Partial Dynamical Symmetries for Transitional Nuclei

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Transitional Nuclei

Shape-phase transition	spherical $\rightarrow$ deformed	Nd-Sm-Gd
Shape coexistence	spherical-deformed	Cd, Sr, Zr, Ni
	prolate-oblate	Kr, Se, Hg
	spherical-prolate-oblate	^{186}Pb

- Shell model approach

  - Multiparticle-multihole intruder excitations across shell gaps
  - Drastic truncation of large SM spaces

- Mean-field approach (EDF)

  - Coexisting shapes associated with different minima of an energy surface
  - Beyond MF methods: restoration of broken symmetries

- Symmetry-based approach

  - Dynamical symmetries $\leftrightarrow$ phases
  - Geometry: coherent (intrinsic) states

Dynamical Symmetry

$$
\begin{array}{ccccccc}
G_{\rm dyn} & \supset & G & \supset & \cdots & \supset & G_{\rm sym} \\
\downarrow & & \downarrow & & & & \downarrow \\
[N] & & \langle \Sigma \rangle & & & & \Lambda
\end{array}
$$

$$\hat{H} = \sum_G a_G \hat{C}_G$$

- *Solvability of the* ***complete*** *spectrum* $E = E_{[N]\langle\Sigma\rangle\ldots\Lambda}$
- *Quantum numbers for* ***all*** *eigenstates* $|[N]\langle\Sigma\rangle\Lambda\rangle$

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- *Quantum numbers for* ***all*** *eigenstates* $|[N]\langle\Sigma\rangle\Lambda\rangle$

- IBM: s (L=0) , d (L=2) bosons, N conserved *(Arima, Iachello 75)*

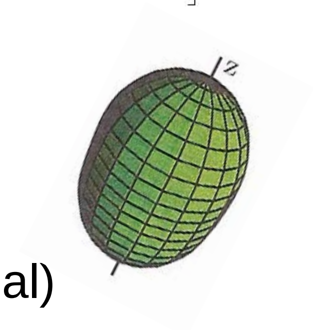
G_{dyn} = U(6), G_{sym} = SO(3)

U(6) ⊃ U(5) ⊃ SO(5) ⊃ SO(3)	$\vert [N]\ n_d\ \tau\ n_\Delta\ L\ \rangle$	Spherical vibrator
U(6) ⊃ SU(3) ⊃ SO(3)	$\vert [N]\ (\lambda,\mu)\ K\ L\rangle$	Prolate-deformed rotor
U(6) ⊃ $\overline{\text{SU(3)}}$ ⊃ SO(3)	$\vert [N]\ (\bar{\lambda},\bar{\mu})\ \bar{K}\ L\rangle$	Oblate-deformed rotor
U(6) ⊃ SO(6) ⊃ SO(5) ⊃ SO(3)	$\vert [N]\ \sigma\ \tau\ n_\Delta\ L\ \rangle$	γ-unstable deformed rotor

Geometry

Coherent state
$$|\,\beta,\gamma;N\rangle \;=\; (N!)^{-1/2}(b_c^\dagger)^N\,|0\,\rangle$$
$$b_c^\dagger \;=\; (1+\beta^2)^{-1/2}\left[\beta\,\cos\gamma\,d_0^\dagger+\beta\,\sin\gamma\,\frac{1}{\sqrt{2}}\left(d_2^\dagger+d_{-2}^\dagger\right)+s^\dagger\right]$$

Energy surface
$$E_N(\beta,\gamma)=\langle\beta,\gamma;N|\hat{H}|\beta,\gamma;N\rangle$$



Global min: equilibrium shape (β_0,γ_0)

$\beta_0 = 0$ spherical
$\beta_0 > 0$ deformed: $\gamma_0 = 0$ (prolate), $\gamma_0 = \pi/3$ (oblate), $0 < \gamma_0 < \pi/3$ (triaxial)

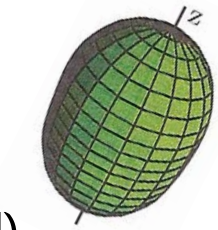
Intrinsic state ground band $|\beta_0,\gamma_0; N\rangle$, L-projected states $|\beta_0,\gamma_0; N,x,L\rangle$

Geometry

Coherent state $|\beta,\gamma;N\rangle = (N!)^{-1/2}(b_c^\dagger)^N|0\rangle$

$$b_c^\dagger = (1+\beta^2)^{-1/2}\left[\beta\cos\gamma\, d_0^\dagger + \beta\sin\gamma\frac{1}{\sqrt{2}}(d_2^\dagger + d_{-2}^\dagger) + s^\dagger\right]$$

Energy surface $E_N(\beta,\gamma) = \langle\beta,\gamma;N|\hat{H}|\beta,\gamma;N\rangle$



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Intrinsic state ground band $|\beta_0,\gamma_0;N\rangle$, L-projected states $|\beta_0,\gamma_0;N,x,L\rangle$

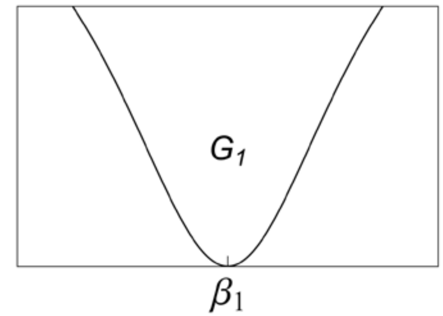
$U(6) \supset \mathbf{G_1} \supset G_2 \supset \ldots SO(3)$	$|N,\lambda_1,\lambda_2,\ldots,L\rangle$

U(5)	$\beta_0 = 0$	$n_d = 0$
SU(3)	$(\beta_0 = \sqrt{2}, \gamma_0 = 0)$	$(\lambda,\mu) = (2N,0)$
$\overline{SU(3)}$	$(\beta_0 = \sqrt{2}, \gamma_0 = \pi/3)$	$(\lambda,\mu) = (0,2N)$
SO(6)	$(\beta_0 = 1, \gamma_0$ arbitrary)	$\bar{\sigma} \equiv N$

- Dynamical symmetry corresponds to a particular shape (β_0,γ_0)
- $|\beta_0,\gamma_0;N\rangle$ lowest (highest) weight state in a particular irrep λ_1 of leading subalgebra $\mathbf{G_1}$
- H: 1-, 2-, 3-body terms $E_N(\beta,\gamma)$: quadratic, quartic, sextic β^2, $\beta^3\cos3\gamma$

Dynamical Symmetry

$U(6) \supset G_1 \supset G_2 \supset \ldots SO(3)$ $\quad |N, \lambda_1, \lambda_2, \ldots, L\rangle \quad (\beta_1, \gamma_1)$



- Complete solvability
- Good quantum numbers for all states
- DS: benchmark for the dynamics of a single shape

[G_1 = U(5), SU(3), $\overline{SU(3)}$, SO(6)]

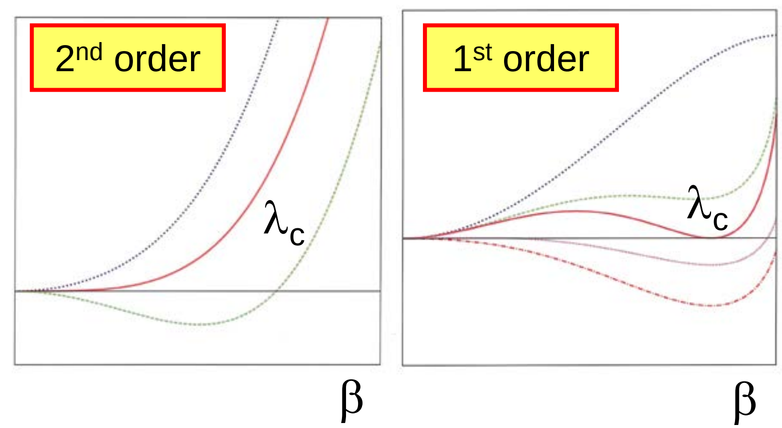
Spherical, prolate- , oblate-, γ-unstable deformed

Quantum Phase Transition (QPT)

$$H(\lambda) = \lambda H_{G1} + (1-\lambda) H_{G2}$$

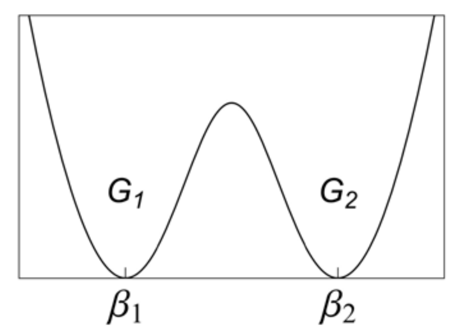
G_1, G_2 incompatible symmetries

Are there any remaining “symmetries” ?



Partial Dynamical Symmetry

- Some states solvable and/or with good quantum numbers
- PDS: benchmark for the dynamics of multiple shapes



Leviatan, Prog. Part. Nucl. Phys. **66**, 93 (2011)

Construction of Hamiltonians with a single PDS

$$G_{\rm dyn} \supset G \supset \cdots \supset G_{\rm sym}$$

$[N]$ $\langle\Sigma\rangle$ Λ

n-particle annihilation operator

$$\hat{T}_{[n]\langle\sigma\rangle\lambda} | [N] \langle\Sigma_0\rangle \Lambda\rangle = 0$$

for **all** possible Λ contained in the irrep $\langle\Sigma_0\rangle$ of G

Equivalently:

$$\hat{T}_{[n]\langle\sigma\rangle\lambda} | [N] \langle\Sigma_0\rangle \rangle = 0$$

| **Lowest weight state** ⟩

- Condition is satisfied if $\langle\sigma\rangle \otimes \langle\Sigma_0\rangle \not\ni [N-n]$

$$\hat{H} = \sum_{\alpha,\beta} u_{\alpha\beta} \hat{T}^\dagger_\alpha \hat{T}_\beta$$

DS is **broken** but **solvability** of states with $\langle\Sigma\rangle = \langle\Sigma_0\rangle$ Is **preserved**

Alhassid, Leviatan, J. Phys. A **25**, L1265 (1992)
Garcia-Ramos, Leviatan, Van Isacker, PRL **102**, 112502 (2009)

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DS is **broken** but
solvability of states with $\langle\Sigma\rangle = \langle\Sigma_0\rangle$ Is **preserved**

- PDS Hamiltonian $\hat{H}' = \hat{H} + \hat{H}_c$ Intrinsic collective resolution

Intrinsic part: $H \,|\, [N]\, \langle\Sigma_0\rangle\, \Lambda\rangle = 0$

Collective part: H_c composed of Casimir operators of conserved $G_i \subset G$ in the chain

SU(3) PDS

U(6) ⊃ SU(3) ⊃ SO(3)

[N] (λ,μ) K L

$\hat{T}^{\dagger}_{[n](\lambda,\mu)\ell m}$

$$\left.\begin{aligned} P^{\dagger}_{0} &= d^{\dagger}\cdot d^{\dagger} - 2\,(s^{\dagger})^2 \\ P^{\dagger}_{2,\mu} &= 2\,s^{\dagger}d^{\dagger}_{\mu} + \sqrt{7}(d^{\dagger}d^{\dagger})^{(2)}_{\mu} \end{aligned}\right\}$$

(λ,μ) = (0,2)

(2,0) ⊗ (2N,0) ∉ [N-2]

$$P_{\ell,\mu}|[N](2N,0)L\rangle \;=\; 0$$

(λ,μ) = (2N,0)

$$P_{\ell,\mu}|N;\beta=\sqrt{2}\,\rangle \;=\; 0$$

Single shape (β=√2,γ=0)

$$H \;=\; h_0 P^{\dagger}_0 P_0 + h_2 P^{\dagger}_2\cdot\tilde{P}_2$$

(λ,μ) = (0,0)⊕(2,2)

$$H(h_0=h_2) = \left[-\hat{C}_{SU(3)} + 2\hat{N}(2\hat{N}+3)\right]$$

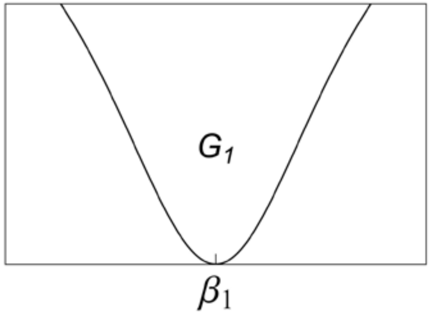
SU(3) PDS $\hat{H}' = h_0\,P^{\dagger}_0 P_0 + h_2\,P^{\dagger}_2\cdot\tilde{P}_2 + \rho\,\hat{C}_2[\mathrm{SO(3)}]$

Empirical manifestation: ^{168}Er (Leviatan, PRL 1996)

Rare earth & actinides (Casten et al. PRL 2014, PRC 2015, 2016)

Multiple PDS and Shape Coexistence

$$\mathrm{U}(6) \supset \mathrm{G}_1 \supset \mathrm{G}_2 \supset \ldots \supset \mathrm{SO}(3) \qquad |N,\, \lambda_1,\, \lambda_2,\, \ldots,\, L\rangle \qquad (\beta_1,\gamma_1)$$



Single PDS
Single shape

$$\hat{H}|\beta_1,\gamma_1;N,\lambda_1=\Lambda_0,\lambda_2,\ldots,L\rangle = 0$$

Multiple PDS and Shape Coexistence

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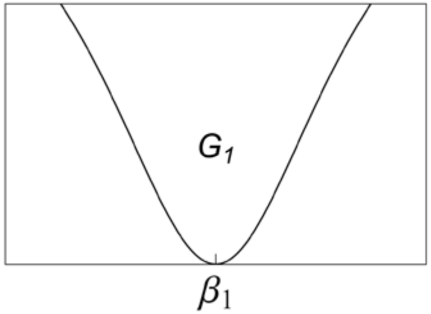
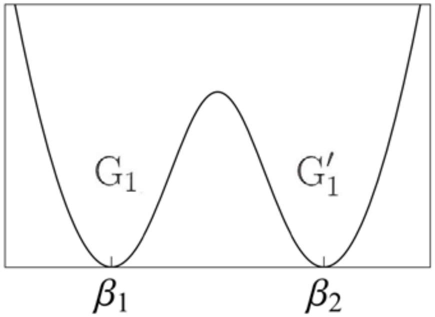


Single PDS
Single shape

$$\hat{H}|\beta_1, \gamma_1; N, \lambda_1 = \Lambda_0, \lambda_2, \ldots, L\rangle = 0$$

$$\mathrm{U}(6) \supset \mathrm{G}_1 \supset \mathrm{G}_2 \supset \ldots \supset \mathrm{SO}(3) \qquad |N,\, \lambda_1,\, \lambda_2,\, \ldots,\, L\rangle \qquad (\beta_1,\gamma_1)$$

$$\mathrm{U}(6) \supset \mathrm{G}_1' \supset \mathrm{G}_2' \supset \ldots \supset \mathrm{SO}(3) \qquad |N,\, \sigma_1,\, \sigma_2,\, \ldots,\, L\rangle \qquad (\beta_2,\gamma_2)$$



Multiple PDS
Multiple shapes

$$\begin{cases} \hat{H}|\beta_1, \gamma_1; N, \lambda_1 = \Lambda_0, \lambda_2, \ldots, L\rangle = 0 \\ \hat{H}|\beta_2, \gamma_2; N, \sigma_1 = \Sigma_0, \sigma_2, \ldots, L\rangle = 0 \end{cases}$$

$$\mathrm{G}_1 \neq \mathrm{G}_1'$$

Critical-point Hamiltonian $\hat{H}' = \hat{H} + \hat{H}_{\mathrm{c}}$

G_1 -PDS & G'_1 -PDS

Intrinsic part: $\hat{H}$ determines $E(\beta,\gamma)$ band structure

Collective part: $\hat{H}_{\mathrm{c}} = \sum_{\mathrm{G_i}} a_{\mathrm{G_i}} \hat{C}_{\mathrm{G_i}}$ rotational splitting

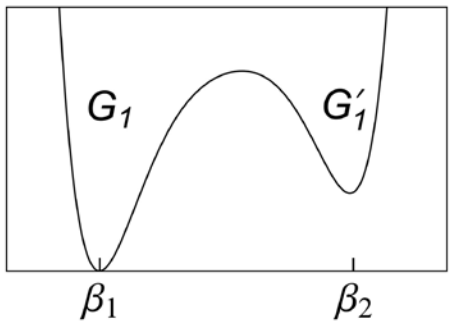
→ conserved G_i in both chains

Departure from the Critical Point

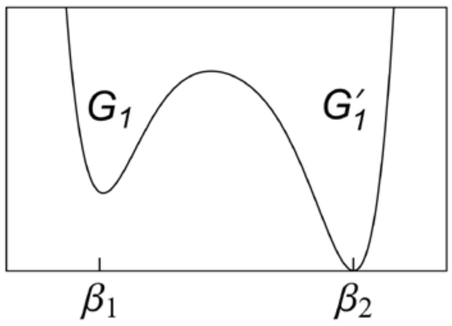
$\mathrm{U}(6) \supset \mathrm{G}_1 \supset \mathrm{G}_2 \supset \ldots \supset \mathrm{SO}(3) \qquad |N,\, \lambda_1,\, \lambda_2,\, \ldots,\, L\rangle \qquad (\beta_1,\gamma_1)$

$\mathrm{U}(6) \supset \mathrm{G}'_1 \supset \mathrm{G}'_2 \supset \ldots \supset \mathrm{SO}(3) \qquad |N,\, \sigma_1,\, \sigma_2,\, \ldots,\, L\rangle \qquad (\beta_2,\gamma_2)$

$\mathrm{G}_1 \neq \mathrm{G}'_1$



$$\hat{H}' = \hat{H}'_{\mathrm{cp}} + \alpha\, \hat{C}[\mathrm{G}_1]$$



$$\hat{H}' = \hat{H}'_{\mathrm{cp}} + \alpha\, \hat{C}[\mathrm{G}'_1]$$

Symmetry-based Approach to Shape-Coexistence

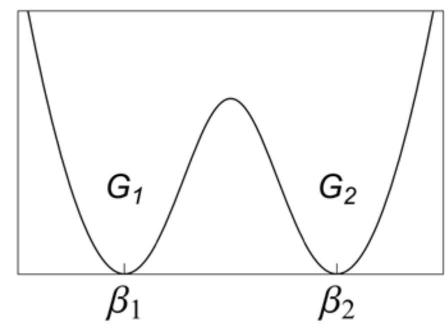
$U(6) \supset U(5) \supset SO(5) \supset SO(3)$	Spherical vibrator	$\beta = 0$
$U(6) \supset SU(3) \supset SO(3)$	Prolate-deformed rotor	$\beta = \sqrt{2},\ \gamma = 0$
$U(6) \supset \overline{SU(3)} \supset SO(3)$	Oblate-deformed rotor	$\beta = \sqrt{2},\ \gamma = \pi/3$
$U(6) \supset SO(6) \supset SO(5) \supset SO(3)$	γ-unstable deformed rotor	$\beta = 1$, γ arbitrary

Multiple PDS and Multiple Shapes

$G_1 = U(5)$ $G_2 = SU(3)$ spherical – prolate ♦

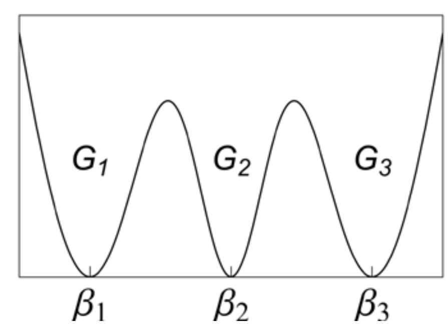
$G_1 = SU(3)$ $G_2 = \overline{SU(3)}$ prolate – oblate ♣

$G_1 = U(5)$ $G_2 = SO(6)$ spherical - γ-unstable ♠



Triple coexistence

$G_1 = U(5)$ $G_2 = SU(3)$ $G_3 = \overline{SU(3)}$ spherical-prolate-oblate ♣



♦ Leviatan, PRL **98,** 242502 (2007); Macek, A.L. **351**, 302 (2014)

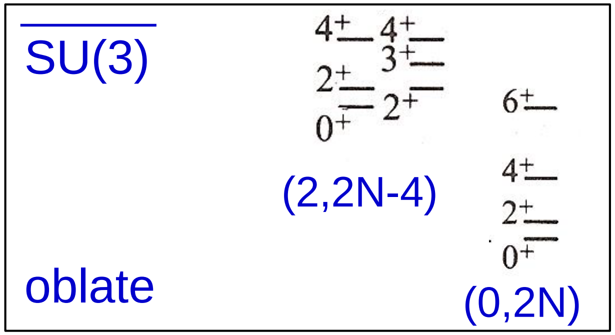
♣ Leviatan, Shapira, PRC **93**, 051302(R) (2016)

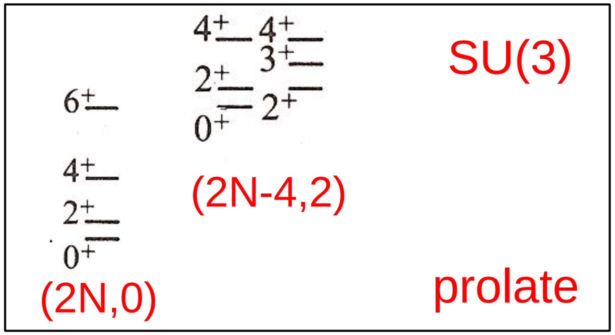
♠ Leviatan, Gavrielov, Phys. Scr. **92**, 114005 (2017)

SU(3) and $\overline{SU(3)}$ Dynamical Symmetries

$U(6) \supset SU(3) \supset SO(3)$ $\quad |[N]\ (\lambda, \mu)\ K\ L\rangle$ $\quad$ Prolate-deformed rotor

$U(6) \supset \overline{SU(3)} \supset SO(3)$ $\quad |[N]\ (\bar{\lambda}, \bar{\mu})\ K\ L\rangle$ $\quad$ Oblate-deformed rotor





DS spectra are identical

Quadrupole moments of corresponding states differ in sign

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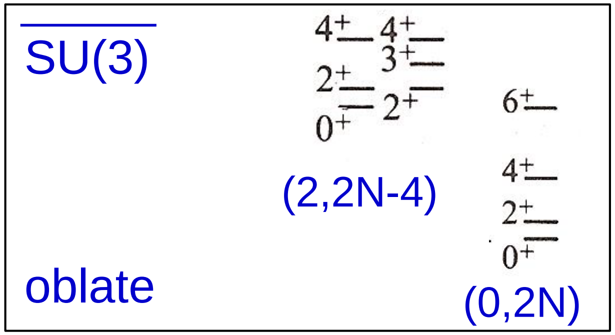

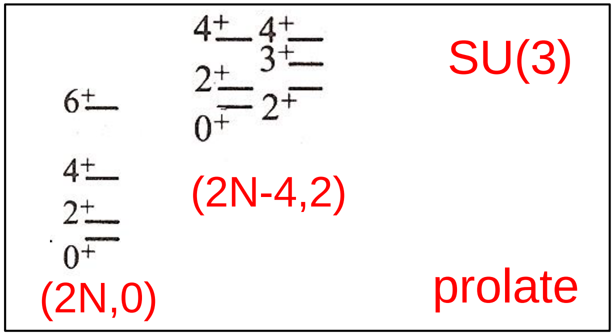

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Quadrupole moments of corresponding states differ in sign

Prolate-Oblate Shape Coexistence

Intrinsic part of C.P. Hamiltonian

$$\begin{cases} \hat{H}|N,\,(\lambda,\mu)=(2N,0),\,K=0,\,L\rangle = 0 \\ \hat{H}|N,\,(\bar{\lambda},\bar{\mu})=(0,2N),\,\bar{K}=0,\,L\rangle = 0 \end{cases}$$

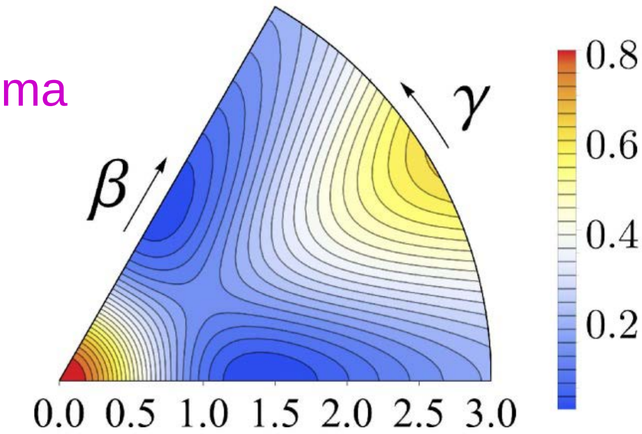
$$\hat{H} = h_0\,P_0^{\dagger}\hat{n}_s P_0 + h_2\,P_0^{\dagger}\hat{n}_d P_0 + \eta_3\,G_3^{\dagger}\cdot\tilde{G}_3 \qquad P_0^{\dagger} = d^{\dagger}\cdot d^{\dagger} - 2(s^{\dagger})^2 \qquad G_{3,\mu}^{\dagger} = \sqrt{7}[(d^{\dagger}d^{\dagger})^{(2)}d^{\dagger}]_{\mu}^{(3)}$$

Energy Surface $\tilde{E}(\beta,\gamma) = z_0 + (1+\beta^2)^{-3}[A\beta^6 + B\beta^6\Gamma^2 + D\beta^4 + F\beta^2]$ $\qquad \Gamma = \cos 3\gamma$

oblate-prolate

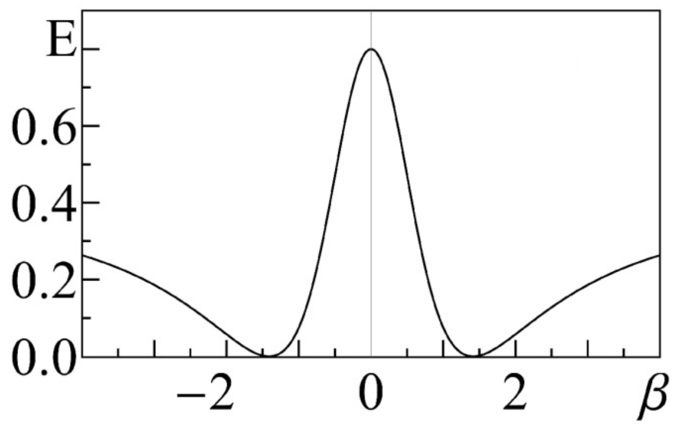
Two degenerate P-O global minima

($\beta=\sqrt{2},\gamma = 0$) and ($\beta=\sqrt{2},\gamma = \pi/3$)



E(β,γ)

Saddle points support a barrier separating the various minima

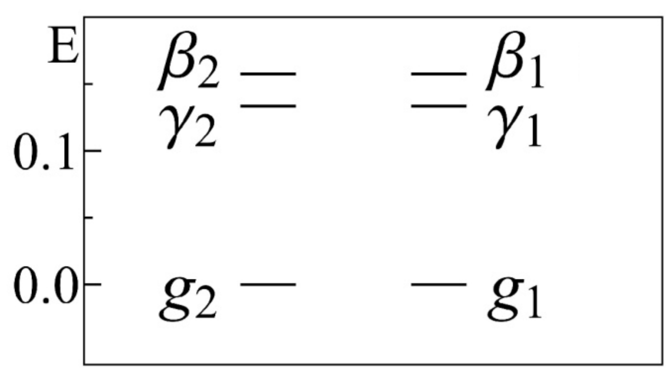


E(β,γ=0)

Normal modes:

$$\epsilon_{\beta 1} = \epsilon_{\beta 2} = \frac{8}{3}(h_0 + 2h_2)N^2$$

$$\epsilon_{\gamma 1} = \epsilon_{\gamma 2} = 4\eta_3 N^2$$

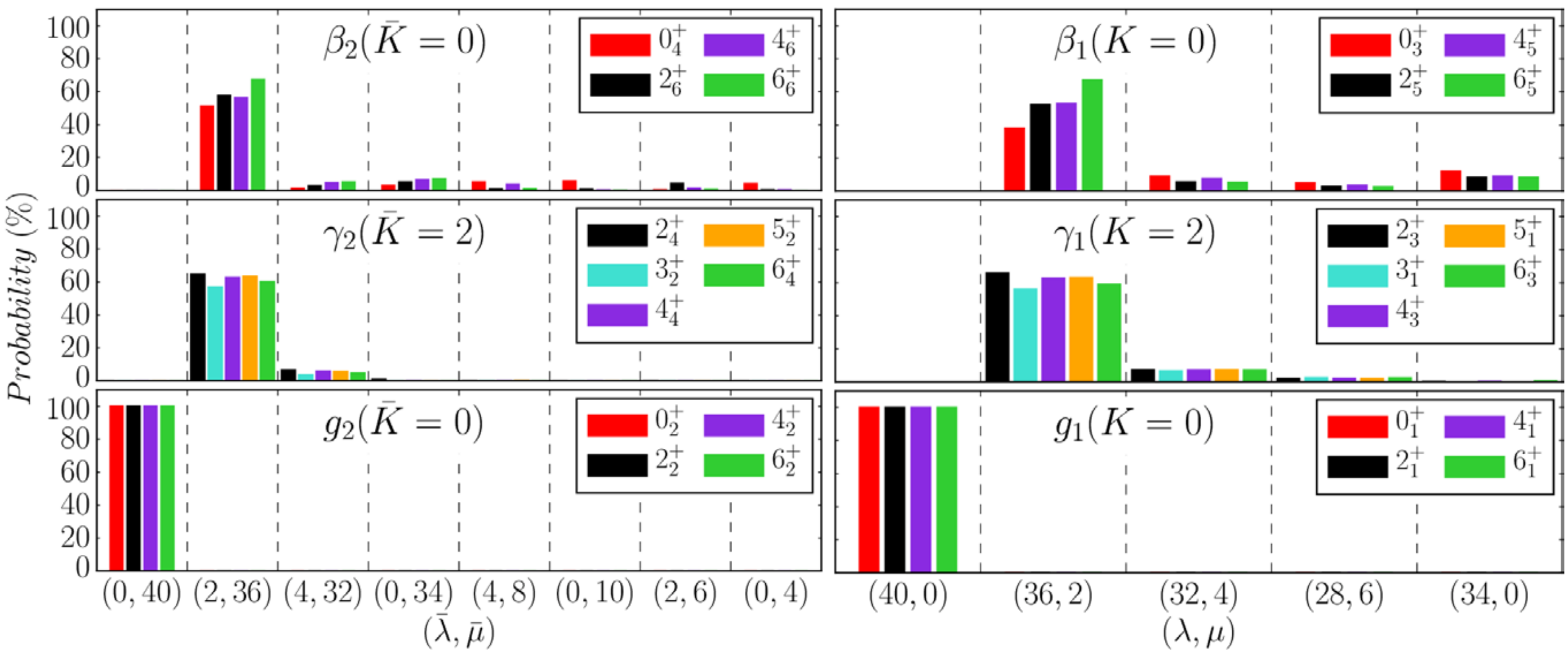


Complete Hamiltonian $\hat{H}' = \hat{H}(h_0, h_2, \eta_3) + \alpha\,\hat{\theta}_2 + \rho\,\hat{C}_2[\mathrm{SO}(3)]$

$$\alpha\,\hat{\theta}_2 = \alpha\,[-\hat{C}_2[SU(3)] + 2\hat{N}(2\hat{N}+3)]$$

$$\tilde{\alpha}(1+\beta^2)^{-2}[(\beta^2-2)^2 + 2\beta^2(2-2\sqrt{2}\beta\Gamma+\beta^2)] \qquad \tilde{\alpha} = \alpha/(N-2)$$

$\overline{\mathrm{SU(3)}}$ decomposition SU(3) decomposition

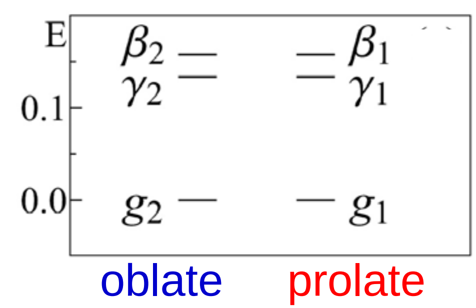


Ground g_1 band: pure SU(3)-DS states (2N,0)

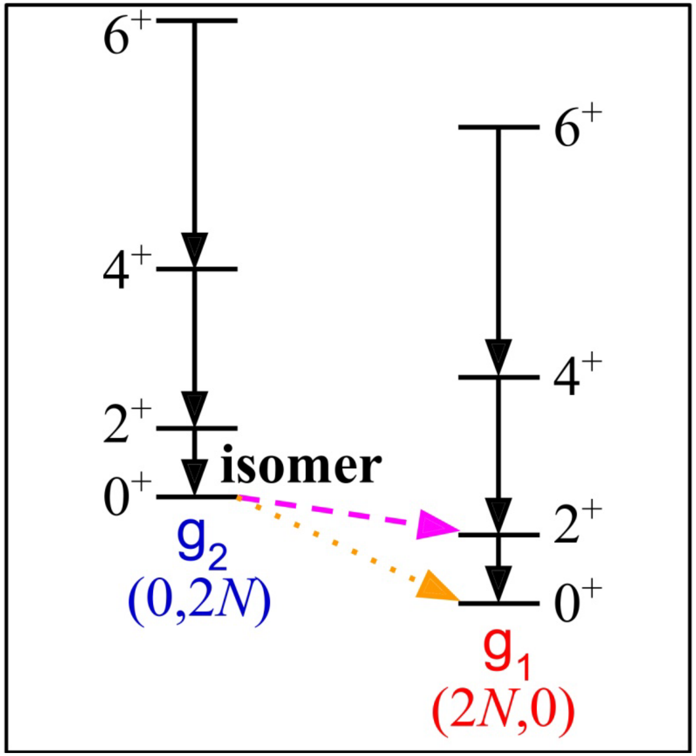
Ground g_2 band: pure $\overline{\mathrm{SU(3)}}$-DS states (0,2N)

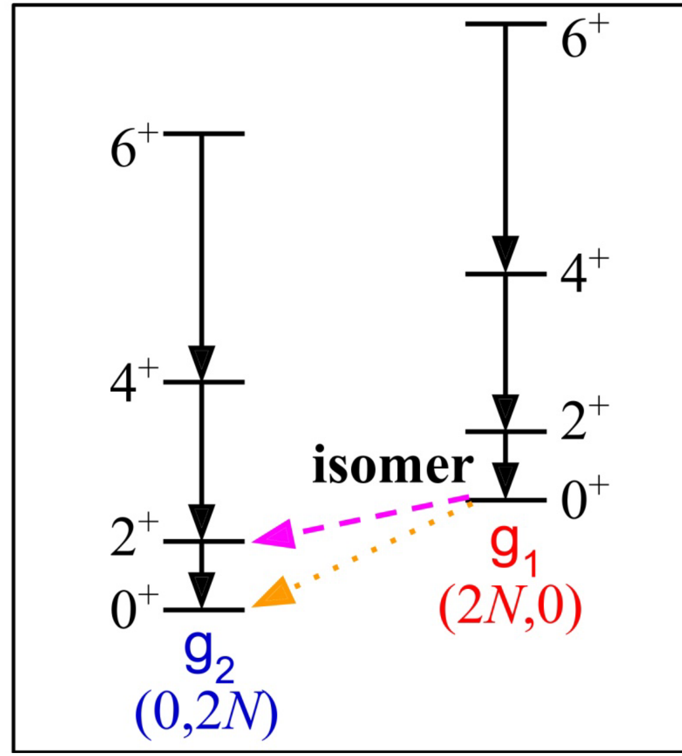
Excited β and γ bands: considerable mixing

⇒ SU(3)-PDS coexisting with $\overline{\mathrm{SU(3)}}$-PDS



P-O coexistence





$T(E2) = e_B(d^\dagger s + s^\dagger \tilde{d})$ (1,1) ⊕ (2,2) tensor

$T(E0) \propto \hat{n}_d$ (0,0) ⊕ (2,2) tensor

E2 selection rule: $g_1 \not\leftrightarrow g_2$

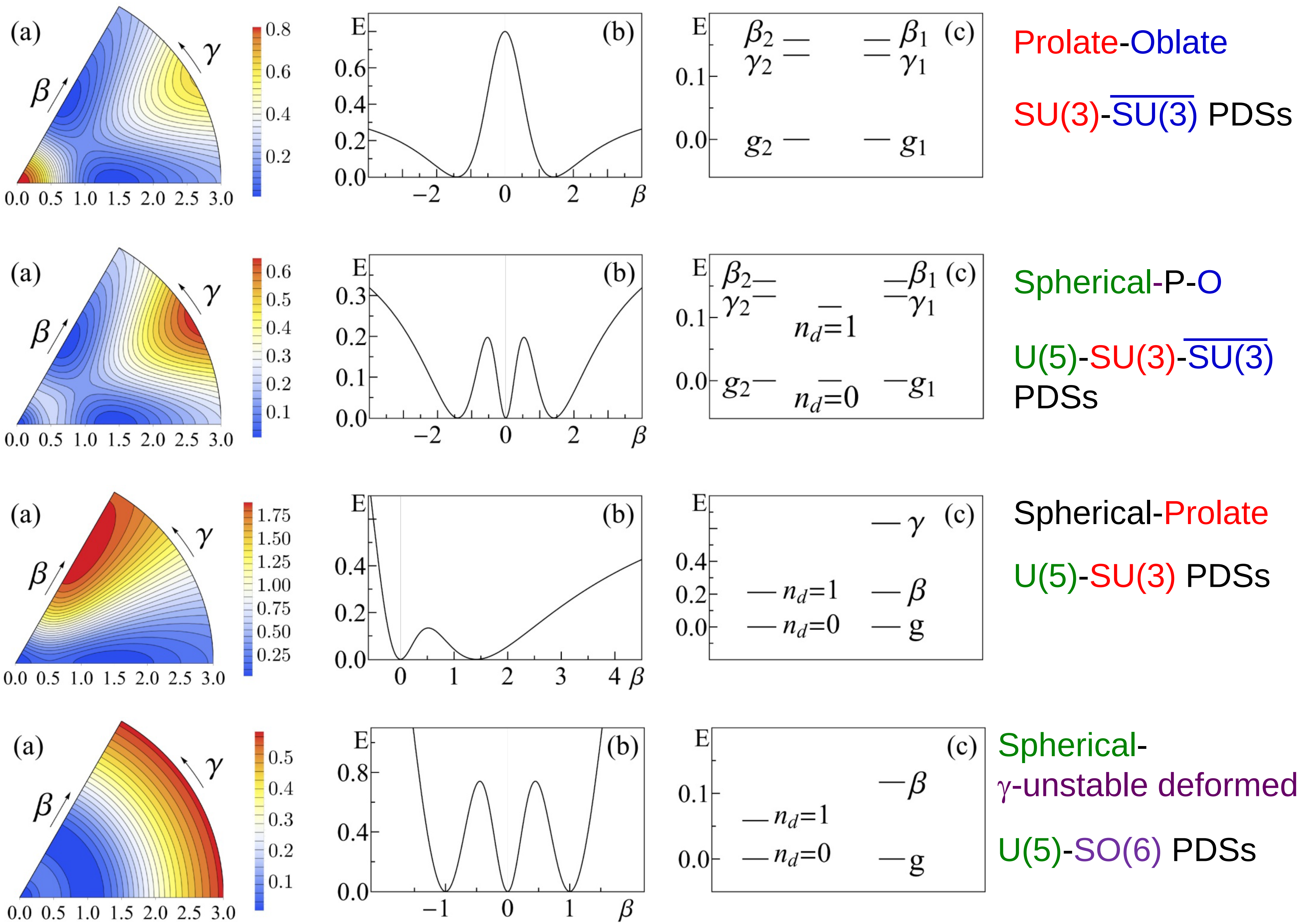
E0 selection rule: $g_1 \not\leftrightarrow g_2$

$$Q_L = \mp e_B \sqrt{\frac{16\pi}{40}} \frac{L}{2L+3} \frac{4(2N-L)(2N+L+1)}{3(2N-1)}$$

$$B(E2; g_i,\ L+2 \to g_i,\ L) =$$

$$e_B^2 \frac{3(L+1)(L+2)}{2(2L+3)(2L+5)} \frac{(4N-1)^2(2N-L)(2N+L+3)}{18(2N-1)^2}$$

ANALYTIC expressions !



(a)
(b)
(c)
Prolate-Oblate
SU(3)-$\overline{SU(3)}$ PDSs
Spherical-P-O
U(5)-SU(3)-$\overline{SU(3)}$ PDSs
Spherical-Prolate
U(5)-SU(3) PDSs
Spherical-
γ-unstable deformed
U(5)-SO(6) PDSs

Shape coexistence near shell closure

- Multiparticle-multihole intruder excitations across shell gaps

- Interacting boson model with configuration mixing (IBM-CM) [Duval, Barrett , PLB 81]

0p-0h, 2p-2h, 4p-4h,… → [N] ⊕ [N+2] ⊕ [N+4]… normal⊕ intruder states

• Hamiltonian

$$\hat{H} = \begin{pmatrix} \hat{H}^{(N)}_{\text{normal}} & \hat{V}_{\text{mix}} \\ \hat{V}_{\text{mix}} & \hat{H}^{(N+2)}_{\text{intruder}} \end{pmatrix}$$

• Geometry

$$E(\beta,\gamma) = \begin{pmatrix} E_N(\beta,\gamma) & W(\beta,\gamma) \\ W(\beta,\gamma) & E_{N+2}(\beta,\gamma) \end{pmatrix}$$

Matrix coherent states

$E_{\pm}(\beta,\gamma)$ Eigenpotentials

[Frank, Van Isacker, Vargas, PRC 2004]

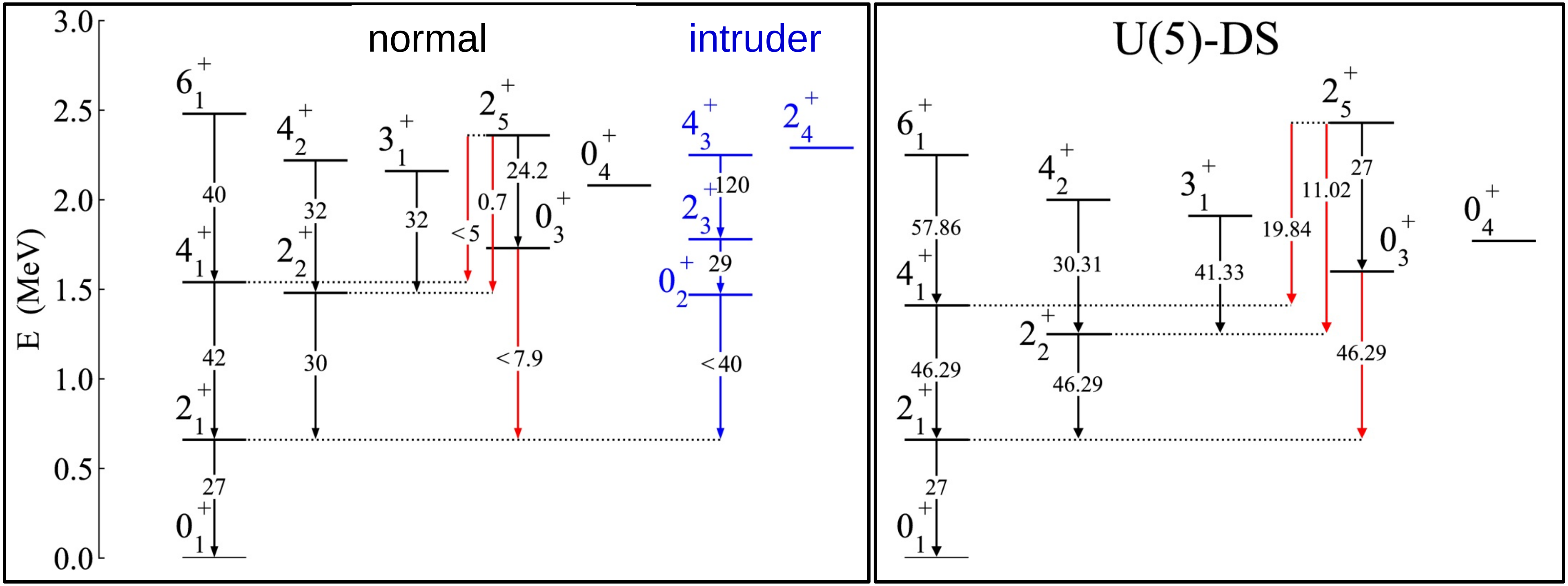
$$E_N(\beta,\gamma) = \langle \beta,\gamma;N|\hat{H}^{(N)}_{\text{normal}}|\beta,\gamma;N\rangle$$

$$E_{N+2}(\beta,\gamma) = \langle \beta,\gamma;N+2|\hat{H}^{(N+2)}_{\text{intruder}}|\beta,\gamma;N+2\rangle$$

$$W(\beta,\gamma) = \langle \beta,\gamma;N|\hat{V}_{\text{mix}}|\beta,\gamma;N+2\rangle$$

Applications: Po, Hg, Pb.. [Garcia-Ramos, Heyde, Van Isacker, Nomura, Robledo…]

^{110}Cd



Most normal states good spherical vibrator $B(E2;\, 2_1 \rightarrow 0_1) = 27.0\ (8)$ W.u.

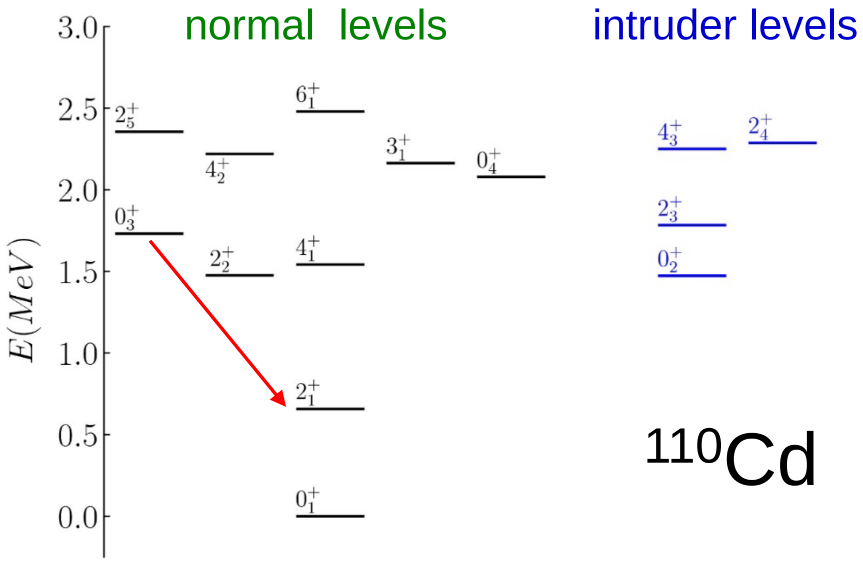
BUT: [W.u.]	EXP	U(5)	
$B(E2;\, 0_3 \rightarrow 2_1)$	< 7.9	46.29 .	$(n_d = 2 \rightarrow n_d = 1)$
$B(E2;\, 2_5 \rightarrow 4_1)$	< 5	19.84	$(n_d = 3 \rightarrow n_d = 2)$
$B(E2;\, 2_5 \rightarrow 2_2)$	$< 0.7^{+0.5}_{-0.6}$	11.02	
$B(E2;\, 0_4 \rightarrow 2_2)$	small BR	57.86	

Garret *et al.* PRC (2012)

- Attempted solution: normal-intruder mixing

$$\hat{H} = \begin{pmatrix} \hat{H}^{(N)}_{U(5)} & \hat{V}_{mix} \\ \hat{V}_{mix} & \hat{H}^{(N+2)}_{SO(6)} \end{pmatrix}$$

Requires **strong** (maximal ~ 50%) **mixing** to reproduce B(E2; $0_3 \rightarrow 2_1$) < 7.9 W.u, but results in discrepancy in the decay pattern of other states

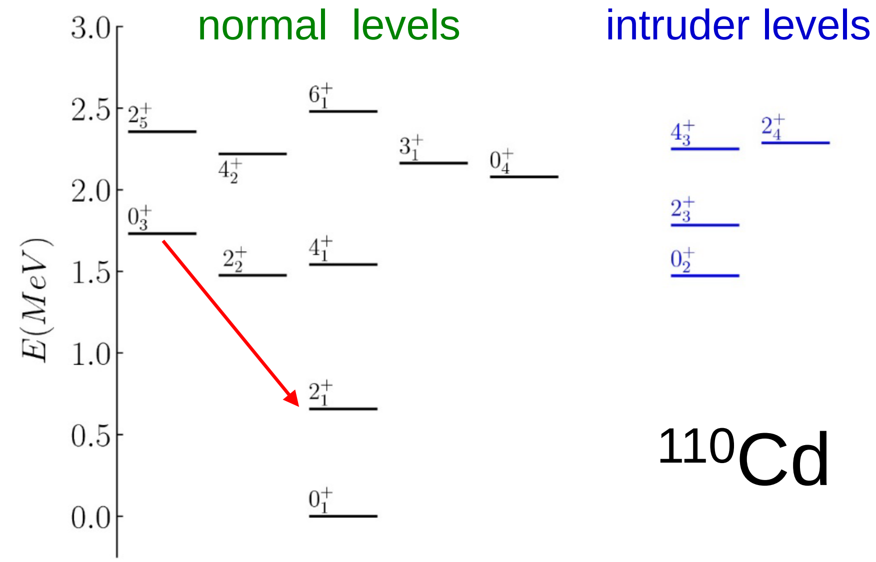


⇒ Strong normal-intruder mixing refuted

- Claims: "Breakdown of vibrational motion in Cd isotopes" (Garrett PRC 2008)
  "Need for a paradigm change"

- Attempted solution: normal-intruder mixing

$$\hat{H} = \begin{pmatrix} \hat{H}^{(N)}_{U(5)} & \hat{V}_{mix} \\ \hat{V}_{mix} & \hat{H}^{(N+2)}_{SO(6)} \end{pmatrix}$$



Requires **strong** (maximal ~ 50%) **mixing** to reproduce B(E2; $0_3 \rightarrow 2_1$) < 7.9 W.u, but results in discrepancy in the decay pattern of other states

$\Rightarrow$ Strong normal-intruder mixing refuted

- Claims: "Breakdown of vibrational motion in Cd isotopes" (Garrett PRC 2008)
  "Need for a paradigm change"

- Alternative approach:

good U(5) — Class A: $n_d = \tau$ = 0,1,2,3 (n_Δ = 0) — $0_1(0)$, $2_1(658)$, $4_1(1542)$, $2_2(1476)$, $6_1(2480)$, $4_2(2220)$, $3_1(2163)$

broken U(5):
- Class B: $n_d = \tau+2$ = 2,3 (n_Δ = 0) — $0_3(1731)$, $2_5(2356)$
- Class C: $n_d = \tau$ = 3 (n_Δ = 1) — $0_4(2079)$

- Some states with good U(5) symmetry
- Some states break U(5) symmetry

$\Rightarrow$ Partial Dynamical Symmetry

U(5) PDS

U(6) ⊃ U(5) ⊃ SO(5) ⊃ SO(3) $|[N]\ n_d\ \tau\ n_\Delta\ L\rangle$

$$\hat{H}_{PDS} = \hat{H}_{DS} + \hat{V}_0$$

$$\hat{V}_0 = r_0\, G_0^\dagger G_0 + e_0 \left(G_0^\dagger K_0 + K_0^\dagger G_0 \right)$$

$$G_0^\dagger = [(d^\dagger d^\dagger)^{(2)} d^\dagger]^{(0)} \qquad K_0^\dagger = s^\dagger (d^\dagger d^\dagger)^{(0)}$$

$$\hat{V}_0\, |N, n_d = \tau, \tau, n_\Delta = 0, L\rangle = 0$$

Class A: solvable Classes B,C: mixed

U(5) PDS-CM

$$\hat{H} = \begin{pmatrix} \hat{H}^{(N)}_{\text{normal}} & \hat{V}_{\text{mix}} \\ \hat{V}_{\text{mix}} & \hat{H}^{(N+2)}_{\text{intruder}} \end{pmatrix}$$

$$\hat{H}^{(N)}_{\text{normal}} = \hat{H}_{\text{PDS}}$$ U(5)-PDS

$$\hat{H}^{(N+2)}_{\text{intruder}} = \kappa \hat{Q} \cdot \hat{Q} + \Delta$$ SO(6)

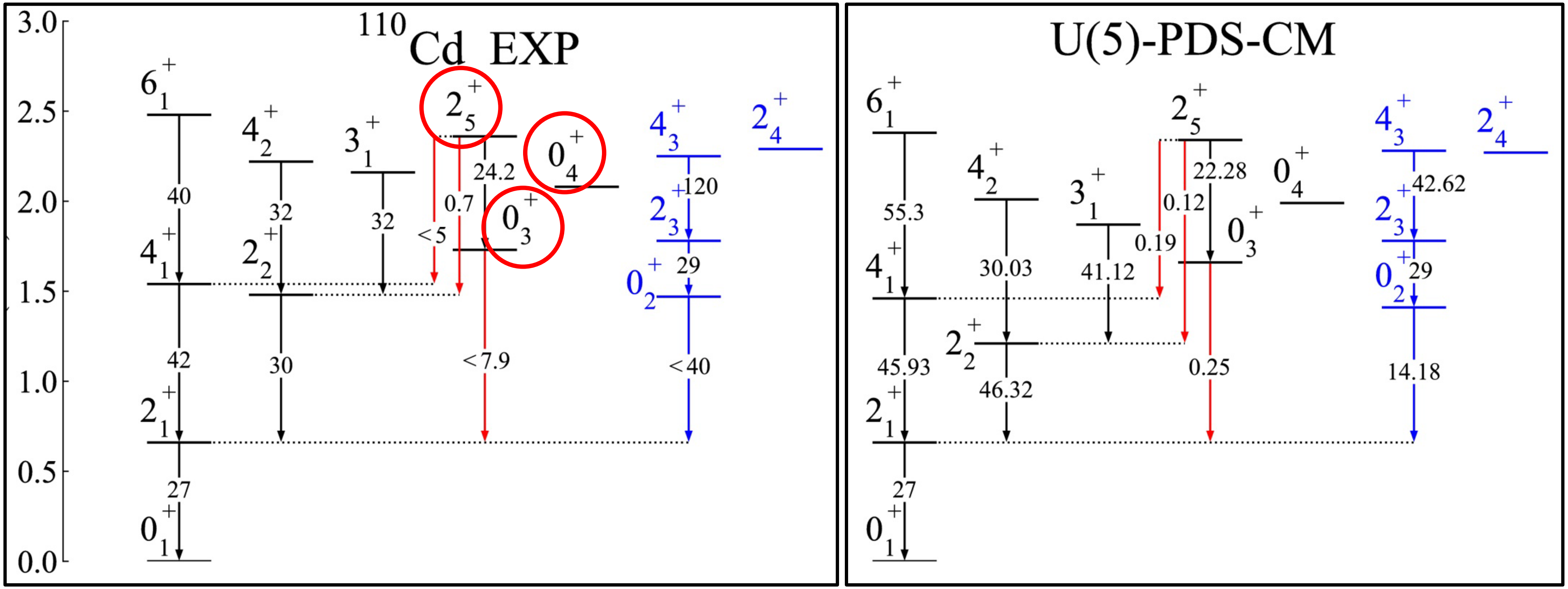
$$\hat{V}_{\text{mix}} = \alpha \left[(d^\dagger d^\dagger)^{(0)} + (s^\dagger)^2 + \text{H.c.} \right]$$

$$|\Psi\rangle = a|\Psi_n^{(N)}\rangle + b|\Psi_i^{(N+2)}\rangle$$

$$\hat{T}(E2) = e_B^{(N)}\, \hat{Q}^{(N)} + e_B^{(N+2)}\, \hat{Q}^{(N+2)}$$

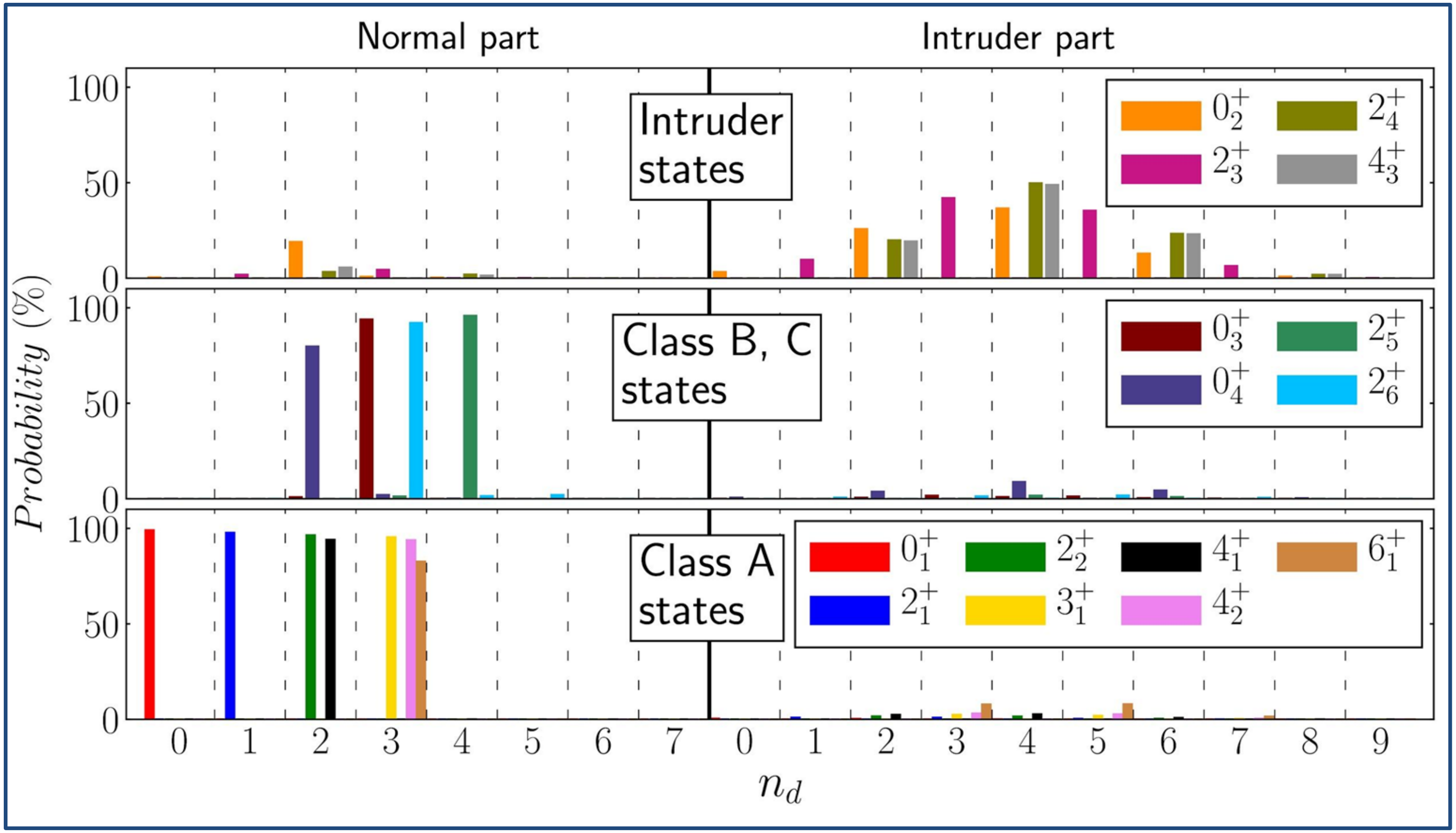
$$\hat{Q} = d^\dagger s + s^\dagger \tilde{d}$$

Normal and intruder levels in ^{110}Cd



- U(5)-PDS-CM: good description of empirical data
- Normal states of class A retain good U(5) symmetry and n_d quantum number
- Non-yrast states of classes B & C [0_3(1731), 0_4(2079), 2_5(2356)]: dramatic changes
- Weak normal-intruder mixing (small b^2) $|\Psi\rangle = a|\Psi_n^{(N)}\rangle + b|\Psi_i^{(N+2)}\rangle$

A. Leviatan, N. Gavrielov, J.E. Garcia Ramos and P. Van Isacker,
Phys. Rev. C **98**, 031302(R) (2018)



U(5)-PDS-CM

U(5)-DS

Majority of normal states (class A) are pure wrt U(5) (> 97%)

Weak normal-intruder mixing

$n_d = \tau$

$0_3(1731)$: (0.9% $n_d = 2$) , (94% $n_d = 3$) , (5.1% intruder) $n_d = 2$

$0_4(2079)$: (79.8% $n_d = 2$) , (2% $n_d = 3$), (18% intruder) $n_d = 3$

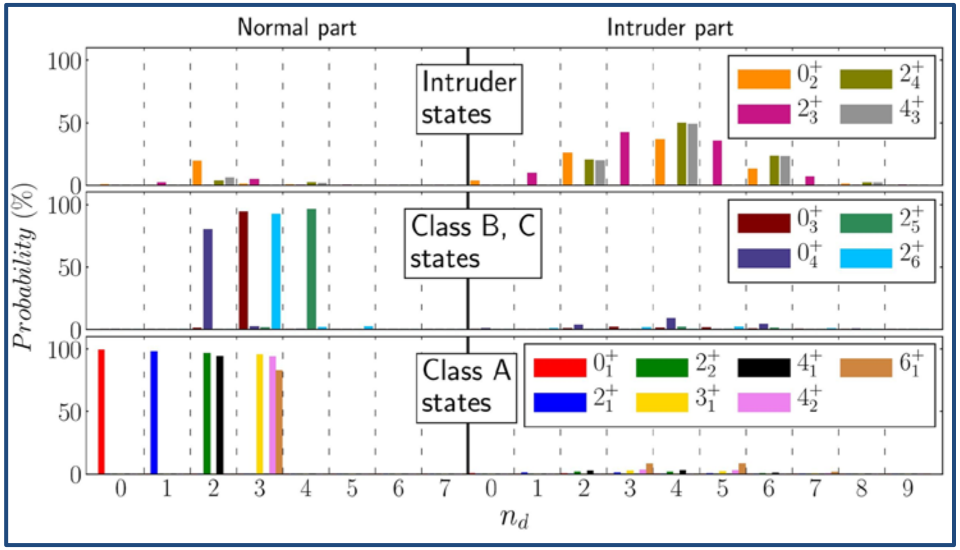
$2_5(2356)$: (1.2% $n_d = 3$) , (95.8% $n_d = 4$) , (2.9% intruder) $n_d = 3$

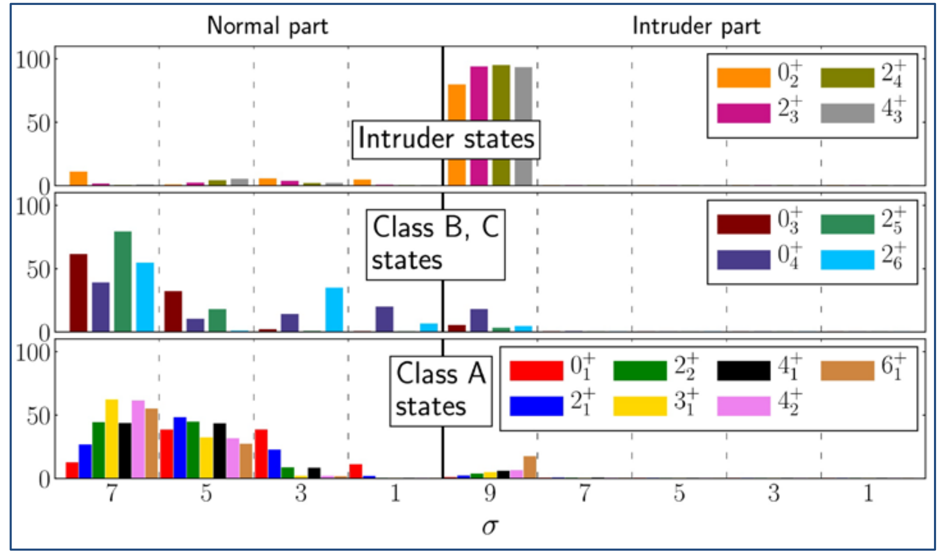
L_i	L_f	EXP	U(5)-DS	U(5)-PDS-CM
2_1^+	0_1^+	27.0 (8)	27.00	27.00
4_1^+	2_1^+	42 (9)	46.29	45.93
2_2^+	2_1^+	30 (5)	46.29	46.32
	0_1^+	1.35 (20); 0.68 (14)^a	0.00	0.00
0_3^+	2_2^+	$< 1680^a$	0.00	55.95
	2_1^+	$< 7.9^a$	46.29	0.25
6_1^+	4_1^+	40 (30); 62 (18)^a	57.86	55.30
	4_2^+	$< 5^a$	0.00	0.00
	$4_{3;i}^+$	14 (10); 36 (11)^a		2.39
4_2^+	4_1^+	12^{+4}_{-6}	27.55	27.45
	2_2^+	32^{+10}_{-14}	30.31	30.03
	2_1^+	$0.20^{+0.06}_{-0.09}$	0.00	0.00
	$2_{3;i}^+$	$< 0.5^a$		0.005
3_1^+	4_1^+	$5.9^{+1.8}_{-4.6}$	16.53	16.48
	2_2^+	32^{+8}_{-24}	41.33	41.12
	2_1^+	$1.1^{+0.3}_{-0.8}$; 0.85 (25)^a	0.00	0.00
	$2_{3;i}^+$	$< 5^a$		0.012
0_4^+	2_2^+	[$< 0.65^a$]	57.86	1.24
	2_1^+	[0.010^a]	0.00	31.76
	$2_{3;i}^+$	[100^a]		16.32
2_5^+	0_3^+	24.2 (22)^a	27.00	22.28
	4_1^+	$<5^a$	19.84	0.19
	2_2^+	$^a0.7^{+0.5}_{-0.6}$	11.02	0.12
	2_1^+	$2.8^{+0.6}_{-1.0}$	0.00	0.00
	$2_{3;i}^+$	$< 5^a$		0.002
	$0_{2;i}^+$	$< 1.9^a$		0.20

E2 transitions in ^{110}Cd

[W.u.]	EXP	U(5)-PDS-CM
$B(E2; 0_3 \rightarrow 2_1)$	< 7.9	0.25
$B(E2; 2_5 \rightarrow 4_1)$	< 5	0.19
$B(E2; 2_5 \rightarrow 2_2)$	$< 0.7^{+0.5}_{-0.6}$	0.12

L_i	L_f	EXP	U(5)-PDS-CM
$0_{2;i}^+$	2_1^+	$< 40^a$	14.18
$2_{3;i}^+$	$0_{2;i}^+$	29 (5)^a	29.00
	0_1^+	$0.31^{+0.08}_{-0.12}$	0.08
	2_1^+	$0.7^{+0.3}_{-0.4}$	0.00
	2_2^+	$< 8^a$	0.96
$2_{4;i}^+$	2_1^+	$0.019^{+0.020}_{-0.019}$	0.10
$4_{3;i}^+$	2_1^+	$0.22^{+0.09}_{-0.19}$	0.49
	2_2^+	$2.2^{+1.4}_{-2.2}$	0.00
	$2_{3;i}^+$	120^{+50}_{-110}	42.62
	4_1^+	$2.6^{+1.6}_{-2.6}$	0.00



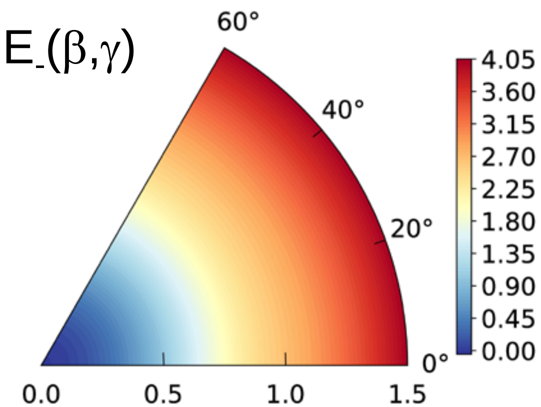


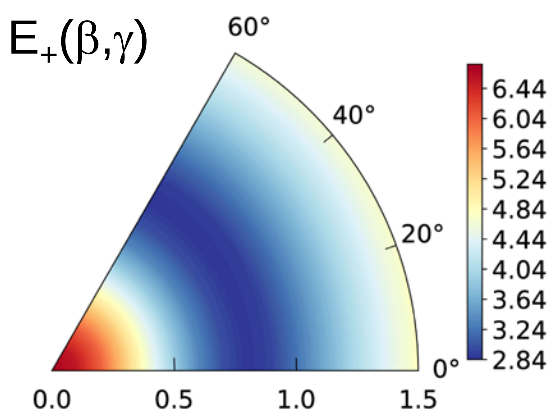
$$\hat{H} = \begin{pmatrix} \hat{H}^{(N)}_{\mathrm{U(5)-PDS}} & \hat{V}_{\mathrm{mix}} \\ \hat{V}_{\mathrm{mix}} & \hat{H}^{(N+2)}_{\mathrm{SO(6)}} \end{pmatrix}$$

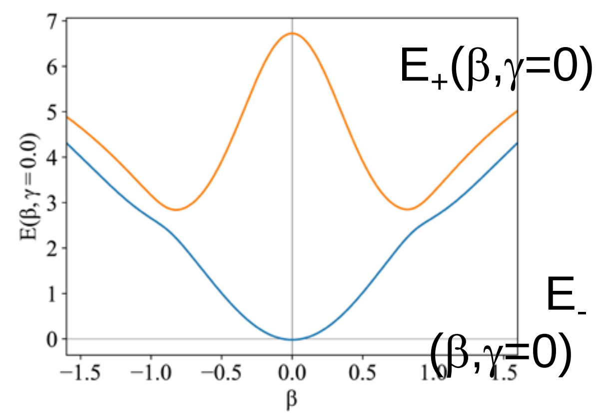
Normal states: U(5)-PDS good n_d for class A states

Intruder states: SO(6) "good" σ

V_{mix} small







Concluding Remarks

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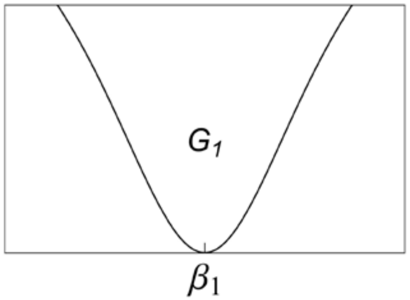
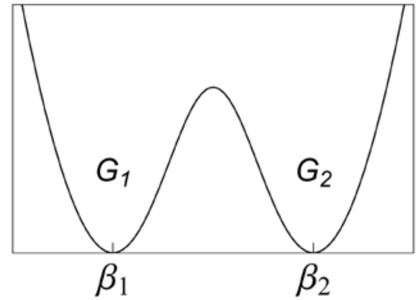
A symmetry-based approach

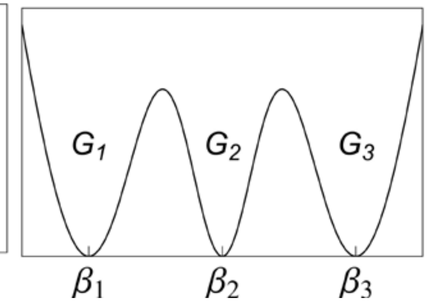
- Quantum phase transitions with multiple shapes



Single DS or PDS → Multiple PDSs

[G_i = U(5), SU(3), $\overline{\mathrm{SU(3)}}$, SO(6)]





- A single number-conserving rotational invariant H; DS preserved in selected bands
- Higher-order terms
- Solvable bands unmixed. Strong band-mixing can destroy the PDS

- Shape coexistence near shell closure

$$\hat{H} = \begin{pmatrix} \hat{H}^{(N)}_{G_1} & \hat{V}_{\rm mix} \\ \hat{V}_{\rm mix} & \hat{H}^{(N+2)}_{G_2} \end{pmatrix} \qquad \begin{pmatrix} \hat{H}^{(N)}_{\rm U(5)} & \hat{V}_{\rm mix} \\ \hat{V}_{\rm mix} & \hat{H}^{(N+2)}_{\rm SO(6)} \end{pmatrix} \longrightarrow \begin{pmatrix} \hat{H}^{(N)}_{\rm U(5)-PDS} & \hat{V}_{\rm mix} \\ \hat{V}_{\rm mix} & \hat{H}^{(N+2)}_{\rm SO(6)} \end{pmatrix}$$

^{110}Cd

- G_1 DS → G_1 PDS
- Strong → weak normal-intruder mixing
- G_1 DS maintained in selected normal states and G_2 DS in intruder states

Thank you