Quadrupole shape fluctuations in nuclei and collective Hamiltonian model within Skyrme EDF

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Introduction: Quadrupole shape fluctuations

Goal: 5 Dim. quadrupole collective Hamiltonian

Method: 3D QRPA with Skyrme energy density functional

Result: Strength functions of triaxial superfluid nucleus

Result: Moment of inertia by local FAM-QRPA

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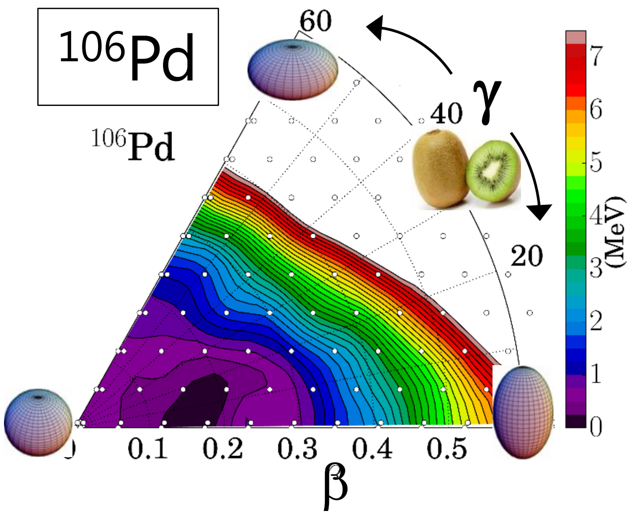
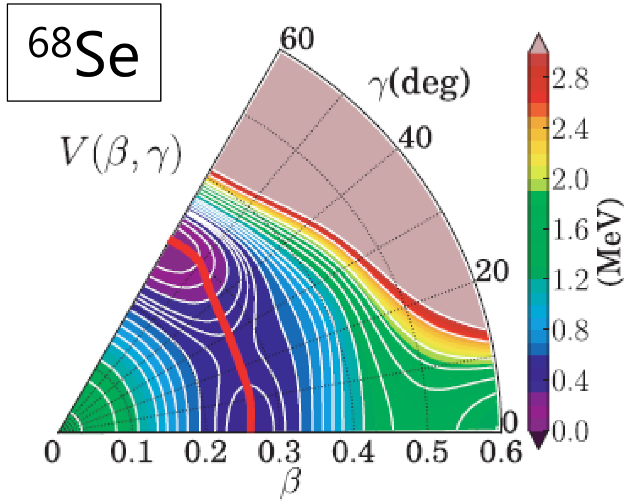
Introduction: Quadrupole shape fluctuation

Shape fluctuation in transitional region, for example:



Potential is flat (V < 2 MeV) at large region

➡ Shape fluctuation



Hinohara et al., PRC82 (2010) 064313

Two minima at oblate and prolate region

➡ Shape coexistence

Description of shape fluctuation **beyond mean-field** is necessary

5D quadrupole collective (Bohr) model

$$\mathcal{H} = T_{\text{vib}} + T_{\text{rot}} + V(\beta, \gamma)$$

$$T_{\text{vib}} = \frac{1}{2} D_{\beta\beta}(\beta, \gamma)\dot{\beta}^2 + D_{\beta\gamma}(\beta, \gamma)\dot{\beta}\dot{\gamma} + \frac{1}{2} D_{\gamma\gamma}(\beta, \gamma)\dot{\gamma}^2$$

$$T_{\text{rot}} = \frac{1}{2} \sum_{k=1}^{3} \mathcal{J}_k(\beta, \gamma)\omega_k^2$$

⬇

Quantization of Hamiltonian

⬇

$$\hat{H}\Psi_{\alpha IM}(\beta, \gamma, \Omega) = E_{\alpha I}\Psi_{\alpha IM}(\beta, \gamma, \Omega)$$

Excitation spectra

5D quadrupole collective (Bohr) model

HFB (V) + Local QRPA ($D_{\beta\beta}$, $D_{\beta\gamma}$, $D_{\gamma\gamma}$, J_k) with P+Q force

Hinohara et al., PRC82 (2010) 064313, 84 (2011) 061302, 85 (2012) 024323

HFB (V) + Cranking mass ($D_{\beta\beta}$, $D_{\beta\gamma}$, $D_{\gamma\gamma}$, J_k)

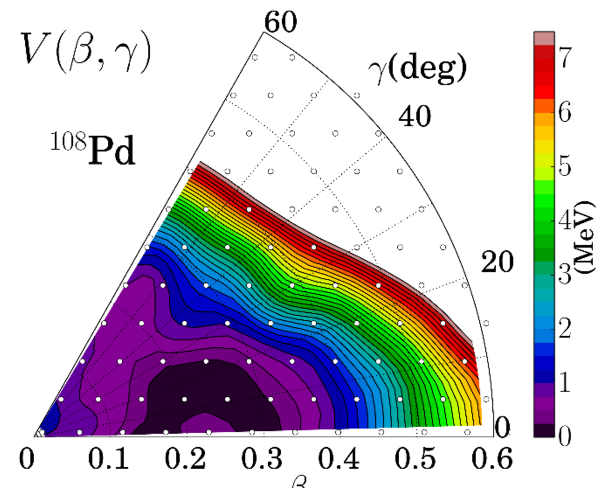
with modern energy density functional (Skyrme, Gogny, Relativistic)

Prochniak et al., NPA730 (2004) 59
Niksic et at., PRC79 (2009) 034303
Delaroche et al., PRC81 (2010) 014303, and more

This work: Combine two approaches

$V(\beta,\gamma)$ — β, γ-constrained Skyrme HFB

$D_{\mu\nu}(\beta,\gamma)$, $\mathcal{J}_k(\beta,\gamma)$ — Local Skyrme QRPA on each β, γ



Aim of this talk

To construct 5D quadrupole collective Hamiltonian with Skyrme EDF

But triaxial (three-dimensional) Skyrme QRPA is NOT available

Step 1: Construct 3D Skyrme QRPA

Finite amplitude method (FAM)

FAM: method for efficiently solving QRPA

Step 2: Local FAM+QRPA at each β, γ

$\Rightarrow$ Collective inertial functions

Finite amplitude Method (FAM)

Nakatsukasa et al., PRC76 (2007) 024318
Avogadro & Nakatsukasa, PRC84(2011)014314
Stoitsov et al., PRC84 (2011) 041305
Liang et al., PRC87 (2013) 054310
Niksic et al., PRC88 (2013) 044327
Pei et al., PRC90 (2014) 051304
Mustonen et al., PRC93 (2016) 014304

Linear response TDDFT

$$(E_\mu + E_\nu - \omega)X_{\mu\nu}(\omega) + \delta H^{20}_{\mu\nu}(\omega) = -F^{20}_{\mu\nu}$$
$$(E_\mu + E_\nu + \omega)Y_{\mu\nu}(\omega) + \delta H^{02}_{\mu\nu}(\omega) = -F^{02}_{\mu\nu}$$

$F_{\mu\nu}$: External perturbation field

(Isoscalar quadrupole moment in this talk)

$Q^{(\pm)}_{2K} \propto r^2(Y_{2+K} \pm Y_{2-K})$

Finite amplitude method (FAM)

$$\delta H_{\mu\nu} = \frac{\partial H_{\mu\nu}}{\partial \mathcal{R}_{\alpha\beta}} \partial \mathcal{R}_{\alpha\beta}$$

$$\Rightarrow \delta H_{\mu\nu} = \frac{1}{\eta}\{H_{\mu\nu}[\mathcal{R}_0 + \eta\delta\mathcal{R}] - H_{\mu\nu}[\mathcal{R}_0]\}$$

Residual part → finite difference form

$\mathcal{R}_0$: Ground state density

$\delta\mathcal{R}$: Fluctuating density

η : Small parameter

Finite amplitude Method (FAM)

Size of problem

	(traditional) QRPA	FAM
Matrix	$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix}$	$\delta H_{\mu\nu}$
Dimension	N^2 x N^2	N x N
Approximation	Cutoff (E_{2QP}) (Simplified interaction)	None

N: number of basis states

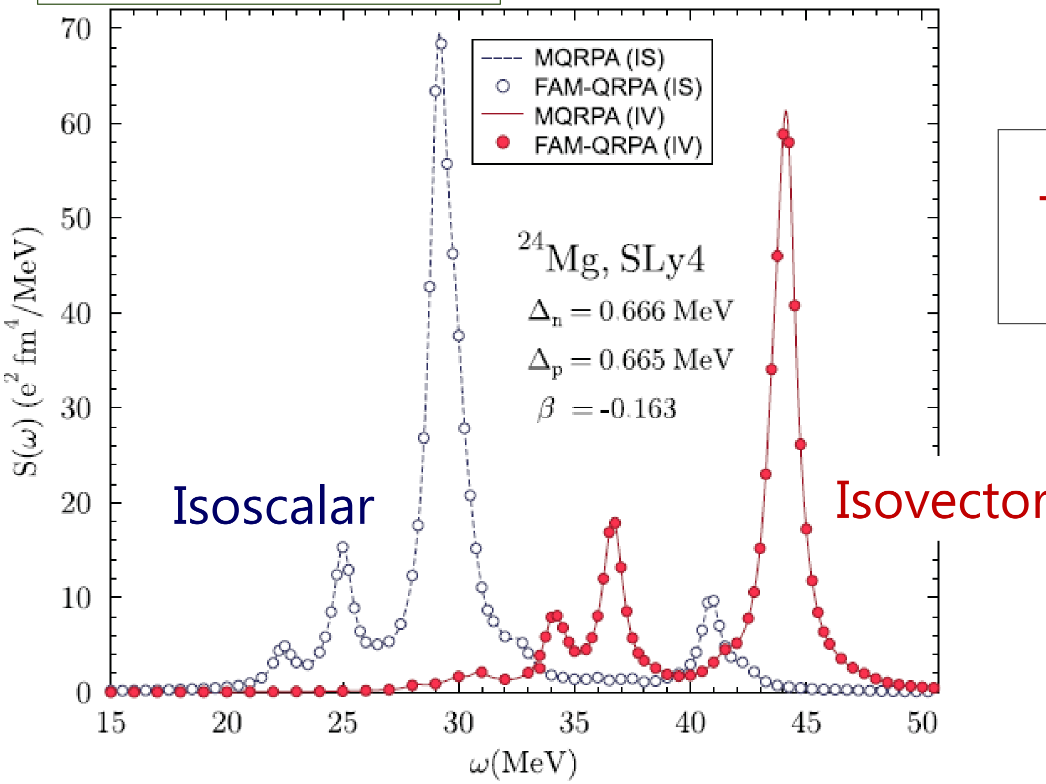
($N \sim 10^3$ for deformed nuclei)

FAM = QRPA in strength functions

Identical strength functions

Stoitsov et al., PRC84 (2011)041305

^{24}Mg, Monopole



Matrix QRPA

FAM-QRPA

Calculation set-up of FAM

- Based on 3D Cartesian coordinate HFB (cr8 code)
- Iteratively solved at each ω

$$X, Y \rightarrow \delta\mathcal{R} \rightarrow \delta H^{20,02} \rightarrow \text{new } X, Y \ (\text{at fixed } \omega)$$

- $\omega \rightarrow \omega + i\gamma$ γ = 0.5 MeV
- Strength function

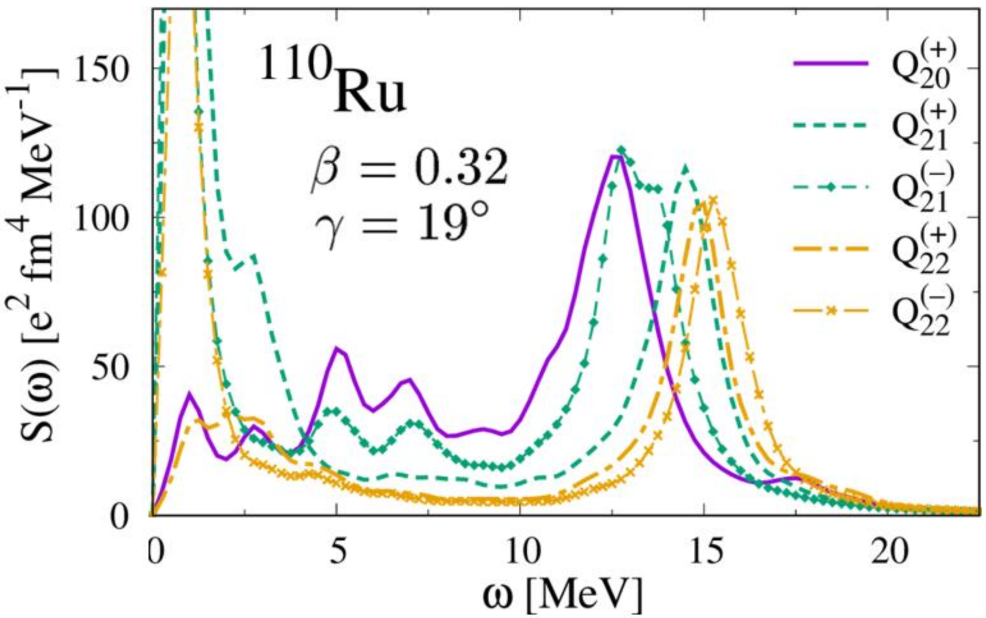
$$S(\omega) = -\frac{1}{\pi}\mathrm{Im}\left(\sum_{\mu<\nu} F^{20*}_{\mu\nu} X_{\mu\nu} + F^{02*}_{\mu\nu} Y_{\mu\nu}\right)$$

Triaxially deformed superfluid nucleus

KW, Nakatsukasa, PRC96, 041304(R) (2017)

Isoscalar quadrupole response

$$Q_{2K}^{(\pm)} \propto r^2 (Y_{2+K} \pm Y_{2-K})$$



SkM*

- Five different strength functions
- Three spurious rotations (x, y, z)
- 98-99% of energy-weighted sum for ω<50MeV

Computation:

15 min/ω (16threads)

3.5 GB memory

Numerical set up

^{110}Ru: 17^3 mesh, R_{max}=14.0fm, 1120 HF states

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Finite amplitude method (FAM)

FAM: method for efficiently solving QRPA

Step 2: Local FAM+QRPA at each β, γ

$\Rightarrow$ Collective inertial functions

Moment of inertia from FAM $T_{\rm rot} = \frac{1}{2}\sum_{k=1}^{3}\mathcal{J}_k(\beta,\gamma)\omega_k^2$

Nambu-Goldstone mode and mass (Thouless-Valatin inertia)

(e.g. Translation, rotation etc.)

$$S^{\rm FAM}(\hat{P}_{\rm NG}, \omega = 0) = \sum_{\mu<\nu} P_{\mu\nu}^{20*} X_{\mu\nu}(0) + P_{\mu\nu}^{02*} Y_{\mu\nu}(0)$$
$$= -M_{\rm NG}$$

Hinohara, PRC92(2015)034321

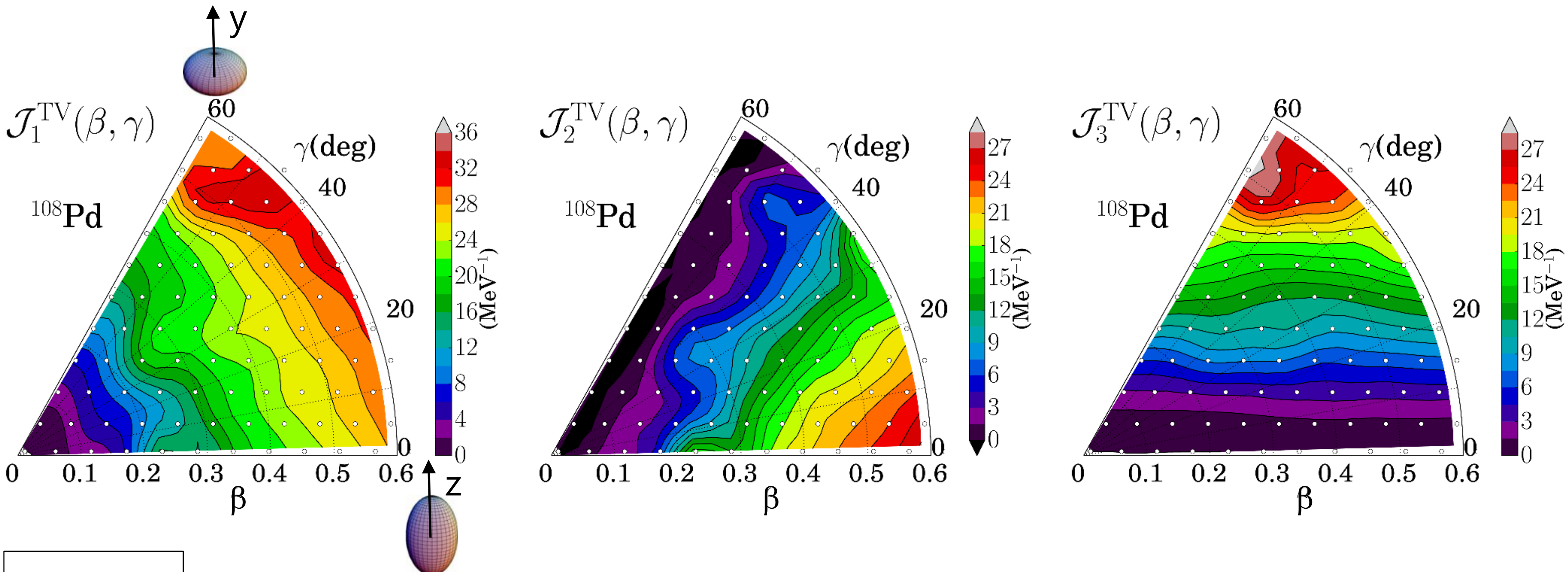
$\hat{P}_{\rm NG} = \hat{J}_k, \ M_{\rm NG} = \mathcal{J}_k^{\rm TV}$ for rotational moment of inertia

Strength at only ω = 0 is necessary

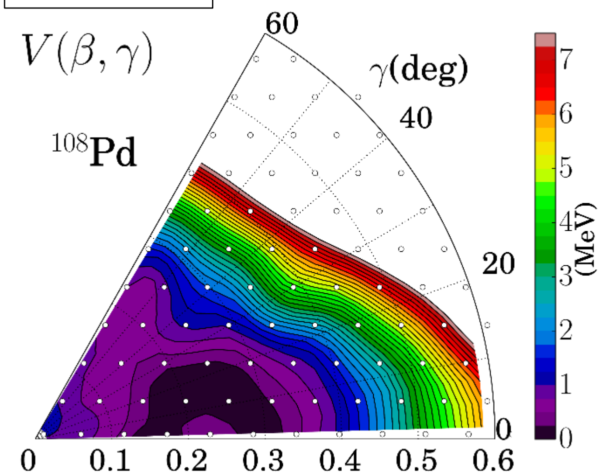
➡ Small computations

(a few minutes with 16 threads for a β–γ point)

Result: Moment of inertia on β, γ plane



Potential



At β–γ 84 points (white dots), constrained HFB+ local FAM-QRPA with interpolation in between

<u>Numerical set up</u>

17^3 mesh, R_{max}=14.0fm, 1120 HF states, volume pairing

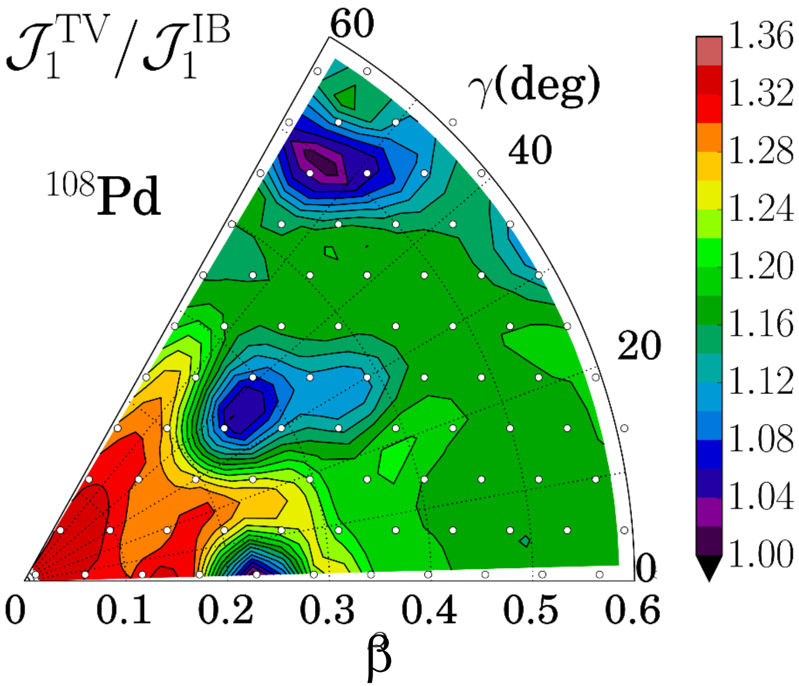
Importance of residual effect

Thouless-Valatin moment of inertia ⟵ Residual interaction included

vs.

Inglis-Belyaev moment of inertia ⟵ Residual interaction neglected

(Cranking mass)



- Moment of inertia is increased by the residual interaction $\mathcal{J}^{TV}/\mathcal{J}^{IB} > 1$
- β–γ dependence is important
- Use of J^{IB} or constant factor on J^{IB} is not enough

KW, Nakatsukasa, arXiv:1803.06828

Vibrational mass from FAM

Hinohara et al., PRC82 (2010) 064313

QRPA for vibrational mass at each β, γ

$$\delta\langle\phi(\beta,\gamma)|[\hat{H}_{\mathrm{CHFB}}(\beta,\gamma),\hat{Q}^i(\beta,\gamma)] - \frac{1}{i}\hat{P}^i(\beta,\gamma)|\phi(\beta,\gamma)\rangle = 0$$

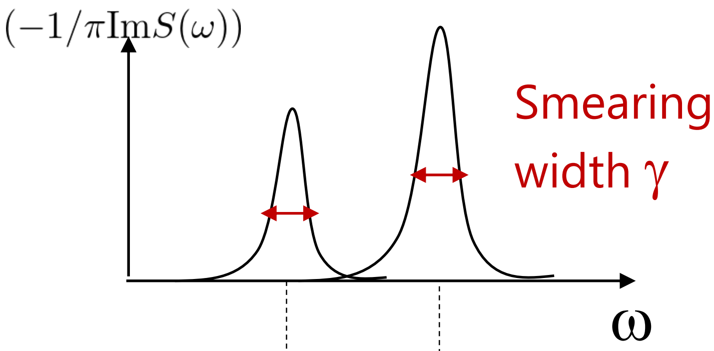
$$\delta\langle\phi(\beta,\gamma)|[\hat{H}_{\mathrm{CHFB}}(\beta,\gamma),\frac{1}{i}\hat{P}^i(\beta,\gamma)] - C_i(\beta,\gamma)\hat{Q}^i(\beta,\gamma)|\phi(\beta,\gamma)\rangle = 0$$

➡ $Q^i_{\mu\nu},\ P^i_{\mu\nu},\ C_i = \Omega_i^2$ Low-lying discrete QRPA solutions are necessary

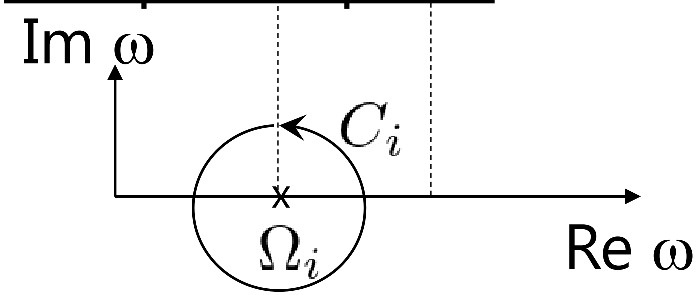
Low-lying discrete states from FAM

Hinohara et al., PRC87.064309(2013)

FAM Strength



Complex ω plane



FAM strength

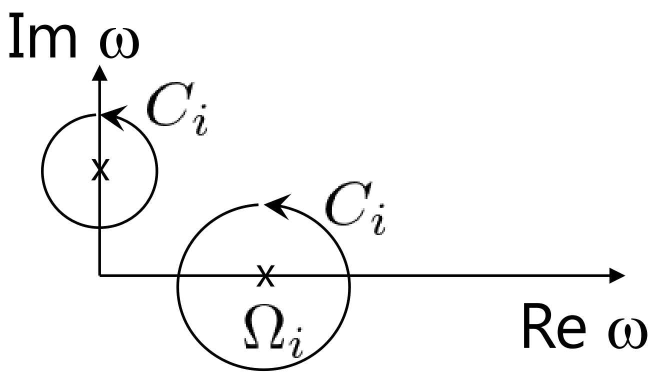
QRPA strength

$$S(\hat{F},\omega) = -\sum_i \left(\frac{|\langle i|\hat{F}|0\rangle|^2}{\Omega_i - \omega} + \frac{|\langle 0|\hat{F}|i\rangle|^2}{\Omega_i + \omega} \right)$$

$$\frac{1}{2\pi i}\oint_{C_i} d\omega S(\hat{F},\omega) = |\langle i|\hat{F}|0\rangle|^2$$

Vibrational mass from FAM

Local QRPA can have both real & imaginary solutions



PQ representation of FAM

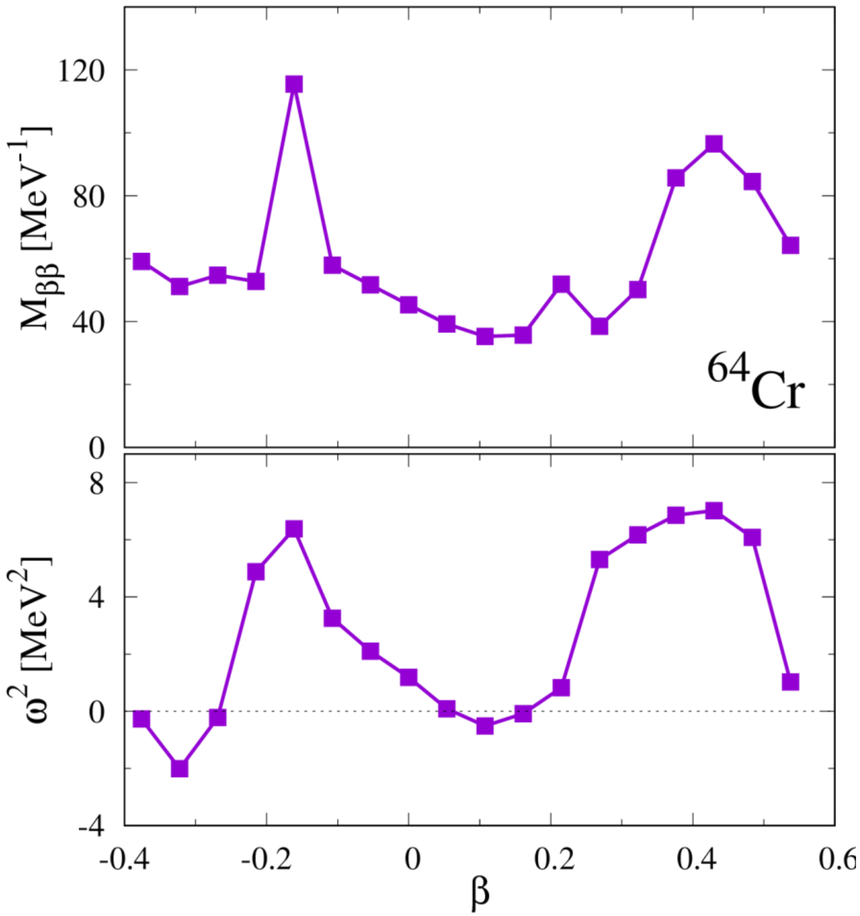
Hinohara, PRC92(2015)034321

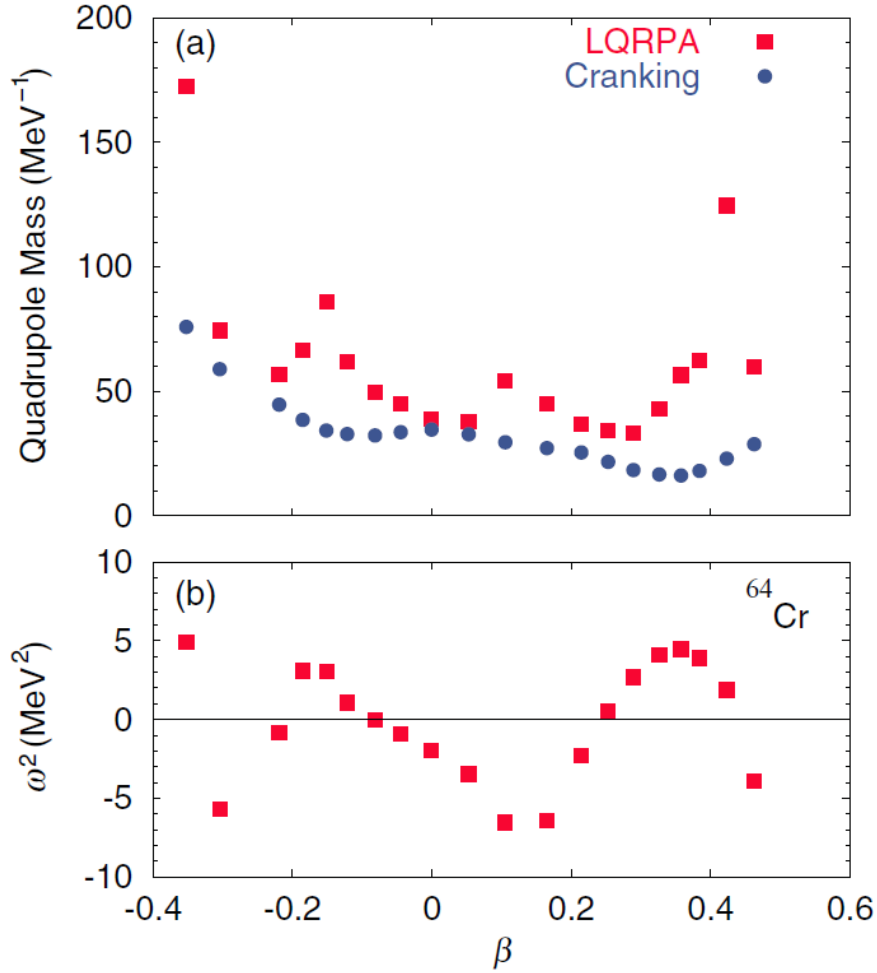
Hinohara, private communication

$$\frac{1}{2\pi i}\oint_{C_i} d\omega\ \omega S(\hat{F},\omega) = \frac{1}{2}|\langle P_i|\hat{F}|0\rangle|^2$$

Vibrational mass from FAM

Test: Mass along β, axial case





Yoshida, Hinohara, PRC83, 061302(2011)

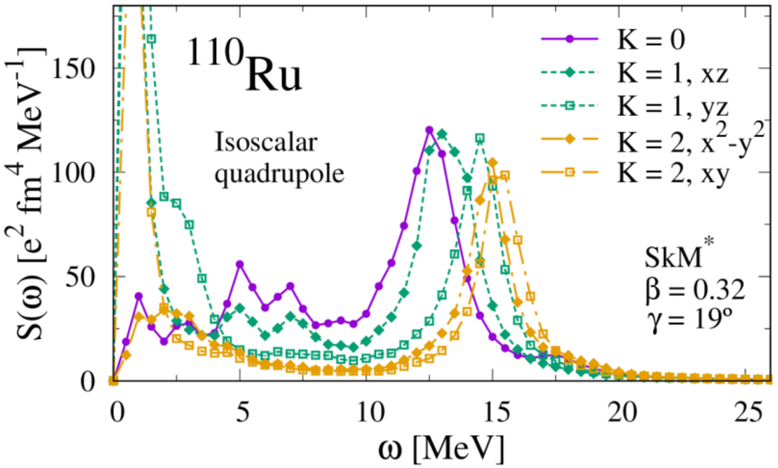
Summary

Shape fluctuation → Large amplitude collective motion

3D FAM+QRPA with Skyrme EDF is ready

FAM for triaxial superfluid nucleui

Moment of inertia by Local FAM+QRPA



Future plan

Local FAM+QRPA $D_{\beta\beta},\ D_{\beta\gamma},\ D_{\gamma\gamma}$

5D quadrupole collective (Bohr) Hamiltonian