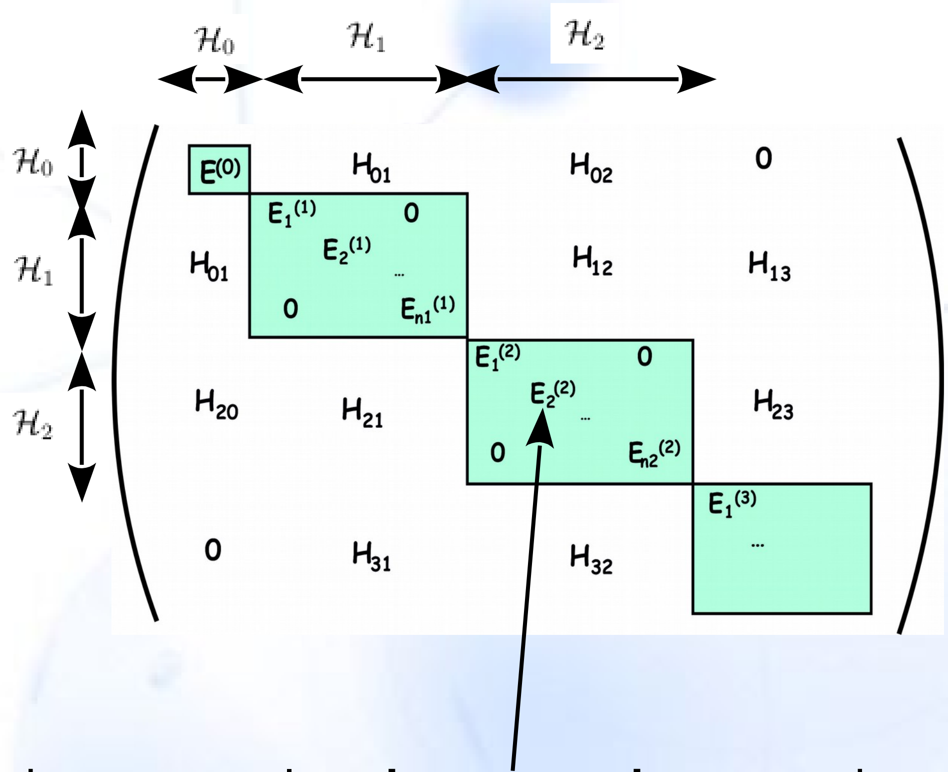
$$O_\nu^\dagger = \sum_{ph} c_{ph}^\nu a_p^\dagger a_{\tilde{h}}$$

Phonons = linear combination of 1p-1h excitations can represent **collective modes**

$$\begin{aligned}
\mathcal{H}_0 &= \{|HF>\} \\
\mathcal{H}_1 &= \{O_{\nu_1}^\dagger |HF>\} \\
\mathcal{H}_2 &= \{O_{\nu_1}^\dagger O_{\nu_2}^\dagger |HF>\} \\
&\;\;. \\
&\;\;. \\
\mathcal{H}_n &= \{O_{\nu_1}^\dagger O_{\nu_2}^\dagger \ldots O_{\nu_n}^\dagger |HF>\}
\end{aligned}$$

# EMPM

$$\mathcal{H}_0 = \{|HF>\}$$
$$\mathcal{H}_1 = \{O^{\dagger}_{\nu_1}|HF>\}$$
$$\mathcal{H}_2 = \{O^{\dagger}_{\nu_1}O^{\dagger}_{\nu_2}|HF>\}$$
$$\cdot$$
$$\cdot$$
$$\mathcal{H}_n = \{O^{\dagger}_{\nu_1}O^{\dagger}_{\nu_2}...O^{\dagger}_{\nu_n}|HF>\}$$




the total **Hamiltonian** mixes configurations from different **Hilbert subspaces**

**Equation of Motion** (EoM) – recursive eq. to solve **eigen-energies** on each **i**-phonon subspace while knowing the (**i-1**)-phonon solution

$$< i,\beta_i|[\hat{H},O^{\dagger}_{\nu}]|i-1,\alpha_{i-1}> = (E^{i}_{\beta_i} - E^{i-1}_{\alpha_{i-1}}) < i,\beta_i|O^{\dagger}_{\nu}|i-1,\alpha_{i-1}>$$

**non-diagonal** blocks of **Hamiltonian** calculated from amplitudes $< i,\beta_i|O^{\dagger}_{\nu}|i-1,\alpha_{i-1}>$

we diagonalize the total **Hamiltonian**

# Ground State Correlations

**NN** interaction - $\chi$ **NNLO**$_{\mathbf{opt}}$

A. Ekström et al., **PRL 110**, 192502 (2013)

**2-phonon** correlations in the **g.s.**

$$|\Psi_{g.s.}>\approx C_{HF}^{g.s.}|HF>+\sum_{\mu_2} C_{\mu_2}^{g.s.}|i=2,\mu_2>$$

**G. De Gregorio, J. Herko, F. Knapp, N. Lo Iudice, P. Veselý, PRC 95**, 024306 (2017)

TABLE I. Binding energies per nucleon. The EMPM value for $^{40}$Ca was obtained for $N_{max} = 8$, which is not an extremal point.

| $^{A}X$ | HF | BE/$A$ (MeV) PT | EMPM | Exp. |
|---|---|---|---|---|
| $^{4}$He | 3.96 | 7.07 | 6.67 | 7.07 |
| $^{16}$O | 3.22 | 8.29 | 6.77 | 7.98 |
| $^{40}$Ca | 4.00 | 9.77 | 7.02 | 8.55 |

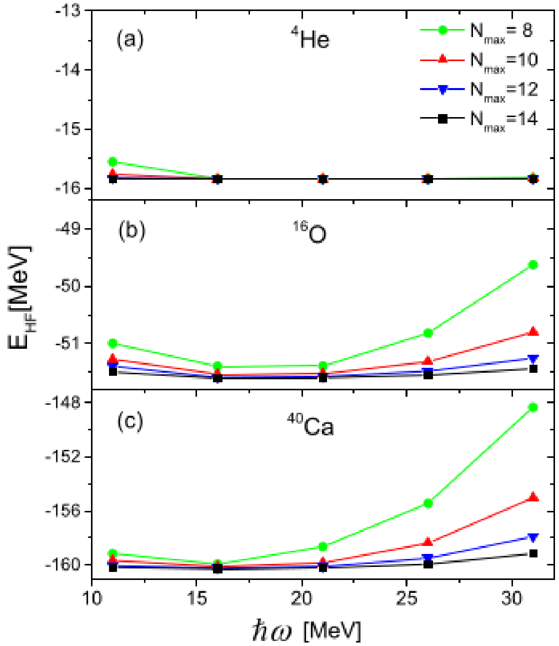



FIG. 1. HF ground-state energy of $^{4}$He (a), $^{16}$O (b), and $^{40}$Ca (c) versus the HO frequencies $\omega$ for different HO space dimensions $N_{max}$.

$E_{HF}$

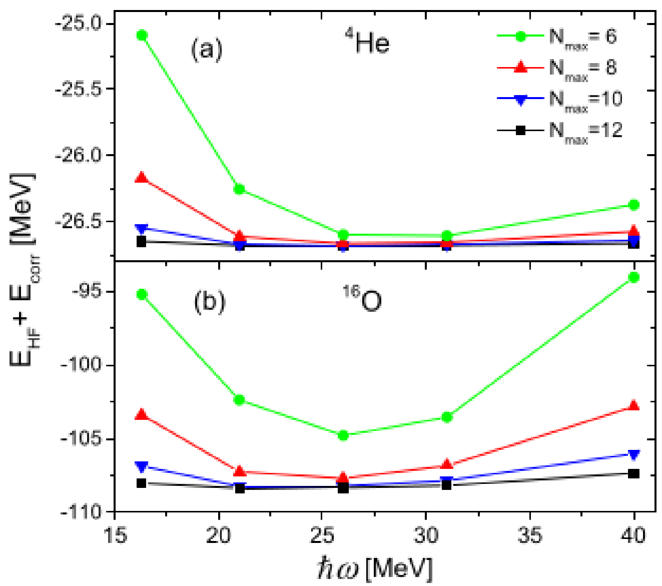



FIG. 4. The EMPM ground-state energy of $^{4}$He (a) and $^{16}$O (b) versus the HO frequency $\omega$ for different $N_{max}$.

$E_{EMPM}$

**N**$_{\mathbf{max}}$ – maximal osc. shell

**h$\omega$** – parameter of basis

Final energy must be converged with respect to **N**$_{\mathbf{max}}$ and for **N**$_{\mathbf{max}}$ big enough independent on **h$\omega$** ...

# Ground State Correlations

**NN** interaction - $\chi$ **NNLO$_{opt}$**

A. Ekström et al., **PRL 110**, 192502 (2013)

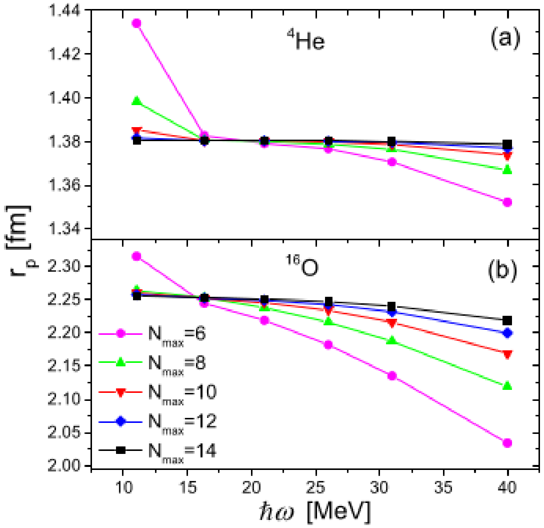



FIG. 5. HF point proton radius versus the HO frequency $\omega$ for different $N_{max}$ in $^4$He (a) and $^{16}$O (b).

“plateau“ dependence on **hω** – parameter

← $(\mathbf{r_p})_{HF}$

small effect of correlations on $\mathbf{r_p}$

**NNN** forces play important role here!

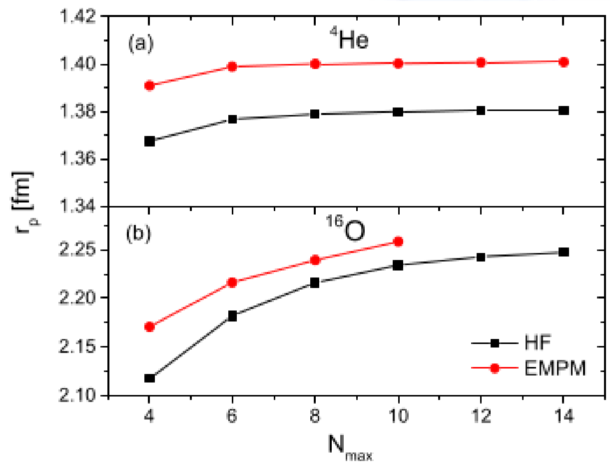



FIG. 7. HF and EMPM point proton radii of $^4$He (a) and $^{16}$O (b) versus $N_{max}$ for fixed frequency ($\hbar\omega = 26$ MeV).

**2-phonon** correlations in the **g.s.**

$$|\Psi_{g.s.}> \approx C^{g.s.}_{HF}|HF> + \sum_{\mu_2} C^{g.s.}_{\mu_2}|i=2,\mu_2>$$

proton **point radii**

$$<r_p^2> = <\Psi_{g.s.}|r_p^2|\Psi_{g.s.}> = <r_p^2>_{HF} + <r_p^2>_{corr.}$$

**G. De Gregorio, J. Herko, F. Knapp, N. Lo Iudice, P. Veselý, PRC 95**, 024306 (2017)

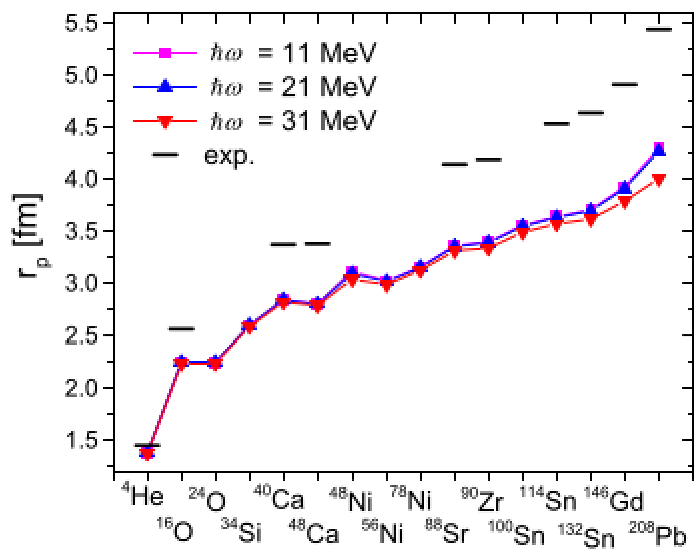



FIG. 6. Systematic of root-mean-square point proton radii computed in HF. The calculations are performed for $N_{max} = 14$ and different HO frequencies $\omega$. The experimental data are from Ref. [49].

| $^AX$ | HF | $r_p$ (fm) EMPM | Exp. |
|---|---|---|---|
| $^4$He | 1.38 | 1.40 | 1.46 |
| $^{16}$O | 2.25 | 2.26 | 2.57 |

# Ground State – NNN Force

**NN+NNN** interaction - $\chi$ **NNLO**$_{sat}$ (Ekström et al. **Phys. Rev. C** 91 (2015) 051301R )

**HO basis** *N = (2n + l)*

**hω** = 16 MeV **N**$_{max}$ up to 12

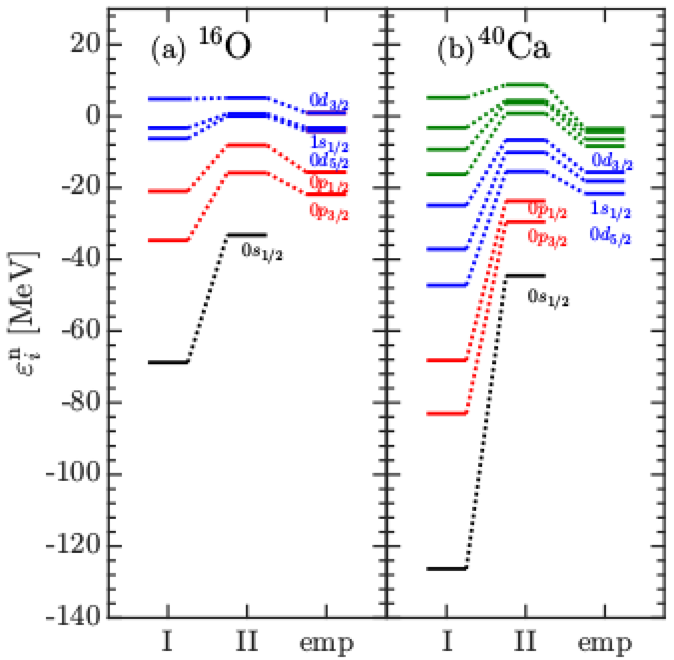



*Fig.* 3: The neutron single-particle energies $\varepsilon_i^n$ of $^{16}$O (a) and $^{40}$Ca (b) calculated with $NN$ (I) and $NN+NNN$ (II) interactions. The empirical data (emp) [24] are shown for comparison.

**charged radii**

Table 1: The charge radii $r_{ch} = \sqrt{< r_{ch}^2 >}$ [fm] of $^{16}$O and $^{40}$Ca calculated with $NN$ and $NN+NNN$ forces are compared with the experimental data (exp) [23].

| $^AX$ | $NN$ | $NN+NNN$ | exp |
|---|---|---|---|
| $^{16}$O | 2.19 | 2.77 | 2.70 |
| $^{40}$Ca | 2.58 | 3.54 | 3.48 |

**HF energy**

Table 2: Binding energies per nucleon $BE/A$ [MeV] calculated with $NN$ and with $NN+NNN$ forces in $^{16}$O and $^{40}$Ca compared to the experimental values (exp).

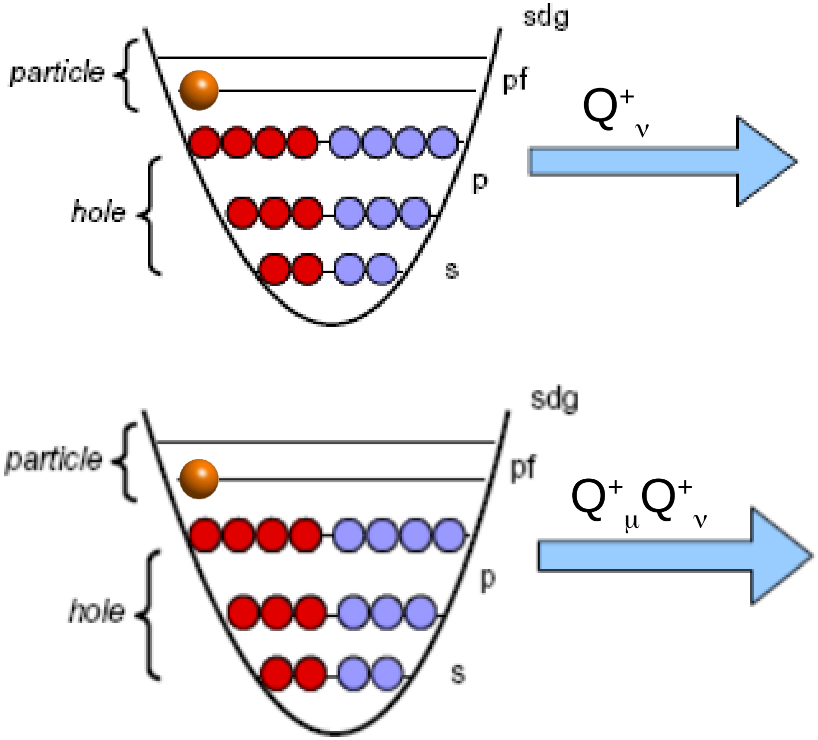
| $^AX$ | $NN$ | $NN+NNN$ | exp |
|---|---|---|---|
| $^{16}$O | 7.36 | 2.66 | 7.98 |
| $^{40}$Ca | 11.65 | 2.31 | 8.55 |

**HF underestimates** g.s. energy
(correlations necessary)

However **NNN** force improves significantly **radii** & **single-particle energies** already at the mean-field level

**P. Veselý, G. De Gregorio, J. Pokorný,** *accepted to* **Phys. Scr.**

# EMPM for Even-Odd Nuclei



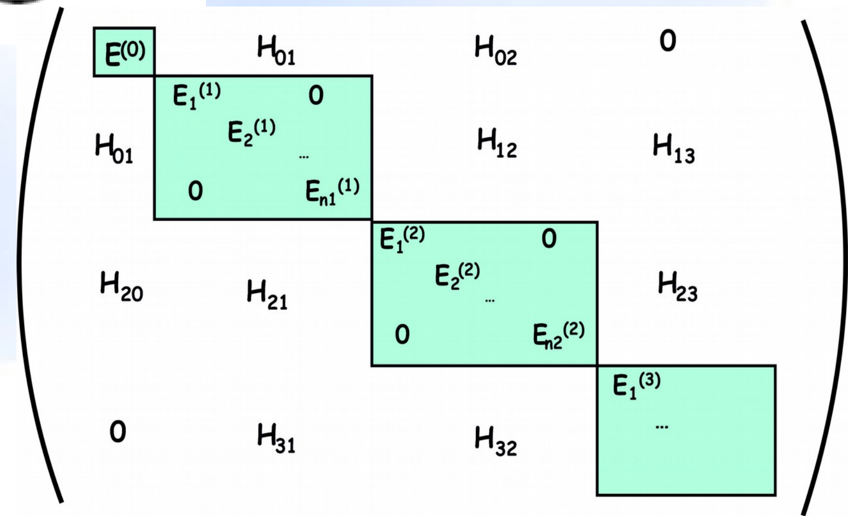

**Hilbert space** – divided into separate **n-phonon** subspaces

$$\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus ... \oplus \mathcal{H}_n$$

$$\begin{aligned}
\mathcal{H}_0 &= \left\{ c^\dagger_m |HF> \right\}, \\
\mathcal{H}_1 &= \left\{ c^\dagger_m Q^\dagger_\nu |HF> \right\}, \\
&\ \ . \\
&\ \ . \\
\mathcal{H}_n &= \left\{ c^\dagger_m Q^\dagger_{\nu_1} ... Q^\dagger_{\nu_n} |HF> \right\}.
\end{aligned}$$

explicit **coupling** of valence **nucleon** to general **excitations** of the **nuclear core**

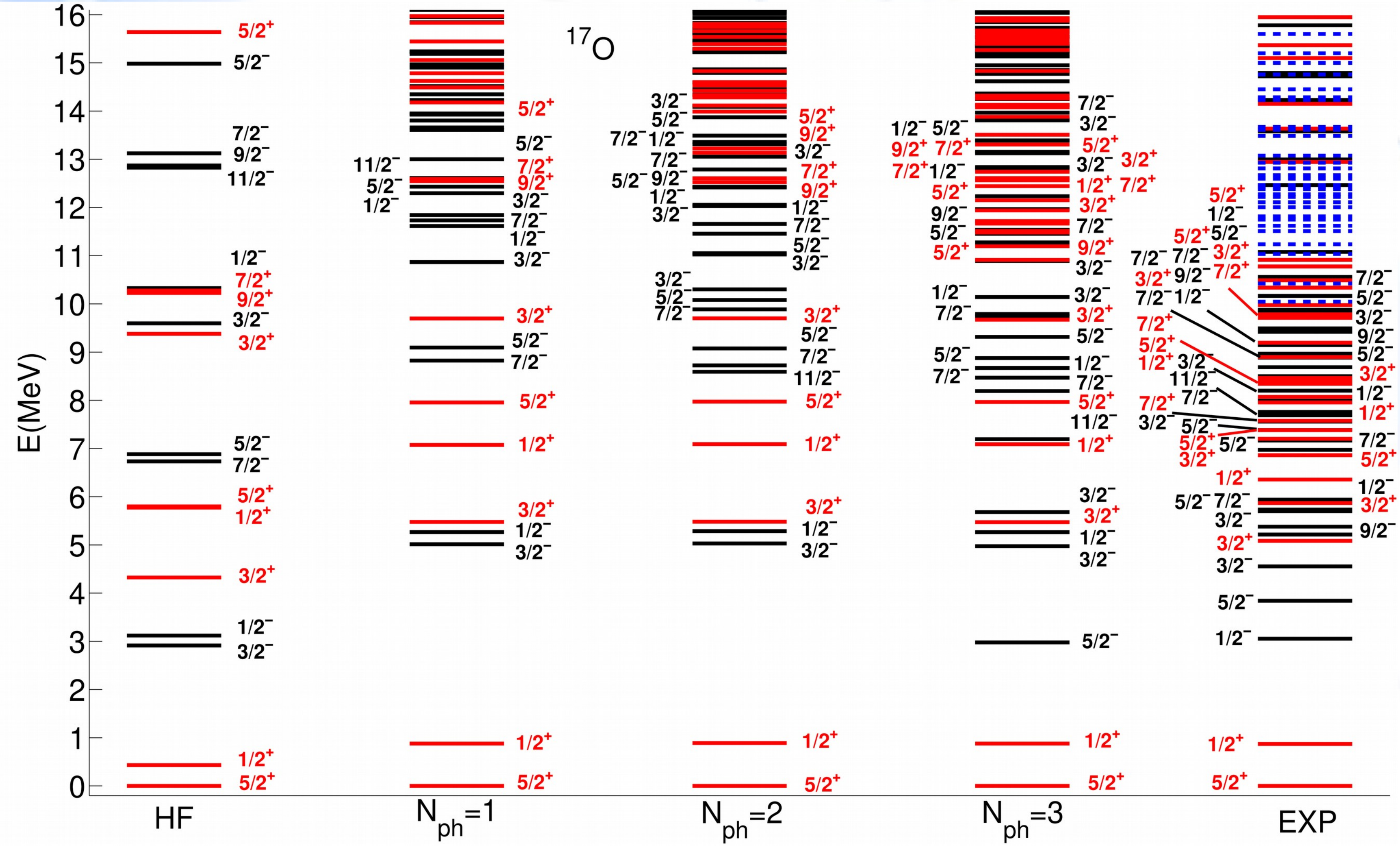
then diagonalization of complete Hamiltonian



# EMPM for Even-Odd Nuclei

$^{17}$**O** - particle coupled to (multi)phonon excitations **NN** interaction - $\chi$ **NNLO**$_{opt}$

First application of **EMPM** on the **odd** nuclear systems:
*G. De Gregorio, F. Knapp, N. Lo Iudice, P. Veselý*, **Phys. Rev. C94**, 061301(R) (2016)

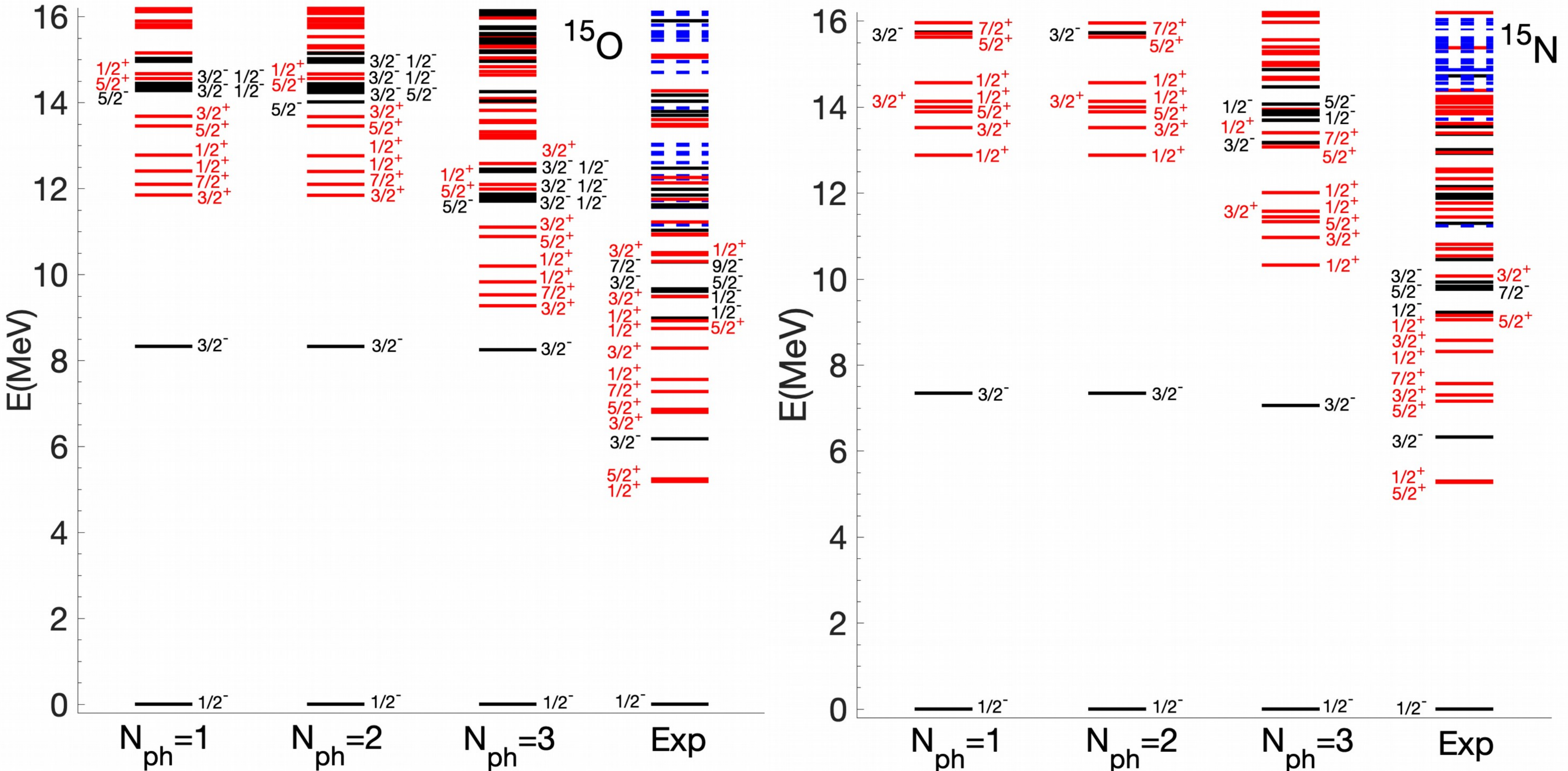


# EMPM for Even-Odd Nuclei

$^{15}$**O**, $^{15}$**N**, $^{21}$**O**, $^{21}$**N**- hole coupled to (multi)phonon excitations **NN** interaction - $\chi$ **NNLO**$_{opt}$

*G. De Gregorio, F. Knapp, N. Lo Iudice, P. Veselý*, *sent to* **Phys. Rev. C** (2018)

Lowest states – predominantly from hole-1phonon configurations
For better description, stronger coupling to more-phonon configs. needed

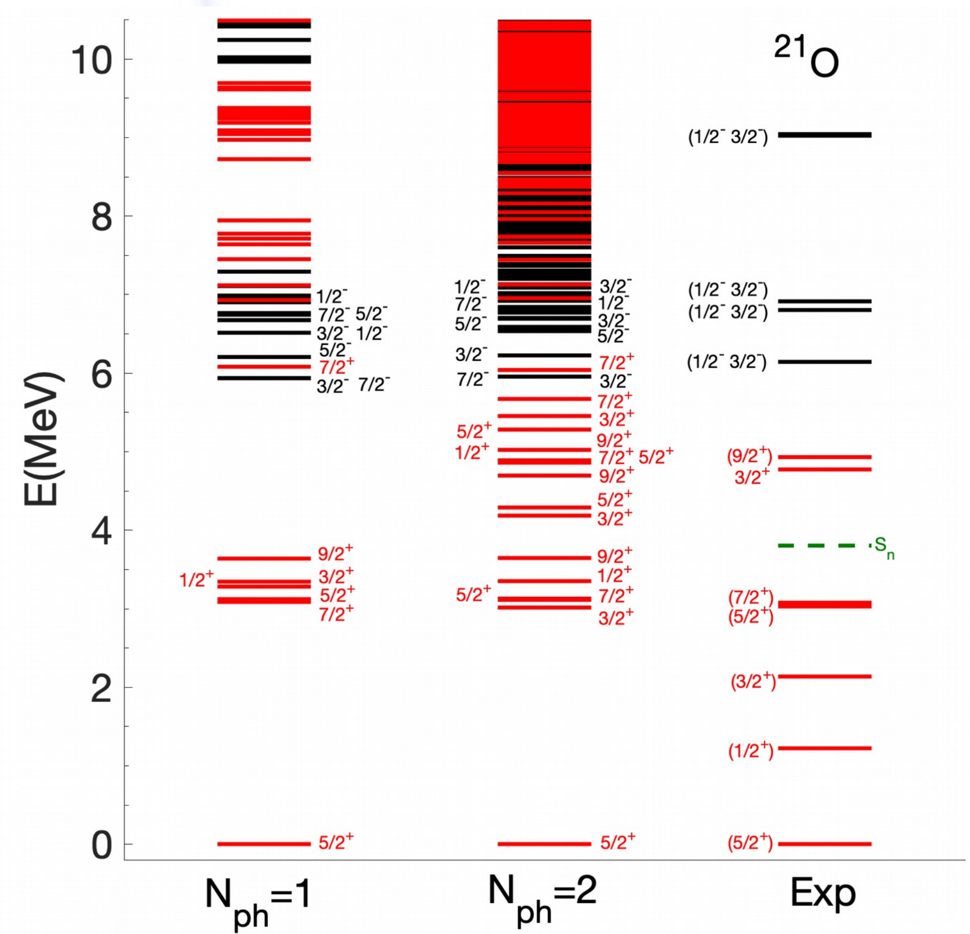


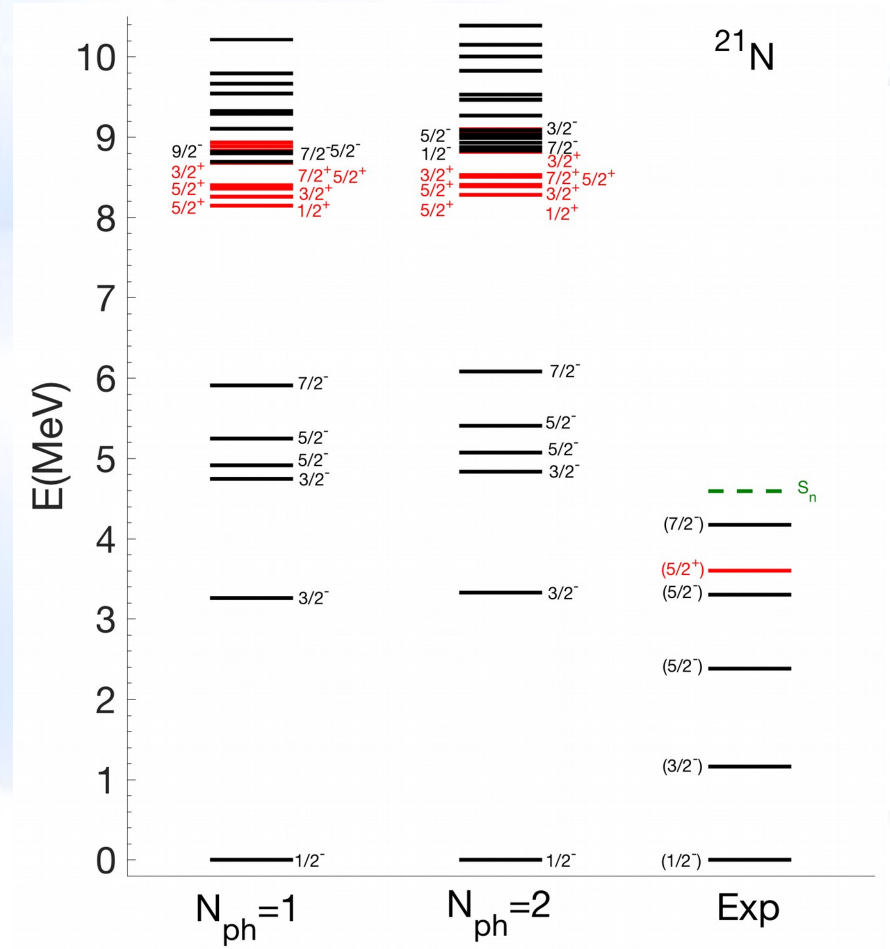
# EMPM for Even-Odd Nuclei

**$^{15}$O, $^{15}$N, $^{21}$O, $^{21}$N**- hole coupled to (multi)phonon excitations **NN** interaction - $\chi$ **NNLO$_{opt}$**

*G. De Gregorio, F. Knapp, N. Lo Iudice, P. Veselý*, *sent to* **Phys. Rev. C** (2018)

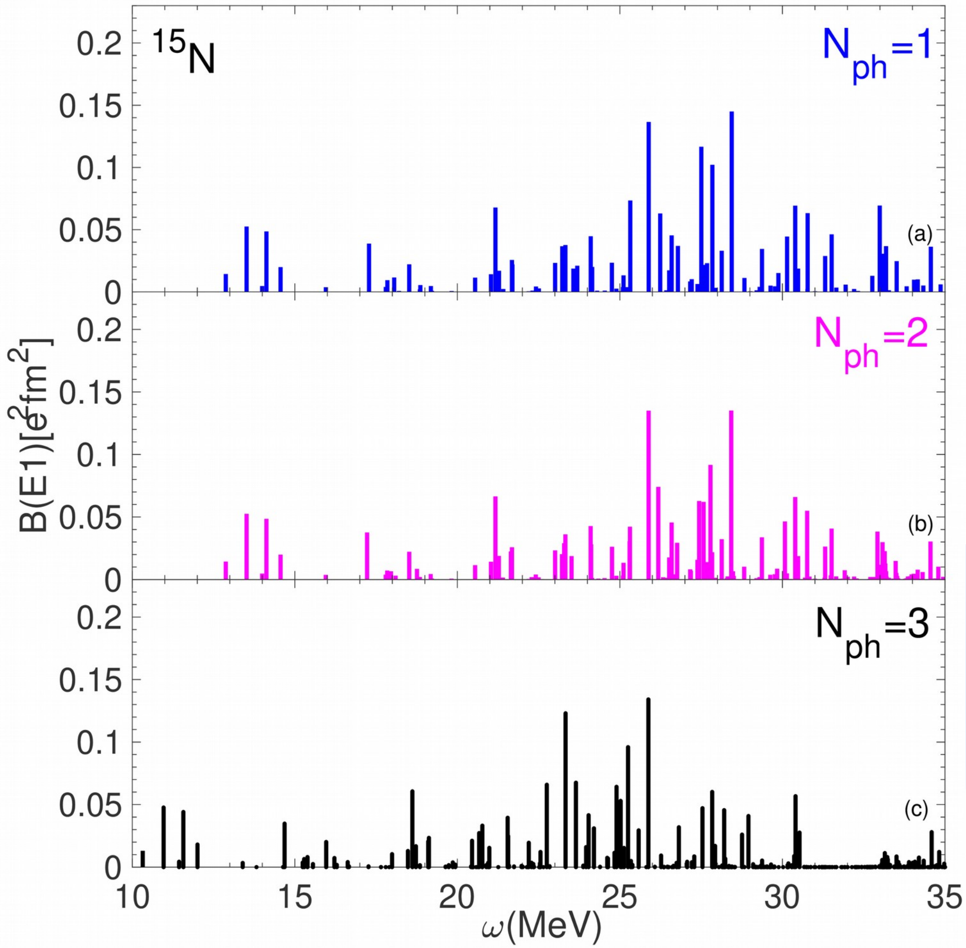
ground states – predominantly the hole-states nature

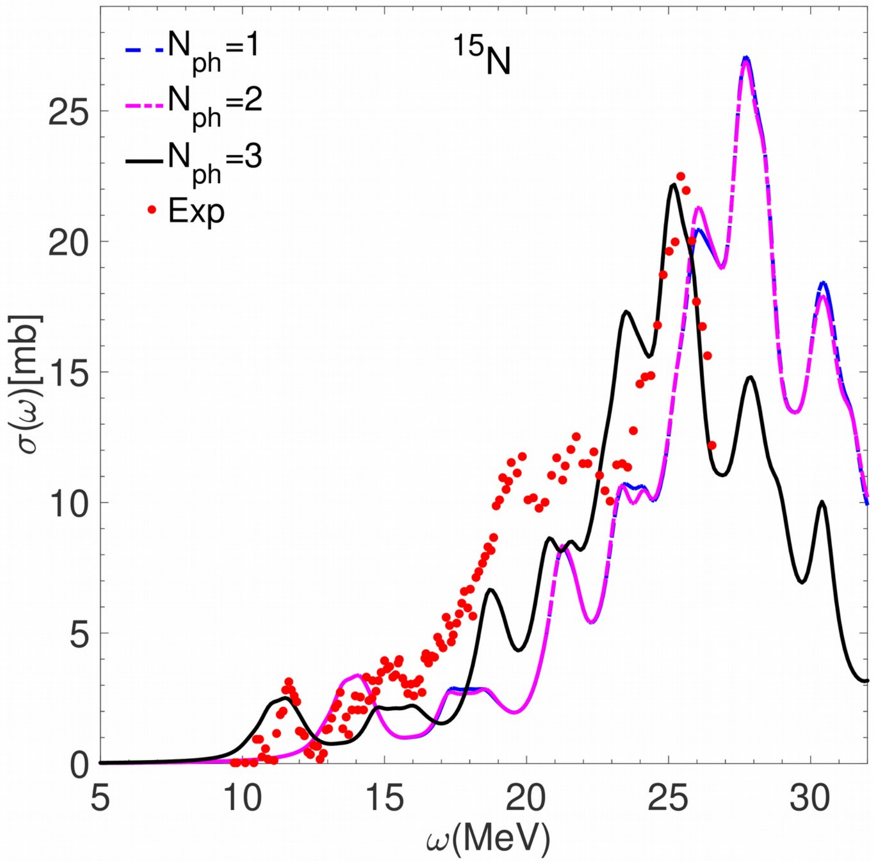


# EMPM for Even-Odd Nuclei

role of **hole-1phon**, **hole-2phon**, **hole-3phon** configurations on **E1** transitions in $^{15}$**N**




~ **112 %** of Thomas-Reike-Kuhn sum rule up to **40 MeV**

~ **50 %** of TRK sum rule up to **26.5 MeV** (experiment ~ **58 %**)

# EMPM for Hypernuclei

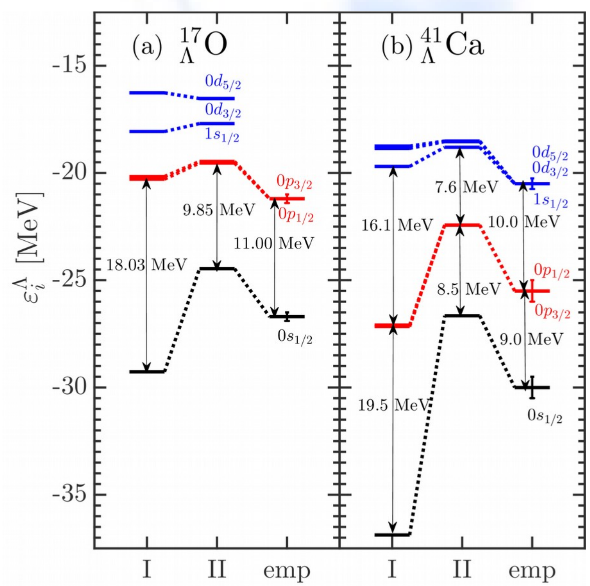
$$\widehat{H} = \widehat{T}_N + \widehat{T}_\Lambda + \widehat{V}^{NN} + \widehat{V}^{NNN} + \widehat{V}^{\Lambda N} + \widehat{V}^{\Lambda NN} - \widehat{T}_{CM}$$

**NN+NNN** interaction - $\chi$ **NNLO$_{sat}$** (Ekström et al. **Phys. Rev. C** 91 (2015) 051301R )
**ΛN** part of **YN** interaction - $\chi$ **LO** (H. Polinder, J. Haidenbauer, U. Meissner, **Nucl. Phys. A** 779 (2006) 244) **cut-off** $\lambda$ = 550 MeV

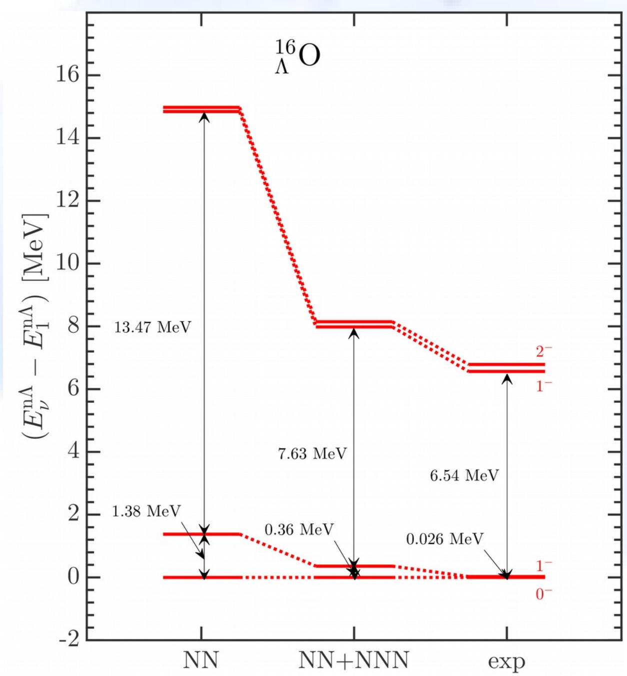
so far implemented:
**extension** of **HF+TDA** formalism on **hypernuclei** → **proton-neutron-Λ HF** + **ΛN TDA**

(replacement of the **nucleon** by **Λ**)

work in progress: - adding **Λ-Σ coupling** and **ΛNN SRG induced** force into the formalism
- **coupling** to (multi)**phonon** configurations




single-particle energies of **Λ**




important role of **NNN** part of **NNLO$_{sat}$**

strong dependence on **cut-off** parameter of **YN** int.

in future testing the quality of **YN** interaction (**spin-dependent** part)

# Outlook

**next goals**:

- study of the role of **NNN** interaction in nuclear ground state properties
- more systematic studies of **odd nuclei** – heavier systems
- further extensions of **EMPM** formalism – **odd-odd nuclei**, **hypernuclei**, ...
- transitions in nuclei – **GDR, M1, GMR, $\beta$** decay (**2**$\beta$ decay) …
- possibly calculations of electro**production** of **hypernuclei**

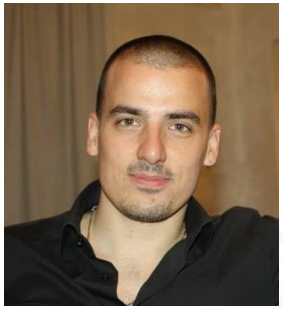
$^{40}$Ca (e,e‘ K$^{+}$) $^{40}_{\Lambda}$K

$^{48}$Ca (e,e‘ K$^{+}$) $^{48}_{\Lambda}$K

# List of Collaborators

## Nuclear Physics Institute, Czech Academy of Sciences





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# Thank you!!